

# Selective Algorithm for Expanded Group Method of Data Handling Applied to Power Amplifier Modeling

Ana Paula P. Machado<sup>1</sup>, Vithor B. Nypwipwy<sup>1</sup>, Caroline de França<sup>1</sup> and Eduardo G. de Lima<sup>1</sup>

<sup>1</sup> Department of Electrical Engineering, Universidade Federal do Paraná (UFPR), Curitiba, Brazil  
e-mail: anaprinccival@ufpr.br

**Abstract**— Power amplifiers (PAs) are electronic devices commonly used in telecommunications that need to transmit information with high energetic efficiency. For this, it is necessary to use data manipulation methods that assist in the linearization of the output signal. Our previous conference paper presented experiments from two codes constructed based on the Group Method of Data Handling (GMDH) and which differ in their way of selecting the best coefficients to be used in the calculations of the neural network. The first method, called Embracing, assumes greater availability of data, while the second, called Selective, selects information from the beginning of the code. This work extends the previous GMDH models by expanding the PA inputs into Laguerre basis functions with a single real pole. The comparison among the different approaches employs experimental data collected from a GaN High Electron Mobility Transistor (HEMT) class AB PA and a Si Laterally Diffused Metal-Oxide-Semiconductor (LDMOS) class AB. The most selective and computationally more complex structure, when searching for identification since from the first layers, expresses minor errors and the best results in the output for both Conventional and Expanded GMDH models, becoming a reasoned option for use in PAs. A normalized mean square error (NMSE) of -35.44 dB was obtained by Expanded GMDH with Selective algorithm and 5 inputs when using the GaN PA, whereas a NMSE of -40.35 dB was obtained by the Expanded GMDH with Selective algorithm and 4 inputs when calculated with the Si LDMOS PA data.

**Index Terms**— Group Method of Data Handling, Laguerre basis functions, modeling, power amplifier, wireless communications.

## I. INTRODUCTION

Wireless communications are constantly evolving to provide higher data transfer rates to an increasing number of users and applications [1]-[2]. A wireless communication system consists basically of three elements: transmitter, medium and receiver [3]. The transmitter converts the information into a form suitable for sending it through an air interface.

PAs are located at the transmitter chain. PAs are devices that basically have the function of amplifying the capacity of the signal applied to their input. They are divided into classes, indicating how much the output signal varies within an operating cycle for a complete input cycle [4].

To keep linearity between the output signal and input signal, radio frequency PAs need to operate in regions with low input power. However, in order to obtain a system with good energetic efficiency, avoiding the exacerbated consumption of battery in isolated systems, operations in non-linear regions (in which greater power is required) are used, but, for this, it is necessary to appropriate methods that guarantee the linearization of the system [5].

There are different types of modeling for system linearization, and among these neural networks stand [6]-[7]. The

Feed Forward topology consists of a network in which data travel in only one direction, from the entrance to the last layer. Each layer is made up of neurons, which manipulate the data according to their programming. Its nomenclature is an analogy to biological neurons, which have potential for cell passage [5].

When it comes to data modeling that has many coefficients, some identification algorithms, such as method of Least Squares (LS), start to show inaccuracies in their results. The GMDH can be exploited to find an efficient method that produces the correct data collection [8], which consists of algorithms capable of identifying, organizing and regressing systems with a large number of inputs.

This paper is an extended version of [9]. In [9], two distinct algorithms, namely Embracing and Selective, were applied to the identification of a GMDH-based PA model. Using experimental data measured on a GaN HEMT PA, a superior performance was reported in [9] by the Selective algorithm that performs LS considering only the neurons that are kept active at the final network topology. In the conventional GMDH-based PA model of [9], present and past samples of the complex PA input are directly inputted to the network. The first contribution of this extended paper is to expand the set of complex PA inputs into a set of Laguerre basis functions prior to their application as GMDH inputs. Such Expanded GMDH has a unique real pole that can be optimized to describe PA dynamics with fewer parameters [10]. The second contribution of this extended paper is to report modeling results for Conventional and Expanded GMDH models trained by Embracing and Selective algorithms based on experimental data measured on a GaN PA and on a Si LDMOS PA.

This work is organized as follows. Section II presents the topology of the GMDH network and how it is organized in the distribution and selection of neurons, layer by layer, in addition to the activation function contained in each one. In Section III it is explained how the GMDH model was used in the construction of the Embracing and Selective methods, adapting the network to work with the PA signal and designing architectures with different scenarios. Section IV presents the expanded model, which details the use of the Laguerre expansion in the previously constructed model. Section V explains in greater detail the operation of codes and how they act, at each layer, in extracting the coefficients, and also the difference between the architectures and their selection criteria. Section VI is focused on the simulation, first using the GaN HEMT PA, and then the Si LDMOS PA. The performances of the Embracing and Selective methods, as well as the Conventional and Expanded models, are compared. Finally, in Section VII there is the conclusion of the training of methods and models presented based on the discussed results.

