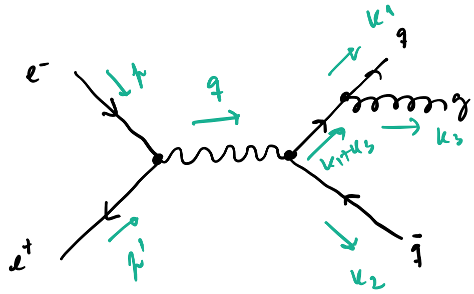
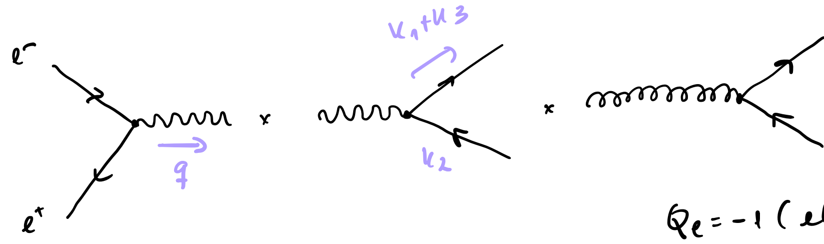
Real amplitude



Para os propagadores dos quarks usamos m_q

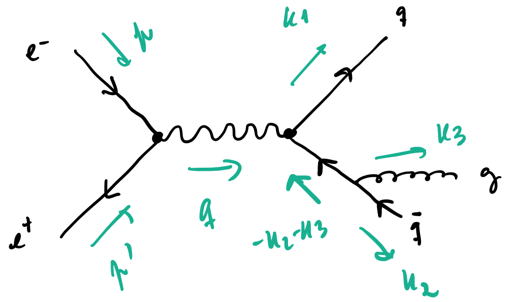
$\mathcal{A} =$ 

$Q_e = -1$ (electron)

$$\mathcal{A}_1 = \bar{v}(p')\left(ieQ_e\gamma^\mu\right)u(p)\left(\frac{-ig^{\mu\nu}}{q^2}\right)\bar{u}(k_1)\left(ig_s\gamma^\alpha t^a\right)$$

$$\times \frac{i}{\not{k}_1+\not{k}_3-m_q}\left(-ieQ_q\gamma_\nu\right)v(k_2)\times\varepsilon^\alpha(k_3)$$

$$\mathcal{A}_1 = -ie^2Q_q g_s t^a\left\{\bar{v}(p')\gamma^\mu u(p)\,\frac{1}{q^2}\,\bar{u}(k_1)\gamma^\alpha\frac{1}{\not{k}_1+\not{k}_3-m_q}\gamma^\mu v(k_2)\,\varepsilon^\alpha(k_3)\right\}$$



$Q_e = -1$

$$\mathcal{A}_2 = \bar{v}(p')\,(ieQ_e\gamma^\mu)\,u(p)\left(\frac{-ig^{\mu\nu}}{q^2}\right)\bar{u}(k_1)\,(-ieQ_q\gamma_\nu)$$

$$\times\ \frac{i}{-\not{k}_2-\not{k}_3-m_q}\,(ig_s\gamma^\alpha t^a)\ v(k_2)\ \times\ \varepsilon^\alpha(k_3)$$

$$\mathcal{A}_2 = i e^2 Q_q g_s t^a \left\{\bar{v}(p')\,\gamma^\mu u(p)\,\frac{1}{q^2}\,\bar{u}(k_1)\,\gamma^\mu\,\frac{1}{\not{k}_2+\not{k}_3+m_q}\,\gamma^\alpha\,v(k_2)\right\}\varepsilon^\alpha(k_3)$$

Juntando a contribuição dos dois diagramas (seguindo "Chong Li", pg 195)

$$\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2$$

$$\mathcal{A} = i e^2 Q_q g_s t^a \Bigg\{ -\bar{v}(p')\,\gamma^\mu u(p)\,\frac{1}{q^2}\,\bar{u}(k_1)\,\gamma^\alpha\,\frac{1}{\not{k}_1+\not{k}_3-m_q}\,\gamma^\mu\,v(k_2)\,\varepsilon^\alpha(k_3)$$

$$+\ \bar{v}(p')\,\gamma^\mu u(p)\,\frac{1}{q^2}\,\bar{u}(k_1)\,\gamma^\mu\,\frac{1}{\not{k}_2+\not{k}_3+m_q}\,\gamma^\alpha\,v(k_2)\,\varepsilon^\alpha(k_3)\Bigg\}$$

$$A = i e^2 Q_q g_s t^a \bar{v}(p') \gamma^\mu u(p) \frac{1}{q^2} \bar{u}(k_1) \cdot$$
$$\left\{ -\gamma^\alpha \frac{1}{\not{k}_1 + \not{k}_3 - m_q} \gamma^\mu + \gamma^\mu \frac{1}{\not{k}_2 + \not{k}_3 + m_q} \gamma^\alpha \right\} v(k_2) \varepsilon^\alpha(k_3)$$

Definindo

$$\Lambda_{\alpha\mu} = -\gamma^\alpha \frac{1}{\not{k}_1 + \not{k}_3 - m_q} \gamma^\mu + \gamma^\mu \frac{1}{\not{k}_2 + \not{k}_3 + m_q} \gamma^\alpha$$

a amplitude fica

$$A = i e^2 Q_q g_s t^a \bar{v}(p') \gamma^\mu u(p) \frac{1}{q^2} \bar{u}(k_1) \cdot \Lambda_{\alpha\mu} v(k_2) \varepsilon^\alpha(k_3)$$

Módulo amplitude real ao quadrado

$$\langle A|A\rangle = A^\dagger A = e^4 Q_q^2 g_s^2 \, tr(t^a t_a) \frac{1}{q^4} \cdot$$

$$\left[\bar{v}(p') \gamma^\mu u(p) \; \bar{u}(k_1) \Lambda_{\alpha\mu} v(k_2) \; \bar{u}(p) \gamma^\nu \bar{v}(p') \; \bar{v}(k_2) \Lambda_{\nu\beta} u(k_1) \right] \varepsilon^\alpha(k_3) \varepsilon^\beta(k_3)$$

e temos

$$tr(t_a t^a) = \frac{4}{3} Tr(\mathbb{1}) = \frac{4}{3} \times 3 = 4$$

Somando sobre as polarizações e os spins

$$\overline{|\mathcal{M}|}^2 = \underbrace{\frac{1}{2}\frac{1}{2}\sum_{r,s}}_{\text{average over initial pol}} \underbrace{\sum_{m,n}}_{\text{sum over final pol}} |\mathcal{M}|^2$$

$$\frac{1}{4} \sum_{r,s} \sum_{m,n} \langle A|A\rangle$$

$$= \frac{1}{4} \sum_{r,s} \sum_{m,n} 4 e^4 Q_q^2 g_s^2 \frac{1}{q^4}$$

$$\left[\bar{v}(p') \gamma^\mu u(p) \; \bar{u}(k_1) \Lambda_{\alpha\mu} v(k_2) \; \bar{u}(p) \gamma^\nu \bar{v}(p') \; \bar{v}(k_2) \Lambda_{\nu\beta} u(k_1) \right] \varepsilon^\alpha(k_3) \varepsilon^\beta(k_3)$$

In the calculation of $|\mathcal{M}|^2 = \mathcal{M}^* \mathcal{M}$, the following identity is needed

$$[\bar{u} \gamma^\mu v]^* = [u^\dagger \gamma^0 \gamma^\mu v]^\dagger = v^\dagger \gamma^{\mu\dagger} \gamma^0 u = \bar{v} \gamma^\mu u$$

$$\Leftrightarrow \boxed{[\bar{u} \gamma^\mu v]^* = \bar{v} \gamma^\mu u}$$

so we have the following amplitude

$$|\overline{\mathcal{M}}|^2 = \frac{1}{4} \sum_{r,s,m,n} \sum_{pol} |\mathcal{M}|^2 =$$

$$= \frac{1}{4} \sum 4 e^4 Q_q^2 g_s^2 \frac{1}{q^4} \times$$

$$\left[\bar{v}(p') \gamma^\mu u(p)\right]\left[\bar{u}(k_1) \Lambda_{\alpha\mu} v(k_2)\right]\left[\bar{u}(p) \gamma^\nu v(p')\right]\left[\bar{v}(k_2) \Lambda_{\nu\beta} u(k_1)\right] \varepsilon^\alpha \varepsilon^{*\beta}$$

<u>Casimir trick</u>

$$\sum_{r,s} \left[\bar{v}_r \gamma^\mu u_s\right]\left[\bar{u}_s \gamma^\nu v_r\right]$$

$$= \sum_{r,s} \left[(\bar{v}_r)_\alpha (\gamma^\mu)_{\alpha\beta} (u_s)_\beta\right]\left[(\bar{u}_s)_\sigma (\gamma^\nu)_{\sigma\delta} (v_r)_\delta\right]$$

$$= \sum_r \left[(v_r)_\delta (\bar{v}_r)_\alpha\right] \sum_s \left[(u_s)_\beta (\bar{u}_s)_\sigma\right] (\gamma^\mu)_{\alpha\beta} (\gamma^\nu)_{\sigma\delta}$$

and we can use the <u>completeness relations</u>

$$\sum_s u^s_\beta \bar{u}^s_\sigma = (\not{p} + m)_{\beta\sigma}$$

$$\sum_r v_\delta \bar{v}_\alpha = (\not{p}' - m)_{\delta\alpha}$$

$m_q \equiv$ physical mass

$\mu \equiv$ regulator mass

$$\not{p} + m$$

$$= \not{p} + \sqrt{m_q^2 + \mu^2} \,, \qquad m^2 = m_q^2 + \mu^2$$

$$\Rightarrow m = \sqrt{m_q^2 + \mu^2}$$

and we obtain

$$(\not{k}' - m')_{\delta\alpha} (\not{k} + m')_{\beta\sigma} (\gamma^\mu)_{\alpha\beta} (\gamma^\nu)_{\sigma\delta}$$

