

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**V SEMESTER B.TECH DEGREE EXAMINATION(S), MAY 2019**

Course Code: CS309

Course Name: GRAPH THEORY AND COMBINATORICS

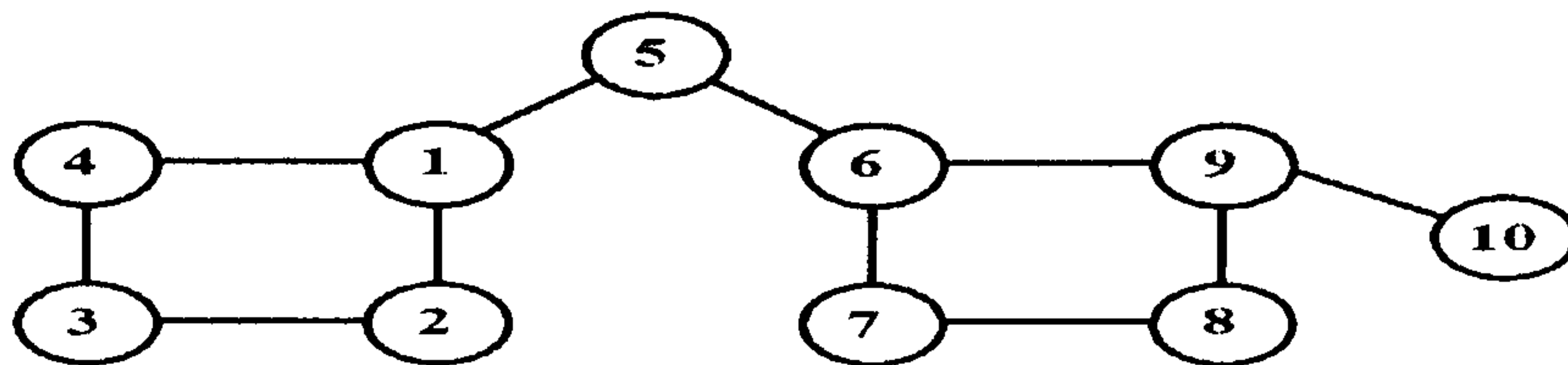
Max. Marks: 100

Duration: 3 Hours

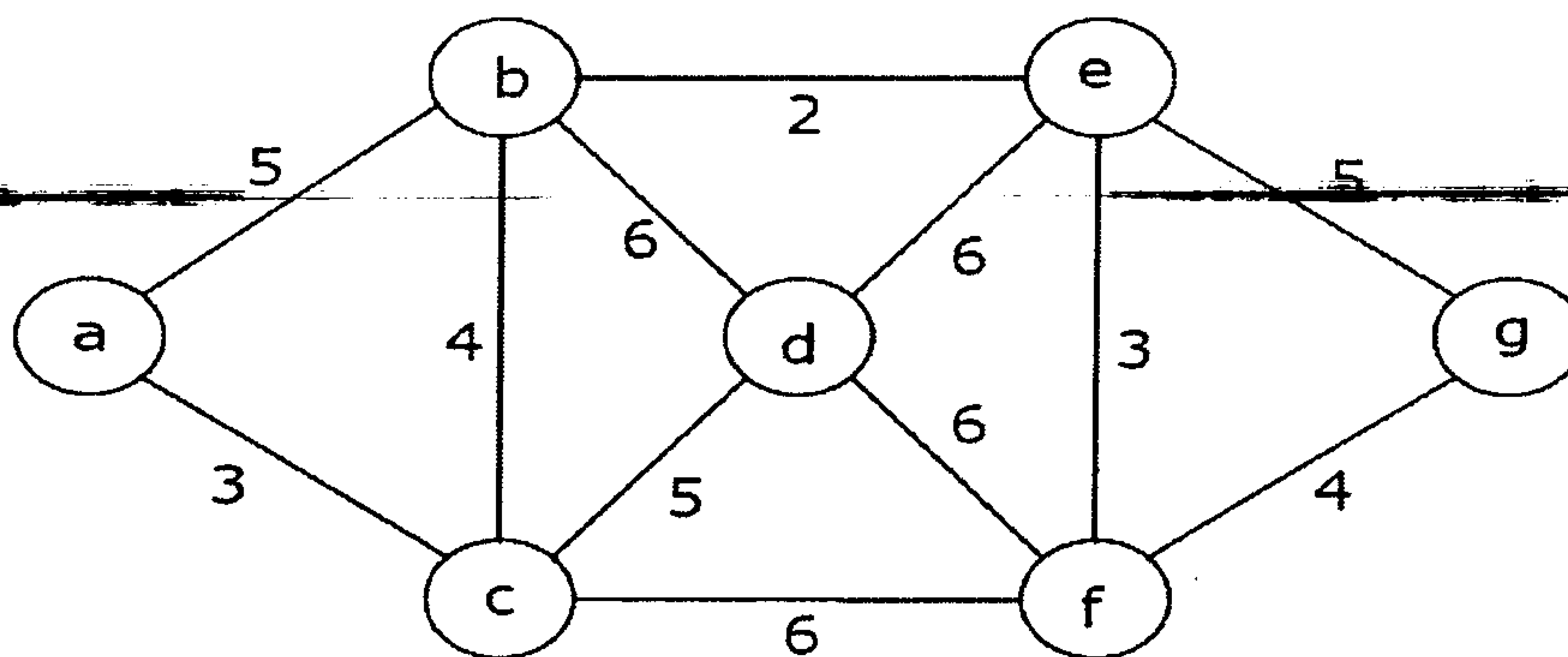
**PART A***Answer all questions, each carries 3 marks.*

