Department of Applied Mechanics and Biomedical Engineering, Indian Institute of Technology, Madras.

Fundamentals of Finite Element Analysis (AM 5450)
Assignment 4 – 1D FE Solutions

Submission Date: 9 Sept 2024

Instructions:

I. Use scientific non-programmable calculator wherever necessary.
II. MATLAB/PYTHON scripts used to prepare plots/figures/results should be attached with your answer.
III. Scan all the pages of your hand-written answers, MATLAB/PYTHON scripts, output results, and figures into a PDF file.

Definitions –

- p-type FE approach – It is an approach where improvement in the solution of the field variable $\tilde{u}(x)$ is sought by implementing higher degree polynomial approximation for the solution.
- h-type FE approach – In order to improve the solution of the field variable $\tilde{u}(x)$ number of finite elements (with the same order) are increased until the desired relative error threshold is achieved.
- Combination of p-type and h-type – For rapid convergence to the solution, number of higher-order finite elements (based on higher degree polynomial approximation) are used; although, this approach requires more computational resources.
- Relative error R_E for the solution $\tilde{u}(x)$ obtained with *N* and *N*-1 finite elements is computed as follows.

$$R_E = \frac{\tilde{u}_N - \tilde{u}_{N-1}}{\tilde{u}_N}$$

- A solution quantity $\tilde{u}(x)$ (or another quantity say strain $\tilde{u}'(x)$) is said to be converged if the Relative error in that quantity attains a desired preset threshold criteria i.e. R_E <= 1e-6. (Use this criteria for this assignment)

Question

Find the displacement field $\tilde{u}(x)$ and strain $\tilde{u}'(x)$ for the governing differential equation of the tapered bar as shown in ***Figure 1*** below using the **three approaches mentioned in A and B** programmed in MATLAB/PYTHON.

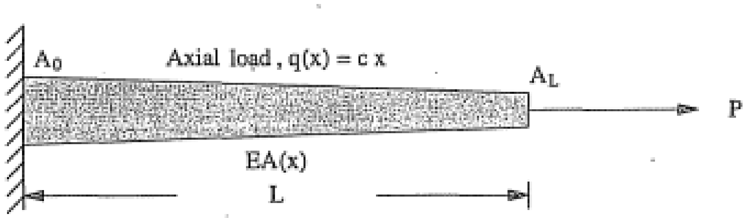

Figure 1 Axially loaded bar with linearly varying cross-section area A(x)

Assume $E = 200$ GPa, $A_0 = 3.1416E-04$ m², $A_L = 1.9635E-05$ m², $L = 2$ m, $P = 2$ kN, and $q(x) = c = 1$ kN/m.

$$\frac{d}{dx}\left(EA(x)\frac{du}{dx}\right) + q(x) = 0$$

with the boundary conditions given as, $u(0) = 0$ and $AEu'(x)|_{x=L} = P$.

A. Use the **Galerkin's method** by assuming polynomials of order 1 to N_P, where $\boldsymbol{N_P}$ is the converged polynomial order at which the residual function $e(x)$ = 1E-6 (approximates to 0) for all values of x in the domain. **(a)** Plot the displacement field $\tilde{u}(x)$ and strain $\tilde{u}'(x)$ as a function of x in MATLAB/PYTHON for each polynomial order 1 to N_P. **(b)** Plot the residual function $e(x)$ by substituting back the obtained $\tilde{u}(x)$ for each polynomial order 1 to N_P.

B. Use the **Galerkin's method with a 2-noded finite element form of assumed solution with the linear approximation** $\tilde{u}(x) = N_1 u_1 + N_2 u_2$. Use the Mapping to convert the interpolation functions N_i to a local element CS ranging over -1 to +1. **(a)** Find out the number of such linear elements $\boldsymbol{N_{LE}}$ required to obtain a converged nodal displacement at the tip point and at the mid-point of the tapered bar. **(b)** Find out the relative error for the strain $\tilde{u}'(x)$ in the elements carrying nodes of the tip and mid-point of the tapered bar when $\boldsymbol{N_{LE}}$ number of elements are used. **(c)** Plot the solutions of displacement field $\tilde{u}(x)$ and strain $\tilde{u}'(x)$ over the length of bar (separate plots in MATLAB/PYTHON) as the number of elements increase upto $\boldsymbol{N_{LE}}$ (Show min. 5 plots).

Which approach among A and B seems to be better? Answer based on following points –

1) Considering the computational complexity and time required.
2) Accuracy of the obtained solution and its derivative with minimum computational effort.