GNF

$A \to a\alpha$

$\alpha \in N^*$

$\alpha \to$ Collectⁿ of non-terminals

GNF $\to$ RHS should start with a terminal.

Lemma 1:-

if $A \to B\alpha$

$B \to \beta$

then $A \to \beta\alpha$

Lemma 2:-

if $A \to \cancel{A\alpha/\beta}$ $A\alpha | \beta \to L$

$\to$ Left recursion

$L' \begin{cases} A \to \beta A' | \beta \\ A' \to \alpha A' | \alpha \end{cases}$

eg: $S \to AB$

$A \to BS | b$

$B \to SA | a$

Rename all non-terminals with A_i. Always $S = A_1$

$A = A_2$

$B = A_3$

$\left.\begin{array}{l} A_1 \to A_2 A_3 \\ A_2 \to A_3 A_1 | b \\ A_3 \to A_1 A_2 | a \end{array}\right\}$ Rewriting the product's

(i) $A_1 \to A_2 A_3$

Applying Lemma 1 since $A_i \to A_j \alpha$ & $i < j$

$A_1 \to A_2 A_3$

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$A_1 \to A_3 A_1 A_3 \mid bA_3$

~~A1~~ Continue this process till $i \leq j$. If $i=j$, apply Lemma 2. If $i>j$, it is in GNF

$\cancel{A_1 \to A_1 A_2 A_1 A_3 \mid aA_3 A_1 A_3 \mid abA_3}$

$A_1 \to A_1 A_2 A_1 A_3 \mid aA_1 A_3 \mid bA_3$

⇒ ~~Now i=j~~ $A \to A\alpha \mid \beta_1 \mid \beta_2$

$A \to \beta_1 A' \mid \beta_2 A' \mid \beta_1 \beta_2$

$A' \to \alpha A' \mid \alpha$

So $A_1 \to aA_1 A_3 Z_1 \mid bA_3 Z_1 \mid aA_1 A_3 \mid bA_3$

$Z_1 \to A_2 A_1 A_3 Z_1 \mid A_2 A_1 A_3$

So A_1 is now in GNF

(ii) $A_2 \to A_3 A_1 \mid b$

$A_2 \to A_1 A_2 A_1 \mid aA_1 \mid b$

$A_2 \to aA_1 A_3 Z_1 A_2 A_1 \mid bA_3 Z_1 A_2 A_1 \mid aA_1 A_3 A_2 A_1 \mid bA_3 A_2 A_1 \mid aA_1 \mid b$

So A_2 is now in GNF

(iii) $A_3 \to A_1 A_2 \mid a$

$A_3 \to aA_1 A_3 Z_1 A_2 \mid bA_3 Z_1 A_2 \mid aA_1 A_3 A_2 \mid bA_3 A_2 \mid a$

So A_3 is now in GNF

$Z_1 \to aA_1 A_3 Z_1 A_2 A_1 A_1 A_3 Z_1 \mid bA_3 Z_1 A_2 A_1 A_1 A_3 Z_1 \mid aA_1 A_3 A_2 A_1 A_1 A_3 Z_1 \mid bA_3 A_2 A_1 A_1 A_3 Z_1 \mid aA_1 A_1 A_3 Z_1 \mid bA_1 A_3 Z_1 \mid aA_1 A_3 Z_1 A_2 A_1 A_1 A_3 \mid bA_3 Z_1 A_2 A_1 A_1 A_3 \mid aA_1 A_3 A_2 A_1 A_1 A_3 \mid bA_3 A_2 A_1 A_1 A_3 \mid aA_1 A_1 A_3 \mid aA_1 A_1 A_3 \mid bA_1 A_3$

① $S \to AB \mid a$

$A \to BBA \mid a$

$B \to b$

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Convert to GNF.

ans ~~A~~ $S = A_1$, $A = A_2$, $B = A_3$

$A_1 \to A_2 A_3 \mid a$

$A_2 \to A_3 A_2 A_2 \mid a$

$A_3 \to b$

$A_1 \to A_2 A_3 \mid a$

$A_1 \to A_3 A_2 A_2 A_3 \mid a A_3 \mid a$

$A_1 \to b A_2 A_2 A_3 \mid a A_3 \mid a$ $\to$ GNF

$A_2 \to A_3 A_2 A_2 \mid a$

$A_2 \to b A_2 A_2 \mid a$ $\to$ GNF

$A_3 \to b$ $\to$ GNF

(2) Convert to GNF

$S \to ASB \mid AB$

$A \to SSB \mid a$

$B \to b$

ans $S = A_1$, $A = A_2$, $B = A_3$

$A_1 \to A_2 A_1 A_3 \mid A_2 A_3$

$A_2 \to A_1 A_1 A_3 \mid a$

$A_3 \to b$

$A_1 \to A_1 A_1 A_3 A_1 A_3 \mid a A_1 A_3 \mid \cancel{A_2 A_3}\ A_1 A_1 A_3 A_3 \mid a A_3$

$A \to A\alpha_1 \mid \beta_1 \mid A\alpha_2 \mid \beta_2$

$A_1' \to \alpha_1 A_1' \mid \alpha_1$.

