Economics 7103 - Homework 3

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1 Description of the problem

Suppose that for a home i, you think the underlying relationship between electricity use and predictor variables is

$$y_i = e^{\alpha}\delta^{d_i}z_i^{\gamma}e^{\eta_i} \tag{1}$$

where

- e is Euler's number or the base of the natural logarithm
- d_i is a binary variable equal to one if home i received the retrofit program
- z_i is a vector of the other control variables
- η_i is unobserved error
- α, δ, γ are parameters to estimate.

Question 1.a

Show that $ln(y_i) = \alpha + ln(\delta)d_i + \gamma ln(z_i) + \eta_i$

Response: The given equation, $y_i = e^{\alpha}\delta^{d_i}z_i^{\gamma}e^{\eta_i}$, represents a power regression model, which is a non-linear regression model. To transform it into a linear regression model, we need to take the natural log of both sides of the equation. We call this log-log transformation. I will use the properties of logarithms and exponential functions in this transformation:

- **Taking log:** $ln(y_i) = ln(e^{\alpha}\delta^{d_i}z_i^{\gamma}e^{\eta_i})$
- **Product Rule:** $ln(y_i) = ln(e^{\alpha}) + ln(\delta^{d_i}) + ln(z_i^{\gamma}) + ln(e^{\eta_i})$
- **Power Rule:** $ln(y_i) = \alpha ln(e) + ln(\delta)d_i + \gamma ln(z_i) + \eta_i ln(e)$
- **The natural logarithm of $e = 1$:** $ln(y_i) = \alpha + ln(\delta)d_i + \gamma ln(z_i) + \eta_i$

Question 1.b

What is the intuitive interpretation of δ ?

Response: If we divide the conditional expectations of the original equation for the treated group by the conditional expectation of the same equation for the untreated group, we will obtain:

$$\frac{[y_i|d_i = 1, z_i]}{[y_i|d_i = 0, z_i]} = \frac{e^{\alpha}\delta^{1}z_i^{\gamma}e^{\eta_i}}{e^{\alpha}\delta^{0}z_i^{\gamma}e^{\eta_i}} = \frac{e^{\alpha}\delta z_i^{\gamma}e^{\eta_i}}{e^{\alpha}z_i^{\gamma}e^{\eta_i}} = \delta \tag{2}$$

So, as we saw δ is the ratio between the expected electricity consumption of the treated and the untreated houses.

Question 1.c

Show that $\frac{\triangle y_i}{\triangle d_i} = \frac{\delta - 1}{\delta^{d_i}} y_i$. What is the intuitive interpretation of $\frac{\triangle y_i}{\triangle d_i}$.

Response:

From previous section, we saw that

$$E[y_i|d_i = 1, z_i] = E[y_i|d_i = 0, z_i]\delta \tag{3}$$

Since d_i is a binary variable, $\frac{\Delta y_i}{\Delta d_i}$ is the difference in the expected value of y_i between the treatment and control group. So,

$$\frac{\Delta y_i}{\Delta d_i} = [y_i|d_i = 1, z_i] - [y_i|d_i = 0, z_i] = [y_i|d_i = 0, z_i]\delta - [y_i|d_i = 0, z_i] = (\delta - 1)[y_i|d_i = 0, z_i]$$

To find what $[y_i|d_i = 0]$ mean, we can apply the potential outcome framework. Namely, y_{0i} is the potential outcome of y_i when $d_i = 0$ and y_{1i} when $d_i = 1$. According to the potential outcome framework, the log-transformed equation could be rewritten the following way

$$\ln(y_i) = \ln(y_{0i}) + [\ln(y_{1i}) - \ln(y_{0i})]d_i$$

As we saw in previous section, $y_{1i}/y_{0i} = \delta$.

$$\ln(\frac{y_i}{y_{0i}}) = d_i \ln(\frac{y_{1i}}{y_{0i}}) = d_i \ln(\delta)$$

$$\frac{y_i}{y_{0i}} = e^{d_i \ln(\delta)} = \delta^{d_i}$$

$$y_{0i} = \frac{1}{\delta^{d_i}} y_i$$

$$\frac{\Delta y_i}{\Delta d_i} = (\delta - 1)[y_i|d_i = 0, z_i] = (\delta - 1)y_{0i} = \frac{\delta - 1}{\delta^{d_i}} y_i$$

$\frac{\Delta y_i}{\Delta d_i}$ shows the change in monthly electricity consumption in kWh if a HH is treated (If a HH received a retrofit).

Question 1.d

Show that $\frac{\partial y_i}{\partial z_i} = \gamma \frac{y_i}{z_i}$. What is the intuitive interpretation of $\frac{\partial y_i}{\partial z_i}$ when z_i is the size of the home in square feet?

Response:

$$\frac{\partial y_i}{\partial z_i} = \frac{\partial(e^{\alpha}\delta^{d_i} z_i^{\gamma} e^{\eta_i})}{\partial z_i} = e^{\alpha}\delta^{d_i}\gamma z_i^{\gamma-1} e^{\eta_i} = e^{\alpha}\delta^{d_i} z_i^{\gamma} e^{\eta_i}\gamma z_i^{-1} = \gamma \frac{y_i}{z_i} \tag{4}$$

$\frac{\partial y_i}{\partial z_i}$ describes the marginal change in the electricity consumption of HHs when the size of the home in square feet changes.

Question 1.e

Response: Estimated parameters and the average marginal effects of z_i and d_i are presented in the table 1. The 95% confidence intervals are displayed in the square brackets.

	Parameter estimates	AME estimates
Constant	-0.769000	
	[-1.93, 0.315]	
=1 if home received retrofit	0.904000	-113.975000
	[0.894, 0.915]	[-127.401, -99.894]
Square feet of home	0.894000	0.629000
	[0.88, 0.908]	[0.617, 0.64]
Outdoor average temperature (°F)	0.281000	3.997000
	[0.048, 0.541]	[0.682, 7.759]
Observations	1000	1000

Table 1: Estimated parameters and AME (Python)

Question 1.f

Response: The Figue 1 presents the average marginal effects of outdoor temperature and square feet of the home with bands.

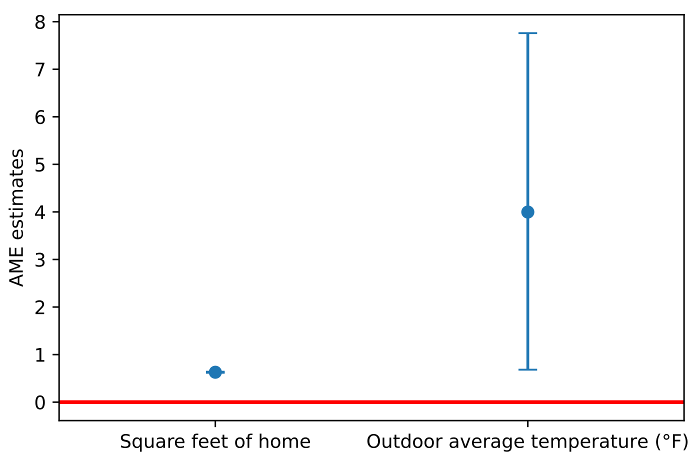

Figure 1: AME of outdoor temperature and square feet of the home with bands for their bootstrapped confidence intervals