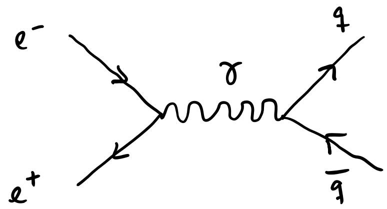
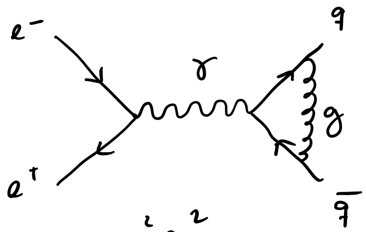
1 - diagrama tree-level



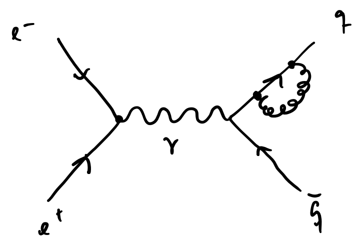
2 - ordem de correção

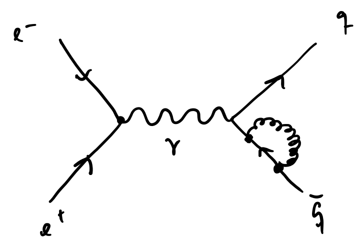
corrections at NLO in QCD $\rightarrow$ 1-loop amplitudes

3 - Diagramas que contribuem para a correção a 1-loop



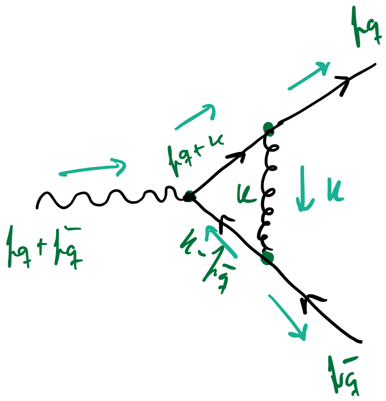
$\sim e^2 g_s^2$

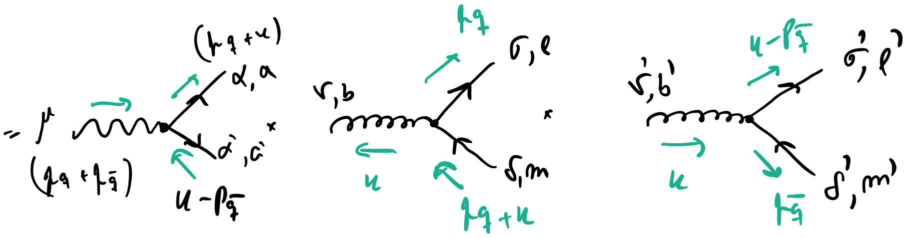




4- Vertex correction (Amplitude using Feynman rules)

We will compute the unregularized amplitude for the one-loop vertex correction subgraph $\gamma^* \to q\bar{q}$



$\mathcal{A} =$ 

QED vertex

$$\mathcal{A} = \left[i Q e \gamma^\mu \right] i g_s \bar{u}^s(p_q) \gamma^\nu_{\sigma\delta} T^b_{\ell m} \; i g_s \gamma^{\nu'}_{\sigma' s'} T^{b'}_{\ell' m'} v^{s'}(p_{\bar{q}})$$

$$\times \left(\frac{i g^{\nu'\nu} \delta^{bb'}}{k^2} \right) \left(i \frac{(\not{p}_q + \not{k} + m)}{(p_q + k)^2 - m^2} \right) \left(i \frac{(\not{k} - \not{p}_{\bar{q}} + m)}{(k - p_{\bar{q}})^2 - m^2} \right)$$

$$\boxed{C \equiv - e Q g_s^2 C_F}$$

$$\therefore \mathcal{A} = \overbrace{\left[e Q \right] (g_s)^2 \underbrace{\left(T^b_{\ell m} T^b_{m \ell} \right)}_{C_F} i^6}^{C}$$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{\bar{u}(p_q) \gamma^\nu (\not{p}_q + \not{k} + m) \gamma^\mu (\not{k} - \not{p}_{\bar{q}} + m) \gamma^\nu v(p_{\bar{q}})}{k^2 (p_q + k)^2 (k - p_{\bar{q}})^2}$$

