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Vicsek model of self-propelled particles with hybrid noise

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In self-propelled particle models, the interactions are generally subject to some type of noise, which is introduced either during or after the interactions between the particles. Here, self-propelled particles are subject to two different types of noise. If the orientation between them is less than a certain rate β, a vectorial noise is introduced in each particle–particle interaction, otherwise, an angular noise is introduced in the average direction of motion, already calculated, of neighboring particles. We vary the rate β over a wide range ($\beta < \pi$) and study the change in the nature of the phase transition. The product-moment correlation coefficient between the order parameter and the hybrid noise, is calculated. We found that in the combination of the two types of noise, the critical noise decreases with increasing the parameter β, and by analysis of the Binder cumulant we estimate the value of β as from which begins to appear the effects of the vectorial noise on the phase transition.

Keywords: Biological systems; collective motion; self-propelled particles; phase transitions; collective behavior rules.

1. Introduction

In nature, many organisms display ordered collective motion. It is a biological phenomenon that arises from the numerous interactions between the members of the group, as for example, flocks of birds, schools of fish, bacterial populations, insect swarms.[1–4] To describe this fascinating behavior, self-propelled particle (SSP) models for collective behavior (without a leader present) have emerged and become an important tool in studies involving dynamics of animal collective motion,[1,5–10] which can produce behaviors qualitatively similar to real biological systems.

Vicsek and collaborators[7] were the pioneers in the scientific investigation of swarms and proposed a model (simplest) of SPPs. They studied swarming behavior in terms of a phase transition induced by noise, where self-propelling particles move at the same constant speed ν and in the average direction of motion of the particles in its neighborhood of radius R. It should be noted that despite its simplicity the model has proved to be a useful tool and efficient.

In general, in self-propelled particle systems, the interactions between nearest neighbors are subject to some type of noise, that represents possible "measurement" errors committed by a particle when evaluating the motion direction of its neighbors. In this respect, there are two ways in which noise can be introduced into the system, either during or after the interactions between the particles. These two types of noise are generally called angular and vectorial noise. Angular noise (originally employed by Vicsek and collaborators) is introduced in the system when particles try to follow the average direction of motion, already perfectly calculated, of neighboring particles, while the vectorial noise (introduced by Gregoire and Chate[9]) is added directly to the interactions between a certain particle and each one of its neighbors, emerging as a randomness from the evaluation of each particle–particle interaction. Such errors are made when a particle does not see its neighbors very well.

In collective motion of self-propelled particles when the direction of motion ($\theta_i^{t+\tau}$) of each individual is disturbed by the presence of an angular noise, it is updated by the following rule:

$$\theta_i^{t+\tau} = \arg\left[\sum_{\langle i,j\rangle} e^{i\theta_j^t}\right] + \eta\varepsilon_i^t\,, \tag{1}$$

where the sum is performed over the ith particle and all its j-neighbors, and ε is a white noise uniformly distributed in the range between $-\pi$ and π, and η is the noise amplitude. In the first term, the interaction between the particles occurs; the particles will try to align themselves and move in a common direction. The second term represents errors in "measurement" committed by the particle i when adjusting its direction to the average motion direction of its neighbors. It is easy to note in Eq. (1) that the average motion direction is computed first and then the noise is added.

When the direction of motion of each individual is disturbed by a vectorial noise, the updating rule for every particle's orientation is given by

$$\theta_i^{t+\tau} = \arg\left[\sum_{\langle i,j\rangle} e^{i\theta_j^t} + \eta n_i e^{i\varepsilon_i^t}\right], \tag{2}$$

where n_i is the current number of neighbors of particle i. Unlike the rule given by Eq. (1), here, the noise is added in a different way, by adding a vector ($\eta e^{i\varepsilon_i^t}$) to the sum. The index, $\langle i,j\rangle$, indicates that summation is performed over the ith particle and all its neighboring j particles.

Driven by noise (angular or vectorial), self-propelled particle systems display a phase transition from a disordered state (random directions) to a state of ordered motion. However, the two types of noise described above can produce different types of phase transitions.[14] The nature of this phase transition, whether continuous or discontinuous, will depend on whether the noise introduced is of the type angular or vectorial. Angular noise leads to a continuous phase transition while vectorial noise makes the transition discontinuous.[14,15]

In self-propelled particle systems the order parameter ψ, that characterizes this transition is the absolute value of the normalized mean velocity,

$$\psi = \frac{1}{N\nu}\left|\sum_{i=1}^{N}\mathbf{v}_i(t)\right|. \tag{3}$$

In general, in the study of self-propelled particle systems, there are various parameters that control the individual behavior. For example, in some studies the interactions between individuals depend on distance,[16] others take into account the field of perception,[17] while name others adopt a speed appropriate distribution,[18] process involving phase transitions and criticality also has been studied.[11–13] In common, all these models consider that during the interaction process, individuals advance without taking into account the difference of orientation between them. This means that whatever the orientation between the velocity vectors, a noisy term (angular or vectorial) is added in the rule that updates the direction of motion. Although phase transitions of systems SSPs, as a function of the noise (angular or vectorial) have been extensively studied, we have not yet found, in the literature, an analysis of the phase transition due to the combinated effect of these two types of noise (which depend on the angle between the velocities). For this reason, the main motivation in this paper is to investigate how the system behaves at different types of noise, or how the phase transition is affected when the noise goes from one type to another.

