Q1) For each of the following pairs of expressions, give instances of relations that show the expressions are not equivalent.

a) $\Pi_A(r - s)$ and $\Pi_A(r) - \Pi_A(s)$.

Let R = {(1, 2)}, S = {(1, 3)}

The result of the left-hand side expression is {(1)}, whereas the result of the right-hand side expression is empty.

b) $\sigma B{<}4(A\gamma max(B) \text{ as } B(r))$ and $A\gamma max(B) \text{ as } B(\sigma B{<}4(r))$.

Let R = {(1, 2), (1, 5)}

The left-hand side expression has an empty result, whereas the right- hand side one has the result {(1, 2)}.

c) In the preceding expressions, if both occurrences of *max* were replaced by *min*, would the expressions be equivalent?

Yes, on replacing the max by the min, the expressions will become equivalent. Any tuple that the selection in the RHS eliminates would not pass the selection on the LHS if it were the minimum value and would be eliminated anyway if it were not the minimum value.

d) $(r \bowtie s) \bowtie t$ and $r \bowtie (s \bowtie t)$ {In other words, the natural right outer join is not associative}.

R = {(1, 2)}, S = {(2, 3)}, T = {(1, 4)}. The left-hand expression gives {(1, 2, null, 4)} whereas the right-hand expression gives {(1, 2, 3, null)}.

e) $\sigma\theta(E1 \bowtie E2)$ and $E1 \bowtie \sigma\theta(E2)$, where θ uses only attributes from $E2$.

Let R be of the schema (A, B) and S of (A, C). Let R = {(1, 2)}, S = {(2, 3)} and let θ be the expression C = 1. The left side expression's result is empty, whereas the right-side expression results in {(1, 2, null)}.

Q2) Consider the two expressions $\sigma\theta(E1 \bowtie E2)$ and $\sigma\theta(E1 \bowtie E2)$.

a) Show using an example that the two expressions are not equivalent in general.

Consider relations *dept*(id, deptname) and *emp*(id,name,dept id) with sample data as follows:

Sample data for *dept*:

501	Finance
502	Administration
503	Marketing
504	Sales

Sample data for *emp*:

1	John 501
2	Martin 503
3	Sarah 504

Now consider the expressions:
        $\sigma deptname{<}'Z'(dept \bowtie emp)$ and $\sigma deptname{<}'Z'(dept \bowtie emp)$.

The result of the first expression is:

501	Finance	1	John
503	Marketing	2	Martin
504	Sales	3	Sarah

The result of second expression is:

501	Finance	1	John
502	Administration	null	null
503	Marketing	2	Martin
504	Sales	3	Sarah

b) Give a simple condition on the predicate θ, which if satisfied will ensure that the two expressions are equivalent.

Considering the same example, if θ included the condition "*name* = 'Einstein'", the two expressions would be equivalent, that is they would always have the same result, since any tuple that is in *dept⟕emp* but not in *dept⋈emp* would not satisfy the condition since its *name* attribute would be null.

Q3) Consider a relation $r(A, B, C)$, with an index on attribute A. Give an example of a query that can be answered by using the index only, without looking at the tuples in the relation. (Query plans that use only the index, without accessing the actual relation, are called *index-only* plans.)

Any query that only involves the attribute A of r can be executed by only using the index. For example, the query

> **select sum(A)**
> **from r**

only needs to use the values of A, and thus does not need to look at r.