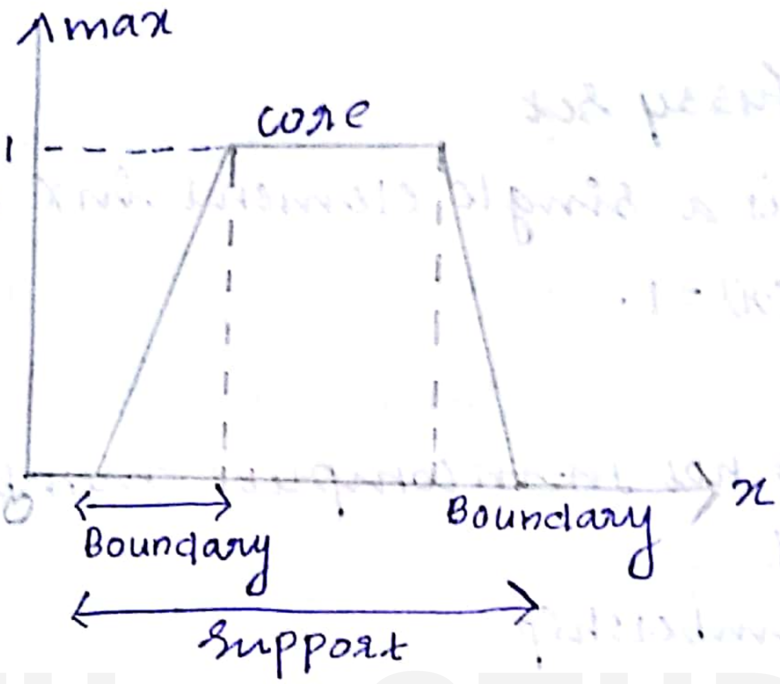
4. Membership Functions



Membership function.

→ Fuzziness in the fuzzy set irrespective of elements

→ Graphical form.

Features of membership function.

$\underset{\sim}{A} = \{(x, \mu A(x)) \mid x \in X\}$

$\mu A(\cdot)$ is the membership function.

$\mu A: X \to M, \mu A(x) \to [0,1]$

core:

- complete membership in the set A
- $\mu A(x) = 1$

- core of a fuzzy set may be empty set.

Support:

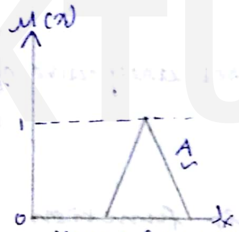
- Non zero membership
- $\mu_A(x) > 0$

Singleton fuzzy set

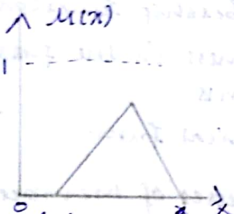
If suppose is a single element in X with $\mu_{\underset{\sim}{A}}(x) = 1$.

Boundary.

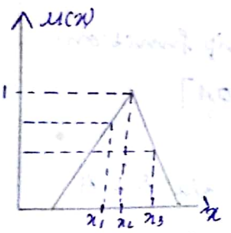
- Non zero but not ~~mem~~ compute membership
- $0 < \mu_A(x) < 1$
- Partial membership



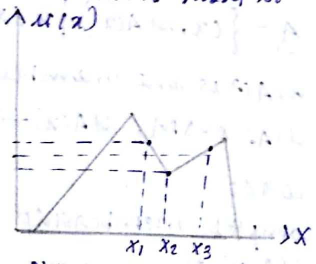
Normal fuzzy set



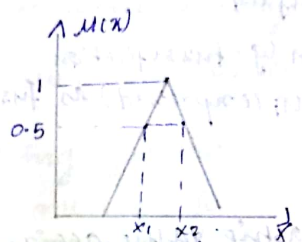
Subnormal fuzzy set



convex normal fuzzy set



Non convex normal fuzzy



crossover point of a fuzzy set

In the case of convex normal fuzzy set.