$$= (\not{k}' - m')_{\delta\alpha} (\gamma^\mu)_{\alpha\beta} (\not{k} + m')_{\beta\sigma} (\gamma^\mu)_{\alpha\beta}$$

$$= \mathrm{Tr}\left[(\not{k}' - m') \gamma^\mu (\not{k} + m') \gamma^\nu \right]$$

$$\sum_{ij} \left[\bar{u}_i(k_1) \Lambda_{\alpha\mu} v_j(k_2) \right] \left[\bar{v}_j(k_2) \Lambda_{\nu\beta} u(k_1) \right]$$

$$= \sum_{ij} \left[(\bar{u}_i^\eta) (\Lambda_{\alpha\mu})^{\eta\lambda} (v_j)^\lambda \right] \left[(\bar{v}_j)_\sigma (\Lambda_{\nu\beta})^{\sigma\delta} (u_i)_\delta \right]$$

$$= \sum_i \left[(u_i)_\delta (\bar{u}_i)_\eta \right] \sum_j \left[(v_j)^\lambda (\bar{v}_j)^\sigma \right] (\Lambda_{\alpha\mu})^{\eta\lambda} (\Lambda_{\nu\beta})^{\sigma\delta}$$

$$= \sum_i \left[(u_i)_\delta (\bar{u}_i)_\eta \right] \sum_j \left[(v_j)^\lambda (\bar{v}_j)^\sigma \right] (\Lambda_{\alpha\mu})^{\eta\lambda} (\Lambda_{\nu\beta})^{\sigma\delta}$$

$$= (\not{k}_1 + m)_{\delta\eta} (\Lambda_{\alpha\mu})^{\eta\lambda} (\not{k}_2 - m)_{\lambda\sigma} (\Lambda_{\nu\beta})^{\sigma\delta}$$

$$= \mathrm{Tr}\left[(\not{k}_1 + m) \Lambda_{\alpha\mu} (\not{k}_2 - m) \Lambda_{\nu\beta} \right]$$

substituindo na amplitude, obtemos

$$|\overline{\mathcal{M}}|^2 = e^4 Q_q^2 g_A^2 \frac{1}{q^4} \, ,$$

$$Tr\left[(\not{k}' - m')\gamma^\mu(\not{k} + m')\gamma^\nu\right] Tr\left[(\not{k}_1 + m)\Lambda_{\alpha\mu}(\not{k}_2 - m)\Lambda_{\nu\beta}\right] \underbrace{\sum_{pol} \varepsilon^\alpha(k_3)\varepsilon^\beta(k_3)^*}_{-g^{\alpha\beta}}$$

$$|\overline{\mathcal{M}}|^2 = -e^4 Q_q^2 g_A^2 \frac{1}{q^4} \underbrace{Tr\left[(\not{k}' - m')\gamma^\mu(\not{k} + m')\gamma^\nu\right]}_{l_{\mu\nu}} \underbrace{Tr\left[(\not{k}_1 + m)\Lambda_{\alpha\mu}(\not{k}_2 - m)\Lambda_{\nu\alpha}\right]}_{G^{\mu\nu}}$$

$$\boxed{|\overline{\mathcal{M}}|^2 = -e^4 Q_q^2 g_A^2 \frac{1}{q^4} \, l^{\mu\nu} G_{\mu\nu}}$$

Espaço fase

$p' + p = q = k_1 + k_2 + k_3$

$$e^+(p') + e^-(p) \longrightarrow q(k_1) + \bar{q}(k_2) + g(k_3)$$

$$\rho = \int \frac{1}{(2\pi)^5} \frac{d^3k_1}{2\omega_1} \frac{d^3k_2}{2\omega_2} \frac{d^3k_3}{2\omega_3} \underbrace{\delta(q - k_1 - k_2 - k_3)}_{\delta(q_0 - \omega_1 - \omega_2 - \omega_3)\ \delta^3(\vec{q} - \vec{k_1} - \vec{k_2} - \vec{k_3})}$$

where $q = p' + p$

choosing the reference frame where $\vec{q} = \vec{p}' + \vec{p} = 0$

gives $\vec{k_1} + \vec{k_2} + \vec{k_3} = 0$

so

$$\rho = \int \frac{1}{(2\pi)^5} \frac{d^3k_1}{2\omega_1} \frac{d^3k_2}{2\omega_2} \frac{1}{2\omega_3} \delta(q_0 - \omega_1 - \omega_2 - \omega_3)$$

Em coordenadas esféricas

$$\begin{cases} d^3k_1 = k_1^2\, d(\cos\theta_1)\, dk_1\, d\phi \iff d^3k_1 = k_1^2\, dk_1\, d\Omega_1 \\ \qquad \text{onde } d\Omega_1 = d(\cos\theta_1)\, d\phi \end{cases}$$

$$\begin{cases} d^3k_2 = k_2^2\, d(\cos\theta_2)\, dk_2\, d\phi \iff d^3k_2 = k_2^2\, dk_2\, d\Omega_2 \\ \qquad \text{onde } d\Omega_2 = d(\cos\theta_2)\, d\phi \end{cases}$$

considerando que o feixe não está polarizado, não há uma direção preferencial nos eixos do referencial para orientar os vetores considerando que $\vec{k_1}$ pode estar orientado para qualquer direção

$$d\Omega_1 = 4\pi$$

e então $\vec{k_2}$ está orientado com o ângulo polar em relação a $\vec{k_1}$.

$$\int d\Omega_2 = \int d(\cos\theta_{12}) \; d\phi = \int d(\cos\theta_3) \; 2\pi$$

Então

$$\rho = \int \frac{1}{(2\pi)^5} \frac{\delta(q_0 - w_1 - w_2 - w_3)}{2w_1 \; 2w_2 \; 2w_3} \; k_1^2 \, dk_1 \; (4\pi) \; k_2^2 \, dk_2 \; d(\cos\theta_2) \; (2\pi)$$

$$\rho = \int \frac{1}{(2\pi)^3} \frac{\delta(q_0 - w_1 - w_2 - w_3)}{2w_1 \; 2w_2 \; w_3} \; k_1^2 \, dk_1 \; k_2^2 \, dk_2 \; d(\cos\theta_2)$$

Assumindo uma massa reguladora μ para o gluon

$$w_3^2 = |\vec{k_3}|^2 + \mu^2 \rightarrow w_3^2 = |\vec{k_1}|^2 + |k_2|^2 + 2 k_1 k_2 \cos\theta_3 + \mu^2$$

$$\cancel{2} w_3 \, dw_3 = \cancel{2} k_1 k_2 \; d(\cos\theta_3)$$

$$w_3 \, dw_3 = k_1 k_2 \; d(\cos\theta_3)$$

$$\Leftrightarrow \; d(\cos\theta_3) = \frac{w_3 \, dw_3}{k_1 k_2}$$

$$\rho = \int \frac{1}{(2\pi)^3} \frac{\delta(q_0 - w_1 - w_2 - w_3)}{2w_1 \; 2w_2 \; \cancel{w_3}} \; k_1^{\cancel{2}} \, dk_1 \; k_2^{\cancel{2}} \, dk_2 \; \frac{\cancel{w_3} \, dw_3}{\cancel{k_1} \, \cancel{k_2}}$$

we have $\quad w_i \, dw_i = k_i \, dk_i$, so

$$\rho = \int \frac{1}{(2\pi)^3} \overbrace{\frac{\delta(q_0 - w_1 - w_2 - w_3)}{2\cancel{w_1} \; 2\cancel{w_2}}} \; \cancel{w_1} \, dw_1 \; \cancel{w_2} \, dw_2 \; \overbrace{dw_3}$$

$$\Leftrightarrow \quad \boxed{\rho = \frac{1}{32\pi^3} \int dw_1 \, dw_2}$$

Dimensionless variables

Defining (for $\vec{q} = 0$)

$$x_1 = \frac{1}{q^2}\left[(k_1 - q)^2 - m^2\right] \quad , \quad m^2 = m_q^2 + \mu^2$$

$\downarrow$

physical mass

$$x_2 = \frac{1}{q^2}\left[(k_2 - q)^2 - m^2\right]$$

$$x_3 = \frac{1}{q^2}\left[(k_3 - q)^2 - \mu^2\right]$$

For $i = 1, 2$

$$X_i = \frac{(k_i - q)^2}{q^2} - \frac{m^2}{q^2} = \frac{(k_i - q)^2}{q^2} - \frac{m_q^2}{q^2} - \frac{\mu^2}{q^2}$$

$$X_i = \cancel{\frac{k_i^2}{q^2}} + 1 - \frac{2 k_i q}{q^2} \cancel{- \frac{m_q^2}{q^2} - \frac{\mu^2}{q^2}}$$

using $k_i^2 = m_q^2 + \mu^2$

$$X_i = 1 - \frac{2 k_i q_i}{q^2} = 1 - \frac{2 k_i^0 q_0 - 2 \vec{k}_1 \cdot \vec{q}}{q_0^2 - |\vec{q}|^2}$$

In a reference frame where $\vec{q} = 0$

$$X_j = 1 - \frac{2 k_i^0 q_0 - \cancel{2 \vec{k}_1 \cdot \vec{q}}^{\,0}}{q_0^2 - \cancel{|\vec{q}|^2}_{\,0}}$$

$$\Leftrightarrow X_i = 1 - \frac{2 w_i \cancel{q_0}}{q_0^{\cancel{2}}} \Leftrightarrow X_i = 1 - \frac{2 w_i}{q_0} \quad , i = 1, 2$$

For $i = 3$, $k_3 = \mu^2$

$$x_3 = \frac{1}{q^2}\left[(k_3 - q)^2 - \mu^2\right] \Leftrightarrow X_3 = 1 - \frac{2 w_3}{q_0}$$

So we obtain

$$\boxed{x_i = 1 - \frac{2 w_i}{q_0}} \quad , \; i=1,2,3$$

Differentiating both sides of the eq.

