

Lecture 6. Transmission Characteristics of Optical Fibers

- Fiber attenuation
- Fiber dispersion
- Group velocity
- Material dispersion
- Waveguide dispersion
- Chromatic dispersion compensation
- Polarization mode dispersion
- Polarization-maintaining fibers

Reading: Senior 3.1-3.4, 3.6, 3.8-3.13
Keiser 3.1 – 3.3

Transmission characteristics of optical fibers

- The transmission characteristics of most interest: **attenuation** (*loss*) and **bandwidth**.
- Now, *silica-based* glass fibers have losses about **0.2 dB/km** (i.e. **95% launched power remains after 1 km of fiber transmission**). This is essentially the *fundamental lower limit* for attenuation in silica-based glass fibers.
- **Fiber bandwidth** is limited by the signal dispersion within the fiber. Bandwidth determines the number of bits of information transmitted in a given time period. Now, fiber bandwidth has reached many 10's Gbit/s over many km's per wavelength channel.

Attenuation

- Signal attenuation within optical fibers is usually expressed in the logarithmic unit of the decibel.

The decibel, which is used for comparing two *power* levels, may be defined for a particular optical wavelength as the *ratio* of the *output optical power* P_o from the fiber to the *input optical power* P_i .

$$\text{Loss (dB)} = -10 \log_{10} (P_o/P_i) = 10 \log_{10} (P_i/P_o)$$

$$(P_o \leq P_i)$$

*In *electronics*, $\text{dB} = 20 \log_{10} (V_o/V_i)$

*The logarithmic unit has the advantage that the operations of *multiplication* (and *division*) reduce to *addition* (and *subtraction*).

In numerical values: $P_o/P_i = 10^{[-\text{Loss(dB)}/10]}$

The attenuation is usually expressed in decibels per unit length (i.e. dB/km):

$$\gamma L = -10 \log_{10} (P_o/P_i)$$

γ (dB/km): signal attenuation per unit length in decibels

L (km): fiber length

dBm

- dBm is a specific unit of power in decibels when the reference power is 1 mW:

$$\text{dBm} = 10 \log_{10} (\text{Power}/1 \text{ mW})$$

e.g. $1 \text{ mW} = 0 \text{ dBm}$; $10 \text{ mW} = 10 \text{ dBm}$; $100 \mu\text{W} = -10 \text{ dBm}$

$$\Rightarrow \text{Loss (dB)} = \text{input power (dBm)} - \text{output power (dBm)}$$

e.g. Input power = 1 mW (0 dBm), output power = 100 μW (-10 dBm)

$$\Rightarrow \text{loss} = -10 \log_{10} (100 \mu\text{W}/1 \text{ mW}) = 10 \text{ dB}$$

$$\text{OR } 0 \text{ dBm} - (-10 \text{ dBm}) = 10 \text{ dB}$$

The dBm Unit

Example 3.2 As Sec. 1.3 describes, optical powers are commonly expressed in units of *dBm*, which is the decibel power level referred to 1 mW. Consider a 30-km long optical fiber that has an attenuation of 0.4 dB/km at 1310 nm. Suppose we want to find the optical output power P_{out} if 200 μW of optical power is launched into the fiber. We first express the input power in dBm units:

$$\begin{aligned} P_{\text{in}}(\text{dBm}) &= 10 \log \left[\frac{P_{\text{in}}(\text{W})}{1 \text{ mW}} \right] \\ &= 10 \log \left[\frac{200 \times 10^{-6} \text{ W}}{1 \times 10^{-3} \text{ W}} \right] = -7.0 \text{ dBm} \end{aligned}$$

From Eq. (3.1c) with $P(0) = P_{\text{in}}$ and $P(z) = P_{\text{out}}$ the output power level (in dBm) at $z = 30 \text{ km}$ is

$$\begin{aligned} P_{\text{out}}(\text{dBm}) &= 10 \log \left[\frac{P_{\text{out}}(\text{W})}{1 \text{ mW}} \right] \\ &= 10 \log \left[\frac{P_{\text{in}}(\text{W})}{1 \text{ mW}} \right] - \alpha z \\ &= -7.0 \text{ dBm} - (0.4 \text{ dB/km})(30 \text{ km}) \\ &= -19.0 \text{ dBm} \end{aligned}$$

In unit of watts, the output power is

$$\begin{aligned} P(30 \text{ km}) &= 10^{-19.0/10} (1 \text{ mW}) = 12.6 \times 10^{-3} \text{ mW} \\ &= 12.6 \mu\text{W} \end{aligned}$$

e.g. When the mean optical power launched into an 8 km length of fiber is $120\ \mu\text{W}$, the mean optical power at the output is $3\ \mu\text{W}$.

Determine:

(a) the overall signal attenuation (or loss) in decibels through the fiber assuming there are no *connectors* or *splices*

(b) the signal attenuation per kilometer for the fiber

(c) the overall signal attenuation for a 10 km optical link using the same fiber with *splices* (i.e. fiber connections) at 1 km intervals, each giving an attenuation of 1 dB

(d) the output/input power ratio in (c).

(a) signal attenuation = $-10 \log_{10}(P_o/P_i) = 16 \text{ dB}$

(b) $16 \text{ dB} / 8 \text{ km} = 2 \text{ dB/km}$

(c) the loss incurred along 10 km fiber = 20 dB.

With a total of 9 *splices* (i.e. fiber connections) along the link, each with an attenuation of 1 dB, the loss due to the splices is 9 dB.

=> the overall signal attenuation for the link = $20 + 9 \text{ dB} = 29 \text{ dB}$.

(d) $P_o/P_i = 10^{(-29/10)} = 0.0013$

fiber attenuation mechanisms:

1. Material absorption
2. Scattering loss
3. Bending loss
4. Radiation loss (due to mode coupling)
5. Leaky modes

1. Material absorption losses in silica glass fibers

- Material absorption is a loss mechanism related to both *the material composition* and the *fabrication process* for the fiber. The optical power is lost as *heat* in the fiber.
- The light absorption can be *intrinsic* (due to the material components of the glass) or *extrinsic* (due to impurities introduced into the glass during fabrication).

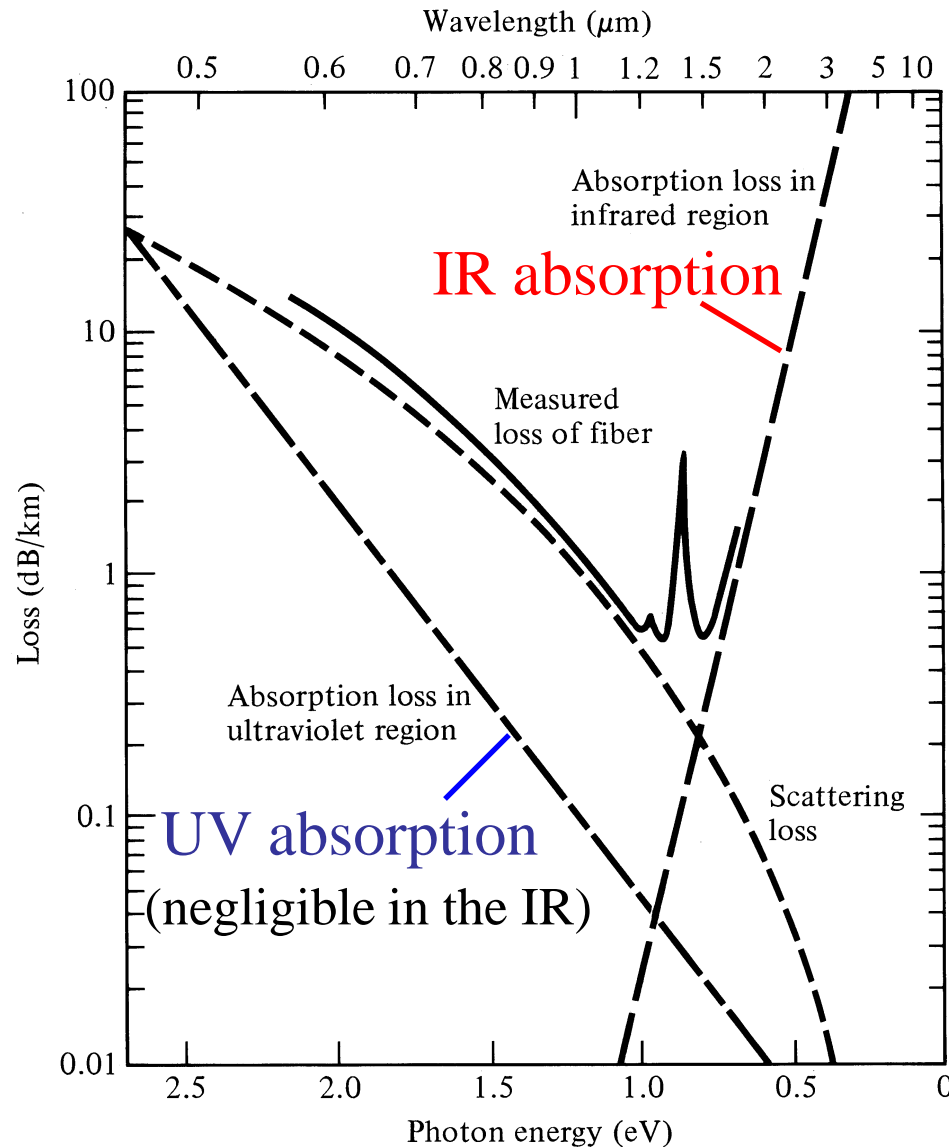
Intrinsic absorption

- Pure silica-based glass has *two* major intrinsic absorption mechanisms at optical wavelengths:

(1) a *fundamental UV absorption* edge, the peaks are centered in the *ultraviolet wavelength region*. This is due to the *electron transitions* within the glass molecules. The tail of this peak may extend into the the shorter wavelengths of the fiber transmission spectral window.

(2) A fundamental *infrared and far-infrared absorption edge*, due to *molecular vibrations* (such as Si-O). The tail of these absorption peaks may extend into the longer wavelengths of the fiber transmission spectral window.

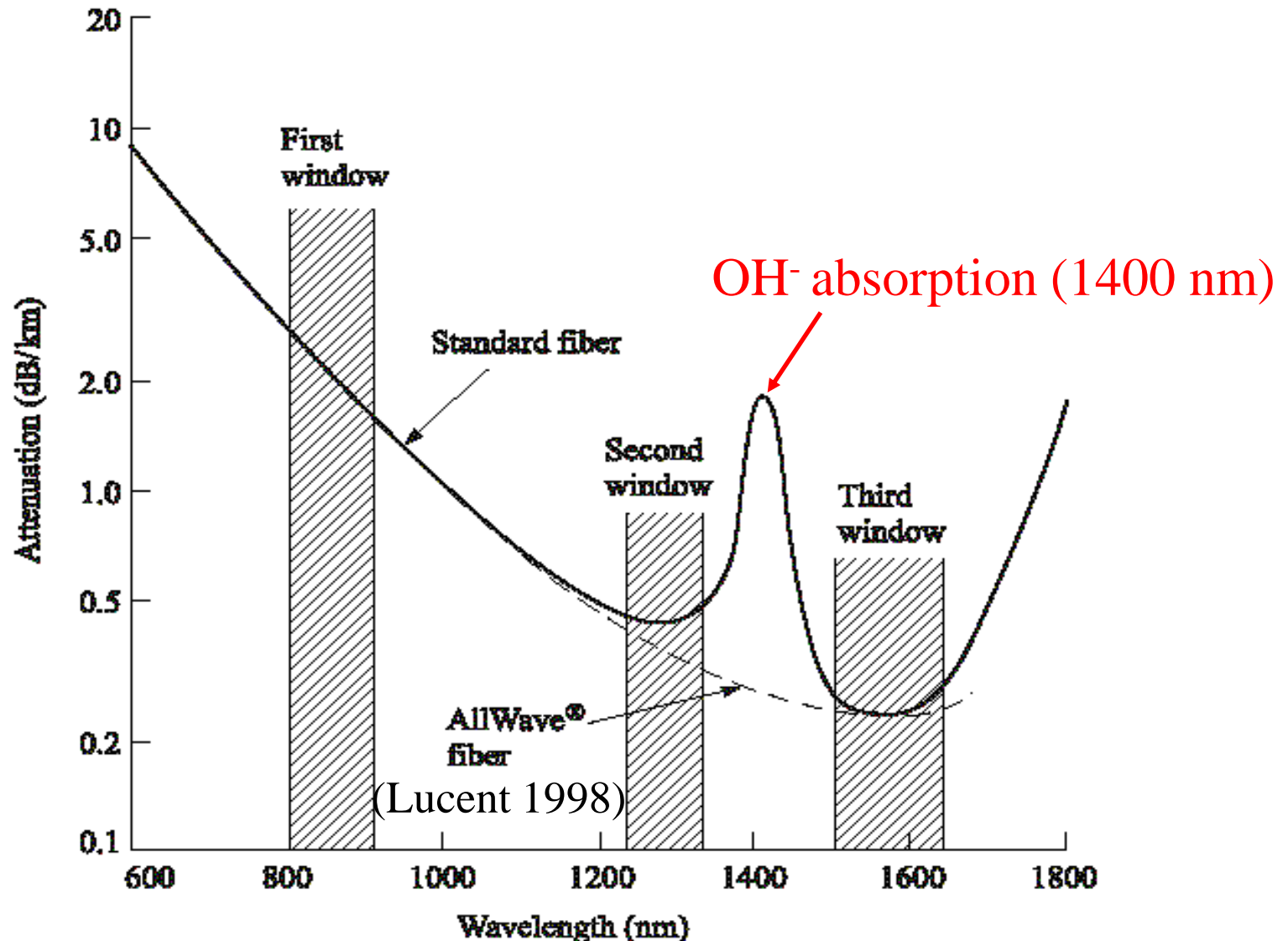
Fundamental fiber attenuation characteristics



Extrinsic absorption

- Major extrinsic loss mechanism is caused by absorption due to *water (as the hydroxyl or OH^- ions)* introduced in the glass fiber during *fiber pulling by means of oxyhydrogen flame*.
- These OH^- ions are bonded into the glass structure and have absorption peaks (due to *molecular vibrations*) at **1.38 μm** .
- Since these OH^- absorption peaks are sharply peaked, narrow spectral windows exist **around 1.3 μm and 1.55 μm which are essentially unaffected by OH^- absorption**.
- The lowest attenuation for typical silica-based fibers occur at **wavelength 1.55 μm at about 0.2 dB/km**, approaching the *minimum possible attenuation* at this wavelength.

1400 nm OH⁻ absorption peak and spectral windows



OFS AllWave fiber: example of a “low-water-peak” or “full spectrum” fiber. Prior to 2000 the fiber transmission bands were referred to as “windows.”

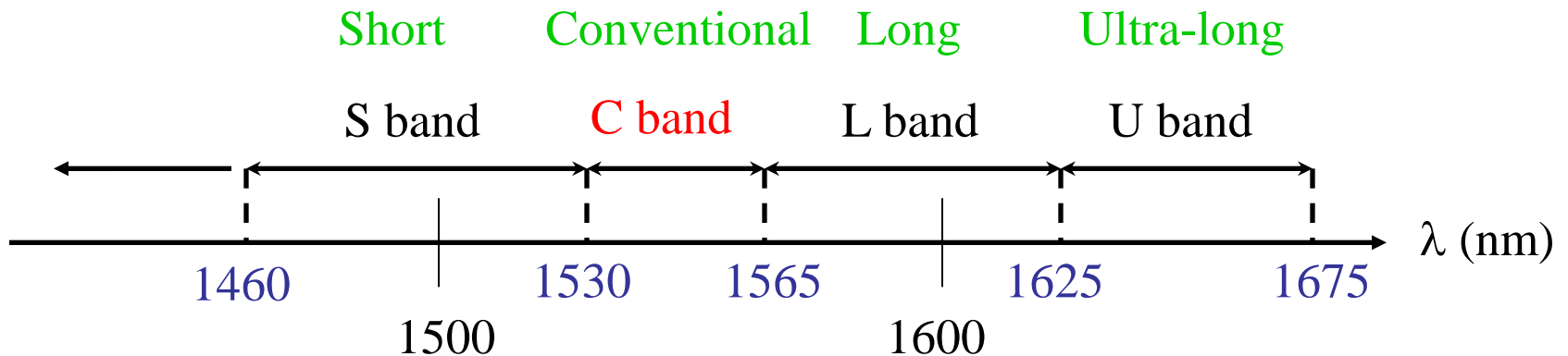
Three major spectral windows where fiber attenuation is low

The 1st window: 850 nm, attenuation 2 dB/km

The 2nd window: 1300 nm, attenuation 0.5 dB/km

The 3rd window: 1550 nm, attenuation 0.3 dB/km

1550 nm window is today's standard **long-haul** communication wavelengths.



Absorption Losses of Impurities

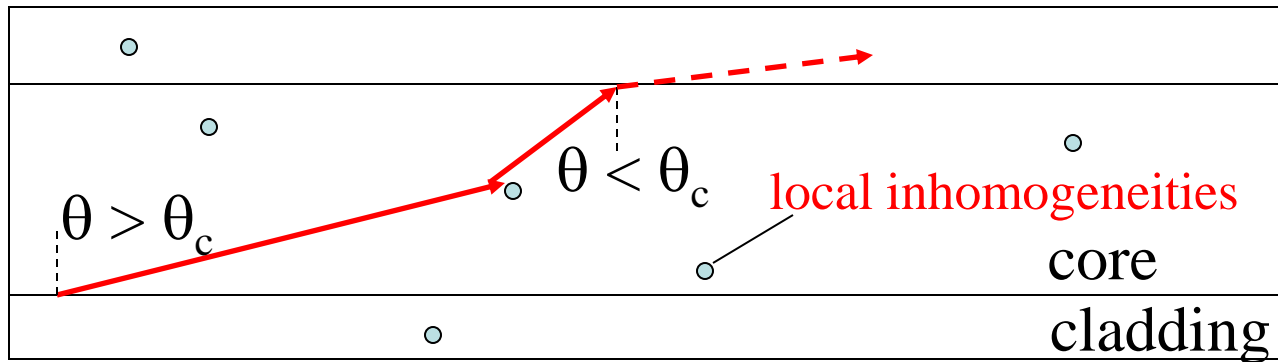
Table 3.1 *Examples of absorption loss in silica glass at different wavelengths due to 1 ppm of water-ions and various transition-metal impurities*

<i>Impurity</i>	<i>Loss due to 1 ppm of impurity (dB/km)</i>	<i>Absorption peak (nm)</i>
Iron: Fe ²⁺	0.68	1100
Iron: Fe ³⁺	0.15	400
Copper: Cu ²⁺	1.1	850
Chromium: Cr ²⁺	1.6	625
Vanadium: V ⁴⁺	2.7	725
Water: OH ⁻	1.0	950
Water: OH ⁻	2.0	1240
Water: OH ⁻	4.0	1380

2. Scattering loss

Scattering results in attenuation (*in the form of radiation*) as the scattered light may not continue to satisfy the total internal reflection in the fiber core.

One major type of scattering is known as *Rayleigh scattering*.



Silica glass
is amorphous.

The scattered ray can escape by refraction according to Snell's Law.

- *Rayleigh scattering* results from **random inhomogeneities** that are small **in size** compared with the wavelength.