## II. CONVENTIONAL GMDH MODEL

The GMDH model was initially introduced by Ivakhnenko in the late 1960s with the aim of studying non-linear relationships between input and output variables [8]. It has consolidated itself as a pioneering method in the matter of self-organization, acting with minimal human intervention and presenting accurate final results, even with a large number of inputs. It proved to be of great importance in the field of electronics and telecommunications by assisting in the linearization of transmission chains [5].

This modeling method is organized in layers, each with a certain number of neurons. Each neuron receives two inputs and results in only one output, which will be used as an input to a next layer using all possible combinations, as shown in Fig. 1.

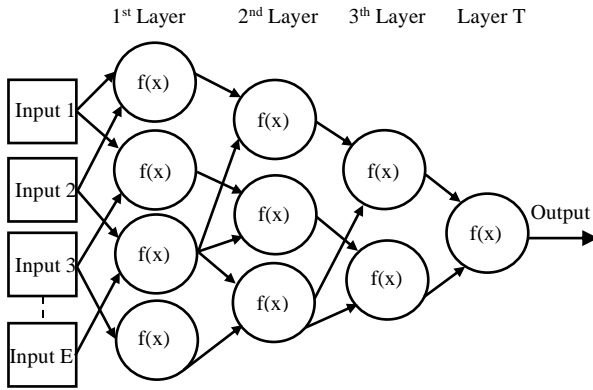


Fig. 1 – Block diagram of GMDH

The number of neurons in each layer is directly defined by the number of real inputs, which, starting from the second layer, is made up of the real outputs of the previous neurons [8]. Each of these neurons has the activation function explained by

$$f(x) = m + nx_i + ox_j + px_i x_j + qx_i^2 + rx_j^2, \quad (1)$$

where the index  $i$  subscribed refers to the first input of the neuron, and the index  $j$  refers to the second input. The coefficients  $m, n, o, p, q$  and  $r$  are adjustable and particular to each neuron, directly influencing the output of each neuron and, therefore, in the calculation of the next layer.

The main points of the method are to draw the topology of the network: number of inputs, number of layers and the best neurons of each one, also analyzing the most advantageous combinations for the accuracy of the final result. The identification and manipulation of the adjustable coefficients of each activation function are also essential for the greater mastery of modeling with GMDH.

## III. GMDH MODEL ADAPTED FOR BEHAVIORAL MODELING OF PAS

When using the GMDH model for the behavioral modeling of PAS it is necessary to adapt the activation function contained in each neuron, since the involved signals are of complex values (real and imaginary parts representing the envelope signal), used for greater accuracy in modeling and reduced complexity, especially when used in telecommunications [5].

For the neuron to process inputs from the complex domain, its activation function must be complex, respecting the condition of being contained in a limited and complex domain [11]. The activation function previously presented is modified to

$$f(x) = mx_i + nx_j + ox_i|x_i| + px_j|x_j| + qx_i|x_j| + rx_j|x_i|, \quad (2)$$

where  $m, n, o, p, q$  and  $r$  are adjustable coefficients,  $i$  and  $j$  remain references to the positions of the inputs, this time complex. From the third term onwards, the complex value module is represented by  $|x|$ , being one of the factors in the multiplication of variables. In (2), each adjustable coefficient is multiplied by a contribution that has exactly one complex input. This was done because only contributions that have odd parity with respect to the complex input are useful to the behavioral modeling of PAS in complex baseband domain [12]. Because of that, the constant term of (1) was removed and the second-order contributions of (1) were modified to be a product between a complex input and the module of a complex input. Since the module has even parity with respect to the complex input, each product between an even and an odd function retains the odd parity property [12].

When GMDH is used for PA modeling there is only one output at the end of the neural network, which depends on the input applied in the current and past moments. The amount of past instants used is directly influenced by the initial number of inputs ( $E$ ), ranging from 0 to  $(E - 1)$ , as shown in Fig. 2.

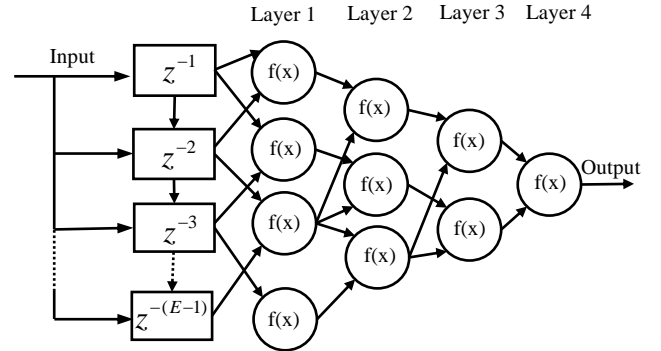


Fig. 2 – Block diagram of Conventional GMDH

In order to always obtain one output, the last layer of the network must converge to just one neuron. In the present work, three layers are used, chosen for simplicity of presentation and clarity of behavior, but the model can be adapted for a much higher number of layers.