$$\boxed{dx_i = -\frac{2}{q_0} dw_i} \quad \Leftrightarrow \quad dw_i = -\frac{q_0}{2} dx_i$$

and the phase space integral in terms of dimensionless variables is

$$\rho = \frac{1}{32\pi^3} \int dw_1 \, dw_2 = \frac{1}{32\pi^3} \frac{q_0^2}{4} \int dx_1 \, dx_2$$

$$\Leftrightarrow \quad \boxed{\rho = \frac{1}{128\pi^3} q_0^2 \int_{x_{1min}}^{x_{1max}} \int_{x_{2min}}^{x_{2max}} dx_1 \, dx_2}$$

$$x_1 + x_2 + x_3 = 1 - \frac{2}{q_0} w_1 + 1 - \frac{2}{q_0} w_2 + 1 - \frac{2}{q_0} w_3$$

$$= 3 - \frac{2}{q_0} (\underbrace{w_1 + w_2 + w_3}_{q_0}) = 3 - 2 \frac{\cancel{q_0}}{\cancel{q_0}} = 1$$

Integrating the real amplitude over the phase space

$$\int d\rho\, |\mathcal{M}|^2 = \frac{q^2}{128\pi^3}\int dx_1\, dx_2\ |\mathcal{M}|^2$$

$$= \frac{q^2}{128\pi^3}\int dx_1 dx_2 \left(-e^4 Q_q^2 g_s^2 \frac{1}{q^4} l^{\mu\nu} G_{\mu\nu}\right)$$

$$\alpha_s = \frac{g_s^2}{4\pi}$$

$$= \frac{-q^2 e^4 Q_q^2 g_s^2}{128\pi^3}\int dx_1 dx_2\ \frac{1}{q^4}\, l^{\mu\nu} G_{\mu\nu}$$

Gauge invariance implies that

$$q_\mu G^{\mu\nu} = q_\nu G^{\mu\nu} = q_\mu l^{\mu\nu} = q_\nu l^{\mu\nu} = 0$$

$$\int G^{\mu\nu} d\rho = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) G(q^2)$$

onde $\quad G(q^2) = \frac{1}{3}\int g_{\mu\nu}\, G^{\mu\nu}\, d\rho$

então

$$\int d\rho\, |\mathcal{M}|^2 = \frac{-q^2 e^4 Q_q^2 g_s^2}{128\pi^3}\int dx_1 dx_2\ \frac{1}{q^4}\, l^{\mu\nu} G_{\mu\nu}$$

$$= -\frac{q^2 e^4 Q_q^2}{32\pi^2}\overset{\alpha}{\left(\frac{g_s^2}{4\pi}\right)}\int \frac{1}{q^4}\, l^{\mu\nu}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) G(q^2)$$

$$= -\frac{q^2 e^4 Q_q^2 \alpha_s}{32\pi^2}\int \frac{1}{q^4}\underbrace{l^{\mu\nu} g_{\mu\nu}}_{-4q^2}\frac{1}{3}\, g_{\alpha\beta}\, G^{\alpha\beta}\, d\rho$$

$$= \frac{\cancel{q^2}\, e^4 Q_q^2 \alpha_s}{24\pi^2} \int \frac{1}{\cancel{q^2}}\, g_{\alpha\beta}\, G^{\alpha\beta}\, d\rho$$

Para calcular $g_{\alpha\beta}\, G^{\alpha\beta} = g_{\alpha\beta}\, \mathrm{Tr}\left[(\not{k}_1 + m)\, \Lambda_{\alpha\beta}\, (\not{k}_2 - m)\, \Lambda_{\beta\alpha}\right]$

$$= \mathrm{Tr}\left[(\not{k}_1 + m)\, \Lambda_{\alpha\beta}\, (\not{k}_2 - m)\, \Lambda_{\beta\alpha}\right]$$

and we defined

$$\Lambda_{\alpha\beta} = \gamma^\alpha \frac{1}{\not{k}_1 + \not{k}_3 - m_q} \gamma^\beta + \gamma^\beta \frac{1}{\not{k}_2 + \not{k}_3 + m_q} \gamma^\alpha$$

and we can write

$$\Lambda_{\alpha\beta} = -\gamma^\alpha \frac{\not{k}_1 + \not{k}_3 + m_q}{(\not{k}_1 + \not{k}_3 - m_q)(\not{k}_1 + \not{k}_3 + m_q)} \gamma^\beta + \gamma^\beta \frac{\not{k}_2 + \not{k}_3 - m_q}{(\not{k}_2 + \not{k}_3 + m_q)(\not{k}_2 + \not{k}_3 - m_q)} \gamma^\alpha$$

$$\Leftrightarrow \Lambda_{\alpha\beta} = \frac{-\gamma^\alpha (\not{k}_1 + \not{k}_3 + m_q)\gamma^\beta}{(k_1 + k_3)^2 - m_q^2} + \frac{\gamma^\beta (\not{k}_2 + \not{k}_3 - m_q)\gamma^\alpha}{(k_2 + k_3)^2 - m_q^2}$$

<u>Note</u>: $(\not{k}_1 + \not{k}_3 + m_q)(\not{k}_1 + \not{k}_3 - m_q)$

$$= \not{k}_1\not{k}_1 + \not{k}_1\not{k}_3 - \cancel{m_q\not{k}_1} + \not{k}_3\not{k}_1 + \not{k}_3\not{k}_3 - \cancel{m_q\not{k}_3} + \cancel{m_q\not{k}_1} + \cancel{m_q\not{k}_3} - m_q^2$$

$$= k_1^2 + k_3^2 + \{\not{k}_1, \not{k}_3\} - m_q^2 = k_1^2 + k_3^2 + 2k_1\cdot k_3 - m_q^2$$

$$= (k_1 + k_3)^2 - m_q^2$$

$$g_{\alpha\beta}\, G^{\alpha\beta} = \mathrm{Tr}\left[(\not{k}_1 + m)\, \Lambda_{\alpha\beta}\, (\not{k}_2 - m)\, \Lambda_{\beta\alpha} \right]$$

$$= \mathrm{Tr}\Bigg[(\not{k}_1 + m) \left[\frac{-\gamma^{\alpha} (\not{k}_1 + \not{k}_3 + m_g)\gamma^{\beta}}{(k_1+k_3)^2 - m_g^2} + \frac{\gamma^{\beta} (\not{k}_2 + \not{k}_3 - m_g)\gamma^{\alpha}}{(k_2+k_3)^2 - m_g^2} \right]$$

$$\cdot (\not{k}_2 - m) \left[\frac{-\gamma^{\beta} (\not{k}_1 + \not{k}_3 + m_g)\gamma^{\alpha}}{(k_1+k_3)^2 - m_g^2} + \frac{\gamma^{\alpha} (\not{k}_2 + \not{k}_3 - m_g)\gamma^{\beta}}{(k_2+k_3)^2 - m_g^2} \right] \Bigg]$$

Separating by the type of denominator:

$$\left[\frac{1}{(k_1+k_3)^2 - m_g^2} \right]^2$$

$$\mathrm{Tr}\left[(\not{k}_1 + m)\gamma^{\alpha} (\not{k}_1 + \not{k}_3 + m_g)\gamma^{\beta} (\not{k}_2 - m)\gamma^{\beta} (\not{k}_1 + \not{k}_3 + m_g)\gamma^{\alpha} \right]$$

using Mathematica to simplify

$$\mathrm{Tr}\left[\overbrace{(\not{k}_1 + m)}^{\mu}\gamma^{\alpha} \overbrace{(\not{k}_1 + \not{k}_3 + m_g)}^{\nu}\gamma^{\beta} \overbrace{(\not{k}_2 - m)}^{\sigma}\gamma^{\beta} \overbrace{(\not{k}_1 + \not{k}_3 + m_g)}^{\delta}\gamma^{\alpha} \right]$$

$$= \mathrm{Tr}\left[(\underbrace{k_1^{\mu}\gamma^{\mu}}_{A} + m)\gamma^{\alpha} \left(\underbrace{(k_1 + k_3)^{\nu}\gamma^{\nu}}_{B} + m_g \right)\gamma^{\beta} (\underbrace{k_2^{\sigma}\gamma^{\sigma}}_{C} - m)\gamma^{\beta} \left(\underbrace{(k_1 + k_3)^{\delta}\gamma^{\delta}}_{D} + m_g \right)\gamma^{\alpha} \right]$$

$$= \mathrm{Tr}\left[(A + m)\gamma^{\alpha} (B + m_g)\gamma^{\beta} (C - m)\gamma^{\beta} (D + m_g)\gamma^{\alpha} \right]$$

$$
\begin{aligned}
&= \mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta C\gamma^\beta D\gamma^\alpha\right] + \mathrm{Tr}\left[\cancelto{0}{m\gamma^\alpha B\gamma^\beta C\gamma^\beta D\gamma^\alpha}\right] + \\
&+ \mathrm{Tr}\left[\cancelto{0}{A\gamma^\alpha m_q\gamma^\beta C\gamma^\beta D\gamma^\alpha}\right] - \mathrm{Tr}\left[\cancelto{0}{A\gamma^\alpha B\gamma^\beta m\gamma^\beta D\gamma^\alpha}\right] + \\
&+ \mathrm{Tr}\left[\cancelto{0}{A\gamma^\alpha B\gamma^\beta C\gamma^\beta m_q\gamma^\alpha}\right] + \mathrm{Tr}\left[m\gamma^\alpha m_q\gamma^\beta C\gamma^\beta D\gamma^\alpha\right] + \\
&- \mathrm{Tr}\left[m\gamma^\alpha B\gamma^\beta m\gamma^\beta D\gamma^\alpha\right] + \mathrm{Tr}\left[m\gamma^\alpha B\gamma^\beta C\gamma^\beta m_q\gamma^\alpha\right] + \\
&- \mathrm{Tr}\left[A\gamma^\alpha m_q\gamma^\beta m\gamma^\beta D\gamma^\alpha\right] + \mathrm{Tr}\left[A\gamma^\alpha m_q\gamma^\beta C\gamma^\beta m_q\gamma^\alpha\right] + \\
&- \mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta m\gamma^\beta m_q\gamma^\alpha\right] - \mathrm{Tr}\left[\cancelto{0}{m\gamma^\alpha m_q\gamma^\beta m\gamma^\beta D\gamma^\alpha}\right] + \\
&- \mathrm{Tr}\left[\cancelto{0}{m\gamma^\alpha B\gamma^\beta m\gamma^\beta m_q\gamma^\alpha}\right] - \mathrm{Tr}\left[\cancelto{0}{A\gamma^\alpha m_q\gamma^\beta m\gamma^\beta m_q\gamma^\alpha}\right] + \\
&+ \mathrm{Tr}\left[\cancelto{0}{m\gamma^\alpha m_q\gamma^\beta C\gamma^\beta m_q\gamma^\alpha}\right] - \mathrm{Tr}\left[m\gamma^\alpha m_q\gamma^\beta m\gamma^\beta m_q\gamma^\alpha\right]
\end{aligned}
$$