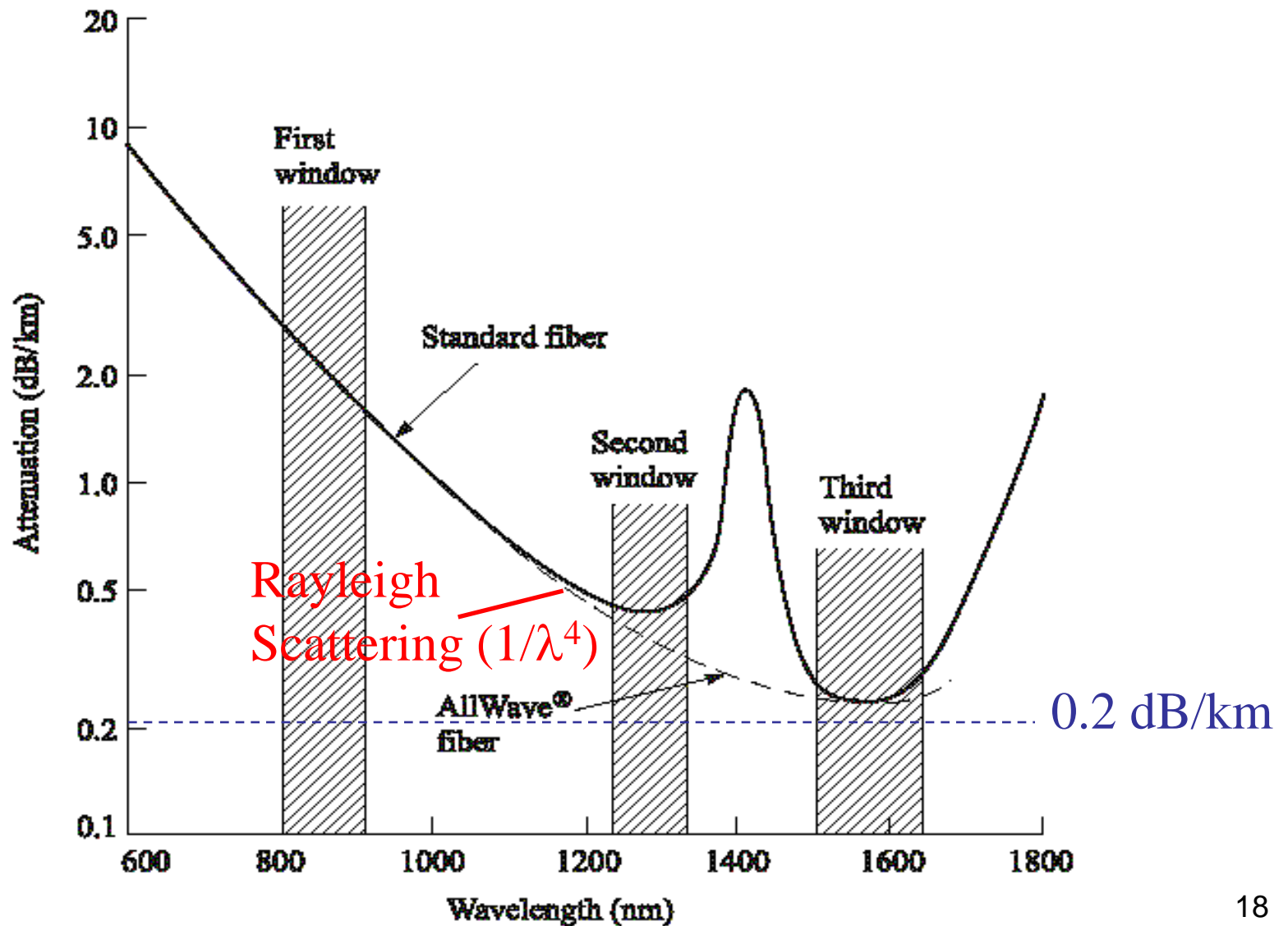
$$\bullet \quad \ll \quad \lambda$$

- These inhomogeneities exist in the form of *refractive index fluctuations* which are frozen into the *amorphous* glass fiber upon fiber pulling. Such fluctuations *always exist and cannot be avoided* !

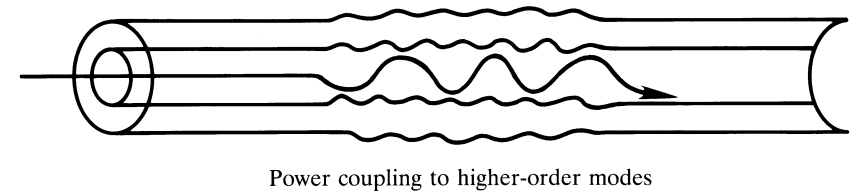
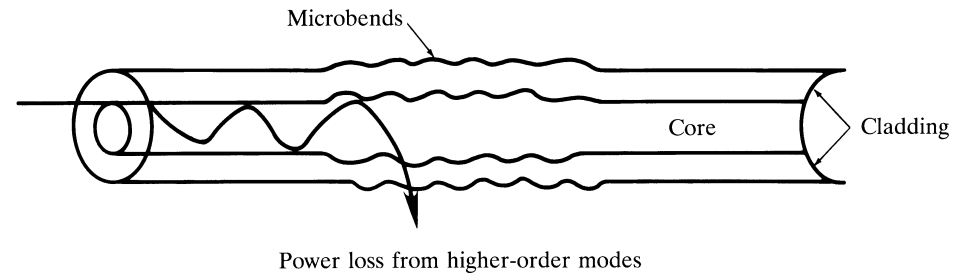
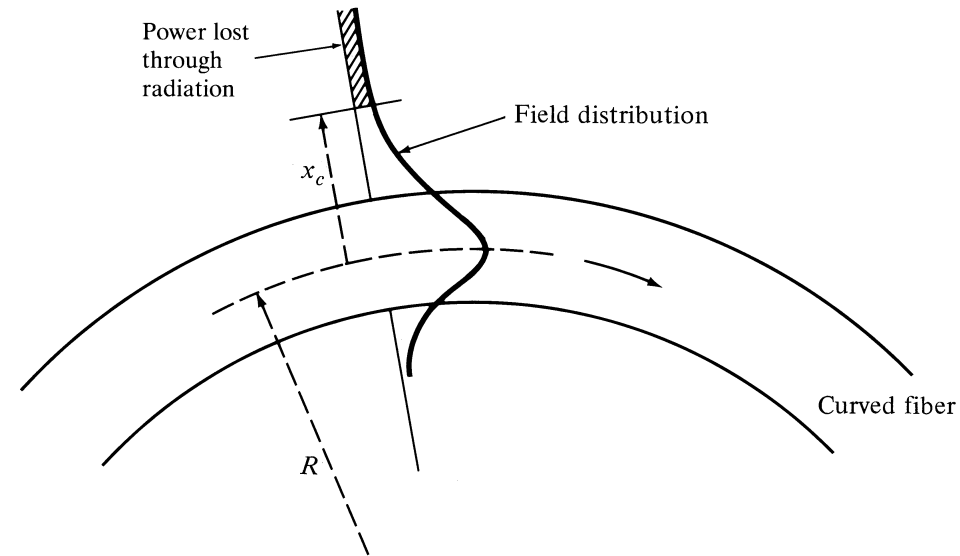
Rayleigh scattering results in an attenuation (dB/km) $\propto 1/\lambda^4$

Where else do we see Rayleigh scattering?

Rayleigh scattering is the dominant loss in today's fibers



Fiber bending loss and mode-coupling to higher-order modes



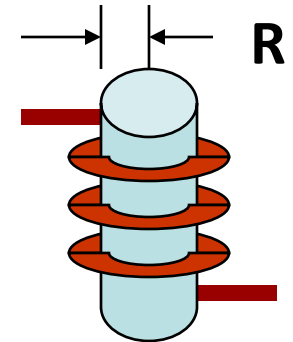
“macrobending”

(how do we measure bending loss?)

“microbending” – power coupling to higher-order modes that are more lossy.

Bending Losses in Fibers (1)

- Optical power escapes from tightly bent fibers
- Bending loss increases at longer wavelengths
 - Typical losses in 3 loops of standard 9- μm single-mode fiber (from: *Lightwave*; Feb 2001; p. 156):
 - 2.6 dB at 1310 nm and 23.6 dB at 1550 nm for $R = 1.15$ cm
 - 0.1 dB at 1310 nm and 2.60 dB at 1550 nm for $R = 1.80$ cm
- Progressively tighter bends produce higher losses
- Bend-loss insensitive fibers have been developed and now are recommended
- Improper routing of fibers and incorrect storage of slack fiber can result in violations of bend radius rules



Test setup for checking bend loss:

N fiber loops on a rod of radius R

Bending Losses in Fibers (2)

The total number of modes that can be supported by a curved fiber is less than in a straight fiber.

$$M_{\text{eff}} = M_{\infty} \left\{ 1 - \frac{\alpha + 2}{2\alpha\Delta} \left[\frac{2a}{R} + \left(\frac{3}{2n_2 k R} \right)^{2/3} \right] \right\}$$

Example 3.6 Consider a graded-index multimode fiber for which the index profile $\alpha = 2.0$, the core index $n_1 = 1.480$, the core-cladding index difference $\Delta = 0.01$, and the core radius $a = 25 \mu\text{m}$. If the radius of curvature of the fiber is $R = 1.0 \text{ cm}$, what percentage of the modes remain in the fiber at a 1300-nm wavelength?

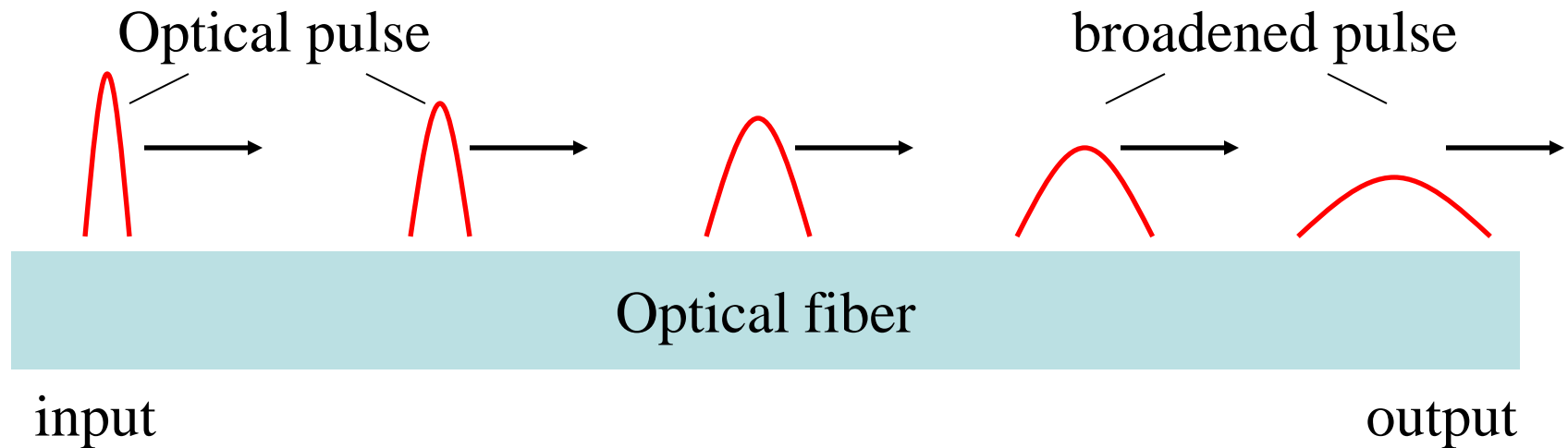
Solution: From Eq. (3.7) the percentage of modes at a given curvature R is

$$\begin{aligned} \frac{M_{\text{eff}}}{M_{\infty}} &= 1 - \frac{\alpha + 2}{2\alpha\Delta} \left[\frac{2a}{R} + \left(\frac{3}{2n_2 k R} \right)^{2/3} \right] \\ &= 1 - \frac{1}{.01} \left[\frac{2(25)}{10000} + \left(\frac{3(1.3)}{2(1.465)2\pi(10000)} \right)^{2/3} \right] \\ &= 0.42 \end{aligned}$$

Thus 42 percent of the modes remain in this fiber at a 1.0-cm bend radius.

Fiber dispersion

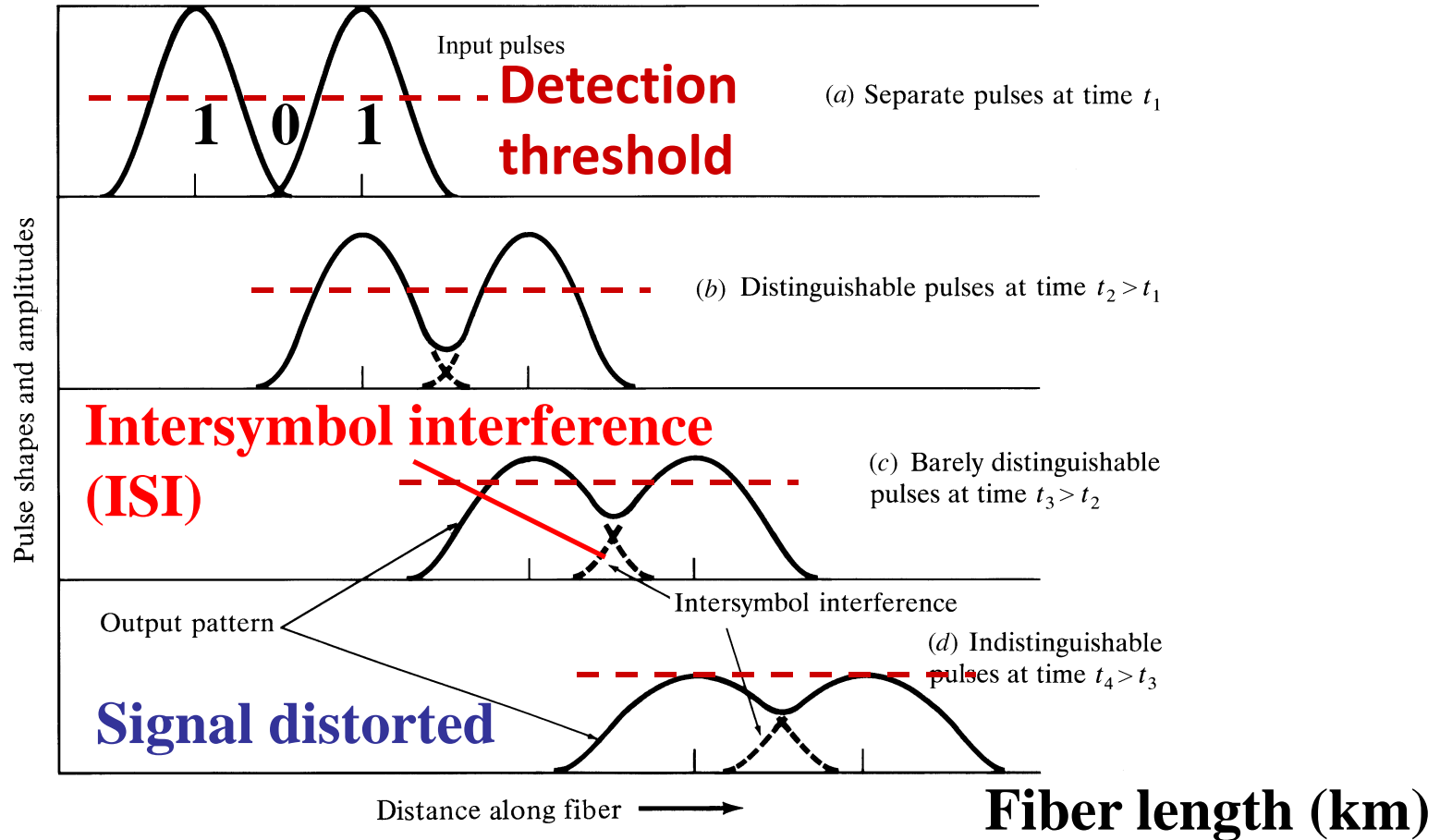
- Fiber dispersion results in *optical pulse broadening* and hence *digital signal degradation*.



Dispersion mechanisms:

1. **Modal** (or *intermodal*) **dispersion**
2. **Chromatic dispersion** (CD)
3. **Polarization mode dispersion** (PMD)

Pulse broadening limits fiber bandwidth (data rate)

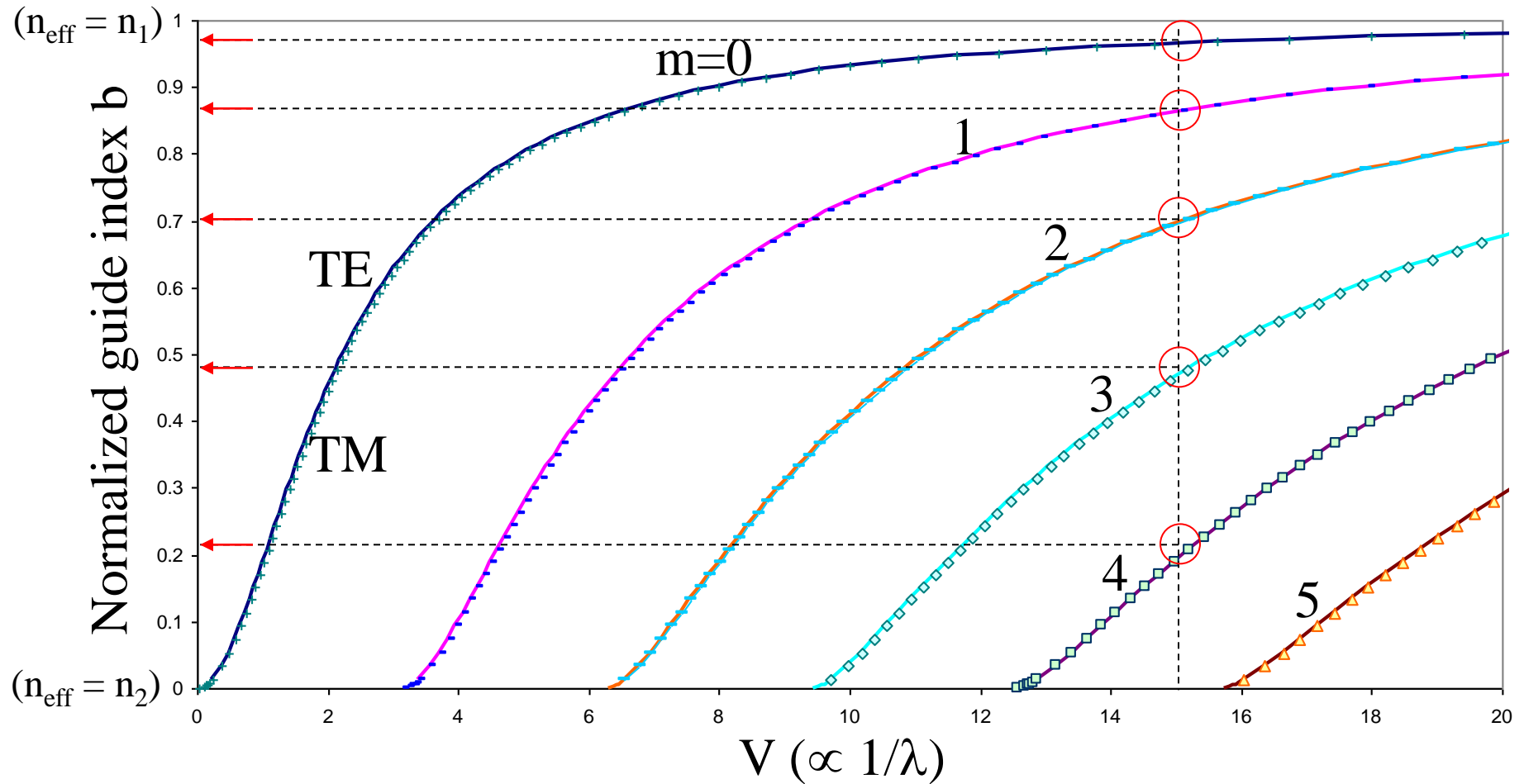


- An *increasing number of errors* may be encountered on the digital optical channel as the ISI becomes more pronounced.

