CH20B012 Anish Anand Pophale

AM5450 Assignment 1

Q1)

MATLAB Code:

```
clc; clear all; close all; format compact; format shortg;

syms x y z;
eqn1 = 2*x - 3*y - z == 5;
eqn2 = 4*x + 4*y - 3*z == 3;
eqn3 = -2*x + 3*y - z ==1;

sol = solve([eqn1, eqn2, eqn3], [x,y,z]);
x = sol.x
y = sol.y
z = sol.z
```

Output:

```
Command Window
x =
-1/2
y =
-1
z =
-3
```

Q2)

MATLAB Code:

```
clc; clear all; close all; format compact; format shortg;

syms x a0 a1 a2 a3 a4 a5

%Degree 5 polynomial
p = poly2sym([a5,a4,a3,a2,a1,a0])

%1st to 5th order derivatives
d1 = diff(p,x,1)
d2 = diff(p,x,2)
d3 = diff(p,x,3)
d4 = diff(p,x,4)
d5 = diff(p,x,5)

%Partial derivatives wrt to ai
da0 = diff(p,a1,1)
da1 = diff(p,a1,1)
da2 = diff(p,a2,1)
da3 = diff(p,a3,1)
da4 = diff(p,a4,1)
da5 = diff(p,a5,1)
```

Output:

a) Polynomial for k = 5

```
p =
a5*x^5 + a4*x^4 + a3*x^3 + a2*x^2 + a1*x + a0
```

b) 1st to 5th order derivatives with respect to x

```
d1 =
5*a5*x^4 + 4*a4*x^3 + 3*a3*x^2 + 2*a2*x + a1
d2 =
20*a5*x^3 + 12*a4*x^2 + 6*a3*x + 2*a2
d3 =
60*a5*x^2 + 24*a4*x + 6*a3
d4 =
24*a4 + 120*a5*x
d5 =
120*a5
```

c) Partial derivatives with respect to a_i:

```
da0 =
x
da1 =
x
da2 =
x^2
da3 =
x^3
da4 =
x^4
da5 =
x^5
```

Q2>

b) $P_n(x) = \sum_{n=0}^{K} a_n x^n$

$\therefore P_5(x) = a_5 x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x_1 + a_0$

$\frac{dP_5}{dx} = 5a_5 x^4 + 4a_4 x^3 + 3a_3 x^2 + 2a_2 x + a_1$

$\frac{d^2P_5}{dx^2} = 20 a_5 x^3 + 12 a_4 x^2 + 6a_3 x + 2a_2$

$\frac{d^3P_5}{dx^3} = 60 a_5 x^2 + 24 a_4 x + 6a_3$

$\frac{d^4P_5}{dx^4} = 120 a_5 x + 24 a_4$

$\frac{d^5P_5}{dx^5} = 120 a_5$

The hand calculated derivatives match with the values using MATLAB above

c) $\frac{\partial P_5}{\partial a_0} = 0$

$\frac{\partial P_5}{\partial a_1} = x$

$\frac{\partial P_5}{\partial a_2} = x^2$

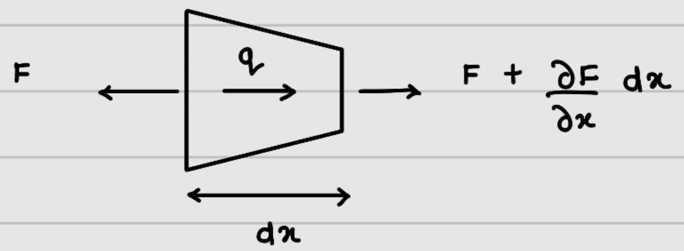
$\frac{\partial P_5}{\partial a_3} = x^3$

$\frac{\partial P_5}{\partial a_4} = x^4$

$\frac{\partial P}{\partial a_5} = x^5$

Again, these hand calculated values match with the ones calculated using MATLAB

Q3) Consider an infinitesimally small part of the bar dx



Using Newton's second law, $\Sigma F = ma$

$$\therefore \quad F + \frac{\partial F}{\partial x} dx - F + q\,dx = ma$$

$$\therefore \quad \frac{\partial F}{\partial x} dx + q\,dx = ma$$

As the cross sectional area varies linearly from A_0 at $x = 0$ to A_L at $x = L$,

$$\therefore \; A(x) = A_0 + \frac{x}{L}(A_L - A_0)$$

For static equilibrium case, $a = \frac{d^2u}{dt^2} = 0$

$$\therefore \quad \frac{\partial F}{\partial x} dx + q\,dx = 0, \quad \frac{\partial F}{\partial x} + q = 0$$

