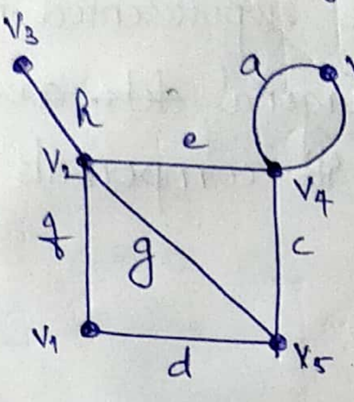
1/11/19

MODULE-5

Matrix Representat^n^ of Graphs

(i) Incidence Matrix (A)

- $n \times e$ matrix where $n \rightarrow$ No. of vertices & $e \rightarrow$ No. of edges
- $A(G) \rightarrow$ incidence matrix of graph G.
- $A(G) = [a_{ij}]$; $a_{ij} = \begin{cases} 1, & \text{if } j\text{th edge is incident on } i\text{th vertex} \\ 0, & \text{otherwise} \end{cases}$



~~A∈~~ $A \rightarrow n \times e$ matrix

$n = 6, e = 8$

$A(G) \rightarrow 6 \times 8$ matrix

$$\begin{array}{c|cccccccc} & a & b & c & d & e & f & g & R \\ \hline V_1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ V_2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ V_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ V_4 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ V_5 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ V_6 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

- Properties

(i) Every column of incidence matrix has exactly 2 1's.

(ii) " row " " " contains as many 1's as the degree of that vertex

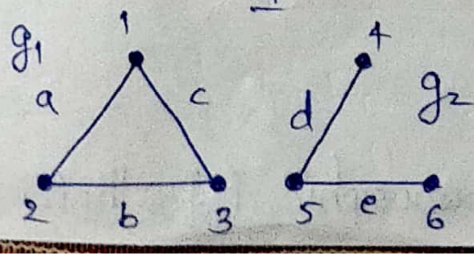
(iii) If a row is completely 0 then the vertex corresponding to that row is an isolated vertex.

(iv) If the graph contains self loop, it can't be represented in the incidence matrix.

(v) If the graph contains parallel edges, colums will be identical

Consider a disconnected graph G with components g_1 & g_2

G



$$A(g_1) = \begin{array}{c|ccc} & a & b & c \\ \hline 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 3 & 0 & 1 & 1 \end{array}$$

$$A(g_2) = \begin{array}{c} \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cc} d & e \end{array} \\ \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$

$$A(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

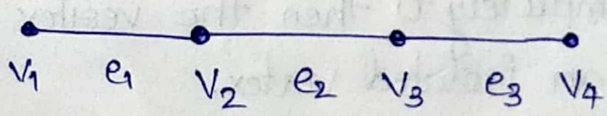
A disconnected graph can be represented in block diagonal form where the diagonal ~~elts are~~ the elements are the incidence matrix of its components.

$$A(G) = \begin{bmatrix} A(g_1) & 0 \\ 0 & A(g_2) \end{bmatrix}$$

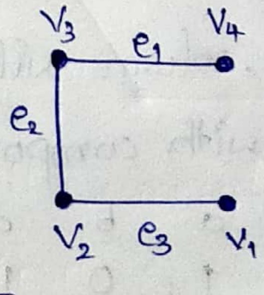
① Draw the corresponding graphs for the incidence matrix

$$A(G_1) = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad A(G_2) = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

ans G_1



G_2



The 2 graphs are isomorphic

Theorem

2 graphs G_1 & G_2 are isomorphic iff their incidence

matrices differ only by permutat's of rows & columns
(exchanging / interchanging)

- Rank of an incidence matrix for a graph G = Rank of G

$r = \begin{cases} n-1, & \text{if } G \text{ is connected} \\ n-k, & \text{if } G \text{ is disconnected} \end{cases}$

- If we remove any 1 row of incidence matrix, A(G) becomes induced incidence matrix A_f

where $A_f \Rightarrow (n-1) \times e$

The vertex corresponding to that deleted row is called ref. vertex.

A_f is always a square matrix of order $(n-1) \times e$ – will be a tree
$= (n-1) \times (n-1)$

- The values of A(G) are either 0 or 1 & hence it is known as (0–1) matrix / binary matrix.
- 1 in A(G) indicates an edge j incident on vertex i & hence it is also known as vertex – edge incident matrix

2/11/19

Scanned by CamScanner