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KTU LECTURE NOTES

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

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CLASSICAL SETS AND FUZZY SETS

LECTURE 4

November 26, 2017

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Classical sets(Crisp sets)

- A *set* is defined as a collection of objects, which share certain characteristics.
- A *classical set* is a collection of distinct objects.
- Each individual entity in a set is called a *member* or an *element* of the set.
- Collection of elements in the universe(U) is called *whole set*.
- Number of elements in U is called *cardinal number*.
- Collection of elements within a set are called *subsets*.
- Classical set is defined as the U is spitted in to two groups: *members and nonmembers*.
- No partial membership exists.

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Defining a set

1. The list of all the members of a set may be given.
$$A = \{2, 4, 6, 8, 10\}$$
2. The properties of the set elements may be specified.
$$A = \{x \mid \textit{x is prime number} < 20\}$$
3. The formula for the definition of a set may be mentioned.
$$A = \{x_i = \frac{x_i + 1}{5},\ \textit{i=1 to 10, where } x_i = 1\}$$
4. The set may be defined on the basis of the results of a logical operation.
$$A = \{x \mid \textit{x is an element belonging to P AND Q}\}$$
5. There exist a membership function, which may also be used to define a set.
$$\mu_A(x) = \begin{cases} \textit{1 if } x \in A \\ \textit{0 if } x \notin A \end{cases}$$

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Operations on classical sets

1. Union

$$A \cup B = \{x | x \in A \ or \ x \in B\}$$

2. Intersection

$$A \cap B = \{x | x \in A \ and \ x \in B\}$$

3. Complement

$$\overline{A} = \{x | x \notin A, \ x \in X\}$$

4. Difference(Subtraction)

$$A|B \ or \ (A - B) = \{x | x \in A \ and \ x \notin B\} = A - (A \cap B)$$
$$B|A \ or \ (B - A) = \{x | x \in B \ and \ x \notin A\} = B - (B \cap A)$$

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Properties of classical sets

- Commutativity

$$A \cup B = B \cup A; A \cap B = B \cap A$$

- Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C; A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributivity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C); A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- Idempotency

$$A \cup A = A; A \cap A = A$$

- Transitivity

$$A \subseteq B \subseteq C, \ then \ A \subseteq C$$

- Identity

$$A \cup \phi = A; A \cap \phi = \phi$$
$$A \cup X = X; A \cap X = X$$

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- Involution

$$\overline{\overline{A}} = A$$

- Law of excluded middle

$$A \cup \overline{A} = X$$

- Law of contradiction

$$A \cap \overline{A} = \phi$$

- DeMorgan's law

$$|\overline{A \cap B}| = \overline{A} \cup \overline{B}$$
$$|\overline{A \cup B}| = \overline{A} \cap \overline{B}$$

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Function mapping of classical sets

- *Mapping* is a rule of correspondence between set theoretic forms and function theoretic forms.
- A classical set is represented by its characteristic function, $\chi(x)$, where x is the element in the universe.

1. *Union*$(A \cup B)$

$$\chi_{A \cup B}(x) = \chi_A(x) \vee \chi_B(x) = max\{\chi_A(x), \chi_B(x)\}$$

2. *Intersection*$(A \cap B)$

$$\chi_{A \cap B}(x) = \chi_A(x) \wedge \chi_B(x) = min\{\chi_A(x), \chi_B(x)\}$$

3. *Complement*$(\overline{A})$

$$\chi_{\overline{A}}(x) = 1 - \chi_A(x)$$

4. *Containment*

$$If\ A \subseteq B,\ then\ \chi_A(x) \leq \chi_B(x)$$

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Introduction to fuzzy logic

- *Fuzzy logic* is a form of multi−valued logic to deal with reasoning that is approximate rather than precise.
- Fuzzy logic variables may have a truth value that ranges between *0* and *1* and is not constrained to the two truth values of classical proposition logic.
- *0.0* represents *absolute falseness* and *1.0* represents *absolute truth*.
- Fuzzy set is characterized by *(0.0,0,1.0)*.

Figure 2.1: A fuzzy logic system accepting imprecise data and providing a decision

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Example

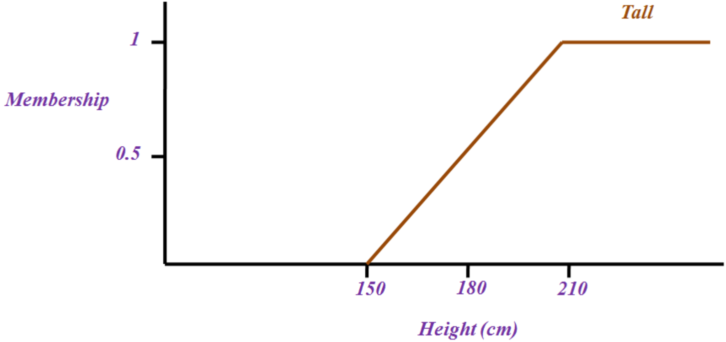

Figure 2.2: Graph showing membership functions for fuzzy set "*tall*"

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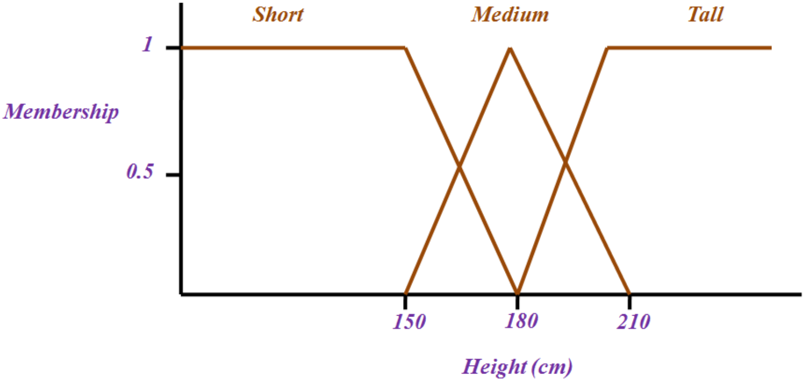

Figure 2.3: Graph showing membership functions for fuzzy sets "*short*", "*medium*", "*tall*"

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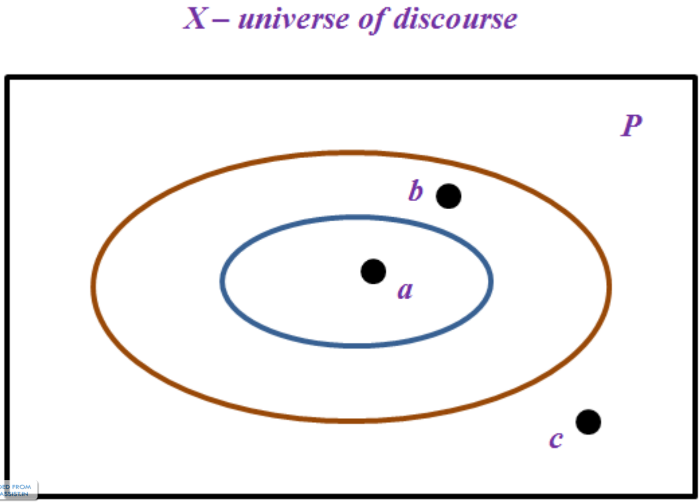
Boundary region of a fuzzy set



