

Instituto Politécnico Nacional

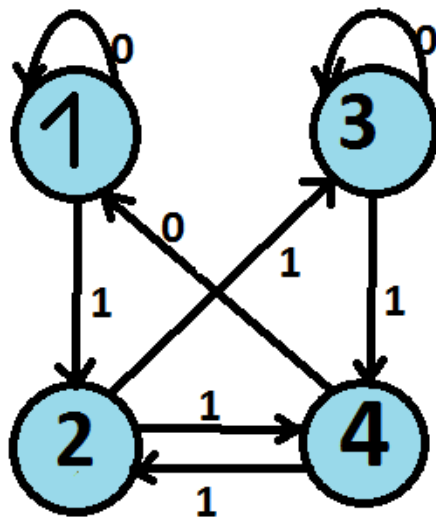
Escuela Superior de Cómputo

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2CM4

Expresiones regulares:

1- Ejercicio 1.



$$k = 0$$

$$R_{11}^0 = 0 + \epsilon = 0$$

$$R_{12}^0 = 1$$

$$R_{13}^0 = \emptyset$$

$$R_{14}^0 = \emptyset$$

$$R_{21}^0 = \emptyset$$

$$R_{22}^0 = \epsilon$$

$$R_{23}^0 = 1$$

$$R_{24}^0 = 1$$

$$R_{31}^0 = \emptyset$$

$$R_{32}^0 = \emptyset$$

$$R_{33}^0 = \epsilon + 0 = 0$$

$$R_{34}^0 = 1$$

$$\begin{aligned}
R_{41}^0 &= 1 \\
R_{42}^0 &= 1 \\
R_{43}^0 &= \oslash \\
R_{44}^0 &= \in
\end{aligned}$$

$$k = 1$$

$$\begin{aligned}
R_{11}^1 &= R_{11}^0 + R_{11}^0(R_{11}^0)^*R_{11}^0 = 0 + 0(0)^*0 = 0^* \\
R_{12}^1 &= R_{12}^0 + R_{11}^0(R_{11}^0)^*R_{12}^0 = 1 + 0^*1 = 0^*1 \\
R_{13}^1 &= R_{13}^0 + R_{11}^0(R_{11}^0)^*R_{13}^0 = \oslash \\
R_{14}^1 &= R_{14}^0 + R_{11}^0(R_{11}^0)^*R_{14}^0 = \oslash
\end{aligned}$$

$$\begin{aligned}
R_{21}^1 &= R_{21}^0 + R_{21}^0(R_{11}^0)^*R_{11}^0 = \oslash \\
R_{22}^1 &= R_{22}^0 + R_{21}^0(R_{11}^0)^*R_{12}^0 = \in \\
R_{23}^1 &= R_{23}^0 + R_{21}^0(R_{11}^0)^*R_{13}^0 = 1 \\
R_{24}^1 &= R_{24}^0 + R_{21}^0(R_{11}^0)^*R_{14}^0 = 1
\end{aligned}$$

$$\begin{aligned}
R_{31}^1 &= R_{31}^0 + R_{31}^0(R_{11}^0)^*R_{11}^0 = \oslash \\
R_{32}^1 &= R_{32}^0 + R_{31}^0(R_{11}^0)^*R_{12}^0 = \oslash \\
R_{33}^1 &= R_{33}^0 + R_{31}^0(R_{11}^0)^*R_{13}^0 = 0 \\
R_{34}^1 &= R_{34}^0 + R_{31}^0(R_{11}^0)^*R_{14}^0 = 1
\end{aligned}$$

$$\begin{aligned}
R_{41}^1 &= R_{41}^0 + R_{41}^0(R_{11}^0)^*R_{11}^0 = 10^* \\
R_{42}^1 &= R_{42}^0 + R_{41}^0(R_{11}^0)^*R_{12}^0 = 10^*1 \\
R_{43}^1 &= R_{43}^0 + R_{41}^0(R_{11}^0)^*R_{13}^0 = \oslash \\
R_{44}^1 &= R_{44}^0 + R_{41}^0(R_{11}^0)^*R_{14}^0 = \in
\end{aligned}$$

$$k = 2$$

$$\begin{aligned}
R_{11}^2 &= R_{11}^1 + R_{12}^1(R_{22}^1)^*R_{21}^1 = 0^* \\
R_{12}^2 &= R_{12}^1 + R_{12}^1(R_{22}^1)^*R_{22}^1 = 0^*1 \\
R_{13}^2 &= R_{13}^1 + R_{12}^1(R_{22}^1)^*R_{23}^1 = 0^*11 \\
R_{14}^2 &= R_{14}^1 + R_{12}^1(R_{22}^1)^*R_{24}^1 = 0^*11
\end{aligned}$$

$$\begin{aligned}
R_{21}^2 &= R_{21}^1 + R_{22}^1(R_{22}^1)^*R_{21}^1 = \oslash \\
R_{22}^2 &= R_{22}^1 + R_{22}^1(R_{22}^1)^*R_{22}^1 = \in \\
R_{23}^2 &= R_{23}^1 + R_{22}^1(R_{22}^1)^*R_{23}^1 = 1 \\
R_{24}^2 &= R_{24}^1 + R_{22}^1(R_{22}^1)^*R_{24}^1 = 1
\end{aligned}$$

$$\begin{aligned}
R_{31}^2 &= R_{31}^1 + R_{32}^1(R_{22}^1)^*R_{21}^1 = \oslash \\
R_{32}^2 &= R_{32}^1 + R_{32}^1(R_{22}^1)^*R_{22}^1 = \oslash \\
R_{33}^2 &= R_{33}^1 + R_{32}^1(R_{22}^1)^*R_{23}^1 = 0 \\
R_{34}^2 &= R_{34}^1 + R_{32}^1(R_{22}^1)^*R_{24}^1 = 1
\end{aligned}$$

$$\begin{aligned}
R_{41}^2 &= R_{41}^1 + R_{42}^1(R_{22}^1)^*R_{21}^1 = 10^* \\
R_{42}^2 &= R_{42}^1 + R_{42}^1(R_{22}^1)^*R_{22}^1 = 10^*1 \\
R_{43}^2 &= R_{43}^1 + R_{42}^1(R_{22}^1)^*R_{23}^1 = 10^*11 \\
R_{44}^2 &= R_{44}^1 + R_{42}^1(R_{22}^1)^*R_{24}^1 = 10^*11
\end{aligned}$$