Marks

- 1 Print a Walk, trail, path and cycle on the graph below. 3



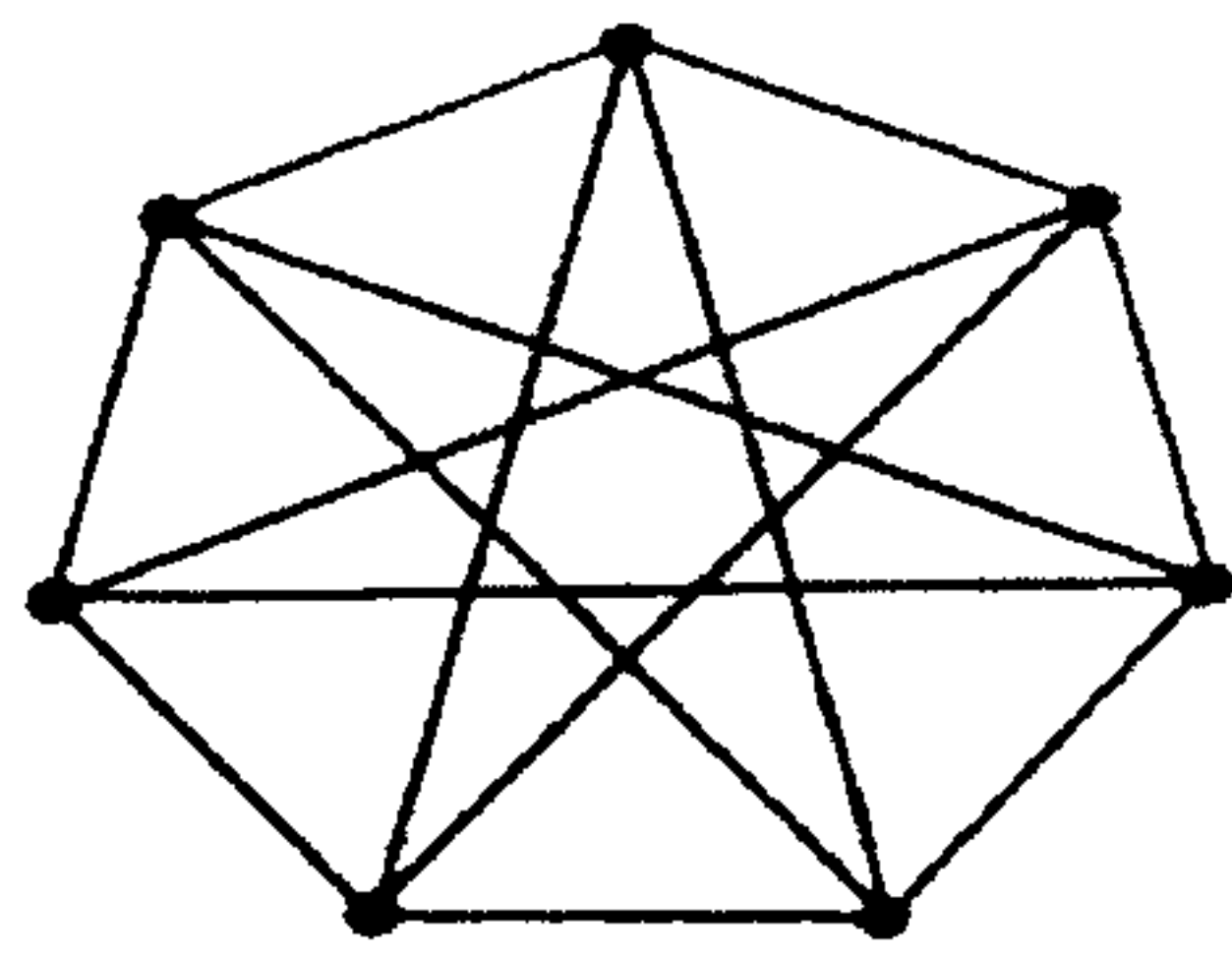
- 2 Define pendant vertex, isolated vertex and null graph with an example each. 3
- 3 State travelling salesman problem. Print a travelling salesman's tour on the graph below. 3



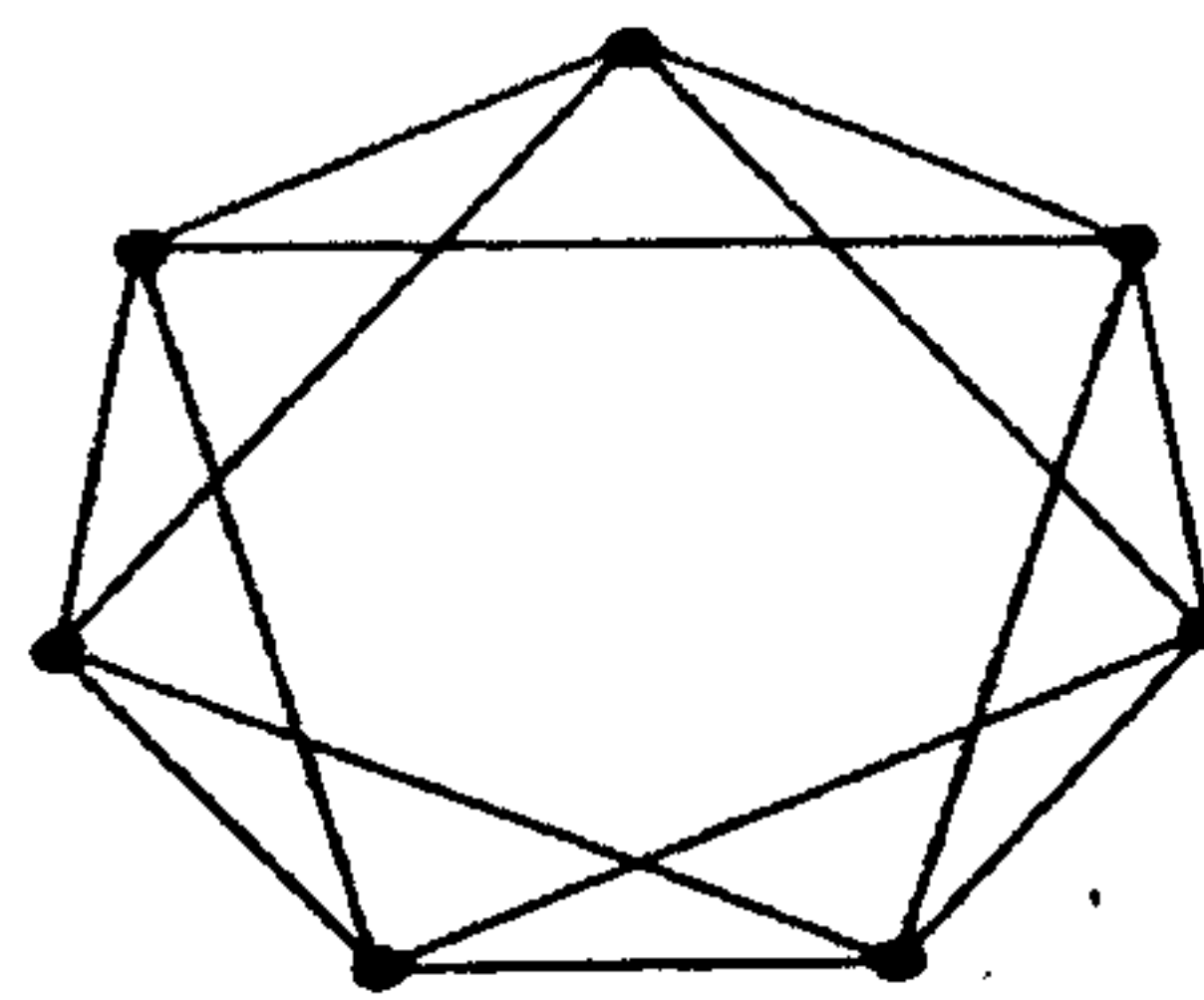
- 4 Prove Dirac's theorem for Hamiltonicity. 3