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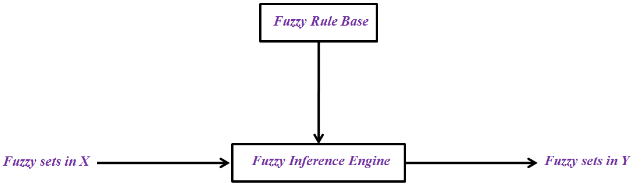
Configuration of a pure fuzzy system

- Fuzziness describes the ambiguity of an event and randomness describes the uncertainity in the occurrence of an event.
- Fuzzy logic consists of *fuzzy inference engine* to perform approximate reasoning.
- Fuzzy sets form the building blocks for fuzzy *IF– THEN rules*.
- A *fuzzy system* is a set of fuzzy rules that converts inputs to outputs.



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Fuzzy sets

- Fuzzy sets may be viewed as an extension and generalization of the basic concepts of crisp sets.
- It allows partial membership.
- A *fuzzy set* is a set having degrees of membership between *1 and 0*.
- Member of one fuzzy set can also be member of other fuzzy sets in the same universe.
- Vagueness is introduced in fuzzy set by eliminating the sharp boundaries that divide members from non−members in the group.
- *Possibility distribution:* A fuzzy set A in the universe of discourse U can be defined as a set of ordered pairs and it is given by,

$$A = \{(x, \mu_A(x)) | x \in U\}$$

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Where, $\mu_A(x)$ is the degree of membership of x in A. $\mu_A(x) \in [0, 1]$

- When U is discrete and finite, fuzzy set A is given as:

$$A = \{\frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \frac{\mu_A(x_3)}{x_3} + ...\} = \{\sum_{i=1}^{n} \frac{\mu_A(x_i)}{x_i}\}$$

  - n is a finite value.
  - The *summation symbol (+)* indicates the collection of each element.
- When U is continuous and infinite, fuzzy set A is given as:

$$A = \{\int \frac{\mu_A(x)}{x}\}$$

  - The *integral sign* $(\int)$ is a continuous function−theoretic union for continuous variables.

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- A fuzzy set is *universal fuzzy set* if and only if the value of membership function is *1* for all members.

$$\mu_U(x) = 1$$

- The universal fuzzy set can also be called *whole fuzzy set*.
- Two fuzzy sets A and B are equal if,

$$\mu_A(x) = \mu_B(x) \text{ } for \text{ } all \text{ } x \in U$$

- A fuzzy set A is an *empty fuzzy set* if and only if value of membership function is *0* for all members.

$$\mu_\phi(x) = 0$$

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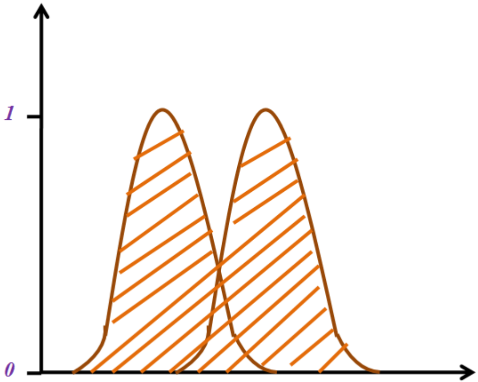
Fuzzy set operations

- *Union*

$$\mu_{A\cup B}(x) = max[\mu_A(x), \mu_B(x)] = \mu_A(x) \vee \mu_B(x), \ for\ all\ x \in U$$



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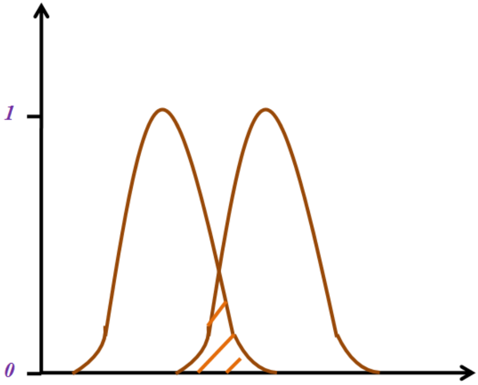
- *Intersection*

$$\mu_{A\cap B}(x) = min[\mu_A(x), \mu_B(x)] = \mu_A(x) \bigcap \mu_B(x), \textit{ for all } x \in U$$



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- *Complement*

$$\mu_{\overline{A}} = 1 - \mu_A(x),\ for\ all\ x \in U$$

- *Algebraic sum*

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)$$

- *Algebraic product*

$$\mu_{A.B}(x) = \mu_A(x).\mu_B(x)$$

- *Bounded sum*

$$\mu_{A \oplus B}(x) = min[1, \mu_A(x) + \mu_B(x)]$$

- *Bounded difference*

$$\mu_{A \odot B}(x) = max[0, \mu_A(x) - \mu_B(x)]$$

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Properties of fuzzy sets

- Commutativity

$$A \cup B = B \cup A; A \cap B = B \cap A$$

- Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C; A \cap (B \cap C) = (A \cap B) \cap C$$

- Distributivity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C); A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- Idempotency

$$A \cup A = A; A \cap A = A$$

- Transitivity

$$A \subseteq B \subseteq C, \text{ then } A \subseteq C$$

- Identity

$$A \cup \phi = A; A \cap \phi = \phi$$

$$A \cup X = X; A \cap X = X$$

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- Involution

$$\overline{\overline{A}} = A$$

- DeMorgan's law

$$|\overline{A \cap B}| = \overline{A} \cup \overline{B}$$
$$|\overline{A \cup B}| = \overline{A} \cap \overline{B}$$

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Problems

(1) Find the power set and cardinality of the given set $X = \{2, 4, 6\}$*. Also find cardinality of power set.*

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(1) Find the power set and cardinality of the given set $X = \{2, 4, 6\}$*. Also find cardinality of power set.*

Set X contains 3 elements, so,

$$n_X = 3$$

The power set of X is,

$$P(X) = \{\phi, \{2\}, \{4\}, \{6\}, \{2, 4\}, \{2, 6\}, \{4, 6\}, \{2, 4, 6\}\}$$