1. Modal dispersion

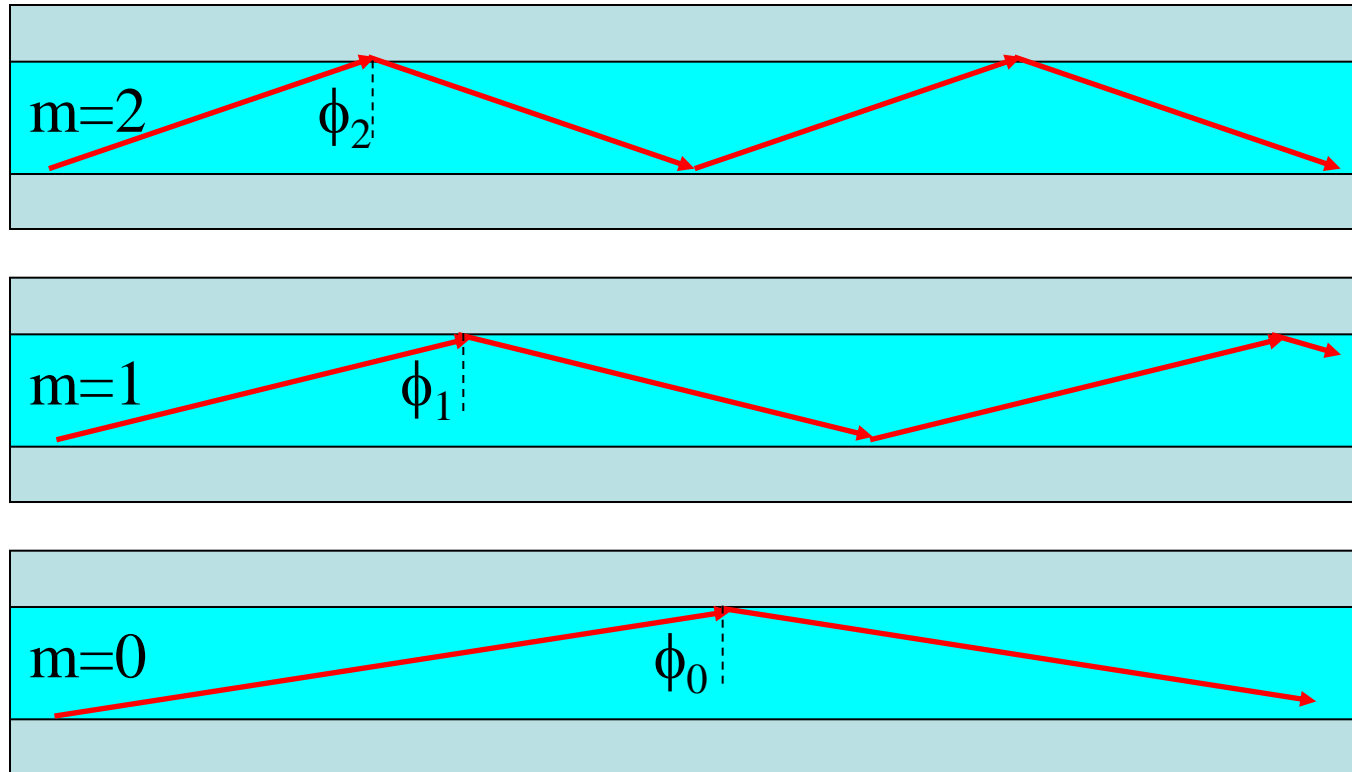
- When numerous waveguide modes are propagating, they all travel with different net velocities with respect to the waveguide axis.
- An input waveform distorts during propagation because its energy is distributed among several modes, each traveling at a different speed.
- Parts of the wave arrive at the output before other parts, spreading out the waveform. This is thus known as **multimode (modal) dispersion**.
- **Multimode dispersion does *not* depend on the source linewidth** (even a *single* wavelength can be simultaneously carried by *multiple modes* in a waveguide).
- **Multimode dispersion would *not* occur if the waveguide allows *only one mode to propagate* - the advantage of *single*-mode waveguides!**²⁴

Modal dispersion as shown from the mode chart of a symmetric slab waveguide



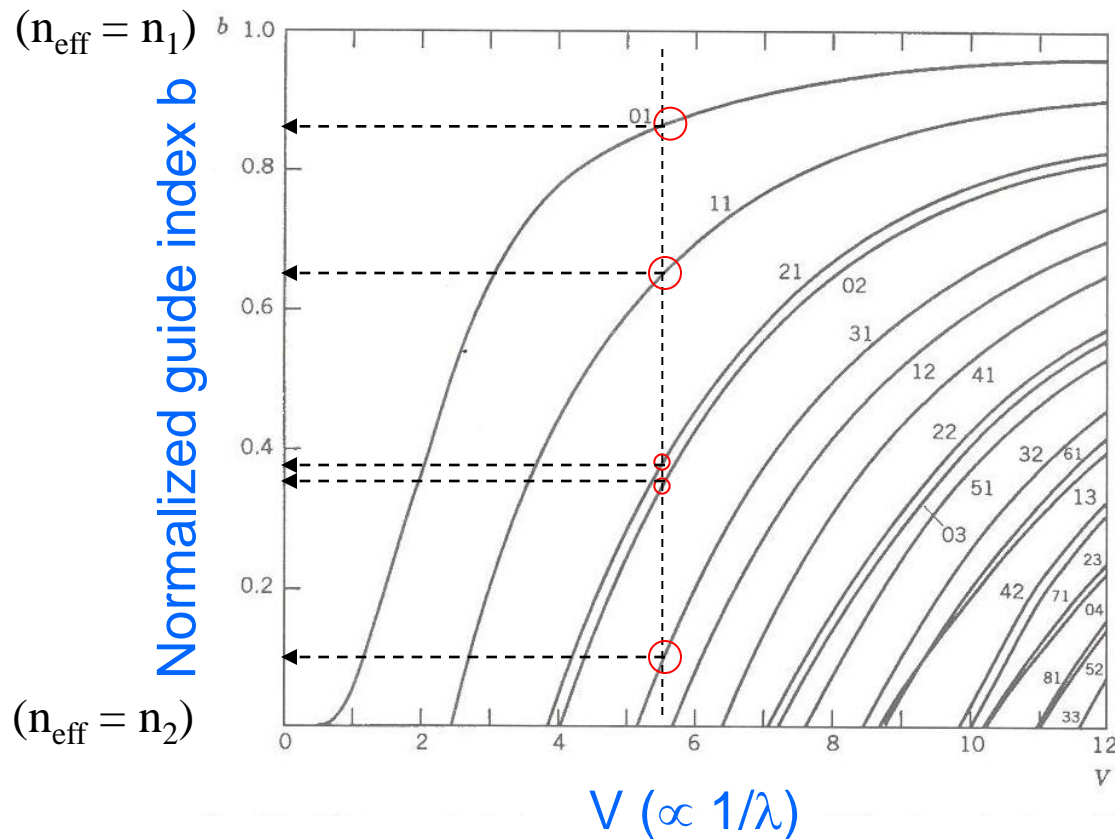
- Phase velocity for mode $m = \omega/\beta_m = \omega/(n_{\text{eff}}(m) k_0)$
(note that $m = 0$ mode is the *slowest* mode)

Modal dispersion in multimode waveguides



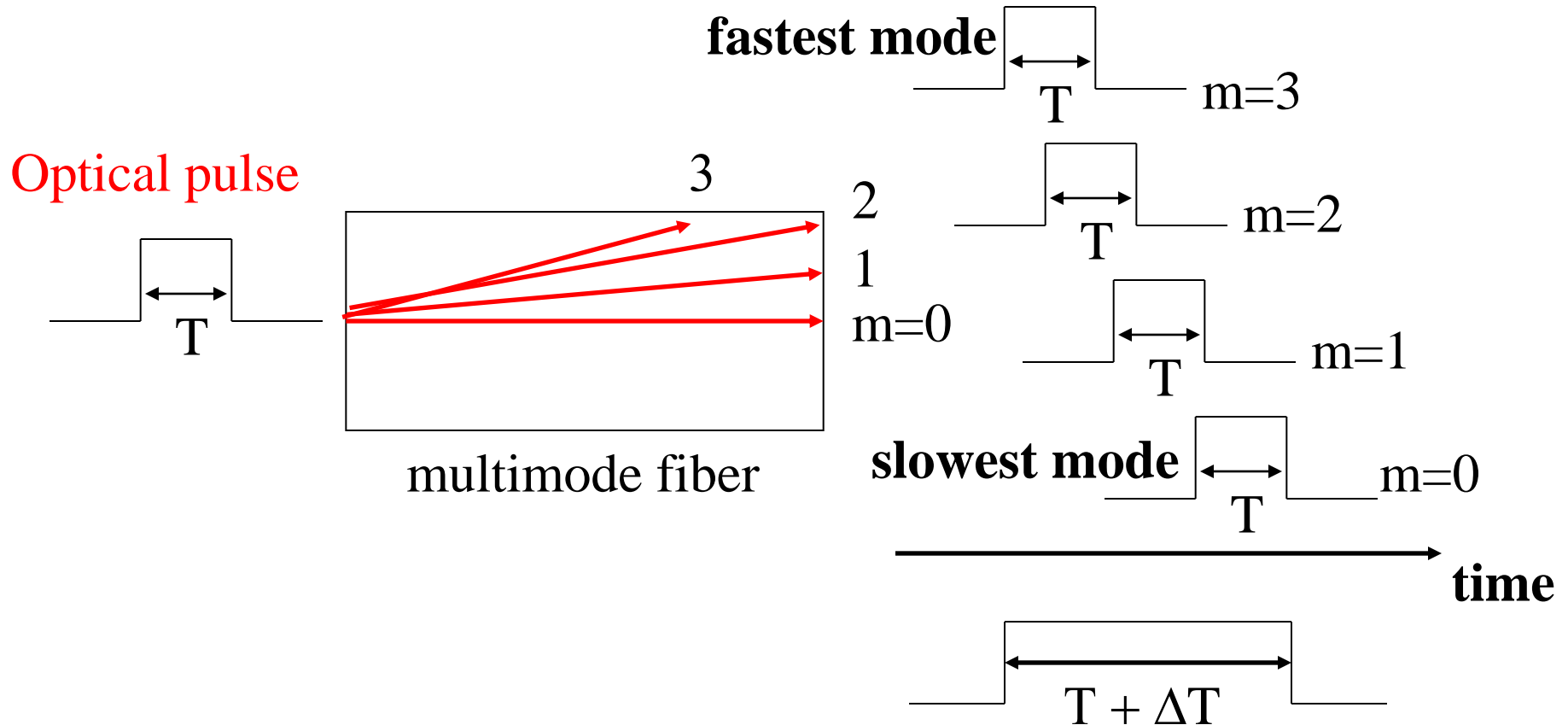
The carrier wave can propagate along all these different “zig-zag” ray paths of *different path lengths*.

Modal dispersion as shown from the LP mode chart of a silica optical fiber



- Phase velocity for LP mode = $\omega/\beta_{lm} = \omega/(n_{\text{eff}}(lm) k_0)$
(note that LP₀₁ mode is the *slowest* mode)

Modal dispersion results in pulse broadening

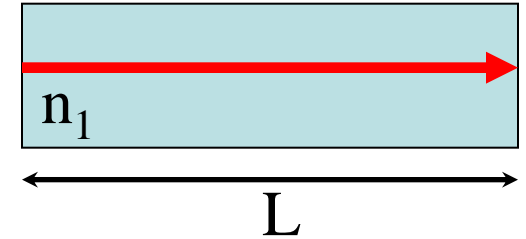


modal dispersion: different modes arrive at the receiver with different delays \Rightarrow pulse broadening

Estimated modal dispersion pulse broadening using phase velocity

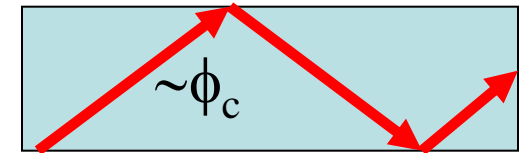
- A zero-order mode traveling near the waveguide axis needs time:

$$t_0 = L/v_{m=0} \approx Ln_1/c \quad (v_{m=0} \approx c/n_1)$$



- The highest-order mode traveling near the critical angle needs time:

$$t_m = L/v_m \approx Ln_2/c \quad (v_m \approx c/n_2)$$



=> the *pulse broadening* due to modal dispersion:

$$\Delta T \approx t_0 - t_m \approx (L/c) (n_1 - n_2)$$

$$\approx (L/2cn_1) NA^2$$

$$(n_1 \sim n_2)$$

e.g. How much will a light pulse spread after traveling along 1 km of a step-index fiber whose $NA = 0.275$ and $n_{\text{core}} = 1.487$?

How does modal dispersion restricts fiber bit rate?

Suppose we transmit at a low bit rate of 10 Mb/s

$$\Rightarrow \text{Pulse duration} = 1 / 10^7 \text{ s} = 100 \text{ ns}$$

Using the above e.g., each pulse will spread up to $\approx 100 \text{ ns}$ (i.e. \approx pulse duration !) every km

\Rightarrow The broadened pulses overlap! (**Intersymbol interference (ISI)**)

*Modal dispersion limits the bit rate of a fiber-optic link to $\sim 10 \text{ Mb/s}$.
(a coaxial cable supports this bit rate easily!)

Bit-rate distance product

- We can relate the pulse broadening ΔT to the *information-carrying capacity* of the fiber measured through the bit rate B .
- Although a precise relation between B and ΔT depends on many details, such as the pulse shape, it is intuitively clear that ΔT *should be less than the allocated bit time slot* given by $1/B$.

\Rightarrow An *order-of-magnitude* estimate of the supported bit rate is obtained from the condition $B\Delta T < 1$.

\Rightarrow *Bit-rate distance product* (limited by modal dispersion)

$$BL < 2c n_{\text{core}} / NA^2$$

This condition provides a rough estimate of a fundamental limitation of step-index multimode fibers.

(the *smaller is the NA, the larger is the bit-rate distance product*)

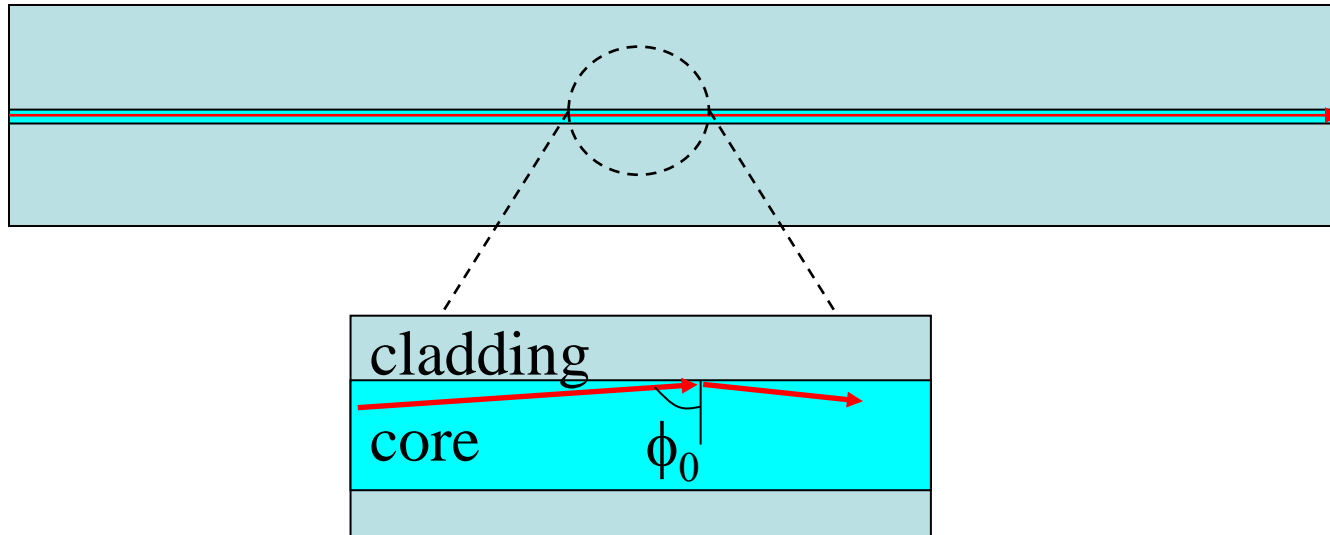
The capacity of optical communications systems is frequently measured in terms of the **bit rate-distance product**.

e.g. If a system is capable of transmitting **10 Mb/s** over a distance of **1 km**, it is said to have a *bit rate-distance* product of **10 (Mb/s)-km**.

This may be suitable for some *local-area networks (LANs)*.

Note that the same system can transmit **100 Mb/s** along **100 m**, or **1 Gb/s** along **10 m**, or **10 Gb/s** along **1 m**, or **100 Gb/s** along **10 cm**, or **1 Tb/s** along **1 cm**

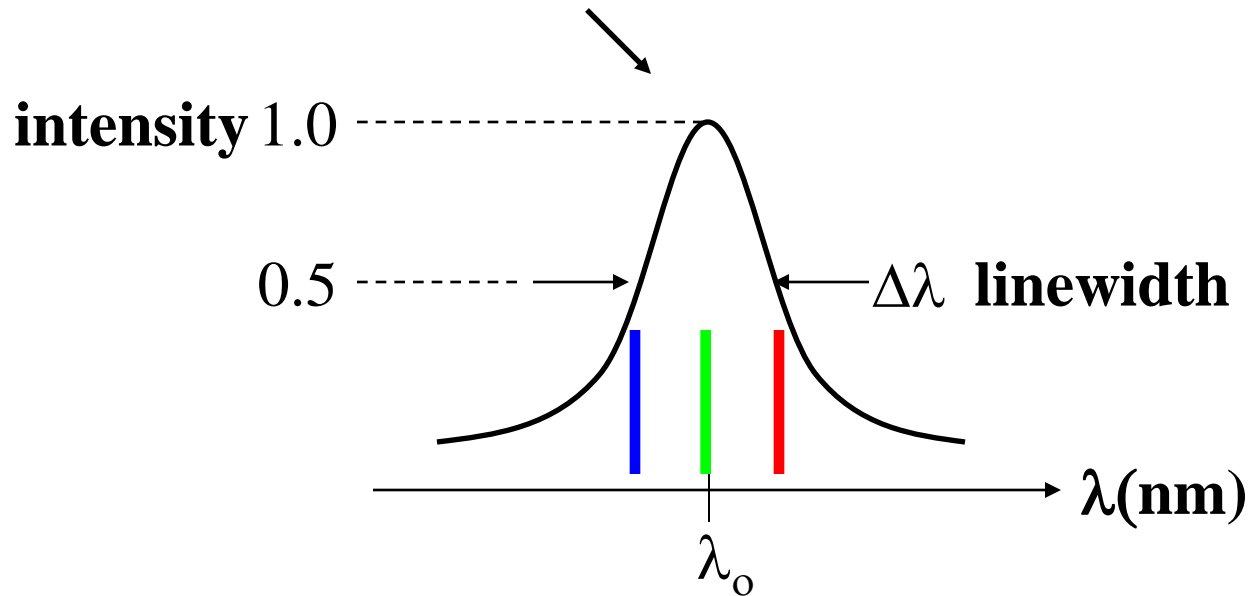
Single-mode fiber eliminates modal dispersion



- The main advantage of *single*-mode fibers is to propagate *only one mode* so that *modal dispersion is absent*.
- However, *pulse broadening does not disappear altogether*. The *group velocity* associated with the fundamental mode is *frequency dependent* within the pulse *spectral linewidth* because of chromatic dispersion.

2. Chromatic dispersion

- Chromatic dispersion (CD) may occur in *all* types of optical fiber. The optical pulse broadening results from the *finite spectral linewidth of the optical source and the modulated carrier*.



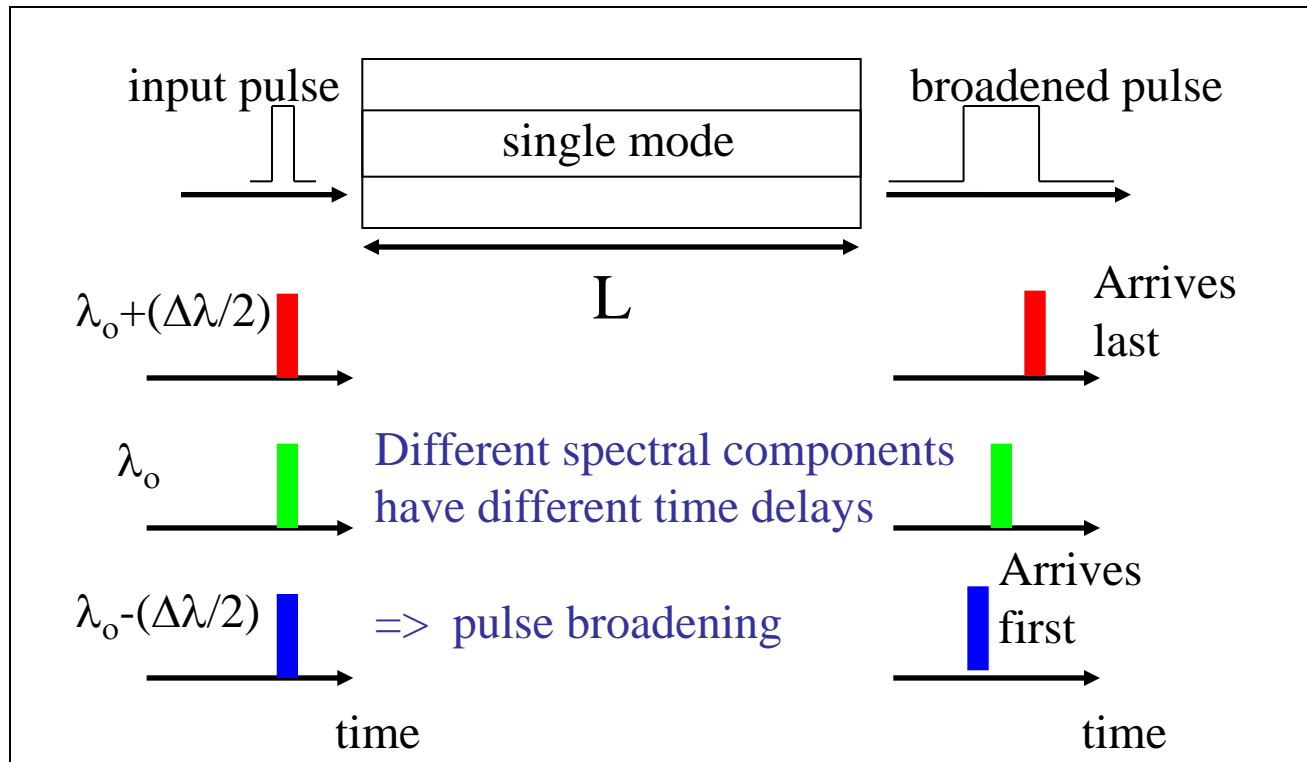
*In the case of the semiconductor laser $\Delta\lambda$ corresponds to only a fraction of % of the centre wavelength λ_0 . For LEDs, $\Delta\lambda$ is likely to be a significant percentage of λ_0 .

