5. FUZZY SETS AND APPROXIMATE REASONING

KTU Students

Truth values and tables in fuzzy logic:

- Linguistic variables

Values are words/sentences

height
- short
- medium
- tall

KTUStudents
Dedication to Success

Characteristics

1. Name of the variable (x)
2. Term set at the variable t(x)
3. Syntactic rule for generating the value of x.
4. Semantic rule for associating each value of term x.

Linguistic hedges [Linguistic modification]
eg: "very tall"
very is a linguistic hedge
very - highly, slightly, moderately, minus, plus, rather

Propositions - text sentence expressed in any language canonical form - z is P.

Scanned by CamScanner

S is the symbol of the subject

P is the predicate designing characteristics of the subject

eg: "London is in united kingdom"

London → Subject

united kingdom → Predicate

Negation - opposite of propositions

Truth table - logic function of two proposition

Consider X, Y → two propositions.

Basic logic operations on proposition.

1. Conjuction ($\wedge$): X AND Y
2. Disjunction ($\vee$): X OR Y
3. Implication / conditional ($\Rightarrow$): If X THEN Y
4. Bidirectional or equivalence ($\Leftrightarrow$) X IF AND ONLY IF Y.

KTUStudents
Dedication to Success

Scanned by CamScanner

Few interface rules

$[X \cap (X \Rightarrow Y)] \Rightarrow Y$

$[\bar{Y} \cap (X \Rightarrow Y)] \Rightarrow \bar{X}$.

$[(X \Rightarrow Y) \wedge (Y \Rightarrow Z)] \Rightarrow (X \Rightarrow Z)$

Tautology

certain propositions always true irrespective to the ~~elements~~ propositions X and Y

→ Truth values of propositions

$[0,1]$

eg: z is A truth value of A

$tv(A) \rightarrow [0,1]$

- $tv(X \text{ AND } Y) = tv(X) \wedge tv(Y)$
  $= \min\{tv(X), tv(Y)\}$ (intersect)
- $tv(X \text{ OR } Y) = tv(X) \vee tv(Y)$
  $= \max\{tv(X), tv(Y)\}$ (union)

$tv(\text{NOT } X) = 1 - tv(X)$ (complement)

$tv(X \Rightarrow Y) = tv(X) \Rightarrow tv(Y) = 1$

$= \max\{1 - tv(X), \min[tv(X), tv(Y)]\}$

Fuzzy propositions:

1. Fuzzy predicates. eg: Peter is tall
2. Fuzzy predicates modifiers
   eg: Climate is moderately cool
3. Fuzzy quantifiers eg: many people are educated
4. Fuzzy qualifiers

→ Fuzzy truth qualification "x is τ"
eg: (Paul is young) is NOT VERY true.

→ Fuzzy probability qualification: "x is λ"
eg: (Paul is young) is likely

→ Fuzzy possibility qualification: "x is π"
eg: (Paul is young) is almost impossible.

→ Fuzzy usuality qualification: " ~~x is usually~~
usually(X) = usually(X is F)

For more study materials>www.ktustudents.in
Scanned by CamScanner

29/9/17 Formation of Rules:

1. Conditional Statements.

If y is very cool THEN STOP
If A is high THEN B is Low ELSE B is not low.

- ~~The~~ If Temprature is high THEN climate is hot.

2. Assignment Statements

y = small
Orange color = orange
Paul is not tall and not very short.

3. Unconditional Statement

Go to sum
Stop
Divide by a
Turn the pressure low.

Canonical form of fuzzy rule based system.

Rule1: If condition $\underset{\sim}{C1}$, then Restriction $\underset{\sim}{R1}$
Rule2: If condition $\underset{\sim}{C2}$, THEN Restriction $\underset{\sim}{R2}$
Rule3: If condition $\underset{\sim}{C3}$, THEN Restriction $\underset{\sim}{R3}$
⋮
Rule n: If condition $\underset{\sim}{Cn}$, THEN Restriction $\underset{\sim}{Rn}$

Decomposition of Rules (Compound Rule).

1. Multiple conjunctive antecedents

If x is $\underset{\sim}{A1}, \underset{\sim}{A2} \dots \underset{\sim}{An}$ THEN y is B_m

$A_m = A_1 \cap A_2 \cap A_3 \dots A_n$

$\mu_{A_m}(x) = \min[\mu_{A_1}(x), \mu_{\underset{\sim}{A_2}}(x), \dots \mu_{\underset{\sim}{A_n}}(x)\}$

$\boxed{\text{IF } A_m \text{ THEN } B_m}$

2. Multiple disjunctive antecedents

IF x is $\underset{\sim}{A1}$ OR A2 OR... An THEN y is Bm

$A_m = \underset{\sim}{A1} \cup \underset{\sim}{A2} \cup \underset{\sim}{A3} \dots \cup \underset{\sim}{An}$

$\mu_{A_m}(x) = \max[\mu_{\underset{\sim}{A_1}}(x), \mu_{\underset{\sim}{A_2}}(x), \dots \mu_{\underset{\sim}{A_n}}(x)\}$

3/10/17 Fuzzy rule based form.
{if antecedent THEN consequent.

