01 - 11 - 2019

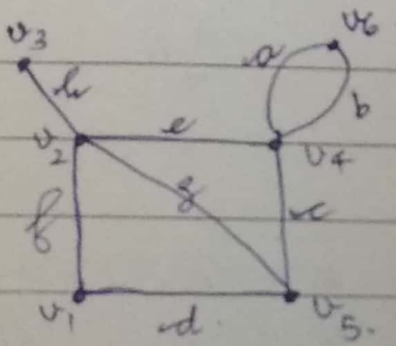
MODULE : V

Matrix representation of graphs.

* Incidence matrix : (A) $n \times e$ matrix. where n is the no. of vertices & e is the no. of edges.

$A(G)$ is the incidence matrix of graph G.

$A(G) = [a_{ij}]$; $a_{ij} = \begin{cases} 1, & \text{if } j^{th} \text{ edge is incident on } i^{th} \text{ vertex.} \\ 0, & \text{otherwise.} \end{cases}$



$A \Rightarrow n \times e$ matrix.

$A(G) \Longrightarrow 6 \times 8$ matrix.

$n = 6$, $e = 8$.

	a	b	c	d	e	f	g	h
v_1	0	0	0	1	0	1	0	0
v_2	0	0	0	0	1	1	1	1
v_3	0	0	0	0	0	0	0	1
v_4	1	1	1	0	1	0	0	0
v_5	0	0	1	1	0	0	1	0
v_6	1	1	0	0	0	0	0	0

* Properties of incidence matrix : (for a connected graph).

(i) Every column of incidence matrix has exactly 2 1's.

(ii) Every row " " " contains as many 1's as the degree of that vertex.

(iii) ~~No. of~~ + If a row is completely 0 then ~~that~~ the vertex corresponding to that row ~~is~~ is an isolated vertex.

*(iv) If the graph contains self loop, it can't be represented in the incidence matrix.

(v) If the graph contains || edges, columns will be identical

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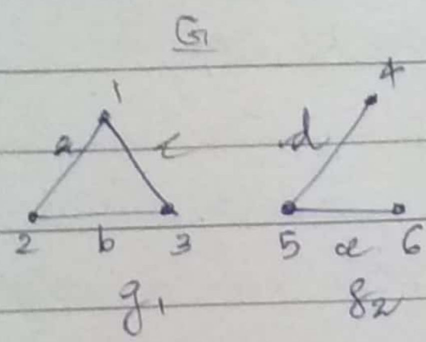
Consider a disconnected graph G with compts. g_1 & g_2



$$A(g_1) = \begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$A(g_2) = \begin{array}{c c} & \begin{array}{cc} d & e \end{array} \\ \begin{array}{c} 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$

$$A(G) = \begin{array}{c c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

A disconnected graph can be represented ~~as~~ in block diagonal form where the diagonal elts. are the incidence ~~graphs~~ matrix of its compts.

$$A(G) = \begin{bmatrix} A(g_1) & 0 \\ 0 & A(g_2) \end{bmatrix}$$

Q. Draw the ~~incidence matrix for~~ the corresponding graph for the incidence matrix.

$$A(G_1) = \begin{array}{c c} & \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$A(G_2) = \begin{array}{c c} & \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

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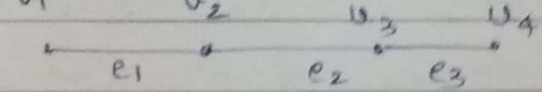
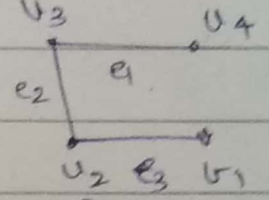
$G_1 =$



The 2 graphs are isomorphic.

$G_2 =$



Theorem: ~~If~~ 2 graphs G_1 & G_2 are isomorphic iff their incidence matrices differ only by permutation of rows & columns
⇓
(exchanging / interchanging)

- Rank of an incidence matrix = Rank of G ~~= n-1~~ for a graph G.

$$r = \begin{cases} n-1, & \text{if } G \text{ is connected} \\ n-k, & \text{if } G \text{ is disconnected.} \end{cases}$$

- If we remove any 1 row of the incidence matrix, $A(G)$ becomes a reduced incidence matrix, A_f, where $A_f \Rightarrow (n-1) \times e$.

The ~~row~~ vertex corresponding to that deleted row is called reference vertex.

A_f is always a square matrix of order $(n-1) \times e$ – will be a tree
$= (n-1) \times (n-1)$

- The values ~~of~~ of A(G) are ~~only~~ either 0 or 1 ~~of A(G)~~ & hence is known as (0–1) matrix / binary matrix.
- 1 in A(G) indicates an edge j incident on vertex i & hence is also known as vertex-edge incident matrix.