The cardinality of power set P(X) is,

$$n_{P(X)} = 2^{n_X} = 2^3 = 8$$

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(2) Consider 2 given fuzzy sets,

$$A = \{\frac{1}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8}\}$$
$$B = \{\frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8}\}$$

Perform,

(a) Union

(b) Intersection

(c) Complement

(d) Difference

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(a) *Union*

$$A \cup B = max\{\mu_A(x), \mu_B(x)\}$$
$$= \{\frac{1}{2} + \frac{0.4}{4} + \frac{0.5}{6} + \frac{1}{8}\}$$

(b) *Intersection*

$$A \cap B = min\{\mu_A(x), \mu_B(x)\}$$
$$= \{\frac{0.5}{2} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.2}{8}\}$$

(c) *Complement*

$$\overline{A} = 1 - \mu_A(x) = \{\frac{0}{2} + \frac{0.7}{4} + \frac{0.5}{6} + \frac{0.8}{8}\}$$
$$\overline{B} = 1 - \mu_B(x) = \{\frac{0.5}{2} + \frac{0.6}{4} + \frac{0.9}{6} + \frac{0}{8}\}$$

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(d) *Difference*

$$A|B = A \cap \overline{B} = \{\frac{0.5}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0}{8}\}$$
$$B|A = B \cap \overline{A} = \{\frac{0}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{0.8}{8}\}$$

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(3) Consider 2 given fuzzy sets,

$$B_1 = \{\frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\}$$
$$B_2 = \{\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}\}$$

Perform,

$(a) B_1 \cup B_2$ $(b) B_1 \cap B_2$
$(c) \overline{B_1}$ $(d) \overline{B_2}$
$(e) B_1 | B_2$ $(f) \overline{B_1 \cup B_2}$
$(g) \overline{B_1 \cap B_2}$ $(h) B_1 \cap \overline{B_1}$
$(i) B_1 \cup \overline{B_1}$ $(j) B_2 \cap \overline{B_2}$
$(k) B_2 \cup \overline{B_2}$

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$$(a)\ B_1 \cup B_2 = \{\frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\}$$
$$(b)\ B_1 \cap B_2 = \{\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}\}$$
$$(c)\ \overline{B_1} = \{\frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0}\}$$
$$(d)\ \overline{B_2} = \{\frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0}\}$$
$$(e)\ B_1|B_2 = B_1 \cap \overline{B_2} = \{\frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\}$$
$$(f)\ \overline{B_1 \cup B_2} = \overline{B_1} \cap \overline{B_2} = \{\frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0}\}$$
$$(g)\ \overline{B_1 \cap B_2} = \overline{B_1} \cup \overline{B_2} = \{\frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0}\}$$

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$$(h)\ B_1 \cap \overline{B_1} = \{\frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0}\}$$

$$(i)\ B_1 \cup \overline{B_1} = \{\frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0}\}$$

$$(j)\ B_2 \cap \overline{B_2} = \{\frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0}\}$$

$$(k)\ B_2 \cup \overline{B_2} = \{\frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0}\}$$

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(4) It is necessary to compare two sensors based upon their detection levels and gain settings. The table of gain settings and sensor detection levels with a standard item being monitored providing typical membership values to represent the detection levels for each sensor is given in table:

Gain setting	Detection level of sensor 1	Detection level of sensor 2
0	0	0
10	0.2	0.35
20	0.35	0.25
30	0.65	0.8
40	0.85	0.95
50	1	1

Perform union, intersection, complement and difference over sensor 1 and sensor 2.

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Given the universe of discourse,

$$X = \{0, 10, 20, 30, 40, 50\}$$

The membership functions for the two sensors in the discrete form as,

$$D_1 = \{\frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50}\}$$

$$D_2 = \{\frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50}\}$$

$$D_1 \Longrightarrow Sensor1$$

$$D_2 \Longrightarrow Sensor2$$

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(a) Union

$$D_1 \cup D_2 = max[D_1, D_2] = \{\frac{0}{0} + \frac{0.35}{10} + \frac{0.35}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50}\}$$

(b) Intersection

$$D_1 \cap D_2 = min[D_1, D_2] = \{\frac{0}{0} + \frac{0.2}{10} + \frac{0.25}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50}\}$$

(c) Complement

$$\overline{D_1} = 1 - D_1 = \{\frac{1}{0} + \frac{0.8}{10} + \frac{0.65}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50}\}$$
$$\overline{D_2} = 1 - D_2 = \{\frac{1}{0} + \frac{0.65}{10} + \frac{0.75}{20} + \frac{0.2}{30} + \frac{0.05}{40} + \frac{0}{50}\}$$

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(d) Difference

$$D_1|D_2 = D_1 \cap \overline{D_2} = \{\frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.2}{30} + \frac{0.05}{40} + \frac{0}{50}\}$$
$$D_2|D_1 = D_2 \cap \overline{D_1} = \{\frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50}\}$$

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(5) Consider 2 given fuzzy sets,

$$A = \{\frac{0.2}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.5}{4}\}$$
$$B = \{\frac{0.1}{1} + \frac{0.2}{2} + \frac{0.2}{3} + \frac{0.1}{4}\}$$

Find,

(a) algebraic sum

(b) algebraic product

(c) bounded sum

(d) bounded difference

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(a) *Algebraic sum*

$$\mu_{A+B}(X) = [\mu_A(x) + \mu_B(x)] - [\mu_A(x) \cdot \mu_B(x)]$$
$$= \{\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{1.5}{4}\} - \{\frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0.5}{4}\}$$
$$= \{\frac{0.28}{1} + \frac{0.44}{2} + \frac{0.52}{3} + \frac{1}{4}\}$$

(b) *Algebraic product*

$$\mu_{A \cdot B}(X) = \mu_A(x) \cdot \mu_B(x)$$
$$= \{\frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0.5}{4}\}$$

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(c) Bounded sum

$$\mu_{A \bigoplus B}(X) = min[1, \mu_A(x) + \mu_B(x)]$$
$$= min\{1, \{\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{1.5}{4}\}\}$$
$$= \{\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{1}{4}\}$$

(d) Bounded difference

$$\mu_{A \ominus B}(X) = max[0, \mu_A(x) - \mu_B(x)]$$
$$= max\{0, \{\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4}\}\}$$
$$= \{\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4}\}$$

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(6) Show the following fuzzy sets satisfy DeMorgan's law:

$$\mu_A(x) = \frac{1}{1+5x}$$
$$\mu_B(x) = \left(\frac{1}{1+5x}\right)^{1/2}$$

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DeMorgan's law:

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

We have,

$$\mu_{A \cup B}(x) = max[\mu_A(x), \mu_B(x)]$$
$$= \frac{\mu_A(x) + \mu_B(x) + |\mu_A(x) - \mu_B(x)|}{2}$$
$$\mu_{A \cap B}(x) = min[\mu_A(x), \mu_B(x)]$$
$$= \frac{\mu_A(x) + \mu_B(x) - |\mu_A(x) - \mu_B(x)|}{2}$$

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$$A \cup B = \mu_{A \cup B}(x)$$

$$= \frac{\mu_A(x) + \mu_B(x) + |\mu_A(x) - \mu_B(x)|}{2}$$

$$= \frac{\mu_A(x) + \mu_B(x) + |-[\mu_B(x) - \mu_A(x)]|}{2}$$

$$(\because \mu_B(x) > \mu_A(x))$$

$$= \frac{\mu_A(x) + \mu_B(x) + [\mu_B(x) - \mu_A(x)]}{2}$$

$$= \frac{\mu_A(x) + \mu_B(x) + \mu_B(x) - \mu_A(x)}{2}$$

$$= \frac{2 \times \mu_B(x)}{2} = \mu_B = (\frac{1}{1+5x})^{1/2}$$

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$$\overline{A \cup B} = 1 - A \cup B = 1 - (\frac{1}{1+5x})^{1/2}$$

$$\overline{A} = 1 - \mu_A(x) = 1 - \frac{1}{1+5x}$$

$$\overline{B} = 1 - \mu_B(x) = 1 - (\frac{1}{1+5x})^{1/2}$$

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$$\overline{A} \cap \overline{B} = \mu_{\overline{A} \cap \overline{B}}(x)$$

$$= \frac{\mu_{\overline{A}}(x) + \mu_{\overline{B}}(x) - |\mu_{\overline{A}}(x) - \mu_{\overline{B}}(x)|}{2}$$

$$= \frac{\mu_{\overline{A}}(x) + \mu_{\overline{B}}(x) - [\mu_{\overline{A}}(x) - \mu_{\overline{B}}(x)]}{2}$$

$$(\because \mu_{\overline{A}}(x) > \mu_{\overline{B}}(x))$$

$$= \frac{\mu_{\overline{A}}(x) + \mu_{\overline{B}}(x) - \mu_{\overline{A}}(x) + \mu_{\overline{B}(x)}}{2}$$

$$= \frac{2 \times \mu_{\overline{B}}(x)}{2} = \mu_{\overline{B}} = 1 - (\frac{1}{1+5x})^{1/2}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B} = 1 - (\frac{1}{1+5x})^{1/2}$$

Hence, DeMorgan's law is satisfied.

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CLASSICAL RELATIONS AND FUZZY RELATIONS

LECTURE 5

November 26, 2017

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Relations

- *Relations* represents mapping between sets and connectives in logic.
- A *classical binary relation* represents the presence or absence of a connection between the elements of two sets.
- An ordered $r-tuple$ is an ordered sequence of $r-elements$ is expressed in the form $(a_1, a_2, a_3.....a_r)$.
- For $r = 2$, the $r-tuple$ is called an *ordered pair*.

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Cartesian product of relation

- For crisp sets,

$$A_1, A_2, ..., A_r$$

the set of all $r-tuples$,

$$(a_1, a_2, a_3, ... a_r)$$

where,

$$a_1 \in A_1, a_2 \in A_2, ..., a_r \in A_r,$$

is called *Cartesian product* and is denoted by,

$$A_1 \times A_2 \times .... \times A_r$$

- If all elements are identical, then the Cartesian product is denoted as, A^r.

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Classical Relation

- An $r\!-\!ary$ relation over $A_1, A_2, ..., A_r$ is a *subset* of the Cartesian product $A_1 \times A_2 \times .... \times A_r$.

r	r−ary relation
2	binary
3	ternary
4	quaternary
5	quinary

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- Consider two universes X *and* Y, their Cartesian product is given by,

$$X \times Y = \{(x, y) | x \in X, y \in Y\}$$

- The *characteristic function*,χ, gives the strength of the relationship between ordered pair of elements in each universe.

$$\chi_{X \times Y}(x, y) = \begin{cases} 1, (x, y) \in X \times Y \\ 0, (x, y) \notin X \times Y \end{cases}$$

- When the sets are finite, then the relation is represented by a matrix called *relation matrix*.

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Example

- Consider,

$$X = \{p, q, r\}$$
$$Y = \{2, 4, 6\}$$

Cartesian product of these two sets, $X \times Y$, is,
$\{(p, 2), (p, 4), (p, 6), (q, 2), (q, 4), (q, 6), (r, 2), (r, 4), (r, 6)\}$
From this set one may select a subset such that,

$$R = \{(p, 2), (q, 4), (r, 4), (r, 6)\}$$

Relation matrix is,

$$\begin{array}{c c} & \begin{array}{ccc} 2 & 4 & 6 \end{array} \\ \begin{array}{c} p \\ q \\ r \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

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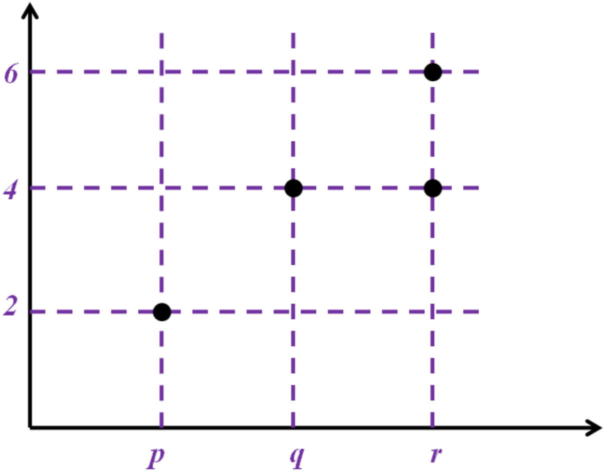
Coordinate diagram of a relation



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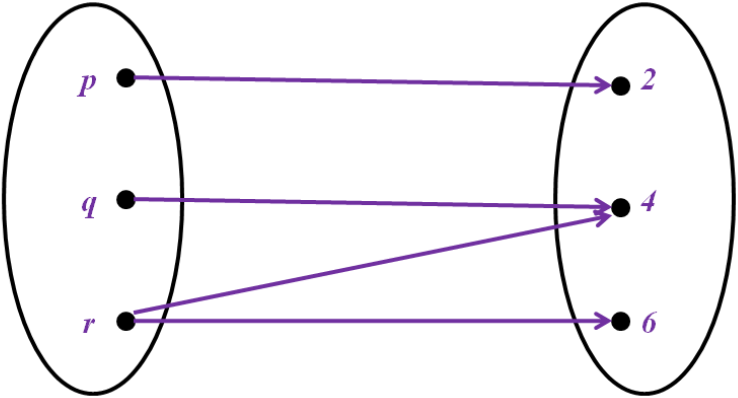
Mapping representation of a relation