onde usamos a notação

$$A \equiv k_1^\mu \gamma^\mu$$

$$B \equiv (k_1+k_3)^\nu \gamma^\nu$$

$$C \equiv k_2^\sigma \gamma^\sigma$$

$$D \equiv (k_1+k_3)^\delta \gamma^\delta$$

$$= Tr\left[A\gamma^\alpha B\gamma^\beta C\gamma^\beta D\gamma^\alpha\right]$$

$$+ mm_q\Big(Tr\left[\gamma^\alpha\gamma^\beta C\gamma^\beta D\gamma^\alpha\right] + Tr\left[\gamma^\alpha B\gamma^\beta C\gamma^\beta\gamma^\alpha\right]$$

$$- Tr\left[A\gamma^\alpha\gamma^\beta\gamma^\beta D\gamma^\alpha\right] - Tr\left[A\gamma^\alpha B\gamma^\beta\gamma^\beta\gamma^\alpha\right]\Big)$$

$$- m^2\, Tr\left[\gamma^\alpha B\gamma^\beta\gamma^\beta D\gamma^\alpha\right]$$

$$+ m_q^2\, Tr\left[A\gamma^\alpha\gamma^\beta C\gamma^\beta\gamma^\alpha\right]$$

$$- m^2 m_q^2\, Tr\left[\gamma^\alpha\gamma^\beta\gamma^\beta\gamma^\alpha\right]$$

Termos sem massa

- $Tr\left[A\gamma^\alpha B\gamma^\beta C\gamma^\beta D\gamma^\alpha\right] =$

$$= k_1^\mu (k_1+k_3)^\nu k_2^\sigma (k_1+k_3)^\delta\, Tr\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\gamma^\sigma\gamma^\beta\gamma^\delta\gamma^\alpha\right]$$

$$\Big\downarrow \text{Mathematica}$$

$$-16\, k_3^2 (k_1\cdot k_2) + 32\, k_1^2 (k_2\cdot k_3) + 32 (k_1\cdot k_3)(k_2\cdot k_3) + 16 k_1^2 (k_1\cdot k_2)$$

Termos proporcionais a $m m_q$ e m^2 e m_q^2

- $Tr\left[\gamma^\alpha \gamma^\beta C \gamma^\beta D \gamma^\alpha\right] = k_2^\sigma (k_1+k_3)^\delta \, Tr\left[\gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\beta \gamma^\delta \gamma^\alpha\right]$ $+ m m_q$
- $Tr\left[\gamma^\alpha B \gamma^\beta C \gamma^\beta \gamma^\alpha\right] = (k_1+k_3)^\nu k_2^\sigma \, Tr\left[\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\sigma \gamma^\beta \gamma^\alpha\right]$ $+ m m_q$
- $Tr\left[A \gamma^\alpha \gamma^\beta C \gamma^\beta \gamma^\alpha\right] = k_1^\mu k_2^\sigma \, Tr\left[\gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\beta \gamma^\alpha\right]$ $+ m_q^2$
- $Tr\left[\gamma^\alpha B \gamma^\beta \gamma^\beta D \gamma^\alpha\right] = (k_1+k_3)^\nu (k_1+k_3)^\delta \, Tr\left[\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\beta \gamma^\delta \gamma^\alpha\right]$ $- m^2$
- $Tr\left[A \gamma^\alpha \gamma^\beta \gamma^\beta D \gamma^\alpha\right] = k_1^\mu (k_1+k_3)^\delta \, Tr\left[\gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\beta \gamma^\delta \gamma^\alpha\right]$ $- m m_q$
- $Tr\left[A \gamma^\alpha B \gamma^\beta \gamma^\beta \gamma^\alpha\right] = k_1^\mu (k_1+k_3)^\nu \, Tr\left[\gamma^\mu \gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\beta \gamma^\alpha\right]$ $- m m_q$

Fazendo a algebra de Dirac no Mathematica e juntando os termos obtemos

$$m m_q \left(-64\, k_1.k_2 + 64\, k_1.k_3 + 64\, k_1^2 - 64\, k_2.k_3\right)$$

$$+ m_q^2 \left(16\, (k_1.k_2)\right)$$

$$- m^2 \left(128\, (k_1.k_3) - 64\, k_1^2 - 64\, k_3^2\right)$$

$$= 64\, m m_q \left(-k_1.k_2 + k_1.k_3 + k_1^2 - k_2.k_3\right) + 16\, m_q^2\, (k_1.k_2)$$

$$+ 64\, m^2 \left(k_1^2 + k_3^2 - 2\, k_1.k_3\right)$$

Separating by the type of denominator:

$$\boxed{\left[\frac{1}{(k_2+k_3)^2 - m_q^2}\right]^2}$$

$A = k_1^\mu \gamma^\mu$

$B = (k_2+k_3)^\nu \gamma^\nu$

$C = k_2^\sigma \gamma^\sigma$

$D = (k_2+k_3)^\delta \gamma^\delta$

$$\mathrm{Tr}\left[(\overbrace{\not{k}_1}^{\mu} + m)\gamma^\beta(\overbrace{\not{k}_2 + \not{k}_3}^{\nu} - m_q)\gamma^\alpha(\overbrace{\not{k}_2}^{\sigma} - m)\gamma^\alpha(\overbrace{\not{k}_2 + \not{k}_3}^{\delta} - m_q)\gamma^\beta\right]$$

$$= \mathrm{Tr}\left[(\underbrace{k_1^\mu\gamma^\mu}_{A} + m)\gamma^\beta(\underbrace{(k_2+k_3)^\nu\gamma^\nu}_{B} - m_q)\gamma^\alpha(\underbrace{k_2^\sigma\gamma^\sigma}_{C} - m)\gamma^\alpha(\underbrace{(k_2+k_3)^\delta\gamma^\delta}_{D} - m_q)\gamma^\beta\right]$$

$$= \mathrm{Tr}\left[(A+m)\gamma^\beta(B-m_q)\gamma^\alpha(C-m)\gamma^\alpha(D-m_q)\gamma^\beta\right]$$

$$= \mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta C\gamma^\beta D\gamma^\alpha\right] + \cancelto{0}{\mathrm{Tr}\left[m\gamma^\alpha B\gamma^\beta C\gamma^\beta D\gamma^\alpha\right]} +$$

$$- \cancelto{0}{\mathrm{Tr}\left[A\gamma^\alpha m_q\gamma^\beta C\gamma^\beta D\gamma^\alpha\right]} - \cancelto{0}{\mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta m\gamma^\beta D\gamma^\alpha\right]} +$$

$$- \cancelto{0}{\mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta C\gamma^\beta m_q\gamma^\alpha\right]} - \mathrm{Tr}\left[m\gamma^\alpha m_q\gamma^\beta C\gamma^\beta D\gamma^\alpha\right] +$$

$$- \mathrm{Tr}\left[m\gamma^\alpha B\gamma^\beta m\gamma^\beta D\gamma^\alpha\right] - \mathrm{Tr}\left[m\gamma^\alpha B\gamma^\beta C\gamma^\beta m_q\gamma^\alpha\right] +$$

$$+ \mathrm{Tr}\left[A\gamma^\alpha m_q\gamma^\beta m\gamma^\beta D\gamma^\alpha\right] + \mathrm{Tr}\left[A\gamma^\alpha m_q\gamma^\beta C\gamma^\beta m_q\gamma^\alpha\right] +$$

$$+ \mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta m\gamma^\beta m_q\gamma^\alpha\right] + \cancelto{0}{\mathrm{Tr}\left[m\gamma^\alpha m_q\gamma^\beta m\gamma^\beta D\gamma^\alpha\right]} +$$

$$+ \cancelto{0}{\mathrm{Tr}\left[m\gamma^\alpha B\gamma^\beta m\gamma^\beta m_q\gamma^\alpha\right]} - \cancelto{0}{\mathrm{Tr}\left[A\gamma^\alpha m_q\gamma^\beta m\gamma^\beta m_q\gamma^\alpha\right]} +$$

$$+ \cancelto{0}{\mathrm{Tr}\left[m\gamma^\alpha m_q\gamma^\beta C\gamma^\beta m_q\gamma^\alpha\right]} - \mathrm{Tr}\left[m\gamma^\alpha m_q\gamma^\beta m\gamma^\beta m_q\gamma^\alpha\right]$$

$$= \mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta C\gamma^\beta D\gamma^\alpha\right]$$

$$+ m m_q \Big(-\mathrm{Tr}\left[\gamma^\alpha\gamma^\beta C\gamma^\beta D\gamma^\alpha\right] - \mathrm{Tr}\left[\gamma^\alpha B\gamma^\beta C\gamma^\beta\gamma^\alpha\right]$$

$$+ \mathrm{Tr}\left[A\gamma^\alpha\gamma^\beta\gamma^\beta D\gamma^\alpha\right] + \mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta\gamma^\beta\gamma^\alpha\right]$$

$$+ m_q^2 \Big(\mathrm{Tr}\left[A\gamma^\alpha\gamma^\beta C\gamma^\beta\gamma^\alpha\right]\Big)$$

$$+ m^2 \Big(-\mathrm{Tr}\left[\gamma^\alpha B\gamma^\beta\gamma^\beta D\gamma^\alpha\right]\Big)$$

$$+ m_q^2 m^2 \Big(-\mathrm{Tr}\left[\gamma^\alpha\gamma^\beta\gamma^\beta\gamma^\alpha\right]\Big)$$

