AM5450 Assignment 8

Q1)

a) **Plots from MATLAB Code using Isoparametric quad elements:**

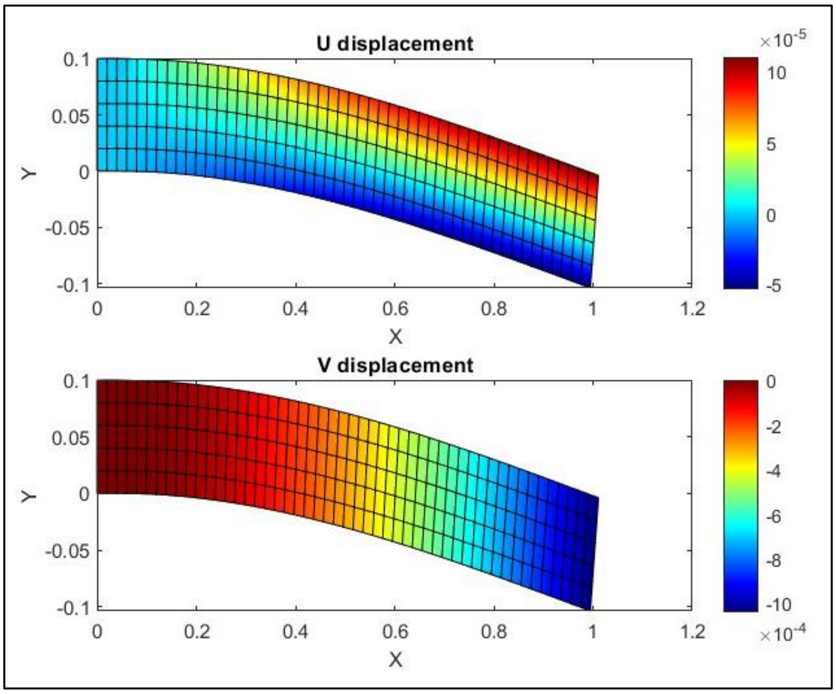

Fig 1: Contours of U and V displacements of the beam using isoparametric quad elements

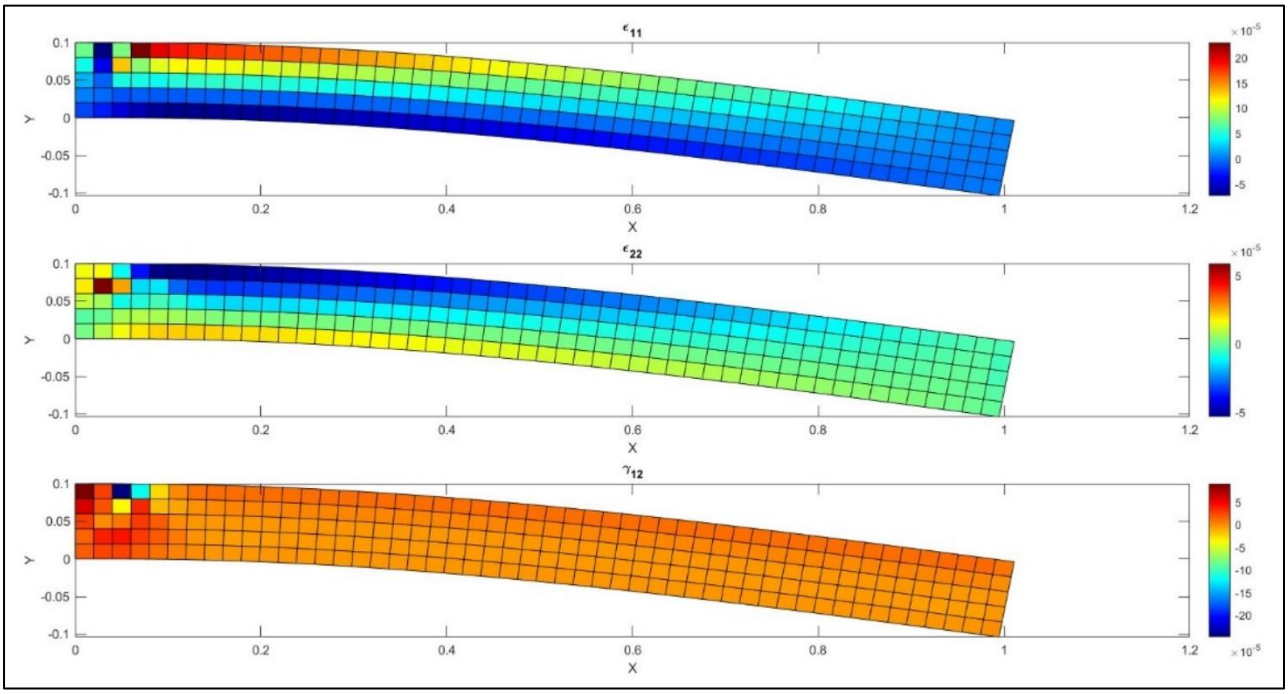

Fig 2: Strain components (ε_{11}, ε_{11}, Υ_{12}) for the deformed beam using isoparametric quad elements