Spectral linewidth

- Real sources emit over a range of wavelengths. This range is the *source linewidth* or *spectral width*.
- The smaller is the linewidth, the smaller is the spread in wavelengths or frequencies, the more *coherent* is the source.
- An *ideal perfectly coherent* source emits light at a single wavelength. It has *zero* linewidth and is perfectly monochromatic.

Light sources	Linewidth (nm)
Light-emitting diodes	20 nm – 100 nm
Semiconductor laser diodes	1 nm – 5 nm
Nd:YAG solid-state lasers	0.1 nm
HeNe gas lasers	0.002 nm

- Pulse broadening occurs because there may be *propagation delay differences* among the *spectral components* of the transmitted signal.



Chromatic dispersion (CD): Different spectral components of a *pulse* travel at different *group velocities*. This is known as *group velocity dispersion (GVD)*.

Phase velocity, group velocity and dispersion

- For a *monochromatic* (i.e. single wavelength) plane optical wave traveling in the z direction, the electric field can be written as

$$\mathbf{E} = \mathbf{E} \exp i(kz - \omega t)$$

where \mathbf{E} is a constant vector independent of space and time. We see a sinusoidal wave whose phase varies with z and t

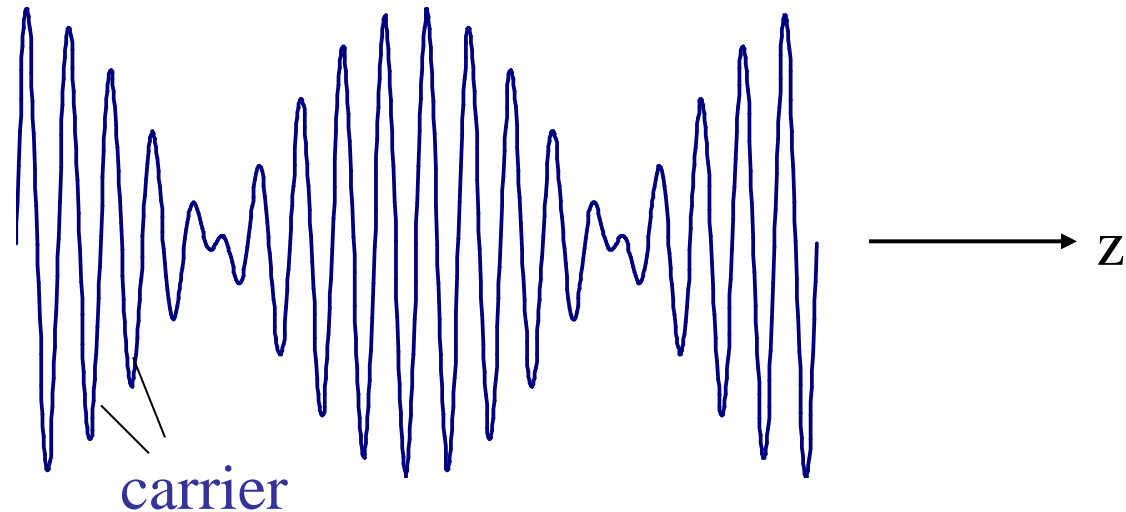
$$\phi = kz - \omega t$$

For a point of constant phase on the space- and time-varying field, $\phi = \text{constant}$ and thus $k dz - \omega dt = 0$. If we track this point of constant phase, we find that it is moving with a velocity of

$$v_p = dz/dt = \omega/k \quad \text{phase velocity}$$

- In reality, a *group of waves* with *closely similar wavelengths* (or *frequencies*) always combine to form a *packet of wave*.

modulation (or envelope)



- For example, we consider a *wave packet* that is composed of two plane waves of equal real amplitude E . The frequencies and propagation constants of the two component plane waves are:

$$\omega_1 = \omega_o + d\omega, k_1 = k_o + dk$$

$$\omega_2 = \omega_o - d\omega, k_2 = k_o - dk$$

- The space- and time-dependent *total real* field of the wave packet is given by

$$\begin{aligned}
 E_{\text{packet}} &= E \exp i(k_1 z - \omega_1 t) + \overset{\text{complex conjugate}}{\text{c.c.}} + E \exp i(k_2 z - \omega_2 t) + \text{c.c.} \\
 &= 2E \{ \cos [(k_o + dk)z - (\omega_o + d\omega)t] + \cos [(k_o - dk)z - (\omega_o - d\omega)t] \} \\
 &= 4E \cos (zdk - td\omega) \cos (k_o z - \omega_o t)
 \end{aligned}$$

- The resultant wave packet has a *carrier*, which has a frequency ω_o and a propagation constant k_o , and an *envelope* $\cos (zdk - td\omega)$.

- Therefore, a fixed point on the envelope is defined by $zdk - td\omega = \text{constant}$, and it travels with a velocity

$$v_g = dz/dt = d\omega/dk \quad \text{group velocity}$$

Remarks on group velocity

- Because the *energy* of a harmonic wave is proportional to the square of its field amplitude, the energy carried by a wave packet that is composed of many frequency components is concentrated in regions *where the amplitude of the envelope is large*.
- Therefore, *the energy in a wave packet is transported at group velocity v_g* .
- The constant-phase wavefront travels at the phase velocity, but the group velocity is the velocity at which *energy* (and *information*) travels.

Any information signal is a wave packet, and thus travels at the group velocity, not at the phase velocity.

Light pulse in a dispersive medium

When a *light pulse* with a spread in frequency $\delta\omega$ and a spread in propagation constant δk propagates in a *dispersive* medium $n(\lambda)$, the *group velocity*:

$$v_g = (d\omega/dk) = (d\lambda/dk) (d\omega/d\lambda)$$

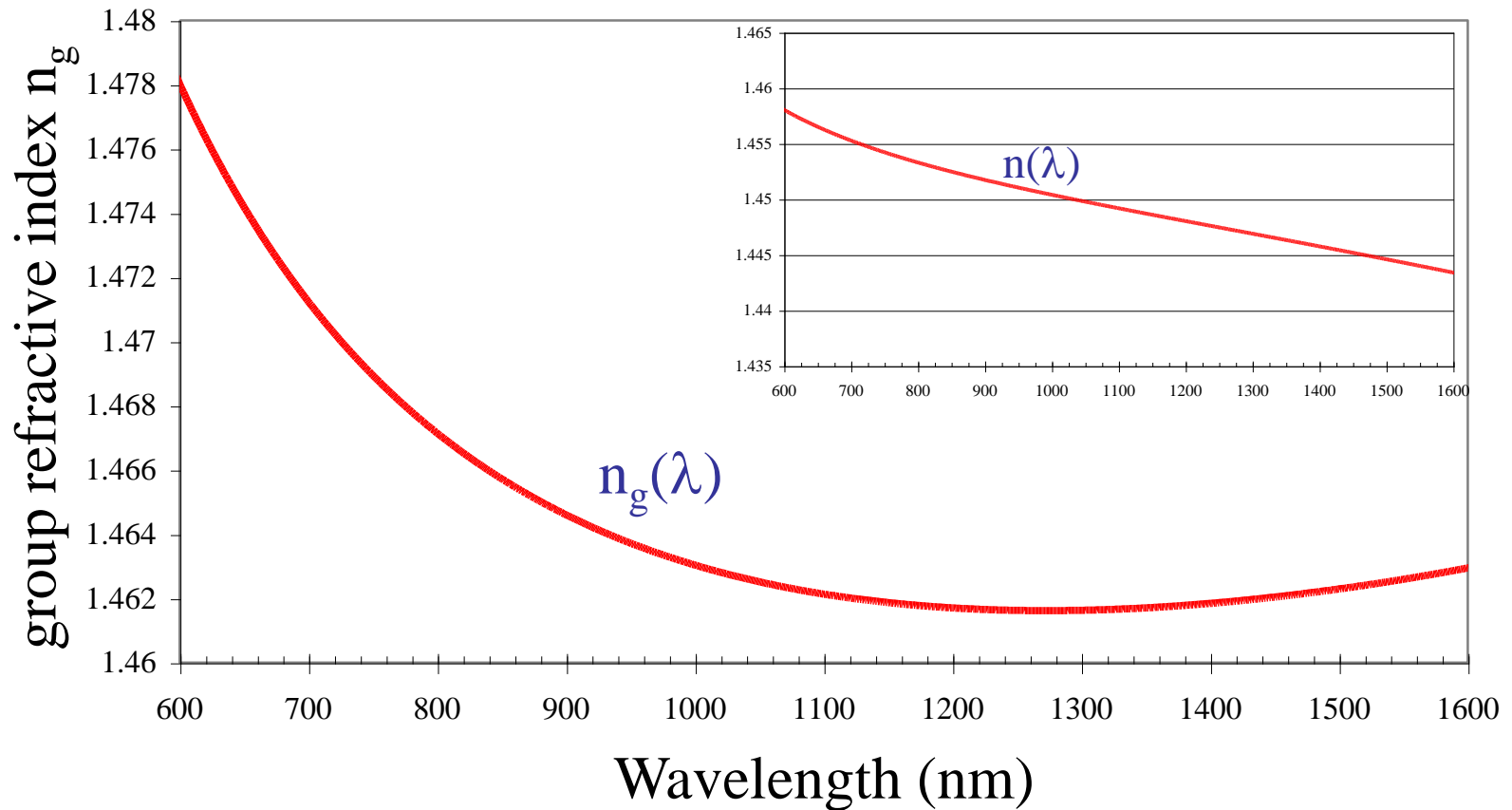
$$k = n(\lambda) 2\pi/\lambda \quad \Rightarrow \quad dk/d\lambda = (2\pi/\lambda) [(dn/d\lambda) - (n/\lambda)]$$

$$\omega = 2\pi c/\lambda \quad \Rightarrow \quad d\omega/d\lambda = -2\pi c/\lambda^2$$

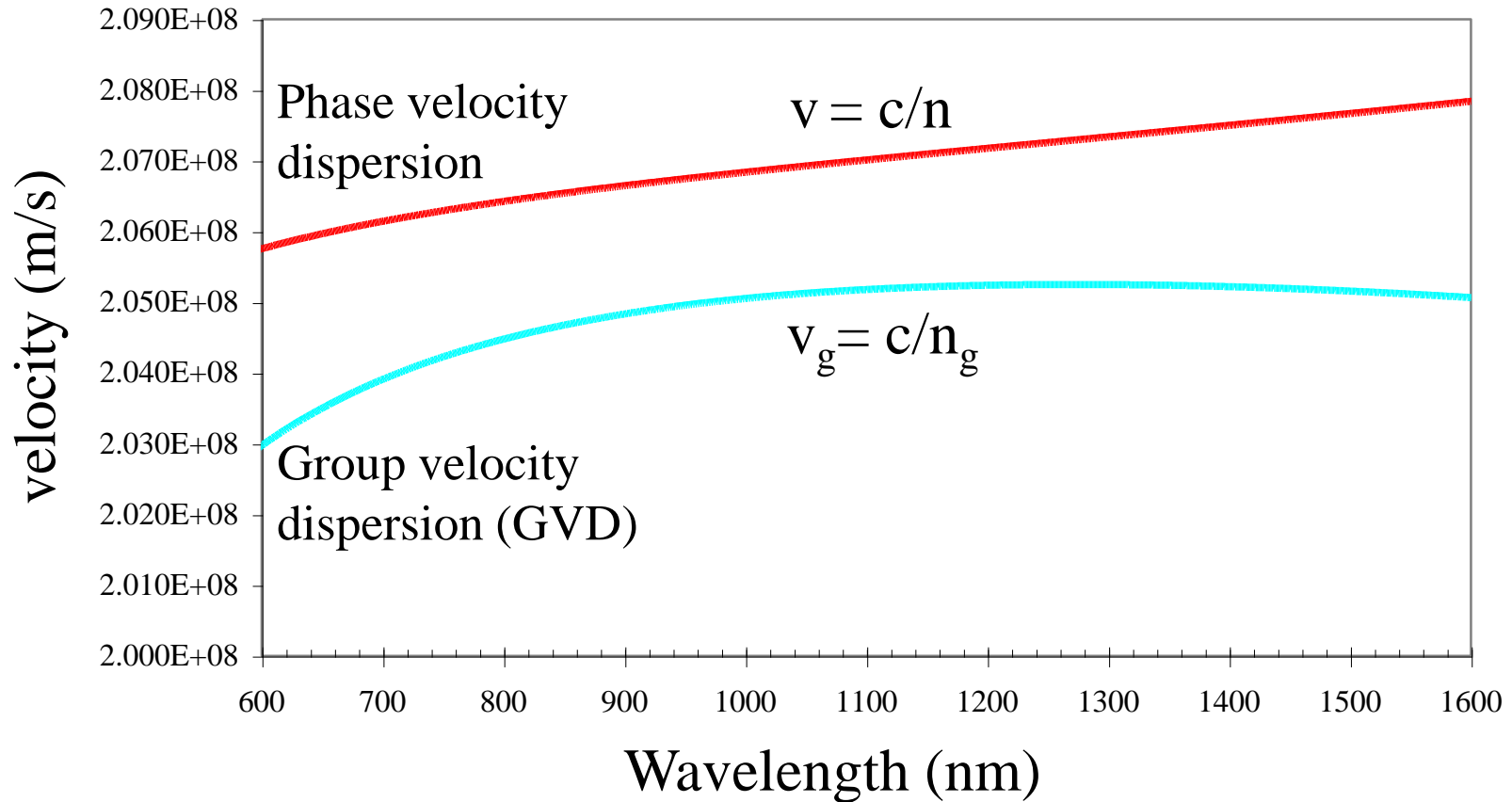
$$\text{Hence} \quad v_g = c / [n - \lambda(dn/d\lambda)] = c / n_g$$

Define the **group refractive index** $n_g = n - \lambda(dn/d\lambda)$

Group refractive index n_g vs. λ for fused silica



Phase velocity c/n and group velocity c/n_g vs. λ for fused silica



Group-Velocity Dispersion (GVD)

Consider a light pulse propagates in a dispersive medium of length L

- A specific spectral component at the frequency ω (or wavelength λ) would arrive at the output end of length L after a *time delay*:

$$T = L/v_g$$

- If $\Delta\lambda$ is the *spectral width* of an optical pulse, the extent of *pulse broadening* for a material of length L is given by

$$\begin{aligned}\Delta T &= (dT/d\lambda) \Delta\lambda = [d(L/v_g)/d\lambda] \Delta\lambda \\ &= L [d(1/v_g)/d\lambda] \Delta\lambda\end{aligned}$$

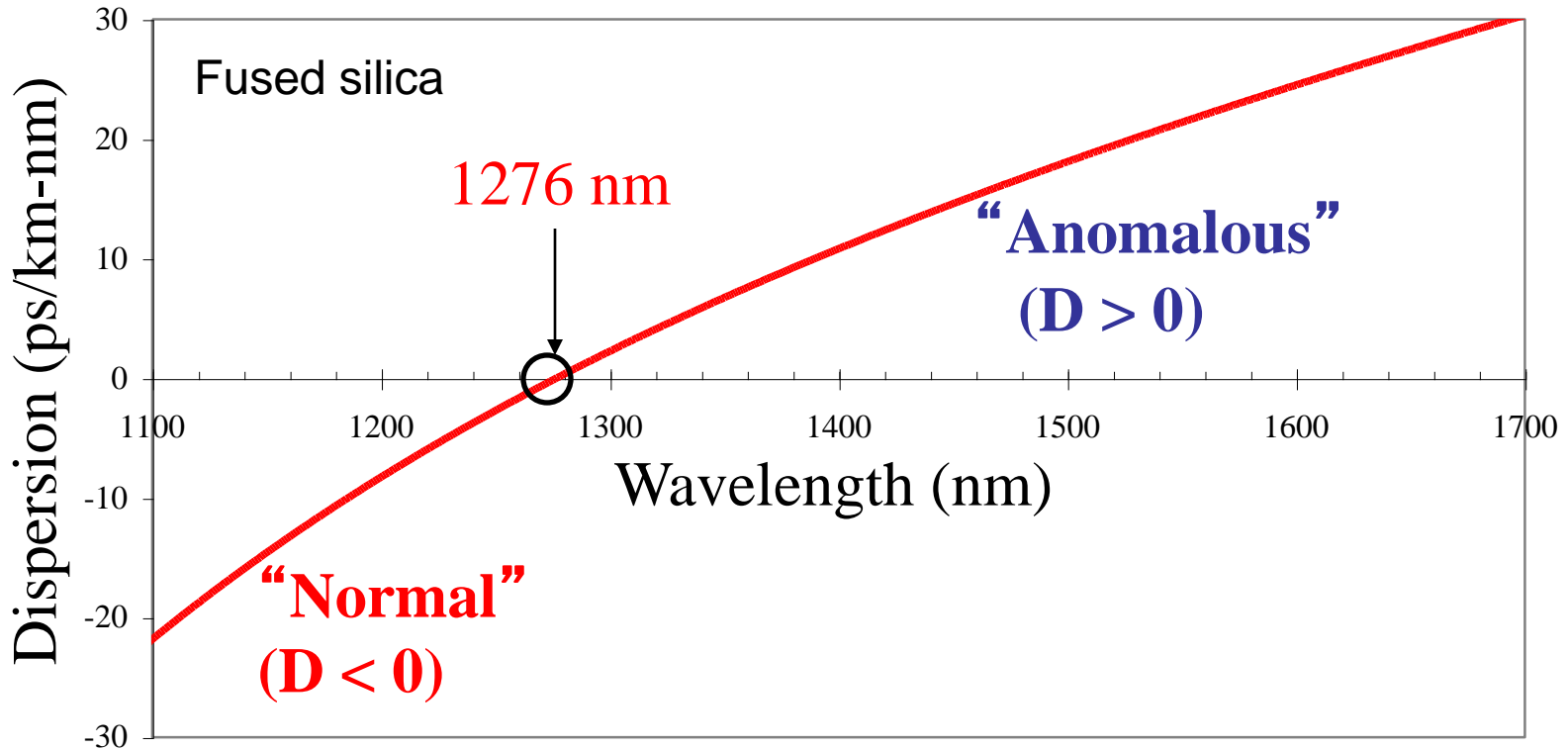
Hence the pulse broadening due to a *differential time delay*:

$$\Delta T = L D \Delta \lambda$$

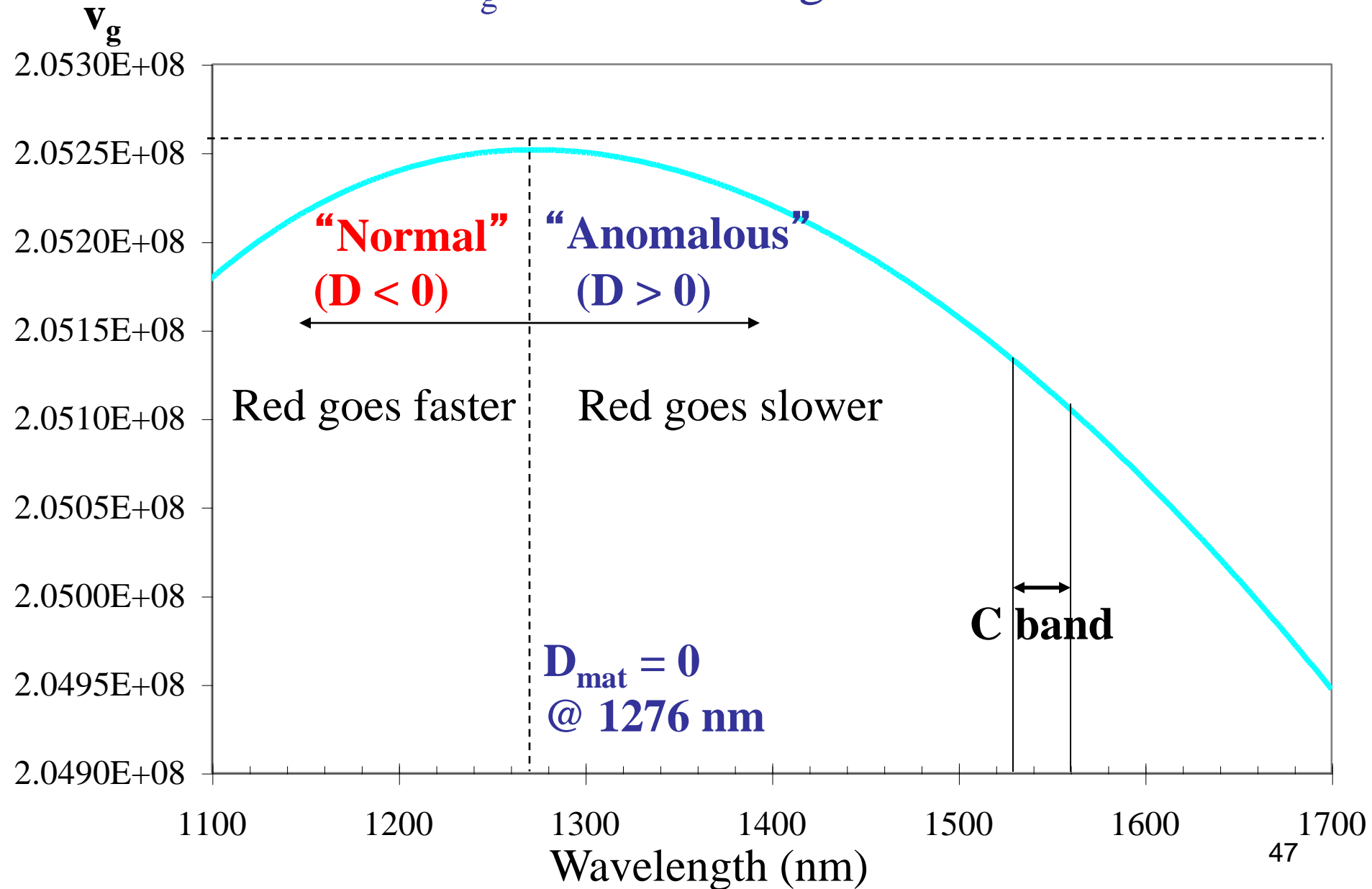
where $D = d(1/v_g)/d\lambda$ is called the *dispersion parameter* and is expressed in units of ps/(km-nm).

