

Influence of the Betweenness Centrality to Characterize the Behavior of Communication in a Group

K. Raya-Díaz, C. Gaxiola-Pacheco, Manuel Castañón-Puga,
L. E. Palafox and R. Rosales Cisneros

Abstract The behavior of the distribution of a rumor must emerge according to the relations between the individuals. Taking as a reference that human society creates links of friendship through random encounters and conscious decisions, therefore, a rumor can be spread considering the degree of grouping that individuals have, also their location in the network and if they decide to cooperate or not. Considering the analysis of the topology that interconnects a set of individuals, relationships are detected between them that allow recognizing their centrality of degree, betweenness, and closeness. For a rumor to spread it requires that the individual has an incentive by which he decides to cooperate or not in the distribution of it. In this chapter, we propose an agent-based model that allows the identification of the central measures of each of the individuals that integrate a group which has a topology based on the Barbell's graph.

1 Introduction

Networks have been used to illustrate a system as a set of nodes joined by links, these nodes could represent persons and the links their relations. In [1] Estrada define a network or graph as a diagrammatic representation of a system. Some networks are defined as the pair $G = (V, E)$, where V is a finite set of nodes and E are the edges this representation is known as simple network, but there are other kind called weighted network defined by $G = (V, W, f)$ where V are the nodes, W are the edges or links that associates the nodes with an specific weight, and f is the mapping which associates some elements of E to a pair elements of V [1, 2]. This topology information of a network help us to describe it behavior, and interactions in micro and macro level.

K. Raya-Díaz (✉) · C. Gaxiola-Pacheco · M. Castañón-Puga · L. E. Palafox
R. Rosales Cisneros

Facultad de Ciencias Químicas e Ingeniería, Universidad Autónoma de Baja California,
14418 Calzada Universidad, 22390 Tijuana, Baja California, Mexico
e-mail: kraya@uabc.edu.mx

[illegible]

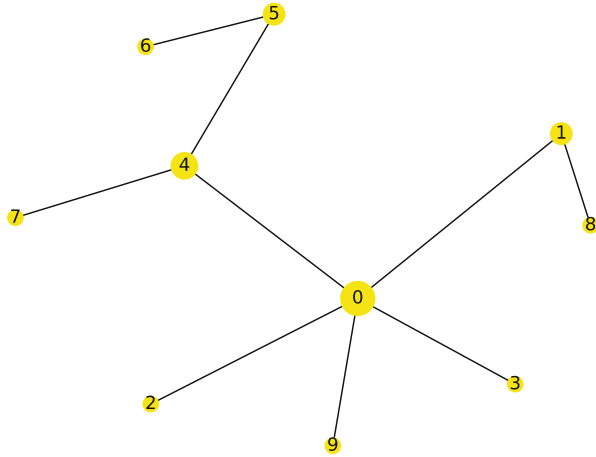
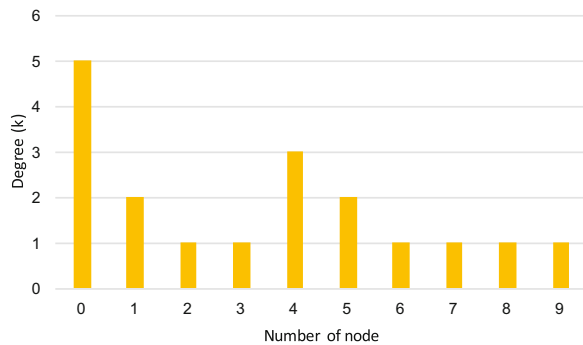


Fig. 2 Scale-free network example follow power law distribution

Fig. 3 Histogram of degree distribution in a random network



link with other node depends of the degree, in other words a node will prefer attach itself to a hub node than a peripheral node. In Fig. 3 shows the histogram with the degree distribution of each node of the network.

Power law is shown in Eq. 1 where $P(k)$ is a fraction of nodes in the network with k links to other nodes and γ is a parameter whose values are in the range $2 < \gamma < 3$ typically.

$$P(k) \sim k^{-\gamma}$$

In Fig. 3 shown the degree distribution in which node zero is the preferred node in complex theory is called hub node. Barabási in [5] found that hubs radically shrink the distance between the nodes in scale-free networks.