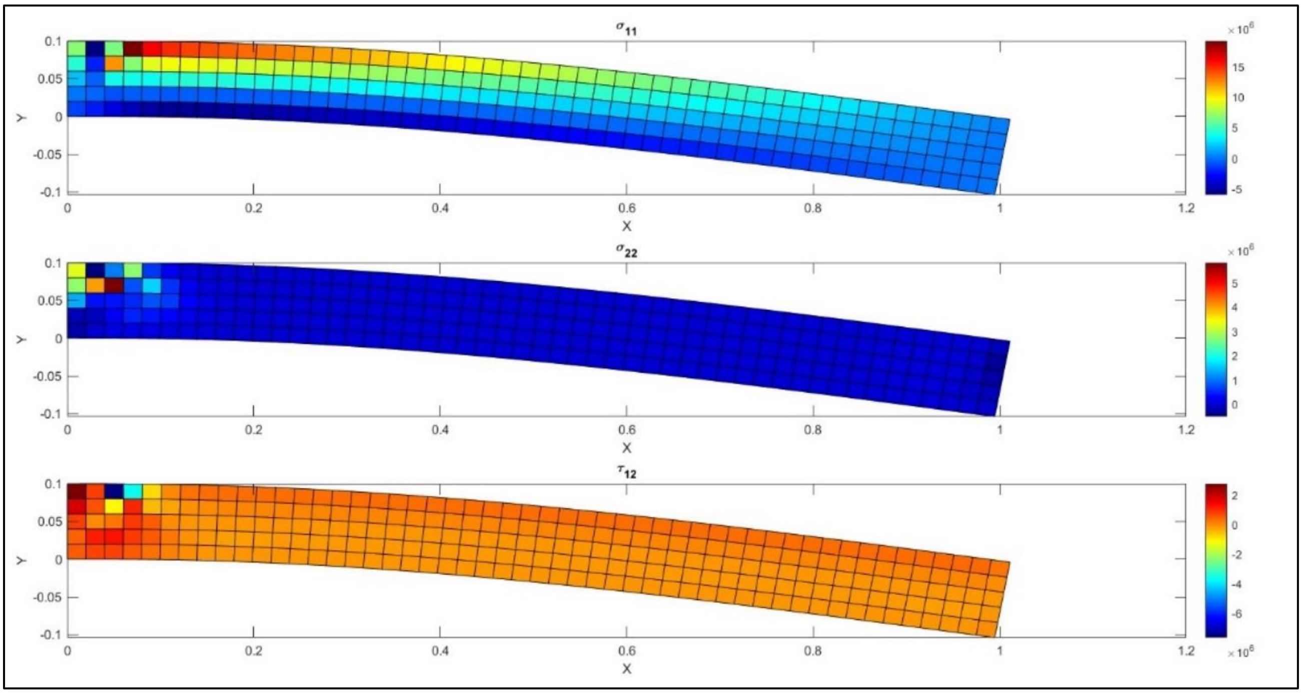

Fig 3: Stress components (σ_{11}, σ_{22}, τ_{12}) for the deformed beam using isoparametric quad elements

b) Plots from MATLAB Code using CST elements (Assignment 7):

- In Assignment 7, a coarse mesh was used for this question, hence for this case a finer mesh is used such that the nodes of the quad and tri meshes correspond to each other

