SOFT COMPUTING ASSIGNMENT

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Fuzzy Reasoning

In fuzzy logic both the antecedents & consequents are allowed to be fuzzy propositions. There exist 4 modes of fuzzy approximate reasoning, which include:

1. **Categorical Reasoning**

Here, the antecedents contain no fuzzy quantifiers & fuzzy probabilities. The antecedents are assumed to be in canonical form.

$\underset{\sim}{L}, \underset{\sim}{M}, \underset{\sim}{N}, \ldots$ = fuzzy variables taking in the universes U, V, W;

$\underset{\sim}{A}, \underset{\sim}{B}, \underset{\sim}{C}$ = fuzzy predicates.

(i) The projection rule of inference is defined by

$$\frac{\underset{\sim}{L}, \underset{\sim}{M} \text{ is } \underset{\sim}{R}}{\underset{\sim}{L} \text{ is } [\underset{\sim}{R} \downarrow \underset{\sim}{L}]}$$

where $[\underset{\sim}{R} \downarrow \underset{\sim}{L}]$ denotes the projection of fuzzy relation $\underset{\sim}{R}$ on $\underset{\sim}{L}$.

(ii) The conjunction rule of inference is given by

$$\underset{\sim}{L} \text{ is } \underset{\sim}{A}, \underset{\sim}{L} \text{ is } \underset{\sim}{B} \Rightarrow \underset{\sim}{L} \text{ is } \underset{\sim}{A} \cap \underset{\sim}{B}$$

$$(\underset{\sim}{L}, \underset{\sim}{M}) \text{ is } \underset{\sim}{A}, \underset{\sim}{L} \text{ is } \underset{\sim}{B} \Rightarrow (\underset{\sim}{L}, \underset{\sim}{M}) \text{ is } \underset{\sim}{A} \cap (\underset{\sim}{B} \times \underset{\sim}{V})$$

$$(\underset{\sim}{L}, \underset{\sim}{M}) \text{ is } \underset{\sim}{A}, (\underset{\sim}{M}, \underset{\sim}{N}) \text{ is } \underset{\sim}{B} \Rightarrow (\underset{\sim}{L}, \underset{\sim}{M}, \underset{\sim}{N}) = (\underset{\sim}{A} \times \underset{\sim}{W}) \cap (\underset{\sim}{U} \times \underset{\sim}{B})$$

(iii) The disjunction rule of inference is given by

$$\underset{\sim}{L} \text{ is } \underset{\sim}{A} \text{ OR } \underset{\sim}{L} \text{ is } \underset{\sim}{B} \Rightarrow \underset{\sim}{L} \text{ is } \underset{\sim}{A} \times \underset{\sim}{B}$$

$$\underset{\sim}{L} \text{ is } \underset{\sim}{A}, \underset{\sim}{M} \text{ is } \underset{\sim}{B} \Rightarrow (\underset{\sim}{L}, \underset{\sim}{M}) \text{ is } \underset{\sim}{A} \times \underset{\sim}{B}$$

(iv) The negative rule of inference is given by

$$\text{NOT}(\underset{\sim}{L} \text{ is } \underset{\sim}{A}) \Rightarrow \underset{\sim}{L} \text{ is } \overline{\underset{\sim}{A}}$$

(v) The compositional rule of inference is given by

$$\underset{\sim}{L} \text{ is } \underset{\sim}{A}, (\underset{\sim}{L}, \underset{\sim}{M}) \text{ is } \underset{\sim}{R} \Rightarrow \underset{\sim}{M} \text{ is } \underset{\sim}{A} \cdot \underset{\sim}{R}$$

where $\underset{\sim}{A} \cdot \underset{\sim}{R}$ denotes the max-min composition of a fuzzy set $\underset{\sim}{A}$ & a fuzzy relation R given by

$$\mu_{\underset{\sim}{A} \cdot \underset{\sim}{R}}(v) = \max_u \min [\mu_{\underset{\sim}{A}}(u), \mu_{\underset{\sim}{R}}(u,v)]$$