**PART B***Answer any two full questions, each carries 9 marks.*

- 5 a) Define isomorphism of graphs. Show that the graphs (a) and (b) are isomorphic. 4

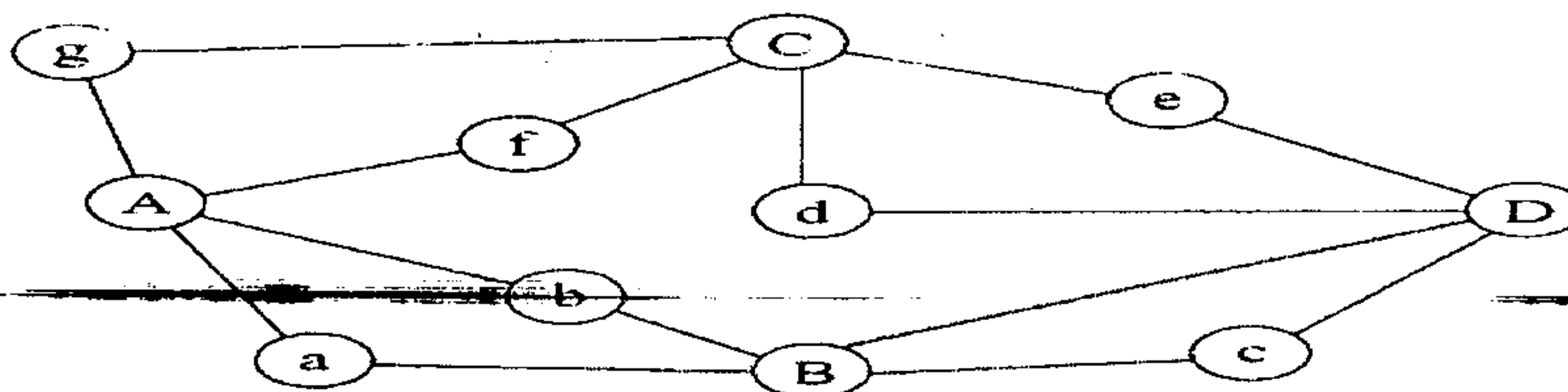


(a)

