KTU LECTURE NOTES

APJ ABDUL KALAM
TECHNOLOGICAL UNIVERSITY

PROPERTIES OF MEMBERSHIP FUNCTIONS, FUZZIFICATION AND DEFUZZIFICATION

LECTURE 6

November 26, 2017

Membership functions

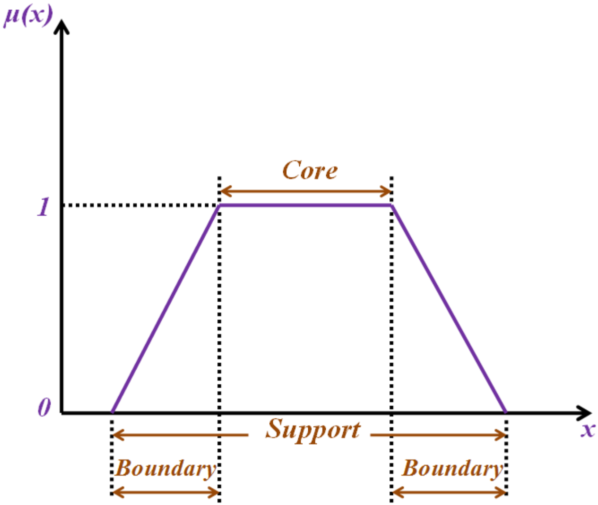
- *Membership function* defines the fuzziness in a fuzzy set irrespective of the elements in the set, which are discrete or continuous.
- They are generally represented in graphical form.
- The rules that describe fuzziness graphically are also fuzzy.

Features of the Membership functions

- The membership function defines all the information contained in a fuzzy set.
- A fuzzy set A in the universe of discourse X can be defined a set of ordered pairs:

$$A = \{(x, \mu_A(x)) | x \in X\}$$

where, $\mu_A(.)$ is called membership function of A.


μ(x)
1
0
Core
Support
Boundary
Boundary
x

Core

- The *core* of a membership function for some fuzzy set A is defined as that region of universe is characterized by complete membership in the set $A.ie$,
$$\mu_A(x) = 1$$
- The core of a fuzzy set may be an empty set.

Support

- The *support* of a membership function for a fuzzy set A is defined as that region of universe is characterized by a nonzero membership in the set $A.ie$,

$$\mu_A(x) > 0$$

- A fuzzy set whose support is a single element in X with

$$\mu_A(x) = 1$$

is referred to as a fuzzy singleton.

Boundary

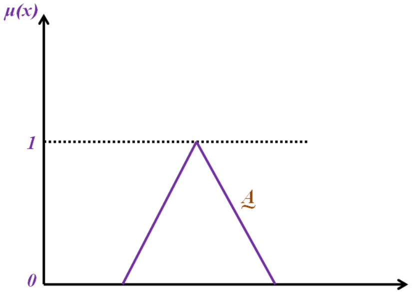
- The *boundary* of a membership function for a fuzzy set A is defined as that region of universe containing elements that have a nonzero but not complete membership.*ie*,

$$0 < \mu_A(x) < 1$$

- The boundary elements are those which possess partial membership in the fuzzy set A.

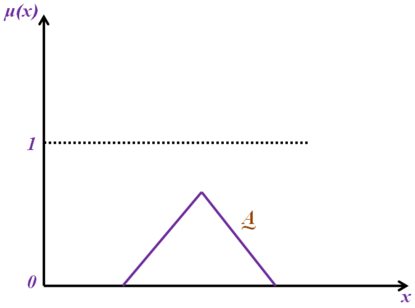
Normal fuzzy set

- A fuzzy set whose membership function has at least one element x in the universe whose membership value is unity is called *normal fuzzy set*.
- The element for which the membership is equal to *1* is called *prototypical element*.



Subnormal fuzzy set

- A fuzzy set where in no membership function has its value equal to *1* is called *subnormal fuzzy set*.



Convex fuzzy set

- A *convex fuzzy set* has a membership function whose membership values are strictly monotonically increasing or strictly monotonically decreasing or strictly monotonically increasing than strictly monotonically decreasing with increasing values for elements in the universe.
- For elements x_1, x_2 and x_3 in a fuzzy set A. If,

$$\mu_A(x_2) \geq min[\mu_A(x_1), \mu_A(x_3)]$$

  then A is said to be convex fuzzy set.
- The intersection between two convex fuzzy sets is also a convex fuzzy set.
- A fuzzy set possessing characteristics opposite to that of convex fuzzy set is called *non convex fuzzy set*.