In this contribution, we take into account both the angle of orientation between two neighboring individuals, and the type of noise to be introduced. If the orientation between its velocity vectors is less than a certain rate $\beta\tau$ (with β in radians), a vectorial noise is introduced in each particle–particle interaction, otherwise, an angular noise is introduced in the average direction of motion, already perfectly calculated, of neighboring particles. We perform a series of simulations changing the rate $\beta\tau$ in order to study the behavior of the order parameter.

2. Model Description and Simulation Parameters

As in the Vicsek model, we consider organisms represented by N identical point-like particles characterized, in time t, by position $\mathbf{x}_i(t)$ and velocity $\mathbf{v}_i(t)$, with $i = 1, 2, 3, \ldots, N$, that move in the direction θ_i in a two-dimensional space, interacting locally by trying to align their directions with their neighbors at each time step τ. New positions are updated according to $\mathbf{x}_i(t+\tau) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\tau$.

In nature, it is reasonable to suppose that an individual modifies their motion direction toward its neighbor by a certain angle. We will denote this angle by α.

We are interested in calculating α because its value will determine which type of noise will be added in the direction of motion. We computed α using the dot-product between the velocities (vectors) and then we take the inverse cosine.

It is well known that the vectorial noise (VN) arises from each particle–particle interaction, and then, when analyzing the interaction rule given by Eq. (2), it can be deduced that the influence of the vectorial noise becomes weaker when the local order is increased. On the other hand, according to Eq. (1), the angular noise (added in the average direction of motion already calculated) depends on the average direction of all particles (with i included), but it does not depend on the local order (since the individual can decide to move in another direction). Based on these observations, we can intuitively infer that the error (or uncertainty) committed by an individual when evaluating the direction of motion of its neighbors, in case the noise is of the vectorial type, can be relatively smaller when compared to interactions with the angular noise (AN). Within this framework, it is reasonable also to consider that the smaller the angle α, the smaller will be the evaluation error committed by a particle when evaluating the motion direction of its neighbors. Based on these assumptions, for α smaller/bigger than a rate $\beta\tau$ follows the algorithm:

if $\alpha < \beta\tau$ then

$\theta_i^{t+\tau} = \arg\left[\sum_{\langle i,j\rangle} e^{i\theta_j^t} + \mathrm{VN}\right]$

else

$\theta_i^{t+\tau} = \arg\left[\sum_{\langle i,j\rangle} e^{i\theta_j^t}\right] + \mathrm{AN}$

end if.

According to the above-presented algorithm, simulations are made using both angular and vectorial noise, given, respectively, by updated rules (1) and (2). Parameters like the interaction radius R, the velocity modulus ν and constant time interval τ, are in the same range as the ones used in Ref. 7, with $R = 1$, $\nu = 0.03$ and time steps $\tau = 1$. Our simulation starts with a random configuration of the positions and directions of each particle. We have adopted periodic boundary conditions, and kept the fixed number of particles $N = 500$. To reach the equilibrium state, we take at least 1×10^5 time steps for all the data obtained. Then, we take more 1.7×10^6 to estimate the average values of the quantities of interest.

3. Quantities of Interest and Results

In Vicsek model,[7] the used order parameter is the absolute value of the normalized mean velocity ψ, given by Eq. (3). This quantity is approximately zero when individual movements are distributed in a random direction, and is nonzero in the ordered state. We are interested in investigating the phase transition in a self-propelled system with hybrid noise. According to the implemented algorithm here, if the angle

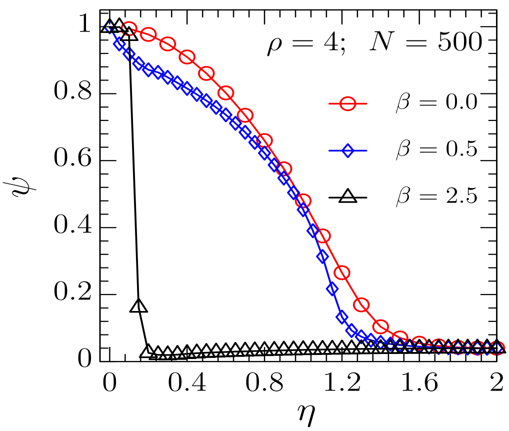

Fig. 1. (Color online) Order parameter ψ as a function of the strength of the noise η for some values of β as indicated in figure. The simulations were performed for $N = 500$ and density $\rho = 4.0$.

