Train In Data

# Assumptions of Linear Models

# Linear Regression Model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_n X_{ni} + \varepsilon_i$$

- Y is the outcome variable
- X are the predictor variables
- $\beta$ are the coefficients
- $\beta_0$ is the intercept
- $\varepsilon_i$ is the difference between the predicted and the observed value of Y for the ith observation

# Linear Model Assumptions

1. **Linearity**: The mean values of the outcome variable for each increment of the predictor(s) lie along a straight line. There is a linear relationship between predictors and target.
2. **No perfect multicollinearity**: There should be no perfect linear relationship between two or more of the predictors.
3. **Normally distributed errors**: the residuals ($\varepsilon_i$) are random, normally distributed with a mean of 0.

5. **Homoscedasticity**: At each level of the predictor variable(s), the variance of the residual terms should be constant.

# When the assumptions are met

- The coefficients and parameters of the regression equation are said to be unbiased.

- The model can be accurately applied.

# When the assumptions are not met

The variables are not good enough to predict accurately the outcome.

Some issues could be:

- Outliers
- Lack of homoscedasticity
- The variables are too skewed

# What can we do?

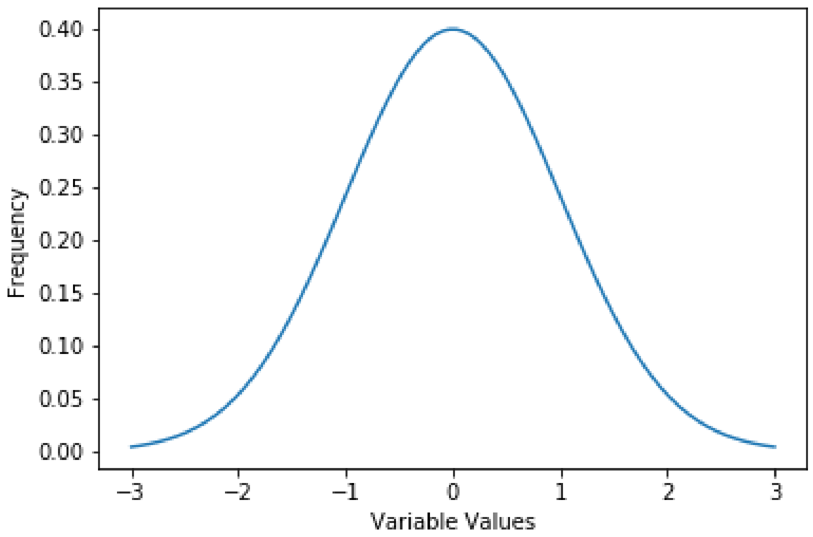
Transforming the data is useful to correct the problems with outliers and homoscedasticity.

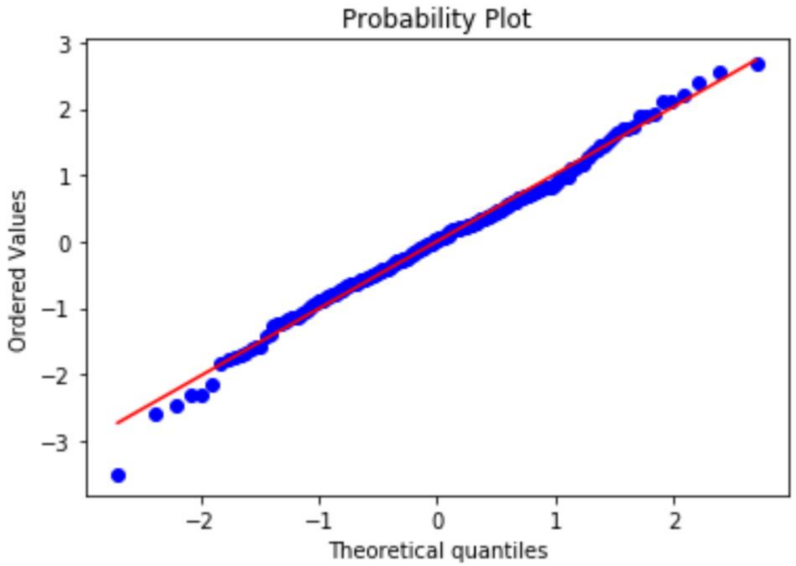
- Mathematical transformations
- Discretisation
- Remove or censor outliers

