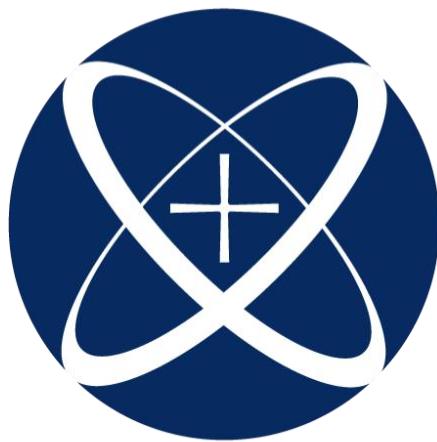


**“PAIRS TRADING WITH COINTEGRATION & KALMAN FILTERS”**

**ITESO**



**ITESO, Universidad  
Jesuita de Guadalajara**

**MICROESTRUCTURA Y SISTEMAS DE TRADING**

**LUIS FELIPE GÓMEZ ESTRADA**

**IVANNA HERRERA IBARRA 744614**

**ANA SOFÍA HINOJOSA BALE 742594**

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# Introduction

In this project, a statistical arbitrage strategy based on pairs trading was developed using 15 years of daily ET and WMB price data, which were selected through correlation analysis, Engle-Granger and Johansen's Cointegration tests. The approach focuses on identifying cointegrated asset pairs that share a long-term equilibrium relationship, allowing the strategy to take advantage of temporary divergences from this equilibrium as trading opportunities. To dynamically model the hedge ratio and generate trading signals, Kalman filters were implemented within Powell's Sequential Decision Analysis (SDA) framework, while the Vector Error Correction Model (VECM) is used to capture short-term deviations around the long-term cointegration relationship and to define the entry and exit signals based on mean reversion dynamics. This framework treats the estimation and trading process as a sequence of prediction, observation, updating, decision, acting, and learning steps, allowing adaptive learning over time. The strategy is evaluated in a realistic backtesting environment, which considers commissions, borrow rate, long and short positions without leverage, rebalances hedge ratios every day, and uses different metrics to measure its performance. The data was divided into train and test sets in order to avoid overfitting and to evaluate the model's performance across different time periods.

## Strategy Description and Rationale

### Overview of pairs trading approach

Pairs trading is a market-neutral statistical arbitrage strategy that seeks to profit from relative price movements between two historically related assets. The idea is that if two assets share a long-term equilibrium relationship, temporary deviations between their prices present potential trading opportunities. When the price spread widens beyond its expected range, the strategy buys the undervalued asset and shorts the overvalued one, expecting the spread to revert to its equilibrium level.

In this project, pairs were selected based on both correlation and cointegration. Correlation ensures that the assets move together in the short term, while cointegration confirms a stable long-term equilibrium between their price levels. Once a cointegrated pair is identified, its spread is modeled and monitored over time. When the spread deviates significantly from the mean, entry and exit signals are generated using the Vector Error Correction Model (VECM) to capture short-term dynamics and mean-reversion tendencies.

### Why cointegration indicates arbitrage opportunity

Cointegration indicates that two or more non-stationary price series share a stable long-term equilibrium relationship, even though each individual price may drift over time. This means that while prices of the two assets can diverge in the short term, their spread remains statistically stationary, fluctuating around a constant mean. In

In the context of pairs trading, this property creates the base for a mean-reversion opportunity. When the spread between the cointegrated assets moves far from its usual level, it shows a short-term price imbalance between the two assets. As prices move back toward equilibrium, the spread is expected to revert to its mean, allowing the strategy to generate profit by simultaneously taking opposite positions in each asset, going long on the undervalued one and short on the overvalued one.

### Justification for Kalman filter use in dynamic hedging

Pairs trading assumes a constant hedge ratio, meaning the relationship between two assets remains fixed over time. However, in real markets, asset relationships are dynamic and can change due to shifts in volatility, liquidity, or market conditions. Using a static hedge ratio can lead to inefficient hedging and reduced strategy performance.

The Kalman filter provides a solution by allowing the hedge ratio to adapt over time. It models the relationship between the two assets using a state-space framework, where the hedge ratio changes over time as new data arrive. It updates its estimate with each new observation, adjusting the hedge ratio to match current market conditions. This adaptive process reduces estimation error and produces smooth, time-varying hedge ratios that make the position more accurate.

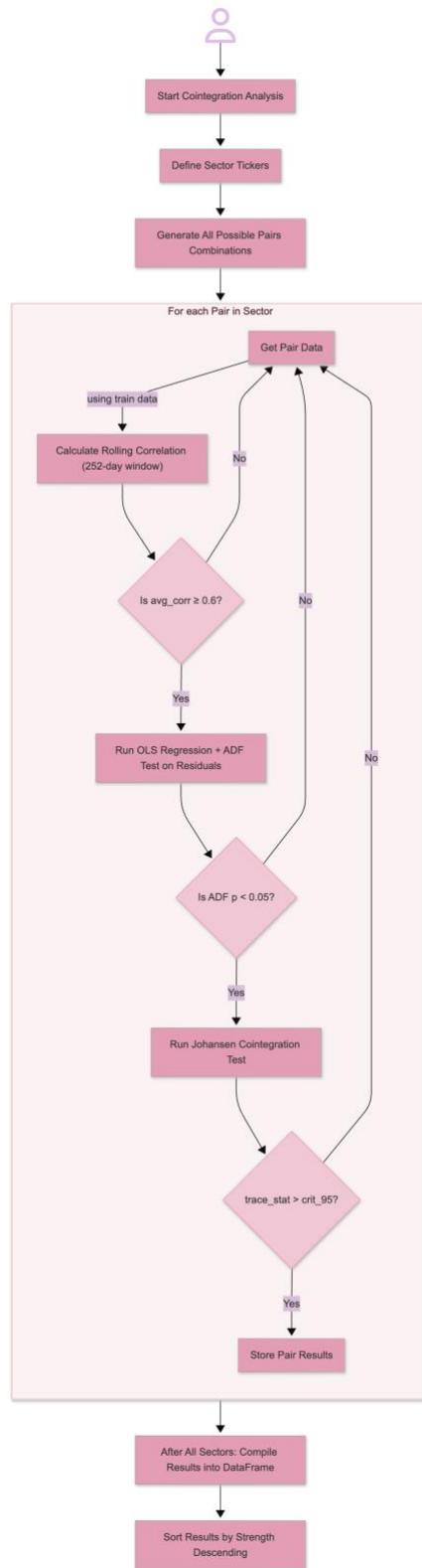
### Expected market conditions for strategy success

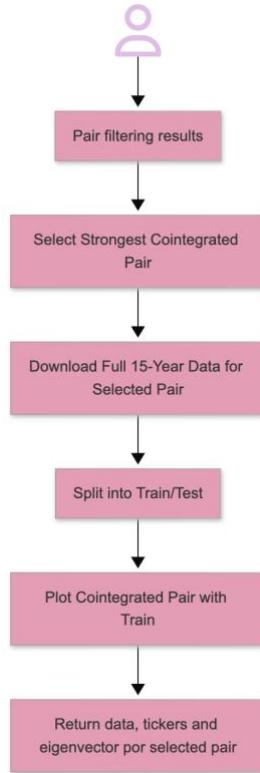
The pairs trading strategy is most effective when markets exhibit moderate volatility, consistent liquidity, and stable correlations between assets, allowing the Kalman filter and VECM to accurately capture and react to short-term deviations around the long-term equilibrium. Under these conditions, temporary divergences between cointegrated assets tend to close as prices revert to their equilibrium relationship, creating profitable trading opportunities. However, the strategy may underperform during strong trending or highly volatile markets, where cointegrated relationships can temporarily break down and price spreads fail to revert.

## Pair Selection Methodology

The pair selection process begins by defining sector tickers and generating all possible pair combinations within each sector. For each pair, training data is gathered to calculate the rolling correlation using a 252-day window, and only those with sufficiently strong relationships are retained. The selected pairs are then tested for cointegration using the Engle-Granger method, and only those that pass this test go through a second confirmation using the Johansen's cointegration method. Finally, pairs that satisfy all statistical criteria are stored, compiled, and ranked by cointegration strength to identify the most robust relationships.

After the strongest cointegrated pair is selected, the 15-year historical data for both assets is downloaded and split into train and test. Finally, the cointegrated pair with train is plotted, and the data, tickers and eigenvector for selected pair is returned.





## Correlation screening criteria and results

To identify potential pairs for the strategy, a correlation screening process was applied to the training data of the selected assets. The purpose was to reduce the list of candidate pairs to those that show strong co-movement in their price behavior over time, serving as an initial indicator of potential long-term relationships.

Same industry sector pairs with a rolling correlation above 0.6 were considered for further testing, as this threshold suggests a consistent relationship between their price movements. This correlation value was set because with higher values almost none of the assets passed to the next test.