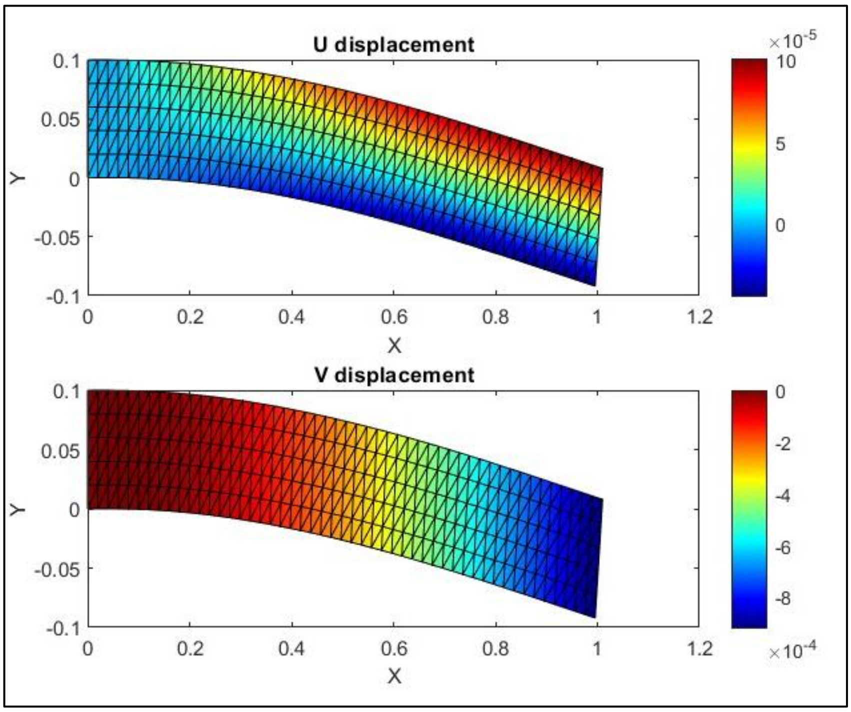

Fig 4: Contours of U and V displacements of the beam using CST elements

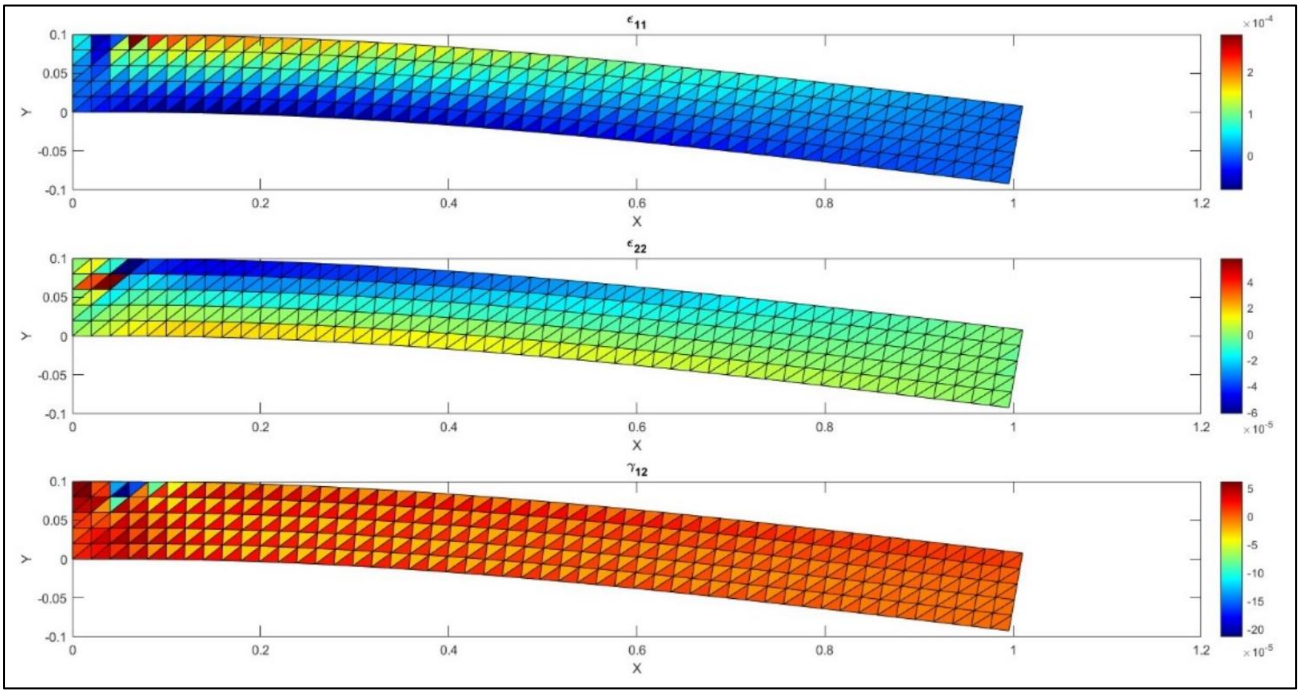

Fig 5: Strain components (ε_{11}, ε_{11}, Υ_{12}) for the deformed beam using CST elements

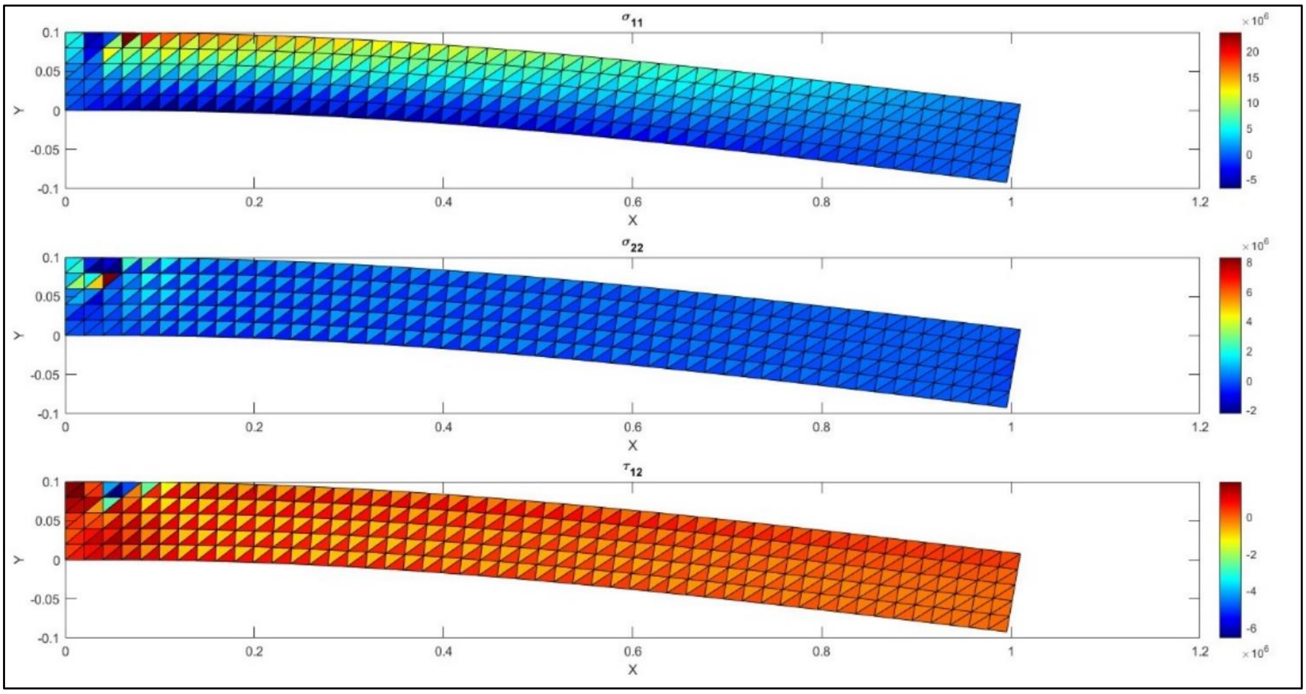

Fig 6: Stress components (σ_{11}, σ_{22}, τ_{12}) for the deformed beam using CST elements

c) Plots from ABAQUS Simulation:

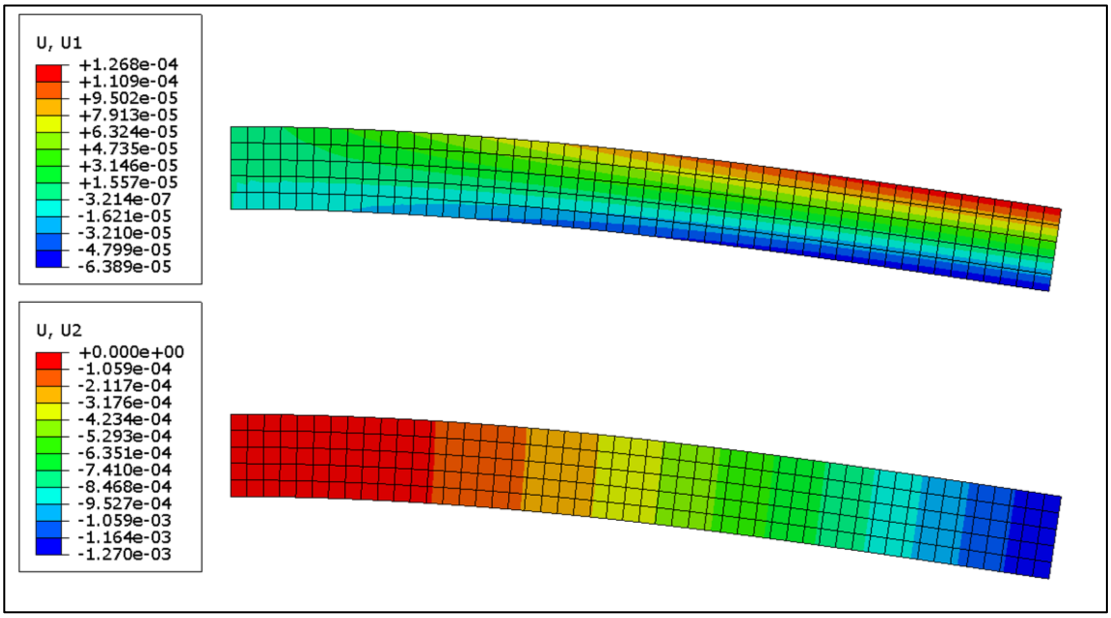

Fig 7: Contours of U and V displacements of the beam using quad elements in ABAQUS

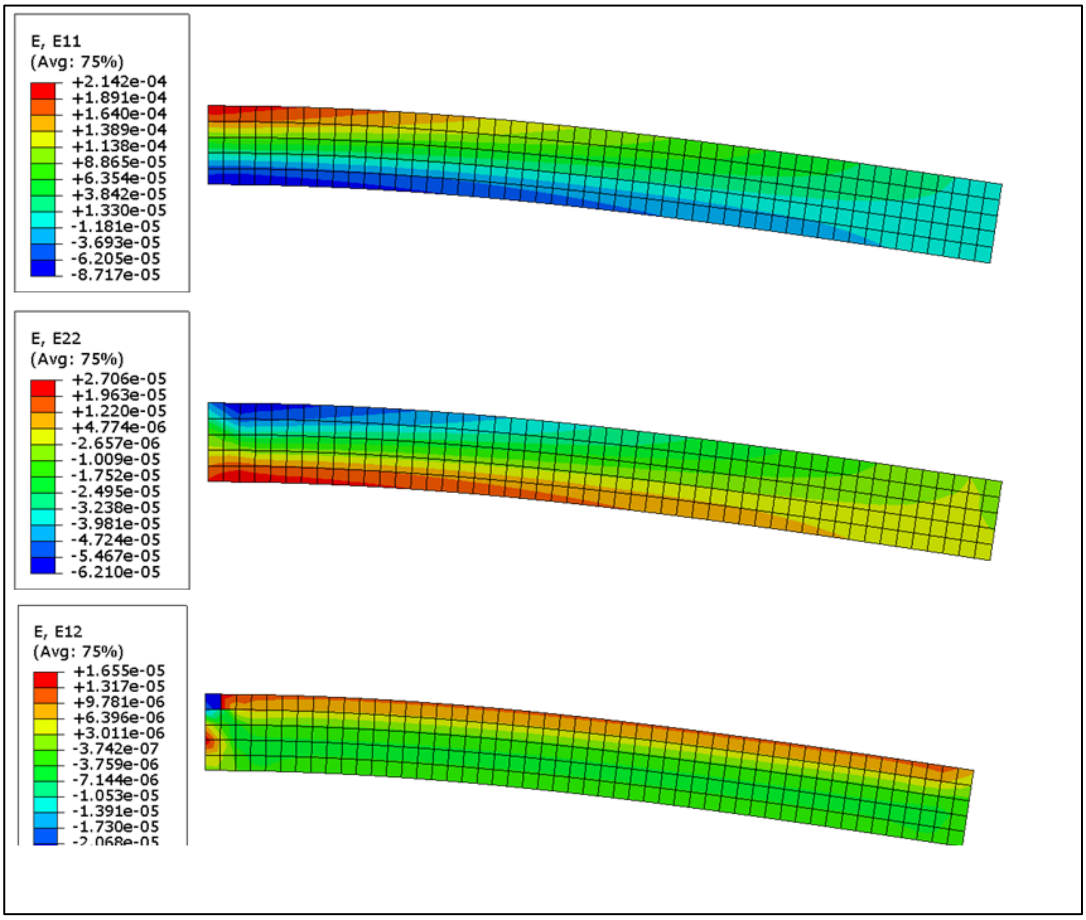

Fig 8: Strain components (E11, E22, E12) for the deformed beam using quad elements in ABAQUS

