* ARCH-M (model) :-

Engel, Lilien and Robins (1987) extended the basic ARCH framework to allow the mean of sequence to depend on its own conditional variance.

$$y_t = \mu_t + \varepsilon_t$$

y_t = excess return (r_t - T treasury bill rate).

μ_t = risk premium necessary to induce the risk-averse investor to hold this risky meet over bond.

ε_t = error/stocks.

$$\mu_t = \beta + \delta h_t \qquad \delta > 0.$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

if $\alpha_i = 0 \ \forall i$ then the model boils down to
$y_t = \beta + \alpha_0 + \varepsilon_t \rightarrow$ constant risk premium model

* I-GARCH Model (Integrated)

$$y_t = \alpha_0 + \beta y_{t-1} + \varepsilon_t.$$

$\varepsilon_t = v_t \sqrt{h_t}$. $\quad V_t \Rightarrow$ white noise.
$h_t \Rightarrow$ conditional volatility

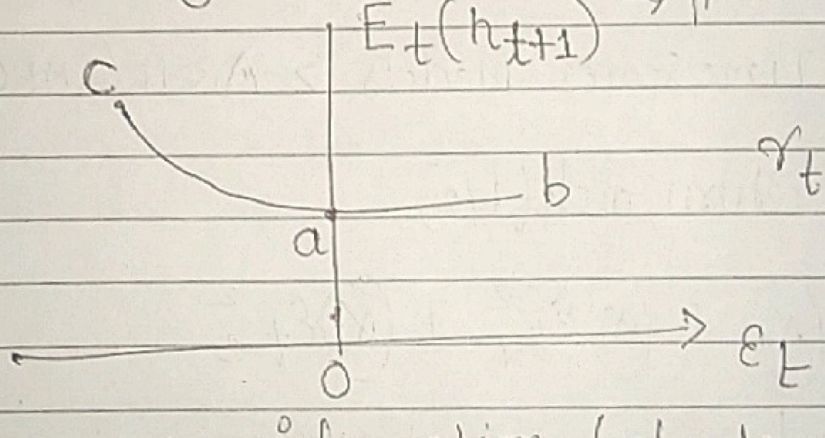
if $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$
& if $\alpha_1 + \beta_1 = 1$ then

$$h_t = \alpha_0 + (1-\beta_1)\varepsilon_{t-1}^2 + \beta_1 h_{t-1}.$$

★ TARCH (Threshold ARCH Model)

Notion of leverage effect →
Tendency for volatility to decline when returns rises
& — // ——— to increase — // — falls

if $\varepsilon_t = 0$ then $E_t(h_{t+1})$ = distance $\overline{0a}$
Any news increases votality
if the news is good $(\varepsilon_t > 0)$
volatility increases $(\overline{ab})$ ⟶ less
if the news is bad $(\varepsilon_t < 0)$.
volatility increases $(\overline{ac})$ → more



$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t$$

new information / shock.

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \lambda D \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

if $\varepsilon_{t-1} < 0$ then $D = 1$
$\varepsilon_{t-1} \geq 0$ then $D = 0$ } Dummy

∴ when $\varepsilon_{t-1} < 0$ (bad news)
$$h_t = \alpha_0 + (\alpha_1 + \lambda)\varepsilon_{t-1}^2 + \beta_1 h_{t-1}.$$

∴ when $\varepsilon_{t-1} \geq 0$ $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$ (good news)

Get data

gent = _n
tsset t
gen return = ln (close / l.close) * 100
tsline ~~return~~ close deaths

gen dg = ln (deaths / l.deaths) * 100
tsline return dg

regress return l.return

estat archlm, lags(3)

Prob > chi2 is lower than 0.05
(at 5 percent level of significance)
we can reject the null.
(null hypo : no arch affects)

stats > TimeSeries Models > ARCH/GARCH Models

arch return, arch(1/2)

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2$$

predict conditional_volatility, variance
tsline conditional_volatility

tsline conditional_volatility dg.

arch return, arch(1) garch(1)

$$h_t^2 = \alpha_0 + \alpha_1 h_{t-1}^2 + \alpha h_{t-2}^2$$

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predict conditional-volatility_1, variance.

tsline conditional-volatility_1.

tsline conditional-volatility_1 dg.

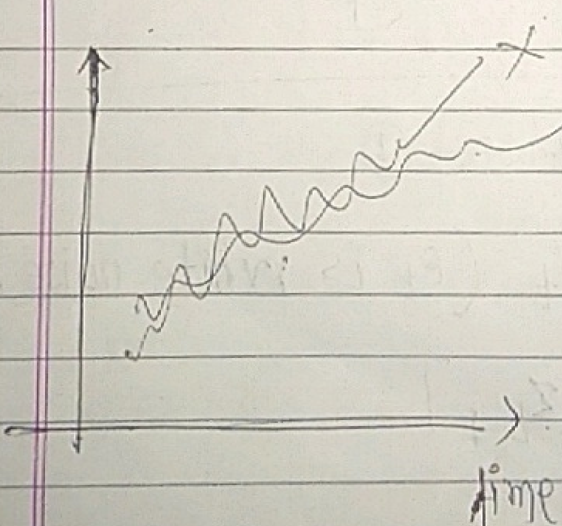
* Cointegration (Market - Comovement).