| Sector                | Ticker1 | Ticker2 | RollingCorr | Correlation ≥ 0.60 |
|-----------------------|---------|---------|-------------|--------------------|
| Midstream_Energy      | ET      | WMB     | 0.702870886 | TRUE               |
| Midstream_Energy      | TRGP    | OKE     | 0.727497685 | TRUE               |
| Midstream_Energy      | TRGP    | KMI     | 0.424209704 | FALSE              |
| Midstream_Energy      | TRGP    | ET      | 0.71728015  | TRUE               |
| Midstream_Energy      | TRGP    | WMB     | 0.640727222 | TRUE               |
| Midstream_Energy      | OKE     | KMI     | 0.308623497 | FALSE              |
| Midstream_Energy      | OKE     | ET      | 0.562100583 | FALSE              |
| Midstream_Energy      | OKE     | WMB     | 0.61105581  | TRUE               |
| Midstream_Energy      | KMI     | ET      | 0.525066078 | FALSE              |
| Midstream_Energy      | KMI     | WMB     | 0.508538851 | FALSE              |
| Industrial_Automation | ROK     | SIEGY   | 0.680904458 | TRUE               |
| Industrial_Automation | ROK     | ETN     | 0.837448418 | TRUE               |
| Industrial_Automation | ROK     | EMR     | 0.75889808  | TRUE               |
| Industrial_Automation | SIEGY   | ETN     | 0.721145916 | TRUE               |
| Industrial_Automation | SIEGY   | EMR     | 0.688987237 | TRUE               |
| Industrial_Automation | ETN     | EMR     | 0.825689319 | TRUE               |
| Banks                 | JPM     | C       | 0.805666867 | TRUE               |
| Banks                 | JPM     | WFC     | 0.722982654 | TRUE               |
| Banks                 | JPM     | USB     | 0.734753902 | TRUE               |
| Banks                 | JPM     | MS      | 0.825039634 | TRUE               |
| Banks                 | JPM     | GS      | 0.81780026  | TRUE               |
| Banks                 | C       | WFC     | 0.620949253 | TRUE               |
| Banks                 | C       | USB     | 0.60778074  | TRUE               |
| Banks                 | C       | MS      | 0.812849645 | TRUE               |
| Banks                 | C       | GS      | 0.856976239 | TRUE               |
| Banks                 | WFC     | USB     | 0.827957433 | TRUE               |
| Banks                 | WFC     | MS      | 0.694569571 | TRUE               |
| Banks                 | WFC     | GS      | 0.724902733 | TRUE               |
| Banks                 | USB     | MS      | 0.633439911 | TRUE               |
| Banks                 | USB     | GS      | 0.632687991 | TRUE               |
| Banks                 | MS      | GS      | 0.905160049 | TRUE               |
| Consumer_Staples      | PEP     | PG      | 0.672031988 | TRUE               |
| Consumer_Staples      | PEP     | K       | 0.373350879 | FALSE              |
| Consumer_Staples      | PEP     | SJM     | 0.450758167 | FALSE              |
| Consumer_Staples      | PEP     | HSY     | 0.54204985  | FALSE              |
| Consumer_Staples      | PEP     | GIS     | 0.516705648 | FALSE              |
| Consumer_Staples      | PEP     | KO      | 0.685453814 | TRUE               |
| Consumer_Staples      | PG      | K       | 0.233912027 | FALSE              |
| Consumer_Staples      | PG      | SJM     | 0.292889134 | FALSE              |
| Consumer_Staples      | PG      | HSY     | 0.590090827 | FALSE              |
| Consumer_Staples      | PG      | GIS     | 0.416493011 | FALSE              |
| Consumer_Staples      | PG      | KO      | 0.546063171 | FALSE              |
| Consumer_Staples      | K       | SJM     | 0.470224289 | FALSE              |
| Consumer_Staples      | K       | HSY     | 0.174173113 | FALSE              |
| Consumer_Staples      | K       | GIS     | 0.555562598 | FALSE              |
| Consumer_Staples      | K       | KO      | 0.267794072 | FALSE              |
| Consumer_Staples      | SJM     | HSY     | 0.30074554  | FALSE              |
| Consumer_Staples      | SJM     | GIS     | 0.691923781 | TRUE               |
| Consumer_Staples      | SJM     | KO      | 0.218867211 | FALSE              |
| Consumer_Staples      | HSY     | GIS     | 0.415560836 | FALSE              |
| Consumer_Staples      | HSY     | KO      | 0.454940645 | FALSE              |
| Consumer_Staples      | GIS     | KO      | 0.324156198 | FALSE              |
| Healthcare_Equipment  | MDT     | SYK     | 0.642273637 | TRUE               |
| Healthcare_Equipment  | MDT     | BSX     | 0.592443803 | FALSE              |
| Healthcare_Equipment  | MDT     | DHR     | 0.62628035  | TRUE               |
| Healthcare_Equipment  | MDT     | ZBH     | 0.746244934 | TRUE               |
| Healthcare_Equipment  | SYK     | BSX     | 0.662244576 | TRUE               |
| Healthcare_Equipment  | SYK     | DHR     | 0.802609533 | TRUE               |
| Healthcare_Equipment  | SYK     | ZBH     | 0.588182588 | FALSE              |
| Healthcare_Equipment  | BSX     | DHR     | 0.603390977 | TRUE               |
| Healthcare_Equipment  | BSX     | ZBH     | 0.548145262 | FALSE              |
| Healthcare_Equipment  | DHR     | ZBH     | 0.547582362 | FALSE              |

As we can see, 35 out of the 62 tested pairs passed the correlation test, as they have a value greater than 0.6, showing a consistent relationship between their price movements. Therefore, all those pairs move on to the next test, which is the Engle-Granger test.

### Simple cointegration test (Engle-Granger method)

The Engle-Granger method tests whether two non-stationary time series share a long-term equilibrium relationship. Although each series is non-stationary on its own, a linear combination of them may be stationary if they are cointegrated. The method first runs a static regression between the two-price series and then applies an Augmented Dickey–Fuller (ADF) test to the residuals to check for unit roots. If the time series is cointegrated, these residuals will be stationary, meaning that although they may drift apart in the short term, they move together in the long run. The test has two hypotheses, null and alternative. The null hypothesis is that no cointegration exists and the alternative hypothesis is that cointegration exists. When the test statistic is less than 0.05, the null hypothesis is rejected, confirming the presence of cointegration between the two series.

| Sector                | Ticker1 | Ticker2 | ADF_pvalue  | Stationary Regression Residuals |
|-----------------------|---------|---------|-------------|---------------------------------|
| Midstream_Energy      | ET      | WMB     | 0.015194486 | TRUE                            |
| Midstream_Energy      | TRGP    | OKE     | 0.624290396 | FALSE                           |
| Midstream_Energy      | TRGP    | KMI     | 0.357092475 | FALSE                           |
| Midstream_Energy      | TRGP    | ET      | 0.357711675 | FALSE                           |
| Midstream_Energy      | TRGP    | WMB     | 0.072157364 | FALSE                           |
| Midstream_Energy      | OKE     | KMI     | 0.757553065 | FALSE                           |
| Midstream_Energy      | OKE     | ET      | 0.964652198 | FALSE                           |
| Midstream_Energy      | OKE     | WMB     | 0.979099859 | FALSE                           |
| Midstream_Energy      | KMI     | ET      | 0.598102429 | FALSE                           |
| Midstream_Energy      | KMI     | WMB     | 0.397222398 | FALSE                           |
| Industrial_Automation | ROK     | SIEGY   | 0.083521356 | FALSE                           |
| Industrial_Automation | ROK     | ETN     | 0.108865825 | FALSE                           |
| Industrial_Automation | ROK     | EMR     | 0.094827094 | FALSE                           |
| Industrial_Automation | SIEGY   | ETN     | 0.056470332 | FALSE                           |
| Industrial_Automation | SIEGY   | EMR     | 0.049738498 | TRUE                            |
| Industrial_Automation | ETN     | EMR     | 0.005322981 | TRUE                            |
| Banks                 | JPM     | C       | 0.163460462 | FALSE                           |
| Banks                 | JPM     | WFC     | 0.959421402 | FALSE                           |
| Banks                 | JPM     | USB     | 0.514087473 | FALSE                           |
| Banks                 | JPM     | MS      | 0.774351365 | FALSE                           |
| Banks                 | JPM     | GS      | 0.879482569 | FALSE                           |
| Banks                 | C       | WFC     | 0.424539685 | FALSE                           |
| Banks                 | C       | USB     | 0.080754587 | FALSE                           |
| Banks                 | C       | MS      | 0.023239007 | TRUE                            |
| Banks                 | C       | GS      | 0.285582652 | FALSE                           |
| Banks                 | WFC     | USB     | 0.514123584 | FALSE                           |
| Banks                 | WFC     | MS      | 0.106950114 | FALSE                           |
| Banks                 | WFC     | GS      | 0.01779077  | TRUE                            |
| Banks                 | USB     | MS      | 0.094475135 | FALSE                           |
| Banks                 | USB     | GS      | 0.124090385 | FALSE                           |
| Banks                 | MS      | GS      | 0.028783457 | TRUE                            |
| Consumer_Staples      | PEP     | PG      | 0.320355073 | FALSE                           |
| Consumer_Staples      | PEP     | K       | 0.798918052 | FALSE                           |
| Consumer_Staples      | PEP     | SJM     | 0.877534656 | FALSE                           |
| Consumer_Staples      | PEP     | HSY     | 0.087222482 | FALSE                           |
| Consumer_Staples      | PEP     | GIS     | 0.812901296 | FALSE                           |
| Consumer_Staples      | PEP     | KO      | 0.001779795 | TRUE                            |
| Consumer_Staples      | PG      | K       | 0.987569624 | FALSE                           |
| Consumer_Staples      | PG      | SJM     | 0.984418135 | FALSE                           |
| Consumer_Staples      | PG      | HSY     | 0.000932518 | TRUE                            |
| Consumer_Staples      | PG      | GIS     | 0.978322908 | FALSE                           |
| Consumer_Staples      | PG      | KO      | 0.050510434 | FALSE                           |
| Consumer_Staples      | K       | SJM     | 0.003371575 | TRUE                            |
| Consumer_Staples      | K       | HSY     | 0.380256582 | FALSE                           |
| Consumer_Staples      | K       | GIS     | 0.066916757 | FALSE                           |
| Consumer_Staples      | K       | KO      | 0.295489584 | FALSE                           |
| Consumer_Staples      | SJM     | HSY     | 0.589542913 | FALSE                           |
| Consumer_Staples      | SJM     | GIS     | 0.006366522 | TRUE                            |
| Consumer_Staples      | SJM     | KO      | 0.321543992 | FALSE                           |
| Consumer_Staples      | HSY     | GIS     | 0.85417227  | FALSE                           |
| Consumer_Staples      | HSY     | KO      | 0.025883267 | TRUE                            |
| Consumer_Staples      | GIS     | KO      | 0.440704119 | FALSE                           |
| Healthcare_Equipment  | MDT     | SYK     | 0.291348018 | FALSE                           |
| Healthcare_Equipment  | MDT     | BSX     | 0.128114919 | FALSE                           |
| Healthcare_Equipment  | MDT     | DHR     | 0.326635247 | FALSE                           |
| Healthcare_Equipment  | MDT     | ZBH     | 0.000608953 | TRUE                            |
| Healthcare_Equipment  | SYK     | BSX     | 0.00358524  | TRUE                            |
| Healthcare_Equipment  | SYK     | DHR     | 0.005017848 | TRUE                            |
| Healthcare_Equipment  | SYK     | ZBH     | 0.498155827 | FALSE                           |
| Healthcare_Equipment  | BSX     | DHR     | 0.046528984 | TRUE                            |
| Healthcare_Equipment  | BSX     | ZBH     | 0.253159392 | FALSE                           |
| Healthcare_Equipment  | DHR     | ZBH     | 0.48855573  | FALSE                           |

As we can see, 15 out of the 62 tested pairs passed the Engle-Granger test, as they have a p-value smaller than 0.05, rejecting the null hypothesis and confirming the presence of cointegration between the two series. However, this is considering all the pairs, not only those that passed the previous test. Therefore, only the pairs that passed both the correlation test and this test, move on to the next and last test, which is the Johansen's cointegration test.