Now, $F = \sigma A(x)$ and $\sigma = E\varepsilon = E\frac{du}{dx}$

$$\therefore \; F(x) = E A(x)\frac{du}{dx} = E\left\{A_0 + \frac{x}{L}(A_L - A_0)\right\}\frac{du}{dx}$$

Also, $q = cx$

$$\therefore \text{ we get } \quad \frac{d}{dx}\left\{E\left[A_0 + \frac{x}{L}(A_L - A_0)\right]\frac{du}{dx}\right\} + cx = 0$$

$$\therefore \; E\left[A_0 + \frac{x}{L}(A_L - A_0)\right]\frac{d^2u}{dx^2} + \frac{E}{L}(A_L - A_0)\frac{du}{dx} + cx = 0,$$

with $0 \leq x \leq L$

As the bar is fixed at $x = 0$,

$u(x=0) = 0$ is the Dirichilet BC at $x = 0$

At $x = L$,

$$F = \sigma A_L = E A_L \varepsilon_L = E A_L \left.\frac{du}{dx}\right|_{x=L} = P$$

$$\therefore \left.\frac{du}{dx}\right|_{x=L} = \frac{P}{EA_L}$$ is the Neumann BC at $x = L$

Q4) For constant cross sectional area, the governing equation becomes

$$EA\frac{d^2u}{dx^2} + cx = 0 \quad ; \quad 0 \leq x \leq L$$

$$\therefore \frac{d^2u}{dx^2} = -\frac{cx}{EA}$$

Integrating, $\displaystyle\int \frac{d^2u}{dx^2}\,dx = \int -\frac{cx}{EA}\,dx$

$$\therefore \frac{du}{dx} = -\frac{cx^2}{2EA} + k_1$$

From the Neumann BC at $x = L$,

$$\left.\frac{du}{dx}\right|_{x=L} = \frac{P}{EA}$$

$$\therefore -\frac{cL^2}{2EA} + k_1 = \frac{P}{EA}$$

$$\therefore k_1 = \frac{2P + cL^2}{2EA}$$

$$\therefore \frac{du}{dx} = -\frac{cx^2}{2EA} + \frac{2P + cL^2}{2EA}$$

Integrating again, $\displaystyle\int \frac{du}{dx}\,dx = \int -\frac{cx^2}{2EA} + \frac{2P + cL^2}{2EA}\,dx$

$$\therefore u(x) = -\frac{cx^3}{6EA} + \frac{(2P + cL^2)}{EA}x + k_2$$

Using the Dirichilet BC $u(0) = 0$ at $x = 0$,

$\therefore K_2 = 0$

$\therefore$ The analytical solution for $u(x)$ and $u'(x)$ are

$$u(x) = -\frac{cx^3}{6EA} + \frac{(2P + cL^2)x}{2EA}$$

$$u'(x) = -\frac{cx^2}{2EA} + \frac{2P + cL^2}{2EA}$$

Values of all constants are substituted in MATLAB and the required plots are generated as follows:

Q4)

MATLAB code for calculating and plotting displacement and strain fields:

```
clc; clear all; close all; format compact; format shortg;

%Constants
E = 200*10^9; d = 20*10^-3; A = pi*d^2/4; P = 2*10^3; L = 2; C = 10^3;

x = 0:0.01:L;

%Calculating displacement and strain fields
u = -(C/(6*E*A))*x.^3 + ((2*P+C*L^2)/(2*E*A))*x;
u_prime = -(C/(2*E*A))*x.^2 + (2*P+C*L^2)/(2*E*A);

%Plotting
figure;
plot(x,u,'-r')
title("u vs x")
xlabel("Length (m)")
ylabel('Displacement (m)')

figure;
plot(x,u_prime,'-b')
title("u' vs x")
xlabel("Length (m)")
ylabel('Strain')
```

```

%Changing the values of c
C_vec = 10^3*[1,3,5,7,9];

figure;
for i = 1:numel(C_vec)
    C = C_vec(i);
    u = -(C/(6*E*A))*x.^3 + ((2*P+C*L^2)/(2*E*A))*x;
    plot(x,u,DisplayName=num2str(C)); hold on
end
title("u vs x")
xlabel("Length (m)")
ylabel('Displacement (m)')
legend("show")

figure;
for i = 1:numel(C_vec)
    C = C_vec(i);
    u_prime = -(C/(2*E*A))*x.^2 + (2*P+C*L^2)/(2*E*A);
    plot(x,u_prime,DisplayName=num2str(C)); hold on
end
title("u' vs x")
xlabel("Length (m)")
ylabel('Strain')
legend("show")
```