# Evaluate model performance

# Errors ~ N(0,σ)

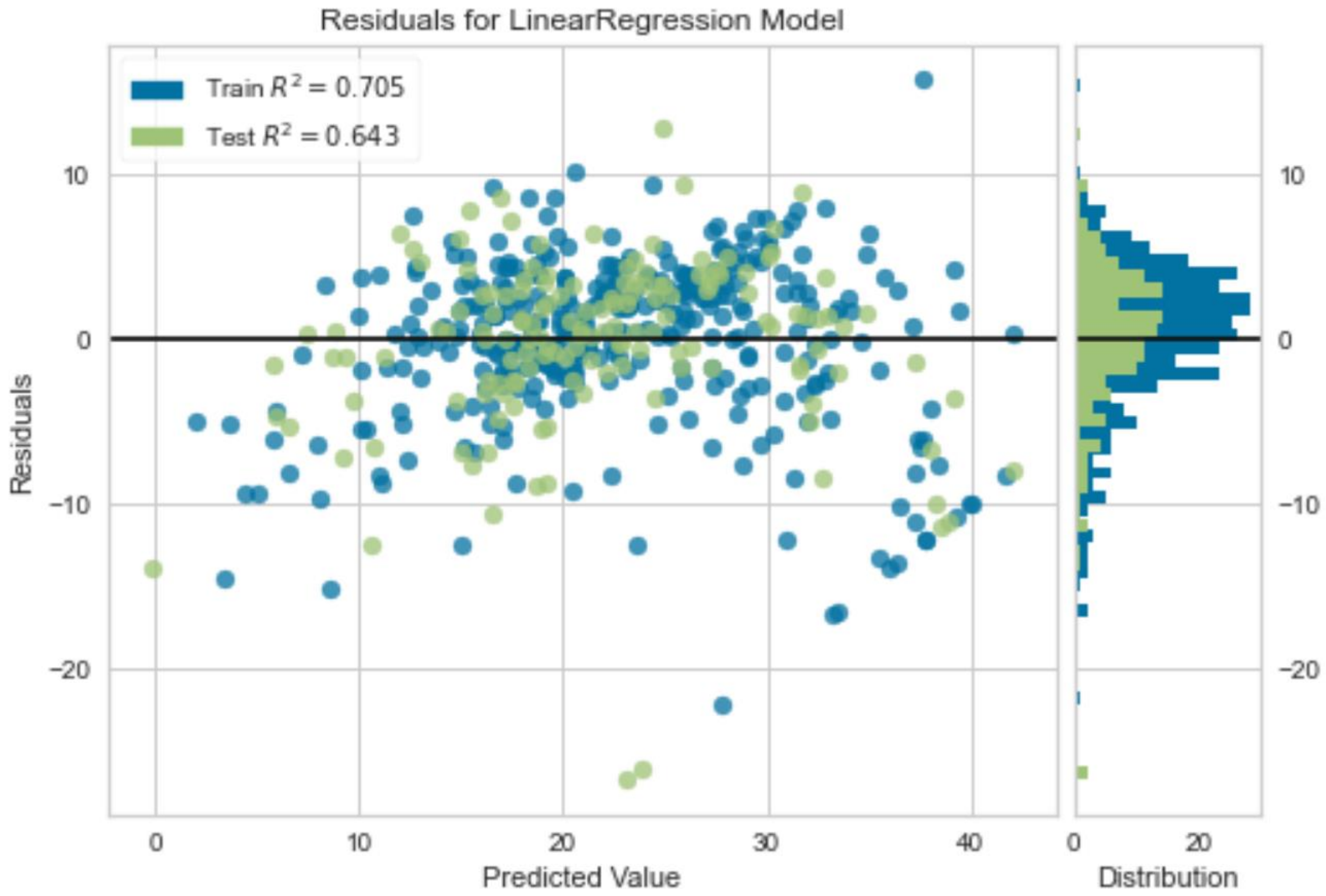
- Normality can be assessed with histograms and Q-Q plots
- Normality can be statistically tested, for example with the Kolmogorov-Smirnov test.