After the analyzes of both kind of networks (random and scale-free) is time to introduce other graphs that reflects the topological structure in a network.

3 Barbell Graph

Barbell is a graph composed of two complete graphs with $m1$ vertices, and one path that connect both graphs containing $m2$ vertices [6]. Where the total number of vertices is equal to $n = 2m1 + m2$, and $(m1 \geq 2, m2 \geq 0)$. The representation of Barbell graph is shown in Fig. 4, where we can see two bells connected by a path.

The adjacency matrix that represents the Barbell graph of Fig. 4, is illustrate in Table 2, where the number of vertices are 15 and the number of edges are 34.

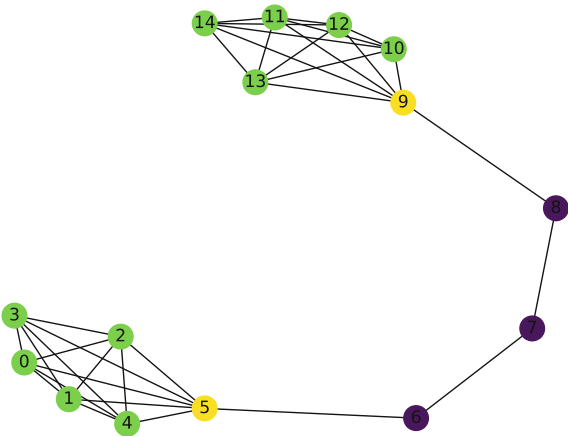


Fig. 4 Example of Barbell graph with $m1 = 6$ and $m2 = 3$

Table 2 Adjacency matrix of Barbell graph of Fig. 4

Nodes	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0
2	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0
3	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0
4	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0
6	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1
10	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1
11	0	0	0	0	0	0	0	0	0	1	1	0	1	1	1
12	0	0	0	0	0	0	0	0	0	1	1	1	0	1	1
13	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1
14	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0

This matrix is used to determine the centrality which is the measure that will be explain in the next section.

4 Types of Centrality Measures

Centrality measure in social networks are frequently use to analyze the efficient communication, also the centrality characterized the behavior of communicating groups and identify the point that control its communications [7].

4.1 Degree Centrality

In social networks centrality was introduced by Bavelas in 1948, he proposed that there is a relationship between structural centrality and influence in group processes [8]. In 1954 Shaw introduce the degree as the value of centrality. Then the degree centrality is defined as the count of the number of nodes_j ($i \neq j$) that are adjacent to node_i. Degree centrality is a way to find the node that is strategical located to communicate or influence a group to propagate the information or not.

4.2 Eigenvector Centrality

Eigenvector centrality of a node_i is determined by adjacency matrix applying Eq. 2. Where x'_i is defined by the sum of the i's centralities of I neighbors, and A_{ij} is an element of the adjacency matrix [9]. The interpretation of eigenvalues tells if a node is growing or shrinking according to the number of neighbors.

$$x'_i = \sum_j A_{ij}x_j$$

This metric is applying to capture the behavior of a network, hierarchy nodes, and detects interactions patterns.

4.3 Closeness Centrality

The closeness centrality of a node_i is the inverse of the sum of the number of hops in the shortest paths from the node_i to the rest of the nodes in the network [9]. This measure as not useful to discriminate or classified nodes in a network, at least in the

Table 3 Results of centrality measures of Barbell graph of Fig. 4

Nodes	Degree	Closeness	Betweenness
0	0.357142857	0.285714286	0
1	0.357142857	0.285714286	0
2	0.357142857	0.285714286	0
3	0.357142857	0.285714286	0
4	0.357142857	0.285714286	0
5	0.428571429	0.35	0.494505495
6	0.142857143	0.378378378	0.527472527
7	0.142857143	0.388888889	0.538461538
8	0.142857143	0.378378378	0.527472527
9	0.428571429	0.35	0.494505495
10	0.357142857	0.285714286	0
11	0.357142857	0.285714286	0
12	0.357142857	0.285714286	0
13	0.357142857	0.285714286	0
14	0.357142857	0.285714286	0

Barbell model many of the nodes that integrate the bells have the same value of closeness it can view in Table 3.