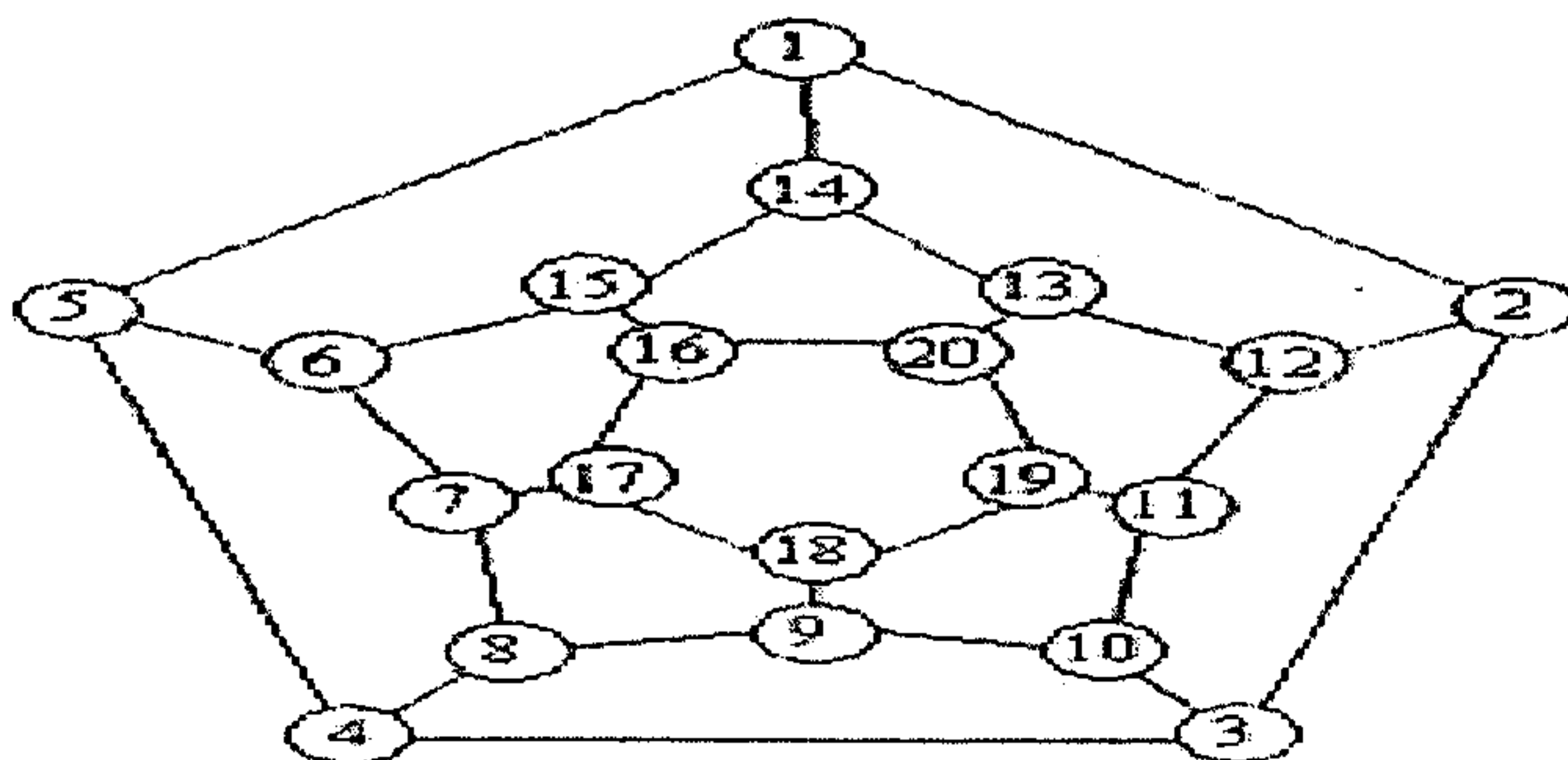


(b)

- b) Define subgraph. Give two subgraphs of the above graph.(Fig. a) 2
- c) Consider a complete graph  $G$  with 11 vertices. 3
1. Find the maximum number of edges possible in  $G$
  2. Find the number of edge-disjoint Hamiltonian circuits in  $G$
- 6 a) Draw a simple disconnected graph with 10 vertices, 4 components and maximum 3
- number of edges.
- b) Explain any two applications of graphs. 2
- c) Check whether the given graph is an Euler graph and if yes, give the Euler line. 4
- Justify your answer.



- 7 a) Prove or disprove: If every vertex of a simple graph  $G$  has degree 2, then  $G$  is a 3
- cycle.
- b) Give Hamiltonian circuit of the following graph. 3

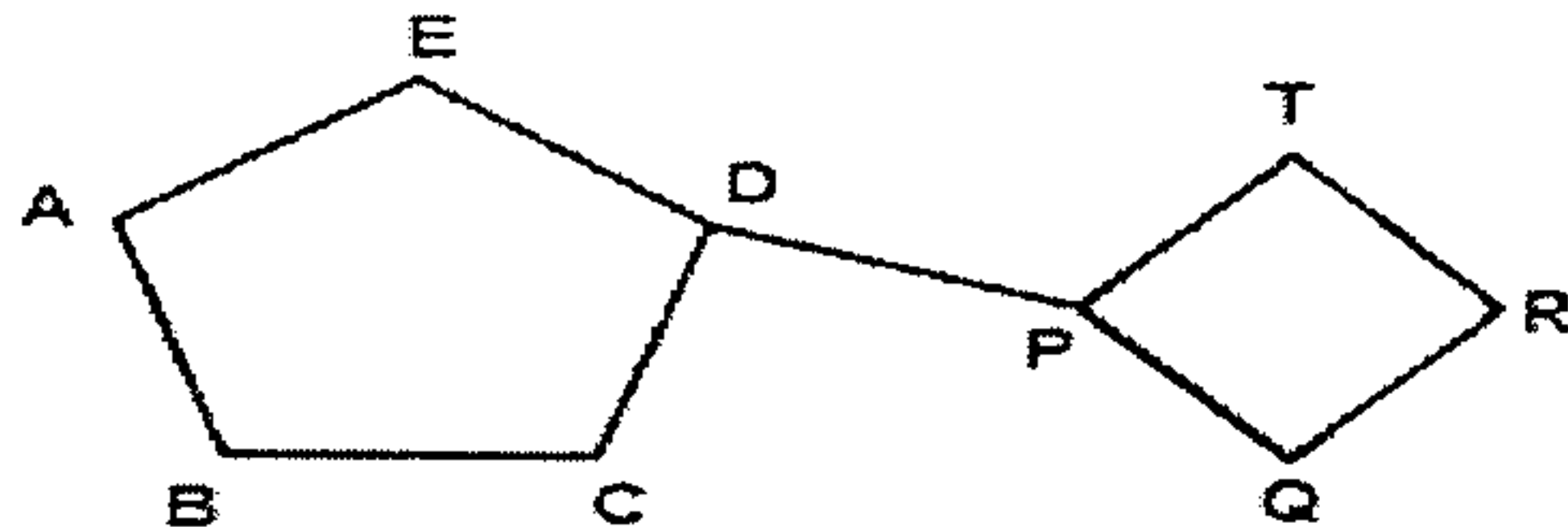


- c) In a graph  $G$  let  $p_1$  and  $p_2$  be two different paths between two given vertices. 3
- Prove that ringsum of  $p_1$  and  $p_2$  is a circuit or a set of circuits.

## PART C

Answer all questions, each carries 3 marks.