Termos sem dependência na massa

- $\mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta C\gamma^\beta D\gamma^\alpha\right]$

$$= k_1^\mu (k_2+k_3)^\nu k_2^\sigma (k_2+k_3)^\delta \,\mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\gamma^\sigma\gamma^\beta\gamma^\delta\gamma^\alpha\right]$$

$$\Big\downarrow \text{Mathematica}$$

$$= 32\, k_2^2 (k_1\cdot k_3) + 32 (k_1\cdot k_3)(k_2\cdot k_3) - 16\, k_3^2 (k_1\cdot k_2) + 16\, k_2^2 (k_1\cdot k_2)$$

Termos proporcionais a m^2, m_q^2 e $m m_q$

- $Tr\left[\gamma^\alpha \gamma^\beta C \gamma^\beta D \gamma^\alpha\right] = k_2^\sigma (k_2+k_3)^\delta \, Tr\left[\gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\beta \gamma^\delta \gamma^\alpha\right]$ $- m m_q$

- $Tr\left[\gamma^\alpha B \gamma^\beta \gamma^\beta D \gamma^\alpha\right] = (k_2+k_3)^\nu (k_2+k_3)^\delta \, Tr\left[\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\beta \gamma^\delta \gamma^\alpha\right]$ $- m^2$

- $Tr\left[\gamma^\alpha B \gamma^\beta C \gamma^\beta \gamma^\alpha\right] = (k_2+k_3)^\nu k_2^\sigma \, Tr\left[\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\sigma \gamma^\beta \gamma^\alpha\right]$ $- m m_q$

- $Tr\left[A \gamma^\alpha \gamma^\beta \gamma^\beta D \gamma^\alpha\right] = k_1^\mu (k_2+k_3)^\delta \, Tr\left[\gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\beta \gamma^\delta \gamma^\alpha\right]$ $+ m m_q$

- $Tr\left[A \gamma^\alpha \gamma^\beta C \gamma^\beta \gamma^\alpha\right] = k_1^\mu k_2^\sigma \, Tr\left[\gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\beta \gamma^\alpha\right]$ $+ m_q^2$

- $Tr\left[A \gamma^\alpha B \gamma^\beta \gamma^\beta \gamma^\alpha\right] = k_1^\mu (k_2+k_3)^\nu \, Tr\left[\gamma^\mu \gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\beta \gamma^\alpha\right]$ $+ m m_q$

Fazendo no Mathematica, obtemos

$$- 64 m^2 \left(2 k_2 \cdot k_3 + k_2^2 + k_3^2 \right)$$

$$+ m_q^2 \left(16 \, k_1 \cdot k_2 \right)$$

$$+ 64 \, m m_q \left(- k_1 \cdot k_2 - k_1 \cdot k_3 + k_2 \cdot k_3 + k_2^2 \right)$$

$$\boxed{\frac{1}{[(k_1+k_3)^2-m^2][(k_2+k_3)^2-m^2]}}$$

$$-\,\mathrm{Tr}\left[(\not k_1+m)\gamma^\alpha(\not k_1+\not k_3+m_q)\gamma^\beta(\not k_2-m)\gamma^\alpha(\not k_2+\not k_3-m_q)\gamma^\beta\right]$$

$$-\,\mathrm{Tr}\left[(\not k_1+m)\gamma^\beta(\not k_2+\not k_3-m_q)\gamma^\alpha(\not k_2-m)\gamma^\beta(\not k_1+\not k_3+m_q)\gamma^\alpha\right]$$

Fazendo troca de índices $\alpha \leftrightarrow \beta$ na 2ª parcela

$$\mu \qquad \nu \qquad \sigma \qquad \delta$$

$$= -2\,\mathrm{Tr}\left[(\not k_1+m)\gamma^\alpha(\not k_1+\not k_3+m_q)\gamma^\beta(\not k_2-m)\gamma^\alpha(\not k_2+\not k_3-m_q)\gamma^\beta\right]$$

$$= -2\,\mathrm{Tr}\left[(\underbrace{k_1^\mu\gamma^\mu}_{A}+m)\gamma^\alpha(\underbrace{(k_1+k_3)^\nu\gamma^\nu}_{B}+m_q)\gamma^\beta(\underbrace{k_2^\sigma\gamma^\sigma}_{C}-m)\gamma^\alpha(\underbrace{(k_2+k_3)^\delta\gamma^\delta}_{D}-m_q)\gamma^\beta\right]$$

$$= -2\,\mathrm{Tr}\left[(A+m)\gamma^\alpha(B+m_q)\gamma^\beta(C-m)\gamma^\alpha(D-m_q)\gamma^\beta\right]$$

$$= -2\Big\{\mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta C\gamma^\alpha D\gamma^\beta\right] + \cancelto{0}{\mathrm{Tr}\left[m\gamma^\alpha B\gamma^\beta C\gamma^\alpha D\gamma^\beta\right]} +$$

$$+\cancelto{0}{\mathrm{Tr}\left[A\gamma^\alpha m_q\gamma^\beta C\gamma^\alpha D\gamma^\beta\right]} - \cancelto{0}{\mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta m\gamma^\alpha D\gamma^\beta\right]} +$$

$$-\cancelto{0}{\mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta C\gamma^\alpha m_q\gamma^\beta\right]} + \mathrm{Tr}\left[m\gamma^\alpha m_q\gamma^\beta C\gamma^\alpha D\gamma^\beta\right] +$$

$$-\,\mathrm{Tr}\left[m\gamma^\alpha B\gamma^\beta m\gamma^\alpha D\gamma^\beta\right] - \mathrm{Tr}\left[m\gamma^\alpha B\gamma^\beta C\gamma^\alpha m_q\gamma^\beta\right] +$$

$$-\,\mathrm{Tr}\left[A\gamma^\alpha m_q\gamma^\beta m\gamma^\alpha D\gamma^\beta\right] - \mathrm{Tr}\left[A\gamma^\alpha m_q\gamma^\beta C\gamma^\alpha m_q\gamma^\beta\right] +$$

$$+\,\mathrm{Tr}\left[A\gamma^\alpha B\gamma^\beta m\gamma^\alpha m_q\gamma^\beta\right] - \cancelto{0}{\mathrm{Tr}\left[m\gamma^\alpha m_q\gamma^\beta m\gamma^\alpha D\gamma^\beta\right]} +$$

$$+\mathrm{Tr}\left[m\gamma^{\alpha}\cancel{B\gamma^{\beta}}^{\,0}\, m\gamma^{\alpha} m_q\gamma^{\beta}\right]+\mathrm{Tr}\left[A\gamma^{\alpha}\cancel{m_q\gamma^{\beta}}^{\,0}\, m\gamma^{\alpha} m_q\gamma^{\beta}\right]+$$

$$-\mathrm{Tr}\left[m\gamma^{\alpha}\cancel{m_q\gamma^{\beta}}^{\,0}\, C\gamma^{\alpha} m_q\gamma^{\beta}\right]+\mathrm{Tr}\left[m\gamma^{\alpha}m_q\gamma^{\beta}m\gamma^{\alpha}m_q\gamma^{\beta}\right]$$

$$=-2\Big\{\mathrm{Tr}\left[A\gamma^{\alpha}B\gamma^{\beta}C\gamma^{\alpha}D\gamma^{\beta}\right]$$

$$+m^2\left(-\mathrm{Tr}\left[\gamma^{\alpha}B\gamma^{\beta}\gamma^{\alpha}D\gamma^{\beta}\right]\right)$$

$$+m_q^2\left(-\mathrm{Tr}\left[A\gamma^{\alpha}\gamma^{\beta}C\gamma^{\alpha}\gamma^{\beta}\right]\right)$$

$$+m\,m_q\Big(\mathrm{Tr}\left[\gamma^{\alpha}\gamma^{\beta}C\gamma^{\alpha}D\gamma^{\beta}\right]-\mathrm{Tr}\left[\gamma^{\alpha}B\gamma^{\beta}C\gamma^{\alpha}\gamma^{\beta}\right]$$

$$-\mathrm{Tr}\left[A\gamma^{\alpha}\gamma^{\beta}\gamma^{\alpha}D\gamma^{\beta}\right]+\mathrm{Tr}\left[A\gamma^{\alpha}B\gamma^{\beta}\gamma^{\alpha}\gamma^{\beta}\right]\Big)$$

$$+m_q^2m^2\left(\mathrm{Tr}\left[\gamma^{\alpha}\gamma^{\beta}\gamma^{\alpha}\gamma^{\beta}\right]\right)\Big\}$$

terms proportional to m^0

- $\mathrm{Tr}\left[A\gamma^{\alpha}B\gamma^{\beta}C\gamma^{\alpha}D\gamma^{\beta}\right]$ $\times(-2)$

$$=k_1^{\mu}(k_1+k_3)^{\nu}\,k_2^{\sigma}(k_2+k_3)^{\delta}\,\mathrm{Tr}\left[\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu}\gamma^{\beta}\gamma^{\sigma}\gamma^{\alpha}\gamma^{\delta}\gamma^{\beta}\right]$$

$\times(-2)$

Mathematica

$$=64(k_1\cdot k_2)^2+64(k_1\cdot k_2)(k_1\cdot k_3)+64(k_1\cdot k_2)(k_2\cdot k_3)+64k_3^2(k_1\cdot k_2)$$