$$k = 3$$

$$\begin{aligned}
R_{11}^3 &= R_{11}^2 + R_{13}^2(R_{33}^2)^*R_{31}^2 = 0^* \\
R_{12}^3 &= R_{12}^2 + R_{13}^2(R_{33}^2)^*R_{32}^2 = 0^*1 \\
R_{13}^3 &= R_{13}^2 + R_{13}^2(R_{33}^2)^*R_{33}^2 = 0^*110^* \\
R_{14}^3 &= R_{14}^2 + R_{13}^2(R_{33}^2)^*R_{34}^2 = 0^*110^*1
\end{aligned}$$

$$\begin{aligned}
R_{21}^3 &= R_{21}^2 + R_{23}^2(R_{33}^2)^*R_{31}^2 = \emptyset \\
R_{22}^3 &= R_{22}^2 + R_{23}^2(R_{33}^2)^*R_{32}^2 = \in \\
R_{23}^3 &= R_{23}^2 + R_{23}^2(R_{33}^2)^*R_{33}^2 = 10^* \\
R_{24}^3 &= R_{24}^2 + R_{23}^2(R_{33}^2)^*R_{34}^2 = 10^*1
\end{aligned}$$

$$\begin{aligned}
R_{31}^3 &= R_{31}^2 + R_{33}^2(R_{33}^2)^*R_{21}^2 = \emptyset \\
R_{32}^3 &= R_{32}^2 + R_{33}^2(R_{33}^2)^*R_{22}^2 = \emptyset \\
R_{33}^3 &= R_{33}^2 + R_{33}^2(R_{33}^2)^*R_{23}^2 = 0^* \\
R_{34}^3 &= R_{34}^2 + R_{33}^2(R_{33}^2)^*R_{24}^2 = 0^*1
\end{aligned}$$

$$\begin{aligned}
R_{41}^3 &= R_{41}^2 + R_{43}^2(R_{33}^2)^*R_{21}^2 = 10^* \\
R_{42}^3 &= R_{42}^2 + R_{43}^2(R_{33}^2)^*R_{22}^2 = 10^*1 \\
R_{43}^3 &= R_{43}^2 + R_{43}^2(R_{33}^2)^*R_{23}^2 = 10^*110^* \\
R_{44}^3 &= R_{44}^2 + R_{43}^2(R_{33}^2)^*R_{24}^2 = 10^*110^*1
\end{aligned}$$

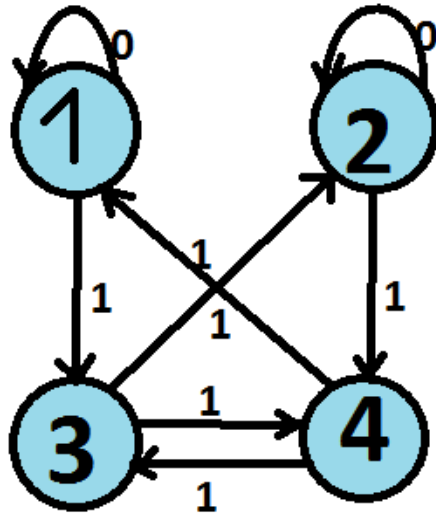
$$\begin{aligned}
k=4 \quad R_{11}^4 &= R_{11}^3 + R_{14}^3(R_{44}^3)^*R_{41}^3 = 0^* + 0^*11 + 0^*110^*1(0^*11 + 0^*110^*1)^*0^* \\
R_{12}^4 &= R_{12}^3 + R_{14}^3(R_{44}^3)^*R_{42}^3 = 0^*1 + 0^*11 + 0^*110^*1(0^*11 + 0^*110^*1)^*0^*1 \\
R_{13}^4 &= R_{13}^3 + R_{14}^3(R_{44}^3)^*R_{43}^3 = 0^*110^* + 0^*11 + 0^*110^*1 \\
&\quad (0^*11 + 0^*110^*1)^*0^*110^* \\
R_{14}^4 &= R_{14}^3 + R_{14}^3(R_{44}^3)^*R_{44}^3 = 0^*11 + 0^*110^*1 + 0^*11 + \\
&\quad 0^*110^*1(0^*11 + 0^*110^*1)^*0^*11 + 0^*110
\end{aligned}$$

$$\begin{aligned}
R_{21}^4 &= R_{21}^3 + R_{24}^3(R_{44}^3)^*R_{41}^3 = \in + 1 + 10^*1(0^*11 + 0^*110^*1)^*0^* \\
R_{22}^4 &= R_{22}^3 + R_{24}^3(R_{44}^3)^*R_{42}^3 = \emptyset + 1 + 10^*1(0^*11 + 0^*110^*1)^*0^*1 \\
R_{23}^4 &= R_{23}^3 + R_{24}^3(R_{44}^3)^*R_{43}^3 = 10^* + 1 + 10^*1(0^*11 + 0^*110^*1)^*0^*110^* \\
R_{24}^4 &= R_{24}^3 + R_{24}^3(R_{44}^3)^*R_{44}^3 = 1 + 10^*1 + 1 + 10^*1(0^*11 + 0^*110^*1)^*0^*11 + 0^*110^*1
\end{aligned}$$

$$\begin{aligned}
R_{31}^4 &= R_{31}^3 + R_{34}^3(R_{44}^3)^*R_{41}^3 = \emptyset + 0^*1(0^*11 + 0^*110^*1)^*0^* \\
R_{32}^4 &= R_{32}^3 + R_{34}^3(R_{44}^3)^*R_{42}^3 = \emptyset + 0^*1(0^*11 + 0^*110^*1)^*0^*1 \\
R_{33}^4 &= R_{33}^3 + R_{34}^3(R_{44}^3)^*R_{43}^3 = 0^* + 0^*1(0^*11 + 0^*110^*1)^*0^*110^* \\
R_{34}^4 &= R_{34}^3 + R_{34}^3(R_{44}^3)^*R_{44}^3 = 0^*1 + \\
&\quad 0^*1(0^*11 + 0^*110^*1)^*0^*11 + 0^*110^*1
\end{aligned}$$

$$\begin{aligned}
R_{41}^4 &= R_{41}^3 + R_{44}^3(R_{44}^3)^*R_{41}^3 = 0^* + 0^*11 + \\
&\quad 0^*110^*1(0^*11 + 0^*110^*1)^*0^* \\
R_{42}^4 &= R_{42}^3 + R_{44}^3(R_{44}^3)^*R_{42}^3 = 0^*1 + 0^*11 + \\
&\quad 0^*110^*1(0^*11 + 0^*110^*1)^*0^*1 \\
R_{43}^4 &= R_{43}^3 + R_{44}^3(R_{44}^3)^*R_{43}^3 = 0^*110^* + 0^*11 + \\
&\quad 0^*110^*1(0^*11 + 0^*110^*1)^*0^*110^* \\
R_{44}^4 &= R_{44}^3 + R_{44}^3(R_{44}^3)^*R_{44}^3 = 0^*11 + 0^*110^*1 + 0^*11 + \\
&\quad 0^*110^*1(0^*11 + 0^*110^*1)^*0^*11 + 0^*110
\end{aligned}$$