$$\mathcal{A} = C \int_k \frac{\bar u(p_q)\,\gamma^\nu(\not p_q + \not k + m)\,\gamma^\mu\,(\not k - \not p_{\bar q} + m)\,\gamma^\nu\, v(p_{\bar q})}{k^2\left[(p_q + k)^2 - m^2\right]\left[(k - p_{\bar q})^2 - m^2\right]}$$

$$\boxed{\frac{1}{p^2} = \frac{1}{k^2\left[(p_q + k)^2 - m^2\right]\left[(k - p_{\bar q})^2 - m^2\right]}}$$

$$\mathcal{A} = \int \frac{C}{P_2} \Big\{ \bar u\,\gamma^\nu \not p_q\,\gamma^\mu \not k\,\gamma^\nu v \quad \text{A}$$

$$- \bar u\,\gamma^\nu \not p_q\,\gamma^\mu \not p_{\bar q}\,\gamma^\nu v \quad \text{B}$$

$$+ \bar u\,\gamma^\nu \not p_q\,\gamma^\mu m\,\gamma^\nu v \quad \text{C}$$

$$+ \bar u\,\gamma^\nu \not k\,\gamma^\mu \not k\,\gamma^\nu v \quad \text{D}$$

$$- \bar u\,\gamma^\nu \not k\,\gamma^\mu \not p_{\bar q}\,\gamma^\nu v \quad \text{E}$$

$$+ \bar u\,\gamma^\nu \not k\,\gamma^\mu m\,\gamma^\nu v \quad \text{F}$$

$$+ \bar u\,\gamma^\nu m\,\gamma^\mu \not k\,\gamma^\nu v \quad \text{G}$$

$$- \bar u\,\gamma^\nu m\,\gamma^\mu \not p_{\bar q}\,\gamma^\nu v \quad \text{H}$$

$$+ \bar u\,\gamma^\nu m\,\gamma^\mu m\,\gamma^\nu v \Big\} \quad \text{I}$$

A − B + C + D − E + F + G − H + I

Desenvolvendo cada termo

D.E.

$\bar{u}\not{k} = m\bar{u}$

$\not{\bar{p}}\, v = -mv$

$\bar{u}\gamma^\nu \not{p}\gamma^\mu \not{k}\gamma^\nu v$ (A)

$= \bar{u}\, p^\alpha k^\beta\, \underbrace{\gamma^\nu\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu}\, v$

$= -2\bar{u}\, p^\alpha k^\beta \gamma^\beta\gamma^\mu\gamma^\alpha v$

$\gamma^\beta\gamma^\mu\gamma^\alpha$

$= \gamma^\beta 2g^{\mu\alpha} - \gamma^\beta\gamma^\alpha\gamma^\mu$

$= 2\gamma^\beta g^{\mu\alpha} - 2g^{\alpha\beta}\gamma^\mu + \gamma^\alpha\gamma^\beta\gamma^\mu$

$= -2\bar{u}\, p^\alpha k^\beta (2\gamma^\beta g^{\mu\alpha} - 2\gamma^\mu g^{\alpha\beta} + \gamma^\alpha\gamma^\beta\gamma^\mu) v$

$= -4\bar{u}\, p^\mu \not{k} v + 4\bar{u}\, p.k\, \gamma^\mu v - 2\bar{u}\not{k}\not{k}\gamma^\mu v$

$= -4\bar{u}\, p^\mu \not{k} v + 4\bar{u}\, p.k\, \gamma^\mu v - 2\bar{u} m\not{k}\gamma^\mu v$

$\bar{u}\gamma^\nu \not{k}\gamma^\mu \not{\bar{p}}\gamma^\nu v$ (B)

$= \bar{u}\, p^\alpha \bar{p}^\beta \gamma^\nu\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu v = -2\bar{u}\, p^\alpha \bar{p}^\beta \gamma^\beta\gamma^\mu\gamma^\alpha v$

$= -2\bar{u}\, p^\alpha \bar{p}^\beta (\gamma^\beta 2g^{\mu\alpha} - \gamma^\mu 2g^{\alpha\beta} + 2g^{\beta\mu}\gamma^\alpha - \gamma^\alpha\gamma^\mu\gamma^\beta) v$