$$\begin{aligned} D &= d(1/v_g)/d\lambda = c^{-1} dn_g/d\lambda = c^{-1} d[n - \lambda(dn/d\lambda)]/d\lambda \\ &= -c^{-1} \lambda d^2n/d\lambda^2 \end{aligned}$$

$$\text{Dispersion parameter } D = - (\lambda/c) d^2n/d\lambda^2$$



Variation of v_g with wavelength for fused silica



Zero-dispersion wavelength

Material dispersion $D_{\text{mat}} = 0$ at $\lambda \sim 1276$ nm for fused silica.

This λ is referred to as the *zero-dispersion wavelength* λ_{ZD} .

Chromatic (or *material*) dispersion $D(\lambda)$ can be zero;

or

negative \Rightarrow longer wavelengths travel *faster* than shorter wavelengths;

or

positive \Rightarrow shorter wavelengths travel *faster* than longer wavelengths.

In fact there are two mechanisms for chromatic dispersion in a fiber:

(a) Silica refractive index *n is wavelength dependent (i.e. $n = n(\lambda)$)*

=> different wavelength components travel at different speeds in silica

This is known as material dispersion.

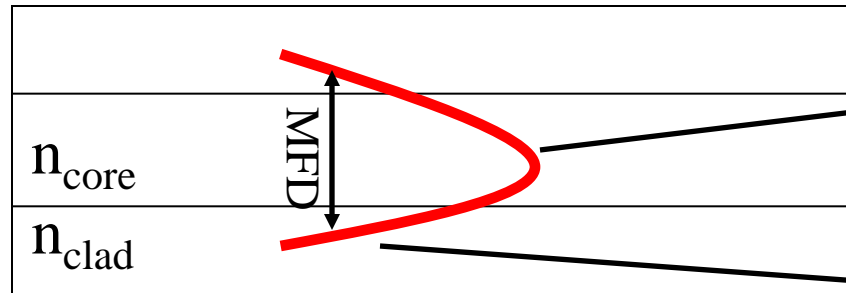
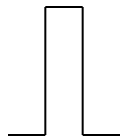
(b) Light energy of *a mode propagates partly in the core and partly in the cladding of a fiber*. The mode power distribution between the core and the cladding *depends on λ* . (Recall the mode field diameter)

This is known as waveguide dispersion.

$$\Rightarrow D(\lambda) = D_{\text{mat}}(\lambda) + D_{\text{wg}}(\lambda)$$

Waveguide dispersion in a single-mode fiber

input pulse



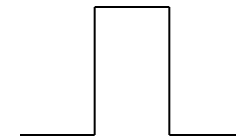
Singlemode fiber

core pulse
slower

cladding pulse
faster

time

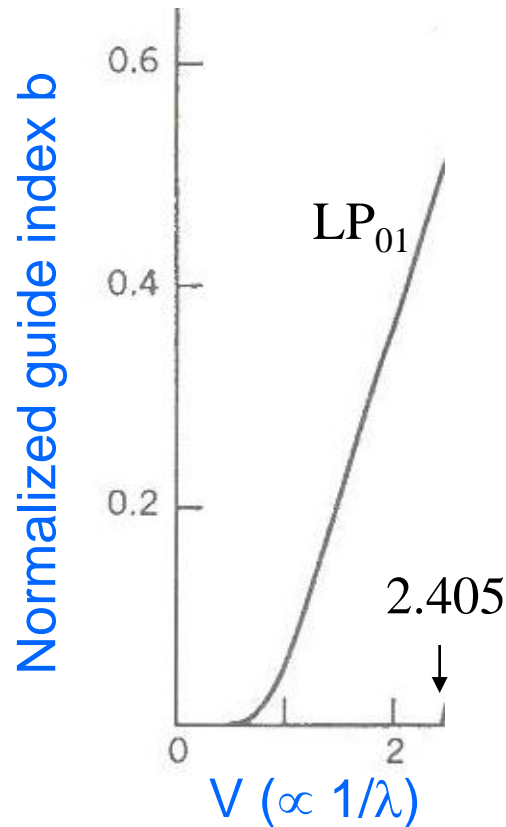
=>



broadened pulse !

Waveguide dispersion depends on the *mode field distribution in the core and the cladding*. (given by the fiber V number)

Waveguide dispersion of the LP_{01} mode



- Different wavelength components λ of the LP_{01} mode see different effective indices n_{eff}

Derivation of waveguide dispersion-induced pulse broadening

- Consider an optical pulse of linewidth $\Delta\lambda$ ($\Delta\omega$) and a corresponding spread of propagation constant $\Delta\beta$ propagating in a waveguide

Group velocity

$$v_{g,\text{eff}} = d\omega/d\beta \quad \text{—— Waveguide propagation constant}$$

$$\text{or} \quad v_{g,\text{eff}}^{-1} = d\beta/d\omega$$

$$= d/d\omega (c^{-1} \omega n_{\text{eff}}) \quad \text{Waveguide effective index}$$

$$= c^{-1} (n_{\text{eff}} + \omega dn_{\text{eff}}/d\omega)$$

$$= c^{-1} (n_{\text{eff}} - \lambda dn_{\text{eff}}/d\lambda) = c^{-1} n_{g,\text{eff}}$$

Time delay after a waveguide of length L : $\tau = L/v_{g,\text{eff}}$

Or *time delay per unit length*: $\tau/L = v_{g,\text{eff}}^{-1}$

- If $\Delta\lambda$ is the spectral width of an optical pulse, the extent of *pulse broadening* for a waveguide of length L is given by

$$\begin{aligned}\Delta\tau &= (d\tau/d\lambda) \Delta\lambda = [d(L/v_{g,\text{eff}})/d\lambda] \Delta\lambda \\ &= L [d(1/v_{g,\text{eff}})/d\lambda] \Delta\lambda \\ &= L D_{\text{wg}} \Delta\lambda\end{aligned}$$

- $D_{\text{wg}} = d(1/v_{g,\text{eff}})/d\lambda$ is called the *waveguide dispersion parameter* and is expressed in units of ps/(km-nm).

$$\begin{aligned}D_{\text{wg}} &= d(1/v_{g,\text{eff}})/d\lambda = c^{-1} dn_{g,\text{eff}}/d\lambda = c^{-1} d[n_{\text{eff}} - \lambda dn_{\text{eff}}/d\lambda]/d\lambda \\ &= -c^{-1} \lambda d^2n_{\text{eff}}/d\lambda^2\end{aligned}$$

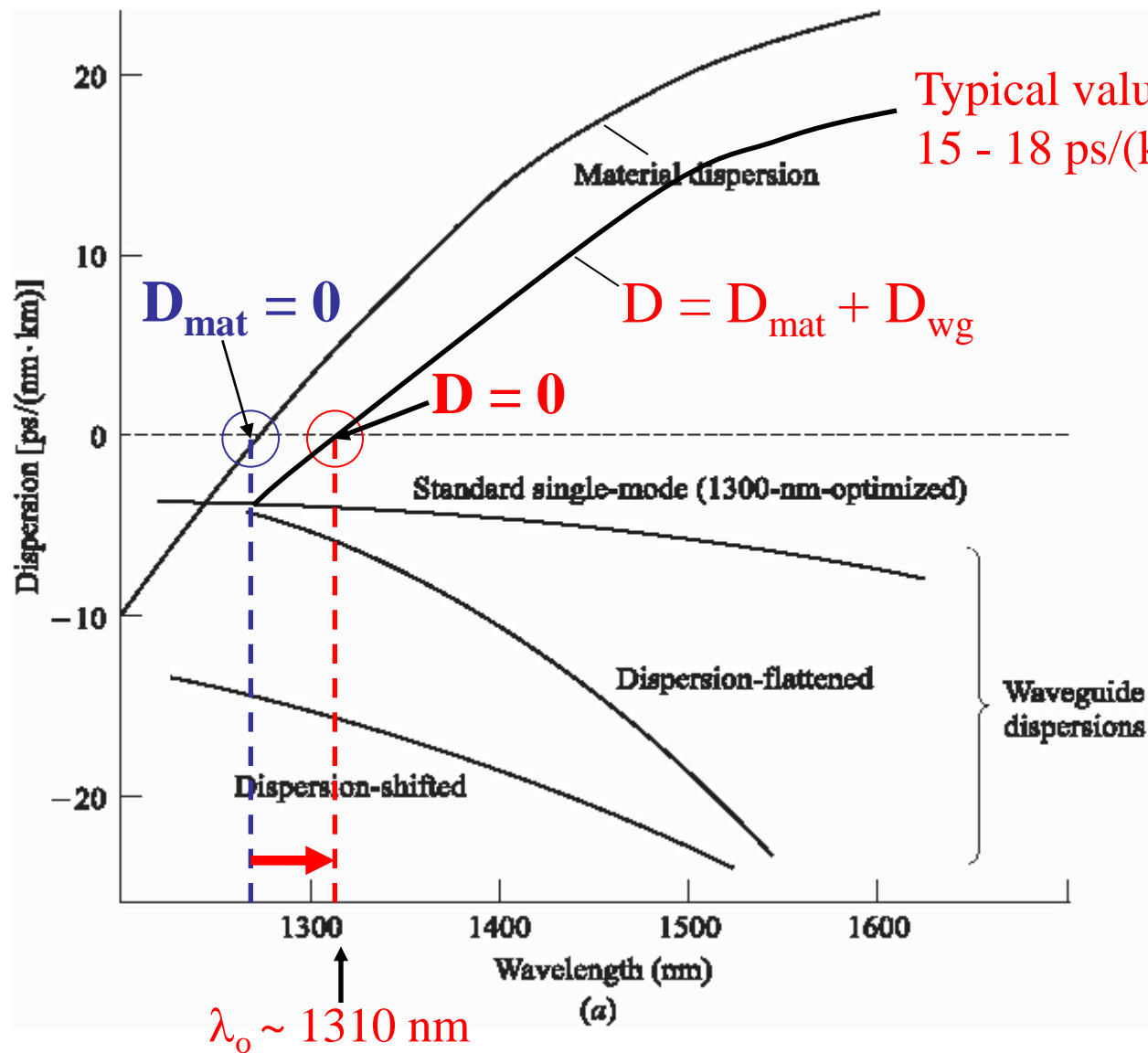
- Recall $v_{g,\text{eff}} = (d\beta/d\omega)^{-1}$ and note that the propagation constant β is a *nonlinear* function of the V number, $V = (2\pi a/\lambda) \text{NA} = a (\omega/c) \text{NA}$
- In the absence of material dispersion* (i.e. when NA is independent of ω), V is directly proportional to ω , so that

$$1/v_{g,\text{eff}} = d\beta/d\omega = (d\beta/dV) (dV/d\omega) = (d\beta/dV) (a \text{NA}/c)$$

- The pulse broadening associated with a source of spectral width $\Delta\lambda$ is related to the time delay $L/v_{g,\text{eff}}$ by $\Delta T = L |D_{\text{wg}}| \Delta\lambda$. The *waveguide dispersion parameter* D_{wg} is given by

$$D_{\text{wg}} = d/d\lambda (1/v_{g,\text{eff}}) = -(\omega/\lambda) d/d\omega (1/v_{g,\text{eff}}) = -(1/(2\pi c)) V^2 d^2\beta/dV^2$$

\Rightarrow The dependence of D_{wg} on λ may be controlled by altering the core radius, the NA, or the V number.



- $D_{\text{wg}}(\lambda)$ compensate some of the $D_{\text{mat}}(\lambda)$ and shifts the λ_{ZD} from about 1276 nm to a *longer* wavelength of about **1310 nm**.

Chromatic dispersion in low-bit-rate systems

Broadening of the light pulse due to Chromatic Dispersion:

$$\Delta T = D L \Delta \lambda$$

Consider the maximum pulse broadening equals to the bit time period $1/B$, then the *dispersion-limited distance*:

$$L_D = 1 / (D B \Delta \lambda)$$

e.g. For $D = 17 \text{ ps}/(\text{km} \cdot \text{nm})$, $B = \underline{2.5 \text{ Gb/s}}$ and $\Delta \lambda = 0.03 \text{ nm}$

$$\Rightarrow L_D = 784 \text{ km}$$

(It is known that dispersion limits a 2.5 Gbit/s channel to roughly 900 km! Therefore, chromatic dispersion is *not* much of an issue in low-bit-rate systems deployed in the early 90s!)

- **When upgrading** from *2.5- to 10-Gbit/s* systems, most technical challenges are less than four times as complicated and the cost of components is usually much less than four times as expensive.
- However, *when increasing the bit rate by a factor of 4, the effect of chromatic dispersion increases by a factor of 16!*

Consider again the dispersion-limited distance:

$$L_D = 1 / (D B \Delta\lambda)$$

Note that spectral width $\Delta\lambda$ is proportional to the modulation of the lightwave!

i.e. Faster the modulation, more the frequency content, and therefore wider the spectral bandwidth $\Rightarrow \Delta\lambda \propto B$

$$\Rightarrow L_D \propto 1 / B^2$$

Chromatic dispersion in high-bit-rate systems

e.g. In standard single-mode fibers for which $D = 17 \text{ ps}/(\text{nm}\cdot\text{km})$ at a signal wavelength of 1550 nm (assuming from the same light source as the earlier example of 2.5 Gbit/s systems), the maximum transmission distance before significant pulse broadening occurs for 10 Gbit/s data is:

$$L_D \sim 784 \text{ km} / 16 \sim 50 \text{ km!}$$

(A more exact calculation shows that 10-Gbit/s (40-Gbit/s) would be limited to approximately 60 km (4 km!).)

*This is why **chromatic dispersion compensation** must be employed for systems operating at 10 Gbit/s (now at 40 Gbit/s and beyond.)*

Zero-dispersion slope

If $D(\lambda)$ is *zero* at a specific $\lambda = \lambda_{\text{ZD}}$, can we eliminate pulse broadening caused by chromatic dispersion?

There are higher-order effects! The derivative

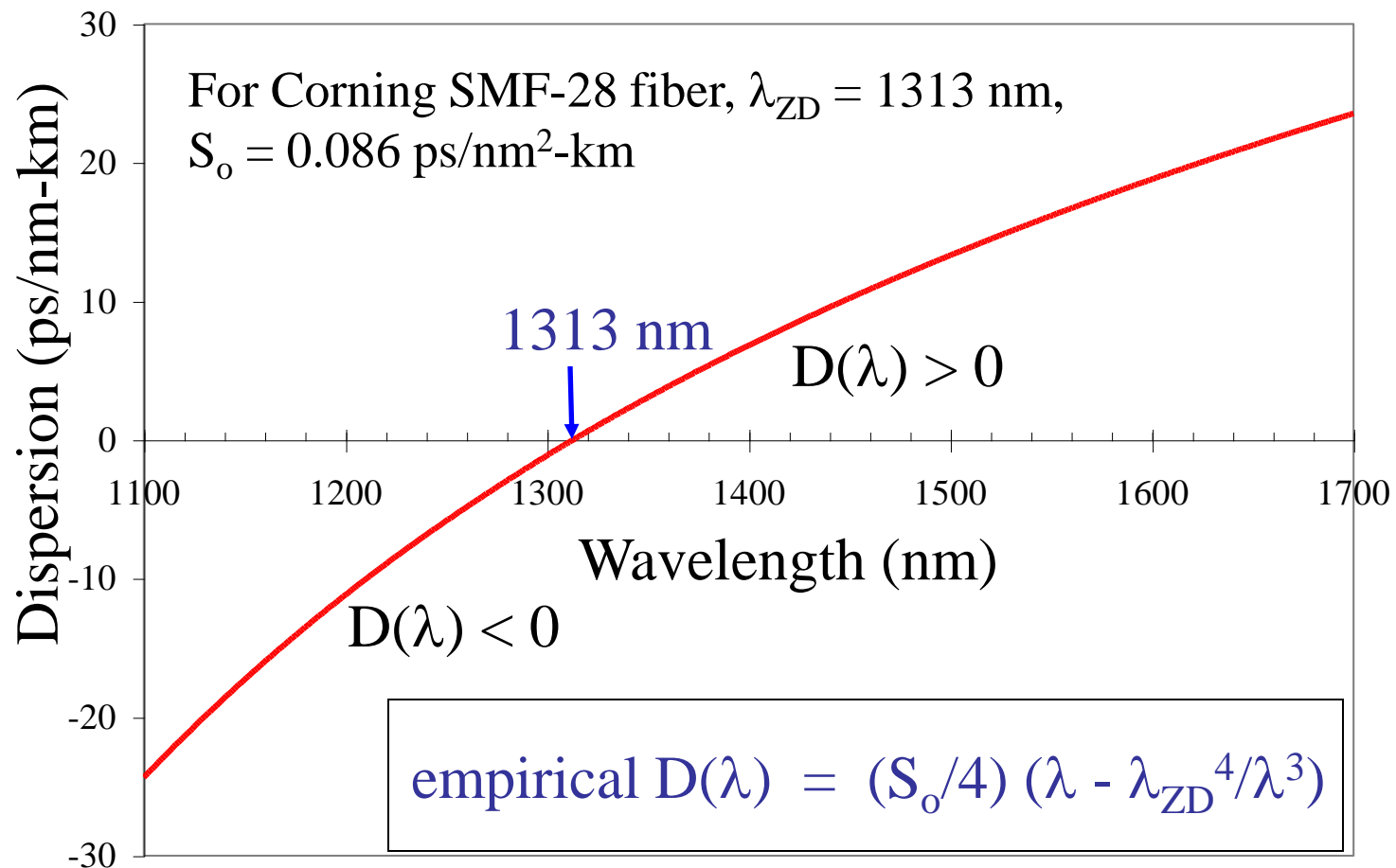
$$dD(\lambda)/d\lambda = S_0$$

needs to be accounted for when the first-order effect is zero (i.e. $D(\lambda_{\text{ZD}}) = 0$).

