Department of Applied Mechanics and Biomedical Engineering, Indian Institute of Technology, Madras.

Fundamentals of Finite Element Analysis (AM 5450)
Assignment 1 – Weighted Residual Methods - I

Submission Date: 26 August 2024

<u>Instructions:</u>

I. Use scientific non-programmable calculator wherever necessary.
II. MATLAB/PYTHON/Python scripts used to prepare plots/figures/results should be attached with your answer.
III. Scan all the pages of your hand-written answers, MATLAB/PYTHON scripts, output results, and figures into a PDF file. Please submit your assignment in Google class room.

1) Find the approximate solution $\tilde{u}(x)$ of the following differential equation using the **least square and Galerkin's methods** by hand calculations. Obtain solutions by assuming polynomials of order 1 to 3. Include both domain and boundary residual as applicable.

$$\frac{d^2u}{dx^2} - 6x = 0$$

with the boundary conditions given as, $u(0) = 0$ and $u(3) = 27$.

a) Plot the solution $\tilde{u}(x)$ and its derivative $\tilde{u}'(x)$ as a function of x in MATLAB/PYTHON for each polynomial order 1 to 3. Also compute the exact solution using analytical method and compare its plot in MATLAB/PYTHON with the solutions obtained by least square method.

b) Plot the residual function $e(x)$ by substituting back the obtained $\tilde{u}(x)$ for each polynomial order 1 to 3.

2) Find the approximate solution $\tilde{u}(x)$ of the following differential equation using the **least square and Galerkin's methods** in MATLAB/PYTHON. Obtain solutions by assuming polynomials of order 1 to N. Include both domain and boundary residue as applicable.

$$\frac{d^2u}{dx^2} + k^2u = 0$$

with the boundary conditions given as, $u(0) = 0$ and $u'(1) = \overline{u}$. Assume $k = 5$ and $\overline{u} = 10$.

a) Find the highest polynomial order N at which the assumed solution $\tilde{u}(x)$ converges to exact solution $u(x)$ and the residual function $e(x)$ = 1E-6 (approximates to 0) for all values of x in the domain.

b) Plot the solution $\tilde{u}(x)$ as a function of x in MATLAB/PYTHON for each polynomial order 1 to N. Also compare the plot of exact solution given by $u(x) = \left(2\, sin(5x)\right)/cos(5)$ in MATLAB/PYTHON with the solutions obtained by least square method.

c) Plot the residual function $e(x)$ by substituting back the obtained $\tilde{u}(x)$ for each polynomial order 1 to N.