α, which gives the orientation between neighboring individuals, is smaller than the rate $\beta\tau$, the noise to be introduced is of the vectorial type, otherwise, an angular noise is introduced. Then it is to be expected that the bigger β is (since $\tau = 1$), the greater will be the possibility for ψ to undergo a discontinuity and so the transition between two different states to exhibit a first-order character. This behavior is well verified in Fig. 1, where the behavior of ψ as a function of the intensity of the hybrid noise, η, is shown. In this, and in all the other figures presented here, the symbols are the data and the connecting lines are guides to the eye.

In the following, we study the behavior of some other quantities of interest: the Binder cumulant C, the susceptibility χ and the product-moment correlation coefficient μ. The Binder cumulant, defined by $C = 1-[\langle\psi^4\rangle]/[3\langle\psi^2\rangle^2]$, is used in the study of the phase transitions to characterize the nature of the transition.[19,20] For a first-order phase transition, C displays a characteristic minimum at the transition, while its monotonous behavior indicates a second-order phase transition. In Fig. 2, it is plotted as a function of the noise intensity η for a wide range of β (as indicated in the figure legend). Note that the Binder cumulant C exhibits the sharp minimum typical of discontinuous phase transitions in the neighborhood of $\beta \approx 1.0$, and from this critical threshold, the effects of the vectorial noise become more evident on the phase transition such that the sharp minimum of C becomes more and more pronounced, and as we fix, within the critical threshold, the value of $\beta = 1.3$, the shape of the Binder cumulant versus η does not changes (see Fig. 3).

The susceptibility $\chi = L^2(\langle\psi^2\rangle - \langle\psi\rangle^2)$, with $(\langle\psi^2\rangle - \langle\psi\rangle^2)$ being the variance of the order parameter, presents a peak around the critical noise for finite systems, the calculation of χ permits us then to compute the critical values of noise η_{crit}. Figure 4(a) shows the dependence of the susceptibility on the noise η, the ordered motion phase corresponds to a low noise $\eta < \eta_{\mathrm{crit}}$, and the disordered motion phase to a high noise $\eta > \eta_{\mathrm{crit}}$.

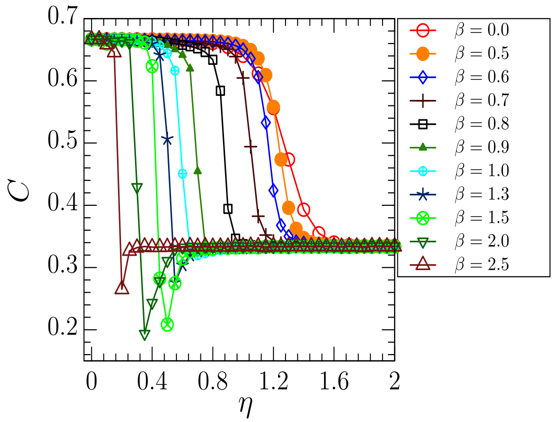

Fig. 2. (Color online) Binder cumulante C plotted as a function of the strength of the noise η for some values of β as indicated in the figure. The simulations were performed for $N = 500$ and density $\rho = 4.0$.

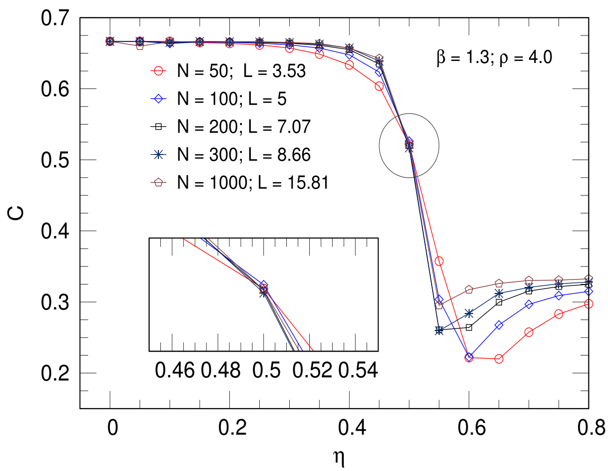

Fig. 3. (Color online) Binder cumulant C plotted as a function of the strength of the noise η for some values of N as indicated in the figure. The simulations were performed for $\beta = 1.3$ and density $\rho = 4.0$.