(vi) The extension principle is defined as

$$\underset{\sim}{L} \text{ is } \underset{\sim}{A} \Rightarrow f(\underset{\sim}{L}) \text{ is } f(\underset{\sim}{A})$$

where "f" is a mapping from u to v so that L is mapped into $f(L)$; & based on the extension principle, the membership function of $f(\underset{\sim}{A})$ is defined as

$$\mu_{f(\underset{\sim}{A})}(v) = \sup_{v = f(u)} \mu_{\underset{\sim}{A}}(u), \quad u \in U, v \in V$$

2. <u>Qualitative Reasoning</u>

Here the input-output relationship of a system is expressed as a collection of fuzzy IF THEN rules where the antecedents & consequents involve fuzzy linguistic variables Qualitative reasoning is widely used in control system analysis. Let $\underset{\sim}{A}$ & $\underset{\sim}{B}$ be the fuzzy input variables & $\underset{\sim}{C}$ be the fuzzy output variable. The relation among $\underset{\sim}{A}, \underset{\sim}{B}$ & $\underset{\sim}{C}$ may be expressed as

IF $\underset{\sim}{A}$ is x_1 AND $\underset{\sim}{B}$ is y_1, THEN $\underset{\sim}{C}$ is z_1

IF $\underset{\sim}{A}$ is x_2 AND $\underset{\sim}{B}$ is y_2, THEN $\underset{\sim}{C}$ is z_2

⋮

IF $\underset{\sim}{A}$ is x_n AND $\underset{\sim}{B}$ is y_n, THEN $\underset{\sim}{C}$ is z_n

where x_i, y_i & z_i, $i = 1$ to n, are fuzzy subsets of their respective universe of discourse.

3. <u>Syllogistic Reasoning</u>

Here, antecedents with fuzzy quantifiers are related to inference rules. A fuzzy syllogism can be expressed as follows:

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$$x = k_1 \text{ A's are B's}$$
$$y = k_2 \text{ C's are D's}$$
$$\overline{\quad z = k_3 \text{ E's are F's} \quad}$$

In the above A, B, C, D, E & F are fuzzy predicates; k_1 & k_2 are the given fuzzy quantifiers & k_3 is the fuzzy quantifier which has to be decided. All the fuzzy predicates provide a collection of fuzzy syllogisms. These syllogisms create a set of inference rules, which combines evidence through conjunction & disjunction. Given below are some important fuzzy syllogisms.

(i) Produce syllogism: $C = A \wedge B$, $F = C \wedge D$

(ii) Chaining syllogism: $C = B$, $F = D$, $E = A$

(iii) Consequent conjunction syllogism: $F = B \wedge D$, $A = C = E$

(iv) Consequent disjunction syllogism: $F = B \vee D$, $A = C = E$

(v) Precondition conjunction syllogism: $E = A \wedge C$, $B = D = F$

(vi) Precondition disjunction syllogism: $E = A \vee C$, $B = D = F$

4. **Dispositional Reasoning**

Here, the antecedents are dispositions that may contain, implicitly or explicitly, the fuzzy quantifier "usually." Usuality plays a major role in dispositional reasoning & it links together the dispositional & syllogistic modes of reasoning. The important inference rules of dispositional reasoning are the following:

(i) Dispositional projection rule of inference:

$$\text{usually } ((L, M) \text{ is } R) \Rightarrow \text{usually } (L \text{ is } [R \downarrow L])$$

where $[R \downarrow L]$ is the projection of fuzzy relation R on L.