2- Ejercicio 2.



$$k = 0$$

$$\begin{aligned} R_{11}^0 &= \in + 0 = 0 \\ R_{12}^0 &= 1 \\ R_{13}^0 &= \emptyset \\ R_{14}^0 &= \emptyset \end{aligned}$$

$$\begin{aligned} R_{21}^0 &= \emptyset \\ R_{22}^0 &= \in \\ R_{23}^0 &= 1 \\ R_{24}^0 &= 1 \end{aligned}$$

$$\begin{aligned} R_{31}^0 &= \emptyset \\ R_{32}^0 &= \emptyset \\ R_{33}^0 &= 0 + \in = 0 \\ R_{34}^0 &= 1 \end{aligned}$$

$$\begin{aligned} R_{41}^0 &= 0 \\ R_{42}^0 &= 1 \\ R_{43}^0 &= \emptyset \\ R_{44}^0 &= \in \end{aligned}$$

$$k = 1$$

$$\begin{aligned} R_{11}^1 &= R_{11}^0 + R_{11}^0 (R_{11}^0)^* R_{11}^0 = 0^* \\ R_{12}^1 &= R_{12}^0 + R_{11}^0 (R_{11}^0)^* R_{12}^0 = 0^* 1 \\ R_{13}^1 &= R_{13}^0 + R_{11}^0 (R_{11}^0)^* R_{13}^0 = \emptyset \\ R_{14}^1 &= R_{14}^0 + R_{11}^0 (R_{11}^0)^* R_{14}^0 = \emptyset \end{aligned}$$

$$\begin{aligned} R_{21}^1 &= R_{21}^0 + R_{21}^0 (R_{11}^0)^* R_{11}^0 = \emptyset \\ R_{22}^1 &= R_{22}^0 + R_{21}^0 (R_{11}^0)^* R_{12}^0 = \in \\ R_{23}^1 &= R_{23}^0 + R_{21}^0 (R_{11}^0)^* R_{13}^0 = 1 \\ R_{24}^1 &= R_{24}^0 + R_{21}^0 (R_{11}^0)^* R_{14}^0 = 1 \end{aligned}$$

$$R_{31}^1 = R_{31}^0 + R_{31}^0 (R_{11}^0)^* R_{11}^0 = \emptyset$$

$$\begin{aligned}
R_{32}^1 &= R_{32}^0 + R_{31}^0(R_{11}^0)^*R_{12}^0 = \oslash \\
R_{33}^1 &= R_{33}^0 + R_{31}^0(R_{11}^0)^*R_{13}^0 = 0 \\
R_{34}^1 &= R_{34}^0 + R_{31}^0(R_{11}^0)^*R_{14}^0 = 1
\end{aligned}$$

$$\begin{aligned}
R_{41}^1 &= R_{41}^0 + R_{41}^0(R_{11}^0)^*R_{11}^0 = 0^* \\
R_{42}^1 &= R_{42}^0 + R_{41}^0(R_{11}^0)^*R_{12}^0 = 0 + 0^*1 = 0^*1 \\
R_{43}^1 &= R_{43}^0 + R_{41}^0(R_{11}^0)^*R_{13}^0 = \oslash \\
R_{44}^1 &= R_{44}^0 + R_{41}^0(R_{11}^0)^*R_{14}^0 = \oslash
\end{aligned}$$

$$k = 2$$

$$\begin{aligned}
R_{11}^2 &= R_{11}^1 + R_{12}^1(R_{22}^1)^*R_{21}^1 = 0^* \\
R_{12}^2 &= R_{12}^1 + R_{12}^1(R_{22}^1)^*R_{22}^1 = 0^*1 \\
R_{13}^2 &= R_{13}^1 + R_{12}^1(R_{22}^1)^*R_{23}^1 = 0^*11 \\
R_{14}^2 &= R_{14}^1 + R_{12}^1(R_{22}^1)^*R_{24}^1 = 0^*11
\end{aligned}$$

$$\begin{aligned}
R_{21}^2 &= R_{21}^1 + R_{22}^1(R_{22}^1)^*R_{21}^1 = \oslash \\
R_{22}^2 &= R_{22}^1 + R_{22}^1(R_{22}^1)^*R_{22}^1 = \in R_{23}^2 = R_{23}^1 + R_{22}^1(R_{22}^1)^*R_{23}^1 = 1 \\
R_{24}^2 &= R_{24}^1 + R_{22}^1(R_{22}^1)^*R_{24}^1 = 1
\end{aligned}$$

$$\begin{aligned}
R_{31}^2 &= R_{31}^1 + R_{32}^1(R_{22}^1)^*R_{21}^1 = \oslash \\
R_{32}^2 &= R_{32}^1 + R_{32}^1(R_{22}^1)^*R_{22}^1 = \oslash \\
R_{33}^2 &= R_{33}^1 + R_{32}^1(R_{22}^1)^*R_{23}^1 = 0 \\
R_{34}^2 &= R_{34}^1 + R_{32}^1(R_{22}^1)^*R_{24}^1 = 1
\end{aligned}$$

$$\begin{aligned}
R_{41}^2 &= R_{41}^1 + R_{42}^1(R_{22}^1)^*R_{21}^1 = 0^* \\
R_{42}^2 &= R_{42}^1 + R_{42}^1(R_{22}^1)^*R_{22}^1 = 0^*1 \\
R_{43}^2 &= R_{43}^1 + R_{42}^1(R_{22}^1)^*R_{23}^1 = 0^*11 \\
R_{44}^2 &= R_{44}^1 + R_{42}^1(R_{22}^1)^*R_{24}^1 = 0^*11
\end{aligned}$$

