

Part 1: The k- ω model

Model used: Wilcox's k- ω model

Boundary Conditions:

Wall -

1. $u = 0$
2. $k = 0$
3. $\omega (y^+ \leq 3) = C_2 * 6 \nu / y^2$

From the given mesh, the first 8 nodes lie in the region of $y^+ \leq 3$, so we apply the above ω boundary condition for nodes 2 to 8. For the wall node, we take the same value as that of node 2.

Channel centerline -

Neumann boundary condition is used for u , k and ω at the channel centerline.

Convergence:

- Under relaxation factor = 0.5 (Code converges to same solution without under relaxation in lesser number of iterations)
- Number of iterations = 93295
- Residual limit = 10^{-4}

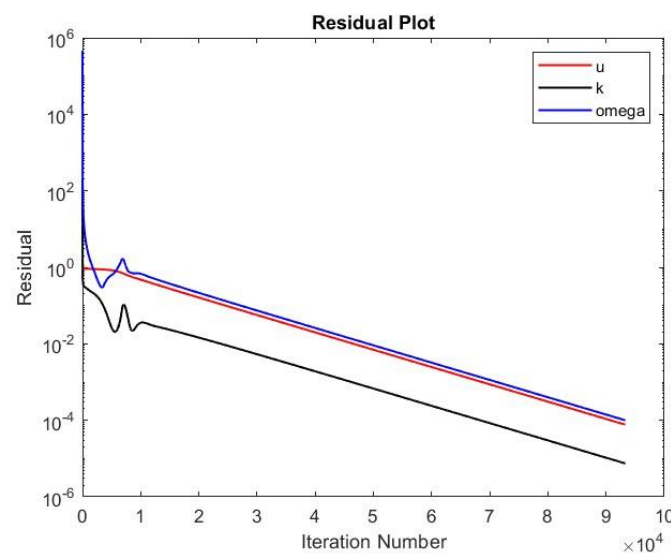


Fig 1: Residual vs Iteration Number for each of the equations

Wall shear stress:

From the calculated velocity profile, the wall shear stress is found out to be 0.9997 from the MATLAB code which is close to the actual value which should be 1.

```
tau_wall =
    0.9997
```

Comparison with DNS Data:

U velocity -

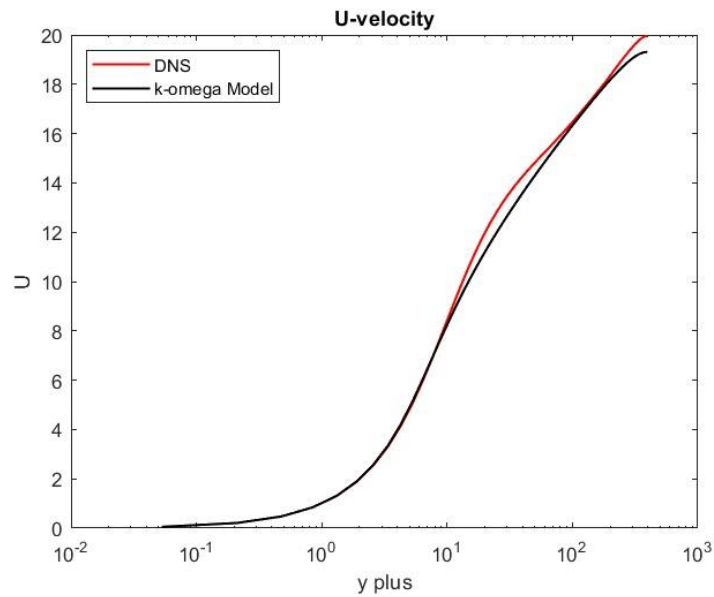


Fig 2: Comparison of U velocity for k- ω model and DNS data

- From the plot, we can see that the k- ω model predicts the velocity well.
- The prediction near to the wall is better than away from the wall where it deviates slightly.