4.4 Betweenness Centrality

Betweenness centrality of a node the fraction of the shortest paths going through node k when considered over all pairs of nodes i and j [9]. Equation 3 define the betweenness of a node as follow.

$$Bc(k) = \sum_i \sum_j \frac{sp(i,k,j)}{sp(i,j)} i \neq j \neq k$$

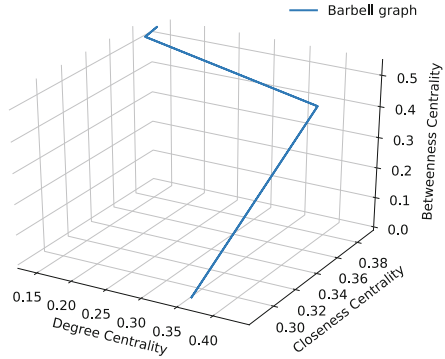
where $sp(i,j)$ is the total number of shortest paths between nodes i and j , and $sp(i,k,j)$ is the number of shortest path that go through node k [1, 9]. This centrality metric helps to detect the role of a node in distributing information.

After all these definitions of centrality in Table 3 illustrate the results of applying the centralities of degree, closeness and betweenness of Barbell network, and Fig. 5 shows that betweenness metric is the indicate better the influence of a node to propagate a message.

Other way to analyze the structure of a network is to calculate different measures of centrality and graph it together to determine which centrality is best to solve the issue.

Complex networks approach is useful to analyze complex problems which cannot be solved by separation of its parts and then put together to get a solution.

Fig. 5 Relationship between centrality measures of degree, closeness and betweenness of barbell graph



5 Agent-Based Modeling

An agent is an autonomous computational individual or object with preferences and actions, this is defined by a computational model. The implementation of agent-based models helps to understand the patterns behavior and emergence of a phenomenon [10]. In complex systems, the interactions of the elements are important features of emergence. The use of computational modeling as agent-based modeling (ABM) enables the simulations of complex systems.

The implementation of ABM allows us to model a set of agents inside of an environment and interact to each other, using a set of rules. Netlogo is a useful and easy language for the implementation of a simulation model. Other benefits of the use an ABM is that allows the programing of heterogeneous agents, in this way an history of interactions of each agent can be observed and found its strategy.

5.1 Proposed Agent-Based Model for Rumor Spreading

The spreading of a rumor depends of the structure of the network which represents the relations between the individuals that composed it. The modeling that we proposed use a set of agents that are in a classroom environment. Agent i is defined by a tuple $Ag_i = \langle d, c, b, s, f \rangle$ where d represents the degree centrality, c the closeness centrality, b the betweenness centrality, s is the status of cooperate that can be zero or one, and f this attribute represent the flow of information, this is set to one when the agent (student) got the message.

The environment where the agents are located is classroom that is integrate by three subgroups α , β , and λ . Figure 6. illustrate the ABM in which we will discover and analyze a set of agents, interconnected by a topology with the distribution of a Barbell graph. Our model calculates three types of centralities, in Fig. 7 the graph of the left shows the degree centrality, mean while the betweenness centrality is show on the right plot, and finally the closeness centrality at the bottom.