(ii) Dispositional chaining hypersyllogism: k_1 A's are B's, k_2 B's are C's, usually $(B \subset A)$

$$\text{usually } (\rightarrow (Q_1 (\cdot) Q_2) \text{ A's are C's})$$

The fuzzy quantifier "usually" is applied to the containment relation $B \subset A$

(iii) Dispositional consequence conjunction syllogism:

usually (A's are B's), usually (A's are C's) $\Rightarrow$ 2 usually (−) 1 (A's are (B and C)'s)

is a specific case of dispositional reasoning

(iv) Dispositional entailment rule of inference:

usually (x is A), $A \subset B \Rightarrow$ usually (x is B)

x is A, usually ($A \subset B$) $\Rightarrow$ usually (x is B)

usually (x is A), usually ($A \subset B$) $\Rightarrow$ usually^2 (x is B)

is the dispositional entailment rule of inference. Here "usually^2" is less specific than "usually".

Neuro-Fuzzy Hybrid Systems

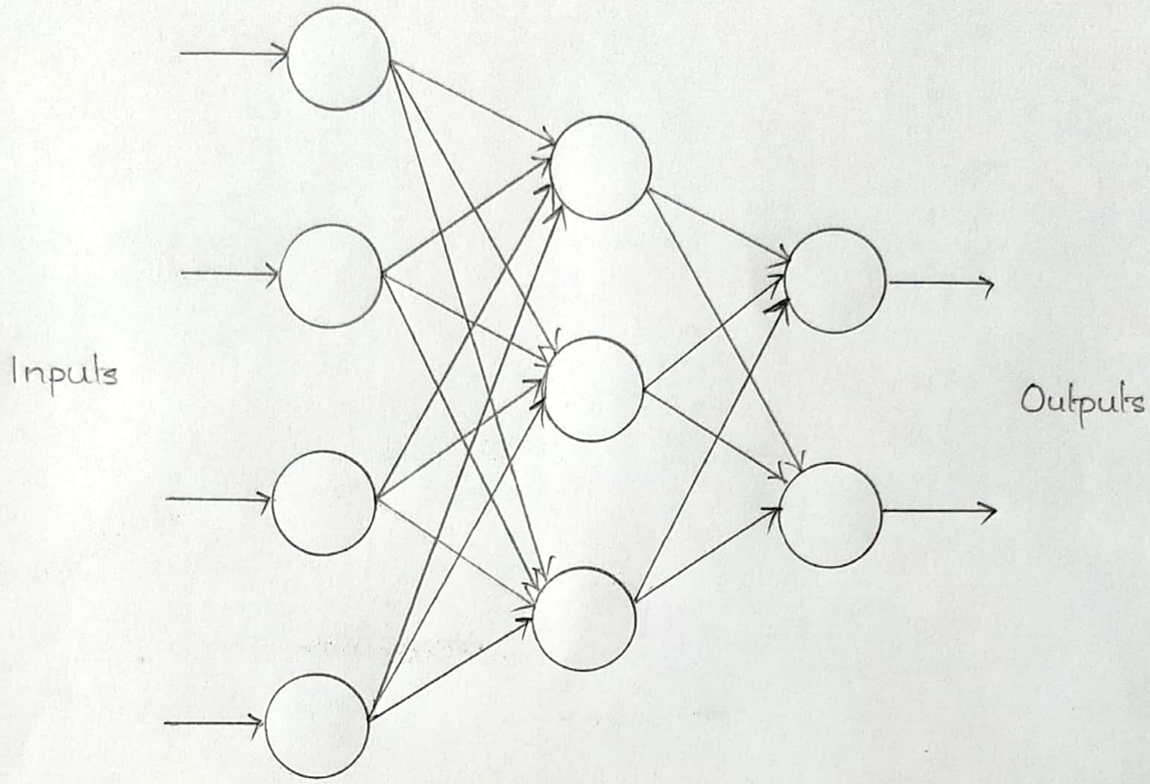
A neuro-fuzzy hybrid system, proposed by J.S.R.Jang, is a learning mechanism that utilizes the training & learning algorithms from neural networks to find parameters of a fuzzy system. It can also be defined as a fuzzy system that determines its parameters by processing data samples by using a learning algorithm derived from or inspired by neural network theory. Alternately, it is a hybrid intelligent system that fuses artificial neural networks & fuzzy logic by combining the learning & connectionist structure of neural networks with human-like reasoning style of fuzzy systems