The number of neurons in the first layer depends directly on the initial entry of inputs, given by

$$C_{r1} = \frac{a!}{b!(a-b)!}, \quad (3)$$

where  $C_{r1}$  is the resulting number of neurons,  $a$  is the total number of inputs to be applied and  $b$  the number of inputs for each neuron (commonly 2, according to the basic model of Ivakhnenko). As each neuron results in only one output, these become the inputs for the second layer. So, for the second layer the amount of neurons is  $C_{r2} = C_{r1}/b!(C_{r1} - b)!$ , and for the third layer is  $C_{r3} = C_{r2}/b!(C_{r2} - b)!$ , following the logic of using the outputs of layer 2 as inputs for layer 3. This mathematical rule follows for the rest of the neural network.

It is important to realize that for layer 1 there are two

possible scenarios. The first, exemplified in Fig. 3, admits that only three neurons are used in calculating the network, since one is repeated in the direction to the second layer. Fig. 4 shows the second scenario, which admits the use of four different neurons. Therefore, the code works with these two propositions, analyzing the order and combination of all possibilities by repeating the use of one neuron, and then analyzes the order and combination admitting the use of all four neurons. After all the comparisons made respecting this criterion, the smallest error is obtained with the identification of the best neurons used in calculation.

The modeling consists of choosing the best neurons to be kept in each layer: which is the only neuron to be kept in third layer, which two neurons to be used in the second layer and which is the most strategic selection in the first layer.

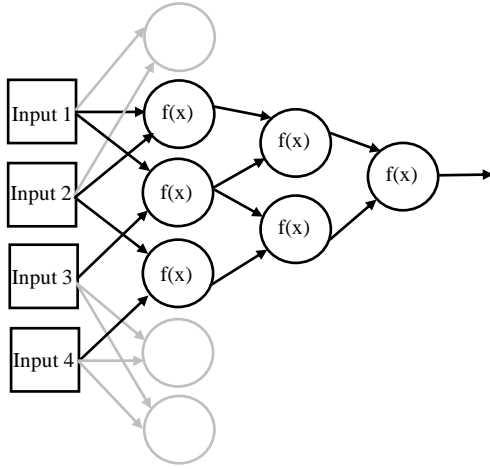


Fig. 3 – First scenario using three neurons in first layer

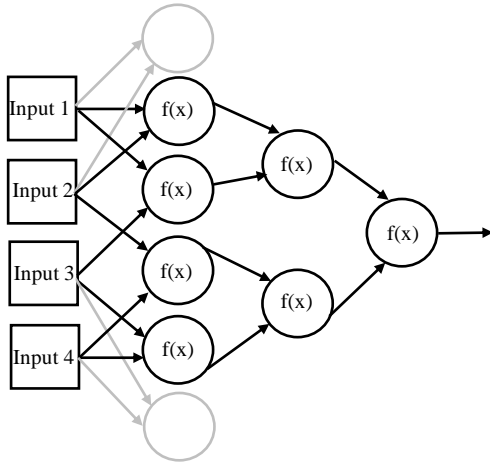


Fig. 4 – Second scenario using four neurons in first layer

#### IV. EXPANDED GMDH MODEL

The set of discrete-time Laguerre basis functions [13] is described by:

$$L_\tau(z, \alpha) = \frac{\sqrt{1-|\alpha|^2}}{1-\alpha z^{-1}} \left\{ \frac{-\alpha + z^{-1}}{1-\alpha z^{-1}} \right\}^\tau; \tau \geq 0, \quad (4)$$

where  $\alpha$  is the real-valued pole of the Laguerre basis functions defined as  $-1 < \alpha < 1$ ,  $\tau$  indicates Laguerre basis functions incorporated in the model and  $z^{-1}$  indicates a delay of one sample in discrete time. If  $\tilde{x}(n)$  and  $\tilde{y}(n)$  represent,

respectively, the input and output complex-valued envelope signals of the Laguerre network of order  $L$ , the linear Laguerre model can be written as:

$$\tilde{y}_1(n) = \sum_{\tau=0}^{L-1} C_\tau \tilde{\delta}_\tau(n), \quad (5)$$

where the order  $L$  is the number of Laguerre basis functions,  $C_\tau$  are the Laguerre regression coefficients and

$$\tilde{\delta}_\tau(n) \doteq L_\tau(z, \alpha) \tilde{x}(n), \quad (6)$$

is the output of the Laguerre model where, depending on the amount of Laguerre basis functions  $L$ , the first state vector is

$$\tilde{\delta}_0(n) \doteq \frac{\sqrt{1-|\alpha|^2}}{1-\alpha z^{-1}} \tilde{x}(n), \quad (7)$$

and the next state vectors of the Laguerre model are defined as

$$\tilde{\delta}_\tau(n) \doteq \frac{-\alpha + z^{-1}}{1-\alpha z^{-1}} \tilde{\delta}_{\tau-1}(n), \tau = 1, 2, \dots, L-1. \quad (8)$$

Since the number of neurons in the first layer of the Conventional GMDH model depends directly on the initial entry of inputs, given by  $C_{r1}$ , which were discussed in Section III, for the Expanded GMDH model the inputs are defined through the outputs of the Laguerre model whose number of the needed Laguerre basis functions is equal to the number of neurons in the first layer of the Conventional GMDH model. Therefore, the state matrix of Laguerre model containing  $L$  vectors applied as input signals in the first layer of the Expanded GMDH model is given by

$$\tilde{\delta}_\tau(n) = [\tilde{\delta}_0(n), \tilde{\delta}_1(n), \dots, \tilde{\delta}_{L-1}(n)]. \quad (9)$$

Fig. 5 shows the block diagram of the Expanded GMDH model.