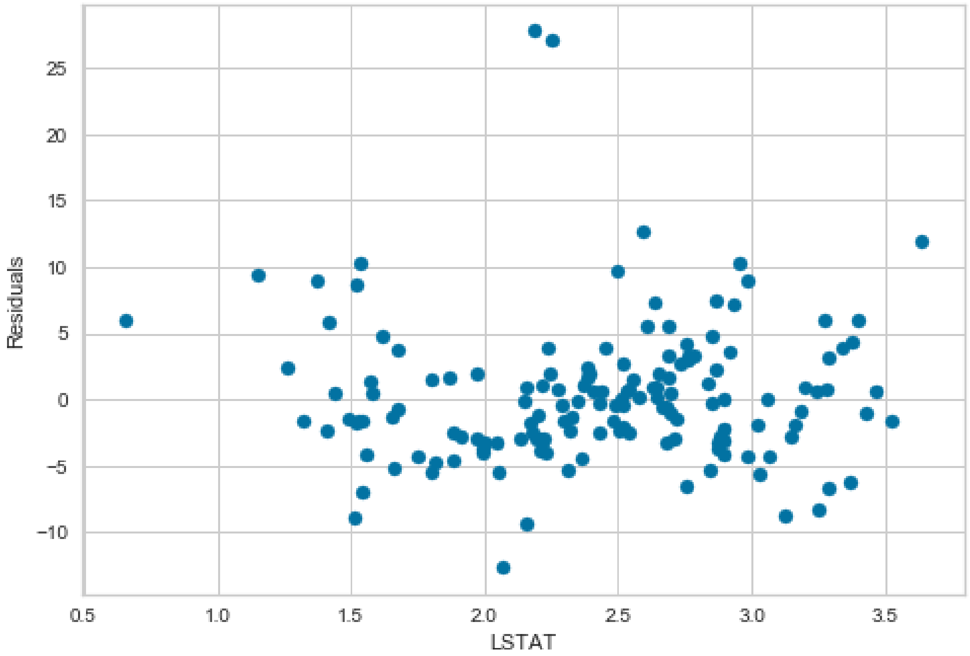



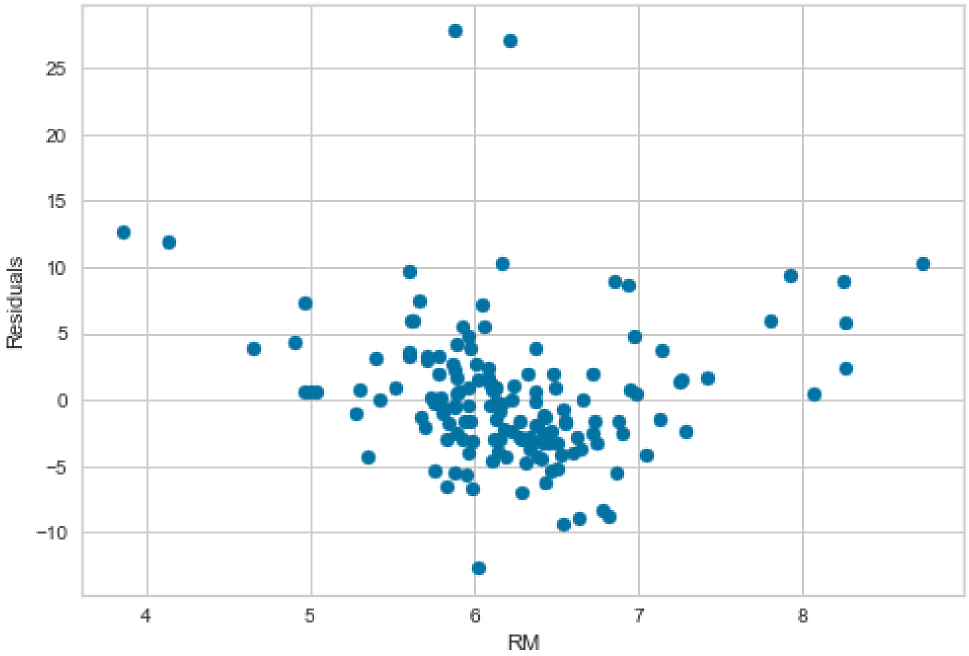
# Homoscedasticity

- There are tests and plots to determine homoscedasticity.
    - Residuals plot
    - Levene's test
    - Barlett's test
    - Goldfeld-Quandt Test
- Visual inspection



# Homoscedasticity



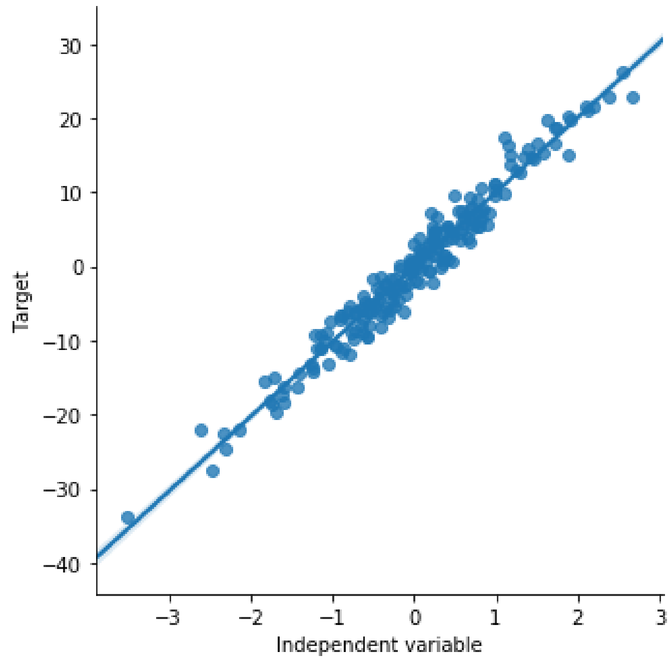




