Escuela de

Administración Pública

Administración

Pública Territorial

Matemáticas II

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segundo semestre

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Práctica de Aplicación

2 Determinación de Excedentes.

Ecuación demanda $\Rightarrow q = f(p) = \dfrac{90}{p} - 2$

Ecuación oferta $\Rightarrow q = g(p) = p - 1$

CS = Excedente del consumidor $= \displaystyle\int_0^{q_0} f(q)\,dq - q_0 p_0$

Equilibrio $f(p) = g(p) \Rightarrow \dfrac{90}{p} - 2 = p - 1$

$90 - 2p = p^2 - p \Rightarrow p^2 + p - 90 = 0$

$(p+10)(p-9) = 0 \Rightarrow p - 9 = 0 \Rightarrow \underline{p_0 = 9}$.

$q_0 = p_0 - 1 \Rightarrow q_0 = 9 - 1 \Rightarrow \underline{q_0 = 8}$

$q = \dfrac{90}{p} - 2 \Rightarrow q + 2 = \dfrac{90}{p} \Rightarrow f(q) = \dfrac{90}{q+2}$

$CS = \displaystyle\int_0^{8} \frac{90}{q+2}\,dq - (9)(8)$

$CS = 90 \ln(q+2)\Big]_0^8 - 72 \Rightarrow CS = 90\ln(10) - 90\ln 2 - 72$

$\boxed{CS = 72.85}$

PS = Excedente del productor $= \int_0^{q_0} [p_0 - g(q)]\,dq$

$$PS = \int_0^8 [9-(q+1)]\,dq \Rightarrow PS = 8q - \frac{q^2}{2}\Big]_0^8$$

$$PS = (8)(8) - \frac{(8)^2}{2} \Rightarrow \boxed{PS = 32}$$

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3 a) Demanda $\Rightarrow p = 20 - 0.8q$

Oferta $\Rightarrow p = 4 + 1.2q$

$$CS = \int_0^{q_0} [f(q) - p_0]\,dq$$

$$20 - 0.8q = 4 + 1.2q \Rightarrow 20 - 4 = 1.2q + 0.8q$$

$$16 = 2q \Rightarrow \underline{q_0 = 8} \Rightarrow p_0 = 20 - 0.8(8) \Rightarrow \underline{p_0 = 13.6}$$

$$CS = \int_0^8 [20 - 0.8q - 13.6]\,dq \Rightarrow CS = 6.4q - 0.4q^2\Big]_0^8$$

$$CS = 6.4(8) - 0.4(8)^2 \Rightarrow \boxed{CS = 25.6}$$

$$PS = \int_0^{q_0} [p_0 - g(q)]\,dq \Rightarrow PS = \int_0^8 [13.6 - 4 - 1.2q]\,dx$$

$$PS = 9.6q - \frac{1.2q^2}{2}\Big]_0^8 \Rightarrow PS = 9.6(8) - 0.6(8)^2$$

$$\boxed{PS = 38.4}$$

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b) Demanda $\Rightarrow p = \frac{50}{q+5}$

Oferta $\Rightarrow p = \frac{q}{10} + 4.5$

$$CS = \int_0^{q_0} [f(q) - P_0]\, dq \qquad \frac{50}{q+5} = \frac{q}{10} + 4.5$$

$$50(10) = (q+5)(q+45) \Rightarrow q^2 + 50q + 225 - 500 = 0$$

$$q^2 + 50q - 275 = 0 \Rightarrow (q+55)(q-5) = 0 \Rightarrow \underline{q_0 = 5} \Rightarrow \underline{P_0 = 5}$$

$$CS = \int_0^5 \left[\frac{50}{q+5} - 5\right] dq \Rightarrow CS = 50\ln(q+5) - 5q \Big]_0^5$$

$$CS = 50\ln(10) - 25 - 50\ln(5) = 50(\ln 10 - \ln 5) - 25$$

$$\Rightarrow \boxed{CS = 50\ln 2 - 25}$$

$$PS = \int_0^{q_0} [P_0 - g(q)]\, dq \Rightarrow PS = \int_0^5 \left[5 - \frac{q}{10} - 4.5\right] dq$$

$$PS = 0.5q - \frac{1}{10}\frac{q^2}{2}\Big]_0^5 \Rightarrow PS = 0.5(5) - \frac{1}{20}(5)^2$$

$$\Rightarrow \boxed{PS = 1.25}$$

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e) Demanda : $q = 100(10-p) \rightarrow p = 10 - \frac{q}{100}$