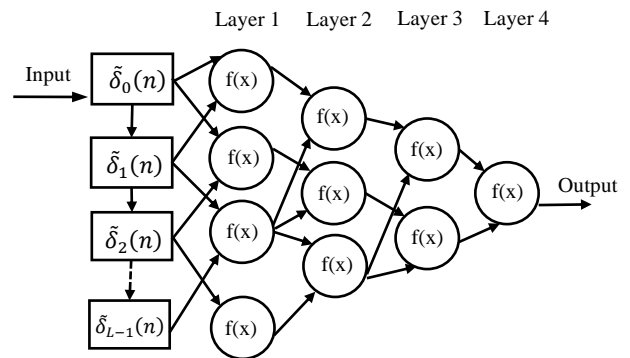


Fig. 5 – Block diagram of the Expanded GMDH

#### V. DESIGN OF THE BEHAVIORAL MODEL OF PA BASED ON THE CONVENTIONAL AND EXPANDED GMDH

The input signal of the Expanded GMDH model can be developed by projecting onto a set of the Laguerre orthonormal basis functions, whose number of Laguerre functions is chosen considering previous knowledge of the GMDH model. In practice, the number of Laguerre basis depends on the complexity and the number of neurons in the first layer of the Conventional GMDH.

The Conventional and Expanded GMDH models are considered non-linear in their parameters, since their adjustable

coefficients are multiplied by one another, reaching powers greater than 1. However, a linear in its parameter model can be obtained if the coefficients of each layer are calculated independently and the pole is known. The work consists of obtaining the coefficients of the first layer expanded by the Laguerre basis functions and, from these, extracting the coefficients of the second layer that are also expanded through the Laguerre basis and, finally, calculating the expanded coefficients of the last layer, analyzing each result in isolation to estimate the next outputs.

Since the expansion of the GMDH model is non-linear in its parameters and in the dynamics of the Laguerre basis functions, the single real-valued pole plays an important role in the quality of the coefficient approximation of the Conventional GMDH model by truncated Laguerre basis functions. The adequate choice of the Laguerre single pole is performed through the optimization of an objective function [14], which depends entirely on the dynamic pole and that results in a single pole close to the fundamental pole of the Expanded GMDH model.

In the Conventional and Expanded GMDH models, the coefficients of each layer are extracted using LS, a method used for linear regression that consists of minimizing the sum of square errors [15]. To define the most strategic neurons (those with the best coefficients), all possible combinations in the neural network of the Conventional GMDH and the Expanded GMDH by Laguerre basis, including its corresponding fundamental pole, are tested. At the end of each layer, the NMSE is calculated, used to access the accuracy of the results, defined by

$$NMSE = 10 \log \frac{\sum_{n=1}^N |e(n)|^2}{\sum_{n=1}^N |yref(n)|^2}, \quad (10)$$

where  $N$  is the total number of samples and  $n$  is the referenced instant. The term  $e(n)$  represents the difference between the desired output (indicated by  $yref(n)$ ) and the estimated output. The best selection of the neural network is the one with the lowest NMSE, in all the methods presented here.

The Conventional and Expanded GMDH designs work with two modeling methods that use Mean Square Error (MSE) on each layer for the extraction of their coefficients, differing in the scope of the selection of neurons.

The first method, here called Embracing, assumes the availability of all neurons in the first layer of the Conventional and Expanded GMDH models, with only one extraction of coefficients at that time. In the same way, it is admitted that all neurons of the second layer in both models are used, and this way the coefficients are also extracted by only a calculation of MSE. As for the third layer of the models it is assumed that there is only one neuron, and the calculation of the MSE is performed according to the number of possibilities of combinations, given by the same value of  $Cr_2$ , that is the number of neurons in the second layer. This happens because two inputs are strictly required in the single neuron of the last layer, so all combinations of 2 to 2 of the available neurons in the second layer of each of the two models are tested, directed to each routine by code construction modeling.