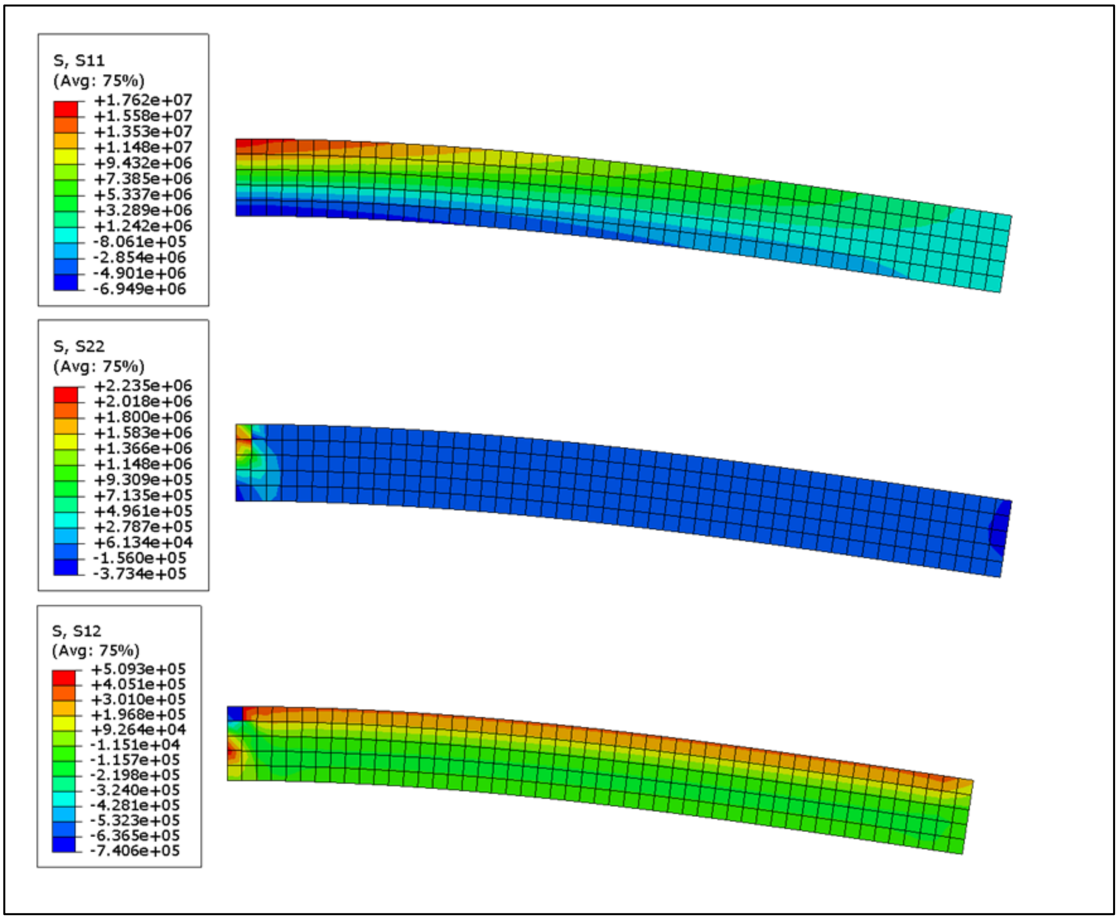

Fig 9: Stress components (S11, S22, S12) for the deformed beam using quad elements in ABAQUS

- The MATLAB codes for the isoparametric quad element and updated code for finer mesh for CST elements is attached with the report along with corresponding mesh details in separate excel sheets.
- The comparison of displacements at each node for the isoparametric quad, CST and ABAQUS simulation with quad elements in given in the Q1_Comparison excel sheet.
- The strain and stress are only compared in a tabular form between the quad elements because it cannot be directly compared for triangular and quad elements as the total number of elements is different.
- From the figures above, qualitatively the variation of the displacement, strain and stress throughout the domain is similar in all the three cases.

Q2)

- The MATLAB code and mesh data from ABAQUS (Q2_QuadMesh) is attached to the submission.
- In Assignment 7 CST elements with a plane stress assumption was used which may not be appropriate for the given case and hence the results were not accurate

- Here, we use the axisymmetric formulation with isoparametric quad elements in the r (or x), z (or y), and θ coordinates. For the axisymmetric case, 11, 22, 33 correspond to the radial, axial and azimuthal components of the strain and stress respectively.