$\mu_{\underset{\sim}{A}}(x) \geq \min[\mu_{\underset{\sim}{A}}(x_1), \mu_A(x_3)]$

Height

Maximum of membership value (≤ 1)

Fuzzification

- process of transforming a crisp set of fuzzy set or fuzzier set
- Translate crisp input values into linguistic variables.
- $\underset{\sim}{A} = \{\mu_i / x_i \mid x_i \in X\}$ - common fuzzy algorithm

[μ_i constant and x_i transformed to fuzzy set $Q(x_i)$]

Support fuzzification (S-fuzzification)

$\underset{\sim}{A} = \mu_1 Q(x_1) + \mu_2 Q(x_2) + \dots \mu_n Q(x_n)$

$Q(x_i)$ is the kernel for fuzzification.

~ means fuzzified

Grade fuzzification (g-fuzzification.

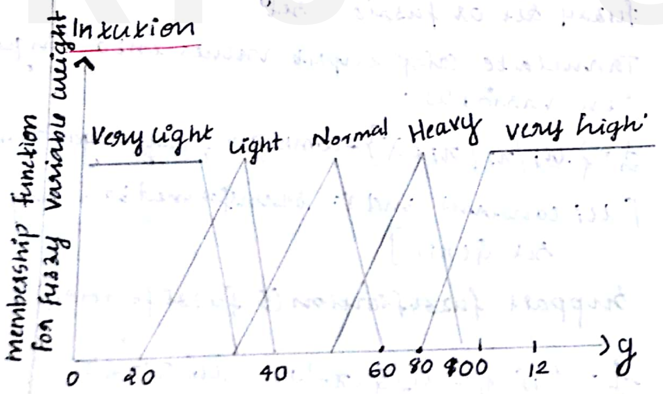
- x_i constant μ_i (expressed) as fuzzy set.

Fuzzification.

→ Methods of membership value assignment.

1. Intuition
2. Inference
3. Rank ordering
4. Angular fuzzy sets
5. Neural networks
6. Genetic algorithm.
7. Inductive reasoning

Intuition

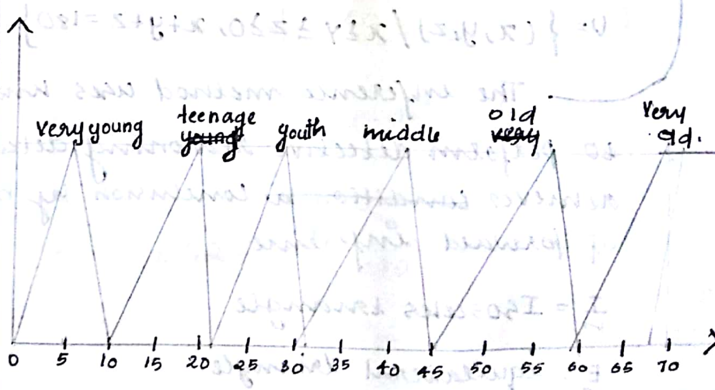


- Intelligent capacity
- $0 < w < 20$ - very light

Q) Using your own intution plot the fuzzy membership function for the age of people.

A) The universe of discourse is age of people. Let A denote age of people in year ~~the~~ The linguistic variable are defined as follow.

below 10 - very young
10 - 21 - young
22 - 32 - middle.
40 - 70 - old
above 70 - very old



VY → A < 12

Y → 10 ≤ A ≤ 22

M → 20 ≤ A ≤ 42

O → 40 ≤ A ≤ 72

VO → 70 < A

Q. using intuition assign the membership function for (1) popupdation of car
(2) library usage.

2. INFERENCE

Consider a triangle
x, y, z are the angles

$x \geq y \geq z \geq 0$

$U = \{(x, y, z) / x \geq y \geq z \geq 0, x+y+z = 180\}$

The inference method uses knowledge to perform detective reasoning detection achieves ~~condition to~~ conclusion by means of forward inference

$\underset{\sim}{I}$ = Isosceles triangle

$\underset{\sim}{E}$ = equilateral triangle.

$\underset{\sim}{R}$ - Right angle triangle

$\underset{\sim}{IR}$ = Isosceles and right angle.