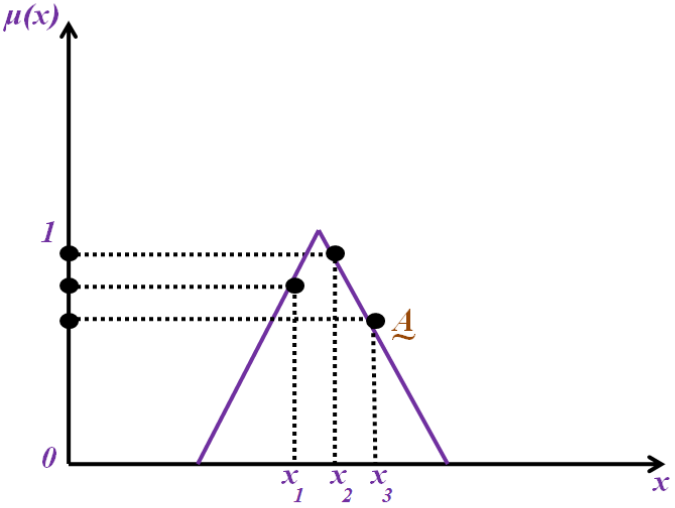


Figure 2.1: Convex normal fuzzy set

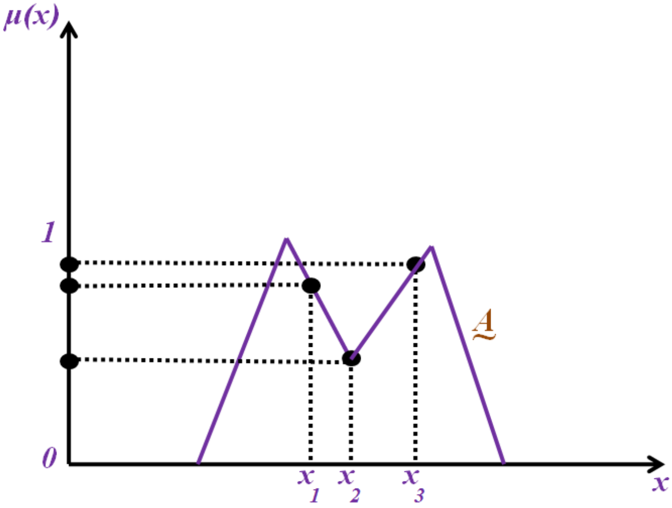


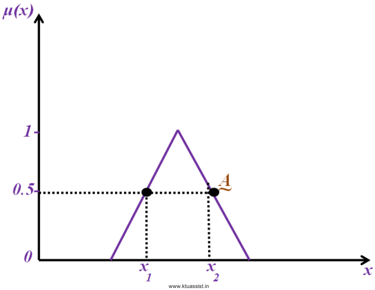
Figure 2.2: Non convex normal fuzzy set

Crossover point of a fuzzy set

- The element in the universe for which a particular fuzzy set A has its value equal to *0.5* is called *crossover point* of a membership function.*ie*,

$$\mu_A(x) = 0.5$$

- There can be more than one crossover point in a fuzzy set.



Height of the fuzzy set

- The maximum value of the membership function in a fuzzy set A is called as the *height* of the fuzzy set.
- For a normal fuzzy set, the height is equal to *1*.
- If the height of a fuzzy set is less than *1*, then the fuzzy set is called subnormal fuzzy set.
- When the fuzzy set A is a convex single point normal fuzzy set, then A is termed as a fuzzy number.

Fuzzification

- *Fuzzification* is the process of transforming a crisp set to a fuzzy set.
- This operation translates accurate crisp input values into linguistic variables.
- They possess uncertainity within themselves.
- The variable is probably fuzzy and can be represented by a membership function.

Kernel of fuzzification

- For a fuzzy set,

$$A = \{\frac{\mu_i}{x_i} | x_i \in X\}$$

- A common fuzzification algorithm is performed by keeping μ_i constant and x_i being transformed to a fuzzy set $Q(x_i)$ depicting the expression about x_i.
- The fuzzy set $Q(x_i)$ is referred to as the *Kernel of fuzzification or support fuzzification or s−fuzzification*.
- The fuzzified set A can be expressed as,

$$\mu_1 Q(x_1) + \mu_2 Q(x_2) + ... + \mu_n Q(x_n)$$

- *Grade fuzzification or g−fuzzification:* where x_i is kept constant and μ_i is expressed as a fuzzy set.

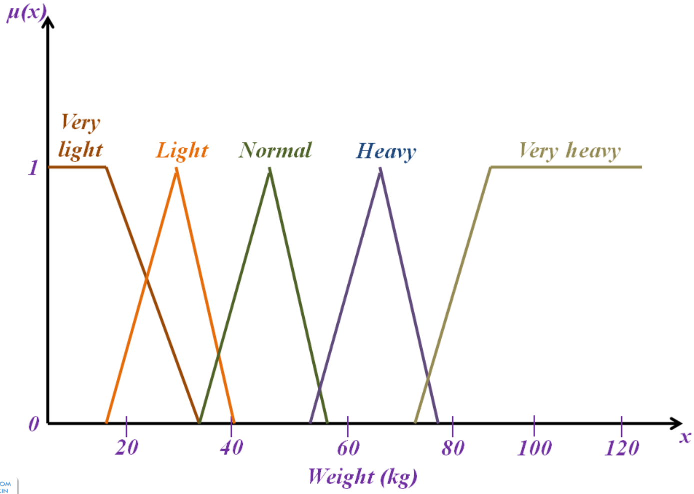
Methods of membership value assignments

1. Intuition
2. Inference
3. Rank ordering
4. Angular fuzzy sets
5. Neural networks
6. Genetic algorithm
7. Inductive reasoning

Intuition

- *Intuition method* is based upon the common intelligence of human.
- It is the capacity of the human to develop membership functions on the basis of their own intelligence and understanding capability.
- There should an in−depth knowledge of the application to which membership value assignment has to be made.

Membership functions for the fuzzy variable "weight"



Problems

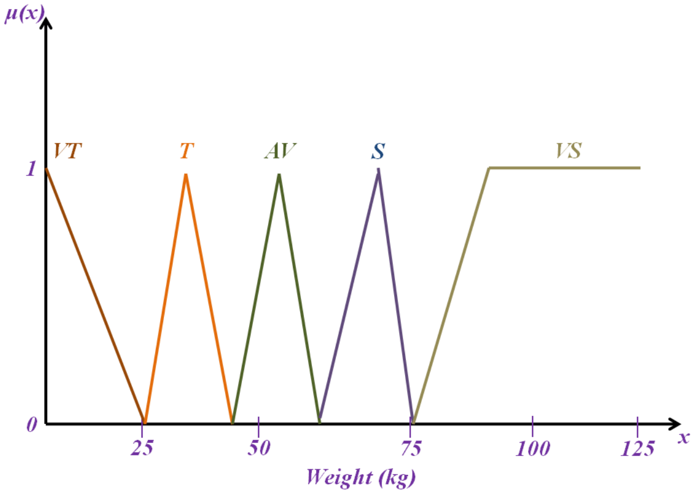
(1) *Using your own intuition and definitions of the universe of discourse, plot fuzzy membership functions for "weight of people".*

$U = weight\ of\ people$

Let the weights be in kilogram.

