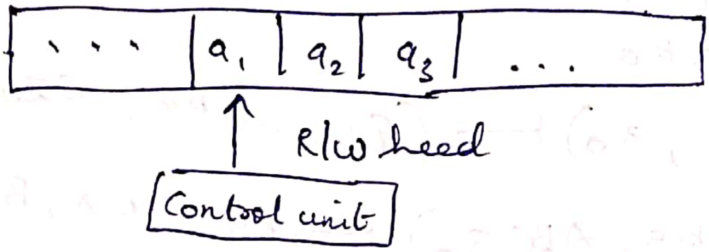
Module 5 : TOC

- **Turing Machine**

is a FA with following components.
Tape, R/w head, Control unit.



Tape: to store information & is infinite on both side. string to be scanned should be stored from leftmost position on the tape.

R/w head: can read symbol from where it is pointing to and it can write into tape to where it points to.
It can move either towards left/right.

Control unit: determines the reading from tape/writing into tape.

$M = (Q, \Sigma, \delta, q_0, F, B, \Gamma)$ 7 tuple

$Q \rightarrow$ set of states

$\Sigma \rightarrow$ set of i/p symbols

$\Gamma \rightarrow$ set of tape symbols.

$\delta \rightarrow$ transition function $Q \times \Gamma \rightarrow Q \times \Gamma \times (L, R)$

$q_0 \rightarrow$ initial state

$B \rightarrow$ blank symbol.

F - set of final states.

Instantaneous Description

An ID of a TM ~~of~~ is a string in $\alpha q \beta$, q - current state. $\alpha\beta$ is a string made from tape symbols denoted by γ. ~~Initi~~

Initial ID is $q\alpha\beta$ q - start state

R/w head points to 1st symbol of α from left. Final ID is $\alpha\beta q B$ where $q \in F$.

R/w head points to B.

Transition function

$$\delta(q_1, a) = (p, b, R)$$

Movement

$$a_1 a_2 a_3 \ldots a_{k-1} q a_k \ldots a_n \vdash a_1 a_2 \ldots a_{k-1} b p a_{k+1} \ldots a_n$$

If the transition function is

$$\delta(q, a) = (p, b, L)$$

Movement

$$a_1 a_2 a_3 \ldots a_{k-1} q a_k a_{k+1} \ldots a_n \vdash a_1 a_2 \ldots a_{k-2} p a_{k-1} b a_{k+1} \ldots a_n$$

Acceptance by TM

$L(M) = \{ w \mid q_0 w \vdash^* \alpha_1 p \alpha_2$ where $w \in \Sigma^*$

$p \in F$ & $\alpha_1, \alpha_2 \in \gamma^*$. $q_0 w$ initial ID $\alpha_1 p \alpha_2$ final ID.

Language accepted by TM is recursively enumerable.

Q.1] Design a TM to accept $L = 0^n 1^n \mid n \geq 1$

$\cancel{\delta(q_0, 0)}$ $\cancel{X}\ \cancel{X}\ \cancel{X}\ Y.$

$\cancel{0}$	$\cancel{0}$	$\cancel{1}$	$\cancel{1}$	B

$\delta(q_0, 0) = (q_1, X, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, 1) = (q_2, Y, L)$