Plots:

a)

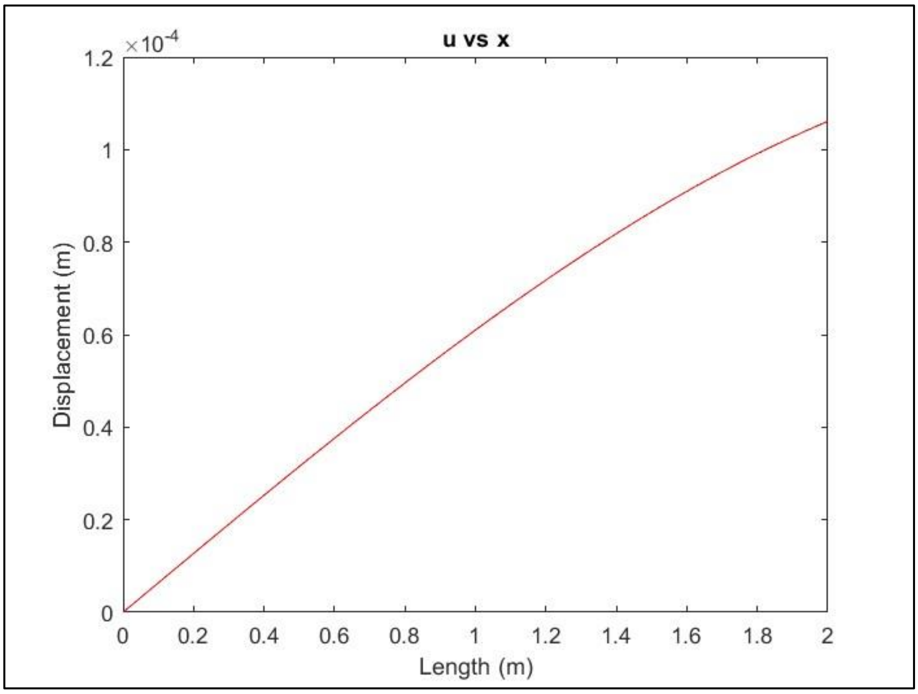


Fig 1: Displacement field vs x for c = 1 kN/m

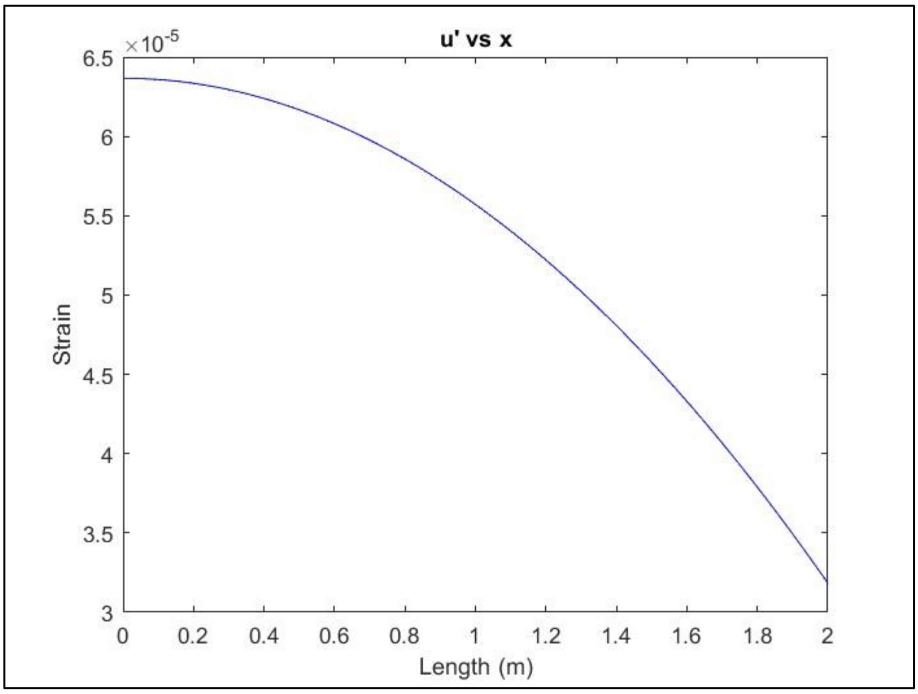


Fig 2: Strain vs x for c = 1 kN/m

b) Comparison of strain and displacement fields for different values of c

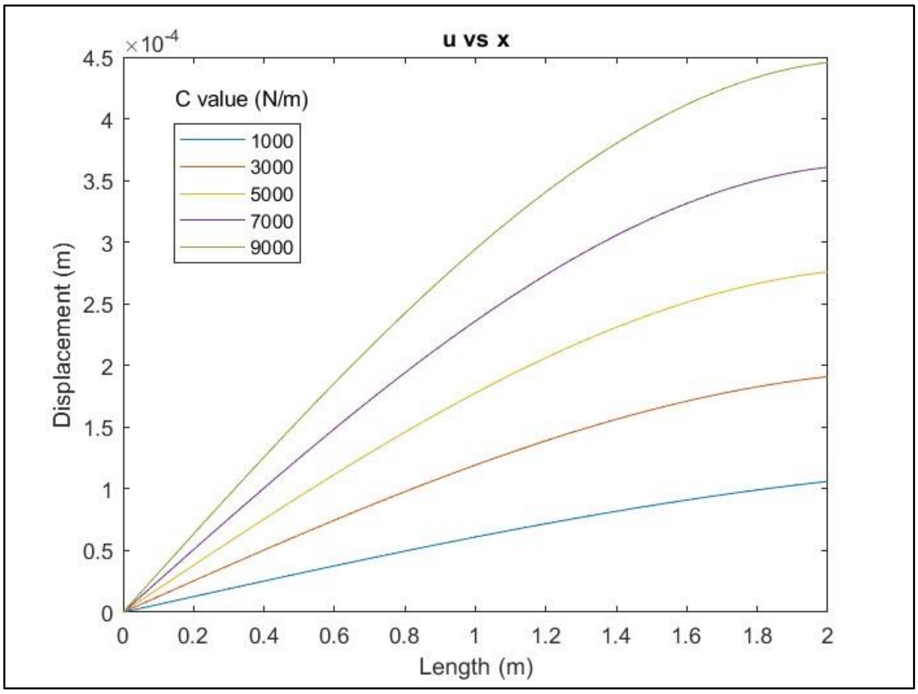


Fig 3: Displacement field vs x for different values of c