Let the linguistic variables be the following:

$$
\begin{aligned}
Very\ thin(VT) &: W \leq 25 \\
Thin(T) &: 25 < W \leq 45 \\
Average(AV) &: 45 < W \leq 60 \\
Stout(S) &: 60 < W \leq 75 \\
Very\ stout(VS) &: W > 75
\end{aligned}
$$


μ(x)
1
0
VT
T
AV
S
VS
25
50
75
100
125
x
Weight (kg)

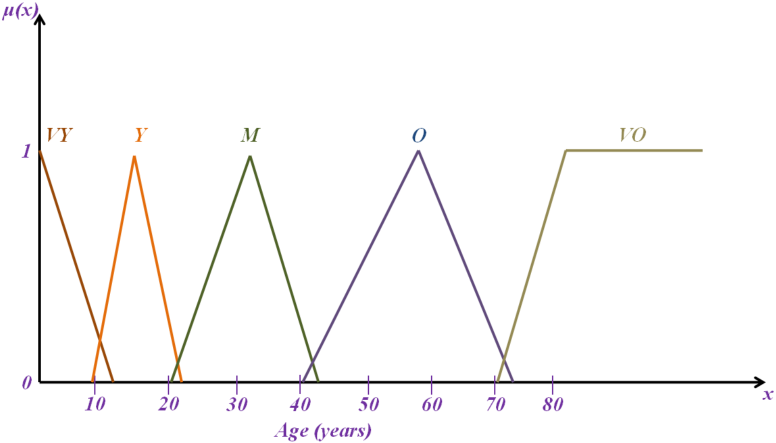
(2) *Using your own intuition and definitions of the universe of discourse, plot fuzzy membership functions for "age of people".*

$U =$ *age of people*

Let A denote age of people in years.

Let the linguistic variables be the following:

$$Very\ young(VY) : A < 12$$
$$Young(Y) : 10 \leq A \leq 22$$
$$Middle\ age(M) : 20 \leq A \leq 42$$
$$Old(O) : 40 \leq A \leq 72$$
$$Very\ old(VO) : 70 < A$$


μ(x)
VY
Y
M
O
VO
1
0
10
20
30
40
50
60
70
80
x
Age (years)

Inference

- The *inference method* uses knowledge to perform deductive reasoning.
- Deduction achieves conclusion by means of forward inference.
- The knowledge of geometrical shapes and geometry is used for defining membership values.
- The membership functions may be defined using various shapes: *triangular, trapezoidal, bell–shaped, Gaussian etc.*
- The inference method here is via triangular shape.

- Consider a triangle, where X, Y and Z are the angles, such that

$$X \geq Y \geq Z \geq 0$$

Let U be the universe of triangles, *ie*,

$$U = \{(X, Y, Z) | X \geq Y \geq Z \geq 0; X + Y + Z = 180\}$$

- There are various types of triangles available:
  1. $I = isosceles\ triangle$
  2. $E = \ equilateral\ triangle$
  3. $R = \ right-angle\ triangle$
  4. $IR = \ isosceles\ and\ right-angle\ triangle$
  5. $T = \ other\ triangles$

- The membership values of approximate isosceles triangle is obtained using,

$$\mu_I(X, Y, Z) = 1 - \frac{1}{60^o} min(X - Y, Y - Z)$$

where,

$$X \geq Y \geq Z \geq 0 \text{ and } X + Y + Z = 180$$

If $X = Y$ or $Y = Z$, then

$$\mu_I(X, Y, Z) = 1$$

If $X = 120^o$ or $Y = 60^o$ and $Z = 0^o$, then

$$\mu_I(X, Y, Z) = 0$$

- The membership value of approximate right angle triangle is given by,

$$\mu_R(X, Y, Z) = 1 - \frac{1}{90^o}|X - 90^o|$$

If $X = 90^o$, then

$$\mu_R(X, Y, Z) = 1$$

If $X = 180^o$, then

$$\mu_R(X, Y, Z) = 0$$

- The membership value of approximate isosceles right angle triangle is obtained by,

$$IR = I \cap R$$

and it is given by,

$$\mu_{IR}(X, Y, Z) = min[\mu_I(X, Y, Z), \mu_R(X, Y, Z)]$$
$$= 1 - max[\frac{1}{60^o} min(X - Y, Y - Z), \frac{1}{90^o}|X - 90^o|]$$

- The membership function for a fuzzy equilateral triangle is given by,

$$\mu_E(X, Y, Z) = 1 - \frac{1}{180^o}|X - Z|$$

- The membership function of other triangles is given by,

$$T = \overline{I \cup R \cup E}$$

By using DeMorgan′s law,

$$T = \overline{I} \cap \overline{R} \cap \overline{E}$$

The membership value can be obtained using,

$$\mu_T(X, Y, Z) = min[1 - \mu_I(X, Y, Z), 1 - \mu_E(X, Y, Z), 1 - \mu_R(X, Y, Z)]$$

Problems

(1) *Using the inference approach, find the membership values for the triangular shapes I,R,E,IR and T for a triangle with angles* $45^o, 55^o$ *and* 180^o.