$= -4\bar{u}\, p^\mu \not{\bar{p}} v + 4\bar{u}\, p.\bar{p}\, \gamma^\mu v - 4\bar{u}\, \bar{p}^\mu \not{k} v + 2\bar{u}\not{k}\gamma^\mu \not{\bar{p}} v$

$= -4\bar{u}\, p^\mu (-m) v + 4\bar{u}\, p.\bar{p}\, \gamma^\mu v - 4\bar{u}\, \bar{p}^\mu m v + 2\bar{u}\, m\gamma^\mu (-m) v$

$= 4m\, p^\mu \bar{u} v + 4p.\bar{p}\, \bar{u}\gamma^\mu v - 4m\, \bar{p}^\mu \bar{u} v - 2m^2 \bar{u}\gamma^\mu v$

$$\bar{u}\, \gamma^\nu \not{k}\, \gamma^\mu m\, \gamma^\nu v \qquad C$$

$$= \bar{u}\, p^\alpha m \underbrace{\gamma^\nu \gamma^\alpha \gamma^\mu \gamma^\nu}_{4 g^{\alpha\mu}} v = 4 m\, p^\mu\, \bar{u} v$$

$$\bar{u}\, \gamma^\nu \not{k}\, \gamma^\mu \not{k}\, \gamma^\nu v \qquad D$$

$$= \bar{u}\, k^\alpha k^\beta \gamma^\nu \gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu v = -2 \bar{u}\, k^\alpha k^\beta \gamma^\beta \gamma^\mu \gamma^\alpha v$$

$$= -2 \bar{u}\, k^\alpha k^\beta \left(\gamma^\beta 2 g^{\mu\alpha} - \gamma^\beta \gamma^\alpha \gamma^\mu \right) v$$

$$= -4 \bar{u}\, k^\mu \not{k}\, v + 2 \bar{u} \not{k} \not{k}\, \gamma^\mu v$$

$$= -4 \bar{u}\, k^\mu \not{k}\, v + 2 \bar{u}\, k^2 \gamma^\mu v$$

$$\bar{u}\, \gamma^\nu \not{k}\, \gamma^\mu \not{\bar{p}}\, \gamma^\nu v \qquad E$$

$$= \bar{u}\, k^\alpha \bar{p}^\beta \underbrace{\gamma^\nu \gamma^\alpha \gamma^\mu \gamma^\beta}_{-2 \gamma^\beta \gamma^\mu \gamma^\alpha} \gamma^\nu v$$

$$-2 \bar{u}\, k^\alpha \bar{p}^\beta \gamma^\beta \gamma^\mu \gamma^\alpha v$$

$$= -2 \bar{u}\, k^\alpha \bar{p}^\beta \left(2 g^{\mu\beta} \gamma^\alpha - 2 g^{\beta\alpha} \gamma^\mu + \gamma^\mu \gamma^\alpha \gamma^\beta \right)$$

$$= -4 \bar{u}\, \bar{p}^\mu \not{k}\, v + 4 \bar{u}\, k \cdot \bar{p}\, \gamma^\mu v - 2 \bar{u}\, \gamma^\mu \not{k} \underbrace{\not{\bar{p}}\, v}_{-m v}$$

$$= -4 \bar{p}^\mu \bar{u} \not{k}\, v + 4 k \cdot \bar{p}\, \bar{u}\, \gamma^\mu v + 2 m\, \bar{u}\, \gamma^\mu \not{k}\, v$$

$$\bar{u}\,\gamma^{\nu}\,\not{k}\,\gamma^{\mu}\,m\,\gamma^{\nu}\,v \quad \text{(F)}$$

$$= m\,\bar{u}\,k^{\alpha}\,\underbrace{\gamma^{\nu}\gamma^{\alpha}\gamma^{\mu}\gamma^{\nu}}_{4g^{\mu\alpha}}\,v = 4m\,k^{\mu}\,\bar{u}\,v$$

$$\bar{u}\,\gamma^{\nu}\,m\,\gamma^{\mu}\,\not{k}\,\gamma^{\nu}\,v \quad \text{(G)}$$