$$k = 3$$

$$\begin{aligned}
R_{11}^3 &= R_{11}^2 + R_{13}^2(R_{33}^2)^*R_{31}^2 = 0^* \\
R_{12}^3 &= R_{12}^2 + R_{13}^2(R_{33}^2)^*R_{32}^2 = 0^*1 \\
R_{13}^3 &= R_{13}^2 + R_{13}^2(R_{33}^2)^*R_{33}^2 = 0^*110^* \\
R_{14}^3 &= R_{14}^2 + R_{13}^2(R_{33}^2)^*R_{34}^2 = 0^*11 + 0^*110^*1
\end{aligned}$$

$$\begin{aligned}
R_{21}^3 &= R_{21}^2 + R_{23}^2(R_{33}^2)^*R_{31}^2 = \oslash \\
R_{22}^3 &= R_{22}^2 + R_{23}^2(R_{33}^2)^*R_{32}^2 = \in \\
R_{23}^3 &= R_{23}^2 + R_{23}^2(R_{33}^2)^*R_{33}^2 = 10^* \\
R_{24}^3 &= R_{24}^2 + R_{23}^2(R_{33}^2)^*R_{34}^2 = 1 + 10^*1
\end{aligned}$$

$$\begin{aligned}
R_{31}^3 &= R_{31}^2 + R_{33}^2(R_{33}^2)^*R_{31}^2 = \oslash \\
R_{32}^3 &= R_{32}^2 + R_{33}^2(R_{33}^2)^*R_{32}^2 = \oslash \\
R_{33}^3 &= R_{33}^2 + R_{33}^2(R_{33}^2)^*R_{33}^2 = 0^* \\
R_{34}^3 &= R_{34}^2 + R_{33}^2(R_{33}^2)^*R_{34}^2 = 0^*1
\end{aligned}$$

$$\begin{aligned}
R_{41}^3 &= R_{41}^2 + R_{43}^2(R_{33}^2)^*R_{31}^2 = 0^* \\
R_{42}^3 &= R_{42}^2 + R_{43}^2(R_{33}^2)^*R_{32}^2 = 0^*1
\end{aligned}$$

$$R_{43}^3 = R_{43}^2 + R_{43}^2(R_{33}^2)^*R_{23}^2 = 0^*110^*$$

$$R_{44}^3 = R_{44}^2 + R_{43}^2(R_{33}^2)^*R_{24}^2 = 0^*11 + 0^*110^*1$$

$$k = 4$$

$$R_{11}^4 = R_{11}^3 + R_{14}^3(R_{44}^3)^*R_{41}^3 = (0^*11 + 0^*110^*1)^*0^*$$

$$R_{12}^4 = R_{12}^3 + R_{14}^3(R_{44}^3)^*R_{42}^3 = (0^*11 + 0^*110^*1)^*0^*1$$

$$R_{13}^4 = R_{13}^3 + R_{14}^3(R_{44}^3)^*R_{43}^3 = (0^*11 + 0^*110^*1)^*0^*110^*$$

$$R_{14}^4 = R_{14}^3 + R_{14}^3(R_{44}^3)^*R_{44}^3 = (0^*11 + 0^*110^*1)^*$$

$$R_{21}^4 = R_{21}^3 + R_{24}^3(R_{44}^3)^*R_{41}^3 = (1 + 10^*1)(0^*11 + 0^*110^*1)^*0^*$$

$$R_{22}^4 = R_{22}^3 + R_{24}^3(R_{44}^3)^*R_{42}^3 = (1 + 10^*1)(0^*11 + 0^*110^*1)^*0^*1$$

$$R_{23}^4 = R_{23}^3 + R_{24}^3(R_{44}^3)^*R_{43}^3 = (1 + 10^*1)(0^*11 + 0^*110^*1)^*0^*110^*$$

$$R_{24}^4 = R_{24}^3 + R_{24}^3(R_{44}^3)^*R_{44}^3 = (1 + 10^*1)(0^*11 + 0^*110^*1)^*$$

$$R_{31}^4 = R_{31}^3 + R_{34}^3(R_{44}^3)^*R_{41}^3 = 0^*(0^*11 + 0^*110^*1)^*0^*$$

$$R_{32}^4 = R_{32}^3 + R_{34}^3(R_{44}^3)^*R_{42}^3 = 0^*(0^*11 + 0^*110^*1)^*0^*1$$

$$R_{33}^4 = R_{33}^3 + R_{34}^3(R_{44}^3)^*R_{43}^3 = 0^*(0^*11 + 0^*110^*1)^*0^*110^*$$

$$R_{34}^4 = R_{34}^3 + R_{34}^3(R_{44}^3)^*R_{44}^3 = 0^*(0^*11 + 0^*110^*1)^*$$

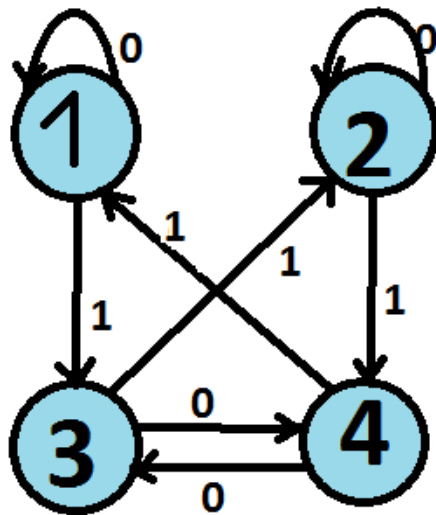
$$R_{41}^4 = R_{41}^3 + R_{44}^3(R_{44}^3)^*R_{41}^3 = (0^*11 + 0^*110^*1)^*0^*$$

$$R_{42}^4 = R_{42}^3 + R_{44}^3(R_{44}^3)^*R_{42}^3 = (0^*11 + 0^*110^*1)^*0^*1$$

$$R_{43}^4 = R_{43}^3 + R_{44}^3(R_{44}^3)^*R_{43}^3 = (0^*11 + 0^*110^*1)^*0^*110^*$$

$$R_{44}^4 = R_{44}^3 + R_{44}^3(R_{44}^3)^*R_{44}^3 = (0^*11 + 0^*110^*1)^*$$

3- Ejercicio 3.