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Cardinality of classical relation

- When the cardinality of,

$$X = n_X \text{ and}$$
$$Y = n_Y,$$

  then the cardinality of relation R between the two universe is,

$$n_{X \times Y} = n_X \times n_Y$$

- The cardinality of the power set is given by,

$$n_{P(X \times Y)} = 2^{(n_X n_Y)}$$

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Operations on classical relations

- Union

$$R \cup S \longrightarrow \chi_{R \cup S}(x,y); \chi_{R \cup S}(x,y) = max[\chi_R(x,y), \chi_S(x,y)]$$

- Intersection

$$R \cap S \longrightarrow \chi_{R \cap S}(x,y); \chi_{R \cap S}(x,y) = min[\chi_R(x,y), \chi_S(x,y)]$$

- Complement

$$\overline{R} \longrightarrow \chi_{\overline{R}}(x,y); \chi_{\overline{R}}(x,y) = 1 - \chi_R(x,y)$$

- Containment

$$R \subset S \longrightarrow \chi_R(x,y); \chi_R(x,y) \leq \chi_S(x,y)$$

- Identity

$$\phi \longrightarrow \phi_R \; and \; X \longrightarrow E_R$$

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Properties of crisp relations

- Commutativity
- Associativity
- Distributivity
- Involution
- Idempotency
- Excluded middle laws
- DeMorgan's law

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Composition of classical relations

- The operation executed on two compatible binary relations to get a single binary relation is called *composition*.
- Let R be a relation that maps elements from X to Y and S be a relation that maps elements from Y to Z. R and S are compatible if,

$$R \subseteq X \times Y \text{ and } S \subseteq Y \times Z$$

- The composition between the two relations is denoted by $R \circ S$.

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Example

- Consider the universal sets,

$$X = \{a_1, a_2, a_3\}$$
$$Y = \{b_1, b_2, b_3\}$$
$$Z = \{c_1, c_2, c_3\}$$

Let the relations R and S be formed as,

$$R = X \times Y = \{(a_1, b_1), (a_1, b_2), (a_2, b_2), (a_3, b_3)\}$$
$$S = Y \times Z = \{(b_1, c_1), (b_2, c_3), (b_3, c_2)\}$$

$$T = R \circ S = \{(a_1, c_1), (a_2, c_3), (a_3, c_2), (a_1, c_3)\}$$

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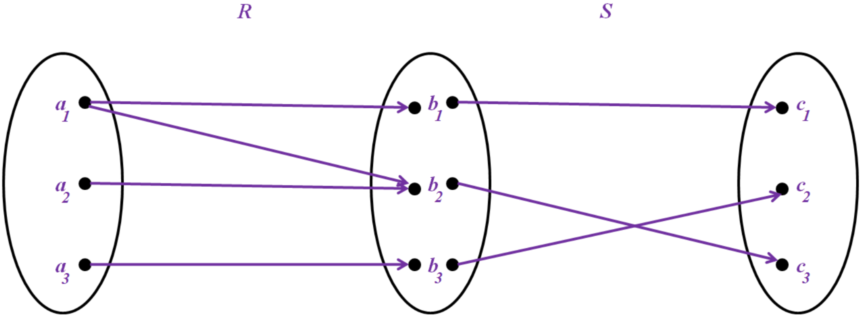
Illustration of relation R and S



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- The composition operations are of two types:
  1. Max–min composition
  2. Max–product composition

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Max–min composition

- The max–min composition is defined by the function theoretic expression as:

$$T = R \circ S$$
$$\chi_T(x,z) = \vee\{\chi_R(x,y) \wedge \chi_S(y,z)\}$$

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Max–product composition

- The max–product composition is defined by the function theoretic expression as:

$$T = R \circ S$$
$$\chi_T(x,z) = \vee\{\chi_R(x,y) \cdot \chi_S(y,z)\}$$

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Few properties of composition operation

Associative	$(R \circ S) \circ M = R \circ (S \circ M)$
Commutative	$R \circ S = S \circ R$
Inverse	$(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

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Fuzzy Relations

- *Fuzzy relations* relate elements of one universe to those of another universe through the Cartesian product of the two universes.
- Based on the concept that everything is related to some extent or unrelated.
- A fuzzy relation is a fuzzy set defined on the Cartesian product of classical sets,

$$\{X_1, X_2, ..., X_n\}$$

where tuples,

$$(x_1, x_2, ..., x_n)$$

may have varying degrees of membership,

$$\mu_R(x_1, x_2, ..., x_n)$$

within the relation.

$$R(X_1, X_2, ..., X_n) = \int_{X_1 \times X_2 \times .... \times X_n} \frac{\mu_R(x_1, x_2, ..., x_n)}{(x_1, x_2, ..., x_n)}, \; x_i \in X_i$$

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Fuzzy matrix

- Let,

$$X = \{x_1, x_2, ..., x_n\} \text{ and } Y = \{y_1, y_2, ..., y_n\}$$

Fuzzy relation $R(x, y)$ can be expressed as an $n \times m$ matrix as:

$$R(x, y) = \begin{bmatrix} \mu_R(x_1, y_1) & \mu_R(x_1, y_2) & ..... & \mu_R(x_1, y_m) \\ \mu_R(x_2, y_1) & \mu_R(x_2, y_2) & ..... & \mu_R(x_2, y_m) \\ ... & ... & ... & \\ ... & ... & ... & \\ \mu_R(x_n, y_1) & \mu_R(x_n, y_2) & ..... & \mu_R(x_n, y_m) \end{bmatrix}$$

- The matrix representing a fuzzy relation is called *Fuzzy matrix*.

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Fuzzy graph

- *Fuzzy graph* is a graphical representation of binary fuzzy relation.
- Each element in X and Y corresponds to a node in the fuzzy graph.
- The connection links are established between the nodes by the elements of $X \times Y$ with nonzero membership grades in $R(X, Y)$.
- The links may also be present in the form of arcs.
- Links are labeled with the membership values as $\mu_R(x_i, y_j)$.

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Bipartite graph

- When $x \neq y$, the link connecting the two nodes is an undirected binary graph called *bipartite graph*.
- Each of the sets X and Y can be represented by a set of nodes such that nodes corresponding to one set are clearly differentiated from the nodes representing the other set.

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Directed graph

- When $x = y$, a node is connected to itself and directed links are used.
- Only one set of nodes corresponding to set X is used.