$\delta(q_2, 0) = (q_2, 0, L)$

$\delta(q_2, X) = (q_0, X, R)$

$\cancel{\delta(q_1, X) =}$

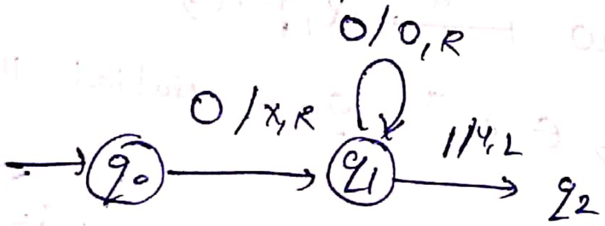
$\delta(q_1, Y) = (q_1, Y, R)$

$\delta(q_2, Y) = (q_2, Y, L)$

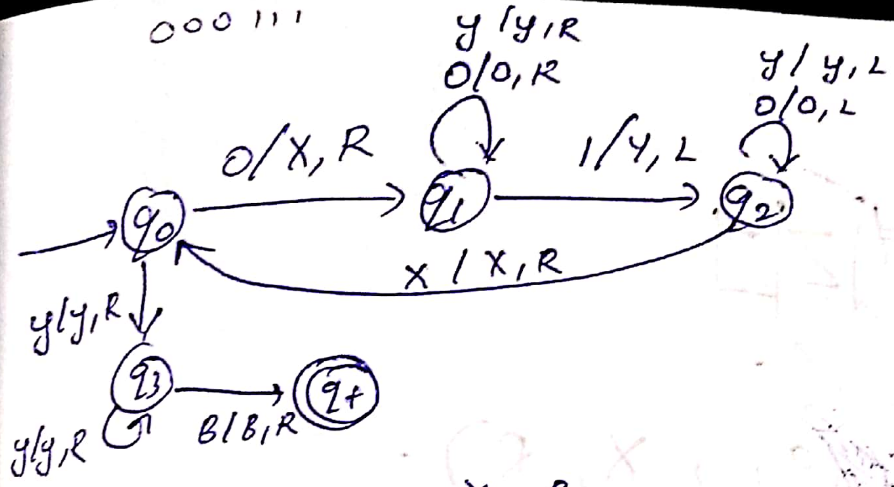
$\underline{\delta(q_0, Y) = (q_3, Y, R)}$ $\quad 0^n 1^n = XY$