$$k = 0$$

$$R_{11}^0 = \epsilon + 0$$

$$R_{12}^0 = \emptyset$$

$$R_{13}^0 = 1$$

$$R_{14}^0 = \emptyset$$

$$\begin{aligned} R_{21}^0 &= \emptyset \\ R_{22}^0 &= 0 + \in \\ R_{23}^0 &= 1 \\ R_{24}^0 &= \emptyset \end{aligned}$$

$$\begin{aligned} R_{31}^0 &= \emptyset \\ R_{32}^0 &= 1 \\ R_{33}^0 &= \in + \emptyset \\ R_{34}^0 &= 0 \end{aligned}$$

$$\begin{aligned} R_{41}^0 &= 1 \\ R_{42}^0 &= \emptyset \\ R_{43}^0 &= 0 \\ R_{44}^0 &= \in + \emptyset \end{aligned}$$

$$k = 1$$

$$\begin{aligned} R_{11}^1 &= R_{11}^0 + R_{11}^0 (R_{11}^0)^* R_{11}^0 = 0^* \\ R_{12}^1 &= R_{12}^0 + R_{11}^0 (R_{11}^0)^* R_{12}^0 = \emptyset \\ R_{13}^1 &= R_{13}^0 + R_{11}^0 (R_{11}^0)^* R_{13}^0 = 1 + 0^* 1 = 0^* 1 \\ R_{14}^1 &= R_{14}^0 + R_{11}^0 (R_{11}^0)^* R_{14}^0 = \emptyset \end{aligned}$$

$$\begin{aligned} R_{21}^1 &= R_{21}^0 + R_{21}^0 (R_{11}^0)^* R_{11}^0 = \emptyset \\ R_{22}^1 &= R_{22}^0 + R_{21}^0 (R_{11}^0)^* R_{12}^0 = (0 + \in) + \emptyset = 0 \\ R_{23}^1 &= R_{23}^0 + R_{21}^0 (R_{11}^0)^* R_{13}^0 = \emptyset \\ R_{24}^1 &= R_{24}^0 + R_{21}^0 (R_{11}^0)^* R_{14}^0 = 1 \end{aligned}$$

$$\begin{aligned} R_{31}^1 &= R_{31}^0 + R_{31}^0 (R_{11}^0)^* R_{11}^0 = \emptyset \\ R_{32}^1 &= R_{32}^0 + R_{31}^0 (R_{11}^0)^* R_{12}^0 = 1 + \emptyset = 1 \\ R_{33}^1 &= R_{33}^0 + R_{31}^0 (R_{11}^0)^* R_{13}^0 = \in + \emptyset + \emptyset = \in \\ R_{34}^1 &= R_{34}^0 + R_{31}^0 (R_{11}^0)^* R_{14}^0 = 0 \end{aligned}$$

$$\begin{aligned} R_{41}^1 &= R_{41}^0 + R_{41}^0 (R_{11}^0)^* R_{11}^0 = 1 + 1(\in + 0)^* (\in + 0)^* = 10^* \\ R_{42}^1 &= R_{42}^0 + R_{41}^0 (R_{11}^0)^* R_{12}^0 = \emptyset + 1(\in + 0)^* \emptyset = \emptyset \\ R_{43}^1 &= R_{43}^0 + R_{41}^0 (R_{11}^0)^* R_{13}^0 = 0 + 1(\in + 0)^* 1 = 0 + 10^* 1 \\ R_{44}^1 &= R_{44}^0 + R_{41}^0 (R_{11}^0)^* R_{14}^0 = \in + 1(\in + 0)^* \emptyset = \in \end{aligned}$$

$$k = 2$$

$$\begin{aligned} R_{11}^2 &= R_{11}^1 + R_{12}^1 (R_{22}^1)^* R_{21}^1 = 0^* \\ R_{12}^2 &= R_{12}^1 + R_{12}^1 (R_{22}^1)^* R_{22}^1 = \emptyset \\ R_{13}^2 &= R_{13}^1 + R_{12}^1 (R_{22}^1)^* R_{23}^1 = 0^* 1 \\ R_{14}^2 &= R_{14}^1 + R_{12}^1 (R_{22}^1)^* R_{24}^1 = \emptyset \end{aligned}$$

$$\begin{aligned} R_{21}^2 &= R_{21}^1 + R_{22}^1 (R_{22}^1)^* R_{21}^1 = \emptyset \\ R_{22}^2 &= R_{22}^1 + R_{22}^1 (R_{22}^1)^* R_{22}^1 = 0^* \\ R_{23}^2 &= R_{23}^1 + R_{22}^1 (R_{22}^1)^* R_{23}^1 = \emptyset \\ R_{24}^2 &= R_{24}^1 + R_{22}^1 (R_{22}^1)^* R_{24}^1 = 0^* 1 \end{aligned}$$

$$\begin{aligned}
R_{31}^2 &= R_{31}^1 + R_{32}^1(R_{22}^1)^*R_{21}^1 = \oslash \\
R_{32}^2 &= R_{32}^1 + R_{32}^1(R_{22}^1)^*R_{22}^1 = 10^* \\
R_{33}^2 &= R_{33}^1 + R_{32}^1(R_{22}^1)^*R_{23}^1 = \in \\
R_{34}^2 &= R_{34}^1 + R_{32}^1(R_{22}^1)^*R_{24}^1 = 0 + 10^*1
\end{aligned}$$

$$\begin{aligned}
R_{41}^2 &= R_{41}^1 + R_{42}^1(R_{22}^1)^*R_{21}^1 = 10^* \\
R_{42}^2 &= R_{42}^1 + R_{42}^1(R_{22}^1)^*R_{22}^1 = \oslash \\
R_{43}^2 &= R_{43}^1 + R_{42}^1(R_{22}^1)^*R_{23}^1 = 0 + 10^*1 \\
R_{44}^2 &= R_{44}^1 + R_{42}^1(R_{22}^1)^*R_{24}^1 = \in
\end{aligned}$$

