2/11/19 Hence it is also known as Vertex - edge incident matrix

(ii) Circuit Matrix (B)

- $q \times e$ matrix where $q \to$ No. of different circuits & $e \to$ No. of edges
- $B(G) \to$ Circuit matrix of G
- $B(G) = [b_{ij}]$; $b_{ij} = \begin{cases} 1, & \text{if } j\text{th edge included in } i\text{th circuit} \\ 0, & \text{otherwise} \end{cases}$

eg: Same graph in A.

Circuits

1. $\{a, b\}$
2. $\{c, e, g\}$
3. $\{d, f, g\}$
4. $\{c, d, f, e\}$

$$B(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$q = 4$, $e = 8$ $\quad q \times e \Rightarrow 4 \times 8$

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• Properties

(i) No. of 1's in each row represents the no. of edges in the circuit

(ii) A column with all 0's represents non-circuit edge

(iii) Each row of B(G) ~~has~~ is circuit vector

(iv) If the graph contains self-loop

(v) Circuit matrix of a disconnected graph can be represented in block diagonal form.

Consider a disconnected graph G with 2 components g_1 & g_2

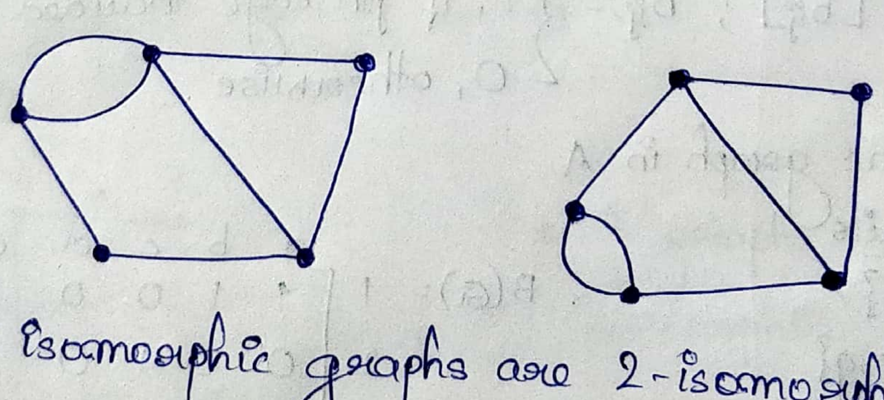
$$B(G) = \left[\begin{array}{c|c} B(g_1) & 0 \\ \hline 0 & B(g_2) \end{array}\right]$$

2 graphs G_1 & G_2 will have the same ckt matrix iff G_1 & G_2 are 2-isomorphic

2-Isomorphism

2 graphs G_1 & G_2 are said to be 2-isomorphic iff they have circuit correspondence. 2 graphs are said to have circuit correspondence if there is 1:1 correspondence btw the edges & 1-to-1 correspondence btw the circuits in G_1 & G_2.

eg. Consider 2 graphs G_1 & G_2



All isomorphic graphs are 2-isomorphic, but the converse is not true.

Relatⁿ btw A & B

Let B & A be respectively the circuit matrix & incidence matrix of a self-loop for graph whose columns are arranged using the same order of edges, then

$$AB^T = BA^T = 0 \pmod 2$$

when module 2 arithmetic is used.

$$A(G) = \begin{array}{c c} & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} & \left[\begin{array}{cccccccc} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]_{6\times 8} \end{array}$$

$$B(G) = \begin{array}{c c} & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccccccc} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{array}\right]_{4\times 8} \end{array}$$

$$B(G)^T = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} & \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]_{8\times 4} \end{array}$$

$$AB^T = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} & \left[\begin{array}{cccc} 0 & 0 & 2 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 2 \\ 0 & 2 & 2 & 2 \\ 2 & 0 & 0 & 0 \end{array}\right]_{6\times 4} \end{array}$$

$$\text{Mod } 2\,(AB^T) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{6\times 4}$$

Fundamental Circuit Matrix (B_f)