**Homoscedasticity**: the error term (that is, the “noise” in the relationship between the independent variables X and the dependent variable Y) is the same across all the independent variables.

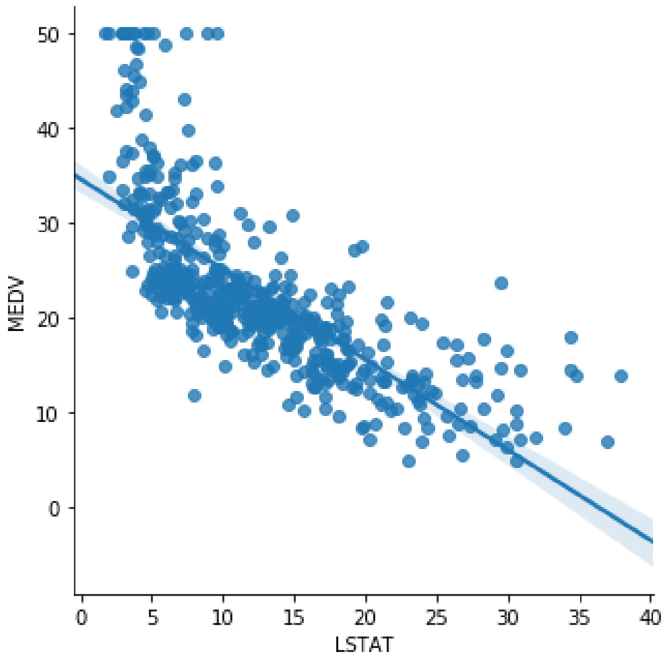
To identify homoscedasticity we need to plot the residuals vs each of the independent variables.