$k = 3$

$$\begin{aligned}
R_{11}^3 &= R_{11}^2 + R_{13}^2(R_{33}^2)^*R_{31}^2 = 0^* \\
R_{12}^3 &= R_{12}^2 + R_{13}^2(R_{33}^2)^*R_{32}^2 = 0^*110^* \\
R_{13}^3 &= R_{13}^2 + R_{13}^2(R_{33}^2)^*R_{33}^2 = 0^*1 \\
R_{14}^3 &= R_{14}^2 + R_{13}^2(R_{33}^2)^*R_{34}^2 = 0^*(0 + 10^*1)
\end{aligned}$$

$$\begin{aligned}
R_{21}^3 &= R_{21}^2 + R_{23}^2(R_{33}^2)^*R_{31}^2 = \oslash \\
R_{22}^3 &= R_{22}^2 + R_{23}^2(R_{33}^2)^*R_{32}^2 = 0^* \\
R_{23}^3 &= R_{23}^2 + R_{23}^2(R_{33}^2)^*R_{33}^2 = \oslash \\
R_{24}^3 &= R_{24}^2 + R_{23}^2(R_{33}^2)^*R_{34}^2 = 0^*1
\end{aligned}$$

$$\begin{aligned}
R_{31}^3 &= R_{31}^2 + R_{33}^2(R_{33}^2)^*R_{21}^2 = \oslash \\
R_{32}^3 &= R_{32}^2 + R_{33}^2(R_{33}^2)^*R_{22}^2 = 10^* \\
R_{33}^3 &= R_{33}^2 + R_{33}^2(R_{33}^2)^*R_{23}^2 = \in \\
R_{34}^3 &= R_{34}^2 + R_{33}^2(R_{33}^2)^*R_{24}^2 = 0 + 10^*1
\end{aligned}$$

$$\begin{aligned}
R_{41}^3 &= R_{41}^2 + R_{43}^2(R_{33}^2)^*R_{21}^2 = 10^* \\
R_{42}^3 &= R_{42}^2 + R_{43}^2(R_{33}^2)^*R_{22}^2 = (0 + 10^*1)10^* \\
R_{43}^3 &= R_{43}^2 + R_{43}^2(R_{33}^2)^*R_{23}^2 = 0 + 10^*1 \\
R_{44}^3 &= R_{44}^2 + R_{43}^2(R_{33}^2)^*R_{24}^2 = (0 + 10^*1)(0 + 10^*1)
\end{aligned}$$

$k = 4$

$$\begin{aligned}
R_{11}^4 &= R_{11}^3 + R_{14}^3(R_{44}^3)^*R_{41}^3 = 0^*(0 + 10^*1)^*10^* \\
R_{12}^4 &= R_{12}^3 + R_{14}^3(R_{44}^3)^*R_{42}^3 = 0^*110^* + 0^*(0 + 10^*1)^*10^* \\
R_{13}^4 &= R_{13}^3 + R_{14}^3(R_{44}^3)^*R_{43}^3 = 0^*1 + 0^*(0 + 10^*1)^* \\
R_{14}^4 &= R_{14}^3 + R_{14}^3(R_{44}^3)^*R_{44}^3 = (0^*(0 + 10^*1))^*
\end{aligned}$$

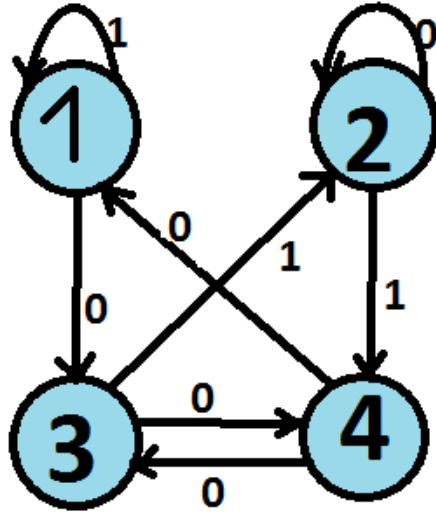
$$\begin{aligned}
R_{21}^4 &= R_{21}^3 + R_{24}^3(R_{44}^3)^*R_{41}^3 = 0^*1(0 + 10^*1)^*10^* \\
R_{22}^4 &= R_{22}^3 + R_{24}^3(R_{44}^3)^*R_{42}^3 = 0^* + 0^*1(0 + 10^*1)^*10^* \\
R_{23}^4 &= R_{23}^3 + R_{24}^3(R_{44}^3)^*R_{43}^3 = 0^*1(0 + 10^*1)^* \\
R_{24}^4 &= R_{24}^3 + R_{24}^3(R_{44}^3)^*R_{44}^3 = 0^*1 + 0^*1(0 + 10^*1)^*
\end{aligned}$$

$$\begin{aligned}
R_{31}^4 &= R_{31}^3 + R_{34}^3(R_{44}^3)^*R_{41}^3 = (0 + 10^*1)^*10^* \\
R_{32}^4 &= R_{32}^3 + R_{34}^3(R_{44}^3)^*R_{42}^3 = 10^* + (0 + 10^*1)^*10^* \\
R_{33}^4 &= R_{33}^3 + R_{34}^3(R_{44}^3)^*R_{43}^3 = (0 + 10^*1)^* \\
R_{34}^4 &= R_{34}^3 + R_{34}^3(R_{44}^3)^*R_{44}^3 = (0 + 10^*1)^*
\end{aligned}$$



$$\begin{aligned}
R_{41}^4 &= R_{41}^3 + R_{44}^3(R_{44}^3)^*R_{41}^3 = 10^* + (0 + 10^*1)^*10^* \\
R_{42}^4 &= R_{42}^3 + R_{44}^3(R_{44}^3)^*R_{42}^3 = (0 + 10^*1)^*10^* \\
R_{43}^4 &= R_{43}^3 + R_{44}^3(R_{44}^3)^*R_{43}^3 = (0 + 10^*1)^* \\
R_{44}^4 &= R_{44}^3 + R_{44}^3(R_{44}^3)^*R_{44}^3 = (0 + 10^*1)^*
\end{aligned}$$