Let the universe of discourse be,

$$U = \{(X, Y, Z) | X = 80^o \geq Y = 55^o \geq Z = 45^o \geq 0, X + Y + Z = 80^o + 55^o + 45^o = 180^o\}$$

Membership value of isosceles triangle, I

$$\mu_I = 1 - \frac{1}{60^o} min(X - Y, Y - Z)$$
$$= 1 - \frac{1}{60^o} min(80^o - 55^o, 55^o - 45^o)$$
$$= 1 - \frac{1}{60^o} min(25^o, 10^o) = 1 - \frac{1}{60^o} \times 10^o = 0.833$$

Membership value of right angle triangle, R

$$\mu_R = 1 - \frac{1}{90^o} |X - 90^o|$$
$$= 1 - \frac{1}{90^o} |80^o - 90^o|$$
$$= 1 - \frac{1}{90^o} \times 10^o = 0.889$$

Membership value of equilateral triangle, E

$$\begin{aligned}\mu_E &= 1 - \frac{1}{180^o}(X - Z)\\ &= 1 - \frac{1}{180^o}(80^o - 45^o)\\ &= 1 - \frac{1}{180^o} \times 35^o = 0.8056\end{aligned}$$

Membership value of isosceles and right angle triangle, IR

$$\begin{aligned}\mu_{IR} &= min(\mu_I, \mu_R)\\ &= min(0.833, 0.889) = 0.833\end{aligned}$$

Membership value of other triangles, T

$$\begin{aligned}\mu_T &= min(1 - \mu_I, 1 - \mu_E, 1 - \mu_R)\\ &= min(0.167, 0.1944, 0.111) = 0.111\end{aligned}$$

Rank Ordering

- On the basis of the preferences made by an individual, a committee, a poll and other opinion methods.
- Pairwise comparisons enables to determine preferences.
- This results in determining the order of the membership.

Angular Fuzzy Sets

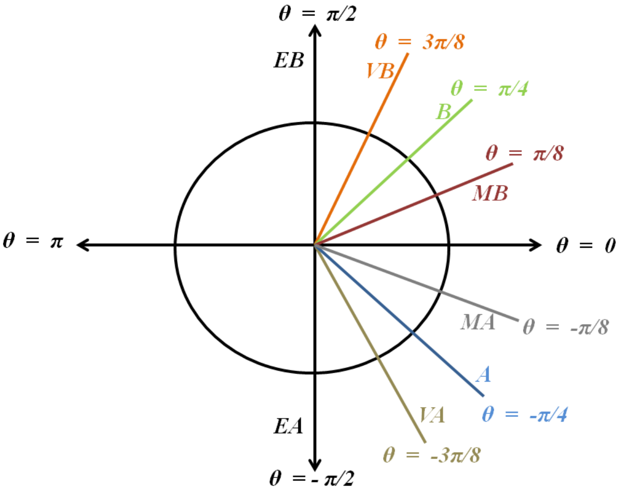
- *Angular fuzzy sets* are defined on a universe of angles, thus repeating the shapes every 2π cycles.
- The truth values of the linguistic variable are represented by angular fuzzy sets.
- The membership value corresponding to the linguistic term can be obtained by,

$$\mu_r(\theta) = t \cdot tan(\theta)$$

where t is horizontal projection of radial vector.*ie*,

$$t = cos(\theta)$$

Model of angular fuzzy set



Defuzzification

- *Defuzzification* is a mapping process from a space of fuzzy control actions defined over an output universe of discourse into a space of crisp control actions.
- It has the capability:
  - to reduce a fuzzy set into a crisp set
  - to convert a fuzzy matrix into a crisp matrix
  - to convert a fuzzy number into a crisp number
- Mathematically, the defuzzification process may also be termed as *rounding it off*.

Lambda−Cuts for Fuzzy Sets

- Consider a fuzzy set A.
- The set $A_\lambda (0 < \lambda < 1)$, called the *$\lambda-$cut or $\alpha-$cut set*, is a crisp set of the fuzzy set.
- It is defined as:

$$A_\lambda = \{x | \mu_A(x) \geq \lambda\}$$

  where, $\lambda \in [0, 1]$
- Any particular fuzzy set A can be transformed into an infinite number of $\lambda-$cut sets.

Properties of $\lambda-$cut sets

1. $(A \cup B)_\lambda = A_\lambda \cup B_\lambda$
2. $(A \cap B)_\lambda = A_\lambda \cap B_\lambda$
3. $(\overline{A})_\lambda \neq (\overline{A}_\lambda)$ *except when* $\lambda = 0.5$
4. *For any* $\lambda \leq \beta$, *where* $0 \leq \beta \leq 1$,*it is true that*,$A_\beta \subseteq A_\lambda$, *where* $A_0 = X$.

Features of membership functions

1. The *core* of A is the $\lambda = 1-$cut set A_1.
2. The *support* of A is the $\lambda-$cut set A_{0^+}, where $\lambda = 0^+$, and it can be defined as,
$$A_{0^+} = \{x | \mu_A(x) > 0\}$$
3. The interval $[A_{0^+}, A_1]$ forms the *boundaries* of the fuzzy set A.

Lambda−Cuts for Fuzzy Relations

- Consider a fuzzy relation R.
- The relation $R_\lambda(0 < \lambda < 1)$, called the *$\lambda-$cut or $\alpha-$cut relation*, is a crisp relation of the fuzzy relation.
- It is defined as:

$$R_\lambda = \{(x, y) | \mu_R(x, y) \geq \lambda\}$$
$$R_\lambda = \{1 | \mu_{R(x,y)} \geq \lambda; 0 | \mu_{R(x,y)} < \lambda\}$$

where, $\lambda \in [0, 1]$

Properties of $\lambda-$cut sets

1. $(R \cup S)_\lambda = R_\lambda \cup S_\lambda$
2. $(R \cap S)_\lambda = R_\lambda \cap S_\lambda$
3. $(\overline{R})_\lambda \neq (\overline{R}_\lambda)$ *except when* $\lambda = 0.5$
4. *For any* $\lambda \leq \beta$, *where* $0 \leq \beta \leq 1$, *it is true that*, $R_\beta \subseteq R_\lambda$.