Turbulence Kinetic energy (TKE) -

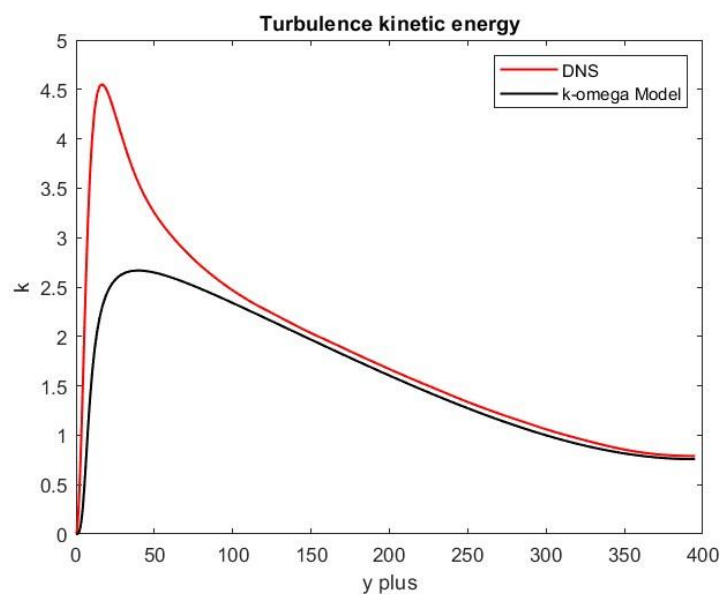


Fig 3: Comparison of TKE for k- ω model and DNS data

- The trend of the TKE vs y^+ for the model is similar to the DNS data, but the near wall gradient of TKE as well as the peak value are under predicted.
- The behaviour away from the wall is similar for the model and the DNS data

Dissipation Rate of TKE (ϵ)

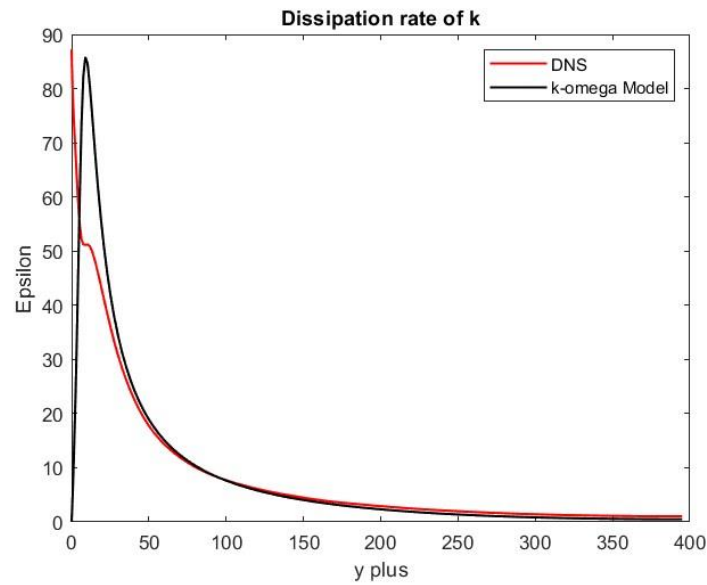


Fig 4: Comparison of ϵ for k- ω model and DNS data

- At the wall, the ϵ model goes to zero, but from DNS data and theory we know that it should go to a maximum value at the wall.
- In the k- ω model, we do not directly calculate ϵ , but it is calculated as $\epsilon = \beta^* k \omega$, as k is zero and ω goes to some large value on the wall, we get ϵ also as zero on the wall
- The model does show a maximum ϵ value close to the wall and then decays similar to the DNS data.