Oferta : $q = 80(p-1) \rightarrow p = \frac{q}{80} + 1$

$100(-p+10) = 80(p-1) \Rightarrow -100p + 1000 = 80p - 80$

$180p = 1080 \Rightarrow \underline{p_0 = 6} \Rightarrow \underline{q_0 = 400}$

$$CS = \int_0^{400} \left[10 - \frac{q}{100} - 6\right] dq \Rightarrow CS = 4q - \frac{q^2}{200}\Big]_0^{400}$$

$$CS = 4(400) - \frac{(400)^2}{200} \Rightarrow \boxed{CS = 800}$$

$$PS = \int_0^{400} \left[6 - \frac{q}{80} - 1\right] dq$$

$$PS = 5q - \frac{q^2}{160}\Big]_0^{400}$$

$$PS = 5(400) - \frac{(400)^2}{160} \Rightarrow \boxed{PS = 1000}$$

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9 Demografía.

Nota: De la única forma que obtenemos la solución sugerida en el documento es tomando las edades entre 36 y 64.

$$N = \text{Número esperado de gente} = \int_{36}^{64} 10.000\sqrt{100-x}\,dx$$

$$u = 100 - x \Rightarrow \quad x = 36 \to u = 64$$
$$du = -dx \qquad x = 64 \to u = 36$$

$$N = \int_{64}^{36} 10000 \cdot u^{1/2} \cdot -du$$

$$N = -10000 \cdot u^{3/2} \left(\tfrac{2}{3}\right) \Big]_{64}^{36}$$

$$N = -6666.66 \left[36^{3/2} - 64^{3/2} \right]$$

$$\Rightarrow \boxed{N = 1973333}$$

10 Costo marginal.

$\frac{dc}{dq} = 0.2q + 3 \Rightarrow C =$ costo total.

$\Delta =$ Incremento : De. $q_0 = 60$ a $q_1 = 70$

$$\Delta = \int_{q_0}^{q_1} \left(\frac{dc}{dq}\right) \cdot dq \Rightarrow \Delta = \int_{60}^{70} (0.2q + 3) dq$$

$$\Delta = 0.2 \frac{q^2}{2} + 3q \Big]_{60}^{70}$$

$$\Delta = 0.1(70)^2 + 3(70) - 0.1(60)^2 - 3(60)$$

$$\Rightarrow \boxed{\Delta = 160 \text{ dólares}}$$

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12 Curva de Lorentz.

Ver gráfico en documento

L = coeficiente de desigualdad

$$L = \frac{\text{Área entre la curva y la diagonal}}{\text{Area bajo la diagonal}} = \frac{A_1}{A}$$

$$A = \int_0^1 x\,dx \Rightarrow A = \frac{x^2}{2} \Big]_0^1$$

$$\Rightarrow A = \frac{1}{2} u^2$$

$$A_1 = \int_0^1 \left[x - \left(\frac{20}{21}x^2 + \frac{1}{21}x \right) \right] dx$$

$$A_1 = \int_0^1 \left(x - \frac{20}{21}x^2 - \frac{1}{21}x \right) dx \Rightarrow A_1 = \int_0^1 \left(\frac{20}{21}x - \frac{20}{21}x^2 \right) dx$$

$$A_1 = \frac{20}{21} \int_0^1 (x - x^2)\, dx$$

$$A_1 = \frac{20}{21} \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 \Rightarrow A_1 = \frac{20}{21} \left[\frac{1}{2} - \frac{1}{3} \right]$$

$$A_1 = \frac{20}{21} \left(\frac{3-2}{6} \right) \Rightarrow A_1 = \frac{20}{126}\, u^2$$

$$L = \frac{A_1}{A} \Rightarrow L = \frac{\frac{20}{126}\, \cancel{u^2}}{\frac{1}{2}\, \cancel{u^2}}$$