Problems

(1) *Consider two fuzzy sets A and B, both defined on X,given as follows:*

$\mu(x_i X)$	x_1	x_2	x_3	x_4	x_5
A	0.2	0.3	0.4	0.7	0.1
B	0.4	0.5	0.6	0.8	0.9

Express the following $\lambda-$ *sets using Zadeh's notation:*

$(a)(\overline{A})_{0.7}$ $(b)(B)_{0.2}$
$(c)(A \cup B)_{0.6}$ $(d)(A \cap B)_{0.5}$
$(e)(A \cup \overline{A})_{0.7}$ $(f)(B \cap \overline{B})_{0.3}$
$(g)(\overline{A \cap B})_{0.6}$ $(h)(\overline{A} \cup \overline{B})_{0.8}$

The two fuzzy sets given are,

$$A = \{\frac{0.2}{x_1} + \frac{0.3}{x_2} + \frac{0.4}{x_3} + \frac{0.7}{x_4} + \frac{0.1}{x_5}\}$$
$$B = \{\frac{0.4}{x_1} + \frac{0.5}{x_2} + \frac{0.6}{x_3} + \frac{0.8}{x_4} + \frac{0.9}{x_5}\}$$

$$(a)(\overline{A}) = 1 - \mu_A(x) = \{\frac{0.8}{x_1} + \frac{0.7}{x_2} + \frac{0.6}{x_3} + \frac{0.3}{x_4} + \frac{0.9}{x_5}\}$$
$$(\overline{A})_{0.7} = \{x_1, x_2, x_5\}$$

$$(b)(B)_{0.2} = \{x_1, x_2, x_3, x_4, x_5\}$$

$$(c)(A \cup B) = max\{\mu_A(x), \mu_B(x)\}$$
$$= \{\frac{0.4}{x_1} + \frac{0.5}{x_2} + \frac{0.6}{x_3} + \frac{0.8}{x_4} + \frac{0.9}{x_5}\}$$
$$(A \cup B)_{0.6} = \{x_3, x_4, x_5\}$$

$$(d)(A \cap B) = min\{\mu_A(x), \mu_B(x)\}$$
$$= \{\frac{0.2}{x_1} + \frac{0.3}{x_2} + \frac{0.4}{x_3} + \frac{0.7}{x_4} + \frac{0.1}{x_5}\}$$
$$(A \cap B)_{0.5} = x_4$$

$$(e)(A \cup \overline{A}) = max\{\mu_A(x), \mu_{\overline{A}}(x)\}$$
$$= \{\frac{0.8}{x_1} + \frac{0.7}{x_2} + \frac{0.6}{x_3} + \frac{0.7}{x_4} + \frac{0.9}{x_5}\}$$
$$(A \cup \overline{A})_{0.7} = \{x_1, x_2, x_4, x_5\}$$

$$(f)(\overline{B}) = 1 - \mu_B(x) = \{\frac{0.6}{x_1} + \frac{0.5}{x_2} + \frac{0.4}{x_3} + \frac{0.2}{x_4} + \frac{0.1}{x_5}\}$$
$$(B \cap \overline{B}) = min\{\mu_B(x), \mu_{\overline{B}}(x)\} = \{\frac{0.4}{x_1} + \frac{0.5}{x_2} + \frac{0.4}{x_3} + \frac{0.2}{x_4} + \frac{0.1}{x_5}\}$$
$$(B \cap \overline{B})_{0.3} = \{x_1, x_2, x_3\}$$

$$(g)(\overline{A \cap B}) = 1 - \mu_{A \cap B}(x) = \{\frac{0.8}{x_1} + \frac{0.7}{x_2} + \frac{0.6}{x_3} + \frac{0.3}{x_4} + \frac{0.9}{x_5}\}$$
$$(\overline{A \cap B})_{0.6} = \{x_1, x_2, x_3, x_5\}$$

$$(h)(\overline{A} \cup \overline{B}) = max\{\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)\}$$
$$= \{\frac{0.8}{x_1} + \frac{0.7}{x_2} + \frac{0.6}{x_3} + \frac{0.3}{x_4} + \frac{0.9}{x_5}\}$$
$$(\overline{A} \cup \overline{B})_{0.8} = \{x_1, x_5\}$$

(2) Consider the discrete fuzzy set defined on the universe, $X = \{a, b, c, d, e\}$ *as,*

$$A = \{\frac{1}{a} + \frac{0.9}{b} + \frac{0.6}{c} + \frac{0.3}{d} + \frac{0}{e}\}$$

Find the $\lambda-$*cut sets for* $\lambda = 1, 0.9, 0.6, 0.3, 0^+$ *and* 0.

$$(a)\lambda = 1, A_1 = \{\frac{1}{a} + \frac{0}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e}\}$$

$$(b)\lambda = 0.9, A_{0.9} = \{\frac{1}{a} + \frac{1}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e}\}$$

$$(c)\lambda = 0.6, A_{0.6} = \{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{0}{d} + \frac{0}{e}\}$$

$$(d)\lambda = 0.3, A_{0.3} = \{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{0}{e}\}$$

$$(e)\lambda = 0^+, A_{0^+} = \{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{0}{e}\}$$

$$(f)\lambda = 0, A_0 = \{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e}\}$$

(3) *Determine the crisp* $\lambda-$*cut relation when* $\lambda = 0.1, 0^+, 0.3$ *and* 0.9 *for the following relation* R:

$$R = \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.3 & 0.7 & 0.1 \\ 0.8 & 0.9 & 1.0 \end{bmatrix}$$

$(a)\lambda = 0.1$

$$R_{0.1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$(b)\lambda = 0^{+}$

$$R_{0^+} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$(c)\lambda = 0.3$

$$R_{0.3} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$(d)\lambda = 0.9$

$$R_{0.9} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Defuzzification Methods