S_0 is known as the zero-dispersion slope measured in **ps/(km-nm²)**.

The chromatic pulse broadening near λ_{ZD} :

$$\Delta T = L S_o |\lambda - \lambda_{\text{ZD}}| \Delta\lambda$$



- Now it becomes clear that at $\lambda = \lambda_{\text{ZD}}$, the dispersion slope S_o becomes the bit rate limiting factor.

We can estimate the limiting bit rate by noting that for a source of spectral width $\Delta\lambda$, the effective value of dispersion parameter becomes

$$D = S_o \Delta\lambda$$

=>

The limiting *bit rate-distance product* can be given as

$$BL |S_o| (\Delta\lambda)^2 < 1 \quad (B \Delta T < 1)$$

*For a *multimode* semiconductor laser with $\Delta\lambda = 2$ nm and a *dispersion-shifted fiber* with $S_o = 0.05$ ps/(km-nm²) at $\lambda = 1.55$ μm , the BL product approaches **5 (Tb/s)-km**. Further improvement is possible by using *single-mode* semiconductor lasers.

Dispersion comparison for a non-dispersion-shifted fiber

Example 3.14 A manufacturer's data sheet states that a non-dispersion-shifted fiber has a zero-dispersion wavelength of 1310 nm and a dispersion slope of 0.092 ps/(nm² · km). Compare the dispersions for this fiber at wavelengths of 1280 nm and 1550 nm.

Solution: Using Eq. (3.47) we find that

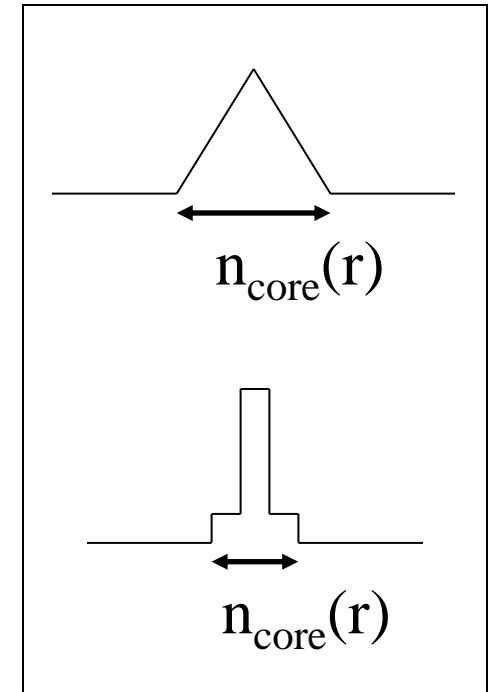
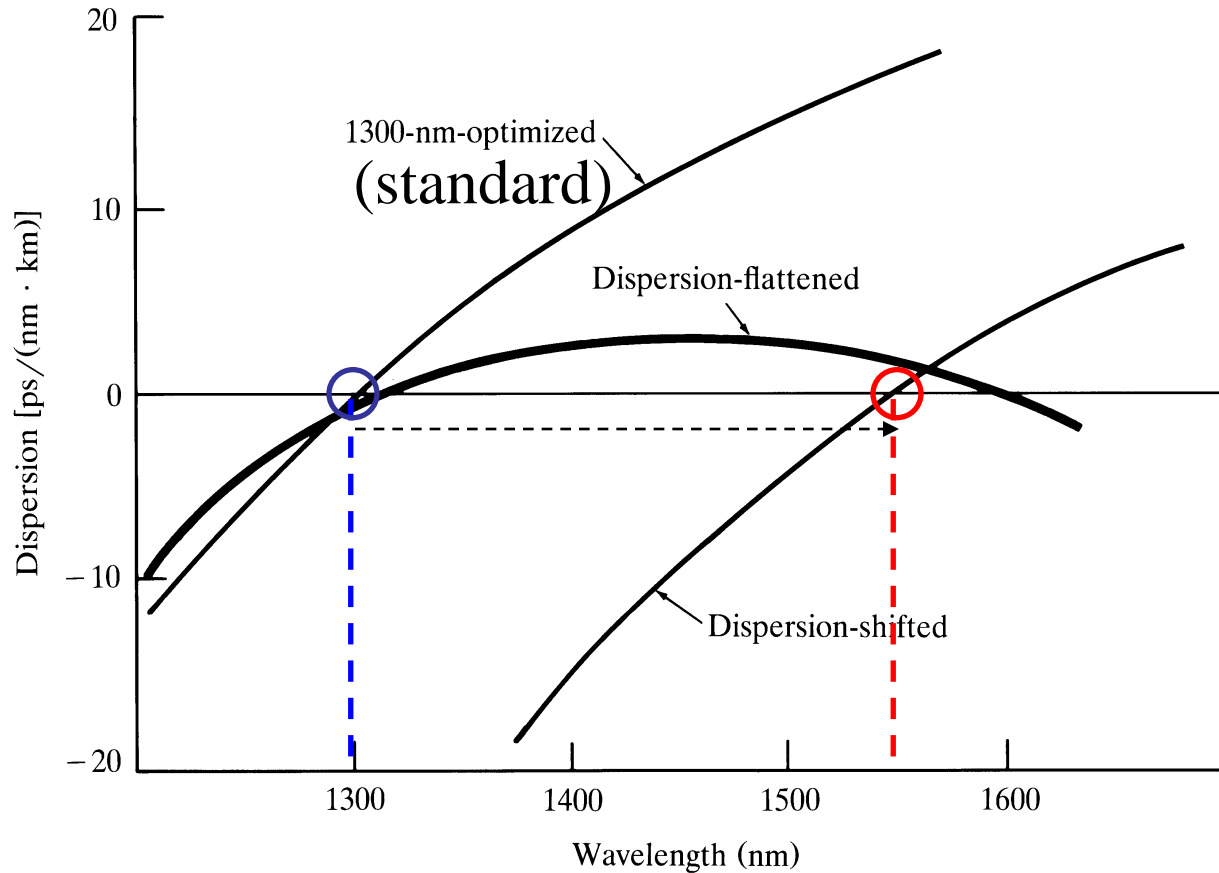
$$\begin{aligned} D(1280) &= \frac{\lambda S_0}{4} \left[1 - \left(\frac{\lambda_0}{\lambda} \right)^4 \right] \\ &= \frac{(1280)(0.092)}{4} \left[1 - \left(\frac{1310}{1280} \right)^4 \right] \\ &= -2.86 \text{ ps/(nm} \cdot \text{km)} \end{aligned}$$

$$\begin{aligned} D(1550) &= \frac{\lambda S_0}{4} \left[1 - \left(\frac{\lambda_0}{\lambda} \right)^4 \right] \\ &= \frac{(1550)(0.092)}{4} \left[1 - \left(\frac{1310}{1550} \right)^4 \right] \\ &= 17.5 \text{ ps/(nm} \cdot \text{km)} \end{aligned}$$

Dispersion tailored fibers

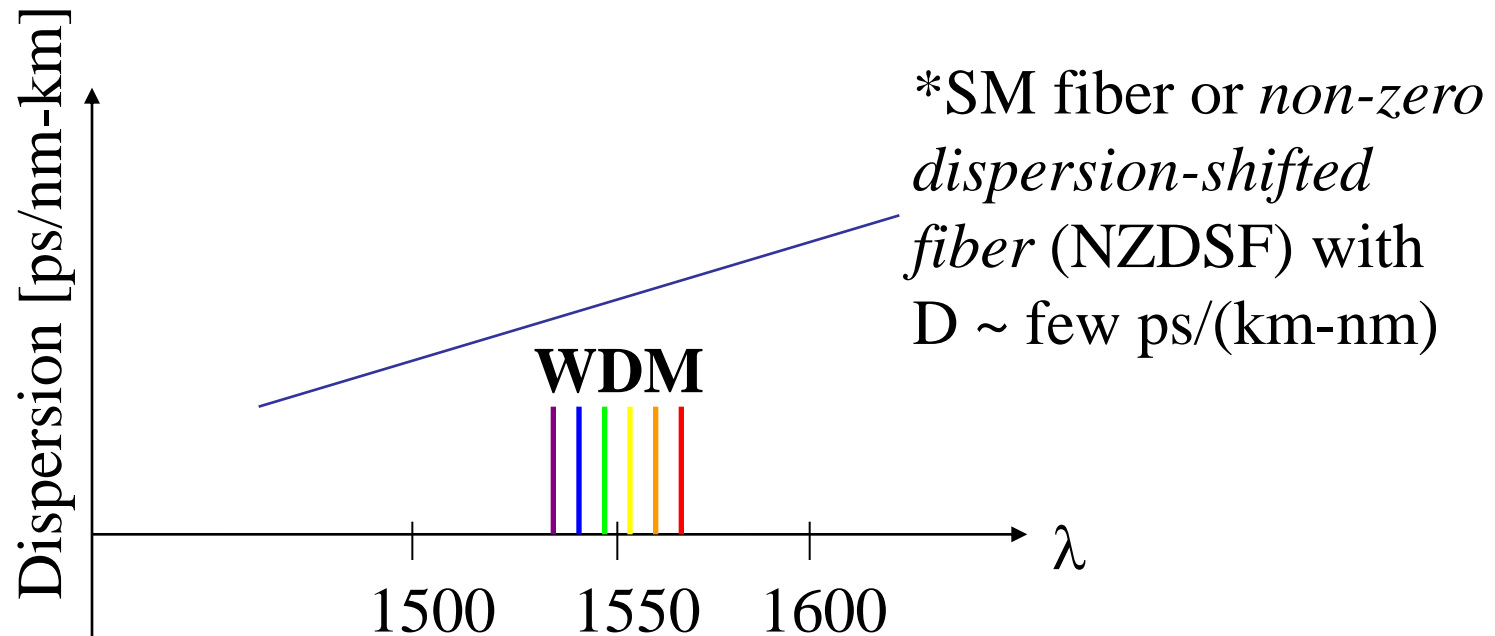
1. As the waveguide contribution D_{wg} depends on the fiber parameters such as the core radius a and the index difference Δ , it is *possible to design the fiber such that λ_{ZD} is shifted* into the neighborhood of $1.55 \mu\text{m}$. Such fibers are called *dispersion-shifted fibers*.
2. It is also possible to tailor the waveguide contribution such that the *total dispersion D is relatively small over a wide wavelength range* extending from 1.3 to $1.6 \mu\text{m}$. Such fibers are called *dispersion-flattened fibers*.

Dispersion-shifted and flattened fibers



- The design of dispersion-modified fibers often involves the use of multiple cladding layers and a tailoring of the refractive index profile.

- Since *dispersion slope* $S > 0$ for singlemode fibers \Rightarrow different *wavelength-division multiplexed (WDM)* channels have different *dispersion values*.



*In fact, for WDM systems, *small amount of chromatic dispersion must be present* to prevent the impairment of **fiber nonlinearity** (i.e. power-dependent interaction between wavelength channels.)

Chromatic Dispersion Compensation

- Chromatic dispersion is ***time independent*** in a *passive* optical link
⇒ allow compensation along the entire fiber span
(**Note** that recent developments focus on *reconfigurable* optical link, which makes chromatic dispersion *time dependent*!)

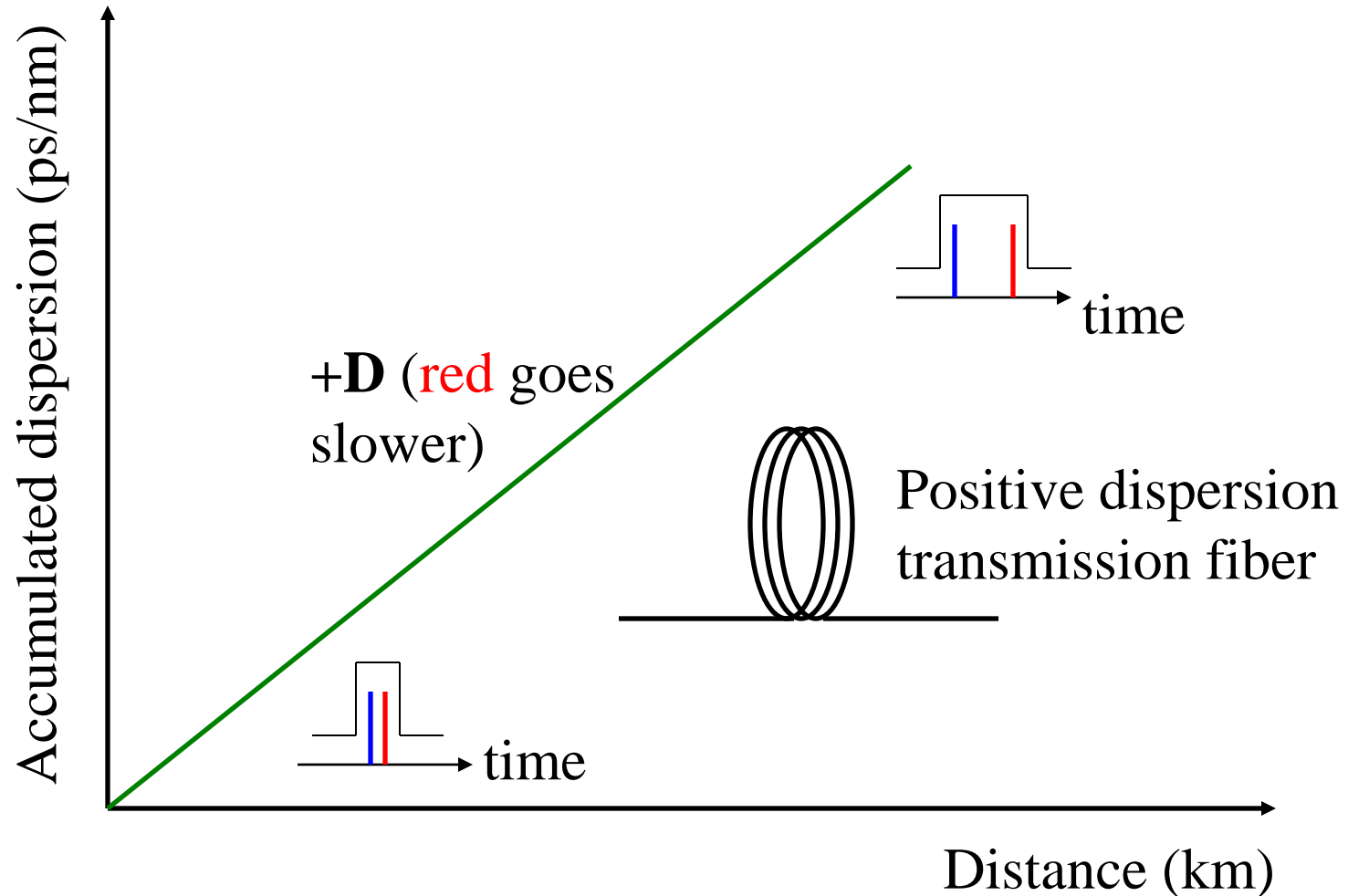
Two basic techniques: (1) **dispersion-compensating fiber DCF**

(2) dispersion-compensating **fiber grating**

- **The basic idea for DCF:** the *positive dispersion* in a conventional fiber (say ~ 17 ps/(km-nm) in the 1550 nm window) can be compensated for by inserting a ***fiber with negative dispersion*** (*i.e.* with large -ve D_{wg}).

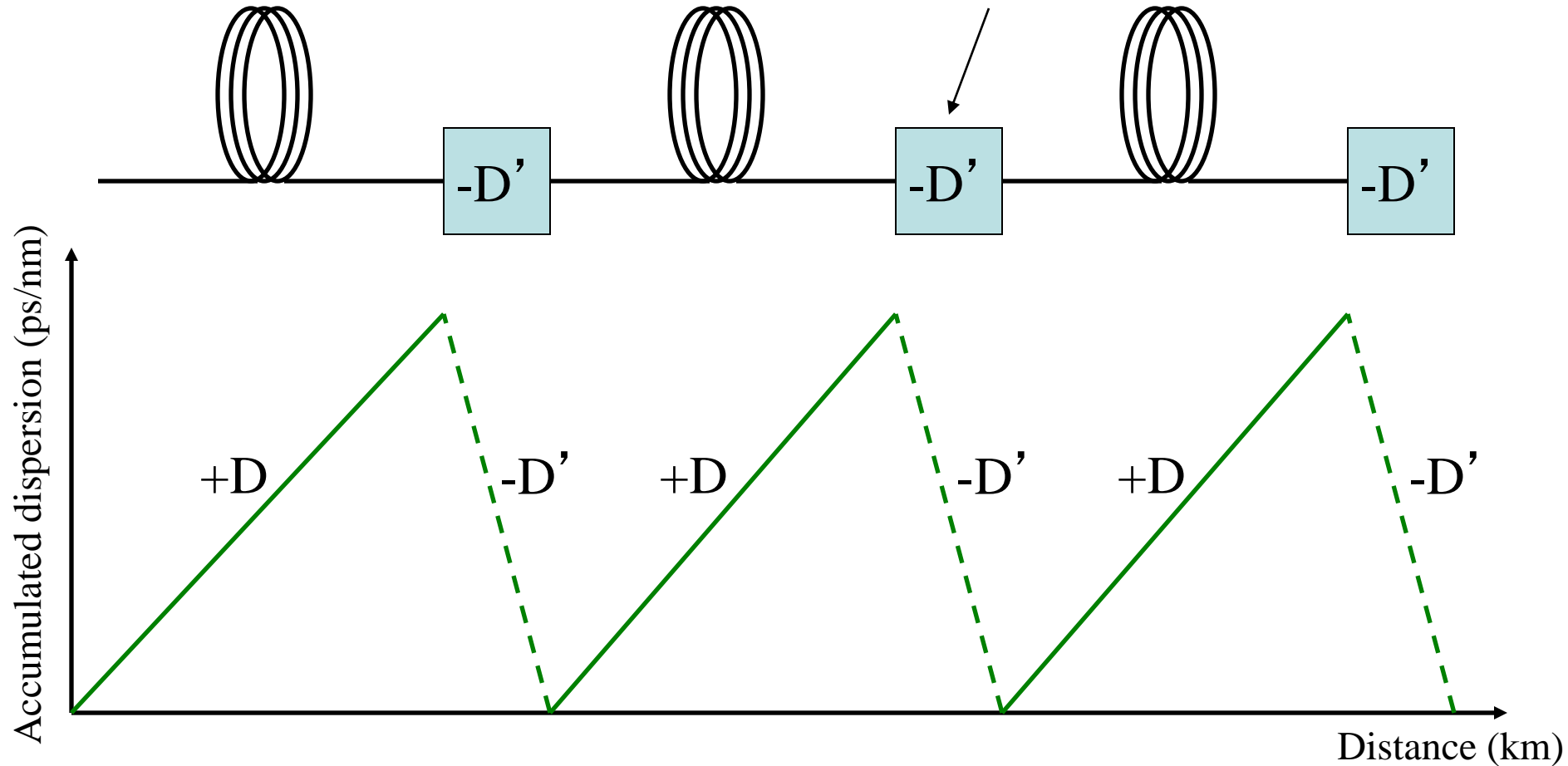
Chromatic dispersion accumulates *linearly* over distance

(recall $\Delta T = D L \Delta \lambda$)



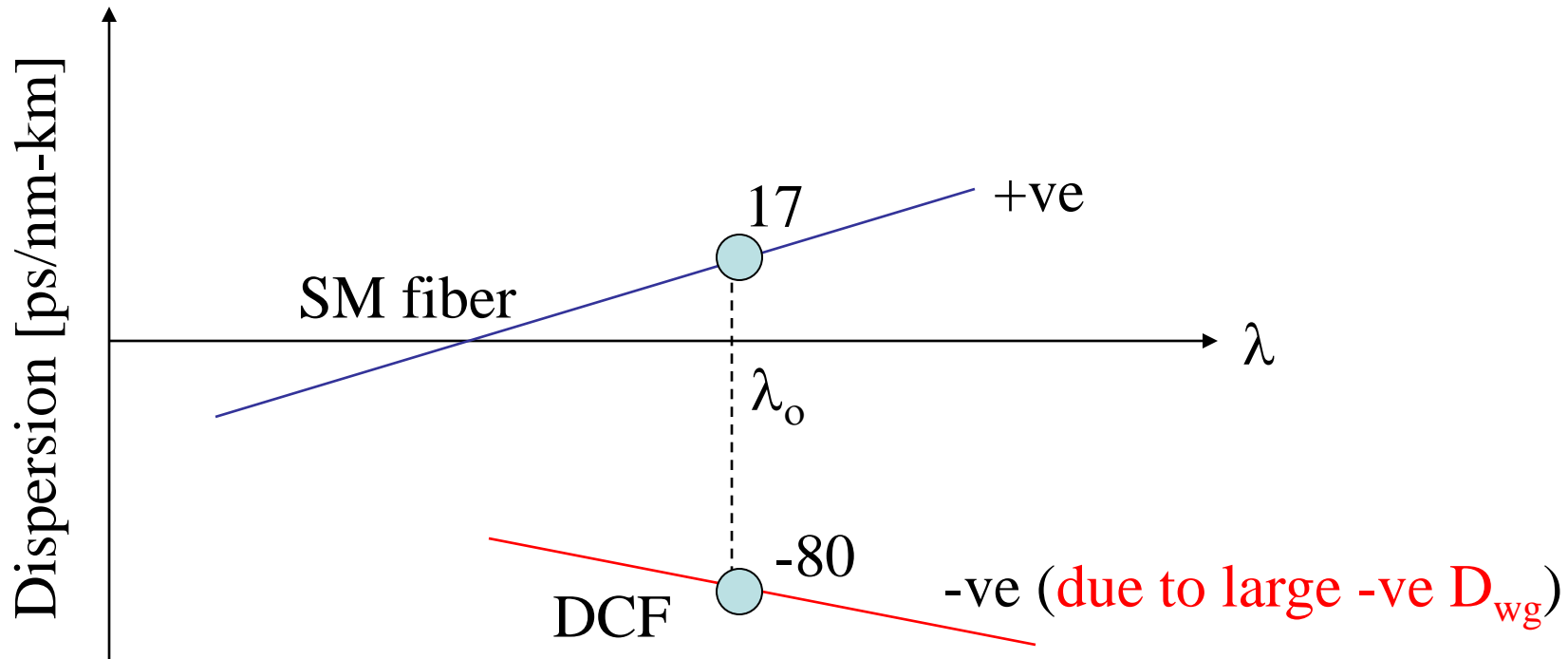
Positive dispersion
transmission fiber

Negative dispersion element



- In a *dispersion-managed* system, positive dispersion transmission fiber *alternates with negative dispersion compensation elements*, such that the total dispersion is zero end-to-end.