$$= \bar{u}\,m\,k^{\alpha}\,\underbrace{\gamma^{\nu}\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu}}_{4g^{\mu\alpha}}\,v = 4m\,k^{\mu}\,\bar{u}\,v$$

$$\bar{u}\,\gamma^{\nu}\,m\,\gamma^{\mu}\,\bar{\not{p}}\,\gamma^{\nu}\,v \quad \text{(H)}$$

$$= m\,\bar{p}^{\alpha}\,\bar{u}\,\gamma^{\nu}\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu}\,v = 4m\,\bar{p}^{\mu}\,\bar{u}\,v$$

$$\bar{u}\,\gamma^{\nu}\,m\,\gamma^{\mu}\,m\,\gamma^{\nu}\,v \quad \text{(I)}$$

$$= m^2\,\bar{u}\,\underbrace{\gamma^{\nu}\gamma^{\mu}\gamma^{\nu}}_{-2\gamma^{\mu}}\,v = -2m^2\,\bar{u}\,\gamma^{\mu}\,v$$

$$A - B + C + D - E + F + G - H + I$$

Juntando tudo

$$
\begin{aligned}
\mathcal{M} = \int \frac{C}{p^2} \Big\{ & -4\bar{u}\, p^\mu \not{k} v + 4\bar{u}\, p\cdot k\, \gamma^\mu v - 2\bar{u} m \not{k} \gamma^\mu v \qquad \text{A} \\
& - \left(4m\, p^\mu \bar{u} v + 4 p\cdot\bar{p}\, \bar{u}\gamma^\mu v - 4m\, \bar{p}^\mu \bar{u} v - 2m^2 \bar{u}\gamma^\mu v \right) \qquad \text{B} \\
& + 4m\, p^\mu \bar{u} v \qquad \text{C} \\
& - 4\bar{u}\, k^\mu \not{k} v + 2\bar{u}\, k^2 \gamma^\mu v \qquad \text{D} \\
& - \left(-4\bar{p}^\mu \bar{u} \not{k} v + 4 k\cdot\bar{p}\, \bar{u}\gamma^\mu v + 2m\bar{u}\gamma^\mu \not{k} v \right) \qquad \text{E} \\
& \overset{\text{F}}{+ 4m\, k^\mu \bar{u} v} \overset{\text{G}}{+ 4m\, k^\mu \bar{u} v} \overset{\text{H}}{- 4m\, \bar{p}^\mu \bar{u} v} \overset{\text{I}}{- 2m^2 \bar{u}\gamma^\mu v} \Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M} = \int \frac{C}{p^2} \Big\{ & -4\bar{u}\, p^\mu \not{k} v + 4\bar{u}\, p\cdot k\, \gamma^\mu v - 2\bar{u} m \not{k} \gamma^\mu v \\
& - \cancel{4m\, p^\mu \bar{u} v} - 4 p\cdot\bar{p}\, \bar{u}\gamma^\mu v + \cancel{4m\, \bar{p}^\mu \bar{u} v} + \cancel{2m^2 \bar{u}\gamma^\mu v}) \\
& + \cancel{4m\, p^\mu \bar{u} v} \\
& - 4\bar{u}\, k^\mu \not{k} v + 2\bar{u}\, k^2 \gamma^\mu v \\
& + 4\bar{p}^\mu \bar{u} \not{k} v - 4 k\cdot\bar{p}\, \bar{u}\gamma^\mu v - 2m\bar{u}\gamma^\mu \not{k} v \\
& \underbrace{+ 4m\, k^\mu \bar{u} v + 4m\, k^\mu \bar{u} v} - \cancel{4m\, \bar{p}^\mu \bar{u} v} - \cancel{2m^2 \bar{u}\gamma^\mu v} \Big\}
\end{aligned}
$$

$$\mathcal{M} = \int \frac{C}{P^2} \Big\{ -4\bar{u}\, p^\mu \not{k}\, v + 4\bar{u}\, p\cdot k\, \gamma^\mu v - 2\bar{u} m \not{k} \gamma^\mu v$$

$$- 4p\cdot\bar{p}\;\bar{u}\gamma^\mu v - 4\bar{u}\, k^\mu \not{k}\, v + 2\bar{u}\, k^2 \gamma^\mu v$$