The second method, called Selective, begins to select neurons from the beginning of the code. It is assumed that in the first layer of the Conventional and Expanded GMDH models only the neurons that will actually be used for the second layer are present, which may be in the amount of 3 or 4. When it is admitted that there is a neuron repetition, totaling 3 neurons in the first layer of the Conventional and Expanded models, there are tested  $n_3$  possibilities, where  $n_3$  is given by three times the number of combinations from 3 to 3 of the total neurons of the first layer,  $Cr_1$ . In this calculation, the number of neurons in the first layer of the models is chosen as the domain of the combinations because it is the one that will leave the other selections for the rest of the neural network; the set of 3 to 3 is selected by the direct number of neurons to be used – in this case, one is repeated; and the final multiplication by 3 is performed in order to analyze all possible positions assumed by each neuron, as shown in Fig. 6. More visually, Fig. 6 shows that three neurons from the domain of the first layer are chosen to provide outputs for the rest of the network: A, B and C. In the second layer there are two neurons, and each of them needs two inputs (which will be A, B and C). For each possibility a distinct neuron is repeated. In a first possibility, the B is repeated, and thus there are the pairs A and B, B and C. In a second possibility, the A is repeated, and so there are the pairs A and B, A and C. In a third possibility, the C is repeated, and so there are the pairs A and C, B and C. So, since there are three neurons, there are three possibilities of organization. Thus, the final multiplication refers to the possibilities of organization for the same trio of neurons from layer 1.

When four different neurons are used, the number of tested possibilities is  $n_4$ , where  $n_4$  is given by three times the number of combinations from 4 to 4 of the  $Cr_1$  value, a calculation whose variables are justified in the same way as for when there is repetition, as shown in Fig. 7. Therefore, the total number of extractions is  $n_{total}$ , the result of the sum of possibilities  $n_3$  and  $n_4$ .

Still on the Selective method, the second layer has only two fixed neurons, each receiving two inputs and resulting in only one output. Then, the coefficients are obtained from each neuron only once, totaling two extractions. Finally, in the third layer there is only one neuron, which receives two inputs and results in an output, with only an extraction of coefficients, which are the most significant of the neural network.

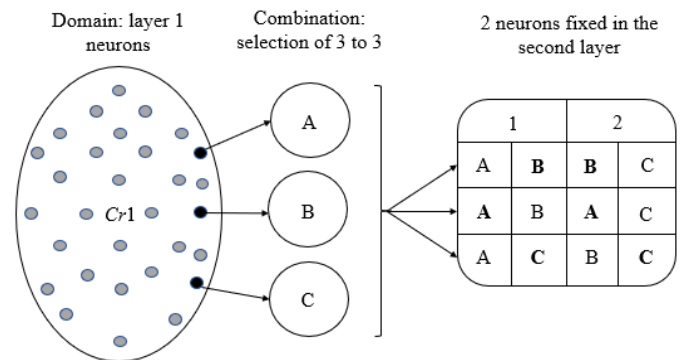


Fig. 6 – Selective method with neuron repetition

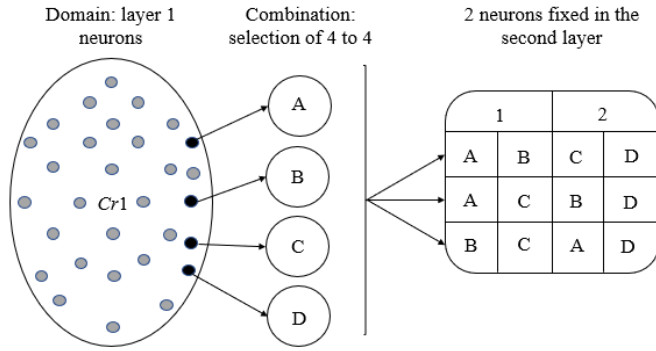


Fig. 7 – Selective method without neuron repetition

## VI. SIMULATION RESULTS

For the tests reported in this section, the MATLAB software was used, in which all codes were built, chosen for its objectivity in terms of modeling [16]. Several commands offered by the program were used, among them the “\” to perform the LS and the “lsqnonlin” to perform the non-linear optimization of the pole value. The samples used are classified as floating-point double precision and were taken ready at the beginning of the work – the main purpose was the data modeling, and it was not worked with their extraction. They were obtained by two PAs. The first PA is a class AB PA that employs HEMT manufactured in GaN technology. A 900 MHz carrier frequency was used, modulated by a Wide-band Code-Division Multiple Access (WCDMA) 3GPP envelope signal with a bandwidth of approximately 3.84 MHz. For the measurement of input and output information, a vector signal analyzer of the Rohde & Schwarz FSQ type was used, with 61.44 MHz as the sampling frequency [17]. The extraction samples are 3,221 in size, while the validation samples are in the order of 2,001. The second PA is a class AB PA that employs a Si LDMOS transistor. A 2 GHz carrier frequency was used, also modulated by a WCDMA 3GPP envelope signal with a bandwidth of approximately 3.84 MHz. For the measurement of input and output information, a vector signal analyzer of the Agilent MXA N9020A type was used, with 30.72 MHz as the sampling frequency [18]. The extraction and validation samples are both 4,500 in size.

The obtained results will be presented first with the GaN HEMT PA in Subsection VI.A, and then with the Si LDMOS PA in Subsection VI.B.

### A. GaN HEMT PA

At this point, the Embracing and Selective methods in the Conventional model, described in Section II, will be compared in training with a HEMT PA.