Fixed (passive) dispersion compensation



SM	DCF	SM	DCF	SM	DCF
+ve	-ve	+ve	-ve	+ve	-ve

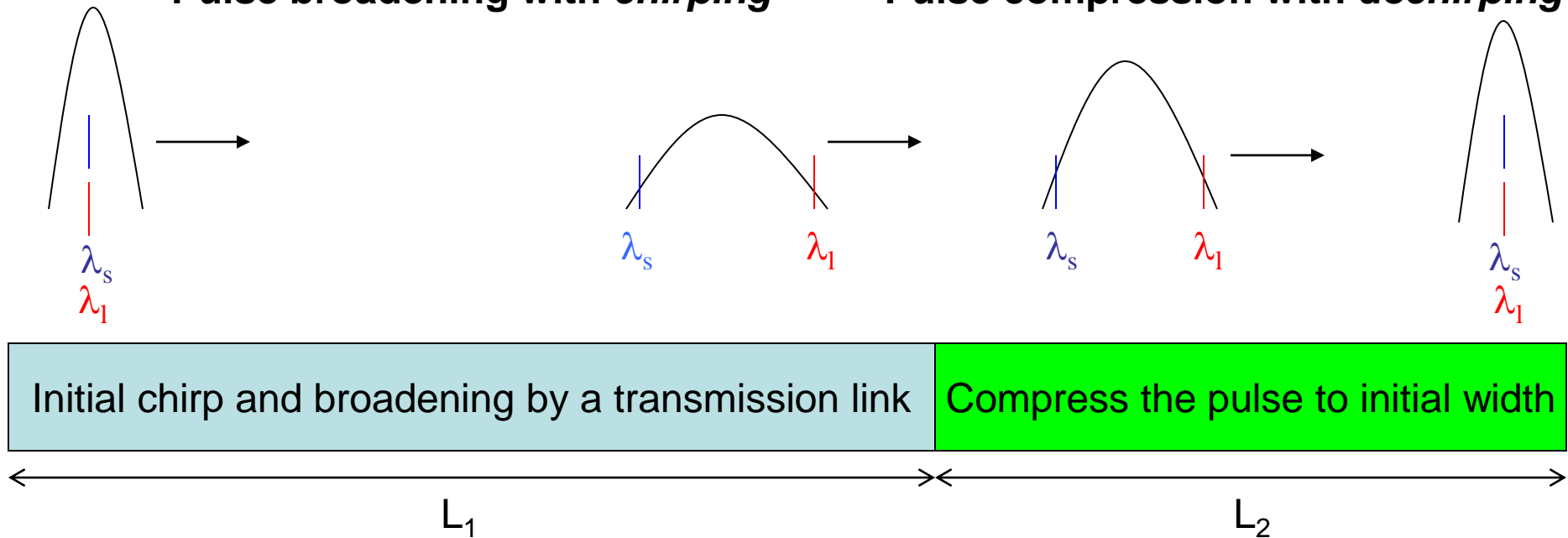
*DCF is a length of fiber producing -ve dispersion *four to five times* as large as that produced by conventional SMF.

Dispersion-Compensating Fiber

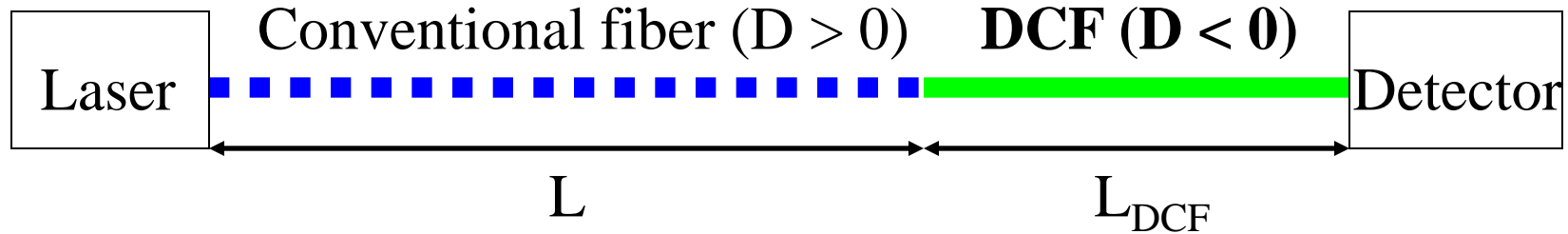
The concept: using a span of fiber to *compress* an initially chirped pulse.

Pulse broadening with *chirping*

Pulse compression with *dechirping*



Dispersion compensated channel: $D_2 L_2 = - D_1 L_1$



e.g. What DCF is needed in order to compensate for dispersion in a conventional single-mode fiber link of 100 km?

Suppose we are using Corning SMF-28 fiber,

=> the dispersion parameter $D(1550 \text{ nm}) \sim 17 \text{ ps}/(\text{km}\cdot\text{nm})$.

=> Pulse broadening $\Delta T_{\text{chrom}} = D(\lambda) \Delta\lambda L \sim 17 \times 1 \times 100 = 1700 \text{ ps}$.

assume the semiconductor (diode) laser linewidth $\Delta\lambda \sim 1 \text{ nm}$.

⇒ The DCF needed to compensate for 1700 ps with a large *negative-dispersion* parameter

i.e. we need $\Delta T_{\text{chrom}} + \Delta T_{\text{DCF}} = 0$

$$\Rightarrow \Delta T_{\text{DCF}} = D_{\text{DCF}}(\lambda) \Delta \lambda L_{\text{DCF}}$$

suppose typical ratio of $L/L_{\text{DCF}} \sim 6 - 7$, we assume $L_{\text{DCF}} = 15 \text{ km}$

$$\Rightarrow D_{\text{DCF}}(\lambda) \sim -113 \text{ ps}/(\text{km-nm})$$

*Typically, **only one wavelength can be compensated exactly.**
Better CD compensation requires **both *dispersion* and *dispersion slope* compensation.**

Compensating the dispersion slope produces the additional requirement:

$$L_2 \, dD_2/d\lambda = - L_1 \, dD_1/d\lambda$$

⇒ The compensating fiber must have a **negative** dispersion slope, and that the *dispersion* and *slope* values need to be compensated for a given length.

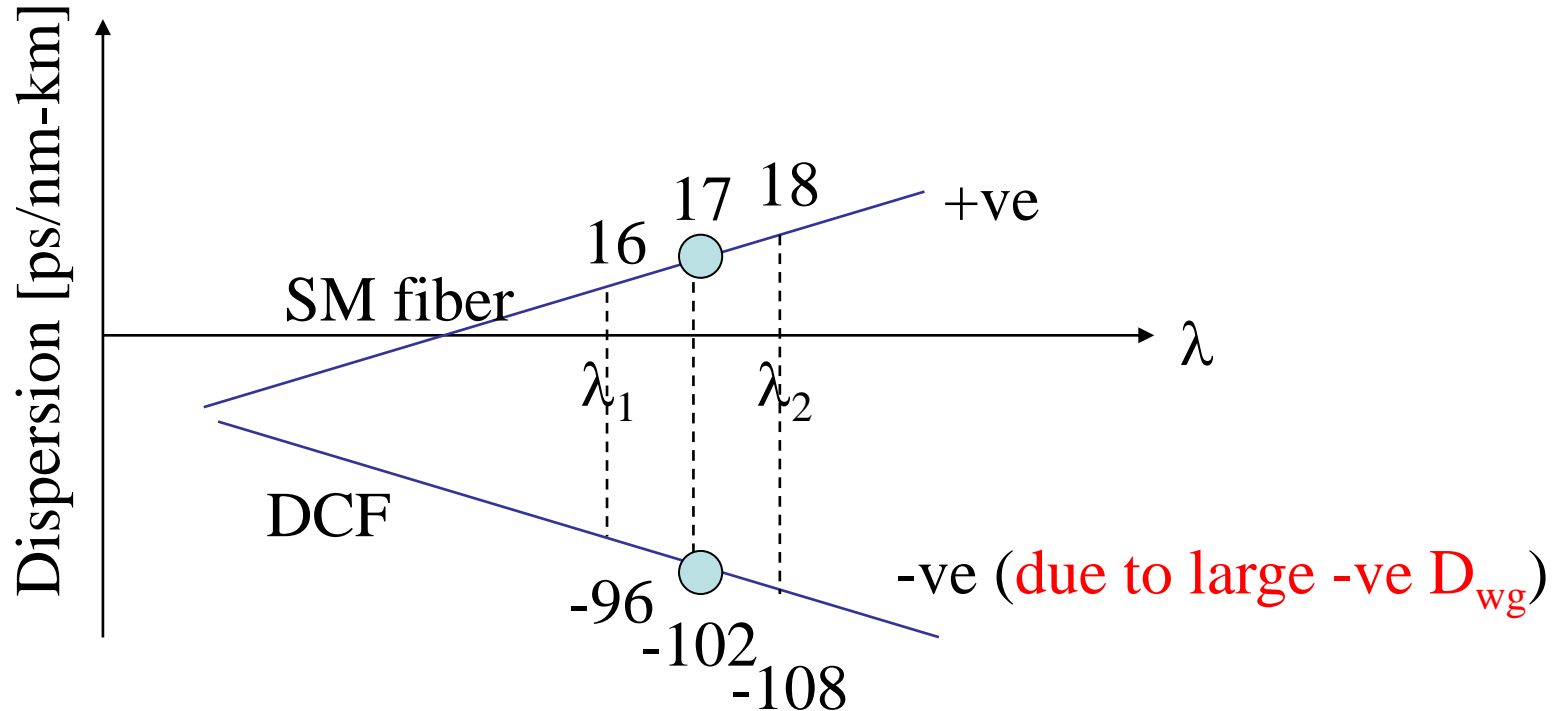
$$D_2 \, L_2 = - D_1 \, L_1$$

$$L_2 \, dD_2/d\lambda = - L_1 \, dD_1/d\lambda$$

⇒ **Dispersion and slope compensation:** $D_2 / (dD_2/d\lambda) = D_1 / (dD_1/d\lambda)$

(In practice, two fibers are used, one of which has negative slope, in which the pulse wavelength is at zero-dispersion wavelength λ_{zD} .)

Dispersion slope compensation



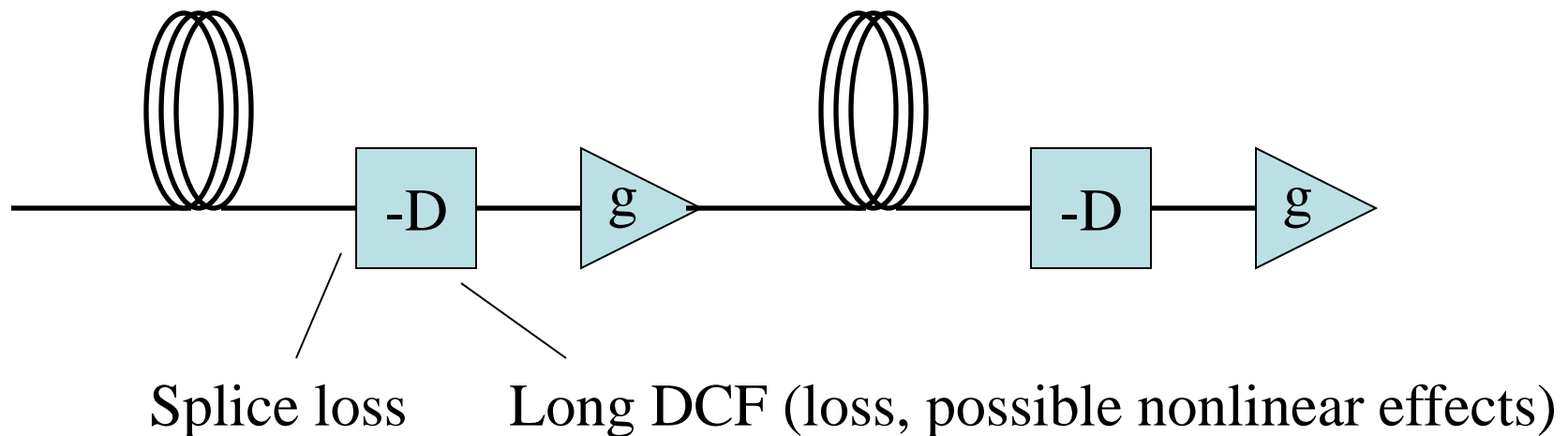
Within the spectral window (λ_1, λ_2) , $D_{DCF}/D_{SM} = -6$

$$S_{DCF} = -12/(\lambda_2 - \lambda_1); S_{SM} = 2/(\lambda_2 - \lambda_1) \Rightarrow S_{DCF}/S_{SM} = -6$$

$$\Rightarrow \text{Dispersion slope compensation: } (D_{SM}/S_{SM}) / (D_{DCF}/S_{DCF}) = 1$$

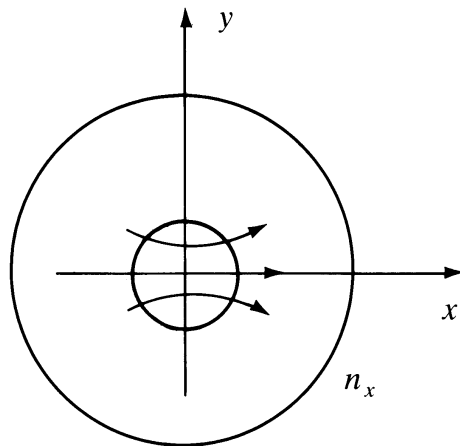
Disadvantages in using DCF

- *Added loss* associated with the increased fiber span
- *Nonlinear effects* may degrade the signal over the long length of the fiber if the signal is of sufficient intensity.
- Links that use DCF often require an *amplifier* stage to compensate the added loss.

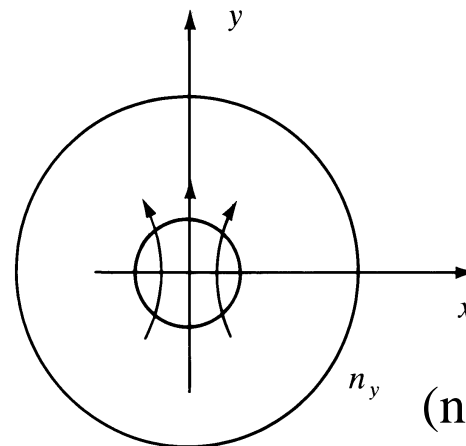


3. Polarization Mode Dispersion (PMD)

- In a single-mode optical fiber, the optical signal is carried by the *linearly polarized* “fundamental mode” LP_{01} , which has *two polarization components that are orthogonal*.



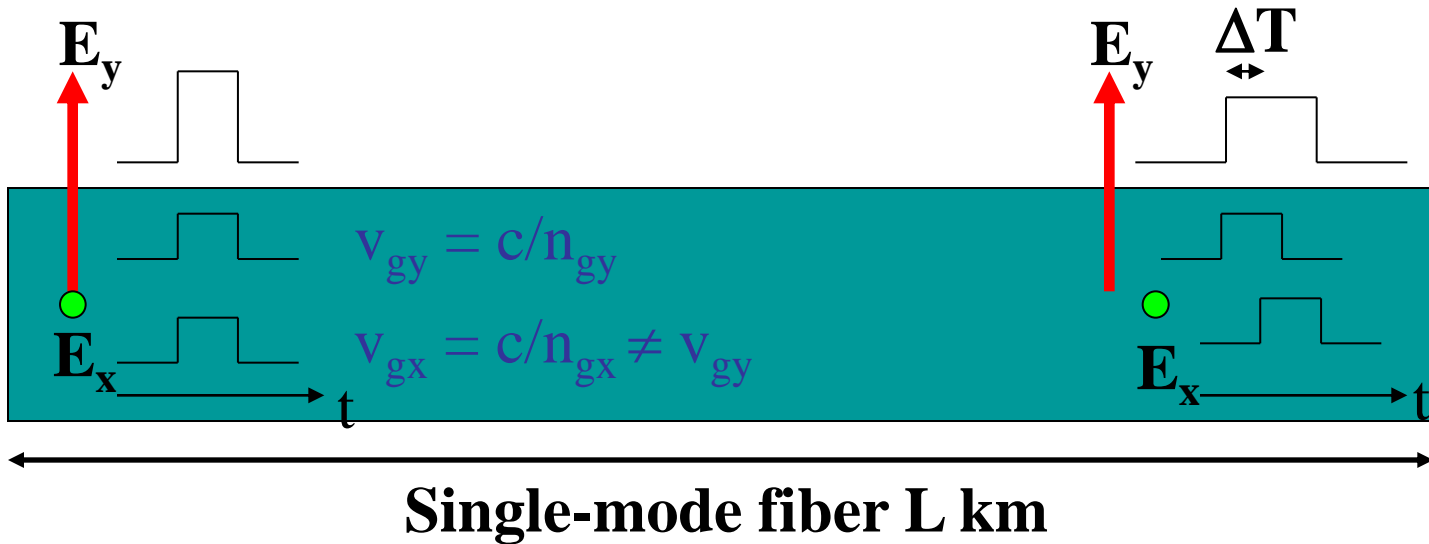
Horizontal mode



Vertical mode

(note that x and y are chosen arbitrarily)

- In a **real fiber** (i.e. $n_{gx} \neq n_{gy}$), the two orthogonal polarization modes propagate at **different group velocities**, resulting in **pulse broadening** – **polarization mode dispersion**.



*1. **Pulse broadening** due to the orthogonal polarization modes
 (The time delay between the two polarization components is characterized as the **differential group delay (DGD)**.)

2. **Polarization varies** along the fiber length

- The *refractive index difference* is known as *birefringence*.