$\delta(q_3, Y) = (q_3, Y, R)$

$\delta(q_3, B) = (q_f, B, R)$



000111



$\delta(q_0, 0) = \cancel{q_1, X, R}$

$q_1,$

Move

$q_0\ 0011B \vdash Xq_1011B \vdash X0q_111B \vdash$

$Xq_20Y1B \vdash q_2X0Y1B \vdash Xq_00Y1B \vdash$

$XXq_1Y1B \vdash XXYq_11B \vdash XXq_2YYB \vdash$

$Xq_2XYYB \vdash XXq_0YYB \vdash XXYq_3YB$

$\vdash XXYYq_3B \vdash XXYYBq_f$

Q.2] $a^n b^n c^n$

~~a~~	~~a~~	Y	Y	Z	Z

$\delta(q_0, a) = (q_1, X, R)$

$\delta(q_1, a) = (q_1, a, R)$

$\delta(q_1, b) = (q_2, Y, R)$

$\delta(q_2, b) = (q_2, b, R)$

$\delta(q_2, c) = (q_3, Z, L)$

$\delta(q_3, b) = (q_3, b, L)$

$\delta(q_3, Y) = (q_3, Y, L)$

$\delta(q_3, a) = (q_3, a, L)$

$\delta(q_3, X) = (q_0, X, R)$

$\delta(q_1, Y) = (q_1, Y, R)$

$\delta(q_2, Z) = (q_2, Z, R)$

$\cancel{\delta(q_3, Z) = (q_3, Z, L)} \Rightarrow \delta(q_3, Z) = (q_3, Z, L)$

5 $\cancel{\delta(q_3, Y) = (q_3, Y, L)}$

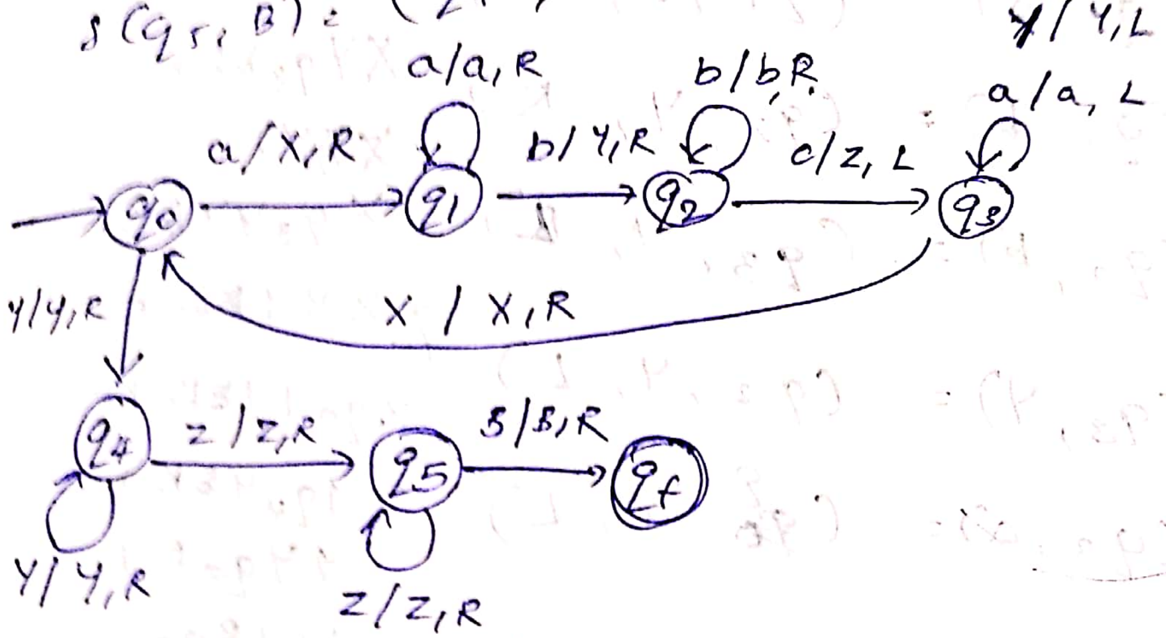
$\delta(q_0, Y) = (q_4, Y, R)$ $\quad a^n b^n c^n = XYZ$

$\delta(q_4, Y) = (q_4, Y, R)$

$\delta(q_4, Z) = (q_5, Z, R)$

$\delta(q_5, Z) = (q_5, Z, R)$

$\delta(q_5, B) = (q_f, B, R)$



~~q_0 aabbccB ⊢~~

$q_0 abcB \vdash X q_1 bcB \vdash XY q_2 cB \vdash X q_3 YZB$

$\vdash q_3 XYZB \vdash X q_0 YZB \vdash XY q_4 ZB \vdash XYZ q_5 B$

$\vdash XYZB q_f$

4

$a^n b^{2n}$:

| X a | X a | Y b | Y b | Y b | Y b | B |

$\delta(q_0, a) = (q_1, X, R)$

$\delta(q_1, a) = (q_1, a, R)$

$\delta(q_1, b) = (q_2, Y, R)$

$\delta(q_2, b) = (q_3, Y, L)$

$\delta(q_3, Y) = (q_2, Y, L)$

$\delta(q_3, a) = (q_3, a, L)$

$\delta(q_3, X) = (q_0, X, R)$

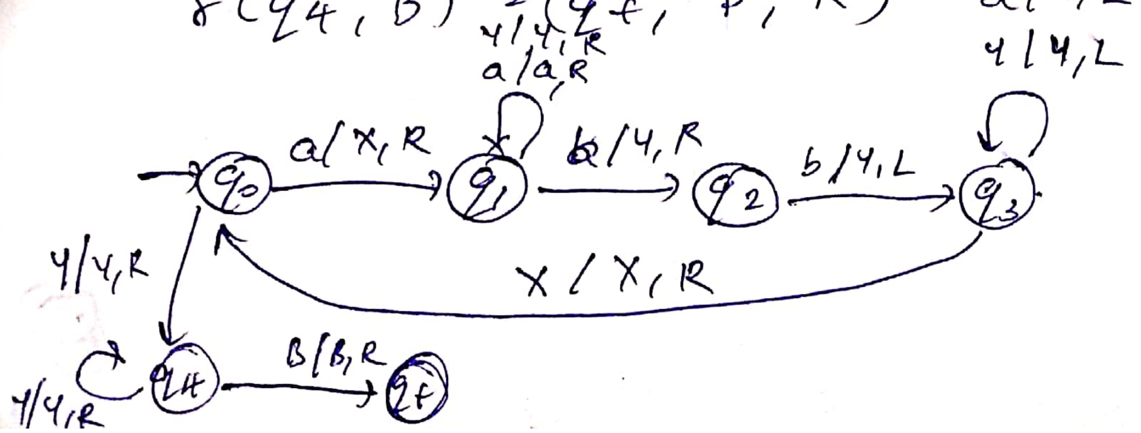
$\delta(q_1, Y) = (q_1, Y, R)$

$\delta(q_0, Y) = (q_4, Y, R)$

$\delta(q_4, Y) = (q_4, Y, R)$

$\delta(q_4, B) = (q_f, B, R)$

$q_0 abbB \vdash$
$X q_1 bbB \vdash$
$XY q_2 bB \vdash$
~~XYq3~~
$X q_3 YYB \vdash$
$q_3 XYYB \vdash$
$X q_0 YYB \vdash$
$XY q_4 YB \vdash$
$XYY q_4 B \vdash$
$XYYB q_f$