Reynolds Shear Stress ($-uv$) -

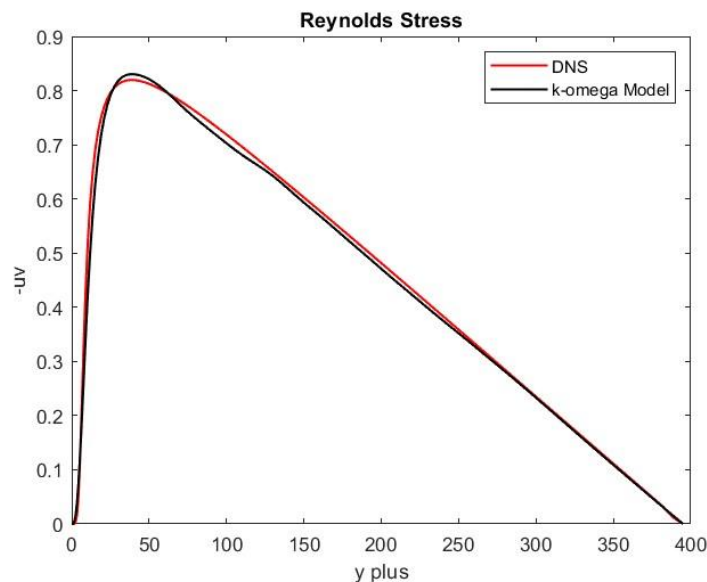


Fig 5: Comparison of Reynolds Shear Stress for k- ω model and DNS data

- For the model, Reynolds shear stress is calculated using the Boussinesq hypothesis as $\nu_t^* du/dy$ which is calculated from the converged ν_t and u values at each node.
- The values from the model and the DNS data are close to each other.

- Turbulence viscosity (ν_t) -

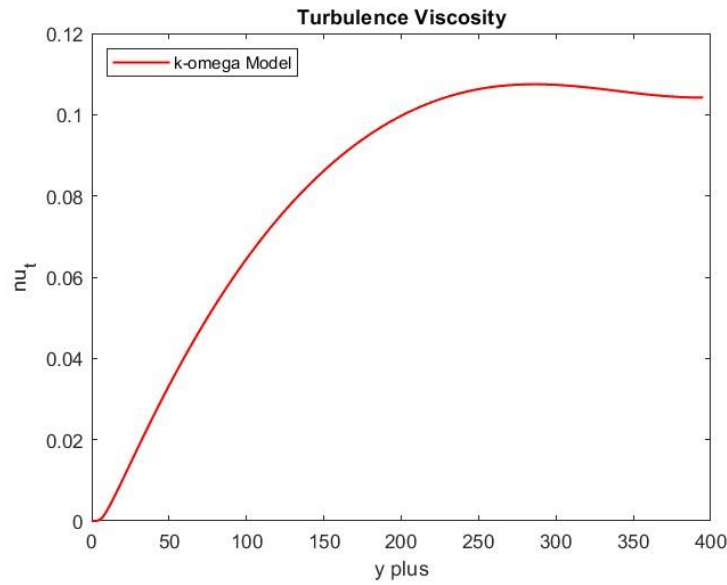


Fig 6: Turbulence viscosity vs y^+ for the k- ω model

- As expected, the turbulent viscosity at the wall is zero and it increases towards the center of the channel.
- For DNS, we do not have turbulent viscosity data as it is only a quantity used for modelling.

Budget of TKE:

Production Rate -

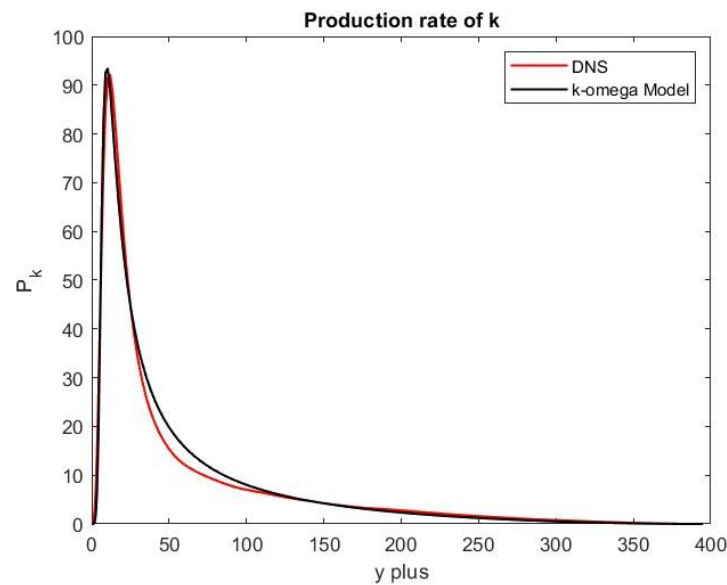


Fig 7: Comparison of Production Rate of TKE for k- ω model and DNS data

- The production rate for the model uses the Boussinesq hypothesis and is given as $\nu_t^*(du/dy)^2$
- The production rate of TKE from the model is zero at the wall, goes to maximum in the buffer layer and then goes to zero towards the center line, which is the same behaviour as DNS data and expected from theory.