Neuro-fuzzy hybridization is widely termed as Fuzzy Neural Network (FNN) or Neuro-Fuzzy System (NFS). The human-like reasoning style of fuzzy systems is incorporated by NFS through the use of fuzzy sets & a linguistic model consisting of a set of IF-THEN fuzzy rules. NFSs are universal approximators with the ability to solicit interpretable IF-THEN rules; this is their main strength. However, the strength of NFSs involves interpretability versus accuracy, requirements that are contradictory in fuzzy modeling.

In the field of fuzzy modeling research, the neuro-fuzzy is divided into 2 areas:

1. Linguistic fuzzy modeling focused on interpretability (mainly the Mamdani model).
2. Precise fuzzy modeling focused on accuracy [mainly the Takagi-Sugeno-Kang (TSK) model].

Characteristics of Neuro-Fuzzy Hybrids



The general architecture of neuro-fuzzy hybrid system is as shown in Figure. A fuzzy system-based NFS is trained by means of a data-driven learning method derived from neural network theory. This heuristic causes local changes in the fundamental fuzzy system. At any stage of the learning process - before, during or after - it can be represented as a set of fuzzy rules. For ensuring the semantic properties of the underlying fuzzy system, the learning procedure is constrained.

An NFS approximates an n-dimensional unknown function, partly represented by training examples. Thus fuzzy rules can be interpreted as vague prototypes of the training data. An NFS is given by a 3-layer feedforward neural network model. The 1st layer corresponds to the input variables, & the 2nd & 3rd layers correspond to the fuzzy rules & output variables, respectively. The fuzzy sets are converted to (fuzzy) connection weights.

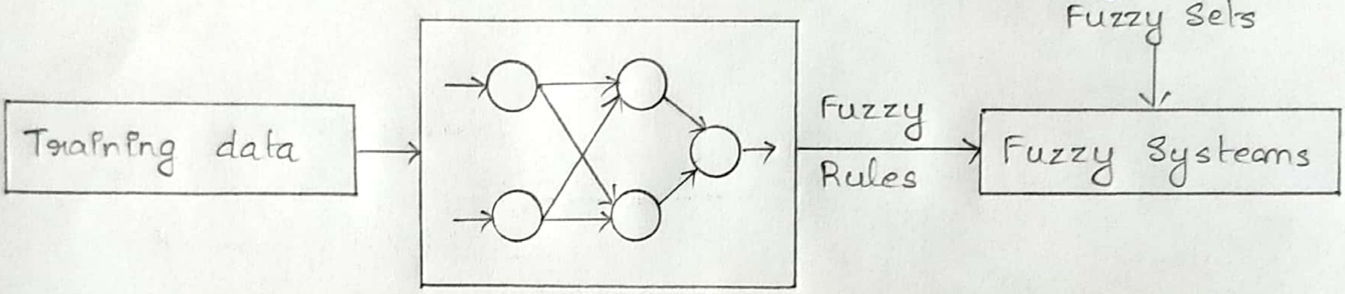
NFS can also be considered as a system of fuzzy rules wherein the system can be initialized in the form of fuzzy rules based on the prior knowledge available.