## Johansen's Cointegration method

The Johansen's cointegration method is used to test cointegrating relationships between several non-stationary time series data. Compared to the Engle-Granger test, the Johansen test allows for more than one cointegrating relationship. This test provides two statistics, the trace statistic and the maximum eigenvalue statistic, to evaluate the presence and number of cointegrating vectors.

The trace test evaluates the number of linear combinations in a time series, where the null hypothesis states that the number of cointegrating relationships ( $K$ ) is equal to a specific value ( $K_0$ ), while the alternative hypothesis suggests that there are more than  $K_0$  cointegrating relationships. The test usually begins with  $K_0 = 0$  to check whether at least one cointegrating relationship exists. If the null hypothesis is rejected, it indicates that there exists a cointegration relationship in the sample. It is expressed as follows:

$$\begin{aligned} H_0: K &= K_0 \\ H_1: K &> K_0 \end{aligned}$$

The maximum eigenvalue test tests whether the number of cointegrating relationships is exactly  $K$  versus  $K_0 + 1$ , focusing on the next possible cointegrating vector. The test usually begins with  $K_0 = 0$  to check whether at least one cointegrating relationship exists. If the null hypothesis is rejected, it indicates that there exists a cointegration relationship in the sample. The hypotheses are as follows:

$$\begin{aligned} H_0: K &= K_0 \\ H_1: K &> K_0 + 1 \end{aligned}$$

| Sector                | Ticker1 | Ticker2 | TraceStat   | Crit_95 | Cointegrated by Johansen's Test | Strength    |
|-----------------------|---------|---------|-------------|---------|---------------------------------|-------------|
| Midstream_Energy      | ET      | WMB     | 17.69307446 | 15.4943 | TRUE                            | 1.141908603 |
| Midstream_Energy      | TRGP    | OKE     | 6.640324863 | 15.4943 | FALSE                           | 0.428565657 |
| Midstream_Energy      | TRGP    | KMI     | 6.350940055 | 15.4943 | FALSE                           | 0.409888801 |
| Midstream_Energy      | TRGP    | ET      | 20.52401584 | 15.4943 | TRUE                            | 1.324617172 |
| Midstream_Energy      | TRGP    | WMB     | 30.27698655 | 15.4943 | TRUE                            | 1.954072565 |
| Midstream_Energy      | OKE     | KMI     | 10.67816078 | 15.4943 | FALSE                           | 0.689167034 |
| Midstream_Energy      | OKE     | ET      | 9.541034945 | 15.4943 | FALSE                           | 0.615777089 |
| Midstream_Energy      | OKE     | WMB     | 12.62549163 | 15.4943 | FALSE                           | 0.814847501 |
| Midstream_Energy      | KMI     | ET      | 7.870858334 | 15.4943 | FALSE                           | 0.507984119 |
| Midstream_Energy      | KMI     | WMB     | 11.77387352 | 15.4943 | FALSE                           | 0.759884185 |
| Industrial_Automation | ROK     | SIEGY   | 9.873198055 | 15.4943 | FALSE                           | 0.63721485  |
| Industrial_Automation | ROK     | ETN     | 7.385459933 | 15.4943 | FALSE                           | 0.476656573 |
| Industrial_Automation | ROK     | EMR     | 9.158105441 | 15.4943 | FALSE                           | 0.591062871 |
| Industrial_Automation | SIEGY   | ETN     | 8.067492239 | 15.4943 | FALSE                           | 0.520674844 |
| Industrial_Automation | SIEGY   | EMR     | 9.065079792 | 15.4943 | FALSE                           | 0.585059008 |
| Industrial_Automation | ETN     | EMR     | 13.65498487 | 15.4943 | FALSE                           | 0.881290853 |
| Banks                 | JPM     | C       | 10.03834065 | 15.4943 | FALSE                           | 0.647873131 |
| Banks                 | JPM     | WFC     | 7.902103314 | 15.4943 | FALSE                           | 0.510000666 |
| Banks                 | JPM     | USB     | 8.097317154 | 15.4943 | FALSE                           | 0.52259974  |
| Banks                 | JPM     | MS      | 5.200886247 | 15.4943 | FALSE                           | 0.335664486 |
| Banks                 | JPM     | GS      | 6.573609719 | 15.4943 | FALSE                           | 0.424259871 |
| Banks                 | C       | WFC     | 6.518134094 | 15.4943 | FALSE                           | 0.420679482 |
| Banks                 | C       | USB     | 8.409212278 | 15.4943 | FALSE                           | 0.542729409 |
| Banks                 | C       | MS      | 10.84041797 | 15.4943 | FALSE                           | 0.699639091 |
| Banks                 | C       | GS      | 6.359856677 | 15.4943 | FALSE                           | 0.410464279 |
| Banks                 | WFC     | USB     | 5.142106981 | 15.4943 | FALSE                           | 0.33187088  |
| Banks                 | WFC     | MS      | 8.047585988 | 15.4943 | FALSE                           | 0.519390098 |
| Banks                 | WFC     | GS      | 12.52730098 | 15.4943 | FALSE                           | 0.80851029  |
| Banks                 | USB     | MS      | 7.954831223 | 15.4943 | FALSE                           | 0.513403718 |
| Banks                 | USB     | GS      | 8.137474227 | 15.4943 | FALSE                           | 0.525191472 |
| Banks                 | MS      | GS      | 11.06105346 | 15.4943 | FALSE                           | 0.713878876 |
| Consumer_Staples      | PEP     | PG      | 4.976091715 | 15.4943 | FALSE                           | 0.321156278 |
| Consumer_Staples      | PEP     | K       | 5.883680981 | 15.4943 | FALSE                           | 0.379731965 |
| Consumer_Staples      | PEP     | SJM     | 5.889650869 | 15.4943 | FALSE                           | 0.38011726  |
| Consumer_Staples      | PEP     | HSY     | 7.978985152 | 15.4943 | FALSE                           | 0.514962609 |
| Consumer_Staples      | PEP     | GIS     | 4.569935478 | 15.4943 | FALSE                           | 0.29494301  |
| Consumer_Staples      | PEP     | KO      | 17.15363651 | 15.4943 | TRUE                            | 1.107093351 |
| Consumer_Staples      | PG      | K       | 5.497139779 | 15.4943 | FALSE                           | 0.354784648 |
| Consumer_Staples      | PG      | SJM     | 5.42784289  | 15.4943 | FALSE                           | 0.350312237 |
| Consumer_Staples      | PG      | HSY     | 19.05860841 | 15.4943 | TRUE                            | 1.230039977 |
| Consumer_Staples      | PG      | GIS     | 5.041606575 | 15.4943 | FALSE                           | 0.325384598 |
| Consumer_Staples      | PG      | KO      | 11.28068416 | 15.4943 | FALSE                           | 0.728053811 |
| Consumer_Staples      | K       | SJM     | 17.63948373 | 15.4943 | TRUE                            | 1.138449864 |
| Consumer_Staples      | K       | HSY     | 5.324159915 | 15.4943 | FALSE                           | 0.343620552 |
| Consumer_Staples      | K       | GIS     | 12.4090807  | 15.4943 | FALSE                           | 0.800880369 |
| Consumer_Staples      | K       | KO      | 5.567391162 | 15.4943 | FALSE                           | 0.359318663 |
| Consumer_Staples      | SJM     | HSY     | 4.543381401 | 15.4943 | FALSE                           | 0.293229213 |
| Consumer_Staples      | SJM     | GIS     | 17.04105713 | 15.4943 | TRUE                            | 1.099827493 |
| Consumer_Staples      | SJM     | KO      | 5.230185325 | 15.4943 | FALSE                           | 0.337555445 |
| Consumer_Staples      | HSY     | GIS     | 3.947528632 | 15.4943 | FALSE                           | 0.254772957 |
| Consumer_Staples      | HSY     | KO      | 11.46654024 | 15.4943 | FALSE                           | 0.740048937 |
| Consumer_Staples      | GIS     | KO      | 4.170606303 | 15.4943 | FALSE                           | 0.26917036  |
| Healthcare_Equipment  | MDT     | SYK     | 4.583347893 | 15.4943 | FALSE                           | 0.295808645 |
| Healthcare_Equipment  | MDT     | BSX     | 6.718674869 | 15.4943 | FALSE                           | 0.433622356 |
| Healthcare_Equipment  | MDT     | DHR     | 4.514863848 | 15.4943 | FALSE                           | 0.291388694 |
| Healthcare_Equipment  | MDT     | ZBH     | 14.9882139  | 15.4943 | FALSE                           | 0.967337272 |
| Healthcare_Equipment  | SYK     | BSX     | 16.40166821 | 15.4943 | TRUE                            | 1.05856142  |
| Healthcare_Equipment  | SYK     | DHR     | 14.01628845 | 15.4943 | FALSE                           | 0.904609337 |
| Healthcare_Equipment  | SYK     | ZBH     | 5.850197653 | 15.4943 | FALSE                           | 0.377570955 |
| Healthcare_Equipment  | BSX     | DHR     | 10.7835936  | 15.4943 | FALSE                           | 0.695971654 |
| Healthcare_Equipment  | BSX     | ZBH     | 6.781460579 | 15.4943 | FALSE                           | 0.437674537 |
| Healthcare_Equipment  | DHR     | ZBH     | 5.675840844 | 15.4943 | FALSE                           | 0.366317991 |

As we can see, only 8 out of the 62 tested pairs passed the Johansen's cointegration test, as their trace stat is greater than the critical value. However, this is considering all the pairs, not only those that passed both previous tests.