Terms proportional to m^2, m_q^2 e $m m_q$ $\times (-2)$

- $Tr\left[\gamma^\alpha \gamma^\beta C \gamma^\alpha D \gamma^\beta\right] = k_2^\sigma (k_2+k_3)^\delta \, Tr\left[\gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\alpha \gamma^\delta \gamma^\beta\right]$ $+ m m_q$
- $Tr\left[\gamma^\alpha B \gamma^\beta \gamma^\alpha D \gamma^\beta\right] = (k_1+k_3)^\nu (k_2+k_3)^\delta \, Tr\left[\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\alpha \gamma^\delta \gamma^\beta\right]$ $- m^2$
- $Tr\left[\gamma^\alpha B \gamma^\beta C \gamma^\alpha \gamma^\beta\right] = (k_1+k_3)^\nu k_2^\sigma \, Tr\left[\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\sigma \gamma^\alpha \gamma^\beta\right]$ $- m m_q$
- $Tr\left[A \gamma^\alpha \gamma^\beta \gamma^\alpha D \gamma^\beta\right] = k_1^\mu (k_2+k_3)^\delta \, Tr\left[\gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\alpha \gamma^\delta \gamma^\beta\right]$ $- m m_q$
- $Tr\left[A \gamma^\alpha \gamma^\beta C \gamma^\alpha \gamma^\beta\right] = k_1^\mu k_2^\sigma \, Tr\left[\gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\alpha \gamma^\beta\right]$ $\sim m_q^2$
- $Tr\left[A \gamma^\alpha B \gamma^\beta \gamma^\alpha \gamma^\beta\right] = k_1^\mu (k_1+k_3)^\nu \, Tr\left[\gamma^\mu \gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\alpha \gamma^\beta\right]$ $+ m m_q$

$\times (-2)$

$\downarrow$ Mathematica

$$= 2m^2 \left(16\, k_1 \cdot k_2 + 16\, k_1 \cdot k_3 + 16\, k_2 \cdot k_3 + 16\, k_3^2 \right)$$

$$+ 2 m_q^2 \left(16\, k_1 \cdot k_2 \right)$$

$$- 2 m m_q \left(-32\, k_1 \cdot k_2 + 16 k_1^2 + 16 k_2^2 \right)$$

Juntando as contribuições de todos os denominadores

$$\left[\frac{1}{(k_1+k_3)^2 - m_q^2}\right]^2$$

$$\Big\{ -16\, k_3^2\,(k_1\cdot k_2) + 32\, k_1^2\,(k_2\cdot k_3) + 32\,(k_1\cdot k_3)(k_2\cdot k_3) + 16 k_1^2\,(k_1\cdot k_2)$$
$$+ 64\, m\, m_q\,(-k_1\cdot k_2 + k_1\cdot k_3 + k_1^2 - k_2\cdot k_3) + 16\, m_q^2\,(k_1\cdot k_2)$$
$$+ 64\, m^2\,(k_1^2 + k_3^2 - 2\, k_1\cdot k_3) - 64\, m^2 m_q^2 \Big\}$$

$$+\left[\frac{1}{(k_2+k_3)^2 - m_q^2}\right]^2$$

$$\Big\{ 32\, k_2^2\,(k_1\cdot k_3) + 32\,(k_1\cdot k_3)(k_2\cdot k_3) - 16\, k_3^2\,(k_1\cdot k_2) + 16\, k_2^2\,(k_1\cdot k_2)$$
$$- 64\, m^2\,(2\, k_2\cdot k_3 + k_2^2 + k_3^2) + m_q^2\,(16\, k_1\cdot k_2)$$
$$+ 64\, m\, m_q\,(-k_1\cdot k_2 - k_1\cdot k_3 + k_2\cdot k_3 + k_2^2) - 64\, m^2 m_q^2 \Big\}$$

$$+ \frac{1}{\left[(k_1+k_3)^2 - m_q^2\right]\left[(k_2+k_3)^2 - m_q^2\right]}$$

$$\Big\{ 64\,(k_1\cdot k_2)^2 + 64\,(k_1\cdot k_2)(k_1\cdot k_3) + 64\,(k_1\cdot k_2)(k_2\cdot k_3) + 64\, k_3^2\,(k_1\cdot k_2)$$
$$2m^2\,(16\, k_1\cdot k_2 + 16\, k_1\cdot k_3 + 16\, k_2\cdot k_3 + 16\, k_3^2)$$
$$+ 2m_q^2\,(16\, k_1\cdot k_2) - 2\, m m_q\,(-32\, k_1\cdot k_2 + 16 k_1^2 + 16 k_2^2) - 128\, m^2 m_q^2$$

Fazendo agora mudanças de variável - reescrever os produtos escalares em termos de x_i

$$k_1 \cdot k_2 = \frac{(k_1+k_2)^2}{2} - \frac{k_1^2}{2} - \frac{k_2^2}{2} \quad , \; k_1^2 = k_2^2 = m^2$$

$$k_1 + k_2 + k_3 = q$$

$$\Leftrightarrow \; k_1 \cdot k_2 = \frac{(k_3 - q)^2}{2} - m^2$$

$$x_i = \frac{(k_i - q)^2}{q^2} - \frac{m^2}{q^2}$$

$$\Leftrightarrow k_1 \cdot k_2 = \frac{q^2 x_3 + \mu^2}{2} - m^2$$

$$x_3 = \frac{(k_3 - q)^2}{q^2} - \frac{\mu^2}{q^2}$$

$$\Leftrightarrow \; q^2 x_3 + \mu^2 = (k_3 - q)^2$$

como $x_1 + x_2 + x_3 = 1$

$$\Leftrightarrow \; x_3 = 1 - x_1 - x_2$$

$$\boxed{k_1 \cdot k_2 = \frac{q^2(1 - x_1 - x_2)}{2} + \frac{\mu^2}{2} - m^2}$$

$$k_1 + k_2 + k_3 = q$$

$$\Leftrightarrow \; k_1 + k_3 = q - k_2$$

$$k_1 \cdot k_3 = \frac{(k_1 + k_3)^2}{2} - \frac{m^2}{2} - \frac{\mu^2}{2}$$

$$x_2 = \frac{(k_2 - q)^2}{q^2} - \frac{m^2}{q^2}$$

$$\Leftrightarrow \; k_1 \cdot k_3 = \frac{(k_2 - q)^2}{2} - \frac{m^2}{2} - \frac{\mu^2}{2}$$

$$q^2 x_2 + m^2 = (k_2 - q)^2$$

$$\Leftrightarrow \; k_1 \cdot k_3 = \frac{q^2 x_2 + \cancel{m^2}}{2} - \frac{\cancel{m^2}}{2} - \frac{\mu^2}{2}$$

$$\Leftrightarrow \; \boxed{k_1 \cdot k_3 = \frac{q^2 x_2 - \mu^2}{2}}$$

$$k_2 \cdot k_3 = \frac{(k_2+k_3)^2}{2} - \frac{m^2}{2} - \frac{\mu^2}{2}$$

$$\Leftrightarrow k_2 \cdot k_3 = \frac{(k_1 - q)^2}{2} - \frac{m^2}{2} - \frac{\mu^2}{2}$$

$$\Leftrightarrow k_2 \cdot k_3 = \frac{q^2 x_1 + \cancel{m^2}}{2} - \frac{\cancel{m^2}}{2} - \frac{\mu^2}{2}$$

$$\Leftrightarrow \boxed{k_2 \cdot k_3 = \frac{q^2 x_1 - \mu^2}{2}}$$

Para os propagadores

$$(k_2 + k_3)^2 - m_q^2 = k_2^2 + k_3^2 + 2k_2 \cdot k_3 - m_q^2$$

$$= m^2 + \cancel{\mu^2} + q^2 x_1 - \cancel{\mu^2} - m_q^2$$

$$= \cancel{m_q^2} + \mu^2 + q^2 x_1 - \cancel{m_q^2}$$

$$(k_2 + k_3)^2 - m_q^2 = q^2 (x_1 + \rho_0) \quad , \quad \rho_0 = \frac{\mu^2}{q^2}$$

Para os propagadores ficamos com

$$\boxed{\begin{aligned} (k_2 + k_3)^2 - m_q^2 &= q^2 (x_1 + \rho_0) \\ (k_1 + k_3)^2 - m_q^2 &= q^2 (x_2 + \rho_0) \end{aligned}}$$

Aplicando a mudança de variável no resultado anterior e usando as condições on-shell $k_3^2 = \mu^2$, $k_1^2 = k_2^2 = m^2$ e colocando tudo no Mathematica para simplificar

$$
\left[\frac{1}{q^2 (x_2 + \mu_0)} \right]^2 \times
$$

$$
\Big\{ -80 m^4 + 128 m^3 m_q - 80 m^2 m_q^2 + 8 m^2 \mu^2 + 8 m^2 q^2 x_1 - 72 m^2 q^2 x_2
+ 8 m^2 q^2 - 32 m m_q \mu^2 + 64 m m_q q^2 x_2 - 32 m m_q q^2 + 8 m_q^2 \mu^2
- 8 m_q^2 q^2 x_1 - 8 m_q^2 q^2 x_2 + 8 m_q^2 q^2 - 8 \mu^2 q^2 + 8 q^4 x_1 x_2 \Big\}
$$

$$
+ \left[\frac{1}{q^2 (x_1 + \mu_0)} \right]^2
$$

$$
\Big\{ -80 m^4 + 128 m^3 m_q - 80 m^2 m_q^2 + 8 m^2 \mu^2 - 72 m^2 q^2 x_1 + 8 m^2 q^2 x_2
+ 8 m^2 q^2 - 32 m m_q \mu^2 + 64 m m_q q^2 x_1 - 32 m m_q q^2 + 8 m_q^2 \mu^2
- 8 m_q^2 q^2 x_1 - 8 m_q^2 q^2 x_2 + 8 m_q^2 q^2 - 8 \mu^2 q^2 + 8 q^4 x_1 x_2 \Big\}
$$

$$
+ \frac{1}{q^2 (x_2 + \mu_0)\, q^2 (x_1 + \mu_0)}
$$

$$
\Big\{ 32 m^4 - 128 m^3 m_q - 160 m^2 m_q^2 - 48 m^2 \mu^2 + 32 m^2 q^2 x_1 + 32 m^2 q^2 x_2
- 48 m^2 q^2 + 32 m m_q \mu^2 - 32 m m_q q^2 x_1 - 32 m m_q q^2 x_1 - 32 m m_q q^2 x_2
+ 32 m m_q q^2 + 16 m_q^2 \mu^2 - 16 m_q^2 q^2 x_1 - 16 m_q^2 q^2 x_2 + 16 m_q^2 q^2 + 16 \mu^4
- 16 \mu^2 q^2 x_1 - 16 \mu^2 q^2 x_2 + 32 \mu^2 q^2 - 16 q^4 x_1 - 16 q^4 x_2 + 16 q^4 \Big\}
$$