Plots using isoparametric quad elements with axisymmetric assumption:

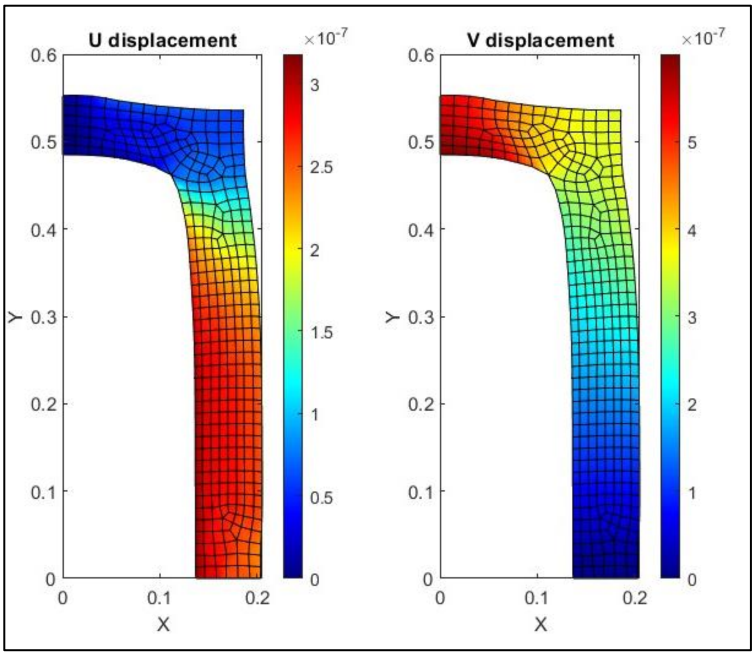

Fig 10: U and V displacements for the pressure vessel using isoparametric quad elements

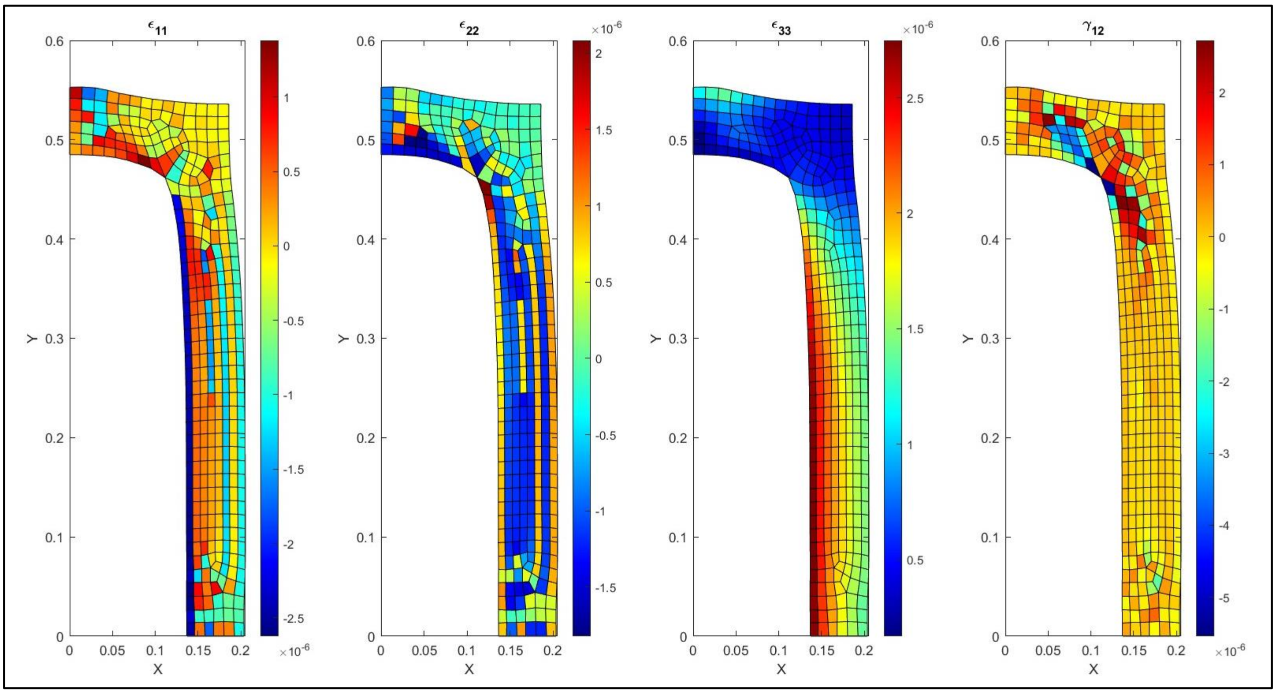

Fig 11: Strain components (ε_{rr}, ε_{zz}, $\varepsilon_{\theta\theta}$, Υ_{rz}) for the deformed pressure vessel using isoparametric quad element