Classification of Neuro-Fuzzy Hybrid Systems

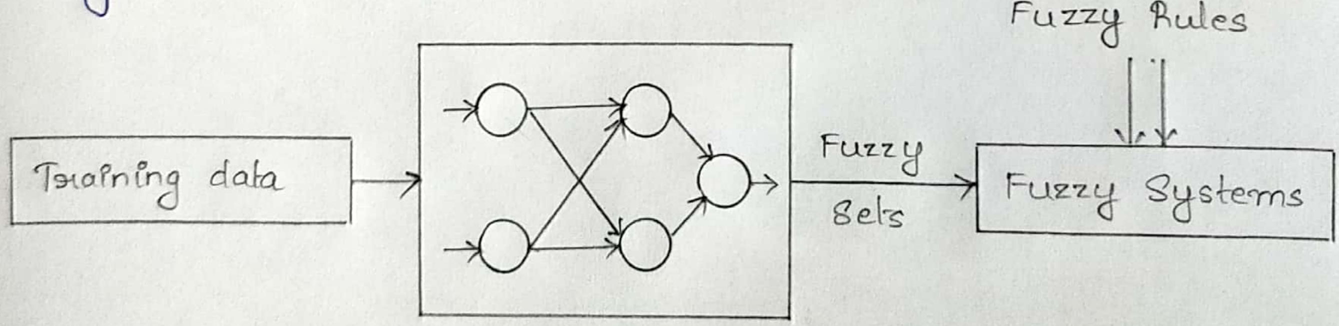
NFSs can be classified into the following 2 systems:

1. Cooperative Neural Fuzzy Systems

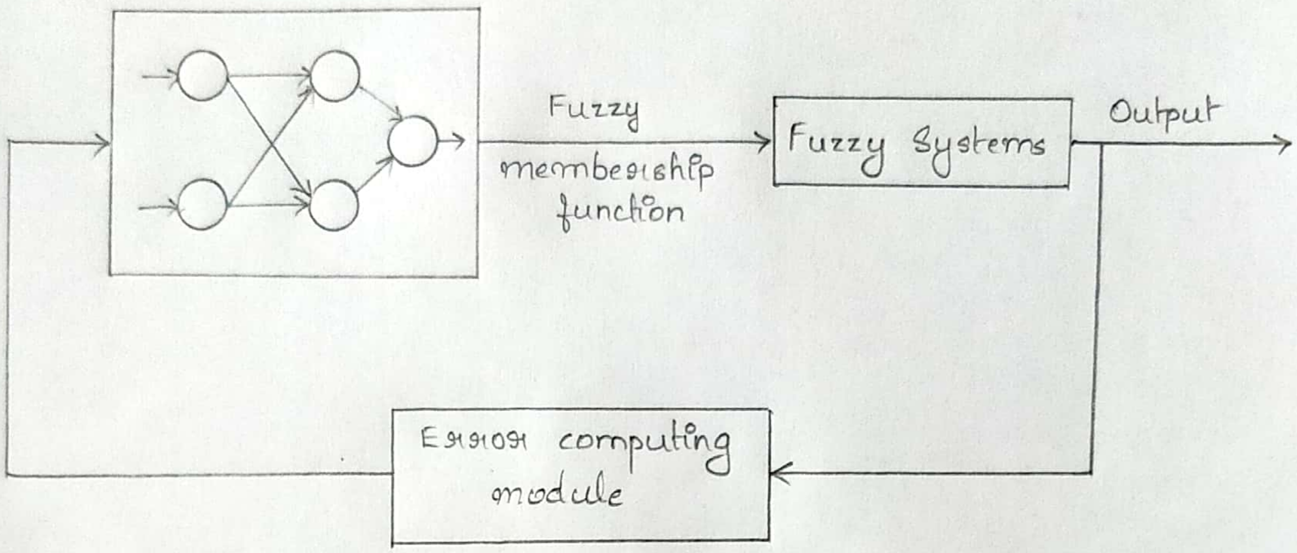
In this type of system, both artificial neural network (ANN) & fuzzy system work independently from each other. The ANN attempts to learn the parameters from the fuzzy system.