- *Defuzzification* is the process of conversion of a fuzzy quantity into a precise quantity.
- The output of a fuzzy process may be union of two or more fuzzy membership functions defined on the universe of discourse of the output variable.
- A fuzzy output process may involve many output parts, and the membership function representing each part of the output can have any shape.
- In general, we have,

$$C_n = \cup_{i=1}^{n} C_i = C$$

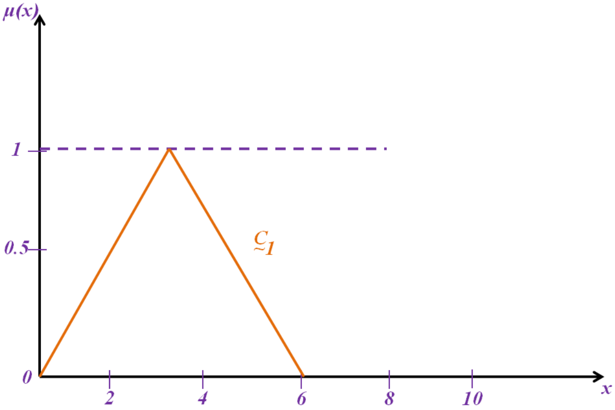


Figure 8.1: C_1, a triangular membership shape

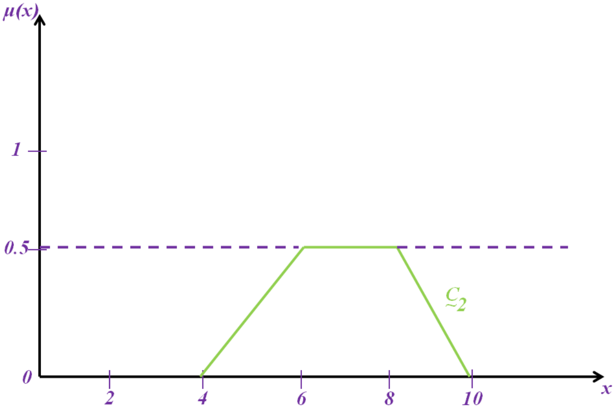


Figure 8.2: C_2, a trapezoidal shape

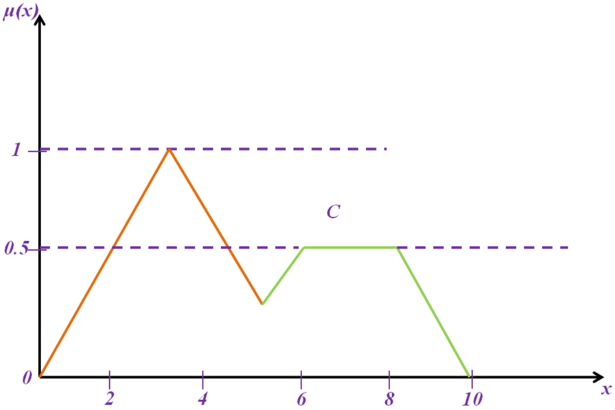


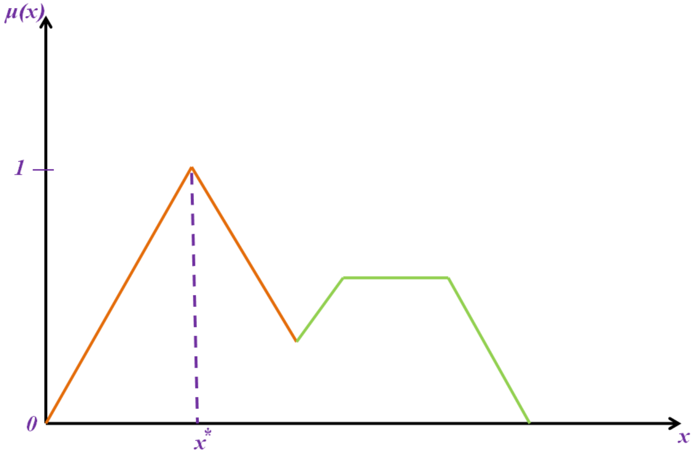
Figure 8.3: $C = C_1 \cup C_2$, which is the outer envelope of the two shapes

- Defuzzification methods include:
  1. *Max–membership principle*
  2. *Centroid method*
  3. *Weighted average method*
  4. *Mean–max membership*
  5. *Center of sums*
  6. *Center of largest area*
  7. *First of maxima, Last of maxima*

Max–Membership Principle

- Also known as *height method*.
- It is limited to peak output functions.
- This method is given by,

$$\mu_C(x^*) \geq \mu_C(x) \text{ for all } x \in X$$

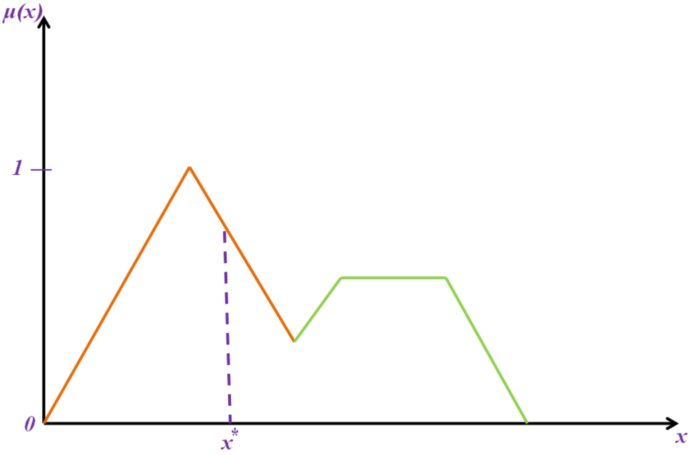

μ(x)
1
0
x*
x

Centroid Method

- Also known as *center of mass*, *center of area* or *Center of gravity* method.
- It is the most commonly used defuzzification method.
- The defuzzified output x^* is defined as,

$$x^* = \frac{\int \mu_C(x) \cdot x dx}{\int \mu_C(x) dx}$$

where the symbol $\int$ denotes algebraic integration.