The number of inputs for the neural network is varied from 3 to 6, and Table I shows the extraction and validation NMSE side by side within each method.

The proximity between the results of extraction and validation proves that there are no problems of noise modeling or ill-conditioning (poor conditioning of the regression matrix), resulting in good results for the codes. However, it is noticeable that the Selective method is outstanding, since it is computationally more complex than the Embracing method. When making MSE calculation a greater number of

times, it results in more accurate results, since it presents the smallest NMSE.

Table I – Comparison between methods in the Conventional model using GaN HEMT PA

Inputs	EMBRACING		SELECTIVE	
	NMSE Ext. (dB)	NMSE Val. (dB)	NMSE Ext. (dB)	NMSE Val. (dB)
3	-31.37	-31.27	-31.37	-31.27
4	-31.08	-31.68	-34.89	-35.39
5	-30.55	-30.84	-36.12	-36.66
6	-27.66	-27.74	-36.53	-37.17

Still looking at Table I, the Selective method is considered the best in terms of cost-benefit, based on its most assertive form of selection and the presentation of the smallest errors. The NMSE, measured on the decibel scale, doubles the error order every 3 units of difference, which reinforces the relevance of this method.

By varying the  $E$  to higher values, the complexity of the calculations and accuracy of results increase, so for a greater number of inputs the number of extractions can vary almost exponentially. The conclusion of the work demands an empirical analysis, considering the advantages of carrying out an additional large number of tests for an improvement of NMSE not so significant. Therefore, considering the performance of the tests and the comparative analysis of results, the best routine performed is the one for 5 inputs applied to the Selective code, as it presents one of the smallest NMSE – with very close extraction and validation – and does not require such complexity to estimate the best coefficients and neurons to be used.

Fig. 8 shows the comparison of the waveforms – estimated by GMDH and desired (experimental measured data) - between the Selective and Embracing methods. As the application of 5 inputs configured the best routine for the Selective method, this routine was used to verify the difference in accuracy between them. About 40 samples were plotted to facilitate visualization, and with that it is remarkable the high accuracy of the designed Conventional GMDH.

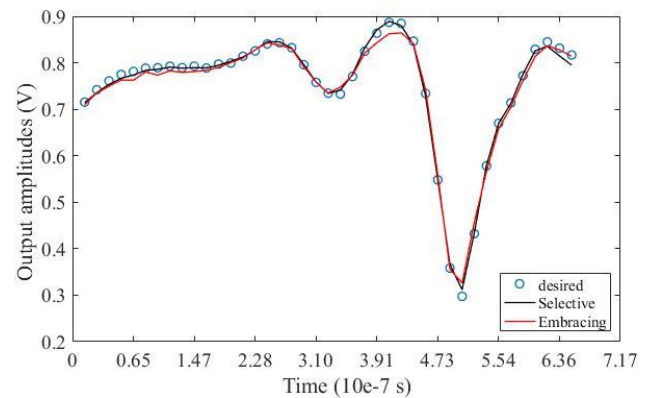


Fig. 8 – Comparison between waveforms for Embracing 5 and Selective 5 with GaN HEMT PA

Fig. 9 presents the waveforms of error amplitudes in dB units of all the inputs applied in the Selective method, illustrating that the models with lower NMSEs have in average lower error amplitudes.



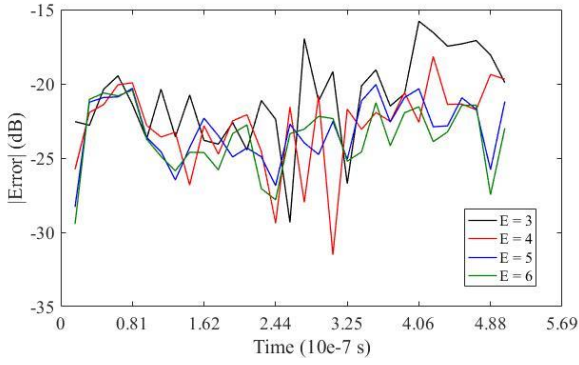


Fig. 9 – Comparison among waveforms of error amplitudes for Selective method using GaN HEMT PA

Then the Embracing and Selective methods are compared, now in the Expanded model, also described in Section IV.

Table II – Comparison between methods in the Expanded model using GaN HEMT PA

Inputs	EMBRACING		SELECTIVE	
	NMSE Ext. (dB)	NMSE Val. (dB)	NMSE Ext. (dB)	NMSE Val. (dB)
3	-33.98	-35.18	-33.98	-35.18
4	-31.79	-31.21	-35.09	-33.98
5	-31.74	-30.43	-35.83	-35.44
6	-27.08	-26.36	-35.61	-33.85

Table II shows a behavior of the Expanded model that is similar to the Conventional model: results close to extraction and validation, which implies good identification performance. Furthermore, again the Selective method excels, since it reaches the smallest error values.