Fig. 6 Interface of the agent-based model in Netlogo

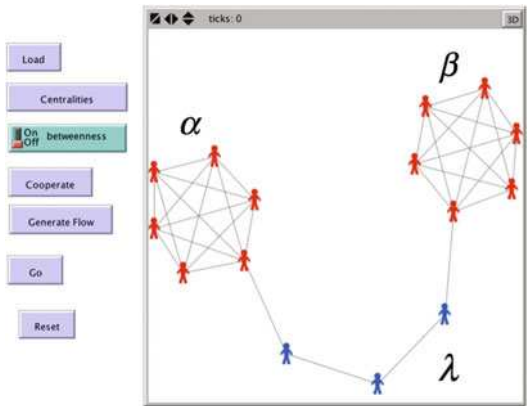
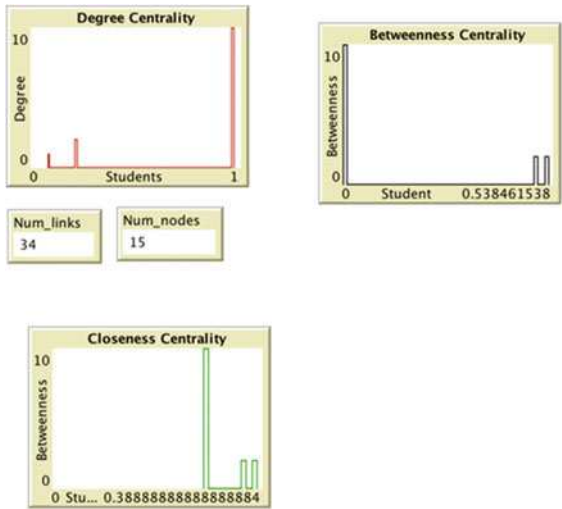


Fig. 7 Results of centralities calculate by the model



After the setup of the world that implements the creation of the topology by importing the adjacency matrix of Barbell graph, the next step is the initialization of the attributes of each agent (student). In Netlogo the properties of an agent are observed by inspecting each node in Fig. 8, turtle 9 is an agent with status cooperation set to cero.

The assignation of s (status of cooperate) is set in a random way, in our model agents with green color will cooperate to spread the rumor, and oranges agents not. Finally, the attribute f is set one by a random selection, this mean that only one agent will start the spreading of the rumor, in this case the color of the agent will set to yellow. Every ABM made in Netlogo has a process traditional called “go”. To explain how this process works we must observe the sequence shows in Figs. 9, 10 and 11.

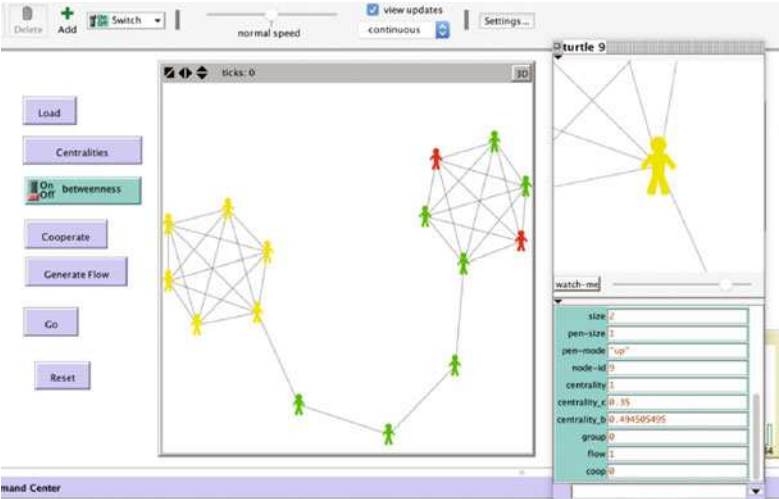
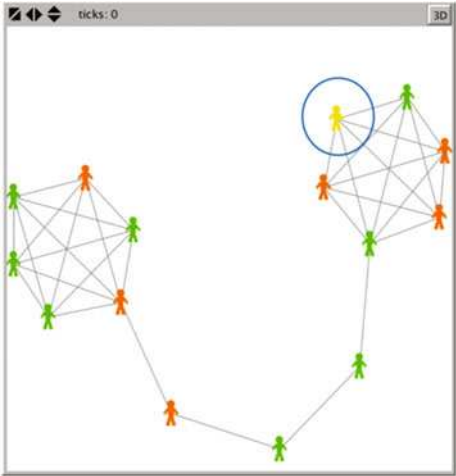


Fig. 8 No cooperation emerges in a pattern where only alpha group got the message

Fig. 9 Setup of the world and start of the rumor in agent number one with yellow color, orange color means no cooperate status, and green agents will cooperate to the spread of the rumor



After one tick the distribution of the rumor depends of two attributes the first one is the status of cooperate and the second is his betweenness centrality. The rule will set if $s == 1$ the agent will spread the rumor only to his neighbors. Figure 10 shows the spreading of the rumor.