$$+ 4\bar{p}^\mu\, \bar{u} \not{k}\, v - 4k\cdot\bar{p}\;\bar{u}\gamma^\mu v - 2m\bar{u}\gamma^\mu \not{k} v + 8m\, k^\mu\, \bar{u} v \Big\}$$

$$\mathcal{M} = \int \frac{C}{P^2} \Big\{ \bar{u}\not{k} v\left(-4p^\mu - 4k^\mu + 4\bar{p}^\mu\right)$$

$$\bar{u}\gamma^\mu v\left(4p\cdot k - 4p\cdot\bar{p} + 2k^2 - 4k\cdot\bar{p}\right)$$

$$\bar{u} v \left(8m\, k^\mu\right)$$

$$\underbrace{-2\bar{u} m \not{k} \gamma^\mu v - 2m\bar{u}\gamma^\mu \not{k} v}$$

$$\to -2m\bar{u}\Big(\underbrace{\not{k}\gamma^\mu + \gamma^\mu\not{k}}_{2k^\mu}\Big) v$$

$$= -4m\, k^\mu\, \bar{u} v$$

$$\mathcal{M} = \int \frac{C}{P^2} \Big\{ \bar{u}\not{k} v\left(-4p^\mu - 4k^\mu + 4\bar{p}^\mu\right)$$

$$\bar{u}\gamma^\mu v\left(4p\cdot k - 4p\cdot\bar{p} + 2k^2 - 4k\cdot\bar{p}\right)$$

$$-4m\, k^\mu\, \bar{u} v$$

$$\mathcal{M} = \int \frac{C}{p^2} \Big\{ \bar{u}\gamma^\alpha v \left(-4k^\alpha p^\mu - 4k^\mu k^\alpha + 4\bar{p}^\mu k^\alpha\right)$$

$$\bar{u}\gamma^\mu v \left(4p\cdot k - 4p\cdot\bar{p} + 2k^2 - 4k\cdot\bar{p}\right)$$

$$-4m k^\mu \bar{u}v \Big\}$$

$$\mathcal{M} = \int C \Big\{ \bar{u}\gamma^\alpha v \left(-4p^\mu I^\alpha - 4 I^{\mu\alpha} + 4\bar{p}^\mu I^\alpha\right)$$

$$\bar{u}\gamma^\mu v \left(4p^\alpha I^\alpha - 4p\cdot\bar{p}\, I + 2 I_2 - 4\bar{p}^\alpha I^\alpha\right)$$

$$- \bar{u}v \left(4m I^\mu\right)$$

Integrais que vêm do vertex correction

Temos que analisar 4 tipos de integrais

$$I = \int \frac{1}{\rho^2} \; ; \quad I^{\mu} = \int \frac{k^{\mu}}{\rho^2} \; ; \quad I^{\mu\nu} = \int \frac{k^{\mu}k^{\nu}}{\rho^2} \; ; \quad I_2 = \int \frac{k^2}{\rho^2}$$

onde

$$\frac{1}{\rho^2} = \frac{1}{k^2\left[(p+k)^2 - m^2\right]\left[(\bar{p}-k)^2 - m^2\right]} \; , \quad \begin{array}{l} p_1 \equiv p \\ p_2 = \bar{p} \end{array}$$

Procedimento para tratar os integrais:

1º) Limite on-shell

2º) Expansão

Por ex.:

$$\frac{1}{(q+k)^2 - m^2} = \frac{1}{\cancel{q^2} + k^2 + 2q.k - \cancel{m^2}} \overset{q^2 = m^2}{=} \frac{1}{k^2 + 2q.k}$$

e introduzo agora uma massa reguladora, ficando

$$\frac{1}{k^2 + 2q.k - \mu^2}$$

e expandindo fica

$$\frac{1}{k^2 - \mu^2} - \frac{2k.q}{(k^2-\mu^2)\left[k^2 + 2k.q - \mu^2\right]}$$

$$\frac{1}{P^2} = \frac{1}{k^2((p+k)^2 - m^2)((\bar{p}-k)^2 - m^2)}$$

limite on-shell

$$= \frac{1}{k^2(\cancel{p^2} + k^2 + 2k\cdot p - \cancel{m^2})(\cancel{\bar{p}^2} + k^2 - 2k\cdot\bar{p} - \cancel{m^2})} \quad , \quad p^2 = \bar{p}^2 = m^2$$