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Domain and range

- The domain of a binary fuzzy relation $R(x, y)$ is the fuzzy set, $dom\ R(x, y)$, having the membership function as,

$$\mu_{domain} R(x) = max_{y \in Y} \mu_R(x, y), \forall x \in X$$

- The range of a binary fuzzy relation $R(x, y)$ is the fuzzy set, $ran\ R(x, y)$, having the membership function as,

$$\mu_{range} R(y) = max_{x \in X} \mu_R(x, y), \forall y \in Y$$

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Example

- Consider,

$$X = \{x_1, x_2, x_3, x_4\}$$

Binary fuzzy relation on X as,

$$\begin{array}{c c} & \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} 0.2 & 0 & 0.5 & 0 \\ 0 & 0.3 & 0.7 & 0.8 \\ 0.1 & 0 & 0.4 & 0 \\ 0 & 0.6 & 0 & 1 \end{bmatrix} \end{array}$$

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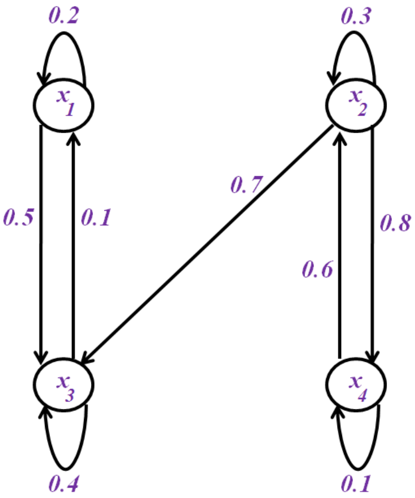
Simple fuzzy graph or Directed graph



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Example

- Let,

$$X = \{x_1, x_2, x_3, x_4\} \textit{ and } Y = \{y_1, y_2, y_3, y_4\}$$

Let R be a relation from X and Y given by,

$$R = \frac{0.2}{(x_1, y_3)} + \frac{0.4}{(x_1, y_2)} + \frac{0.1}{(x_2, y_2)} + \frac{0.6}{(x_2, y_3)} + \frac{1.0}{(x_3, y_3)} + \frac{0.5}{(x_3, y_1)}$$

Fuzzy matrix for relation R is,

$$\begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0 & 0.4 & 0.2 \\ 0 & 0.1 & 0.6 \\ 0.5 & 0 & 1.0 \end{bmatrix} \end{array}$$

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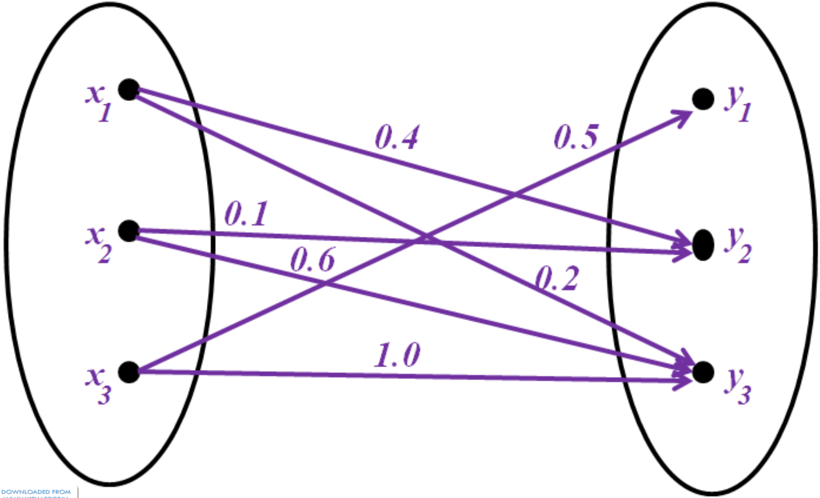
Bipartite graph



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Operations on fuzzy relations

- Union

$$\mu_{R \cup S}(x, y) = max\{\mu_R(x, y), \mu_S(x, y)\}$$

- Intersection

$$\mu_{R \cap S}(x, y) = min\{\mu_R(x, y), \mu_S(x, y)\}$$

- Complement

$$\mu_{\overline{R}}(x, y) = 1 - \mu_R(x, y)$$

- Containment

$$R \subset S \Longrightarrow \mu_R(x, y) \leq \mu_S(x, y)$$

- Inverse

$$R^{-1}(y, x) = R(x, y) \text{ for all pairs } (y, x) \in Y \times X$$

- Projection

$$\mu_{[R \downarrow Y]}(x, y) = max_x \mu_R(x, y)$$

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Properties of fuzzy relations

- Commutativity
- Associativity
- Distributivity
- Identity
- Idempotency
- DeMorgan′s law

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Fuzzy composition

- Let A be a fuzzy set on universe X and B be a fuzzy set on universe Y.
- the cartesian product over A and B results in fuzzy relation $R.ie$,

  $$A \times B = R$$

  where

  $$R \subset X \times Y$$

- The membership function is given by,

  $$\mu_R(x, y) = \mu_{A \times B}(x, y) = min[\mu_A(x), \mu_B(y)]$$

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Fuzzy composition techniques

- *Max–min composition*
  - Let R be fuzzy relation on cartesian space $X \times Y$ and S be fuzzy relation on cartesian space $Y \times Z$.
  - Max–min composition of $R(X, Y)$ and $S(Y, Z)$,

$$\mu_T(x, z) = \mu_{R \circ S}(x, z)$$
$$= max_{y \in Y}\{min[\mu_R(x, y), \mu_S(y, z)]\}$$
$$= \vee_{y \in Y}[\mu_R(x, y) \wedge \mu_S(y, z)] \forall x \in X, z \in Z$$

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- *Min−max composition*
  - Let R be fuzzy relation on cartesian space $X \times Y$ and S be fuzzy relation on cartesian space $Y \times Z$.
  - Max−min composition of $R(X, Y)$ and $S(Y, Z)$,

$$\mu_T(x, z) = \mu_{R \circ S}(x, z)$$
$$= min_{y \in Y}\{max[\mu_R(x, y), \mu_S(y, z)]\}$$
$$= \wedge_{y \in Y}[\mu_R(x, y) \vee \mu_S(y, z)] \forall x \in X, z \in Z$$

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- *Max–product composition*
  - Let R be fuzzy relation on cartesian space $X \times Y$ and S be fuzzy relation on cartesian space $Y \times Z$.
  - Max–min composition of $R(X, Y)$ and $S(Y, Z)$,

$$\begin{aligned} \mu_T(x, z) &= \mu_{R \circ S}(x, z) \\ &= max_{y \in Y}\{\mu_R(x, y) \cdot \mu_S(y, z)\} \\ &= \vee_{y \in Y}[\mu_R(x, y) \cdot \mu_S(y, z)] \forall x \in X, z \in Z \end{aligned}$$

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Properties of fuzzy composition

1. $R \circ S = S \circ R$
2. $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$
3. $(R \circ S) \circ M = R \circ (S \circ M)$

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Tolerance and Equivalence Relations

- The three characteristic properties of relations are: *reflexivity, symmetry and transitivity*.
  1. A relation is said to be *reflexive* if every vertex in the graph originates a single loop.
  2. A relation is said to be *symmetric* if for every edge pointing from vertex i to vertex j, there is an edge pointing in the opposite direction,*ie*, from vertex j to i.
  3. A relation is said to be *transitive* if for every pair of edges– one pointing from vertex i to vertex j and the other pointing from vertex j to vertex k, then there is an edge pointing from vertex i to vertex k.