3. Conditional Statements (with ELSE and UNLESS)

If $\underset{\sim}{A1}$ THEN ($\underset{\sim}{B1}$ ELSE $\underset{\sim}{B_2}$)
can be decomposed into simple canonical forms using "OR".

Scanned by CamScanner

IF $\underset{\sim}{A1}$ THEN $\underset{\sim}{B1}$

OR

IF NOT $\underset{\sim}{A}$ THEN $\underset{\sim}{B2}$

IF $\underset{\sim}{A1}$ (THEN $\underset{\sim}{B1}$) UNLESS $\underset{\sim}{B2}$

can be decomposed as

IF $\underset{\sim}{A1}$ THEN $\underset{\sim}{B1}$

OR

IF $\underset{\sim}{A1}$ THEN NOT $\underset{\sim}{B1}$.

IF $\underset{\sim}{A1}$ THEN B_1 ELSE $\underset{\sim}{A2}$ THEN $\underset{\sim}{B2}$

4. Nested If - then rule

If $\underset{\sim}{A_1}$ THEN (THEN IF $\underset{\sim}{A2}$ THEN $\underset{\sim}{B1}$) can be of the form.

IF $\underset{\sim}{A1}$ AND $\underset{\sim}{A2}$ THEN $\underset{\sim}{B1}$.

10/11 AGGREGATION OF FUZZY RULES

~~1. Conjunctive System of Rules~~

Aggregation of rules is the process of obtaining the overall consequence from the individual consequence provided by each rule

1. Conjunctive system of rules

$y = y_1$ and y_2 and .... and y_n

$y = y_1 \cap y_2 \cap \dots \cap y_n$

$\mu_y(y) = \min[\mu_{y_1}(y), \mu_{y_2}(y) \dots \mu_{y_n}(y)]$

2. Disjunctive system of rules

$y = y_1$ OR y_2 OR ... OR y_n

$y = y_1 \cup y_2 \cup \dots \cup y_n$

$\mu_y(y) = \max[\mu_1(y), \mu_2(y), \dots \mu_n(y)]$

FUZZY INTERFACE SYSTEM (FIS)

(Fuzzy rule based system / Fuzzy expert system / Fuzzy model)

Mamdani Controller

If x_1 is $\underset{\sim}{A_1}$ and x_2 is $\underset{\sim}{A_2}$ THEN y is $\underset{\sim}{B}$. If x_1 is $\underset{\sim}{A_1^k}$ and x_2 is $\underset{\sim}{A_2^k}$ THEN y^k is B^k

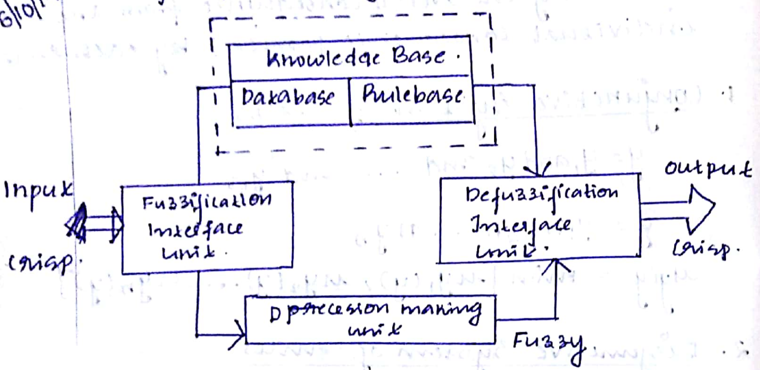
- If x_1 is A_1^1 and x_2 is A_2^1 THEN y^1 is $\underset{\sim}{B^1}$
- If x_1 is A_1^2 and x_2 is A_2^2 THEN y^2 is $\underset{\sim}{B^2}$

For more study materials>www.ktustudents.in

Scanned by CamScanner

6/10/17

Fuzzy Inference System:



Block diagram of FIS

FIS is a key unit of fuzzy logic S/m. The primary work of the s/m is decision making. FIS uses IF-THEN rules along with the connectors 'AND' or 'OR' for making necessary decision rules.

CONSTRUCTION AND WORKING Principle

1. A rule base : that contains numerous fuzzy if-then rules
2. A database : that defines the membership functions of fuzzy sets used in fuzzy rules

database and rule base are collectively called knowledgebased

3. decision making unit : that perform operatn on the rules.
4. fuzzificatn interface unit that converse crisp quantities into fuzzy quantities.
5. defuzzification interface unit that converse fuzzy quantities into crisp quantities.

Methods Of Crisp
1. Mandani FIS (1975)
2. Sugeno FIS (1985)

Mandani FIS

Step1: Determines set of fuzzy rules

Step2: making i/p fuzzy using i/p membership functn.