## Statistical evidence for selected pairs for both Engle-Granger and Johansen

The table below presents only the pairs that passed all three tests, which were 4 out of the 62 tested pairs.

| Sector                                  | Midstream_Energy         | Consumer_Staples         | Consumer_Staples        | Healthcare_Equipment     |
|---|--------------------------|--------------------------|-------------------------|--------------------------|
| <b>Ticker1</b>                          | ET                       | PEP                      | SJM                     | SYK                      |
| <b>Ticker2</b>                          | WMB                      | KO                       | GIS                     | BSX                      |
| <b>RollingCorr</b>                      | 0.702870886              | 0.685453814              | 0.691923781             | 0.662244576              |
| <b>Correlation ≥ 0.60</b>               | TRUE                     | TRUE                     | TRUE                    | TRUE                     |
| <b>ADF_pvalue</b>                       | 0.015194486              | 0.001779795              | 0.006366522             | 0.00358524               |
| <b>Stationary Regression Residuals</b>  | TRUE                     | TRUE                     | TRUE                    | TRUE                     |
| <b>TraceStat</b>                        | 17.69307446              | 17.15363651              | 17.04105713             | 16.40166821              |
| <b>Crit_95</b>                          | 15.4943                  | 15.4943                  | 15.4943                 | 15.4943                  |
| <b>Cointegrated by Johansen's Test</b>  | TRUE                     | TRUE                     | TRUE                    | TRUE                     |
| <b>Strength</b>                         | 1.141908603              | 1.107093351              | 1.099827493             | 1.05856142               |
| <b>Eigenvector</b>                      | [0.73910823 -0.45799696] | [0.22018441 -0.76320763] | [0.1770437 -0.39047545] | [0.13082706 -0.54697572] |
| <b>ADF_pvalue_1</b>                     | 0.319323839              | 0.956119626              | 0.277630504             | 0.98865308               |
| <b>ADF_pvalue_2</b>                     | 0.215825443              | 0.845247879              | 0.485234431             | 0.985272541              |
| <b>Non Stationary Individual Prices</b> | TRUE                     | TRUE                     | TRUE                    | TRUE                     |
| <b>Passed 3 Tests</b>                   | TRUE                     | TRUE                     | TRUE                    | TRUE                     |

In addition to identifying which pairs passed the three tests, we also analyzed which one had the greatest strength in order to select the most cointegrated pair.

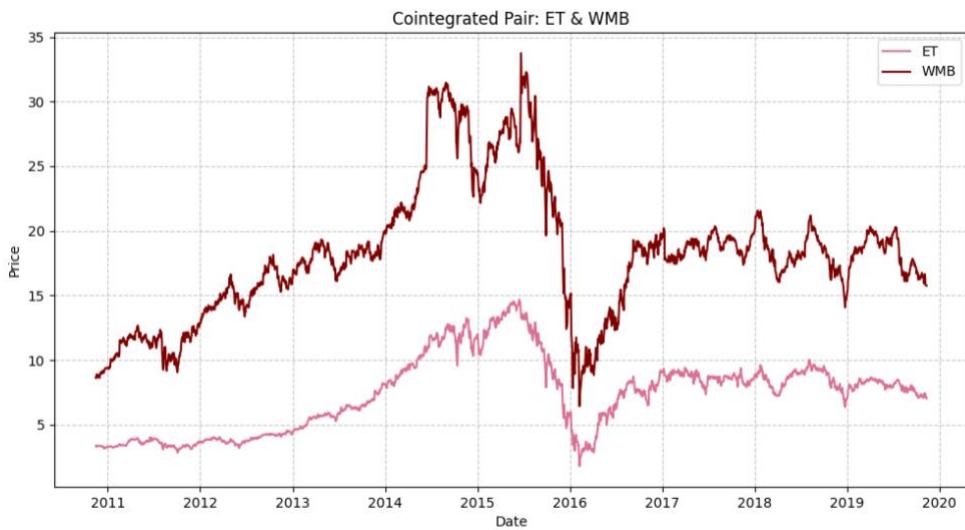
| Sector                                  | Midstream_Energy         |
|---|--------------------------|
| <b>Ticker1</b>                          | ET                       |
| <b>Ticker2</b>                          | WMB                      |
| <b>RollingCorr</b>                      | 0.702870886              |
| <b>Correlation ≥ 0.60</b>               | TRUE                     |
| <b>ADF_pvalue</b>                       | 0.015194486              |
| <b>Stationary Regression Residuals</b>  | TRUE                     |
| <b>TraceStat</b>                        | 17.69307446              |
| <b>Crit_95</b>                          | 15.4943                  |
| <b>Cointegrated by Johansen's Test</b>  | TRUE                     |
| <b>Strength</b>                         | 1.141908603              |
| <b>Eigenvector</b>                      | [0.73910823 -0.45799696] |
| <b>ADF_pvalue_1</b>                     | 0.319323839              |
| <b>ADF_pvalue_2</b>                     | 0.215825443              |
| <b>Non Stationary Individual Prices</b> | TRUE                     |

The selected pair was ET and WMB, both from the midstream energy sector. As we can see, they have a correlation of 0.70, an ADF p-value of 0.015, and a trace statistic higher than the critical value, which confirms that this pair passes all three tests performed. In addition to passing the tests, we observed that it had the highest

strength value, which was 1.14. We also verified that both individual price series are non-stationary, a necessary condition for valid cointegration testing, ensuring that the stationary behavior arises only through their long-term relationship and not from the properties of each series alone.

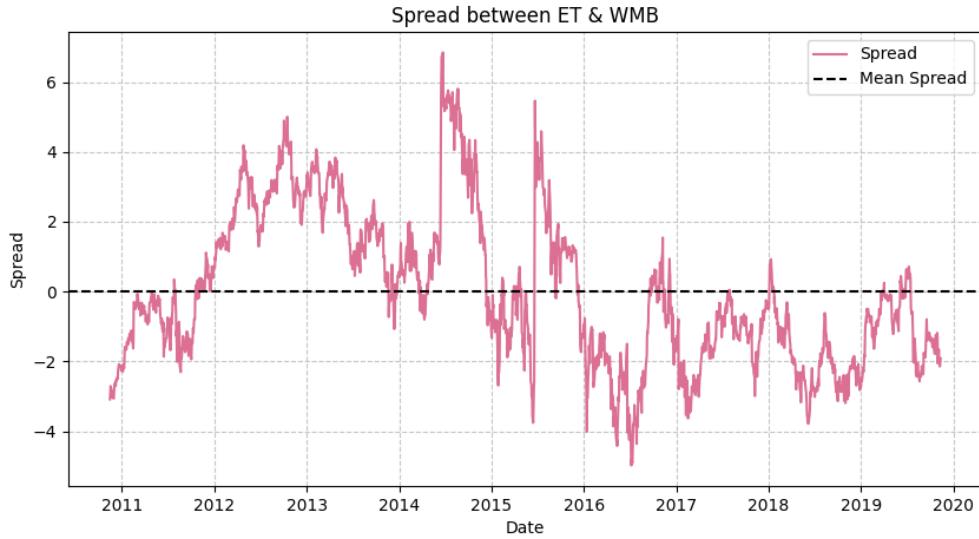
Energy Transfer (ET) and Williams Companies (WMB) form an ideal pair for pairs trading because they are two of the largest and most structurally similar midstream natural gas infrastructure companies in the U.S., operating extensive pipeline networks, gathering and processing facilities, and long-term fee-based contracts. Their business models and revenue streams are highly aligned, making them respond similarly to the same fundamental drivers, such as natural gas production volumes, pipeline demand, and regulated tariff frameworks. Both companies exhibit stable, predictable cash flows and mature operations, which results in parallel equity price movements and a statistically significant long-term relationship, or cointegration. At the same time, subtle differences in capital structure, leverage, liquids exposure, and investor base create temporary divergences in pricing, which allow for mean-reversion opportunities critical to pairs trading strategies. Their deep liquidity, tight bid-ask spreads, and heavy institutional ownership further support reliable execution and risk management. Historical data confirm that their prices move in tandem over time, yet short-term deviations consistently revert to the long-term equilibrium, providing a robust and repeatable framework for capturing relative-value opportunities in the midstream energy sector.

## Price relationships and spread evolution



As we can see, there is a strong relationship between the prices of ET and WMB, as both companies belong to the midstream energy sector and tend to move together during important fluctuations in the market. From 2011 to 2015, both stocks followed an upward trend, but WMB experienced a more pronounced increase, creating a temporary divergence in the pair. This gap closed significantly in 2016, when both prices dropped sharply and returned to their long-term equilibrium. After 2017, the

two stocks stabilized and showed smaller but frequent co-movements, supporting the idea of a mean-reverting spread suitable for pairs-trading strategies.



The spread between ET and WMB shows clear mean-reverting behavior from 2010 to about 2016, with large swings that consistently return toward the average, suggesting a strong and tradable relationship during that period. However, around 2016, the spread's behavior changes noticeably, as volatility drops, the spread spends more time below the mean, and the oscillations become narrower and noisier. This shift indicates a possible structural break in the relationship between the two stocks, possibly due to fundamental changes in the energy sector or company-specific events. These observations highlight the importance of using adaptive models such as the Kalman Filter and rolling VECM estimation, which can adjust to evolving market conditions and mitigate the impact of structural changes in the relationship.