~~$a^n b^m \quad n>0 \quad m>0$.~~

1's complement

1 0 1 1 1 0 1 0 0 B

0 1 0 0 0 1 0 1 1

$q_0 101B \vdash 0q_0 01B$

$\vdash 01q_0 1B \vdash$

$010q_0 B \vdash$

$010Bq_f$.

$\delta(q_0, 1) = (q_0, 0, R)$

$\delta(q_0, 0) = (q_0, 1, R)$

$\delta(q_0, B) = (q_f, B, R)$

Addition of 2 numbers

$x + y = 2 + 3 = 5$

$110111B = 11111 0$

Read 1 Read 0

$\delta(q_0, 1) = (q_0, 1, R)$

$\delta(q_0, 0) = (q_1, 1, R)$

$\delta(q_1, 1) = (q_1, 1, R)$

$\delta(q_1, B) = (q_2, B, L)$

$\delta(q_2, 1) = (q_3, 0, R)$

$\delta(q_3, B) = (q_f, B, R)$

$q_0 1011B \vdash 1q_0 011B$

$\vdash$ ~~$1q_1 011B \vdash$~~

$\vdash 11q_1 11B \vdash 111q_1 1B$

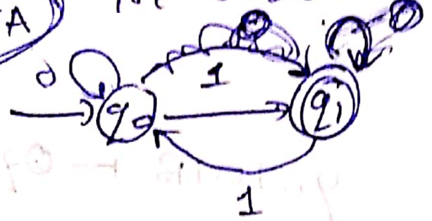
$\vdash 1111q_1 B \vdash 111q_2 1B$

$\vdash 1110q_3 B \vdash 1110Bq_f$

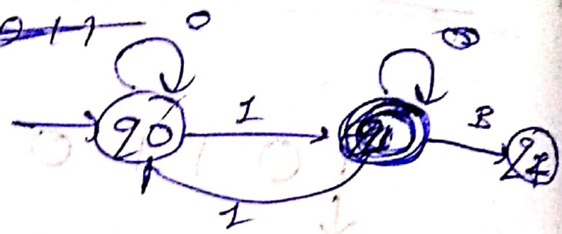
Binary input string of ODD PARITY.
odd no. of 1s.

~~odd no. of 1s~~
1st draw DFA

DFA



~~1011~~



$\delta(q_0, 1) = (q_1, 1, R)$

$\delta(q_1, 1) = (q_0, 1, R)$

$\delta(q_0, 0) = (q_0, 0, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, B) = (q_f, B, R)$

$q_0 10111B \vdash 1 q_1 0111B \vdash 10 q_1 111B$

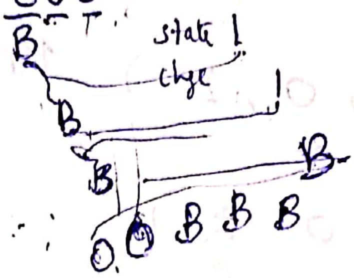
$\vdash 101 q_0 \cancel{1} 1B \vdash 1011 q_1 1B \vdash$

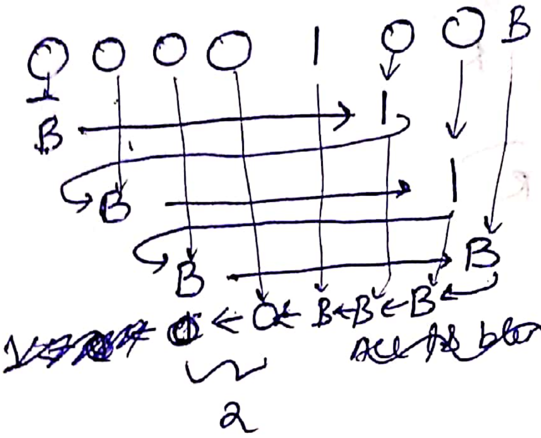
$10111 q_0 B \vdash$

Subtraction of 2 nos.

$$X - Y = 4 - 2 = 2$$

0000 1 00 = 00





~~Read 0 & 1 blank too.~~

~~All 1s blank & all blanks 0~~

Make all 1s B(lank)

Read 0 & make 1st B(lank) to 0.