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Classical Equivalence Relation

- Let relation R on a universe X be a relation from X to X, is an *equivalence relation* if the following *3* properties are satisfied:

1. *Reflexivity*

$$\chi_R(x_i, x_i) = 1 or (x_i, x_i) \in R$$

2. *Symmetry*

$$\chi_R(x_i, x_j) = \chi_R(x_j, x_i)$$
$$ie, (x_i, x_j) \in R \Rightarrow (x_j, x_i) \in R$$

3. *Transitivity*

$$\chi_R(x_i, x_j) and \chi_R(x_j, x_k) = 1, \; so, \; \chi_R(x_i, x_k) = 1$$

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Classical Tolerance Relation

- A *tolerance relation* R_1 on universe X is one where the only properties of reflexivity and symmetry are satisfied.
- It can also be called as *proximity relation*.
- An equivalence relation, R, can be formed from tolerance relation R_1 by $(n-1)$ compositions within itself:

$$R_1^{n-1} = R_1 \circ R_1 \circ \ldots.. \circ R_1 = R$$

where n is the cardinality of the set that defines R_1.

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Fuzzy Equivalence Relation

- The *equivalence relation* can also be called *similarity relation.*
- Let relation R on a universe X be a relation from X to X, is an *fuzzy equivalence relation* if the following *3* properties are satisfied:
  1. *Reflexivity*

     $$\mu_R(x_i, x_i) = 1 \forall x \in X$$

     If this is not the case for few $x \in X$, then $R(X, X)$ is said to be *irreflexive.*
  2. *Symmetry*

     $$\mu_R(x_i, x_j) = \mu_R(x_j, x_i) \forall x_i, x_j \in X$$

     If this is not the case for few $x_i, x_j \in X$, then $R(X, X)$ is called *asymmetric.*
  3. *Transitivity*

     $$\mu_R(x_i, x_j) = \lambda_1 and \mu_R(x_j, x_k) = \lambda_2 \Rightarrow \mu_R(x_i, x_k) = \lambda$$

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Fuzzy Tolerance Relation

- A binary fuzzy relation that possesses the properties of reflexivity and symmetry is called *fuzzy tolerance relation*.
- It can also be called as *resemblance relation*.
- A fuzzy equivalence relation, R, can be formed from fuzzy tolerance relation R_1 by $(n-1)$ compositions within itself:

$$R_1^{n-1} = R_1 \circ R_1 \circ ..... \circ R_1 = R$$

where n is the cardinality of the set that defines R_1.

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Non−interactive Fuzzy sets

- The independent events in probability theory are analogous to non−interactive fuzzy sets in fuzzy theory.
- Defining fuzzy set A on the cartesian space $X = X_1 \times X_2$. Set A is separable into two non−interactive fuzzy sets called *orthogonal projections*, if and only if,

$$A = OPr_{X_1}(A) \times OPr_{X_2}(A)$$

where,

$$\mu_{OPr_{X_1}(A)}(x_1) = max_{x_2 \in X_2} \mu_A(x_1, x_2), \forall x_1 \in X_1$$
$$\mu_{OPr_{X_2}(A)}(x_2) = max_{x_1 \in X_1} \mu_A(x_1, x_2), \forall x_2 \in X_2$$

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Problems

(1) *Consider the following two fuzzy sets:*

$$A = \{\frac{0.3}{x_1} + \frac{0.7}{x_2} + \frac{1}{x_3}\} \textit{ and}$$
$$B = \{\frac{0.4}{y_1} + \frac{0.9}{y_2}\}$$

Perform the cartesian product over these given fuzzy sets.

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$$\mu_R(x_1, y_1) = min[\mu_A(x_1), \mu_B(y_1)] = min[0.3, 0.4] = 0.3$$
$$\mu_R(x_1, y_2) = min[\mu_A(x_1), \mu_B(y_2)] = min[0.3, 0.9] = 0.3$$
$$\mu_R(x_2, y_1) = min[\mu_A(x_2), \mu_B(y_1)] = min[0.7, 0.4] = 0.4$$
$$\mu_R(x_2, y_2) = min[\mu_A(x_2), \mu_B(y_2)] = min[0.7, 0.9] = 0.7$$
$$\mu_R(x_3, y_1) = min[\mu_A(x_3), \mu_B(y_1)] = min[1, 0.4] = 0.4$$
$$\mu_R(x_3, y_2) = min[\mu_A(x_3), \mu_B(y_2)] = min[1, 0.9] = 0.9$$

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$$\mu_R(x_1, y_1) = min[\mu_A(x_1), \mu_B(y_1)] = min[0.3, 0.4] = 0.3$$
$$\mu_R(x_1, y_2) = min[\mu_A(x_1), \mu_B(y_2)] = min[0.3, 0.9] = 0.3$$
$$\mu_R(x_2, y_1) = min[\mu_A(x_2), \mu_B(y_1)] = min[0.7, 0.4] = 0.4$$
$$\mu_R(x_2, y_2) = min[\mu_A(x_2), \mu_B(y_2)] = min[0.7, 0.9] = 0.7$$
$$\mu_R(x_3, y_1) = min[\mu_A(x_3), \mu_B(y_1)] = min[1, 0.4] = 0.4$$
$$\mu_R(x_3, y_2) = min[\mu_A(x_3), \mu_B(y_2)] = min[1, 0.9] = 0.9$$

$$R = A \times B = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.3 & 0.3 \\ 0.4 & 0.7 \\ 0.4 & 0.9 \end{bmatrix} \end{array}$$

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(2) *Two fuzzy relations are given by,*

$$R = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array} \text{ and}$$

$$S = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} y_1 \\ y_2 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Obtain fuzzy relation T as a composition between the fuzzy relations R and S.