$$= \frac{1}{k^2(k^2 + 2k\cdot p)(k^2 - 2k\cdot\bar{p})}$$

introduzindo uma massa reguladora

$$\frac{1}{P^2} = \frac{1}{(k^2-\mu^2)(k^2 + 2k\cdot p - \mu^2)(k^2 - 2k\cdot\bar{p} - \mu^2)}$$

$$I = \int \frac{1}{P^2}$$ → finito (coloquei package-x)

$$I^\mu = \int \frac{k^\mu}{P^2}$$ → finito (coloquei package-x)

$$I^{\mu\nu} = \int \frac{k^\mu k^\nu}{P^2} = \frac{k^\mu k^\nu}{(k^2-\mu^2)\underbrace{(k^2 + 2k\cdot p - \mu^2)}_{\text{expandir}}(k^2 - 2k\cdot\bar{p} - \mu^2)}$$

$$I^{\mu\nu} = \underbrace{\int \frac{k^\mu k^\nu}{(k^2-\mu^2)^2 (k^2 - 2k\cdot\bar{p} - \mu^2)}}_{\circledast} - \underbrace{\int \frac{k^\mu k^\nu (2k\cdot p)}{(k^2-\mu^2)^2 [k^2 + 2k\cdot p - \mu^2][k^2 - 2k\cdot\bar{p} - \mu^2]}}_{\text{finito}}$$

$$\circledast \quad \underbrace{\int \frac{k^\mu k^\nu}{(k^2-\mu^2)^3}}_{I_{log}^{\mu\nu}} + \underbrace{\int \frac{k^\mu k^\nu (2k\cdot\bar{p})}{(k^2-\mu^2)^3 (k^2 - 2k\cdot\bar{p} - \mu^2)}}_{\text{finito}}$$

então

$$I^{\mu\nu} = I_{log}^{\mu\nu} + \text{fin} = \frac{I_{log}(\mu^2)}{4} g^{\mu\nu} + \text{finitos}$$

Package-X

$$I_2 = \frac{\cancel{k^2}}{\cancel{k^2}\,(k^2+2k\cdot p-\mu^2)(k^2-2k\cdot\bar{p}-\mu^2)} = \frac{1}{\underbrace{(k^2+2k\cdot p-\mu^2)}_{\text{expandir}}(k^2-2k\cdot\bar{p}-\mu^2)}$$

$$= \int \frac{1}{\underbrace{(k^2-\mu^2)[k^2-2k\cdot\bar{p}-\mu^2]}_{*}} - \int \frac{2k\cdot p}{\underbrace{(k^2-\mu^2)(k^2-2k\cdot\bar{p}-\mu^2)(k^2+2k\cdot p-\mu^2)}_{\text{finito}}}$$

$$(*) = \underbrace{\int \frac{1}{(k^2-\mu^2)^2}}_{I_{log}(\mu^2)} + \int \frac{2k\cdot\bar{p}}{\underbrace{(k^2-\mu^2)^2(k^2-2k\cdot\bar{p}-\mu^2)}_{\text{finito}}}$$

$$I_2 = I_{log}(\mu^2) + \text{finito}$$

Package-X

Temos a amplitude

$$\mathcal{M} = \int C \Big\{ \bar{u} \gamma^\alpha v \left(-4 p^\mu I^\alpha - 4 I^{\mu\alpha} + 4 \bar{p}^\mu I^\alpha\right)$$
$$\bar{u} \gamma^\mu v \left(4 p^\alpha I^\alpha - 4 p \cdot \bar{p}\, I + 2 I_2 - 4 \bar{p}^\alpha I^\alpha\right)$$
$$- \bar{u} v \left(4 m I^\mu\right)$$

A amplitude está avaliada no Mathematica. O resultado final está nas variáveis T1, T2

T1 → Termos proporcionais a $\bar{u} v$

T2 → Termos proporcionais a $\bar{u} \gamma^\nu v$

A11 → proporcional a $\bar{u} v$

A12 → proporcional a $\bar{u} \gamma^\nu v$

A2 → proporcional a $\bar{u} \gamma^\nu v$

A3 → [illegible] a $\bar{u} v$