# Linear Relationship – Scatter plots

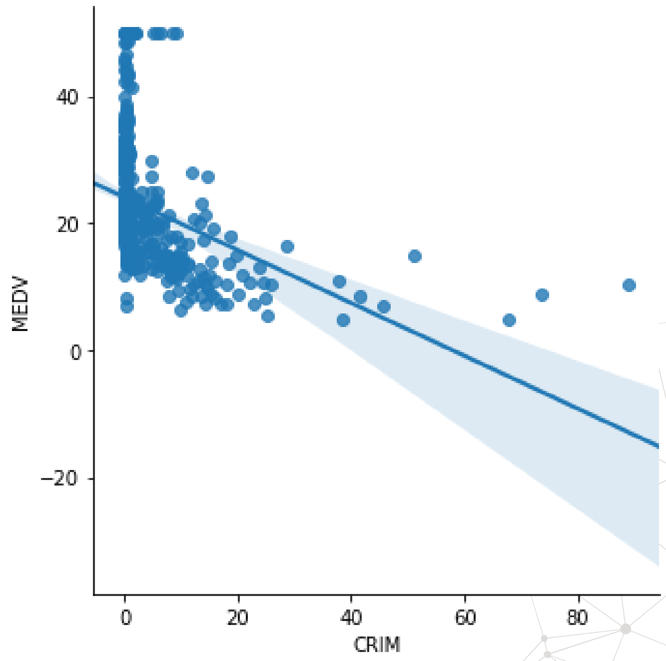
Expected – Simulated data



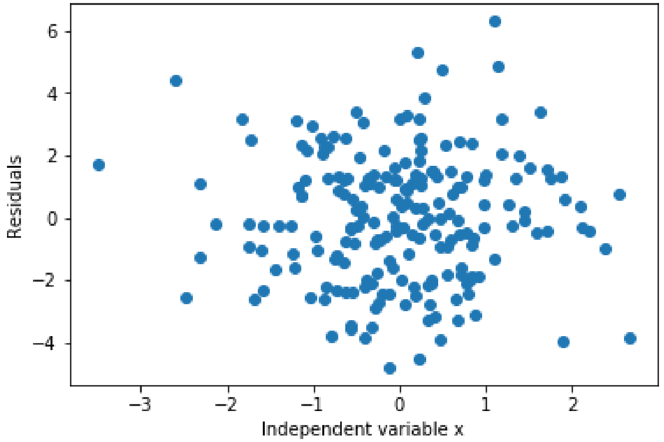

Somewhat linear relationship

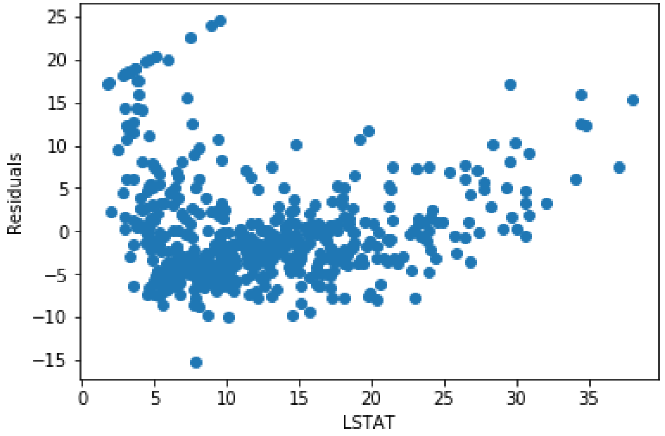


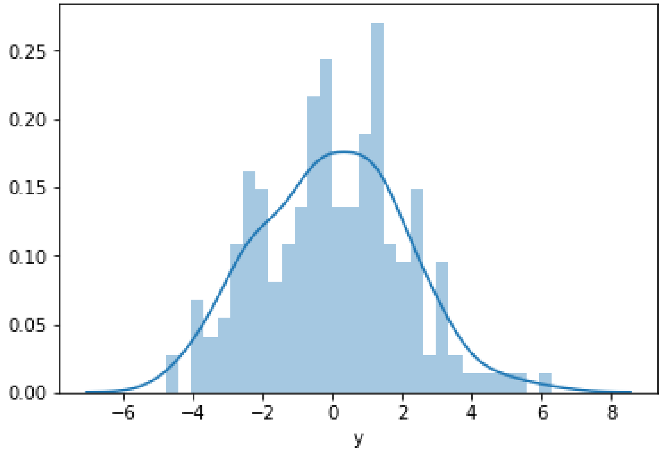