In Fig. 4(b), we determine the phase diagram of the model in the plane of noise versus the parameter β. The general features of the phase diagram show that the critical point (which characterize the ordered/disordered motion) decreases when β increases, indicating a first-order transition.

As the noise here varies from one type to another during the interactions, we calculate the correlation coefficient μ, or more specifically, Pearson product-moment correlation coefficient[21–23] that is a statistical method of quantifying the association or correlation between two related variables. In our case, $\mu_{(\psi,\eta)}$ will measure the strength of the correlation between the order parameter ψ and the hybrid noise η.

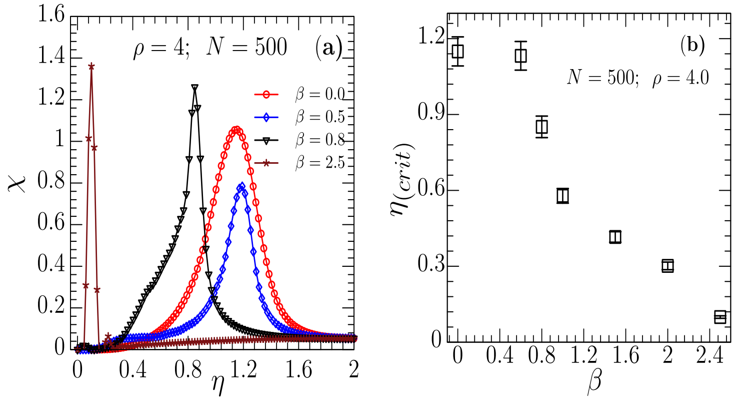

Fig. 4. (Color online) (a) Susceptibility χ versus noise η for some values of β and (b) variation of the critical noise $\eta_{\rm crit}$ as a function of the parameter β.

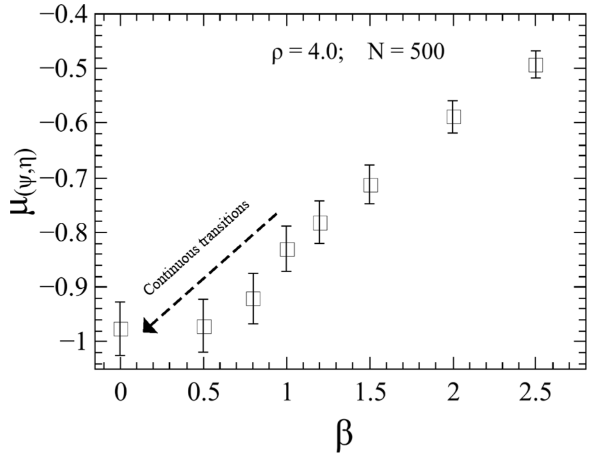

Fig. 5. (Color online) Correlation coefficient $\mu_{(\psi,\eta)}$ as a function of the parameter β. Lower value of β means higher amount of angular noise. The opposite implies higher amount of vectorial noise.

The coefficient $\mu_{\psi,\eta}$ can be either negative or positive, and it ranges from -1 to $+1$. If $\mu_{(\psi,\eta)} = 1$, it indicates a strong positive relationship, and if $\mu_{(\psi,\eta)} = -1$, it indicates a strong negative relationship. As the order parameter ψ vanishes with the increase of noise η, then $\mu_{(\psi,\eta)} = -1$ means that an increase in η is associated with a decrease in ψ. The correlation coefficient is defined by

$$\mu_{(\psi,\eta)} = \frac{\sum(\eta - \bar{\eta})(\psi - \bar{\psi})}{\sqrt{\sum(\eta - \bar{\eta})^2}\sqrt{\sum(\psi - \bar{\psi})^2}}. \tag{4}$$

Figure 5 shows the behavior of the correlation coefficient, $\mu_{(\psi,\eta)}$, between the order parameter ψ and the noise η, as a function of the parameter β. In this figure, we can observe that in the region of small β, $\mu_{(\psi,\eta)}$ reaches a maximum correlation for $\beta \leq 1.0$ (threshold associated with continuous transitions, see the Binder cumulant calculation in Fig. 2), indicating that in the region with predominate continuous transitions, correlation effects are stronger.

4. Conclusions

In this paper, we investigated the phase transition in self-propelled particle system when, in the velocity update rule, both vectorial noise and angular noise are taken into account. We considered that if the angle α between the velocities is less than a certain rate $\beta\tau$, a vectorial noise is introduced in each particle–particle interaction, otherwise it is introduced an angular noise. Simulations were made for different values of β. We found that the effects of the vectorial noise on the phase transition begin to appear for β around 1.0. From a biological point of view, this means that from this threshold value the transition from a disordered state, with motion in random directions, to an ordered motion state occurs faster, as clearly observed by the shifting in the susceptibility peaks, which shows that the critical noise decreases with the increase of β.

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