4 Ejercicio 4.



$k = 1$

$$\begin{aligned}
R_{11}^1 &= R_{11}^0 + R_{11}^0(R_{11}^0)^*R_{11}^0 = (1 + \epsilon) + (1 + \epsilon)(1 + \epsilon)^*(1 + \epsilon) = (1 + \epsilon) + (1 + \epsilon)(1)^*(1 + \epsilon) = 1^* \\
R_{12}^1 &= R_{12}^0 + R_{11}^0(R_{11}^0)^*R_{12}^0 = (1 + \epsilon)(1 + \epsilon)^* = (1)^* \\
R_{13}^1 &= R_{13}^0 + R_{11}^0(R_{11}^0)^*R_{13}^0 = 0 + (1 + \epsilon)(1 + \epsilon)^*0 = 0 + 1^*0 = 1^*0 \\
R_{14}^1 &= R_{14}^0 + R_{11}^0(R_{11}^0)^*R_{14}^0 = \emptyset
\end{aligned}$$

$$\begin{aligned}
R_{21}^1 &= R_{21}^0 + R_{21}^0(R_{11}^0)^*R_{11}^0 = (1 + \epsilon)^*(1 + \epsilon) = 1^* \\
R_{22}^1 &= R_{22}^0 + R_{21}^0(R_{11}^0)^*R_{12}^0 = 0 + \epsilon + (1 + \epsilon)^* = 0 + \epsilon = 0 \\
R_{23}^1 &= R_{23}^0 + R_{21}^0(R_{11}^0)^*R_{13}^0 = (1 + \epsilon)^*0 = 1^*0 \\
R_{24}^1 &= R_{24}^0 + R_{21}^0(R_{11}^0)^*R_{14}^0 = 1 + (1 + \epsilon)^* = 1^*
\end{aligned}$$

$$\begin{aligned}
R_{31}^1 &= R_{31}^0 + R_{31}^0(R_{11}^0)^*R_{11}^0 = (1 + \epsilon)^*(1 + \epsilon) = 1^* \\
R_{32}^1 &= R_{32}^0 + R_{31}^0(R_{11}^0)^*R_{12}^0 = 1 + (1 + \epsilon)^* = 1^* \\
R_{33}^1 &= R_{33}^0 + R_{31}^0(R_{11}^0)^*R_{13}^0 = \epsilon + (1 + \epsilon)^*0 = 1^*0 \\
R_{34}^1 &= R_{34}^0 + R_{31}^0(R_{11}^0)^*R_{14}^0 = 0 + \epsilon + (1 + \epsilon)^* = 01^*
\end{aligned}$$

$$\begin{aligned}
R_{41}^1 &= R_{41}^0 + R_{41}^0(R_{11}^0)^*R_{11}^0 = 0 + 0(1 + \epsilon)^*(1 + \epsilon) = 0 + 01^* = 01^* \\
R_{42}^1 &= R_{42}^0 + R_{41}^0(R_{11}^0)^*R_{12}^0 = 0(1 + \epsilon)^* \\
R_{43}^1 &= R_{43}^0 + R_{41}^0(R_{11}^0)^*R_{13}^0 = 0 + 0(1 + \epsilon)^*0 = 0 + 01^*0 \\
R_{44}^1 &= R_{44}^0 + R_{41}^0(R_{11}^0)^*R_{14}^0 = \epsilon + 0(1 + \epsilon)^* = 01^*
\end{aligned}$$

$k = 2$

$$\begin{aligned}
R_{11}^2 &= R_{11}^1 + R_{12}^1(R_{22}^1)^*R_{21}^1 = 1^* + (0 + \epsilon)^* = 1^* \\
R_{12}^2 &= R_{12}^1 + R_{12}^1(R_{22}^1)^*R_{22}^1 = (0 + \epsilon)^*(0 + \epsilon) \\
R_{13}^2 &= R_{13}^1 + R_{12}^1(R_{22}^1)^*R_{23}^1 = 1^*0 + (0 + \epsilon)^* = 1^*0 \\
R_{14}^2 &= R_{14}^1 + R_{12}^1(R_{22}^1)^*R_{24}^1 = (0 + \epsilon)^*1
\end{aligned}$$

$$\begin{aligned}
R_{21}^2 &= R_{21}^1 + R_{22}^1(R_{22}^1)^*R_{21}^1 = (0+ \in)(0+ \in)^* \\
R_{22}^2 &= R_{22}^1 + R_{22}^1(R_{22}^1)^*R_{22}^1 = (0+ \in) + (0+ \in)(0+ \in)^*(0+ \in) = 0^* \\
R_{23}^2 &= R_{23}^1 + R_{22}^1(R_{22}^1)^*R_{23}^1 = (0+ \in)(0+ \in)^* \\
R_{24}^2 &= R_{24}^1 + R_{22}^1(R_{22}^1)^*R_{24}^1 = 1 + (0+ \in)(0+ \in)^*1 = 1 + 0^*1 = 0^*1
\end{aligned}$$

$$\begin{aligned}
R_{31}^2 &= R_{31}^1 + R_{32}^1(R_{22}^1)^*R_{31}^1 = 1(0+ \in)^* \\
R_{32}^2 &= R_{32}^1 + R_{32}^1(R_{22}^1)^*R_{22}^1 = 1 + 1(0+ \in)^*(0+ \in) = 1 + 10^* = 10^* \\
R_{33}^2 &= R_{33}^1 + R_{32}^1(R_{22}^1)^*R_{23}^1 = \in + 1(0+ \in)^* = \in \\
R_{34}^2 &= R_{34}^1 + R_{32}^1(R_{22}^1)^*R_{24}^1 = 0 + 1(0+ \in)^*1 = 0 + 10^*1
\end{aligned}$$

$$\begin{aligned}
R_{41}^2 &= R_{41}^1 + R_{42}^1(R_{22}^1)^*R_{21}^1 = 01^* \\
R_{42}^2 &= R_{42}^1 + R_{42}^1(R_{22}^1)^*R_{22}^1 = \\
R_{43}^2 &= R_{43}^1 + R_{42}^1(R_{22}^1)^*R_{23}^1 = 0 + 01^*0 \\
R_{44}^2 &= R_{44}^1 + R_{42}^1(R_{22}^1)^*R_{24}^1 = \in
\end{aligned}$$

$k = 3$

$$\begin{aligned}
R_{11}^3 &= R_{11}^2 + R_{13}^2(R_{33}^2)^*R_{31}^2 = 1^* = 1^* \\
R_{12}^3 &= R_{12}^2 + R_{13}^2(R_{33}^2)^*R_{32}^2 = 1^*0(\in)^*10^* = 1^*010^* \\
R_{13}^3 &= R_{13}^2 + R_{13}^2(R_{33}^2)^*R_{33}^2 = 1^*0 + 1^*0 \in \in = 1^*0 \\
R_{14}^3 &= R_{14}^2 + R_{13}^2(R_{33}^2)^*R_{34}^2 = 1^*0 \in (0 + 10^*1) = 1^*00 + 1^*010^*
\end{aligned}$$