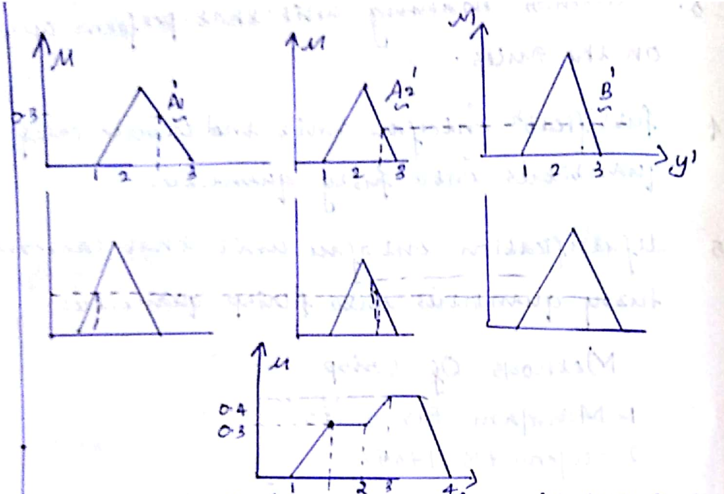
Step3: combined fuzzified input according to fuzzy rules for establishing a rule strength.

Step4: Determine the consequent of a rule by combining the rule strength and the o/p membership functn

Step5: combine all the consequence to get an o/p distribution

Step6: Finally a defuzzified o/p distribution is obtained.

For more study materials>www.ktustudents.in
Scanned by CamScanner



consider a two i/p mamdani FIS with two rules. The model fuzzifies the two inputs by finding the intersection of two input & crisp value with the input membership function. The minimum operation is used to compute the fuzzy input and 'AND' for combining the two fuzzified inputs to obtain a rule strength. The o/p membership funct^n is clipped at the rule strength. Finally max operator is used to compute the fuzzy o/p or for combining o/p of two rules.

Takagi - Sugeno Fuzzy model (T-S method)

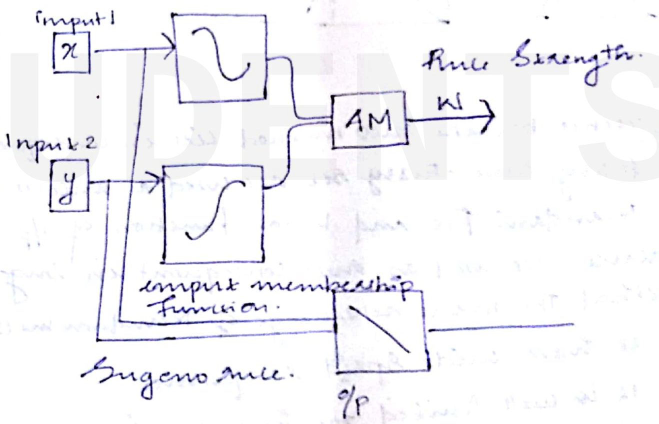
If x is A and y is B Then $z = f(x,y)$

The format of a fuzzy rule of a Sugeno fuzzy model is given by

if x is A and y is B then $z = f(x,y)$

where A, B are fuzzy sets in the antecedent and $z = f(x,y)$ is a crisp function in a consequent

If $f(x,y)$ is first order polynomial we get First order Sugeno model. If f is a constant we get zero order Sugeno model.



→ Steps of fuzzy inference process:-

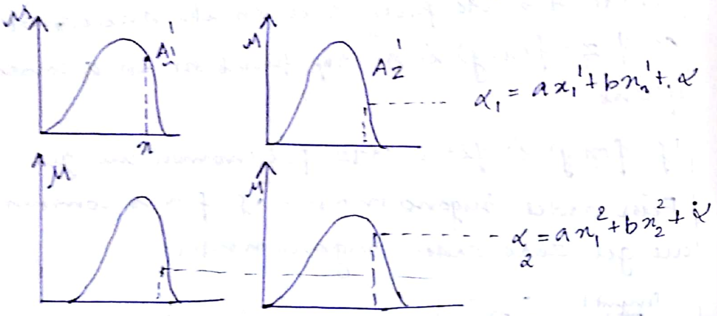
Fuzzifying the input

Scanned by CamScanner

4. Applying the fuzzy operator.

If x_1 is A_1 and x_2 is A_2 Then y is $f(x,y)$.

If x_1^k is A_1^k and x_2^k is A_2^k Then y^k is $f(x^k, y^k)$.



Difference b/w the two method lies in consequent of fuzzy rules. Fuzzy sets are used as rule " in Mandani FIS and Linear function of i/p variables are used as rule consequent in Sugeno method. The main advantage of Mandani method is it have wide spread acceptance.

It is well suited for human inputs

It is intuitive.

Advantage of Sugeno

It is computationally efficient.

2. It is compact and works well with linear technique, premise technique adaptive method.

3. It is best suited for mathematical analysis.

4. It has a ~~gurang~~ guaranteed continuity of the o/p surface.

Graphical Inference methods of FIS

1) Mandani 2) Sugeno.

KTUStudents Dedication to Success

Scanned by CamScanner