- 8 Prove that in a graph  $G$ , if there is exactly one path between every pair of vertices, then  $G$  is a tree. 3
- 9 Given a spanning tree of a graph, how will you find out all spanning trees? 3
- 10 List all cut sets of the following graph. 3



- 11 Prove that every circuit has an even number of edges in common with any cut set. 3

## PART D

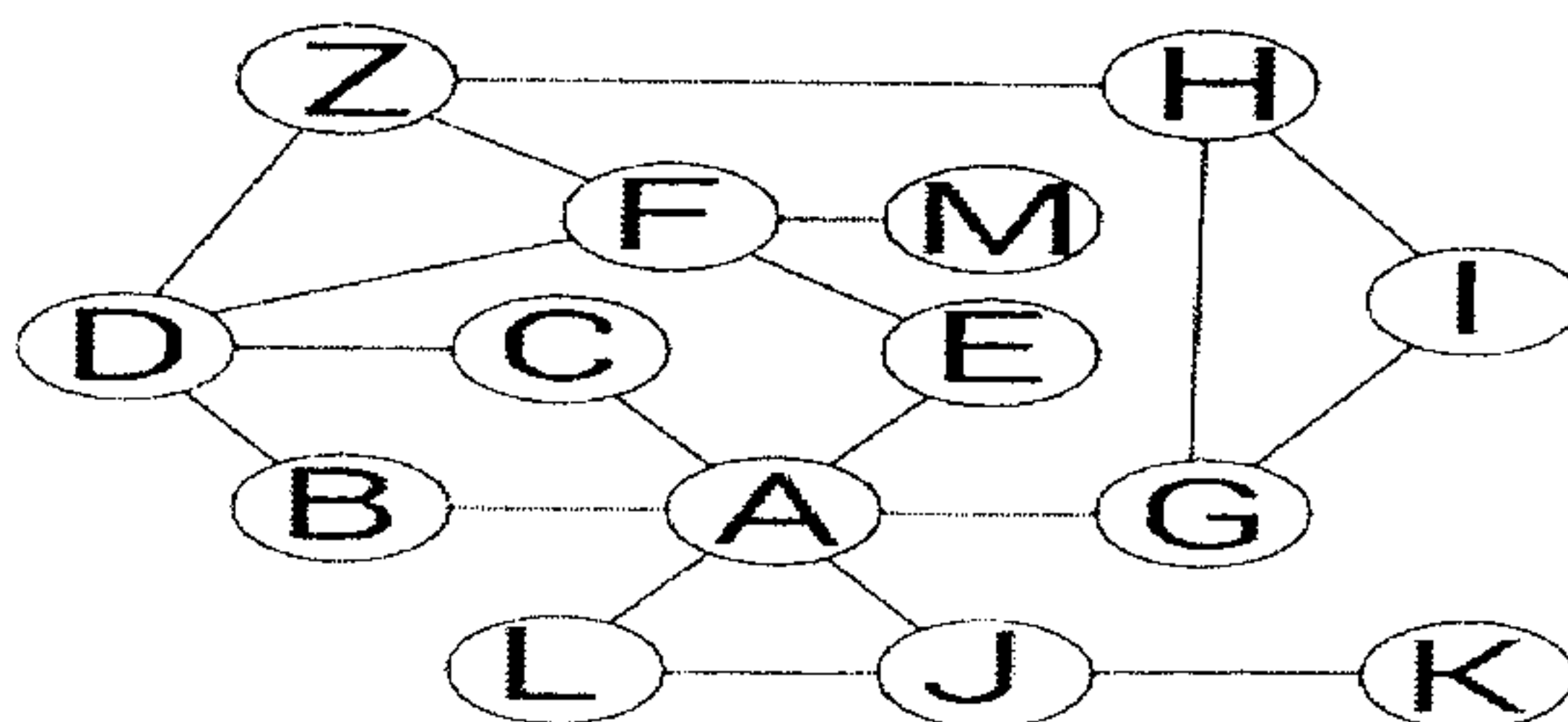
Answer any two full questions, each carries 9 marks.

- 12 a) Define a tree. Give any 4 properties of trees. 3
- b) Prove that a graph is a tree if and only if it is loop-less and has exactly one spanning tree. 3
- c) Prove that every circuit has an even number of edges in common with any cut set. 3
- 
- 13 a) Prove that every tree has either one or two centers. 3
- b) Write short notes on geometric dual and combinatorial dual. 6
- 14 a) Draw a connected graph  $G$  and find two spanning trees  $T_1$  and  $T_2$  of  $G$  such that the distance  $(T_1, T_2) = 3$ . Find the branch set, chord set, rank and nullity of  $T_1$ . 4
- b) Construct a graph  $G$  with the following properties: Edge connectivity = 4, Vertex connectivity = 3 and degree of every vertex of  $G$  is greater than or equal to 5. 5