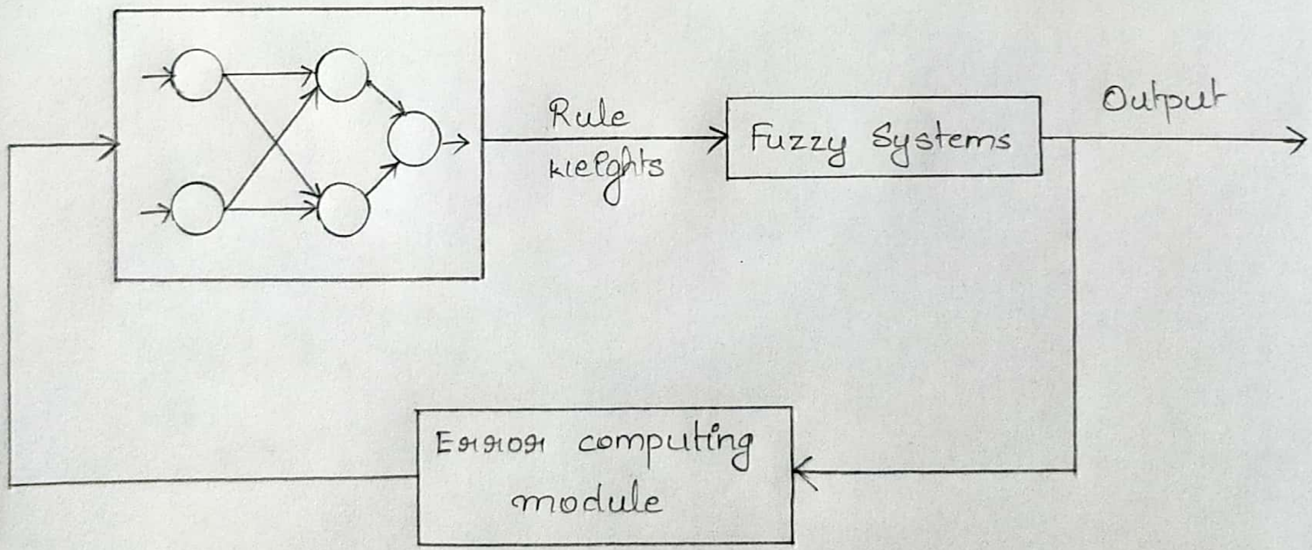
The FNN in Figure learns fuzzy set from the given training data. This is done, usually, by fitting membership functions with a neural network; the fuzzy sets then being determined offline. This is followed by their utilization to form the fuzzy system by fuzzy rules that are given, & not learned.



The NFS in Figure determines, by a neural network, the fuzzy rules from the training data. Here again, the neural networks learn offline before the fuzzy system is initialized. The rule learning happens usually by clustering on self-organizing feature maps. There is also the possibility of applying fuzzy clustering methods to obtain rules.