Viscous Diffusion of TKE -

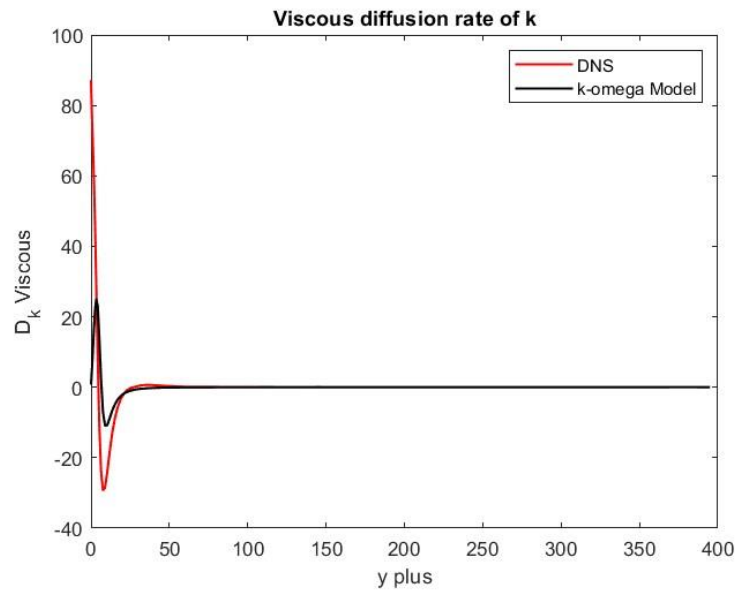


Fig 8: Comparison of Viscous Diffusion Rate of TKE for k- ω model and DNS data

- The viscous diffusion rate from the model is zero at the wall while the DNS data shows that viscous dissipation is maximum at the wall although the overall trend after that is similar.
- The viscous diffusion rate is calculated from the k values as $v^*(dk/dy)$
- The values from the model go to zero at the wall and are smaller than DNS data close to the wall. This is because the TKE values from the model are underpredicted and the near wall gradient of TKE is also smaller for the model compared to the DNS data.
- Away from the wall, as TKE values also are closer to each other, the viscous dissipation for model and DNS data shows close values.

Turbulent Diffusion of TKE -

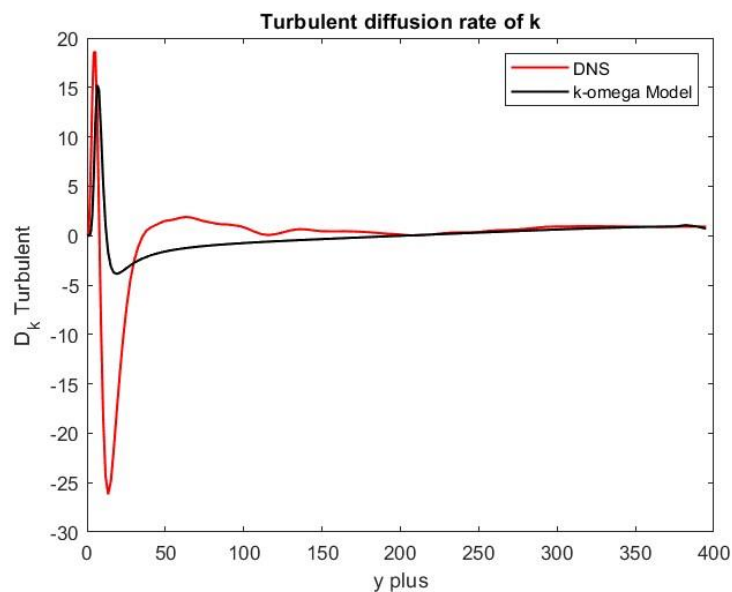


Fig 9: Comparison of Turbulent Diffusion Rate of TKE for k- ω model and DNS data

- The turbulent dissipation rate from the model shows similar trend as the DNS data but the values from the model are lower in magnitude than the DNS data
- It is zero at the wall, goes to a maximum value near the wall, then drops to a minimum and goes to zero away from the wall.