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$A_2' \to \alpha_2 A_2' \mid \alpha_2$

$A \to \beta_1 A_1' \mid \beta_2 A_1' \mid \beta_1 A_2' \mid \beta_2 A_2' \mid \beta_1 \mid \beta_2$

So $A_1 \to aA_1A_3A_1A_3A_1A_3Z_1 \mid aA_3A_1A_3A_1A_3A_1A_3Z_1 \mid$

$aA_1A_3A_1A_3A_3Z_2 \mid aA_3A_1A_3A_3Z_2 \mid aA_1A_3 \mid aA_3$

$Z_1 \to A_1A_3A_1A_3Z_1 \mid A_1A_3A_1A_3$

$Z_2 \to A_1A_3A_3Z_2 \mid A_1A_3A_3$

$A_2 \to aA_1A_3A_1A_3A_1A_3Z_1A_1A_3 \mid aA_3A_1A_3A_1A_3A_1A_3Z_1A_1A_3 \mid$

$aA_1A_3A_1A_3A_3Z_2A_1A_3 \mid aA_3A_1A_3A_3Z_2A_1A_3 \mid aA_1A_3A_1A_3 \mid$

$aA_3A_1A_3 \mid a$

$A_3 \to b$

~~$Z_1 \to aA_1A_3A_1A_3A_1A_3Z_1A_3A_1A_3Z_1 \mid aA_1A_3A_1A_3A_1A_3A_1A_3A_1A_3$~~

~~$\mid aA_3A_1A_3A_1A_3A_1A_3A_1A_3A_1A_3Z_1 \mid aA_3A_1A_3A_1A_3A_1A_3A_1A_3$~~

~~$\mid aA_1A_3A_1A_3A_3A_1A_3A_3Z_2A_1A_3 \mid aA_1A_3A_1A_3A_3A_1A_3A_3A_1A_3 \mid$~~

~~$aA_3A_1A_3A_3A_1A_3A_3Z_2A_1A_3 \mid aA_3A_1A_3A_3A_1A_3A_3A_1A_3 \mid aA_1A_3A_1$~~

~~$A_3 \mid aA_3A_1A_3 \mid a$~~

$Z_1 \to aA_1A_3A_1A_3A_1A_3Z_1A_3A_1A_3Z_1 \mid aA_3A_1A_3A_1A_3A_1A_3Z_1$

$A_3A_1A_3Z_1 \mid aA_1A_3A_1A_3A_3Z_2A_3A_1A_3 \mid aA_3A_1A_3A_3Z_2A_3A_1A_3$

$\mid aA_1A_3A_3A_1A_3Z_1 \mid aA_3A_3A_1A_3Z_1 \mid aA_1A_3A_1A_3A_1A_3Z_1A_3A_1A_3$

$\mid aA_3A_1A_3A_1A_3A_1A_3Z_1A_3A_1A_3 \mid aA_1A_3A_1A_3A_3Z_2A_3A_1A_3 \mid aA_3A_1$

$A_3A_3Z_2A_3A_1A_3 \mid aA_1A_3A_3A_1A_3 \mid aA_3A_3A_1A_3$

$Z_2 \to aA_1A_3A_1A_3A_1A_3Z_1A_3A_3Z_2 \mid aA_3A_1A_3A_1A_3A_1A_3Z_1A_3A_3Z_2$
$\mid aA_1A_3A_1A_3A_3Z_2A_3A_3Z_2 \mid aA_3A_1A_3A_3Z_2A_3A_3Z_2 \mid aA_1A_3A_3$
$A_3Z_2 \mid aA_3A_3A_3Z_2 \mid aA_1A_3A_1A_3A_1A_3Z_1A_1A_3A_3 \mid aA_3A_1A_3A_1$
$A_3A_1A_3Z_1\cancel{A_1}A_3A_3 \mid aA_1A_3A_1A_3A_3A_3A_3 \mid aA_3A_1A_3A_3A_3A_3 \mid$
$aA_1A_3A_3A_3 \mid aA_3A_3A_3$