Fast Block LMS:

Note: X: data block vector
X_0: extended data block vector
X_{OF}: FFT of X_0

- filter order: M
- block size: L

extended block to use FFT: $N = L + M - 1$

$X_0 = [X_{\text{previous block, } M-1 \text{ points}} \; ; \; X_{\text{current block, } L \text{ points}}]$

$W_0 = [W, M \text{ points} \; ; \; 0_{\text{vector, } L-1 \text{ points}}]$

$$Y_0 = IFFT\{ \underbrace{FFT\{X_0\}}_{X_{OF}} .* \underbrace{FFT\{W_0\}}_{W_{OF}} \}$$

.* : element-wise multiplication

Filter output:

$Y = Y_0$, last L elements

$e = d - Y$

$e_0 = [0_{\text{vector, } M-1 \text{ points}} \; ; \; e] \rightarrow e_{OF} = FFT\{e_0\}$

Weights update:

$$W_{OF} = W_{OF} + 2 \cdot \mu \cdot X_{OF}^{*} .* e_{OF}$$

X_{OF}^{*}: conjugate of X_{OF}

.* : element-wise multiplication.

W_{OF}, last $L-1$ points $= 0$;

Repeat at updating new X_0

→ This is wrong: It should be: $W_0 = IFFT\{W_{OF}\}$

W_0, last $L-1$ points $= 0$; → make sure last $L-1$ taps are zeros in Time domain

$W_{OF} = FFT\{W_0\}$

Example: $M = 2$, $L = 4$ $\rightarrow$ $N = 2 + 4 - 1 = 5$

input block: $x_0 = [4 \; 5 \; 6 \; 7 \; 8]$

4 ↓ previous $M-1$ point; $[5 \; 6 \; 7 \; 8]$ current block data, L points

Weight vector: $w_0 = [1 \; 1 \; 0 \; 0 \; 0]$

$[0 \; 0 \; 0]$ 0 vector, $L-1$ point

output filter: FFT ← circular convolution.

$$\begin{bmatrix} \bullet\bullet\circ\circ\circ \\ y_5 = 5w_1 + 4w_2 \\ y_6 = 6w_1 + 5w_2 \\ y_7 = 7w_1 + 6w_2 \\ y_8 = 8w_1 + 7w_2 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 7 & 6 & 5 \\ 5 & 4 & 8 & 7 & 6 \\ 6 & 5 & 4 & 8 & 7 \\ 7 & 6 & 5 & 4 & 8 \\ 8 & 7 & 6 & 5 & 4 \end{bmatrix} \begin{bmatrix} w_1 = 1 \\ w_2 = 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

↑ circular matrix of input block x_0

↓ The last L elements are output filter, which is the linear convolution of $[5 \; 6 \; 7 \; 8]$ and $[1 \; 1]$