Non-linear relationship



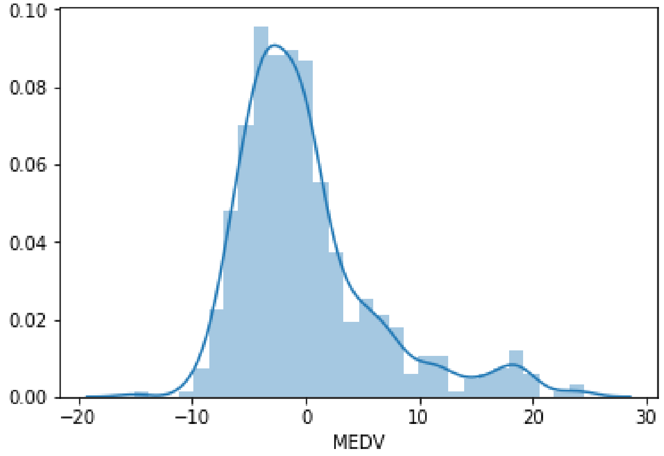
# Linear Relationship – Residual plots




Expected – Simulated data



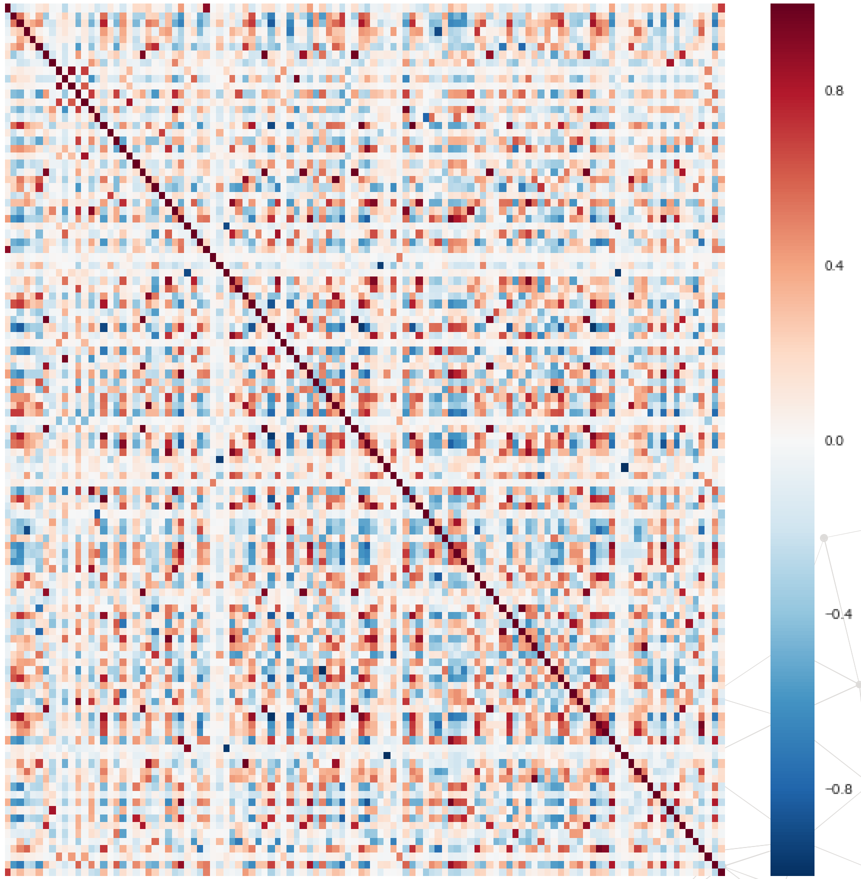

Somewhat linear relationship

- If relationship between X and y is linear, residuals should be