$\delta(q_0, 0) = (q_1, B, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, 1) = (q_2, 1, R)$

$\delta(q_2, 0) = (q_3, 1, L)$

$\delta(q_3, 1) = (q_3, 1, L)$

~~$\delta(q_2, B) = (q_2,$~~

~~$\delta(q_2, B) = (q$~~

$\delta(q_3, 0) = (q_3, 0, L)$

$\delta(q_3, B) = (q_0, B, R)$

$\delta(q_2, 1) = (q_2, 1, R)$

$\delta(q_4, B) = (q_4, B, L)$

$\delta(q_4, 1) = (q_4, B, L)$

$\delta(q_4, 0) = (q_4, 0, L)$ $m>n$

$\delta(q_4, B) = (q_5, 0, R)$

$\delta(q_5, 0) = (q_5, 0, R)$

$\delta(q_5, B) = (q_f, B, R)$

$m<n$

$m=n$

$4-4=0$ Pull blank

$2-4=-$

001000

BB 1 1

$\delta(q_0, 1) = \underline{q_6}, B, R$

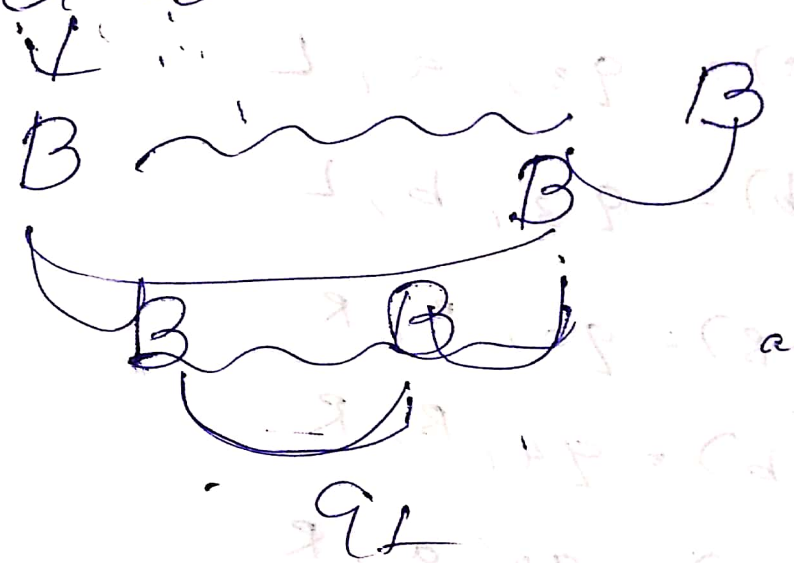
$\delta(q_6, 1) = q_6, B, R$

$\delta(q_6, 0) = q_6, B, R$

$\delta(q_6, B) = q_f, B, R$

$\omega\,\omega^R$

a b b a



abB/Bba

a aba abaa

B ——→ ~~B~~ B

~~B~~ B

WW^R

$\delta(q_0, a) = q_1, B, R$

$\delta(q_1, a) = q_1, a, R$

$\delta(q_1, b) = q_1, b, R$

$\delta(q_1, B) = q_2, B, L$

$\delta(q_2, a) = q_3, B, L$

$\delta(q_3, a) = q_3, a, L$

$\delta(q_3, b) = q_3, b, L$

$\delta(q_3, B) = q_0, B, R$

$\delta(q_0, b) = q_4, B, R$

$\delta(q_4, a) = q_4, a, R$

$\delta(q_4, b) = q_4, b, R$

$\delta(q_4, B) = q_5, B, L$

$\delta(q_5, b) = \underline{q_3}, B, L$

$\delta(q_3, B) = q_f, B, R$

Pumping Lemma for CFL

$L = \{a^p \mid p$ is prime$\}$ not CF?