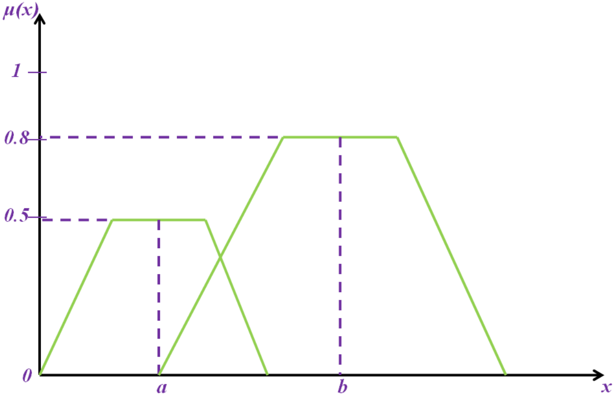

μ(x)
1
0
x*
x

Weighted Average Method

- Each membership function is weighted by its maximum membership value.
- This method is valid for symmetrical output membership functions only.
- The defuzzified output x^* is defined as,

$$x^* = \frac{\sum \mu_C(\overline{x}_i) \cdot \overline{x}_i}{\sum \mu_C(\overline{x}_i)}$$

where $\sum$ denotes algebraic sum and $\overline{x}_i$ is the maximum of the i^{th} membership function.



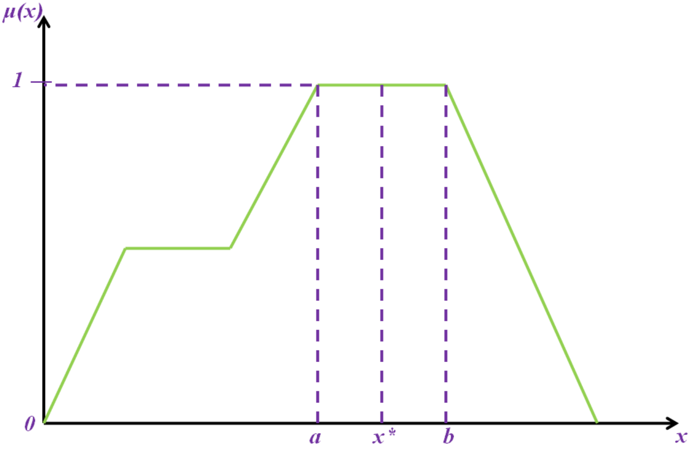
$$x^* = \frac{0.5a + 0.8b}{0.5 + 0.8}$$

where a and b are the means of their respective shapes.

Mean−Max Membership

- Also known as *middle of the maxima*.
- It is closely related to max−membership method except that the locations of the maximum membership can be non−unique.
- The defuzzified output x^* is defined as,

$$x^* = \frac{\sum_{i=1}^{n} \overline{x}_i}{n}$$

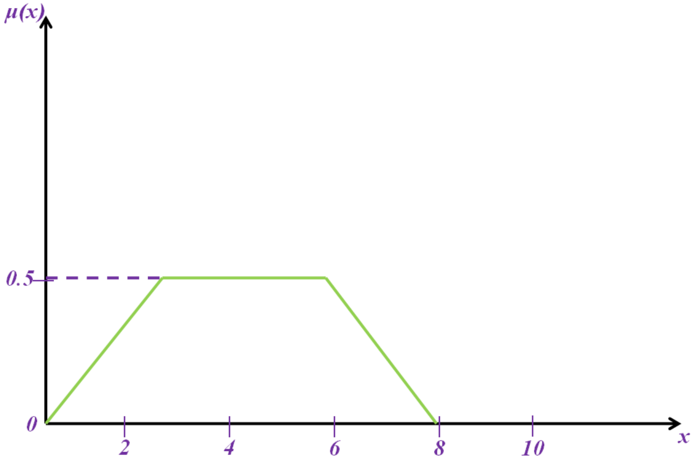


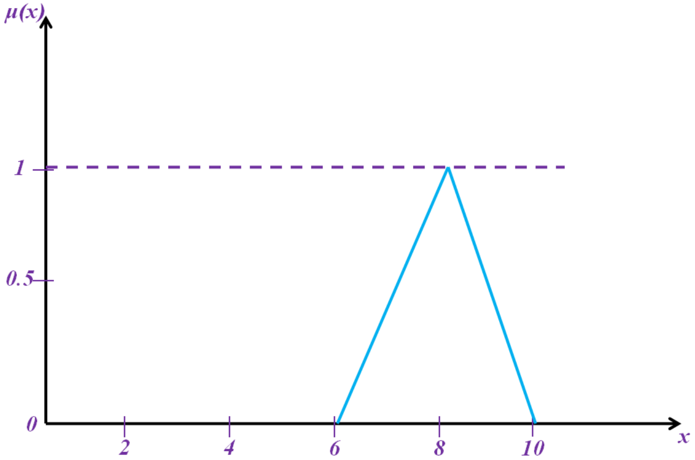
$$x^* = \frac{a+b}{2}$$

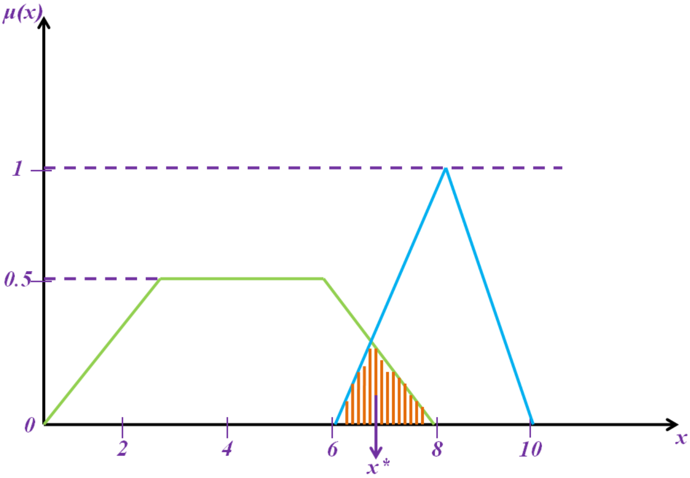
Center of Sums

- This method employs the algebraic sum of the individual fuzzy subsets instead of their union.
- The weights are the areas of the respective membership functions.
- The defuzzified output x^* is defined as,

$$x^* = \frac{\int_x x \sum_{i=1}^{n} \mu_{C_i}(x)\,dx}{\int_x \sum_{i=1}^{n} \mu_{C_i}(x)\,dx}$$