normally distributed and centred around 0
- Residuals are the difference between the predictions and the real value y.

# Multicolinearity

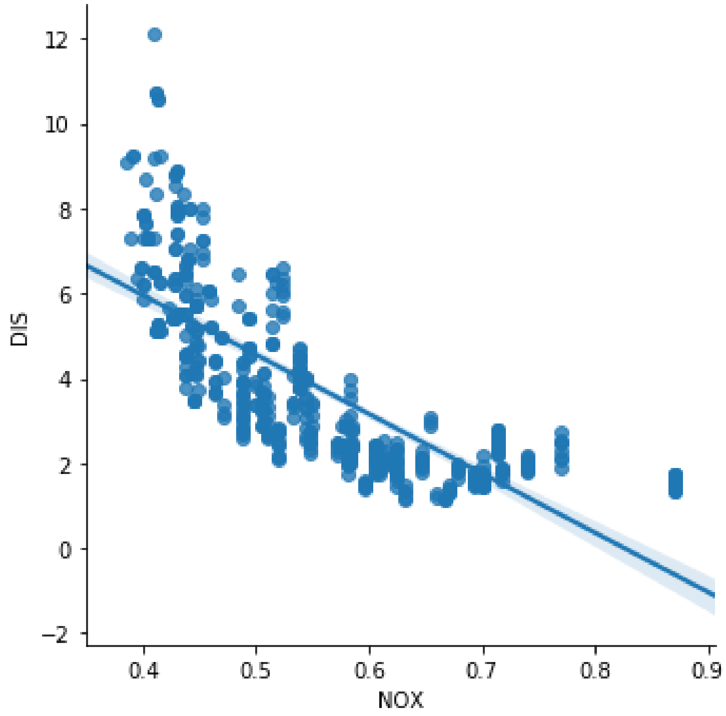
- Multicollinearity occurs when the independent variables are correlated with each other