$\underset{\sim}{T}$ = Other triangle

Isosceles Triangle: $\mu_{\underset{\sim}{I}}(x, y, z) = 1 - \frac{1}{60} \min(x-y, y-z)$

$x = y$ (2 sides equal)

$x - y = 0$

$\therefore \mu_{\underset{\sim}{I}}(x, y, z) = 1$, it is isosceles triangle.

eg: $x = 120$, $y = 60$, $z = 0$

$\mu_{\underset{\sim}{I}}(x, y, z) = 1 - \frac{1}{60} \min(120-60, 60-0)$

$= 1 - \frac{1}{60} \times \min(60, 60)$

$= 1 - \frac{1}{60} \times 60 = 0$

$\mu_{\underset{\sim}{R}}(x, y, z) = 1 - \frac{1}{90} |x - 90°|$ - Right angle triangle.

$\mu_{\underset{\sim}{E}}(x, y, z) = 1 - \frac{1}{180} |x - z|$ - Equilateral.

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10/8/2017 Isosceles and right angle triangle.

$\underset{\sim}{I}\underset{\sim}{R} = \underset{\sim}{I} \cap \underset{\sim}{R}$

$\mu_{\underset{\sim}{IR}}(x, y, z) = \min[\mu_{\underset{\sim}{I}}(x, y, z), \mu_{\underset{\sim}{R}}(x, y, z)]$

Equilateral triangle

$\mu_{\underset{\sim}{E}}(x, y, z) = 1 - \frac{1}{180}|x - z|$

other triangles

$\underset{\sim}{T} = \overline{\underset{\sim}{I} \cup \underset{\sim}{R} \cup \underset{\sim}{E}}$

$\underset{\sim}{T} = \overline{\underset{\sim}{I}} \cap \overline{\underset{\sim}{R}} \cap \overline{\underset{\sim}{E}}$

$= 1-\underset{\sim}{I} \cap 1-\underset{\sim}{R} \cap 1-\underset{\sim}{E}$

$= \min(1-\underset{\sim}{I}, 1-\underset{\sim}{R}, 1-\underset{\sim}{E})$

Q Using the inference approach find the membership values for E, I, R, IR and T. For the triangle with angle 45°, 55°, 80°

A Let $U = \{(x, y, z) \mid x = 80° \geq y = 55° \geq z = 45°\}$

$\mu_{\underset{\sim}{I}}(\overset{80}{x}, \overset{55}{y}, \overset{45}{z}) = 1 - \frac{1}{60}\min(x-y, y-z)$

$= 1 - \frac{1}{60}\min(80-55, 55-45)$

$= 1 - \frac{1}{60}\min(25°, 10) =$

$= 1 - \frac{1}{60} \cancel{\min} \times 10$

$= 1 - 0.1667$

$= 0.8333$

Equilateral

$\mu_{\underset{\sim}{E}}(x, y, z) = 1 - \frac{1}{180}|x - z|$

$= 1 - \frac{1}{180}|80 - 45|$

$= 1 - \frac{1}{180}|35|$

$= \cancel{34.80}$ 0.8055

Right angle

$\mu_{\underset{\sim}{R}}(x, y, z) = 1 - \frac{1}{90}|x - 90°|$

$= 1 - \frac{1}{90}|80 - 90|$

$= 1 - \frac{1}{90}|10|$

$= 0.888$

Isosceles & Right angle triangle.

$\underset{\sim}{I}\underset{\sim}{R} = \underset{\sim}{I} \cap \underset{\sim}{R}$

$\mu_{\underset{\sim}{IR}}(x, y, z) = \min[\mu_{\underset{\sim}{I}}(x, y, z), \mu_{\underset{\sim}{R}}(x, y, z)]$

$= \min(0.833, 0.88)$

$= 0.833$

Other triangle.

$\underset{\sim}{I} = \underset{\sim}{I} \cup \underset{\sim}{R} \cup \underset{\sim}{E}$

$= \min(1-\underset{\sim}{I}, 1-\underset{\sim}{R}, 1-\underset{\sim}{E})$

$= \min(0.167, 0.111, 0.1945)$

$= 0.111$

RANK ORDERING:

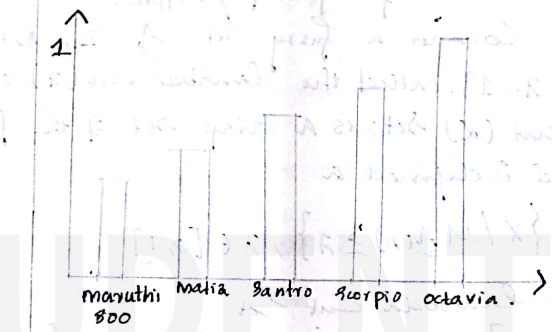
Suppose 1000 people respond to questions about their pairwise preferences among 5 car

$X = \{$maruthi 800 Scorpio, matiz, Santro octavia$\}$.