## Correlation and cointegration statistics for all tested pairs

| Sector                | Ticker1 | Ticker2 | RollingCorr  | Correlation > 0.60 | ADF_pvalue   | Stationary Regression Residuals | TmaxStat     | Crit_95 | Cointegrated by Johansen's Test | Strength     | Eigenvector  | ADF_pvalue_1 | ADF_pvalue_2 | Non_Stationary | Individual Prices | Passed 3 Tests |
|-----------------------|---------|---------|--------------|--------------------|--------------|---------------------------------|--------------|---------|---------------------------------|--------------|--------------|--------------|--------------|----------------|-------------------|----------------|
| Midstream_Energy      | ET      | WMB     | 0.702870886  | TRUE               | 0.015194486  | TRUE                            | 17.9930746   | 15.4943 | TRUE                            | 1.131908603  | [0.3910823   | 0.3193238896 | 0.215825443  | TRUE           | TRUE              |                |
| Midstream_Energy      | TRGP    | OKX     | 0.727497685  | TRUE               | 0.624290396  | FALSE                           | 6.6402324863 | 15.4943 | FALSE                           | 0.42856557   | [0.6792041   | -0.12345157  | 0.31721791   | 0.843537961    | TRUE              |                |
| Midstream_Energy      | TRGP    | KMI     | 0.424209704  | FALSE              | 0.357092475  | TRUE                            | 6.350940055  | 15.4943 | FALSE                           | 0.049888801  | [0.8824747   | -0.14773678  | 0.349581029  | 0.543071387    | TRUE              |                |
| Midstream_Energy      | TRGP    | ET      | 0.71728015   | TRUE               | 0.355711675  | FALSE                           | 20.52401584  | 15.4943 | TRUE                            | 1.324617172  | [0.9608236   | -0.15153717  | 0.317321745  | 0.328382416    | TRUE              |                |
| Midstream_Energy      | TRGP    | WMB     | 0.640272222  | TRUE               | 0.072157364  | FALSE                           | 30.27698655  | 15.4943 | TRUE                            | 1.9540723565 | [0.13090922  | -0.44433867  | 0.317321866  | 0.219476091    | TRUE              |                |
| Midstream_Energy      | OKX     | KMI     | 0.3086223497 | FALSE              | 0.75753065   | FALSE                           | 10.67816078  | 15.4943 | FALSE                           | 0.689167034  | [0.4875507   | 0.24966842   | 0.901369165  | 0.543071298    | TRUE              |                |
| Midstream_Energy      | OKX     | ET      | 0.562100583  | FALSE              | 0.964652198  | FALSE                           | 9.541034945  | 15.4943 | FALSE                           | 0.615777089  | [0.399888    | 0.39426061   | 0.85042358   | 0.31932363     | TRUE              |                |
| Midstream_Energy      | OKX     | WMB     | 0.61105581   | TRUE               | 0.970998959  | FALSE                           | 12.62549163  | 15.4943 | FALSE                           | 0.814847501  | [0.1962293   | -0.20270343  | 0.850423693  | 0.215282542    | TRUE              |                |
| Midstream_Energy      | KMI     | ET      | 0.525066078  | FALSE              | 0.598102429  | TRUE                            | 7.87085334   | 15.4943 | FALSE                           | 0.507984119  | [0.00519607  | 0.34500993   | 0.543070996  | 0.31996664     | TRUE              |                |
| Midstream_Energy      | KMI     | WMB     | 0.5088538851 | FALSE              | 0.397222398  | FALSE                           | 11.77387352  | 15.4943 | FALSE                           | 0.759848185  | [0.16845395  | 0.23346627   | 0.789239286  | 0.397694528    | TRUE              |                |
| Industrial_Automation | ROK     | SIEGY   | 0.680904458  | TRUE               | 0.083521356  | FALSE                           | 9.87319805   | 15.4943 | FALSE                           | 0.63721485   | [0.0496237   | 0.28959968   | 0.789239007  | 0.653737611    | TRUE              |                |
| Industrial_Automation | ROK     | EMR     | 0.837448418  | TRUE               | 0.108865825  | FALSE                           | 7.385459933  | 15.4943 | FALSE                           | 0.47661251   | [0.08619156  | 0.23346627   | 0.789239007  | 0.653737611    | TRUE              |                |
| Industrial_Automation | ROK     | EMR     | 0.75889800   | TRUE               | 0.094827094  | FALSE                           | 9.158105441  | 15.4943 | FALSE                           | 0.591062871  | [0.06441463  | 0.27774819   | 0.789239007  | 0.653737611    | TRUE              |                |
| Industrial_Automation | SIEGY   | ETN     | 0.721145916  | TRUE               | 0.056470732  | FALSE                           | 8.067492239  | 15.4943 | FALSE                           | 0.502674844  | [0.12247125  | 0.27774819   | 0.789239007  | 0.653737611    | TRUE              |                |
| Industrial_Automation | SIEGY   | EMR     | 0.688987237  | TRUE               | 0.0497398498 | TRUE                            | 9.065079792  | 15.4943 | FALSE                           | 0.58509508   | [0.26125685  | 0.23752341   | 0.397694040  | 0.67056754     | TRUE              |                |
| Industrial_Automation | ETN     | EMR     | 0.825689319  | TRUE               | 0.050322981  | TRUE                            | 13.65498487  | 15.4943 | FALSE                           | 0.881790853  | [0.08614369  | 0.18734662   | 0.865373789  | 0.67057781     | TRUE              |                |
| Banks                 | JPM     | C       | 0.805668687  | TRUE               | 0.183460462  | FALSE                           | 10.03844065  | 15.4943 | FALSE                           | 0.647973131  | [0.08567503  | 0.24411664   | 0.989603628  | 0.813362404    | TRUE              |                |
| Banks                 | JPM     | WFC     | 0.72292654   | TRUE               | 0.959421042  | FALSE                           | 7.90210334   | 15.4943 | FALSE                           | 0.510000666  | [0.0102067   | 0.20673372   | 0.989603628  | 0.538092989    | TRUE              |                |
| Banks                 | JPM     | USB     | 0.734735920  | TRUE               | 0.514087473  | FALSE                           | 8.097317154  | 15.4943 | FALSE                           | 0.52259974   | [0.0725234   | 0.29977916   | 0.989603640  | 0.844727849    | TRUE              |                |
| Banks                 | JPM     | MS      | 0.8250919634 | TRUE               | 0.774351365  | FALSE                           | 5.200886247  | 15.4943 | FALSE                           | 0.335664846  | [0.39474355  | 0.15289053   | 0.989602899  | 0.67056754     | TRUE              |                |
| Banks                 | JPM     | GS      | 0.81780026   | TRUE               | 0.879482569  | TRUE                            | 5.57809719   | 15.4943 | FALSE                           | 0.424259871  | [0.08834465  | 0.20991943   | 0.989603576  | 0.67056754     | TRUE              |                |
| Banks                 | WFC     | USB     | 0.677078033  | TRUE               | 0.104253153  | FALSE                           | 6.514087473  | 15.4943 | FALSE                           | 0.647973131  | [0.08567503  | 0.24411664   | 0.989603628  | 0.813362404    | TRUE              |                |
| Banks                 | C       | USB     | 0.72292654   | TRUE               | 0.056454587  | FALSE                           | 8.40921278   | 15.4943 | FALSE                           | 0.542729409  | [0.181356304 | 0.278478991  | 0.813362404  | 0.844727849    | TRUE              |                |
| Banks                 | C       | MS      | 0.812849645  | TRUE               | 0.023139097  | TRUE                            | 10.84041797  | 15.4943 | FALSE                           | 0.696399091  | [0.38011691  | 0.18415562   | 0.813362811  | 0.850928151    | TRUE              |                |
| Banks                 | C       | GS      | 0.856976239  | TRUE               | 0.28552652   | FALSE                           | 6.35986677   | 15.4943 | FALSE                           | 0.104642779  | [0.2798581   | 0.15289053   | 0.989603640  | 0.67056754     | TRUE              |                |
| Banks                 | WFC     | USB     | 0.827857433  | TRUE               | 0.5142123584 | FALSE                           | 5.1421006981 | 15.4943 | FALSE                           | 0.33187088   | [0.28076103  | 0.24857012   | 0.53809259   | 0.844727926    | TRUE              |                |
| Banks                 | WFC     | MS      | 0.694569571  | TRUE               | 0.106950114  | FALSE                           | 8.04758988   | 15.4943 | FALSE                           | 0.519390908  | [0.220731    | 0.16119477   | 0.538092534  | 0.844727926    | TRUE              |                |
| Banks                 | WFC     | GS      | 0.724902733  | TRUE               | 0.01779077   | TRUE                            | 12.52730098  | 15.4943 | FALSE                           | 0.40851029   | [0.24876538  | 0.20530604   | 0.80932099   | 0.67056754     | TRUE              |                |
| Banks                 | USB     | MS      | 0.633439911  | TRUE               | 0.094475135  | FALSE                           | 7.95831223   | 15.4943 | FALSE                           | 0.513403718  | [0.13666334  | 0.25975763   | 0.844727242  | 0.850928216    | TRUE              |                |
| Banks                 | USB     | GS      | 0.632687991  | TRUE               | 0.1240490385 | FALSE                           | 8.137472427  | 15.4943 | FALSE                           | 0.525191472  | [0.2880857   | 0.25898761   | 0.844727301  | 0.67056754     | TRUE              |                |
| Banks                 | MS      | GS      | 0.905160049  | TRUE               | 0.028783457  | TRUE                            | 11.06105346  | 15.4943 | FALSE                           | 0.713878876  | [0.14452965  | 0.11387311   | 0.850921927  | 0.67056754     | TRUE              |                |
| Consumer_Staples      | PEP     | PG      | 0.672031988  | TRUE               | 0.320355073  | FALSE                           | 4.976091715  | 15.4943 | FALSE                           | 0.311562778  | [0.15837022  | 0.20195359   | 0.993728758  | 0.67056754     | TRUE              |                |
| Consumer_Staples      | PEP     | ETN     | 0.573350879  | TRUE               | 0.798918052  | FALSE                           | 5.883680981  | 15.4943 | FALSE                           | 0.379731965  | [0.0374966   | 0.19860749   | 0.951619605  | 0.376904207    | TRUE              |                |
| Consumer_Staples      | PEP     | SIM     | 0.450758167  | TRUE               | 0.877534456  | FALSE                           | 5.889650869  | 15.4943 | FALSE                           | 0.380117276  | [0.0264283   | 0.0771048    | 0.951619605  | 0.27763052     | TRUE              |                |
| Consumer_Staples      | PEP     | HY      | 0.542049858  | TRUE               | 0.087222482  | FALSE                           | 7.97985152   | 15.4943 | FALSE                           | 0.514962609  | [0.1341050   | 0.13284021   | 0.951619514  | 0.9123145      | TRUE              |                |
| Consumer_Staples      | PEP     | GO      | 0.516705648  | TRUE               | 0.812901296  | FALSE                           | 4.569934748  | 15.4943 | FALSE                           | 0.29449301   | [0.3032551   | 0.18476147   | 0.951619605  | 0.48523484     | TRUE              |                |
| Consumer_Staples      | PEP     | KO      | 0.685453814  | TRUE               | 0.001779795  | TRUE                            | 17.15636151  | 15.4943 | TRUE                            | 1.107093351  | [0.2020841   | 0.17632076   | 0.845247879  | 0.67056754     | TRUE              |                |
| Consumer_Staples      | PEP     | K       | 0.233912113  | TRUE               | 0.987569624  | FALSE                           | 5.497139779  | 15.4943 | FALSE                           | 0.354784648  | [0.0280497   | 0.16413199   | 0.993728743  | 0.376904288    | TRUE              |                |
| Consumer_Staples      | PG      | GS      | 0.292889134  | TRUE               | 0.984418135  | FALSE                           | 5.42784289   | 15.4943 | FALSE                           | 0.305012373  | [0.08614369  | 0.278478991  | 0.993728899  | 0.27763052     | TRUE              |                |
| Consumer_Staples      | PG      | HY      | 0.590900827  | TRUE               | 0.000932518  | TRUE                            | 10.958608481 | 15.4943 | TRUE                            | 1.230309977  | [0.26365842  | 0.186558019  | 0.993728751  | 0.912314658    | TRUE              |                |
| Consumer_Staples      | PG      | GO      | 0.416493011  | TRUE               | 0.978322908  | FALSE                           | 5.041606575  | 15.4943 | FALSE                           | 0.325384586  | [0.06131069  | 0.18492086   | 0.993728724  | 0.485234944    | TRUE              |                |
| Consumer_Staples      | PG      | KO      | 0.546063171  | TRUE               | 0.050510434  | FALSE                           | 11.28068416  | 15.4943 | FALSE                           | 0.770853811  | [0.18821036  | 0.256254536  | 0.993728767  | 0.485248331    | TRUE              |                |
| Consumer_Staples      | K       | SIM     | 0.547224249  | TRUE               | 0.003371575  | TRUE                            | 17.63948373  | 15.4943 | TRUE                            | 1.138449864  | [0.38024136  | 0.14783357   | 0.993728869  | 0.277630771    | TRUE              |                |
| Consumer_Staples      | GS      | MS      | 0.553625298  | TRUE               | 0.569976462  | FALSE                           | 5.324159582  | 15.4943 | FALSE                           | 0.32451552   | [0.08614369  | 0.25898761   | 0.993728751  | 0.67056754     | TRUE              |                |
| Consumer_Staples      | GS      | GO      | 0.592443774  | TRUE               | 0.006166522  | TRUE                            | 12.406175    | 15.4943 | FALSE                           | 0.0008807    | [0.12625351  | 0.1739451    | 0.376904289  | 0.485234455    | TRUE              |                |
| Consumer_Staples      | GS      | KO      | 0.592443774  | TRUE               | 0.006166522  | TRUE                            | 5.59371162   | 15.4943 | FALSE                           | 0.559188681  | [0.1894251   | 0.1286451    | 0.376904289  | 0.485234455    | TRUE              |                |
| Consumer_Staples      | GS      | HY      | 0.500745154  | TRUE               | 0.592443774  | FALSE                           | 5.453811051  | 15.4943 | FALSE                           | 0.393229113  | [0.0584959   | 0.00443273   | 0.37670954   | 0.485234455    | TRUE              |                |
| Consumer_Staples      | GS      | MS      | 0.691293781  | TRUE               | 0.006166522  | TRUE                            | 17.0410713   | 15.4943 | TRUE                            | 1.09982749   | [0.17041348  | 0.13047745   | 0.37670904   | 0.67056754     | TRUE              |                |
| Healthcare_Equipment  | MDT     | SYX     | 0.642277367  | TRUE               | 0.291348018  | FALSE                           | 4.58337893   | 15.4943 | FALSE                           | 0.269048377  | [0.13500576  | 0.049839462  | 0.912313963  | 0.485247989    | TRUE              |                |
| Healthcare_Equipment  | MDT     | SXH     | 0.642277367  | TRUE               | 0.291348018  | FALSE                           | 4.58337893   | 15.4943 | FALSE                           | 0.269048377  | [0.13500576  | 0.049839462  | 0.912313963  | 0.485247989    | TRUE              |                |
| Healthcare_Equipment  | MDT     | DH      | 0.626280155  | TRUE               | 0.326635247  | FALSE                           | 4.514863848  | 15.4943 | FALSE                           | 0.291388694  | [0.19317111  | 0.06057488   | 0.952292663  | 0.988512129    | TRUE              |                |
| Healthcare_Equipment  | MDT     | SYX     | 0.642277367  | TRUE               | 0.000608935  | TRUE                            | 14.9882139   | 15.4943 | FALSE                           | 0.96733774   | [0.18042148  | 0.15477299   | 0.376904289  | 0.67056754     | TRUE              |                |
| Healthcare_Equipment  | SYX     | BXK     | 0.662244576  | TRUE               | 0.00358524   | TRUE                            | 16.40166821  | 15.4943 | TRUE                            | 1.05861642   | [0.13082706  | 0.054697572  | 0.98865308   | 0.985272541    | TRUE              |                |
| Healthcare_Equipment  | SYX     | DHR     | 0.802609533  | TRUE               | 0.005017848  | TRUE                            | 14.01628845  | 15.4943 | FALSE                           | 0.90460894   | [0.1413323   | 0.16446554   | 0.988512134  | 0.67056754     | TRUE              |                |
| Healthcare_Equipment  | SYX     | ZBH     | 0.588182588  | TRUE               | 0.498855827  | FALSE                           | 5.850197653  | 15.4943 | FALSE                           | 0.37750955   | [0.03508423  | 0.0803833    | 0.988653074  | 0.68641833     | TRUE              |                |
| Healthcare_Equipment  | SYX     | BXR     | 0.659330977  | TRUE               | 0.046528894  | TRUE                            | 10.7835936   | 15.4943 | FALSE                           | 0.65951873   | [0.13513643  | 0.12303643   | 0.988512134  | 0.67056754     | TRUE              |                |
| Healthcare_Equipment  | SYX     | ZBH     | 0.548145622  | TRUE               | 0.253159392  | FALSE                           | 6.718674869  | 15.4943 | FALSE                           | 0.43362753   | [0.18042148  | 0.284086451  | 0.985272541  | 0.68641833     | TRUE              |                |
| Healthcare_Equipment  | DHR     | ZBH     | 0.547582362  | TRUE               | 0.48855573   | FALSE                           | 5.675840844  | 15.4943 | FALSE                           | 0.366317991  | [0.04050029  | 0.060982455  | 0.985812107  | 0.67056754     | TRUE              |                |