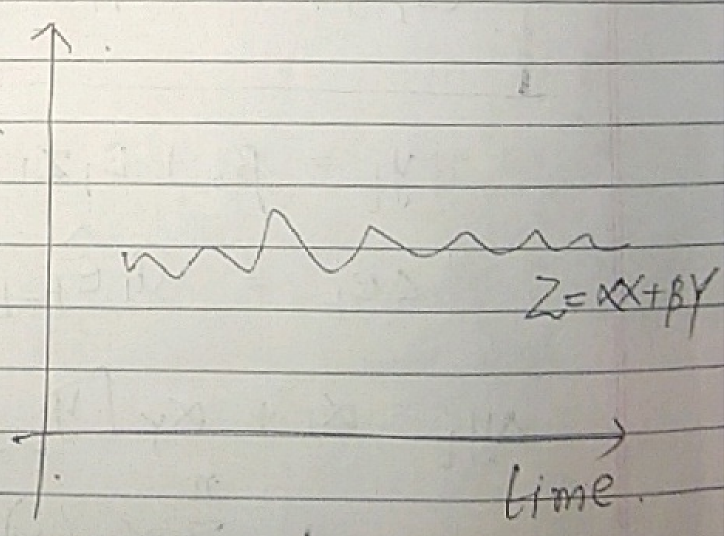
If X and Y are two stocks/indices and both X and Y are non-stationary then a linear combination of X and Y can generate a stationary variable

i.e.. $\alpha X + \beta Y = Z$ (where Z is a linear combination of X and Y)

As, Z is stationary, X and Y (non-stationary) are referred as cointegrated.

It means there is long term, stable, equilibrium relationship b/w X and Y.





Shrikrupa

guage if error reverts back to equilibrium or not.

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Error Correction Model :-

y_t & z_t are 2 variables

continued ....

How to check cointegration?

(I) Unit root test of X and Y (to check non stationarity)

(II) Regress Y over X

$\longrightarrow Y = A + \beta X + \varepsilon$

equivalent to $\alpha X + \beta Y$.

since Y has coefficient 1

X has coefficient β. (Ignore A here).

(III) $Y = \cancel{X\beta} + \varepsilon + A$

$Y - X\beta = \varepsilon'$

or $1 \cdot Y + (-\beta) X = \varepsilon'$ $(\varepsilon' = \varepsilon + A)$.

if ε' is stationary then X and Y are cointegrated.

$(1, -\beta)$ is reffered as cointegrated vector.

$$y_t = \beta_0 + \beta_1 z_t + e_t \quad \text{--- (1)}$$

$$\Delta e_t = \hat{a}_1 \hat{e}_{t-1} + \varepsilon_t$$ (ε_t is white noise term

$$\Delta y_t = \alpha_1 + \alpha_Y [y_{t-1} - \beta_1 z_{t-1}] + \sum_{i=1}^{n} \alpha_{11}(i) \Delta y_{t-i} + \sum_{i=1}^{m} \alpha_{12}(i) \Delta z_{t-i} + \varepsilon_{Y_t}$$

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↑ speed of adjustment coefficient

α_Y is the coefficient of $[Y_{t-1} - \beta_1 z_{t-1} = e_{t-1}]$
past error of regression model (1).

If $|\alpha_Y| = 1$ then old error impacts change in y and ΔY_t changes 100% for unit change in e_{t-1}

it means adjustment happens to maximum possible limit.

if $\alpha_Y = 0$ then response to Y (ΔY) has not been impacted by old error (e_{t-1}).

Ideally $0 \leq |\alpha_Y| \leq 1$ and it detects the degree of adjustment.

Sign of α_Y should be (-ve) because if past error increases then deviation in equilibrium relation is high and to bring back this equilibrium change in Y should drop or fall.

Hence α_Y (denoting response of ΔY to unit change in e_{t-1}) should bear a negative sign. This method is referred as Engel-Granger Cointegration and ECM (error correction model)

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if $\alpha_Y = -0.9$ then there is 90% adjustment error (or correction is happening by 90%) in current period vis-a-vis previous period.

~~S~~tata

* shell extraction (cheaper oil producing method).

gen time = _n
tsset time
tsline gold oil

(check for stationarity) step I

regress d.gold l.gold
regress d.oil l.oil
tsline oil
tsline gold

step II (regress one variable over other)

regress gold oil

for all data

gold = 11.38669 oil + 191.68

predict goldhat.
gen error = gold - goldhat.
tsline error.
(check if error is stationary or not).
if yes cointegration exists.
else cointegration does not exist.

→ regress gold oil if time<200. for first few observations.
gold = -3.706037 oil + 426.6259.

→ similar.

→ regress gold oil time if time < 200
(incorporate trend part also)
predict yhat.
gen e = gold - yhat if time < 200
tsline e.
if e is stationary.
linear combination of gold, oil, time is stationary
⇒ gold, oil, time are cointegrated; i.e there is stable equilibrium.

Now check for error correction.
varsoc d.gold d.oil
regress d.gold l.e l.d.gold l.d.oil