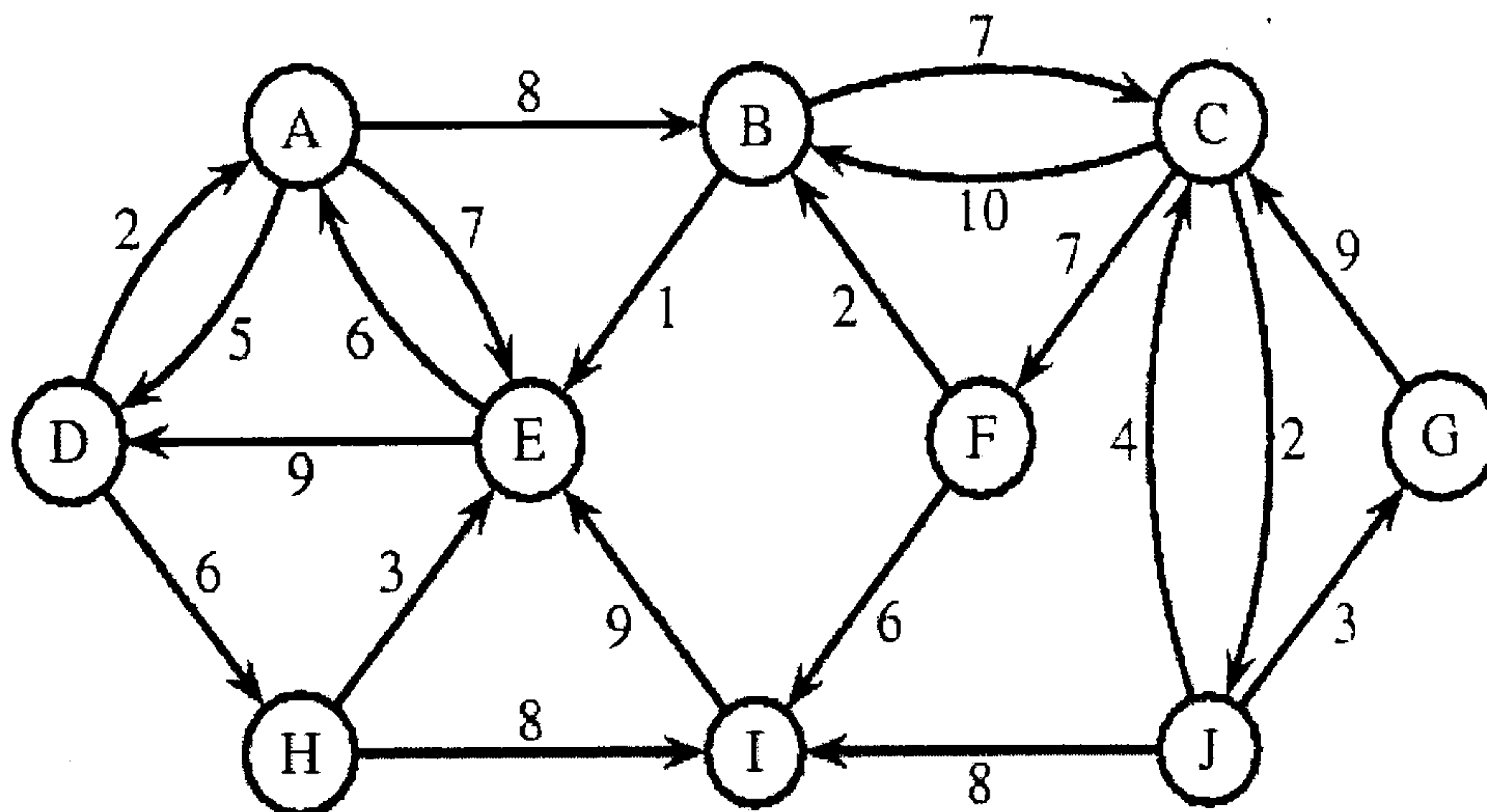
## PART E

Answer any four full questions, each carries 10 marks.