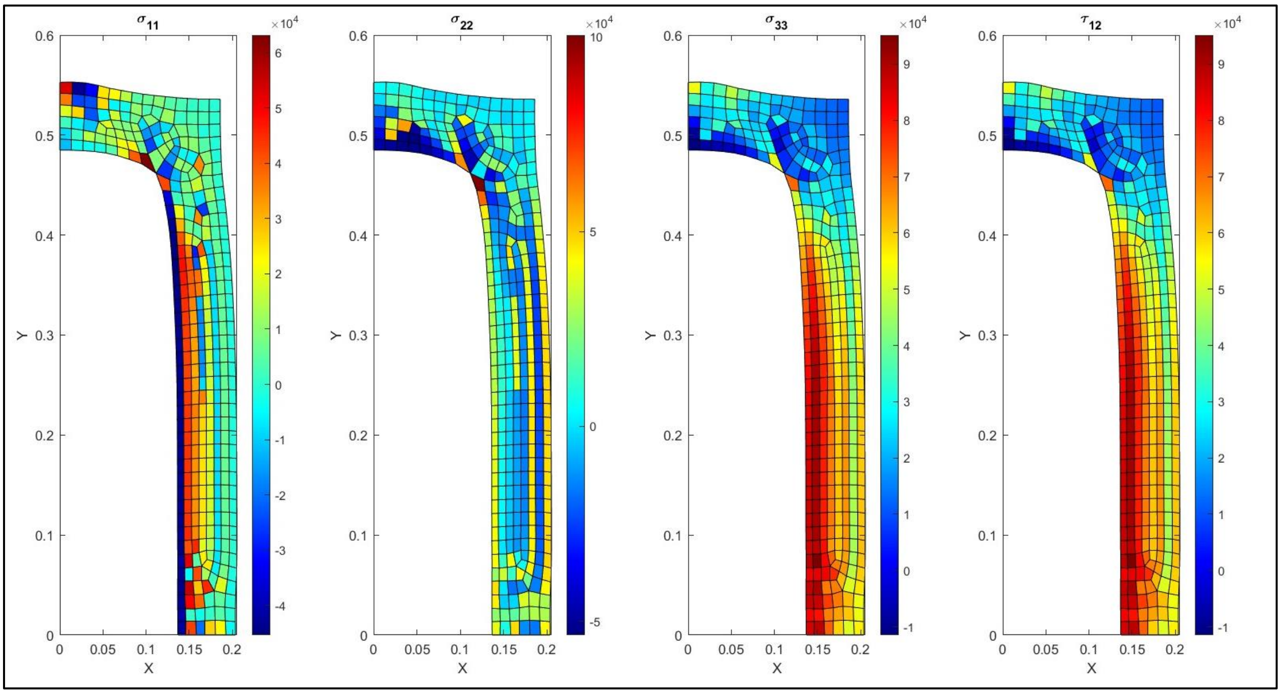

Fig 12: Stress components (σ_{rr}, σ_{zz}, $\sigma_{\theta\theta}$, τ_{rz}) for the deformed pressure vessel using isoparametric quad elements

Plots using tri elements with plane stress assumption (Assignment 7):