## Sequential Decision Analysis Framework

$B_t$ : Belief State

In the specific case that is being analyzed:

$$S_t = \left( P_t^{(1)}, P_t^{(2)}, w_t \right)$$

where:

$$w_t: (w_0, w_1)$$

and:

- $P_t^{(1)}$ : Market Price for first asset at  $t$
- $P_t^{(2)}$ : Market Price for second asset at  $t$
- $w_0$ : Kalman Filter estimate for intercept
- $w_1$ : Kalman Filter estimate for hedge ratio

## 2. Decision Variables

$$\begin{aligned} X^\pi(S_t) &\rightarrow x_t \\ x_t &\in X_t \\ x_t &= K_t(y_t - \hat{y}_t) \end{aligned}$$

where:

- $x_t$ : Decision
- $X^\pi(S_t)$ : Policy for decision
- $K_t$ : Kalman Filter gain
- $y_t - \hat{y}_t$ : Prediction error

## 3. Exogenous Information

$$W_{t+1} = \left( \hat{P}_{t+1}^{(1)}, \hat{P}_{t+1}^{(2)} \right)$$

where:

- $\hat{P}_{t+1}^{(1)}$ : New Market Price for first asset at  $t + 1$
- $\hat{P}_{t+1}^{(2)}$ : New Market Price for second asset at  $t + 1$

## 4. Transition Function

$$S_{t+1} = S^M(S_t, x_t, W_{t+1})$$

where:

- $S^M(\cdot)$ : State transition model
- $S_{t+1}$ : State after transition model at  $t + 1$
- $W_{t+1}$ : Exogenous information
- $x_t$ : Decision
- $S_t$ : State variables

## 5. Objective Function

$$\begin{aligned} & \min C(S_t, X^\pi(S_t) | S_0) \\ & \min_{\pi} E[(y_t - \hat{y}_t)^2] \end{aligned}$$

## Six step modeling process

### 1. Narrative:

This model uses the Kalman Filter to determine how much of one asset should be bought or sold relative to the other by estimating a dynamic hedge ratio that evolves over time. Instead of relying on a fixed beta, the hedge ratio is treated as a latent state that adjusts with every new price observation. Through the predict–observe–update sequence, the filter continuously refines this estimate, ensuring that position sizing reflects the most current equilibrium relationship between the assets. This adaptive process filters out short-term noise, stabilizes the spread, and provides a precise real-time measure of how to weight each leg of the pair in every trade.

### 2. Core Elements:

- Metrics: Minimize mean squared error between observed and predicted values for model.
- Decision: Update the model for estimates.
- Uncertainties: Random price movement and estimation noise in Kalman filter weights.
- Exogenous information: New observed prices.

### 3. Mathematical Process:

#### 1. State Variables

$$\begin{aligned} S_t &= (R_t, I_t, B_t) \\ S_t &= (P_t^{(1)}, P_t^{(2)}, W_t) \end{aligned}$$

#### 2. Decision Variables

$$\begin{aligned} X^\pi(S_t) &\rightarrow x_t \\ x_t &\in X_t \\ x_t &= K_t(y_t - \hat{y}_t) \end{aligned}$$

#### 3. Exogenous Information

$$W_{t+1} = (\hat{P}_{t+1}^{(1)}, \hat{P}_{t+1}^{(2)})$$

#### 4. Transition Function

$$S_{t+1} = S^M(S_t, x_t, W_{t+1})$$

## 5. Objective Function

$$\begin{aligned} & \min C(S_t, X^\pi(S_t) | S_0) \\ & \min_{\pi} E[(y_t - \hat{y}_t)^2] \end{aligned}$$

## 4. Uncertainty Model:

Does not apply.

## 5. Designing Policies:

$$w_t = w_{t-1} + K_t(y_t - \hat{y}_t)$$

## 6. Evaluating Policies:

Does not apply.

## Description of sequential process

At each time step, the model begins by observing the current state, defined by the two asset prices and the Kalman-filtered parameter estimates. Using these values, the system predicts the next observation and compares it with the newly arriving market prices. The difference between the prediction and the actual data drives the Kalman Filter update, which adjusts the parameters  $w_0$  and  $w_1$ . These updated estimates, together with the new prices, form the next state through the transition function. The cycle then repeats: observe prices, predict, compute error, update the weights, and transition forward, all while minimizing the expected squared prediction error. This iterative process enables the model to continuously refine its hedge ratio and intercept as new information arrives.

## Kalman gain calculation and interpretation

The Kalman gain is the core mechanism that lets a Kalman filter balance prediction and measurement at every step. It decides how much the updated estimate should move toward the new observation versus how much it should stick to the model's prior prediction. When the model is uncertain (its predicted variance is large), the Kalman gain becomes high, meaning the filter gives strong weight to the new measurement. When the measurement is noisy (its error variance is large), the Kalman gain becomes small, meaning the filter trusts its internal prediction more than the observed data. In this way, the gain acts like an optimal, dynamically changing weight that adapts to whatever information is more reliable at each moment.

It is calculated as follows:

$$K_t = P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R)^{-1}$$

where:

$K_t$ : Kalman gain at time  $t$

$$\begin{aligned}
 P_{t|t-1} &: \text{Predicted covariance} \\
 H_t &: \text{Observation matrix} \\
 R &: \text{Observation covariance matrix}
 \end{aligned}$$

## Q and R matrix selection with justification

For the first implementation of the Kalman Filter, as the process noise matrix Q control how quickly the hedge ratio adapts over time, while the measurement noise R reflects how much confidence there is in each observed price relationship, the matrix Q was set to:

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

This was chosen because there is a high process noise, and it is highly time-varying, hedge ratios can change rapidly due to shifts in volatility regimes, liquidity conditions, or temporary breakdowns in cointegration. A higher Q prevents the filter from becoming too rigid and allows it to respond quickly when market dynamics change.