In Fig. 10 we can observe that one of the member of λ got the message and even all the members of β . The next step is determining if the rest of the agent will get the message or not. Figure 11 illustrate the last member of λ got the messages, but him will not propagate because his status of cooperate is zero. Finally, after three

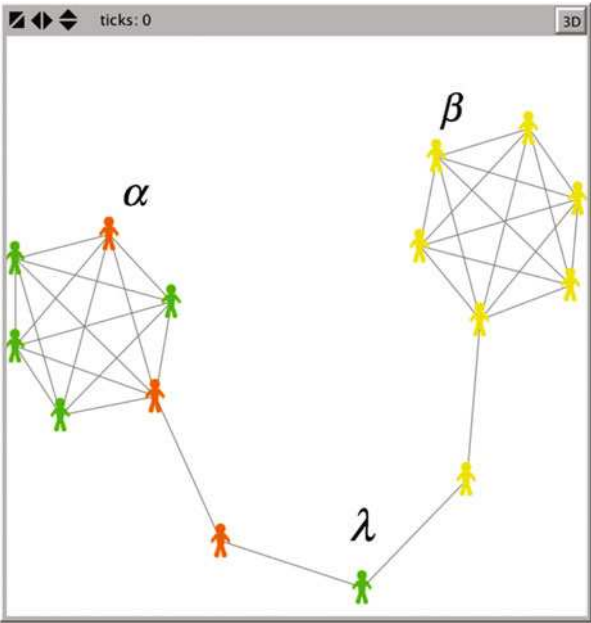


Fig. 10 The spreading of the rumor during the tick number two

Fig. 11 The last agent that got the message is inside of blue circle

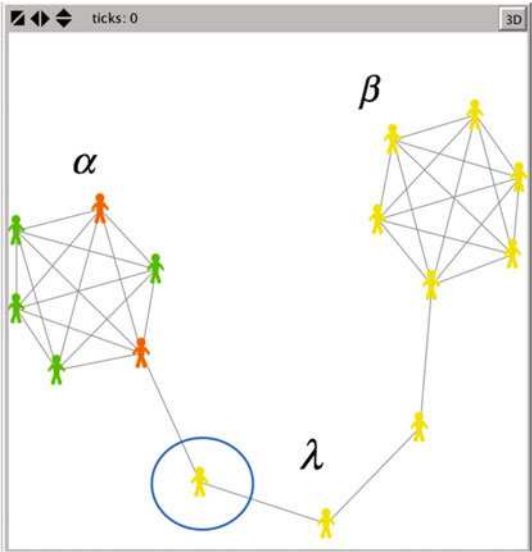
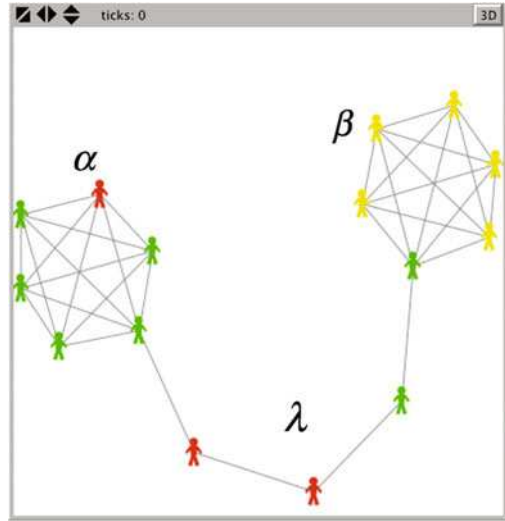


Fig. 12 Only the agent with the same betweenness centrality received the message



ticks the rumor was got for all the members of λ and β , understanding that no one of α group could get it.