Pressure Diffusion of TKE –

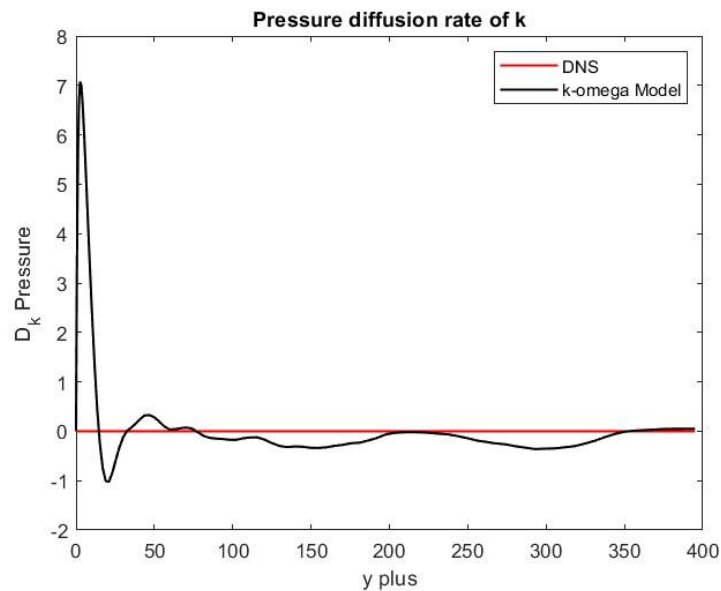


Fig 10: Comparison of Pressure Diffusion Rate of TKE for k- ω model and DNS data

- This term is ignored in the model and set to zero.
- From the DNS data, this is justified as the values of the pressure diffusion rates are small in magnitude compared to the turbulent and viscous diffusion rates

Budget of TKE (DNS Data):