Number who prefered.

	Maruthi800	Scorpio	maltiz	Santra	octavia
maruthi 800	—	192	246	592	621
Scorpio	408	—	621	540	391
Martiz	235	336	—	797	492
Santro	523	364	417	—	608
octavia	616	584	746	726	—

Total	Percentage	Rank
1651	16.5	5
1955	19.6	2
1860	18.6	4
1912	19.1	3
2622	26.1	1



13/09/2017

Defuzzification:- is a mapping process from a space of fuzzy control actions defined over an o/p universe of discourse into a space of crisp (non-fuzzy) control actions.

→ **Lambda for fuzzy sets (Alpha cut)**

Fuzzy set $\underset{\sim}{A}$:

Set A_λ $(0 < \lambda < 1)$ – lambda cut set (X) or

Alpha cut set (α)

$A_\lambda = \{x / \mu_{\underset{\sim}{A}}(x) \geq \lambda\}, \lambda \in [0, 1]$

The defuzzification process has the capability to reduce a fuzzy set into a crisp single valued quantity or into crisp set; to convert a fuzzy set matrix into a crisp set or a to convert a fuzzy number to crisp number.

Lambda cuts for fuzzy / Alpha

Consider a fuzzy set $\underset{\sim}{A}$. The set $A_\lambda [0 < \lambda < 1]$, called the lambda cut (λ) or alpha cut (α) set; is a crisp set of the fuzzy set and is defined a

$$A_\lambda = \{ x / \mu_{\underset{\sim}{A}}(x) \geq \lambda \}, \lambda \in [0,1]$$

Weak lambda cut set.

The set A_λ is called weak lambda cut set if it consists of all the elements of a fuzzy set whose membership functn have value greater than or equal to specific value.

$$A_\lambda = \{ x / \mu_A(x) \geq \lambda \}; \lambda \in [0,1]$$

Strong Lambda Cut Set.

The set A_λ the strong lamber cut set if it consist of all the element of a fuzzy set whose membership function have values of strictly greater than a specific value.

$$A_\lambda = \{ x / \mu_A(x) > \lambda \} \quad \lambda \in [0,1]$$

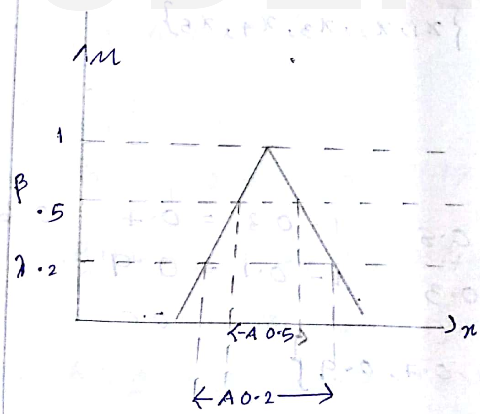
Properties of λ cut set.

1) $(\underset{\sim}{A} \cup \underset{\sim}{B})_\lambda = A_\lambda \cup B_\lambda$

2) $(\underset{\sim}{A} \cap \underset{\sim}{B})_\lambda = A_\lambda \cap B_\lambda$

3) $(\bar{\underset{\sim}{A}})_\lambda \neq \bar{A}_\lambda$ except when $\lambda = 0.5$.

4) For any $\lambda \leq \beta$ where $0 \leq \beta \leq 1$, it is true that $A_\beta \subseteq A_\lambda$ when $A_0 = X$.



Consider two fuzzy set $\underset{\sim}{A}$ and $\underset{\sim}{B}$ both defined on X, given as follows.