In the ABM proposed we can activate the betweenness centrality button has another attribute to verify when the propagation of a rumor start. This scenery is shows in Fig. 12, where the rumor start in one of the member of β group and to spread the rumor only if him status of cooperate is set to one, and the betweenness of his neighbors is equal to himself.

6 Results

The configuration of the status of cooperate is the key for the propagation of a rumor when the betweenness centrality button is off. Figure 13 shows the status of each group and individuals (students) that received the message, the yellow color of the student mean that the rumor was gotten.

To spread the rumor to every agent that belong to the classroom is necessary observe Fig. 14, in this figure we identified that the members of λ group have status of cooperate set to one.

The λ group in a classroom have the control of the rumor distribution, when is necessary to broadcast a message in a group with Barbell topology.

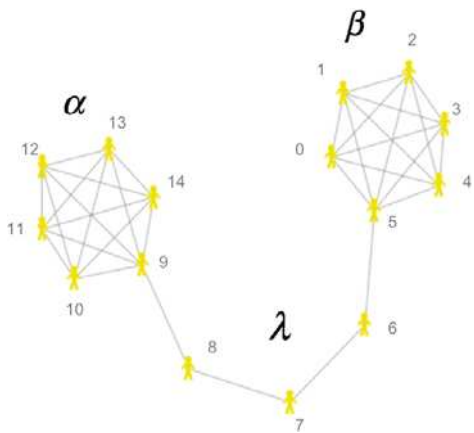


Fig. 13 Status of the network when everyone got the message

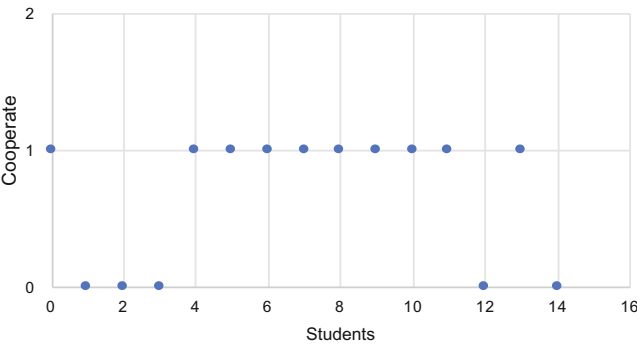


Fig. 14 Configuration of cooperation state of the agents (students)

7 Conclusions

The dynamics of rumor propagation are complex, as the interactions between the individuals that make up the classroom are observed. The simulation with intelligent agents allows the detection of emerging local and global behaviors, with only few study variables such as measures of centrality. The calculation of betweenness centrality in a network is relevant when distribution of information is analyzing. Finally, the centrality measure helps to the detection of leaderships in a group.

8 Future Work

As a future work is necessary to incorporate a traditional model for the spread of disease as SIR (Susceptible, Individuals, Recovered).

References

1. Estrada E (2011) The structure of complex networks: theory and applications. Oxford University Press Inc, New York
2. Barrat A, Barthélemy M, Pastor-Satorras R, Vespignani A (2004) The architecture of complex weighted networks. PNAS 101:3747–3752
3. Chung Fan, Lu L (2006) Complex graphs and networks (CBMS regional conference series in mathematics). American Mathematical Society, Boston
4. Watts DJ, Strogatz SH (1998) Collective dynamics of “small-world” networks. Nature 393:440–442. <https://doi.org/10.1038/30918>
5. Barabasi A-L, Pósfai M (2016) Network science graph theory. Cambridge University Press, Cambridge
6. Aldous D, Fill J (1999) Reversible Markov chains and random walks on graphs, vol 2002, pp 1–516
7. Freeman LC (1977) A set of measures of centrality based on betweenness. Sociometry 40:35–41
8. Freeman LC (1979) Centrality in social networks conceptual clarification. Soc Netw 1:215–239
9. Meghanathan N (2015) Use of eigenvector centrality to detect graph isomorphism. Comput Sci Inf Technol (CS IT) 5:01–09. <https://doi.org/10.5121/csit.2015.51501>
10. Wilensky U, Rand W (2015) An introduction to agent-based modeling