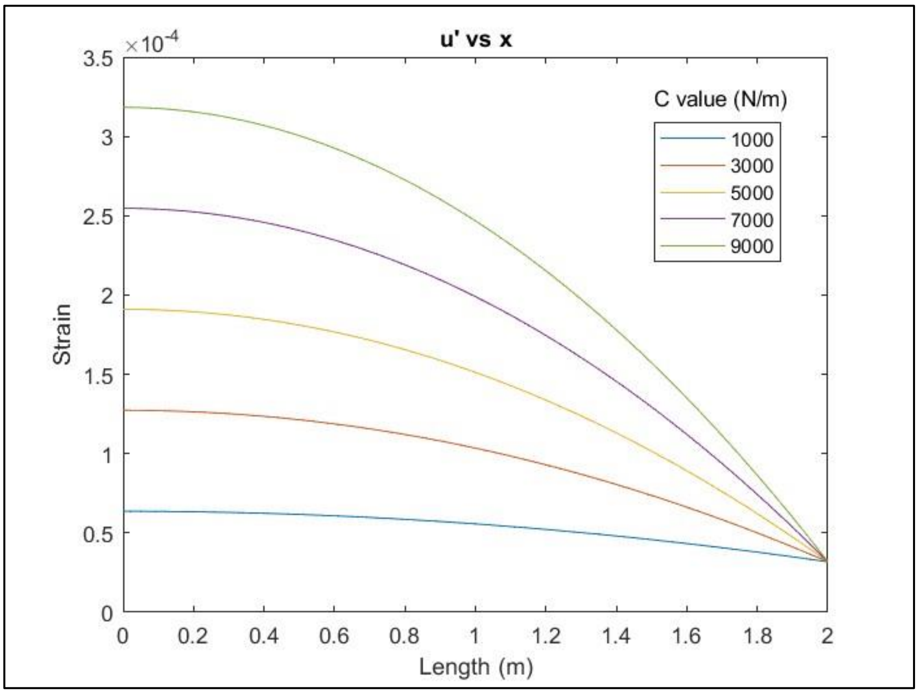


Fig 4: Strain vs x for different values of c

- From the above plots, we can observe that the variation (or nature of the curves) of the displacement as well as the strain with the length x remains the same for all values of c.
- But as C increases, the values of displacement and the strain also increase.