$\mu(x_i X)$	x_1	x_2	x_4	x_5	x_3
$\underset{\sim}{A}$	0.2	0.3	0.7	0.1	0.4
$\underset{\sim}{B}$	0.4	0.5	0.8	0.9	0.6

Express the following λ cut set using Zadeh's notation

a) $\bar{\underset{\sim}{A}}_{0.7}$ (b) $(\underset{\sim}{B})_{0.2}$ (c) $(\underset{\sim}{A} \cup \underset{\sim}{B})_{0.6}$ d) $(\underset{\sim}{A} \cap \underset{\sim}{B})_{0.5}$.

e) $(\underset{\sim}{A} \cup \bar{\underset{\sim}{A}})_{0.7}$ (f) $(\underset{\sim}{B} \cap \bar{\underset{\sim}{B}})_{0.3}$ (g) $(\underset{\sim}{A} \cap \bar{\underset{\sim}{B}})_{0.6}$

h) $(\bar{\underset{\sim}{A}} \cup \bar{\underset{\sim}{B}})_{0.8}$

A (b) $(\underset{\sim}{B})_{0.2} = \{x_1, x_2, x_3, x_4, x_5\}$

(a) $\bar{\underset{\sim}{A}}_{0.7}$

$1 - A = \bar{A}$

$1 - 0.2 = 0.8$ $\quad 1 - 0.3 = 0.7$ $\quad 0.6$

$1 - 0.7 = 0.3$ $\quad 1 - 0.1 = 0.9$

0.6

$\bar{\underset{\sim}{A}}_{0.7} = \{0.8, 0.7, 0.9\}$

$\bar{\underset{\sim}{A}}_{0.7} = \{x_1, x_2, x_5\}$.

(c) $(\underset{\sim}{A} \cup \underset{\sim}{B})_{0.6}$

$\underset{\sim}{A} \cup \underset{\sim}{B} = \{\overset{x_1}{0.4}, \overset{x_2}{0.5}, \overset{x_4}{0.8}, \overset{x_5}{0.9}, \overset{x_3}{0.6}\}$

$(\underset{\sim}{A} \cup \underset{\sim}{B})_{0.6} = \{x_4, x_5, x_3\}$

d) $(\underset{\sim}{A} \cap \underset{\sim}{B})_{0.5}$

$\underset{\sim}{A} \cap \underset{\sim}{B} = \{0.2, 0.3, 0.4, 0.7, 0.1\}$

$(\underset{\sim}{A} \cap \underset{\sim}{B})_{0.5} = \{0.7\} = \{x_4\}$.

e) $(\underset{\sim}{A} \cup \bar{\underset{\sim}{A}}) = \{0.8, 0.7, 0.7, 0.3, 0.9\}$

$(\underset{\sim}{A} \cup \bar{\underset{\sim}{A}})_{0.7} = \{0.8, 0.7, 0.7, 0.9\}$

f) $(\underset{\sim}{B} \cap \bar{\underset{\sim}{B}})_{0.3}$

$\bar{\underset{\sim}{B}} = \{0.6, 0.5, 0.2, 0.1, 0.4\}$

$(\underset{\sim}{B} \cap \bar{\underset{\sim}{B}})_{0.3} = \{0.4, 0.5, 0.4, 0.2,$

$= \{x_1, x_2, x_3\}$

g) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = \{0.8, 0.7, 0.6, 0.3, 0.9\}$

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$(\overline{\underset{\sim}{A} \cap \underset{\sim}{B}})_{0.6} = \{x_1, x_2, x_3, x_5\}$

(h) $(\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}})_{0.6} = \{x_1, x_5\}$

using Zadeh's notation determine the lambda cut set for the given fuzzy set.

$\underset{\sim}{S_1} = \left\{ \frac{0}{0} + \frac{0.5}{20} + \frac{0.65}{40} + \frac{0.85}{60} + \frac{1.0}{80} + \frac{1.0}{100} \right\}$

$\underset{\sim}{S_2} = \left\{ \frac{0}{0} + \frac{0.45}{20} + \frac{0.6}{40} + \frac{0.8}{60} + \frac{0.95}{80} + \frac{1.0}{100} \right\}$

Express the following for $\lambda = 0.5$.