When working with Laguerre, it is noteworthy that the Embracing method becomes more costly in terms of routine training, since it has a greater number of coefficients to be processed and requires more computational effort to present results. When comparing the values obtained in Table II, it is concluded that the application of 5 inputs in the Selective method also provides the best processing, as it presents the smallest error in data extraction. Fig. 10 shows the amplitude transfer characteristic of the Laguerre expansion in the application of 5 inputs in the Selective method, with the Expanded model.

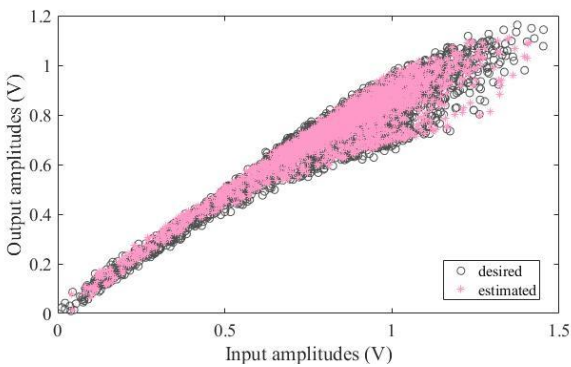


Fig. 10 – Laguerre Expansion using GaN HEMT PA

It is also important to note that for both the Conventional model and the Expanded model, the application of 3 inputs

configures the same extraction and validation NMSE in both methods. This is because, in 3 inputs, the only scenario available to the Selective method is the one with repetition, offering only 3 neurons. This makes the architecture very similar to the Embracing method, which implies equal results.

Finally, Table III shows the comparison between the Conventional and Expanded models.

Table III – Comparison between models using HEMT PA

Routine	CONVENTIONAL MODEL		EXPANDED MODEL	
	NMSE Ext. (dB)	NMSE Val. (dB)	NMSE Ext. (dB)	NMSE Val. (dB)
Embrac. 3	-31.37	-31.27	-33.98	-35.18
Embrac. 4	-31.08	-31.68	-31.79	-31.21
Embrac. 5	-30.55	-30.84	-31.74	-30.43
Embrac. 6	-27.66	-27.74	-27.08	-26.36
Selective 3	-31.37	-31.27	-33.98	-35.18
Selective 4	-34.89	-35.39	-35.09	-33.98
Selective 5	-36.12	-36.66	-35.83	-35.44
Selective 6	-36.53	-37.17	-35.61	-33.85

When analyzing the results of each model, it is seen that the Expanded one has the potential to improve the results, since it makes a more complete scan of the data and thus has a predisposition to extract smaller error values. Laguerre's expansion model searches for a global minimum within its performance, and thus it can be considered that the Conventional model is a particular case of the Expanded model, in which the pole is 0.

However, in some routines - such as Embracing 6, Selective 5 and Selective 6 - the error value when extracting with the Expanded model is no better than the value obtained with the Conventional model. This is because the non-linear extraction method does not guarantee the attainment of the global minimum, being able to stick to local minima depending on initial estimates. In addition, the Laguerre expansion has infinite memory and, as there is no exact knowledge of all the passed information, it is possible that the Laguerre with pole 0 will perform differently from the Conventional one.

#### B. Si LDMOS PA

In the same way, the previously presented comparisons will be made, now with the Si LDMOS PA.

First, the Embracing and Selective methods in Conventional model are compared, as shown in Table IV.

Although both methods have small errors, it is possible to notice that the previous behavior is repeated: the Selective is superior to the Embracing in terms of results, as it presents significantly smaller errors from the fourth routine on. The proximity between validation and extraction values again indicates good identification performance, without modeling generalization problems.

However, a greater number of samples were extracted from this PA, totaling 4,500 extraction positions. Because of this, the computational effort used to obtain results in each routine was higher, which increased the need for a cost-benefit analysis, and thus the best training remains the one that results in the smallest errors with the easiest to obtain.

Table IV – Comparison between methods in the Conventional model using Si LDMOS PA

Inputs	EMBRACING		SELECTIVE	
	NMSE Ext. (dB)	NMSE Val. (dB)	NMSE Ext. (dB)	NMSE Val. (dB)
3	-36.98	-37.47	-36.98	-37.47
4	-24.82	-26.09	-39.42	-39.67
5	-19.04	-19.41	-39.99	-40.12
6	-23.16	-23.04	-40.04	-40.12

Since the application of 5 inputs again presents the best results, this time about forty samples will be plotted using this new dataset, with the results from Embracing 5 and Selective 5, in Fig. 11. It can be noticed, when comparing the waveforms, the superiority and accuracy of the Selective method.

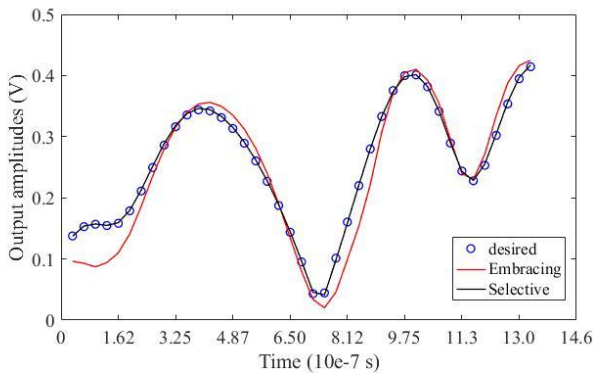


Fig. 11 – Comparison between waveforms for Embracing 5 and Selective 5 with Si LDMOS PA

Then the Embracing and Selective methods are trained in the Expanded model, and the results are shown in Table V.