$w = a^5$

$$\begin{array}{c|c|c|c|c} a & a & a & a & a \\ u & v & w & x & y \end{array}$$

$$\boxed{u v^i w x^i y}$$

when $i = 1$ $\begin{array}{c|c|c|c|c} a & a & a & a & a \\ u & v & w & x & y \end{array}$ $a^5 \in L$

$i = 2$ $\begin{array}{c|c|c|c|c} a & aa & a & aa & a \\ u & v & w & x & y \end{array}$ $a^7 \in L$

$i = 3$ $\begin{array}{c|c|c|c|c} a & aaa & a & aaa & a \\ u & v & w & x & y \end{array}$ $a^9 \in L$

But 9 is not prime.

ww^R

1) $L = \omega\omega$

$L = a^n b^n c^n$

wxw

$L = a^n b^n c^n$

$n = 2$

a	a	b	b	cc
u	v	w	x	y

uv^iwx^iy

$i=1$

$i=2$

a	aa	b	bb	cc
v	v	w	x	y

$a^3b^3c^2 \neq a^nb^n$

$L = wxw$

$w = abc$

a	b	c a	b	c
u	v	w	x	y

abc

$i=2$

a	bb	ca	bb	c
u	v	w	x	y

$i=3$

a bbb ca bbbc

a	bc	a	b	c
u	v	w	x	y

i = 2 a bc bc a b b c

Pumping lemma for CFL

Let L be the CFL, then there is a constant n, which depends only upon L, such that there exist a string $w \in L$ and $|w| \geq n$ where $w = uvwxy$ such that $|vw| \geq 1$, $|vwx| \leq n$ for all $i \geq 0$ $uv^iwx^iy \in L$

Proof

Basic : If $i = 1$

Let G contains the rule $S \to a$ where length of the derived string is 1 i.e $i^0 = 1$ length of the derived string should be $\leq 2^{i-1}$ i.e $2^0 = 1$ Now word length ~~....~~ This language is CFL since $|w| = $ ~~....~~

Induction

Let w be a string which is derived by grammar G. Let k be a variable such that $n = 2k$ $|w| \geq n$. then $|w| > 2^{k-1}$ while deriving w string we may get nonterminals of CFG can be repeated for no of times to get w.

If we pump the sub string to w such that path length of this newly formed string w' (w + pumped string = w') is i and the word length of w' is $\leq 2^{i-1}$.

then grammar G deriving w' is called a CFG

Consider $G = \{(A,B,C)\ \{a\}, \{A \to BC, B \to BA,$
$C \to BA, A \to a, B \to b\}, A\}$

thus $A \stackrel{*}{\Rightarrow} bba = w$. i.e. path length $i = 3$

$|w| \leq 2^{i-1}$ i.e. $3 \leq 2^2$. If we pump a substring into 'w' which satisfies the condn as $p \leq |w| \leq 2^{i-1} \leq n$ the grammar producing w is a CF.

Application

to show that certain languages are not CFL.

1. Assume L is CFL
2. select a string say z & break it into substrings u, v, w, x, y such that $z = uvwxy$ where $|vwx| \leq n$ & $|vx| \geq 1$
3. find any i such that $uv^iwx^iy \notin L$

Scanned by CamScanner

Context Sensitive Grammar (type 1)

Productions are of the form

$\alpha \to \beta$

where $|\beta| \geq |\alpha|$ $\alpha \& \beta \in (V \cup T)^+$ $\alpha \neq \varepsilon$ $\beta \neq \varepsilon$

Language generated → context sensitive language.

Language recognizer → Linear bounded automata.

eg: $S \to aAb$

$aA \to bAA$

$bA \to aa$.

Linear Bounded Automata.

Here TM is bounded based on the length of the input string.

$M = (Q, \Sigma, \Gamma, \delta, q_0, ¢, \$, F)$

¢ - left end marked

\$ - right end marked

LBA has no moves left from ¢ or right from \$.

Language acceptance

$L(M) = \{ \omega \mid \omega$ is in $(\Sigma - (¢, \$))^*$ and $q_0 ¢ \omega \$ \vdash_M^* \alpha q \beta$ for some q in F.