(a) $\underset{\sim}{S_1} \cup \underset{\sim}{S_2}$

A) $\left\{ \frac{0.5}{20} + \frac{0.65}{40} + \frac{0.85}{60} + \frac{1.0}{80} + \frac{1.0}{100} \right\}$

b) $\underset{\sim}{S_1} \wedge \underset{\sim}{S_2}$

A) $\left\{ \frac{0.6}{40} + \frac{0.8}{60} + \frac{0.95}{80} + \frac{1.0}{100} \right\}$

(c) $\overline{\underset{\sim}{S_1}}$

A) $1 - \underset{\sim}{S_1} = \left\{ \frac{1}{0} + \frac{0.5}{20} + \frac{0.35}{40} + \frac{0.15}{60} + \frac{0}{100} + \frac{0}{100} \right\}$

$\overline{\underset{\sim}{S_1}} = \left\{ \frac{1}{0} + \frac{0.5}{20} \right\}$

d) $(\overline{\underset{\sim}{S_2}})_{0.5} = \left\{ \frac{0.4}{40} + \frac{0.2}{60} + \frac{0.05}{80} + \frac{0}{100} \right\}$

e) $(\overline{S_1 \cup S_2})_{0.5} = \left\{ \frac{0.5}{20} + \frac{0.35}{40} + \frac{0.15}{60} + \frac{0}{80} + \frac{0}{100} \right\}$

Q Consider the discrete fuzzy set defined on the universe.

$X = \{a, b, c, d, e\}$

$\underset{\sim}{A} = \left\{ \frac{1}{a} + \frac{0.9}{b} + \frac{0.6}{c} + \frac{0.3}{d} + \frac{0}{e} \right\}$

Using Zadeh's notation find the lambda cut set for $\lambda = 1, 0.9, 0.6, 0.3, 0^+$ and 0.

A) $A\lambda = \{x / \mu(x) \geq \lambda\}$

a) $\lambda = 1$, $A_1 = \left\{ \frac{1}{a} + \frac{0}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e} \right\}$

b) $\lambda = .9$, $A_{.9} = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e} \right\}$

c) $\lambda = 0.6$ $A_{.6} = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{0}{d} + \frac{0}{e} \right\}$

d) $\lambda = 0.3$ $A_{.3} = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{0}{e} \right\}$

e) $\lambda = 0^+ \rightarrow \left\{ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{0}{e} \right\}$

f $\lambda = 0$ $A_0 = \left\{ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} \right\}$

Lambda cuts

Let $\underset{\sim}{R}$ be a fuzzy relation where each row of the relational matrix is considered a fuzzy set. The j^{th} row in a fuzzy relation matrix R denotes a discrete membership functn for a fuzzy set R. A fuzzy relation can be converted to crisp relation in the following manner.

$$R_\lambda = \{ (x,y) / \mu_{\underset{\sim}{R}}(x,y) \geq \lambda \}$$

properties of lambda cut fuzzy relation

Similar to the Properties of fuzzy cut set.

Q Determine the crisp lambda cut relation when $\lambda = .1, 0^+, .3, .9$ for the following relation R

$$\underset{\sim}{R} \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.3 & 0.7 & 1.0 \\ 0.8 & 0.9 & 1.0 \end{bmatrix}$$

$$R_\lambda = \{(x,y) / \mu_{\underset{\sim}{R}}(x,y) \geq \lambda\}$$

$$= \begin{cases} 1/(x,y) / \mu_{\underset{\sim}{R}}(x,y) \geq \lambda \\ 0/(x,y) / \mu_{\underset{\sim}{R}}(x,y) < \lambda \end{cases}$$

$$R_{0.1} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad R_{0.3} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R_{0^+} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad R_{0.9} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

15/9/17

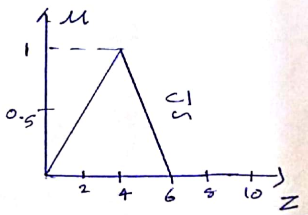
Defuzzification of Scalars.

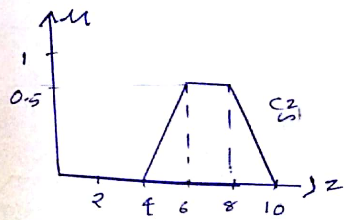
Defuzzification is the process of conversion of a fuzzy quantity into a precise quantity. The output of a fuzzy process may be union of 2/ more fuzzy membership function defined on the universe of discourse of the output variable.