- 15 a) Give incidence matrix of the following graph. 3

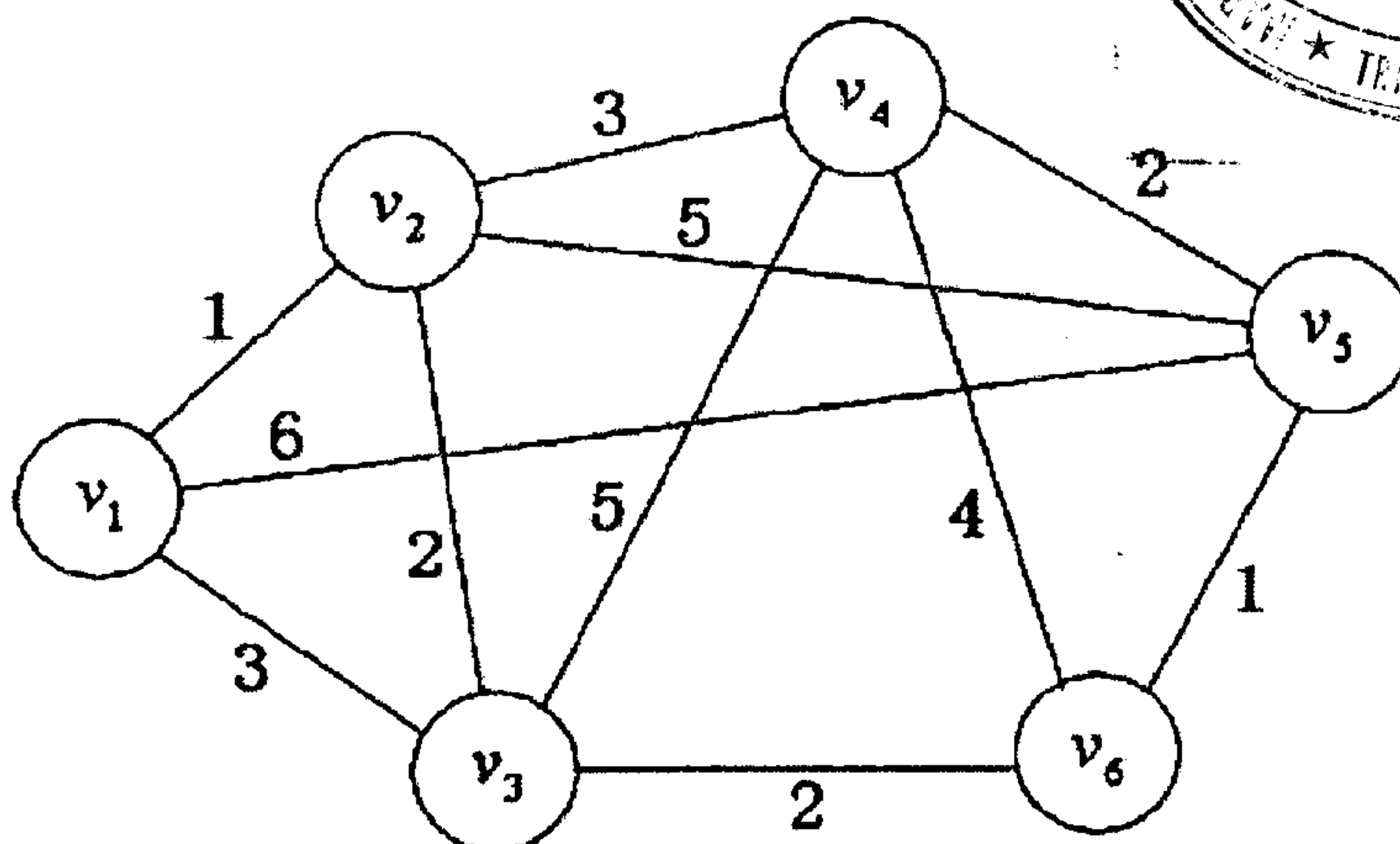
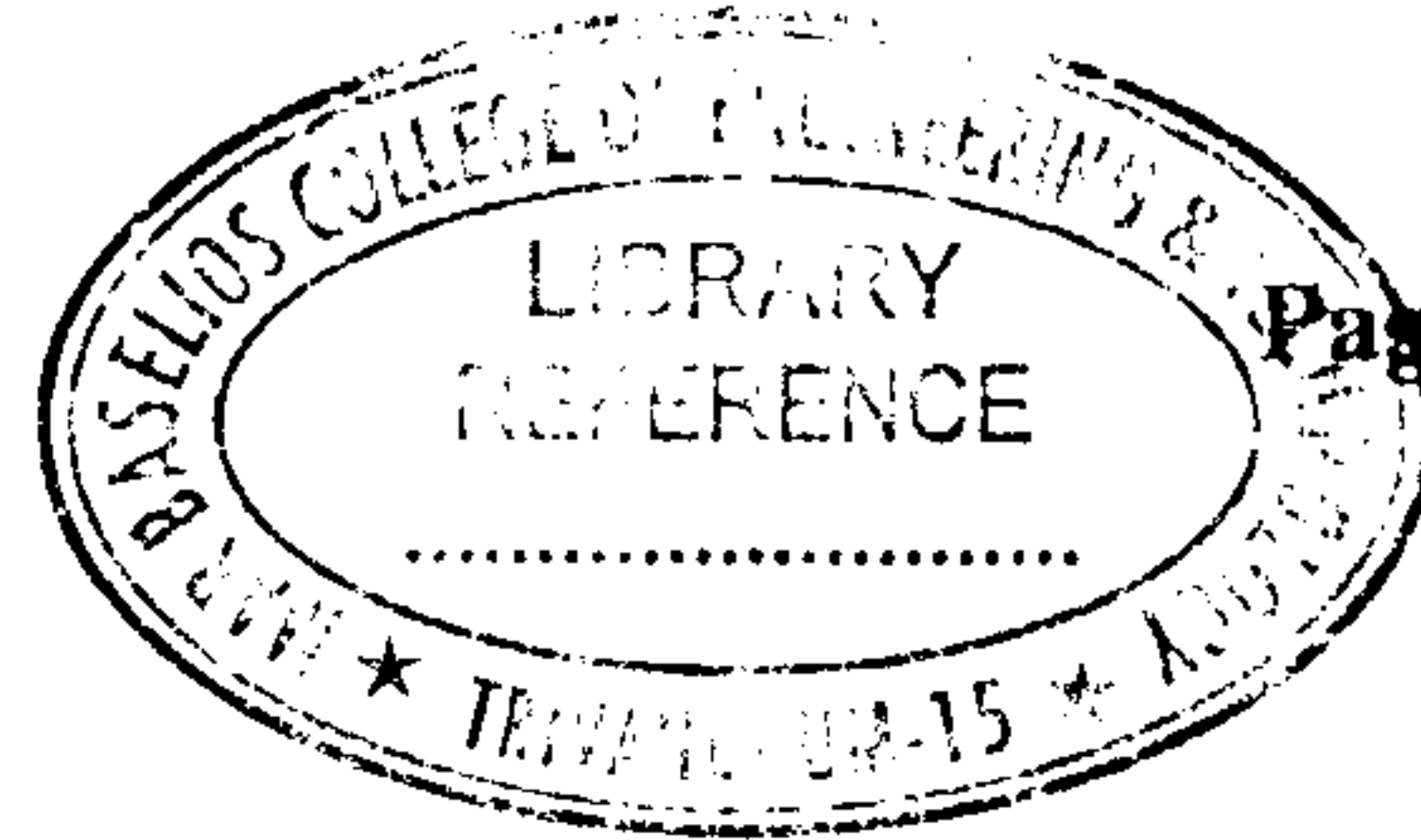