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(a) Max–min composition

$$
\begin{aligned}
\mu_T(x_1, z_1) &= max\{min[\mu_R(x_1, y_1), \mu_S(y_1, z_1)], \\
&\qquad min[\mu_R(x_1, y_2), \mu_S(y_2, z_1)]\} \\
&= max\{min[0.6, 1], min[0.3, 0.8]\} \\
&= max\{0.6, 0.3\} = 0.6 \\
\mu_T(x_1, z_2) &= max\{min[\mu_R(x_1, y_1), \mu_S(y_1, z_2)], \\
&\qquad min[\mu_R(x_1, y_2), \mu_S(y_2, z_2)]\} \\
&= max\{min[0.6, 0.5], min[0.3, 0.4]\} \\
&= max\{0.5, 0.3\} = 0.5 \\
\mu_T(x_1, z_3) &= max\{min[\mu_R(x_1, y_1), \mu_S(y_1, z_3)], \\
&\qquad min[\mu_R(x_1, y_2), \mu_S(y_2, z_3)]\} \\
&= max\{min[0.6, 0.3], min[0.3, 0.7]\} \\
&= max\{0.3, 0.3\} = 0.3
\end{aligned}
$$

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$$
\begin{aligned}
\mu_T(x_2, z_1) &= max\{min[\mu_R(x_2, y_1), \mu_S(y_1, z_1)], \\
&\qquad min[\mu_R(x_2, y_2), \mu_S(y_2, z_1)]\} \\
&= max\{min[0.2, 1], min[0.9, 0.8]\} \\
&= max\{0.2, 0.8\} = 0.8 \\
\mu_T(x_2, z_2) &= max\{min[\mu_R(x_2, y_1), \mu_S(y_1, z_2)], \\
&\qquad min[\mu_R(x_2, y_2), \mu_S(y_2, z_2)]\} \\
&= max\{min[0.2, 0.5], min[0.9, 0.4]\} \\
&= max\{0.2, 0.4\} = 0.4 \\
\mu_T(x_2, z_3) &= max\{min[\mu_R(x_2, y_1), \mu_S(y_1, z_3)], \\
&\qquad min[\mu_R(x_2, y_2), \mu_S(y_2, z_3)]\} \\
&= max\{min[0.2, 0.3], min[0.9, 0.7]\} \\
&= max\{0.2, 0.7\} = 0.7
\end{aligned}
$$

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$$T = R \circ S = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

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(b) *Max–product composition*

$$
\begin{aligned}
\mu_T(x_1, z_1) &= max\{[\mu_R(x_1, y_1) \bullet \mu_S(y_1, z_1)], \\
&\qquad [\mu_R(x_1, y_2) \bullet \mu_S(y_2, z_1)]\} \\
&= max\{[0.6 \times 1], [0.3 \times 0.8]\} \\
&= max\{0.6, 0.24\} = 0.6 \\
\mu_T(x_1, z_2) &= max\{[\mu_R(x_1, y_1) \bullet \mu_S(y_1, z_2)], \\
&\qquad [\mu_R(x_1, y_2) \bullet \mu_S(y_2, z_2)]\} \\
&= max\{[0.6 \times 0.5], [0.3 \times 0.4]\} \\
&= max\{0.3, 0.12\} = 0.3 \\
\mu_T(x_1, z_3) &= max\{[\mu_R(x_1, y_1) \bullet \mu_S(y_1, z_3)], \\
&\qquad [\mu_R(x_1, y_2) \bullet \mu_S(y_2, z_3)]\} \\
&= max\{[0.6 \times 0.3], [0.3 \times 0.7]\} \\
&= max\{0.18, 0.21\} = 0.21
\end{aligned}
$$

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$$
\begin{aligned}
\mu_T(x_2, z_1) &= max\{[\mu_R(x_2, y_1) \bullet \mu_S(y_1, z_1)], \\
&\qquad [\mu_R(x_2, y_2) \bullet \mu_S(y_2, z_1)]\} \\
&= max\{[0.2 \times 1], [0.9 \times 0.8]\} \\
&= max\{0.2, 0.72\} = 0.72 \\
\mu_T(x_2, z_2) &= max\{[\mu_R(x_2, y_1) \bullet \mu_S(y_1, z_2)], \\
&\qquad [\mu_R(x_2, y_2) \bullet \mu_S(y_2, z_2)]\} \\
&= max\{[0.2 \times 0.5], [0.9 \times 0.4]\} \\
&= max\{0.1, 0.36\} = 0.36 \\
\mu_T(x_2, z_3) &= max\{[\mu_R(x_2, y_1) \bullet \mu_S(y_1, z_3)], \\
&\qquad [\mu_R(x_2, y_2) \bullet \mu_S(y_2, z_3)]\} \\
&= max\{[0.2 \times 0.3], [0.9 \times 0.7]\} \\
&= max\{0.06, 0.63\} = 0.63
\end{aligned}
$$

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$$T = R \circ S = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix} \end{array}$$

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(3) *For a speed control of DC motor, the membership functions of series resistance, armature current and speed are given as follows:*

$$SR = \{\frac{0.4}{30} + \frac{0.6}{60} + \frac{1.0}{100} + \frac{0.1}{120}\}$$
$$I = \{\frac{0.2}{20} + \frac{0.3}{40} + \frac{0.6}{60} + \frac{0.8}{80} + \frac{1.0}{100} + \frac{0.2}{120}\}$$
$$N = \{\frac{0.35}{500} + \frac{0.67}{1000} + \frac{0.97}{1500} + \frac{0.25}{1800}\}$$

Compute relation T for relating series resistance to motor speed. Perform max–min composition only.

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$$R = SR \times I = \begin{array}{c c} & \begin{array}{cccccc} 20 & 40 & 60 & 80 & 100 & 120 \end{array} \\ \begin{array}{c} 30 \\ 60 \\ 100 \\ 120 \end{array} & \begin{bmatrix} 0.2 & 0.3 & 0.4 & 0.4 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.6 & 0.6 & 0.6 & 0.2 \\ 0.2 & 0.3 & 0.6 & 0.8 & 1.0 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

$$S = I \times N = \begin{array}{c c} & \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{array}{c} 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \end{array} & \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 & 0.25 \\ 0.35 & 0.6 & 0.6 & 0.25 \\ 0.35 & 0.67 & 0.8 & 0.25 \\ 0.35 & 0.67 & 0.97 & 0.25 \\ 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

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$$T = R \circ S = \begin{array}{c} \\ 30 \\ 60 \\ 100 \\ 120 \end{array} \begin{array}{c} \begin{array}{cccc} 500 & 1000 & 1500 & 1800 \end{array} \\ \begin{bmatrix} 0.35 & 0.4 & 0.4 & 0.25 \\ 0.35 & 0.6 & 0.6 & 0.25 \\ 0.35 & 0.67 & 0.97 & 0.25 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

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(4) *Which of the following are equivalence relations?*

No.	**Set**	**Relation on the set**
(a)	People	is the brother of
(b)	People	has the same parents as
(c)	Points on a map	is connected by a road to
(d)	Lines in plane geometry	is perpendicular to
(e)	Positive Integers	for some integer k equals 10^k times

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END