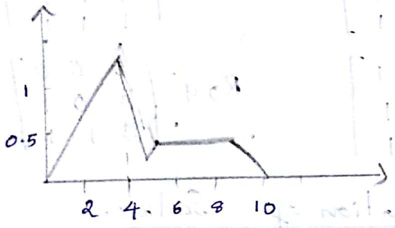
Consider a fuzzy output

$$\underset{\sim}{C} = \underset{\sim}{C_1} \cup \underset{\sim}{C_2}$$

Here $\underset{\sim}{C_1}$ is a triangular membership shape. $\underset{\sim}{C_2}$ is a trapezoidal shape.



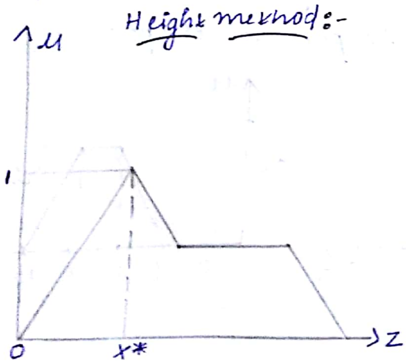




$$C_n = \bigcup_{i=1}^{n} \underset{\sim}{C}_i = \underset{\sim}{C}$$

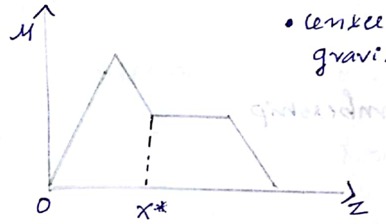
1. max-membership principle.
2. centroid method
3. Weighted average method.
4. Min-max membership
5. center of sums
6. center of largest area.
7. First of maxima, last of maxima.

Height method:-



$$\mu_{\underset{\sim}{C}}(x^*) \geq \mu_{\underset{\sim}{C}}(x) \;\forall x \in X$$

Centroid method

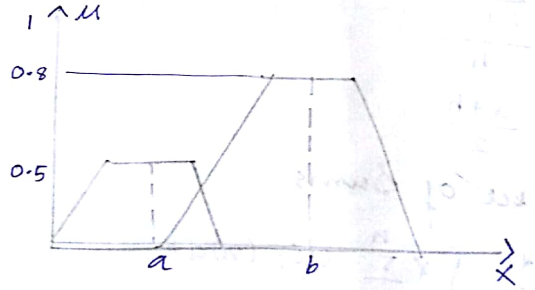


- centre of mass/centre of gravity/centre of area.

$$x^* = \frac{\int \mu_{\underset{\sim}{C}}(x) \cdot x\,dx}{\int \mu_{\underset{\sim}{C}}(x)\,dx}$$

$\int$ -> Algebraic integration

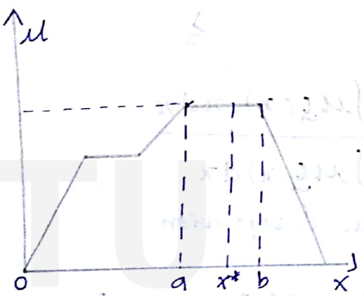
-> Weighted average method:-



$$X^* = \frac{\sum \mu_{\underset{\sim}{C}}(\bar{x}_i) \cdot \bar{x}_i}{\sum \mu_{\underset{\sim}{C}}(\bar{x}_i)}$$

$$x^* = \frac{.8b + .5a}{0.8 + 0.5}$$

Max-Min membership.

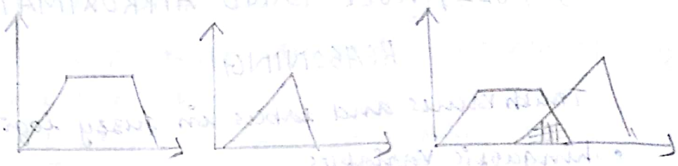


$$x^* = \frac{\sum_{i=1}^{n} \bar{x}_i}{n}$$

$$x^* = \frac{a+b}{2}$$

Centre of Sums

$$x^* = \frac{\int_x x \sum_{i=1}^{n} \mu_{\underset{\sim}{C}_i}(x)\,dx}{\sum_{i=1}^{n} \mu_{\underset{\sim}{C}_i}(x)\,dx}$$



Centre of largest area.

$$x^* = \frac{\int \mu_{\underset{\sim}{C}_i}(x) \cdot x\,dx}{\int \mu_{\underset{\sim}{C}_i}(x)\,dx}$$