μ(x)
0.5
0
2
4
6
8
10
x


μ(x)
1
0.5
0
2
4
6
8
10
x

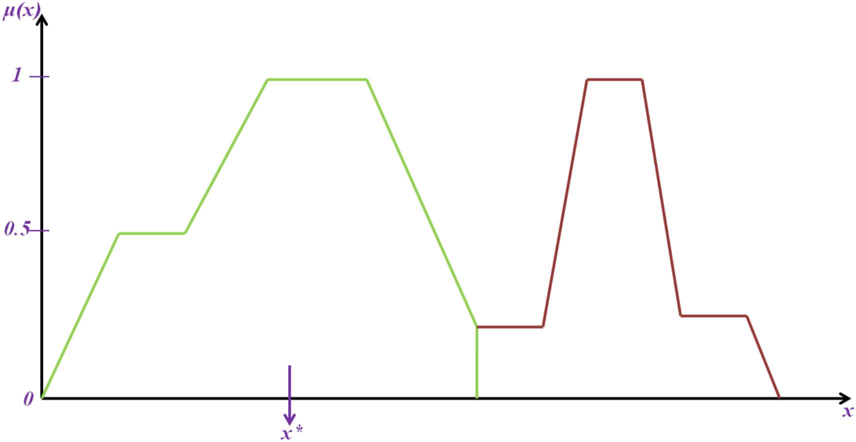

μ(x)
1
0.5
0
2
4
6
8
10
x
x*

Center of Largest Area

- This method can be adopted when the output consists of at least two convex fuzzy subsets which are not overlapping.
- The output in this case is biased towards a side of one membership function.
- The defuzzified output x^* is defined as,

$$x^* = \frac{\int \mu_{C_i}(x) \cdot x dx}{\int \mu_{C_i}(x) dx}$$

where C_i is the convex subregion that has the largest area.

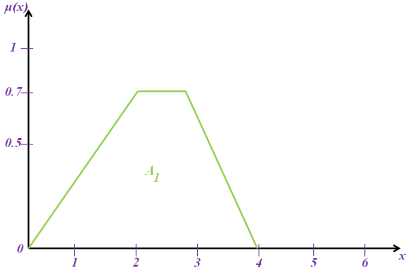

μ(x)
1
0.5
0
x
x*

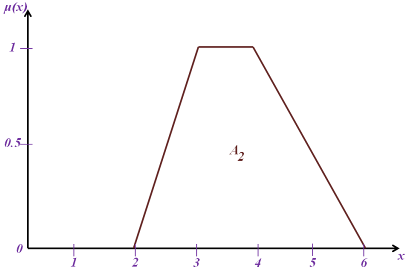
First of Maxima (Last of Maxima)

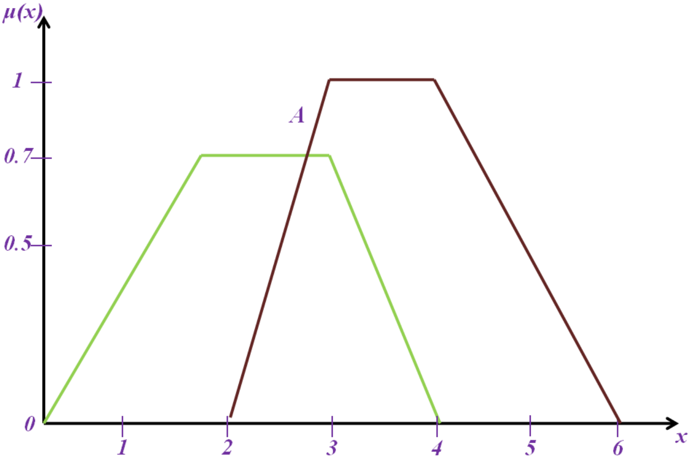
- This method uses the overall output or union of all individual output fuzzy sets for determining the smaller value of the domain with maximized membership.
- The steps for obtaining x^* are:
  1. Initially, the maximum height in the union is found:
     $$hgt(C_i) = \sup_{x \in X} \mu_{C_i}(x)$$
     where sup is supremum,ie,the least upper bound.
  2. Then the first of maxima is found:
     $$x^* = \inf_{x \in X}\{x \in X | \mu_{C_i}(x) = hgt(C_i)\}$$
     where inf is infimum,ie,the greatest lower bound.
  3. After this the last of maxima is found:
     $$x^* = \sup_{x \in X}\{x \in X | \mu_{C_i}(x) = hgt(C_i)\}$$
     where sup is supremum,ie,the least upper bound.

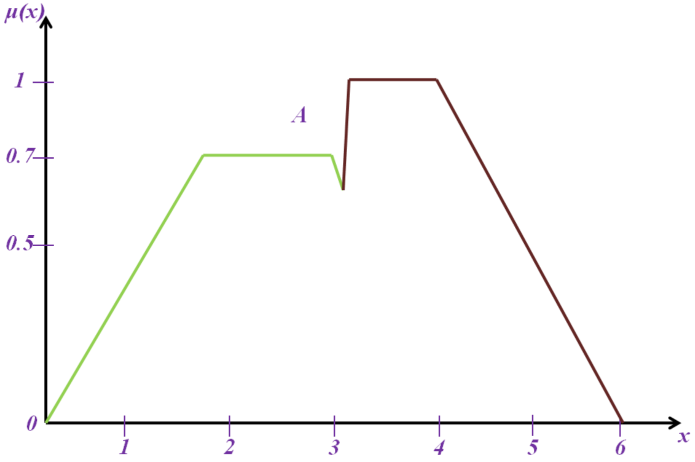
Problems

(1) *For the given membership functions as shown in figure, determine the defuzzified output value by seven methods.*






μ(x)
1
0.7
0.5
0
A
1
2
3
4
5
6
x


μ(x)
1
0.7
0.5
0
A
1
2
3
4
5
6
x

END