On the other side, R was chosen as:

$$R = [0.001]$$

This way, in combination with matrix Q, a hedge-ratio estimate that is *adaptive*, responsive, and able to track short-term deviations is produced, which is desirable when constructing spreads for high-frequency or daily adjusted pairs trading strategies, where fast adjustment improves the stability of the spread and the performance of mean-reversion signals.

## Worked example showing state evolution over several periods



The estimated price of asset 2 constructed using the Kalman-filtered hedge parameters closely tracks the real price over most of the sample, indicating that the

filter has successfully learned and stabilised the underlying relationship between the assets. Early deviations reflect the filter's initial calibration phase, while the strong alignment from 2021 to 2024 demonstrates effective convergence and reliable estimation of both the level and hedge ratio. Some divergence appears toward the end of the period during sharp price increases, which is typical when market volatility temporarily outpaces the filter's adjustment speed. Overall, the results show a well-functioning model that captures the main dynamics of the price with only limited short-term discrepancies.

## Kalman 2

### Detailed mathematical formulation of state-space model

#### 1. State Variables

$$S_t = (R_t, I_t, B_t)$$

where:

$R_t$ : Physical State

$I_t$ : Information State

$B_t$ : Belief State

In the specific case that is being analyzed:

$$S_t = (P_t^{(1)}, P_t^{(2)}, E_t)$$

where:

$P_t^{(1)}$ : Market Price for first asset at  $t$

$P_t^{(2)}$ : Market Price for second asset at  $t$

$E_t = [e_{1t} \ e_{2t}]$ : Eigenvector from Johansen's test at  $t$

#### 2. Decision Variables

$$X^\pi(S_t) \rightarrow x_t$$

$$x_t \in X_t$$

$$x_t = K_t(y_t - \hat{y}_t)$$

where:

$x_t$ : Decision

$X^\pi(S_t)$ : Policy for decision

$K_t$ : Kalman Filter gain

$y_t - \hat{y}_t$ : Prediction error

#### 3. Exogenous Information

$$W_{t+1} = (\hat{P}_{t+1}^{(1)}, \hat{P}_{t+1}^{(2)}, \hat{E}_{t+1})$$

where:

$\hat{P}_{t+1}^{(1)}$ : Market Price for first asset at  $t + 1$

$$\hat{P}_{t+1}^{(2)}: \text{Market Price for second asset at } t+1$$

$$\hat{E}_{t+1}: = [e_{1,t+1} \ e_{2,t+1}]: \text{Eigenvector form Johansen's test at } t+1$$

#### 4. Transition Function

$$S_{t+1} = S^M(S_t, x_t, W_{t+1})$$

where:

$S^M(\cdot)$ : State transition model

$S_{t+1}$ : State after transition model at  $t+1$

$W_{t+1}$ : Exogenous information

$x_t$ : Decision

$S_t$ : State variables

#### 5. Objective Function

$$\min C(S_t, X^\pi(S_t) | S_0)$$

$$\min_{\pi} E[(y_t - \hat{y}_t)^2]$$

### Six step modeling process

#### 1. Narrative:

This model uses the Kalman Filter to dynamically estimate the eigenvector components  $e_1$  and  $e_2$ , which define the cointegrating relationship required by the VECM. By updating these parameters as new data arrive, the model adapts to time-varying market conditions and captures changes in the long-run equilibrium between the assets. This produces more accurate and stable inputs for the VECM, allowing the strategy to generate cleaner and more reliable trading signals. With appropriately tuned Q and R matrices, the filter effectively balances responsiveness and noise reduction, improving signal quality and ultimately enhancing the risk-adjusted performance of the overall strategy.

#### 2. Core elements:

- Metrics: Minimize mean squared error between observed and predicted values for model.
- Decision: Update the model for estimates.
- Uncertainties: Random price movement and estimation noise in Kalman filter weights.
- Exogenous information: New observed prices and eigenvector form Johansen's test.

#### 3. Mathematical Process:

1. State Variables

$$S_t = (R_t, I_t, B_t)$$

$$S_t = (P_t^{(1)}, P_t^{(2)}, E_t)$$

2. Decision Variables

$$X^\pi(S_t) \rightarrow x_t$$

$$x_t \in X_t$$

$$x_t = K_t(y_t - \hat{y}_t)$$

3. Exogenous Information

$$W_{t+1} = (\hat{P}_{t+1}^{(1)}, \hat{P}_{t+1}^{(2)}, \hat{E}_{t+1})$$

4. Transition Function

$$S_{t+1} = S^M(S_t, x_t, W_{t+1})$$

5. Objective Function

$$\min C(S_t, X^\pi(S_t) | S_0)$$

$$\min_{\pi} E[(y_t - \hat{y}_t)^2]$$

4. Uncertainty Model:

Does not apply.

5. Designing Policies:

$$w_t = w_{t-1} + K_t(y_t - \hat{y}_t)$$

6. Evaluating Policies:

Does not apply.

### Description of sequential process

At each time step, the model begins by observing the current state, which includes the two asset prices and the VECM-based error-correction terms. Using these inputs, the model generates a prediction of the next state and compares it with the new incoming market information. The discrepancy between the predicted and observed values, given by the prediction error, is then used by the Kalman Filter to update the decision variables. These updated decisions feed into the state-transition function, producing the next state. This cycle continues iteratively: observe, predict, measure error, update beliefs, and transition to the next period. Across time, the process minimizes prediction error while adapting to new prices and cointegration signals.

## Kalman gain calculation and interpretation

The Kalman gain is the core mechanism that lets a Kalman filter balance prediction and measurement at every step. It decides how much the updated estimate should move toward the new observation versus how much it should stick to the model's prior prediction. When the model is uncertain (its predicted variance is large), the Kalman gain becomes high, meaning the filter gives strong weight to the new measurement. When the measurement is noisy (its error variance is large), the Kalman gain becomes small, meaning the filter trusts its internal prediction more than the observed data. In this way, the gain acts like an optimal, dynamically changing weight that adapts to whatever information is more reliable at each moment.

It is calculated as follows:

$$K_t = P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R)^{-1}$$

where:

$K_t$ : Kalman gain at time  $t$   
 $P_{t|t-1}$ : Predicted covariance  
 $H_t$ : Observation matrix  
 $R$ : Observation covariance matrix

## Q and R matrix selection with justification

For the second implementation of the Kalman Filter, used to estimate the dynamic eigenvector weights of the cointegrating relationship, the process noise matrix Q was set to:

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 0.001 \end{bmatrix}$$

In this case, the choice reflects the fact that the first eigenvector component is allowed to adjust more freely over time, while the second component is assumed to be much more stable. This setup is justified because, in a cointegration-based framework, one leg of the eigenvector is usually more sensitive to short-term adjustments in the long-run equilibrium relationship, whereas the other tends to be more persistent. By using a larger value for the first diagonal entry of Q, the filter permits faster adaptation in that component, while the smaller value for the second entry keeps the system stable and prevents excessive oscillation in the long-run eigenvector direction.

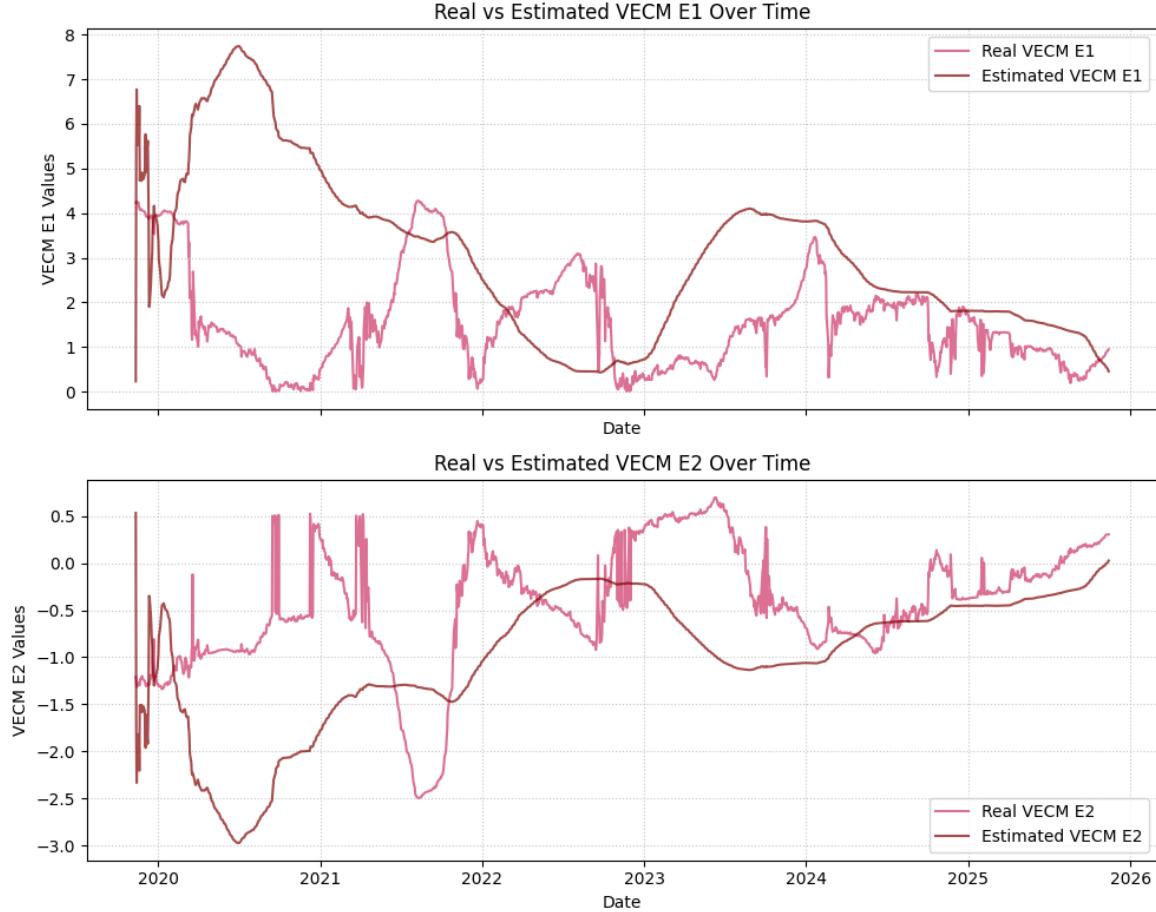
On the other side, R was chosen as:

$$R = [0.001]$$

This way, in combination with matrix Q, this configuration allows the Kalman Filter to provide an eigenvector estimate that is smooth, stable, and consistent with long-run

cointegration while still being flexible enough to adjust to short-term changes in the underlying spread dynamics.