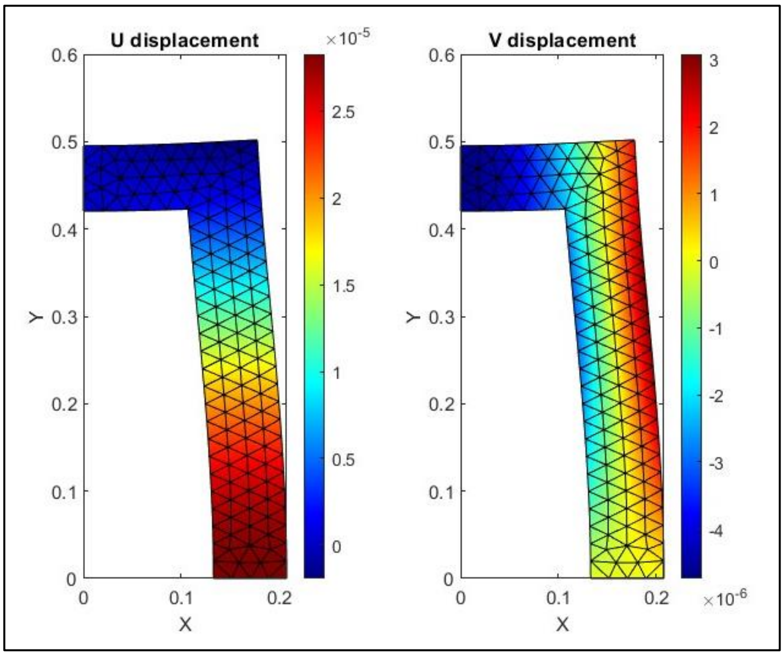

Fig 13: U and V displacements for the pressure vessel using CST elements

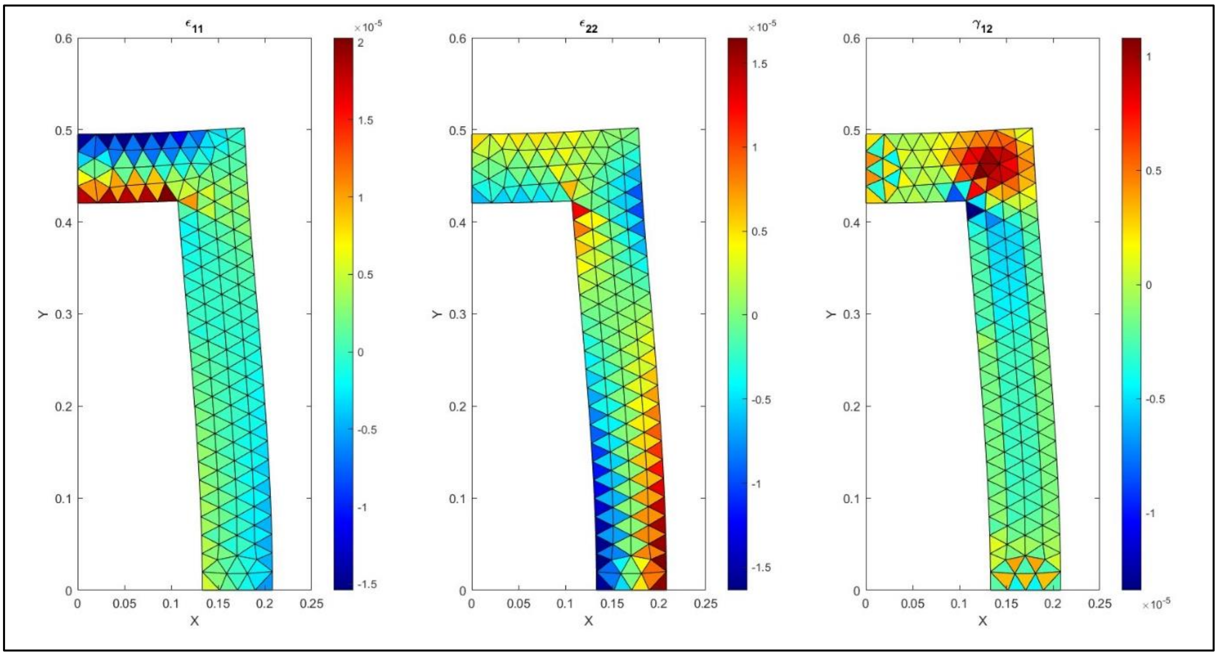

Fig 14: Strain components (ε_{11}, ε_{11}, Υ_{12}) for the deformed pressure vessel

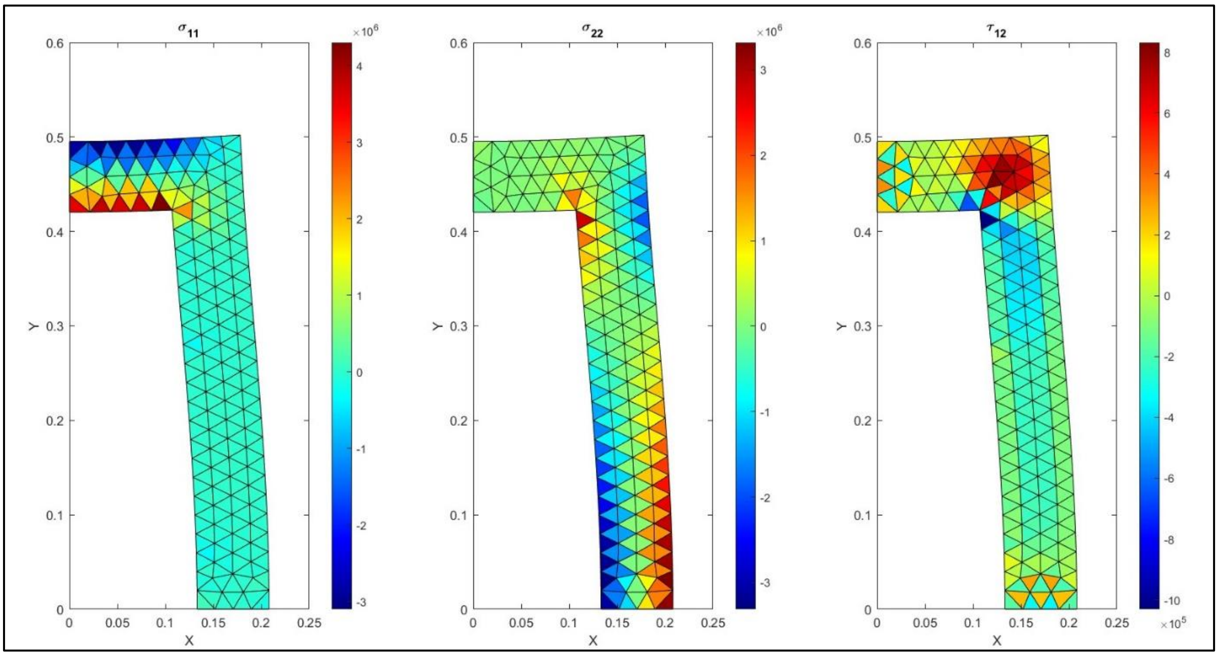

Fig 15: Stress components (σ_{11}, σ_{22}, τ_{12}) for the deformed pressure vessel

- We calculate the hoop stress as the maximum absolute value of the σ_{11} or σ_{rr} stress component in the domain and similarly the longitudinal stress is calculated as maximum absolute value of the σ_{22} or σ_{zz}.
- Using the theory of thin-walled cylindrical pressure vessels, we can calculate the hoop and longitudinal stresses as Pt/2d and Pt/4d respectively. Following is the code output which shows a comparison of both

CST Element Results (Plane Stress):

```
Command Window
The hoop stress calculated is 3511018.219436 N
The longitudinal stress calculated is 2439734.355765 N
The analytical hoop stress assuming thin walled vessel is 133333.333333 N
The analytical longitudinal stress assuming thin walled vessel is  66666.666667 N
fx >>
```

Quad Element Results (Axisymmetric):

```
The hoop stress calculated is 63164.403247 N
The longitudinal stress calculated is 100474.887269 N
The analytical hoop stress assuming thin walled vessel is 133333.333333 N
The analytical longitudinal stress assuming thin walled vessel is  66666.666667 N
fx >>
```

- The results using the isoparametric quad elements with axisymmetric assumption give better results than the CST elements with plane stress elements.
- Both the results are a lot different than each other because of the different mesh density, element type and most importantly the plane stress vs the axisymmetric assumption.
- But still, they are not very close to the analytical solution as the cylinder does not satisfy the criteria for a thin-walled vessel.
- The d/t ratio for this vessel (5.33) is less than 20, so it does not satisfy the criteria for a thin-walled vessel (d/t > 20). Hence, the equation for a thick-walled pressure vessel should be used.

If we use a finer mesh by increasing the number of elements and calculate the analytical stresses using the formulation for a thick cylindrical pressure vessel, more accurate results can be obtained.