$$\Rightarrow \boxed{L = \frac{20}{63}}$$

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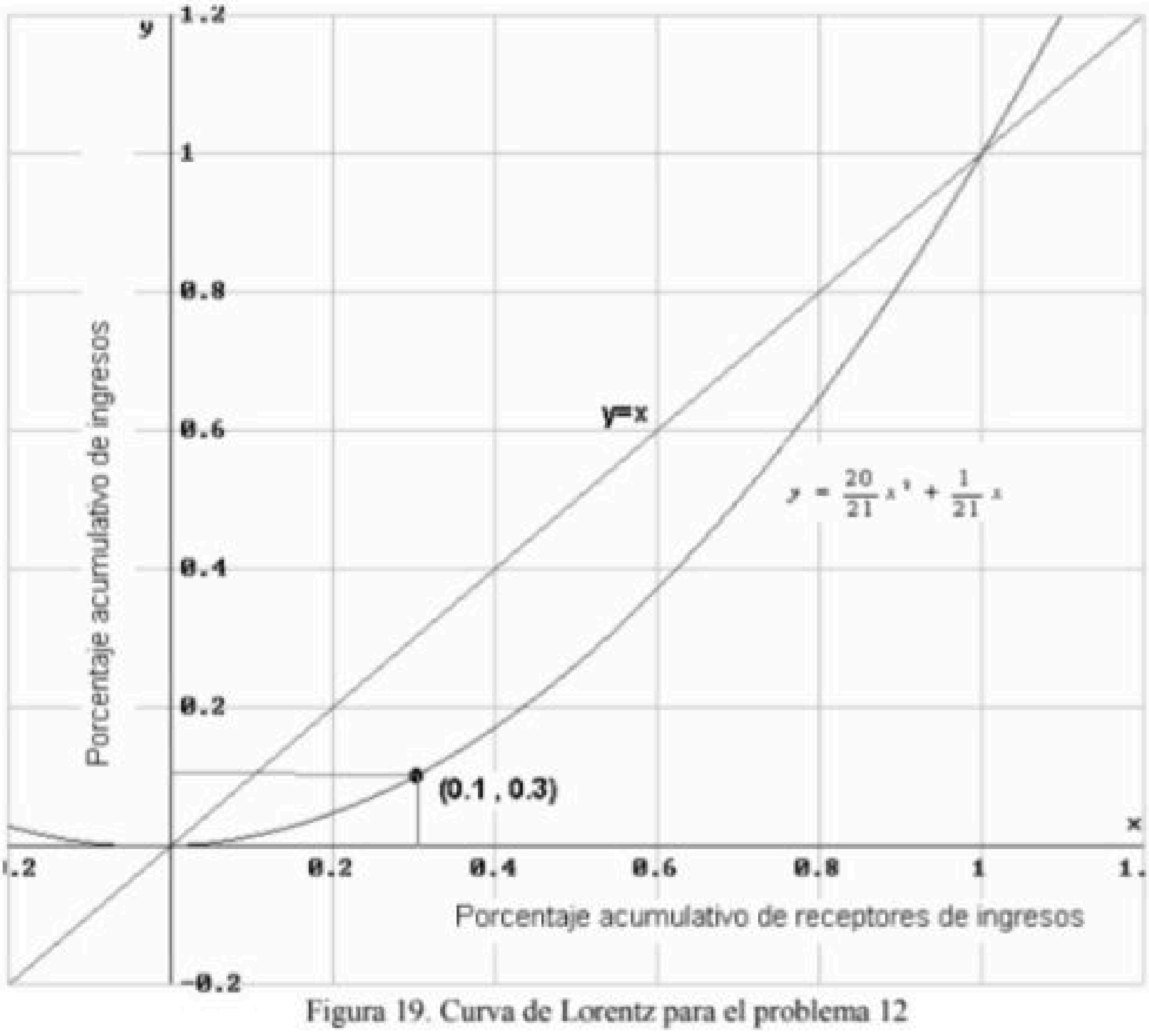

Figura 19. Curva de Lorentz para el problema 12