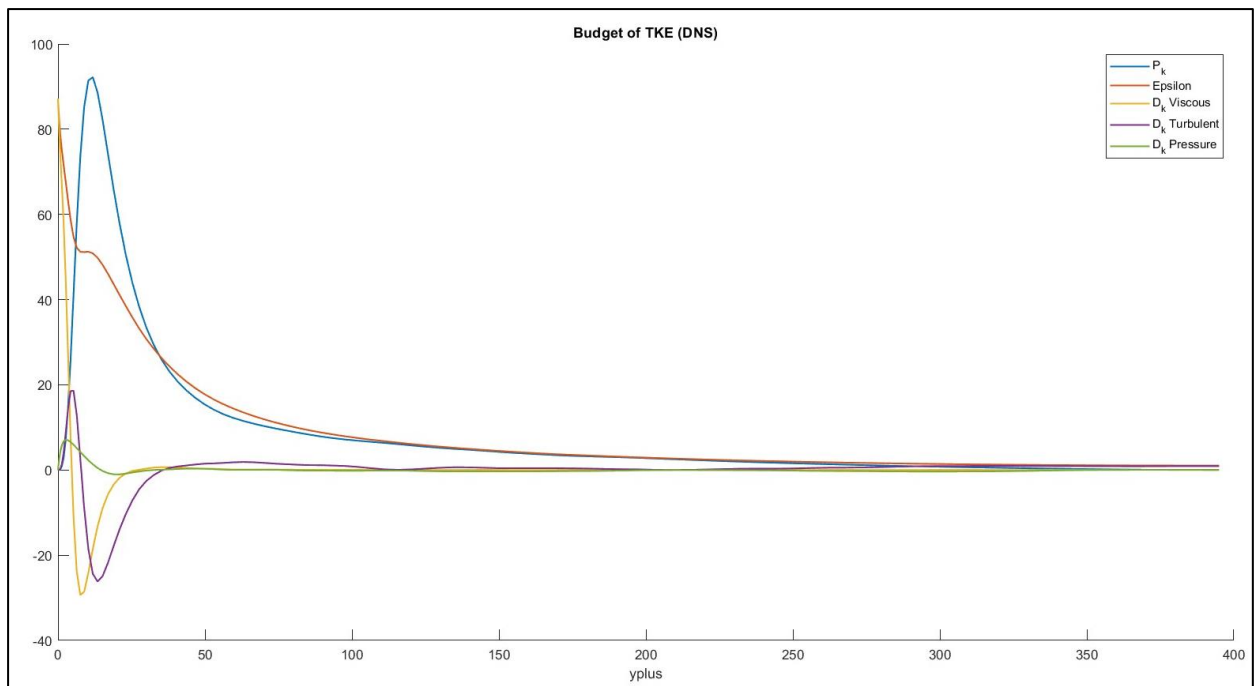


Fig 11: Budget of TKE from DNS data

Budget of TKE (k- ω model):

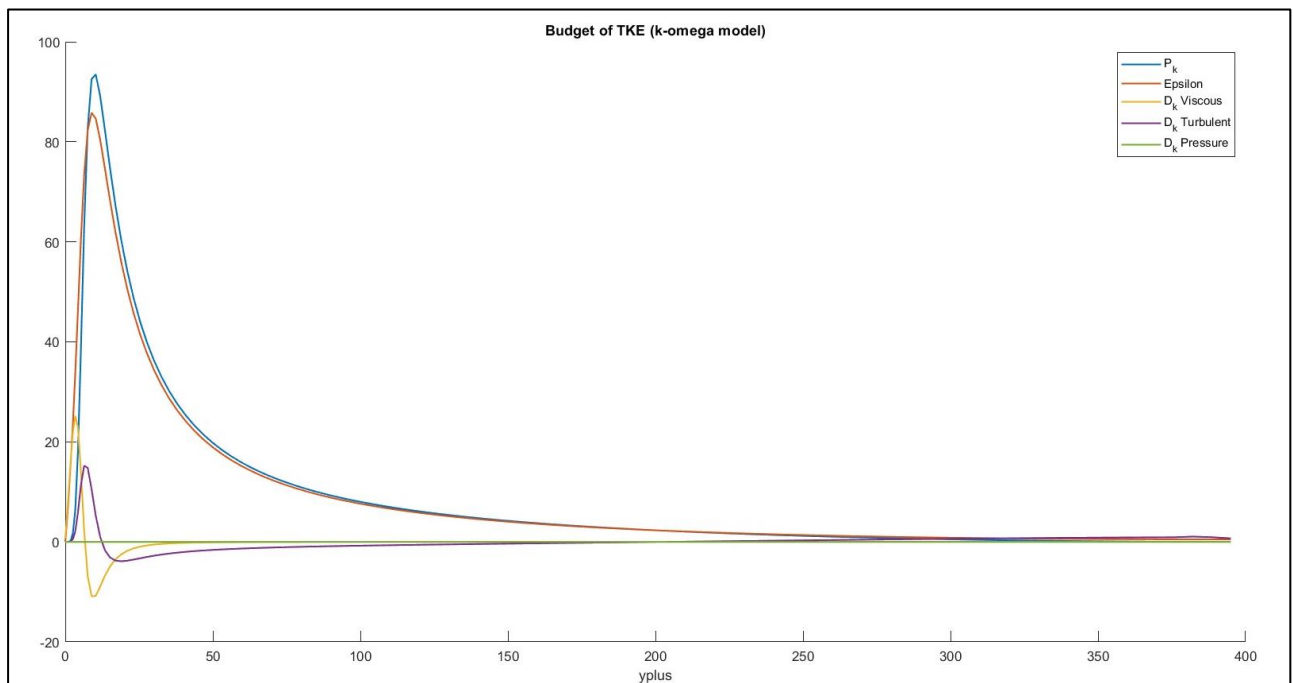


Fig 12: Budget of TKE from k- ω model