$$\begin{aligned}
R_{21}^3 &= R_{21}^2 + R_{23}^2(R_{33}^2)^*R_{31}^2 = \\
R_{22}^3 &= R_{22}^2 + R_{23}^2(R_{33}^2)^*R_{32}^2 = 0^* \\
R_{23}^3 &= R_{23}^2 + R_{23}^2(R_{33}^2)^*R_{33}^2 = \\
R_{24}^3 &= R_{24}^2 + R_{23}^2(R_{33}^2)^*R_{34}^2 = 0^*1
\end{aligned}$$

$$\begin{aligned}
R_{31}^3 &= R_{31}^2 + R_{33}^2(R_{33}^2)^*R_{31}^2 = \in \in \in \in \\
R_{32}^3 &= R_{32}^2 + R_{33}^2(R_{33}^2)^*R_{32}^2 = 10^* + 10^* = 10^* \\
R_{33}^3 &= R_{33}^2 + R_{33}^2(R_{33}^2)^*R_{33}^2 = \in \\
R_{34}^3 &= R_{34}^2 + R_{33}^2(R_{33}^2)^*R_{34}^2 = 0 + 10^*1
\end{aligned}$$

$$\begin{aligned}
R_{41}^3 &= R_{41}^2 + R_{43}^2(R_{33}^2)^*R_{31}^2 = 10^* + (0 + 10^*1)\epsilon^* = 10^* \\
R_{42}^3 &= R_{42}^2 + R_{43}^2(R_{33}^2)^*R_{32}^2 = (0 + 01^*0)(10^*) = 010^* + 01^*010^* \\
R_{43}^3 &= R_{43}^2 + R_{43}^2(R_{33}^2)^*R_{33}^2 = 0 + 01^*0 \\
R_{44}^3 &= R_{44}^2 + R_{43}^2(R_{33}^2)^*R_{34}^2 = \in + (0 + 01^*0)(0 + 10^*1) \\
&= \in + 00 + 010^*1 + 01^*00 + 01^*010^*1
\end{aligned}$$

$k = 4$

$$\begin{aligned}
R_{11}^4 &= R_{11}^3 + R_{14}^3(R_{44}^3)^*R_{41}^3 = 0^* + (1^* + (1^*00 + 1^*010^*1)(\in + 00 + 010^*1 + 01^*00 + 01^*010^*1)^*01^* \\
R_{12}^4 &= R_{12}^3 + R_{14}^3(R_{44}^3)^*R_{42}^3 = 1^*010^* + (1^*00 + 1^*010^*1)(\in + 00 + 010^*1 + 01^*00 + 01^*010^*1)^*(010^* + 01^*010^*) \\
R_{13}^4 &= R_{13}^3 + R_{14}^3(R_{44}^3)^*R_{43}^3 = 1^*0 + (1^*00 + 1^*010^*1)(\in + 00 + 010^*1 + 01^*00 + 01^*010^*1)^*(0 + 01^*0) \\
R_{14}^4 &= R_{14}^3 + R_{14}^3(R_{44}^3)^*R_{44}^3 = 1^*00 + 1^*010^*1 \\
&= (1^*00 + 1^*010^*1)(\in + 00 + 010^*1 + 01^*00 + 01^*010^*1)^*(\in + 00 + 010^*1 + 01^*00 + 01^*010^*1) \\
&= (1^*00 + 1^*010^*1)(\in + 00 + 010^*1 + 01^*00 + 01^*010^*1)^*
\end{aligned}$$

$$\begin{aligned}
R_{21}^4 &= R_{21}^3 + R_{24}^3(R_{44}^3)^*R_{41}^3 = 0^*1(\in + 00 + 010^*1 + 01^*00 + 01^*010^*1)^*01^* \\
R_{22}^4 &= R_{22}^3 + R_{24}^3(R_{44}^3)^*R_{42}^3 \\
&= 0^* + 0^*1(\in + 00 + 010^*1 + 01^*00 + 01^*010^*1)^*(010^* + 01^*010^*) \\
R_{23}^4 &= R_{23}^3 + R_{24}^3(R_{44}^3)^*R_{43}^3 = 0^*1(\in + 00 + 010^*1 + 01^*00 + 01^*010^*1)^*(0 + 01^*0) \\
R_{24}^4 &= R_{24}^3 + R_{24}^3(R_{44}^3)^*R_{44}^3
\end{aligned}$$

$$= 0^*1 + 0^*1(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)$$

$$\begin{aligned} R_{31}^4 &= R_{31}^3 + R_{34}^3(R_{44}^3)^*R_{41}^3 = (0 + 10^*1)(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*01^* \\ R_{32}^4 &= R_{32}^3 + R_{34}^3(R_{44}^3)^*R_{42}^3 = 10^* + (0 + 10^*1)(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(010^* + 01^*010^*) \\ R_{33}^4 &= R_{33}^3 + R_{34}^3(R_{44}^3)^*R_{43}^3 \\ &= \in + (0 + 10^*1)(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(0 + 01^*0) \\ R_{34}^4 &= R_{34}^3 + R_{34}^3(R_{44}^3)^*R_{44}^3 = (0 + 10^*1) \\ &+ (0 + 10^*1)(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(\in +00 + 010^*1 + 01^*00 + 01^*010^*1) \\ &= (0 + 10^*1)(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^* \end{aligned}$$

$$\begin{aligned} R_{41}^4 &= R_{41}^3 + R_{44}^3(R_{44}^3)^*R_{41}^3 \\ &= (01^*) + (\in +00 + 010^*1 + 01^*00 + 01^*010^*1)(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(01^*) \\ &= (\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(01^*) \\ R_{42}^4 &= R_{42}^3 + R_{44}^3(R_{44}^3)^*R_{42}^3 = (010^* + 01^*010^*) \\ &+ (\in +00 + 010^*1 + 01^*00 + 01^*010^*1)(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(010^* + 01^*010^*) \\ &= (\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(010^* + 01^*010^*) \\ R_{43}^4 &= R_{43}^3 + R_{44}^3(R_{44}^3)^*R_{43}^3 = (0 + 01^*0) \\ &+ (\in +00 + 010^*1 + 01^*00 + 01^*010^*1)(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(0 + 01^*0) \\ &= (\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^*(0 + 01^*0) \\ R_{44}^4 &= R_{44}^3 + R_{44}^3(R_{44}^3)^*R_{44}^3 = (\in +00 + 010^*1 + 01^*00 + 01^*010^*1) \\ &+ (\in +00 + 010^*1 + 01^*00 + 01^*010^*1)(\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^* \\ &= (\in +00 + 010^*1 + 01^*00 + 01^*010^*1)^* \end{aligned}$$