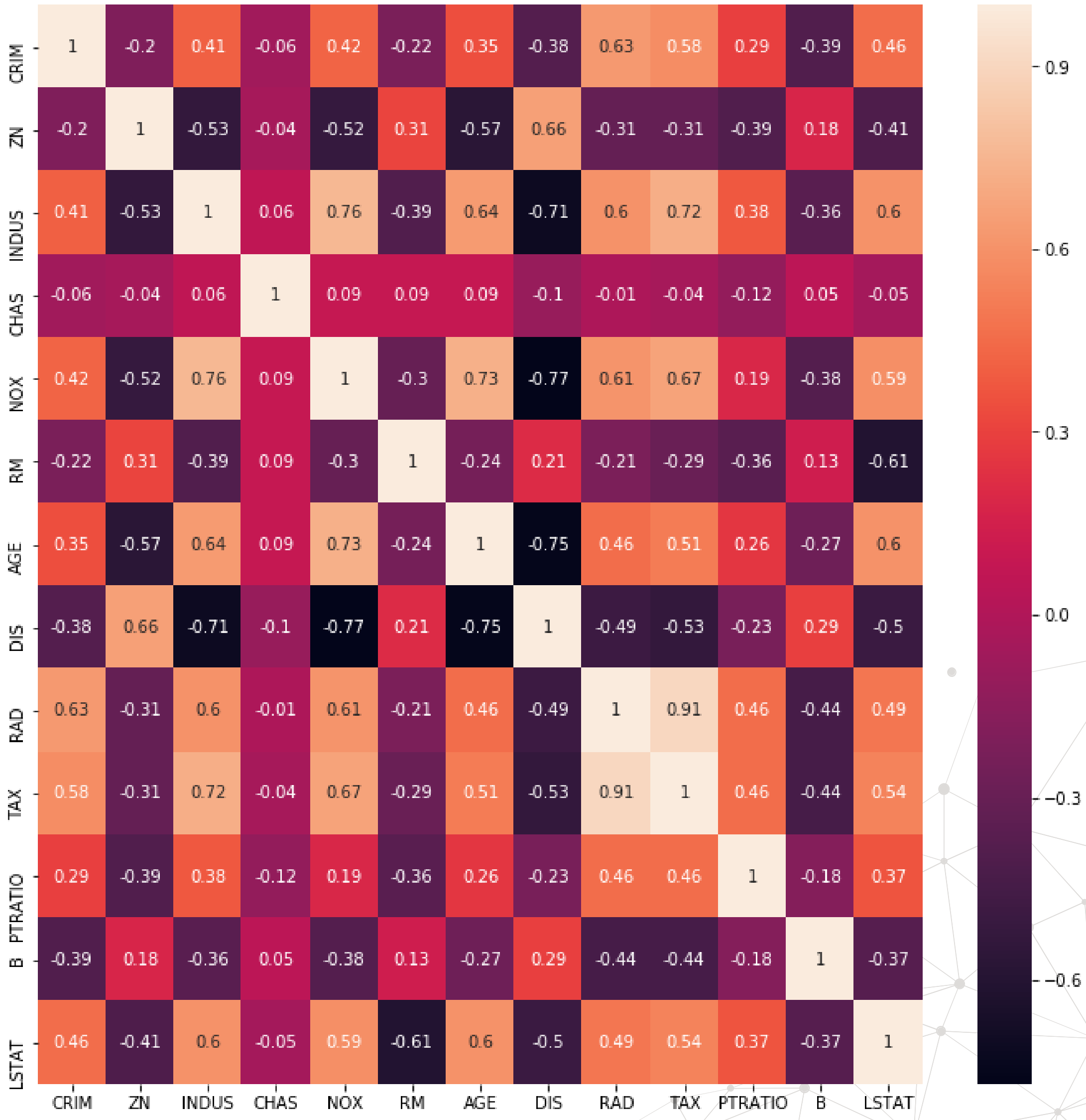
- Multicollinearity can be assessed with a correlation matrix or the variance inflation factor (VIF)
  - Outside of the scope of this course
  - Check the course Feature Selection for Machine Learning



# Multi Co-linearity

Evaluated by correlation



# Accompanying Jupyter Notebook


- Read the accompanying Jupyter Notebook
- Full demonstration of the linear assumptions and the influence of non-linear transformations

# Appendix

More Linear Model assumptions

# Linear Model Assumptions

1. **Variable types**: All predictor variables must be quantitative or categorical (with two categories), and the outcome variable must be quantitative, continuous and unbounded.

2. **Non-zero variance**: The predictors should have some variation in value (i.e., they do not have variances of 0).

3. **No perfect multicollinearity**: There should be no perfect linear relationship between two or more of the predictors. So, the predictor variables should not correlate too highly.

4. **Linearity**: The mean values of the outcome variable for each increment of the predictor(s) lie along a straight line. In plain English this means that it is assumed that the relationship we are modelling is a linear one.

# Linear Model Assumptions

5. **Normally distributed errors**: It is assumed that the residuals in the model are random, normally distributed variables with a mean of 0.
6. **Homoscedasticity**: At each level of the predictor variable(s), the variance of the residual terms should be constant. Independent errors: For any two observations the residual terms should be uncorrelated (or independent)
7. **Independence**: all of the values of the outcome variable are independent
8. **Independent errors**: For any two observations the residual terms should be uncorrelated (or independent). This is sometimes described as a lack of autocorrelation.

# THANK YOU

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