Primeiro denominador

Fazendo $m^2 = m_q^2 + \mu^2$; $m^4 = (m_q^2+\mu^2)^2$; $m = \sqrt{m_q^2+\mu^2}$

$$\left[\frac{1}{q^2(\chi_2+\mu_0)}\right]^2 \cdot$$

$$\Big\{ -160\, m_q^4 + 64\, m_q q^2 \chi_2 \sqrt{m_q^2+\mu^2} - 32\, m_q q^2 \sqrt{m_q^2+\mu^2} - 224\, m_q^2 \mu^2$$
$$+ 96\, m_q \mu^2 \sqrt{m_q^2+\mu^2} - 80 m_q^2 q^2 \chi_2 + 16 m_q^2 q^2 + 128 m_q^3 \sqrt{m_q^2+\mu^2} - 72 \mu^4$$
$$+ 8 \mu^2 q^2 \chi_1 - 72 \mu^2 q^2 \chi_2 + 8 q^4 \chi_1 \chi_2 \Big\}$$

$$\left[\frac{1}{q^2(\chi_2+\mu_0)}\right]^2 \cdot$$

$$\Big\{ -160\, m_q^4 - 32\, m_q q^2 \sqrt{m_q^2+\mu^2} - 224\, m_q^2 \mu^2 + 96\, m_q \mu^2 \sqrt{m_q^2+\mu^2}$$
$$+ 16 m_q^2 q^2 + 128 m_q^3 \sqrt{m_q^2+\mu^2} - 72 \mu^4$$
$$64\, m_q q^2 \chi_2 \sqrt{m_q^2+\mu^2} - 80 m_q^2 q^2 \chi_2 + 8 \mu^2 q^2 \chi_1 - 72 \mu^2 q^2 \chi_2 + 8 q^4 \chi_1 \chi_2 \Big\}$$

$$\left[\frac{1}{q^2(\chi_2+\mu_0)}\right]^2 \cdot$$

$$\Big\{ -160\, m_q^4 - 32\, m_q\, q^2 \sqrt{m_q^2+\mu^2} - 224\, m_q^2 \mu^2 + 96\, m_q \mu^2 \sqrt{m_q^2+\mu^2}$$

$$+ 16\, m_q^2 q^2 + 128\, m_q^3 \sqrt{m_q^2+\mu^2} - 72\, \mu^4$$

$$64\, m_q q^2 \chi_2 \sqrt{m_q^2+\mu^2} - 80\, m_q^2 q^2 \chi_2 + 8 \mu^2 q^2 \chi_1 - 72 \mu^2 q^2 \chi_2 + 8 q^4 \chi_1 \chi_2 \Big\}$$

$$\left[\frac{1}{(\chi_2+\mu_0)}\right]^2 \cdot \qquad \text{seja } \mu_0 = \frac{\mu^2}{q^2} \; ; \; m_{q_0} = \frac{m_q^2}{q^2}$$

$$-160\, m_{q_0}^2 - 32\, \frac{m_q}{q} \sqrt{\frac{m_q^2}{q^2} + \frac{\mu^2}{q^2}} - 224\, m_{q_0} \mu_0 + 96\, \frac{m_q}{q} \mu_0 \sqrt{\frac{m_q^2}{q^2} + \frac{\mu^2}{q^2}}$$

$$+ 16\, m_{q_0} + 128\, \frac{m_q^3}{q^3} \sqrt{\frac{m_q^2}{q^2} + \frac{\mu^2}{q^2}} - 72 \mu_0^2 + 64\, \frac{m_q}{q}\, \chi_2 \sqrt{\frac{m_q^2}{q^2} + \frac{\mu^2}{q^2}}$$

$$- 80\, m_{q_0} \chi_2 + 8 \mu_0 \chi_1 - 72 \mu_0 \chi_2 + 8\, \chi_1 \chi_2 \Big\}$$

$$\text{seja } \sqrt{\frac{m_q^2}{q^2} + \frac{\mu^2}{q^2}} = \sqrt{m_{q_0} + \mu_0} \equiv \gamma \qquad \text{e} \left\{ \begin{array}{l} \dfrac{m_q}{q} = \sqrt{\dfrac{m_q^2}{q^2}} = \sqrt{m_{q_0}} \\[2ex] \left(\dfrac{m_q}{q}\right)^3 = \left(\sqrt{m_{q_0}}\right)^3 = m_{q_0}^{3/2} \end{array} \right.$$

$$\left[\frac{1}{(\chi_2+\mu_0)}\right]^2$$

$$-160\, m_{q_0}^2 - 32 \sqrt{m_{q_0}}\, \gamma - 224\, m_{q_0} \mu_0 + 96 \sqrt{m_{q_0}}\, \mu_0\, \gamma + 16\, m_{q_0} + 128\, m_{q_0}^{3/2}\, \gamma$$

$$- 72 \mu_0^2 + 64 \sqrt{m_{q_0}}\, \chi_2\, \gamma - 80\, m_{q_0} \chi_2 + 8 \mu_0 \chi_1 - 72 \mu_0 \chi_2 + 8\, \chi_1 \chi_2 \Big\}$$

$$\left[\frac{1}{(\chi_2+\mu_0)}\right]^2$$

$$-160\, m_{g0}^2 - 32\sqrt{m_{g0}}\,\gamma + 16\, m_{g0} + 128\, m_{g0}^{3/2}\,\gamma - 144\, m_{g0}\,\mu_0 + 32\sqrt{m_{g0}}\,\mu_0\,\gamma$$

$$-80\, m_{g0}(\chi_2+\mu_0) + 64\sqrt{m_{g0}}\,\gamma(\chi_2+\mu_0) - 72\mu_0(\mu_0+\chi_2) + 8X_1(\mu_0+\chi_2)$$

$$= \left(\frac{1}{(\chi_2+\mu_0)}\right)^2$$

$$\left(-160\, m_{g0}^2 - 32\sqrt{m_{g0}}\,\gamma + 16\, m_{g0} + 128\, m_{g0}^{3/2}\,\gamma - 144\, m_{g0}\,\mu_0 + 32\sqrt{m_{g0}}\,\mu_0\,\gamma\right)$$

$$+ \frac{1}{\chi_2+\mu_0}\left(-80\, m_{g0} + 64\sqrt{m_{g0}}\,\gamma - 72\mu_0 + 8X_1\right)$$

Segundo denominador

Pode obter-se do 1º denominador fazendo $x_1 \leftrightarrow x_2$, e obtém-se

$$= \left(\frac{1}{(x_1+\mu_0)}\right)^2 \left(-160\, m_{\varphi_0}^2 - 32\sqrt{m_{\varphi_0}}\,\gamma + 16\, m_{\varphi_0} + 128\, m_{\varphi_0}^{3/2}\gamma - 144\, m_{\varphi_0}\mu_0 + 32\sqrt{m_{\varphi_0}}\,\mu_0\gamma\right)$$

$$+ \frac{1}{x_1+\mu_0}\left(-80\, m_{\varphi_0} + 64\sqrt{m_{\varphi_0}}\,\gamma - 72\mu_0 + 8x_2\right)$$

Terceiro denominador

$$\frac{1}{q^2(\chi_2+\mu_0)\,q^2(\chi_1+\mu_0)}$$

$$\Big\{ 32m^4 - 128m^3 m_q - 160m^2 m_q^2 - 48m^2\mu^2 + 32m^2q^2\chi_1 + 32m^2q^2\chi_2$$

$$-48m^2q^2 + 32 m m_q \mu^2 - 32 m m_q q^2\chi_1 - 32 m m_q q^2 \chi_1 - 32 m m_q q^2 \chi_2$$

$$+32 m m_q q^2 + 16 m_q^2\mu^2 - 16 m_q^2 q^2 \chi_1 - 16 m_q^2 q^2 \chi_2 + 16 m_q^2 q^2 + 16\mu^4$$

$$-16\mu^2q^2\chi_1 - 16\mu^2 q^2\chi_2 + 32\mu^2 q^2 - 16q^4\chi_1 - 16q^4\chi_2 + 16q^4 \Big\}$$