- b) Prove that two graphs  $G_1$  and  $G_2$  are isomorphic if and only if their incidence matrices  $A(G_1)$  and  $A(G_2)$  differ only by permutations of rows and columns. 2
- c) Give Dijkstra's algorithm to find shortest path between a vertex pair. Use it to find shortest path between A and G. 5

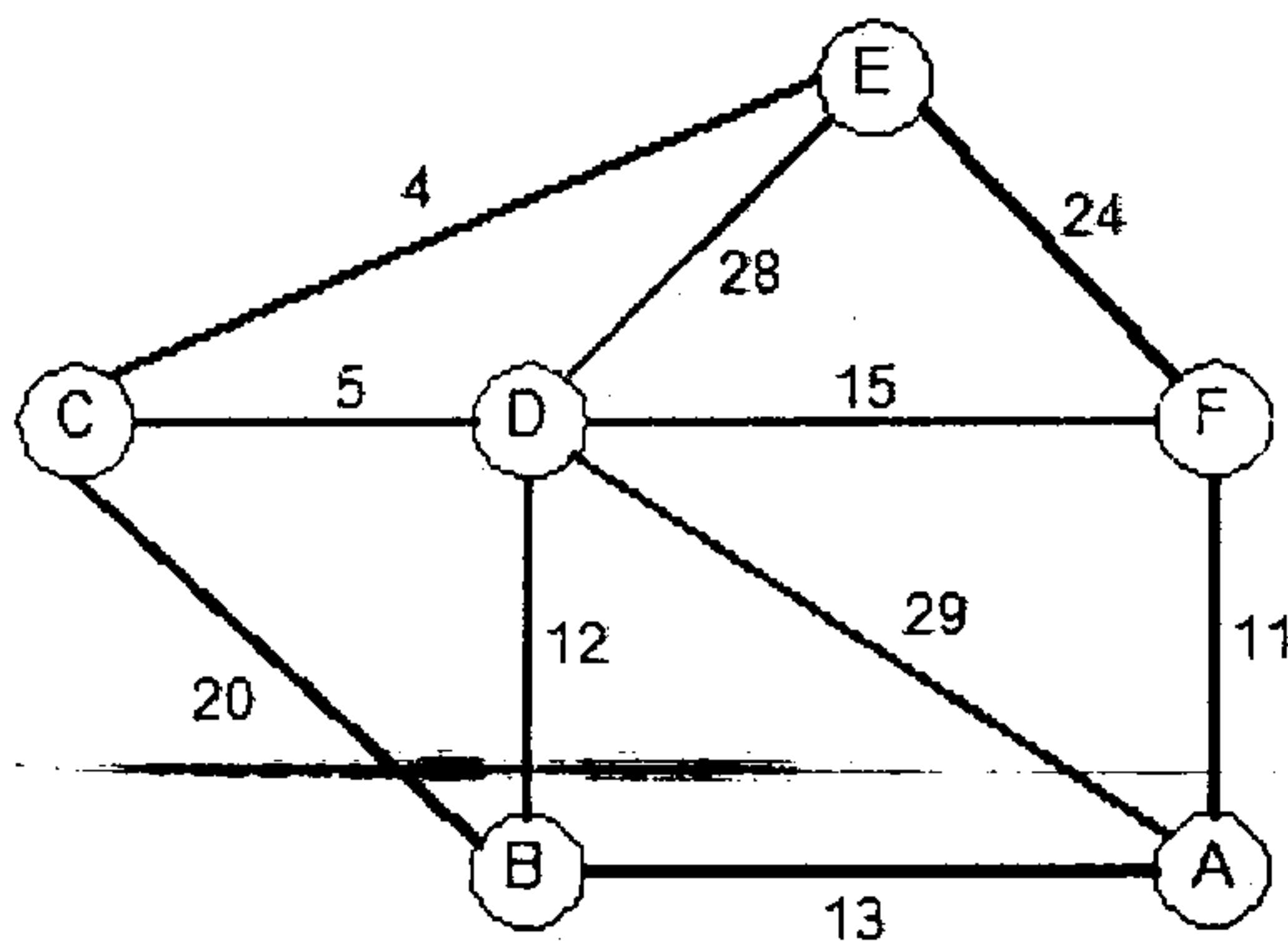


- 16 a) Prove that if  $B$  is a circuit matrix of a connected graph  $G$  with  $n$  vertices and  $e$  edges, then rank of  $B$  is  $e-n+1$ . 3
- b) How will you get fundamental circuit matrix from a circuit matrix. Derive the rank of a fundamental circuit matrix. 2
- c) Explain successor listing and incidence matrix methods used in computer representation of a graph? 5
- 17 a) Prove that the rank of cut set matrix  $C(G)$  is equal to rank of the incidence matrix  $A(G)$ , which equals the rank of the graph  $G$ . 3
- b) Define path matrix. What is the disadvantage of path matrix compared to other matrices. 2
- c) Find a minimum spanning tree of the following graph. Also give its rank and nullity. 5

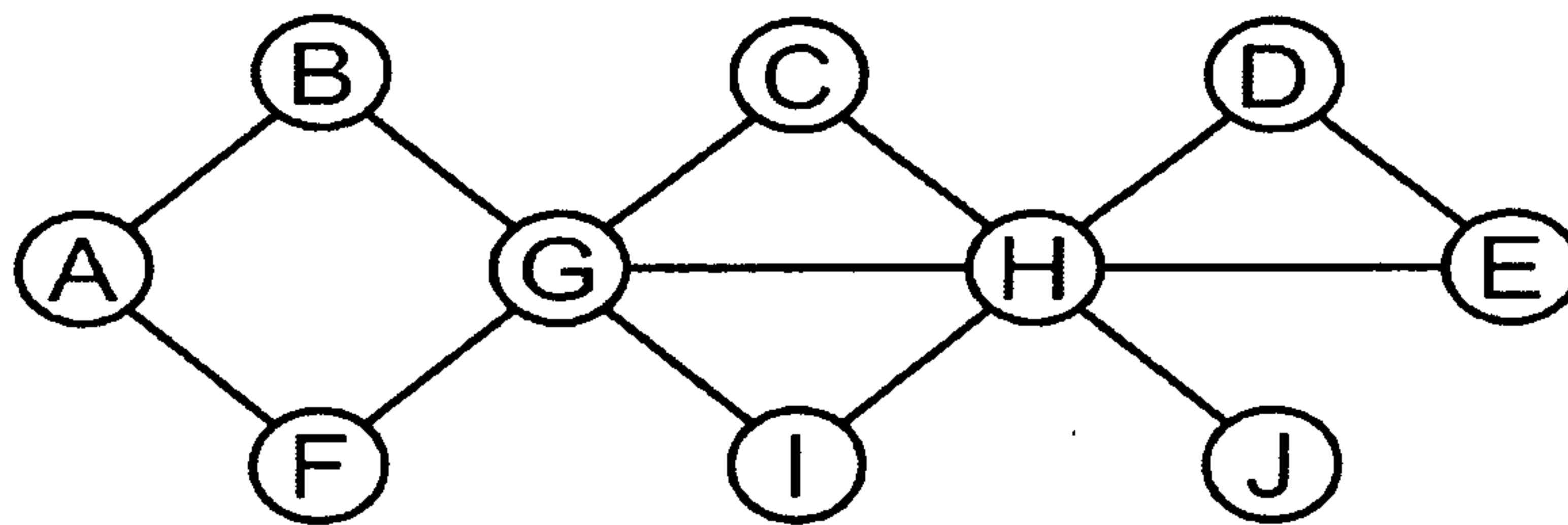




- 18 a) If  $A(G)$  is an incidence matrix of a connected graph  $G$  with  $n$  vertices, then the rank of  $A(G)$  is  $n-1$ . 5
- b) How is Kruskal's algorithm used to find minimum cost spanning tree of a graph. 5  
Find a minimum spanning tree in the graph below.



- 19 a) Write cut set matrix of the following graph. Give its rank. 5



- b) Give an algorithm to check whether a graph is connected or not. How it can be implemented with an adjacency matrix. 5
- 20 a) Give any five properties of circuit matrix. 5
- b) How are edge listing and linear arrays used in computer representation of a graph? 5