Table V – Comparison between methods in the Expanded model using Si LDMOS PA

Inputs	EMBRACING		SELECTIVE	
	NMSE Ext. (dB)	NMSE Val. (dB)	NMSE Ext. (dB)	NMSE Val. (dB)
3	-40.40	-40.28	-40.40	-40.28
4	-36.03	-36.69	-39.94	-40.35
5	-34.25	-27.28	-39.95	-40.06
6	-25.33	-19.87	-39.35	-40.28

As expected, the Selective method continues to perform better. Furthermore, errors remain small and extraction and validation results are consistent, which indicates good performance of the Expanded model. Fig. 12 shows the amplitude transfer characteristic of the Laguerre expansion in the application of 4 inputs in the Selective method, with Expanded model.

And finally, Table VI shows the comparison between the Conventional and Expanded models, presenting the result of all the trained routines.

It is important to note that the Expanded model codes were programmed to look, during the scan, for the best extraction results. Thus, comparisons between models are made by first analyzing the NMSE values of the extraction, which do not necessarily imply the lowest validation NMSE. That said, it is concluded, analyzing Table VI, that again the Expanded model has a predisposition to improve the results, with only

two trained routines - Selective 5 and Selective 6 - that do not surpass the Conventional model, and such behavior is justified in the same way as in Subsection VI.A.

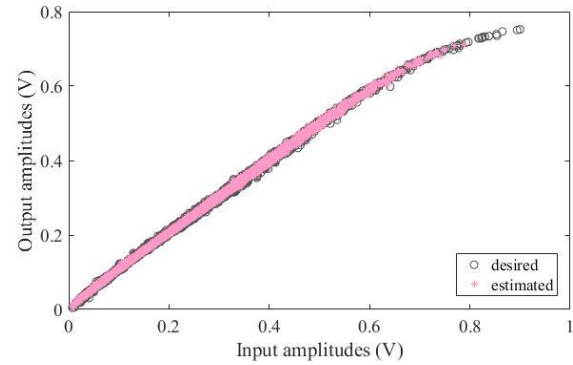


Fig. 12 – Laguerre Expansion using Si LDMOS PA

Table VI – Comparison between models using Si LDMOS PA

Routine	CONVENTIONAL MODEL		EXPANDED MODEL	
	NMSE Ext. (dB)	NMSE Val. (dB)	NMSE Ext. (dB)	NMSE Val. (dB)
Embrac. 3	-36.98	-37.47	-40.40	-40.28
Embrac. 4	-24.82	-26.09	-36.03	-36.69
Embrac. 5	-19.04	-19.41	-34.25	-27.28
Embrac. 6	-23.16	-23.04	-25.33	-19.89
Selective 3	-36.98	-37.47	-40.40	-40.28
Selective 4	-39.42	-39.67	-39.94	-40.35
Selective 5	-39.99	-40.12	-39.95	-40.06
Selective 6	-40.04	-40.12	-39.35	-40.28

It can also be noted that the application of 6 inputs in the Embracing method presents the least accurate results in both models. This can happen because with 6 inputs there is a greater number of possibilities to be tested with the method, in addition to a low neuron selection rigor (compared to the selective one).

## VII. CONCLUSIONS

When realizing the need for linearization of PA output signals, the study of data manipulation methods is also turned to its application in the field of telecommunications. The work presented the results obtained from the construction of two programming codes based on the Conventional and Expanded GMDH models that search to transfer signals through low energy levels. These differ in the way they select the best coefficients to be maintained in the neural network: one is more embracing than the other.

After carrying out several tests, the most selective method stands out, which makes a more careful analysis about the neurons and outputs to be maintained at each layer and, therefore, presents better final results and minor errors. Its logic is applicable to use in PAs, since the solutions resulting from the applied inputs are all concrete.

The Expanded model was built after training both methods in the Conventional one. Knowing the usual behavior of routines, the Expanded model was designed in order to improve the results from a more careful data scan – obtained with the previously defined methods – based on the Laguerre

expansion. At the end of all the tests, it is seen that the Expanded model actually tends to improve the results of the Conventional model, consolidating the accuracy of all the PA signal modeling presented in this work.

In a practical scenario of PA linearization, a digital circuit with adders and multipliers that implements the introduced GMDH model is put in the transmitter chain. The baseband envelope to be transmitted is modified in real-time by such digital circuit, then upconverted to the carrier frequency and finally inputted to the PA. If the GMDH model is previously trained to estimate the PA inverse behavior, such cascade connection between the GMDH model and the PA provides a linear transfer characteristic.

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