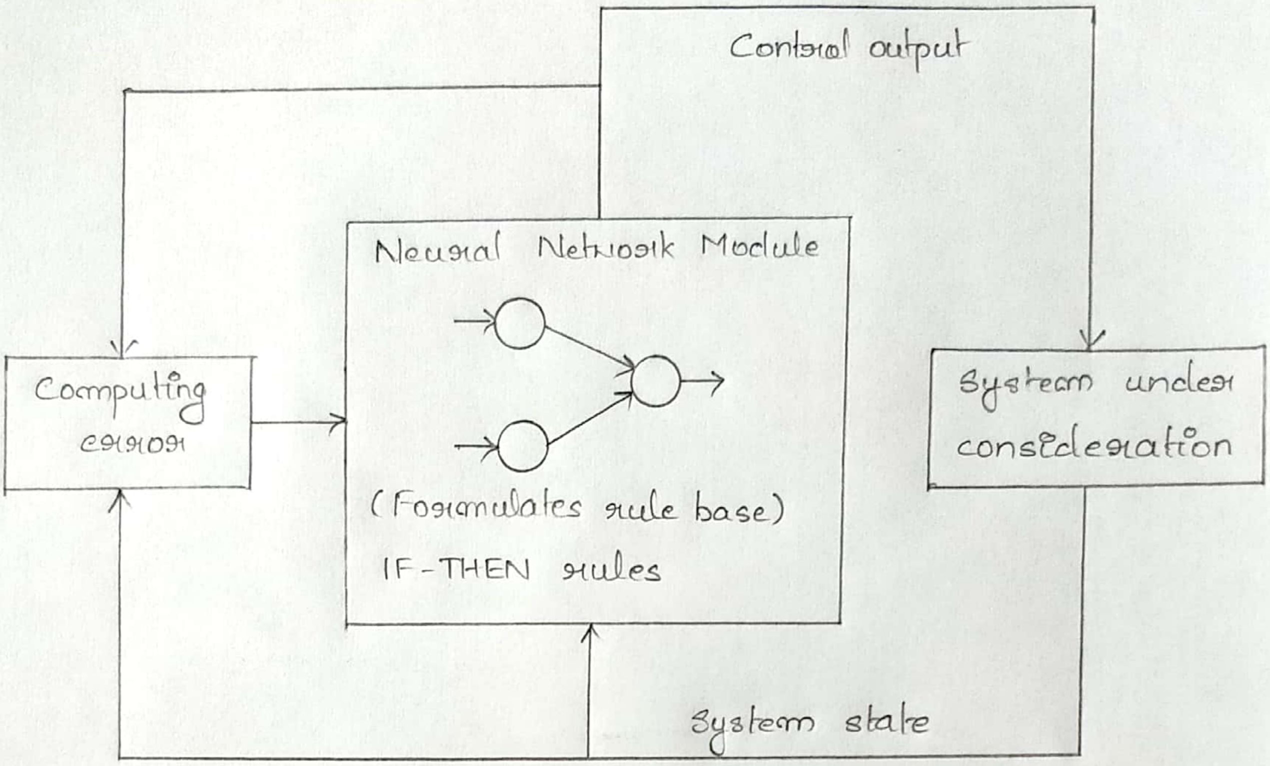


For the neuro-fuzzy model shown in Figure, the parameters of membership function are learnt online, while the fuzzy system is applied. This means that, initially, fuzzy rules & membership functions must be defined beforehand. Also, in order to improve & guide the learning step, the error has to be measured.



The model shown in Figure determines the rule weights for all fuzzy rules by a neural network. A rule is determined by its rule weight - interpreted as the influence of a rule. They are then multiplied with the rule output.

2. General Neuro-Fuzzy Hybrid Systems (General NFHS)



General neuro-fuzzy hybrid systems (NFHS) resemble neural networks where a fuzzy system is interpreted as a neural network of special kind. The architecture of general NFHS gives it an advantage because there is no communication between fuzzy system & neural network. In the figure the rule base of a fuzzy system is assumed to be a neural network; the fuzzy sets are regarded as weights & the rules & the input & output variables as neurons. The choice to include or discard neurons can be made in the learning step. Also, the fuzzy knowledge base is represented by the neurons of the neural network; this overcomes the major drawbacks of both underlying systems.

Membership functions expressing the linguistic terms of the inference rules should be formulated for building a fuzzy controller. However, in fuzzy systems, no formal approach exists to define these functions. Any shape, such as Gaussian or triangular or bell shaped or trapezoidal, can be considered as a membership function with an arbitrary set of parameters. Thus for fuzzy systems, the optimization of these functions in terms of generalizing the data is very important; this problem can be solved by using neural networks.

Using learning rules, the neural network must optimize the parameters by fixing a distinct shape of the membership functions; for example, triangular. But regardless of the shape of the membership functions, training data should also be available.

The neuro fuzzy hybrid systems can also be modeled in another method. In this case, the training data is grouped into several clusters & each cluster is designed to represent a particular rule. These rules are defined by the crisp data points & are not defined linguistically. Hence a neural network, in this case, might be applied to train the defined clusters. The testing can be carried out by presenting a random testing sample to the trained neural network. Each & every output unit will return a degree which extends to fit to the antecedent of rule.