Fazendo

$m^2 = m_q^2 + \mu^2$, seja $\gamma = \sqrt{\frac{m_q^2}{q^2} + \frac{\mu^2}{q^2}}$

$$\frac{m_q}{q} = \sqrt{\frac{m_q^2}{q^2}} = \sqrt{m_{q_0}}$$

$$\left(\frac{m_q}{q}\right)^3 = \left(\sqrt{m_{q_0}}\right)^3 = m_{q_0}^{3/2}$$

$$\frac{1}{(\chi_2+\mu_0)\quad(\chi_1+\mu_0)} \cdot$$

$$\Big\{ -128 m_{q_0}^2 - 32\sqrt{m_{q_0}}\,\chi_1\gamma - 32\sqrt{m_{q_0}}\,\chi_2\gamma + 32\sqrt{m_{q_0}}\,\gamma - 128\, m_{q_0}\mu_0$$

$$-96\sqrt{m_{q_0}}\,\mu_0\gamma + 16\, m_{q_0}\,\chi_1 + 16\, m_{q_0}\,\chi_2 - 32\, m_{q_0} - 128\, m_{q_0}^{3/2}\gamma$$

$$+16\mu_0\,\chi_1 + 16\mu_0\,\chi_2 - 16\mu_0 - 16\,\chi_1 - 16\,\chi_2 + 16 \Big\}$$

$$\frac{1}{(\chi_2+\mu_0)\ (\chi_1+\mu_0)} \cdot$$

$$-128 m_{g_0}^2 + 32\sqrt{m_{g_0}}\,\gamma - 32 m_{g_0} - 128 m_{g_0}^{3/2}\gamma - 160 m_{g_0}\mu_0$$

$$+ 16 m_{g_0}(\chi_1+\mu_0) + 16 m_{g_0}(\chi_2+\mu_0) - 32\sqrt{m_{g_0}}\,\mu_0\gamma$$

$$- 32\sqrt{m_{g_0}}\,\gamma(\chi_1+\mu_0) - 32\sqrt{m_{g_0}}\,\gamma(\chi_2+\mu_0)$$

$$+ 16\left(\mu_0\chi_1 + \mu_0\chi_2 - \mu_0 - \chi_1 - \chi_2 + 1\right)$$

$$\frac{1}{(\chi_2+\mu_0)\ (\chi_1+\mu_0)}$$

$$\cdot\left\{-128 m_{g_0}^2 + 32\sqrt{m_{g_0}}\,\gamma - 32 m_{g_0} - 128 m_{g_0}^{3/2}\gamma - 160 m_{g_0}\mu_0 - 32\sqrt{m_{g_0}}\,\mu_0\gamma\right\}$$

$$+ \frac{1}{\chi_2+\mu_0}\left\{16 m_{g_0} - 32\sqrt{m_{g_0}}\,\gamma\right\} + \frac{1}{\chi_1+\mu_0}\left\{16 m_{g_0} - 32\sqrt{m_{g_0}}\,\gamma\right\}$$

$$+ 16\mu_0\frac{(\chi_1+\chi_2+1)}{(\chi_1+\mu_0)(\chi_2+\mu_0)} - \frac{16}{\chi_2+\mu_0} - \frac{16}{\chi_1+\mu_0} + \frac{16}{(\chi_1+\mu_0)(\chi_2+\mu_0)}$$

juntando a contribuição dos 3 denominadores

$$\left(\frac{1}{(x_2+\mu_0)}\right)^2$$

$$\left(-160\, m_{g_0}^2 - 32\sqrt{m_{g_0}}\,\gamma + 16\, m_{g_0} + 128\, m_{g_0}^{3/2}\,\gamma - 144\, m_{g_0}\,\mu_0 + 32\sqrt{m_{g_0}}\,\mu_0\,\gamma\right)$$

$$\frac{1}{x_2+\mu_0}\left(-80\, m_{g_0} + 64\sqrt{m_{g_0}}\,\gamma - 72\,\mu_0 + 8\, x_1\right)$$

$$+\left(\frac{1}{(x_1+\mu_0)}\right)^2$$

$$\left(-160\, m_{g_0}^2 - 32\sqrt{m_{g_0}}\,\gamma + 16\, m_{g_0} + 128\, m_{g_0}^{3/2}\,\gamma - 144\, m_{g_0}\,\mu_0 + 32\sqrt{m_{g_0}}\,\mu_0\,\gamma\right)$$

$$+\ \frac{1}{x_1+\mu_0}\left(-80\, m_{g_0} + 64\sqrt{m_{g_0}}\,\gamma - 72\,\mu_0 + 8\, x_2\right)$$

$$+\ \frac{1}{(x_2+\mu_0)\ (x_1+\mu_0)}$$

$$\cdot\left\{-128\, m_{g_0}^2 + 32\sqrt{m_{g_0}}\,\gamma - 32\, m_{g_0} - 128\, m_{g_0}^{3/2}\,\gamma - 160\, m_{g_0}\,\mu_0 - 32\sqrt{m_{g_0}}\,\mu_0\,\gamma\right\}$$

$$+\left(\frac{1}{x_2+\mu_0} + \frac{1}{x_1+\mu_0}\right)\left\{16\, m_{g_0} - 32\sqrt{m_{g_0}}\,\gamma - 16\right\}$$

$$+\ 16\,\mu_0\,\frac{(x_1+x_2+1)}{(x_1+\mu_0)(x_2+\mu_0)} + \frac{16}{(x_1+\mu_0)(x_2+\mu_0)}$$

$$\left[\left(\frac{1}{(x_2+\mu_0)}\right)^2+\left(\frac{1}{(x_1+\mu_0)}\right)^2\right]\cdot$$

$$\left(\overbrace{-160\,m_{\xi_0}^2-32\sqrt{m_{\xi_0}}\,\gamma+16\,m_{\xi_0}+128\,m_{\xi_0}^{3/2}\,\gamma-144\,m_{\xi_0}\mu_0+32\sqrt{m_{\xi_0}}\,\mu_0\,\gamma}^{A}\right)$$

$$\left[\frac{1}{x_2+\mu_0}+\frac{1}{x_1+\mu_0}\right]\left(\underbrace{-64\,m_{\xi_0}+32\sqrt{m_{\xi_0}}\,\gamma}_{B}-72\mu_0-16\right)$$

$$+\frac{8x_1}{x_2+\mu_0}+\frac{8x_2}{x_1+\mu_0}$$

$$+\frac{1}{(x_2+\mu_0)\ (x_1+\mu_0)}$$

$$\cdot\left\{\overbrace{-128\,m_{\xi_0}^2+32\sqrt{m_{\xi_0}}\,\gamma-32\,m_{\xi_0}-128\,m_{\xi_0}^{3/2}\,\gamma-160\,m_{\xi_0}\mu_0-32\sqrt{m_{\xi_0}}\,\mu_0\,\gamma}^{C}+\right.$$

$$\left.+\,96+16\mu_0\,(x_1+x_2+1)\right\}$$

= integrais "tod" +

$$\left[\left(\frac{1}{(x_2+\mu_0)} \right)^2 + \left(\frac{1}{(x_1+\mu_0)} \right)^2 \right] \cdot A$$

$$\left[\frac{1}{x_2+\mu_0} + \frac{1}{x_1+\mu_0} \right] \cdot B + \frac{1}{(x_2+\mu_0)\ (x_1+\mu_0)} \cdot C$$

colocando no Mathematica a expressão e simplificando

= integrais "tod"

+

$$\frac{-256\, m_q^2 - 192\, m_q \mu}{(x_1+\mu_0)\ (x_2+\mu)} - 32\, m_q \left(\frac{1}{x_1+\mu_0} + \frac{1}{x_2+\mu_0} \right)$$

$$+ \left(-16\, m_q - 32\, m_q^2 - 112\, m_q \mu \right) \left[\frac{1}{(x_1+\mu_0)^2} + \frac{1}{(x_2+\mu_0)^2} \right]$$

Avaliando no Mathematica

$$\int dx_1 \int dx_2 \, \frac{1}{(x_1+\mu_0)^2} = \int dx_1 \int dx_2 \, \frac{1}{(x_2+\mu_0)^2} = 0$$

Então

= integrais "to d"

+

$$\frac{-256\, m_q^2 - 192\, m_q \mu}{(x_1+\mu_0)\,(x_2+\mu)} - 32\, m_q \left(\frac{1}{x_1+\mu_0} + \frac{1}{x_2+\mu_0} \right)$$

Integrais que preciso de avaliar no Mathematica

$$\frac{1}{x_1+\mu_0} \; ; \quad \frac{1}{x_2+\mu_0} \; ; \quad \frac{1}{(x_1+\mu_0)^2} \; ; \quad \frac{1}{(x_2+\mu_0)^2}$$

$$\frac{1}{(x_1+\mu_0)\,(x_2+\mu_0)}$$

$$\int_{x_{1m}}^{x_{1M}}\int_{x_{2m}}^{x_{2M}} \frac{1}{x_1+\mu_0} =$$ ✓ Mathematica

$$= \int_{x_{1m}}^{x_{1M}} \frac{\sqrt{(-4m_0\mu_0+(\mu_0-x_1)^2)(-4m_0+(-1+x_1)^2)}}{(m_0+x_1)(\mu_0+x_1)}$$

Expandindo em Torno de μ_0 e guardando só até aos termos de 1º ordem

$$\frac{\sqrt{x_1^2(1-4m_0-2x_1+x_1^2)}}{x_1(m_0+x_1)} - \frac{2\sqrt{x_1^2(1-4m_0-2x_1+x_1^2)}\,\mu_0}{x_1^3}$$

$$= \pm \cancel{x_1}\frac{\sqrt{(1-4m_0-2x_1+x_1^2)}}{\cancel{x_1}(m_0+x_1)} \pm \frac{2\cancel{x_1}\sqrt{1-4m_0-2x_1+x_1^2}\,\mu_0}{x_1^{\cancel{3}\,2}}$$

$$= \pm\left[\frac{\sqrt{(x-1)^2-4m_0}}{(x_1+m_0)} - \frac{2\sqrt{(x-1)^2-4m_0}}{x_1^2}\mu_0\right]$$

$$= \pm\left[\sqrt{(x-1)^2-4m_0}\left(\frac{1}{x_1+m_0} - \frac{2\mu_0}{x_1^2}\right)\right]$$

Mathematica

$$\int_{x_{1m}}^{x_{1M}} \sqrt{(x-1)^2 - 4m_0}\left(\frac{1}{x_1 + m_0} - \frac{2\mu_0}{x_1^2}\right)$$

$\downarrow$ resultado que aparece na variável "To1" do notebook