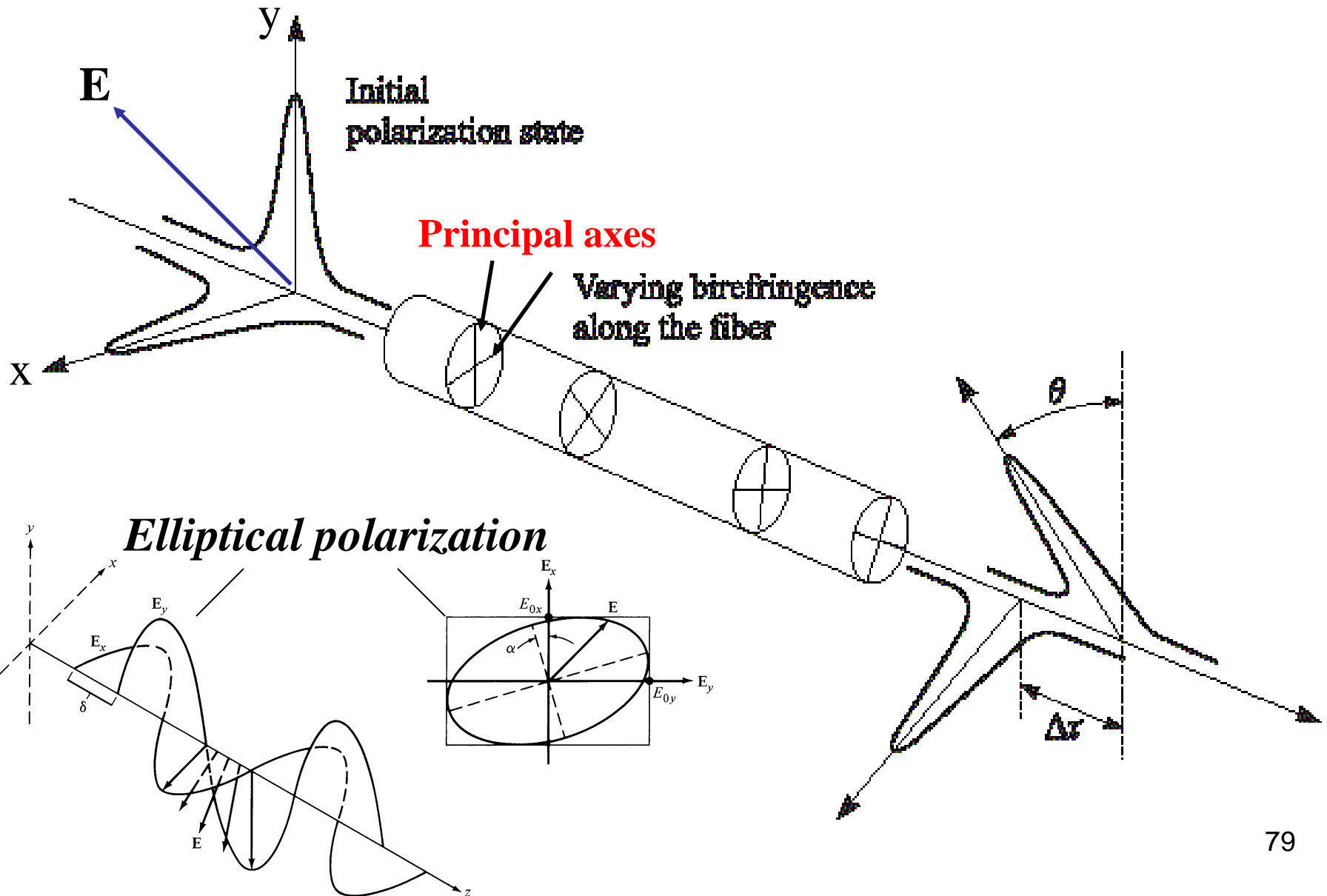
$$B = n_x - n_y \quad (\sim 10^{-6} - 10^{-5} \text{ for single-mode fibers})$$

assuming $n_x > n_y \Rightarrow$ y is the *fast axis*, x is the *slow axis*.

*B varies *randomly* because of *thermal and mechanical stresses over time* (due to *randomly varying* environmental factors in submarine, terrestrial, aerial, and buried fiber cables).

\Rightarrow *PMD is a statistical process !*

Randomly varying birefringence along the fiber



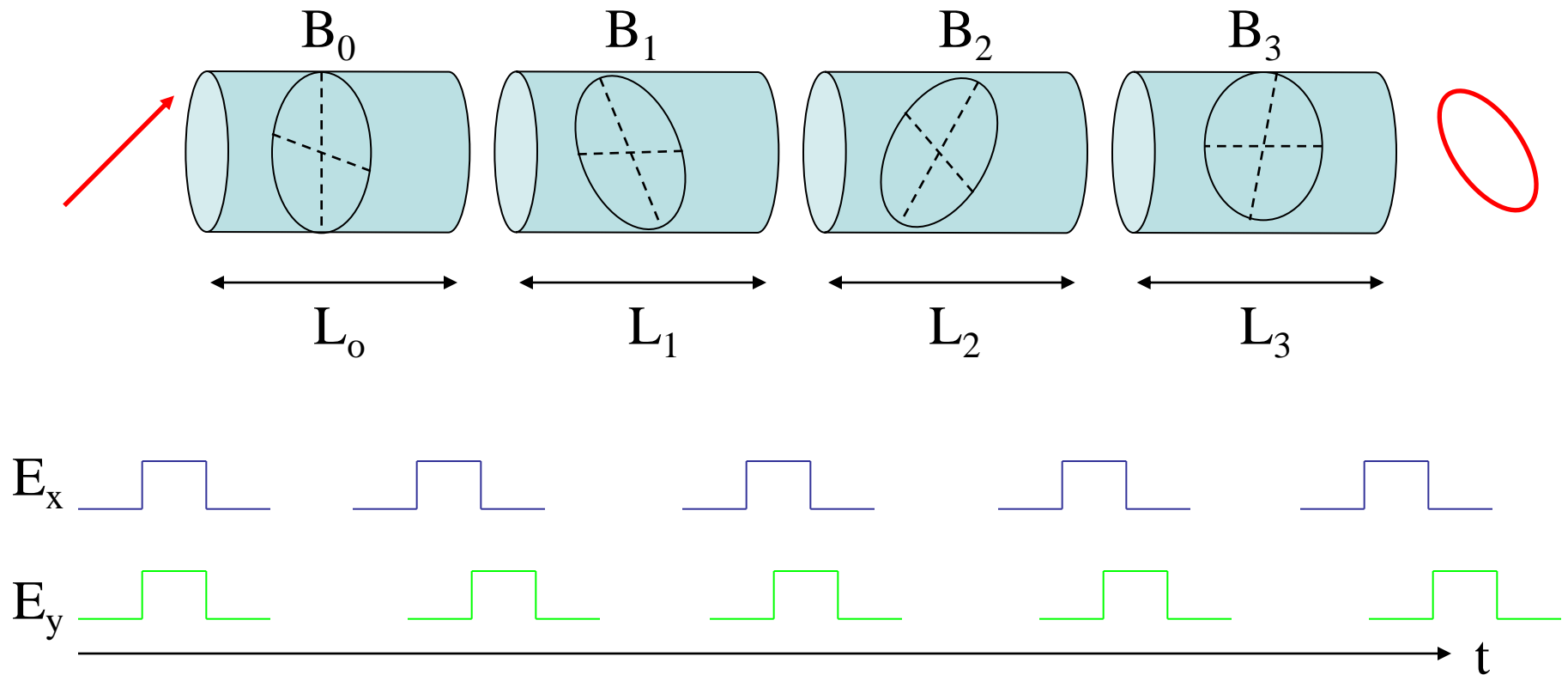
- The polarization state of light propagating in fibers with *randomly varying birefringence* will generally be *elliptical and would quickly reach a state of arbitrary polarization*.

*However, the final polarization state is *not* of concern for most lightwave systems as *photodetectors are insensitive to the state of polarization*.

(**Note:** recent technology developments in “*Coherent Optical Communications*” do require polarization state to be analyzed.)

- A simple model of PMD divides the fiber into a large number of segments. Both the *magnitude of birefringence B* and the *orientation of the principal axes* remain constant in each section but *changes randomly from section to section*.

A simple model of PMD



Randomly changing differential group delay (DGD)

- Pulse broadening caused by a *random* change of fiber polarization properties is known as polarization mode dispersion (PMD).

$$\text{PMD pulse broadening} \quad \Delta T_{\text{PMD}} = D_{\text{PMD}} \sqrt{L}$$

D_{PMD} is the PMD parameter (coefficient) measured in **ps/ $\sqrt{\text{km}}$** .

\sqrt{L} models the “random” nature (like “random walk”)

* D_{PMD} **does not depend on wavelength** (first order) ;

*Today’ s fiber (since 90’ s) PMD parameter is 0.1 - 0.5 ps/ $\sqrt{\text{km}}$.

(Legacy fibers deployed in the 80 ’s have $D_{\text{PMD}} > 0.8$ ps/ $\sqrt{\text{km}}$.)

e.g. Calculate the pulse broadening caused by PMD for a singlemode fiber with a PMD parameter $D_{\text{PMD}} \sim 0.5 \text{ ps}/\sqrt{\text{km}}$ and a fiber length of 100 km. (i.e. $\Delta T_{\text{PMD}} = 5 \text{ ps}$)

Recall that pulse broadening due to chromatic dispersion for a 1 nm linewidth light source was $\sim 15 \text{ ps/km}$, which resulted in 1500 ps for 100 km of fiber length.

\Rightarrow **PMD** pulse broadening is *orders of magnitude less* than chromatic dispersion !

*PMD is relatively small compared with chromatic dispersion. But when one operates at **zero-dispersion** wavelength (or *dispersion compensated wavelengths*) with narrow spectral width, **PMD** can become a significant component of the total dispersion.

So why do we care about PMD?

Recall that chromatic dispersion can be compensated to ~ 0 ,
(at least for single wavelengths, namely, by designing proper
-ve waveguide dispersion)

but there is no simple way to eliminate PMD completely.

=> It is PMD that limits the fiber bandwidth after chromatic dispersion is compensated!

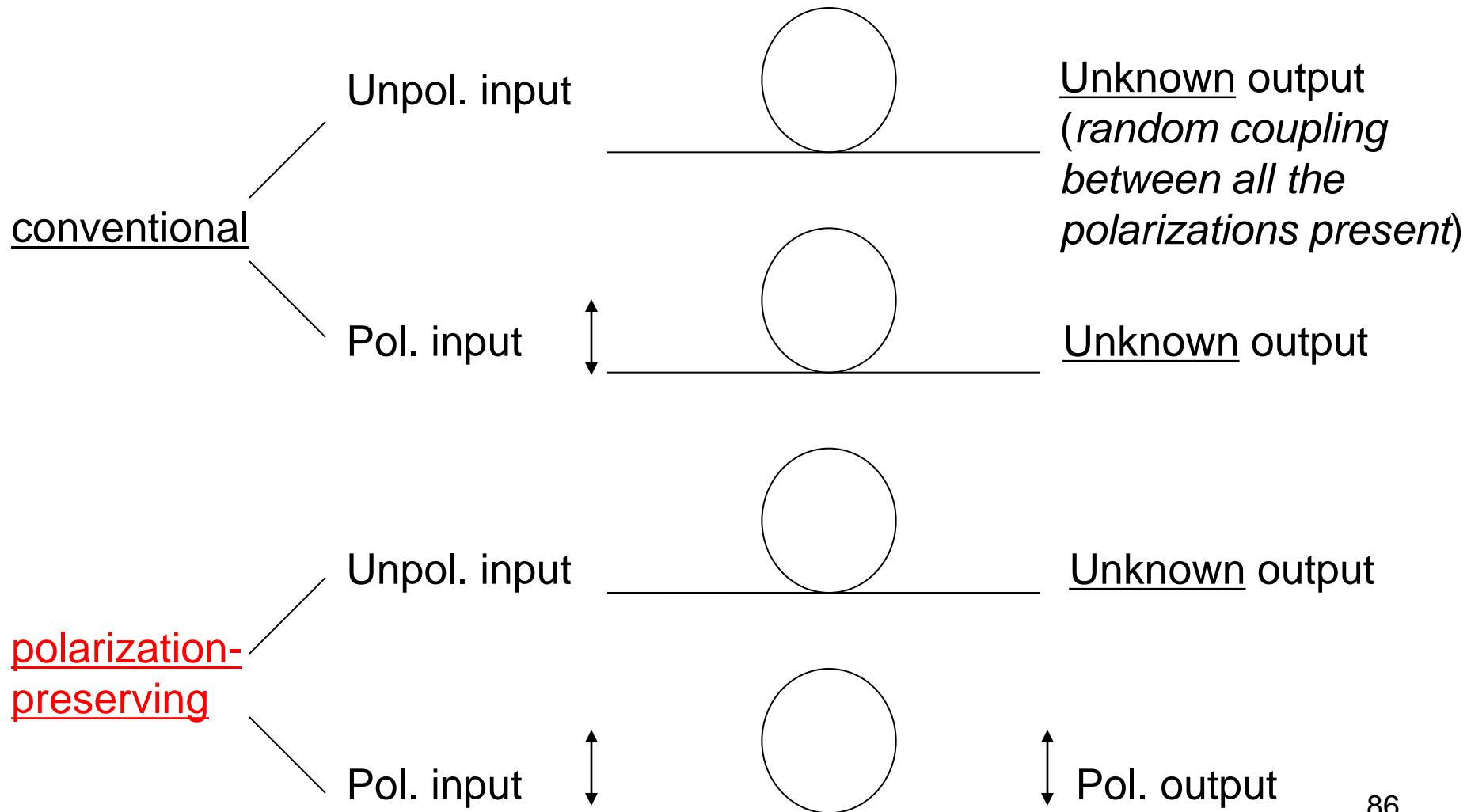
- PMD is of lesser concern in lower data rate systems. At lower transmission speeds (*up to and including 10 Gb/s*), networks have higher tolerances to all types of dispersion, including PMD.

As data rate increases, the dispersion tolerance reduces significantly, creating a need to control PMD as much as possible at the current **40 Gb/s** system.

e.g. The pulse broadening caused by PMD for a singlemode fiber with a PMD parameter of $0.5 \text{ ps}/\sqrt{\text{km}}$ and a fiber length of 100 km \Rightarrow **5 ps**.

However, this is comparable to **the 40G bit period = 25 ps !**

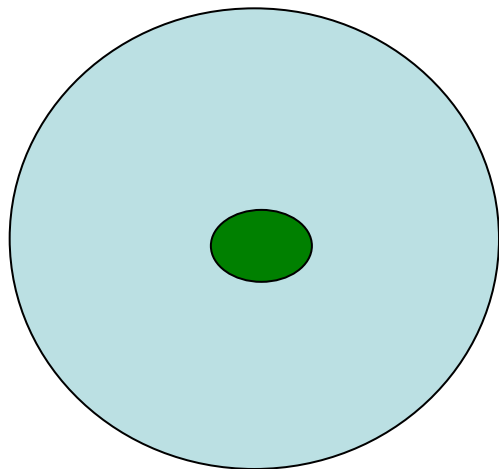
Polarizing effects of conventional / polarization-preserving fibers



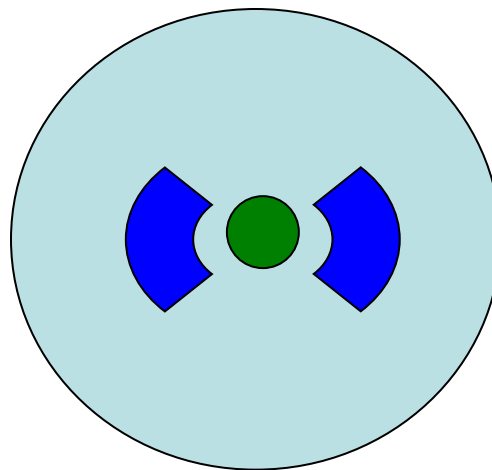
Polarization-preserving fibers

- The fiber birefringence is enhanced in single-mode *polarization-preserving* (*polarization-maintaining*) fibers, which are designed to maintain the polarization of the launched wave.
- *Polarization is preserved* because the two possible waves have significantly different propagation characteristics. This keeps them from exchanging energy as they propagate through the fiber.
- Polarization-preserving fibers are constructed by designing *asymmetries* into the fiber. Examples include fibers with elliptical cores (which cause waves polarized along the *major* and *minor* axes of the ellipse to have different effective refractive indices) and fibers that contain nonsymmetrical stress-producing parts.

Polarization-preserving fibers



Elliptical-core fiber



bow-tie fiber

- The shaded region in the bow-tie fiber is highly doped with a material such as boron. Because the thermal expansion of this doped region is so different from that of the pure silica cladding, a *nonsymmetrical stress* is exerted on the core. This produces a large *stress-induced birefringence*, which in turn *decouples the two orthogonal modes of the singlemode fiber*.

Multimode Fiber Transmission Distances

- The possible transmission distances when using fibers with different core sizes and bandwidths for Ethernet, Fibre Channel, and SONET/SDH applications.

Table 3.3 *Transmission distances in meters in multimode fibers using an 850-nm VCSEL*

Application	Data rate (Gb/s)	50- μm core		62.5- μm core	
		500 MHz.km	2000 MHz.km	160 MHz.km	200 MHz.km
Ethernet	1	550	860	220	275
	10	82	300	26	33
Fibre Channel	1	500	860	250	300
	2	300	500	120	150
	10	82	300	26	33
SONET/SDH	10	85	300	25	33

Examples of Specialty Fibers

Table 3.4 *Examples of specialty fibers and their applications*

<i>Specialty fiber type</i>	<i>Application</i>
Erbium-doped fiber	Gain medium for optical fiber amplifiers
Photosensitive fibers	Fabrication of fiber Bragg gratings
Bend-insensitive fibers	Tightly looped connections in device packages
Termination fiber	Termination of open optical fiber ends
Polarization-preserving fibers	Pump lasers, polarization-sensitive devices, sensors
High-index fibers	Fused couplers, short- λ sources, DWDM devices
Photonic crystal fibers	Switches; dispersion compensation

ITU-T Recommendations for Fibers (2)

<i>ITU-T rec. no.</i>	<i>Title and description</i>
G.655 (Edition 5, Nov. 2009)	<p><i>Title: Characteristics of a Non-Zero Dispersion-Shifted Single-Mode Optical Fiber and Cable</i></p> <p><i>Description:</i> For applications in long-haul links; describes single-mode optical fiber with chromatic dispersion greater than zero throughout the 1530-to-1565-nm wavelength range</p>
G.656 (Edition 2, Dec. 2006)	<p><i>Title: Characteristics of a Fiber and Cable with Non-Zero Dispersion for Wideband Optical Transport</i></p> <p><i>Description:</i> Low chromatic dispersion fiber for expanded WDM applications; can be used for both CWDM and DWDM systems throughout the wavelength region between 1460 and 1625 nm</p>
G.657 (Edition 2, Nov. 2009)	<p><i>Title: Characteristics of a bending loss insensitive single-mode optical fiber and cable for the access network</i></p> <p><i>Description:</i> Addresses use of single-mode fiber for broadband access networks; includes issues such as sensitivity to tight bending conditions for in-building use</p>

ITU-T Recommendations for Fibers (1)

Table 3.2 *Recommendations for fibers used in telecom, access, and enterprise networks*

<i>ITU-T rec. no.</i>	<i>Title and description</i>
G.651.1 (Edition 1, July 2007); Addendum (Dec. 2008)	<p><i>Title: Characteristics of a 50/125 μm multimode graded index optical fiber cable for the optical access network</i></p> <p><i>Description:</i> Gives the requirements of a silica 50/125 μm multimode graded index optical fiber cable for use in the 850-nm or 1300-nm regions, either individually or simultaneously</p>
G.652 (Edition 8, Nov. 2009)	<p><i>Title: Characteristics of a Single-Mode Optical Fiber and Cable</i></p> <p><i>Description:</i> Discusses single-mode fiber optimized for O-band (1310-nm) use, but which also can be used in the 1550-nm region</p>
G.653 (Edition 6, Dec. 2006)	<p><i>Title: Characteristics of a Dispersion-Shifted Single-Mode Optical Fiber and Cable</i></p> <p><i>Description:</i> Discusses single-mode optical fiber with the zero-dispersion wavelength shifted into the 1550 nm region. Describes chromatic dispersion for the 1460-to-1625-nm range for CWDM applications</p>
G.654 (Edition 7, Dec. 2006)	<p><i>Title: Characteristics of a Cut-Off Shifted Single-Mode Optical Fiber and Cable</i></p> <p><i>Description:</i> Undersea applications; discusses single-mode optical fiber with a zero-dispersion wavelength around 1300 nm and with cutoff wavelength shifted to around 1550 nm</p>