### Worked example showing state evolution over several periods



The first chart illustrates how the Kalman Filter estimates the evolution of the first error-correction term (E1). Early in the sample, the estimated series displays sizable fluctuations and only loosely follows the real underlying signal. This is typical during the learning phase, when the filter is still calibrating itself to the dynamics of the system. As the series evolves, the estimated E1 begins to trace the broader direction and turning points of the real E1 much more closely. While short-term deviations persist, the filter reliably captures the long-run behavior of the state. This indicates that the model is successfully learning the structural correction mechanism embedded within E1.

The second chart presents the evolution of the second error-correction term (E2). Compared to E1, the early-period volatility is even more pronounced: the estimated E2 series reacts strongly to noise and displays abrupt jumps. This again highlights the challenges associated with estimating states driven by shorter-term market fluctuations. As time progresses, however, the estimated E2 begins to align more clearly with the overall trajectory of the real E2. Although the filter still exhibits more

sensitivity and short-term noise than in the E1 case, it nevertheless demonstrates meaningful convergence: long-term patterns, sign changes, and major trend reversals are increasingly captured by the model.

## Trading Signal Generation

### Detailed mathematical formulation of state-space model

#### 1. State Variables

$$S_t = (R_t, I_t, B_t)$$

where:

- $R_t$ : Physical State
- $I_t$ : Information State
- $B_t$ : Belief State

In the specific case that is being analyzed:

$$S_t = (P_t^{(1)}, P_t^{(2)}, \beta_{1t}, VECM_t, \theta)$$

where:

- $P_t^{(1)}$ : Market Price for first asset at  $t$
- $P_t^{(2)}$ : Market Price for second asset at  $t$
- $\beta_{1t}$ : Estimate for hedge ratio from Kalman filter 1 at  $t$
- $VECM_t$ : Vector Error Correlation Model at  $t$
- $\theta$ :  $z$ -score threshold

#### 2. Decision Variables

$$\begin{aligned} X^\pi(S_t) &\rightarrow x_t \\ x_t &\in X_t \\ x_t &= K_t(y_t - \hat{y}_t) \\ X^\pi(S_t) &= \begin{cases} \varphi(VECM) > \theta \\ \varphi(VECM) < -\theta \\ |\varphi(VECM)| < 0.05 \\ \text{otherwise} \end{cases} \end{aligned}$$

$$\begin{aligned} F^\pi(\theta) &= C(S_t, X^\pi(S_t), \theta | S_0) \\ \theta &= [0.5, 1.5] \end{aligned}$$

where:

- $x_t$ : Decision
- $X^\pi(S_t)$ : Policy for decision
- $K_t$ : Kalman Filter gain
- $y_t - \hat{y}_t$ : Prediction error
- $\varphi(VECM)$ : VECM normalized

$\theta$ : *z-score threshold*

$F^\pi(\theta)$ : *Objective function that maps each threshold to the resulting cost*

### 3. Exogenous Information

$$W_{t+1} = (\hat{P}_{t+1}^{(1)}, \hat{P}_{t+1}^{(2)}, \widehat{VECM}_{t+1}, \hat{\beta}_{1_{t+1}})$$

where:

$\hat{P}_{t+1}^{(1)}$ : *Market Price for first asset at t + 1*

$\hat{P}_{t+1}^{(2)}$ : *Market Price for second asset at t + 1*

$\widehat{VECM}_{t+1}$ : *Vector Error Correction Model at t + 1*

$\hat{\beta}_{1_{t+1}}$ : *Estimate for hedge ratio from Kalman filter 1 at t + 1*

### 4. Transition Function

$$S_{t+1} = S^M(S_t, x_t, W_{t+1})$$

where:

$S^M(\cdot)$ : *State transition model*

$S_{t+1}$ : *State after transition model at t + 1*

$W_{t+1}$ : *Exogenous information*

$x_t$ : *Decision*

$S_t$ : *State variables*

### 5. Objective Function

$$\max E[C(S_t, X^\pi(S_t) | S_t)]$$
$$\max_\pi(s(\pi))$$

where:

$s$ : *Sortino Ratio*

## Six step modeling process

### 1. Narrative:

This model aims to evaluate a statistical arbitrage strategy based on cointegrated assets. The goal is to generate trading signals using deviations of the equilibrium spread estimated through a VECM and adjusted dynamically with a Kalman filter. At every time step, the model observes new price data and updates the hedge ratio and the spread estimate. Based on the standardized deviation of the VECM signal, the policy decides whether to open, close, or hold long–short positions. The objective is to maximize the Sortino Ratio by improving returns while controlling downside risk. The process captures how decisions, updated state variables, and new market information interact over time to shape the performance of the strategy.

## 2. Core Elements:

- Metrics: Maximize Sortino Ratio
- Decision: Generation of trading signals based on standardized deviations of equilibrium spread.
- Uncertainties: Estimation noise in cointegration parameters and estimation noise in Kalman filter weights.
- Exogenous information: New observed prices, Vector Error Correlation Model (VECM) and hedge ratio.

## 3. Mathematical Process:

### 1. State Variables

$$S_t = (R_t, I_t, B_t)$$

$$S_t = (P_t^{(1)}, P_t^{(2)}, \beta_{1t}, VECM_t, \theta)$$

### 2. Decision Variables

$$X^\pi(S_t) \rightarrow x_t$$

$$x_t \in X_t$$

$$x_t = K_t(y_t - \hat{y}_t)$$

$$X^\pi(S_t) = \begin{cases} \varphi(\text{VECM}) > \theta \\ \varphi(\text{VECM}) < -\theta \\ |\varphi(\text{VECM})| < 0.05 \\ \text{otherwise} \end{cases}$$

$$F^\pi(\theta) = C(S_t, X^\pi(S_t), \theta | S_0)$$

$$\theta = [0.5, 1.5]$$

### 3. Exogenous Information

$$W_{t+1} = (\hat{P}_{t+1}^{(1)}, \hat{P}_{t+1}^{(2)}, \widehat{VECM}_{t+1}, \hat{\beta}_{1t+1})$$

### 4. Transition Function

$$S_{t+1} = S^M(S_t, x_t, W_{t+1})$$

### 5. Objective Function

$$\max E[C(S_t, X^\pi(S_t)) | S_t]$$

$$\max_\pi(s(\pi))$$

## 4. Uncertainty Model:

Does not apply.

## 5. Designing Policies:

$$X^\pi(S_t) = \begin{cases} \varphi(\text{VECM}) > \theta & \rightarrow \text{long asset } y \text{ and short asset } x \\ \varphi(\text{VECM}) < -\theta & \rightarrow \text{short asset } y \text{ and long asset } x \\ |\varphi(\text{VECM})| < 0.05 & \rightarrow \text{close positions} \\ \text{otherwise} & \rightarrow \text{hold} \end{cases}$$

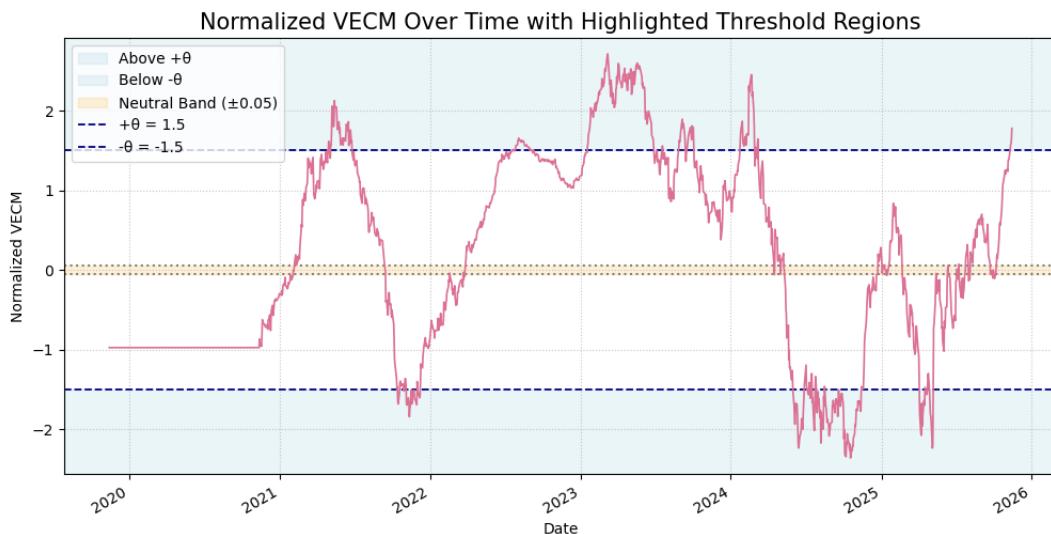
## 6. Evaluating Policies:

$$\text{Sortino Ratio} = \frac{\text{Average return}}{\text{Downside volatility}}$$

### Description of sequential process

At each step, the model observes the latest asset prices, updates the hedge ratio through the Kalman filter, and recalculates the equilibrium spread using the VECM framework. This spread is then normalized to produce a z-score, which is compared against a threshold to generate trading decisions, entering long–short positions when deviations are large, closing positions when the spread reverts, or holding otherwise. New exogenous information, including updated prices, VECM estimates, and hedge ratios, feeds into the state transition to form the next state. The process iterates forward, continuously refining parameter estimates and trading signals with the objective of maximizing the Sortino Ratio by enhancing risk-adjusted performance while controlling downside volatility.

### Worked example showing state evolution over several periods



The previous graph visually illustrates the logic used to generate trading signals. The light-blue shaded regions highlight periods when the normalized VECM exceeds the selected positive z-score threshold or drops below its negative counterpart, indicating potential entry points. The yellow band marks the zone where the normalized VECM lies between  $-0.05$  and  $0.05$ , representing the neutral range in

which positions are closed. This visual structure clearly shows how signals emerge as the VECM moves in and out of predefined thresholds.

## Kalman Filter Implementation

### Initialization procedures

In the first stage (Kalman 1), the goal is to determine how much of one asset should be bought or sold relative to the other. The Kalman Filter is initialized using a static linear regression between the two asset prices:

$$Y_t = \beta_0 + \beta_1 X_t$$

Here, the price of asset 1 and price of asset 2 are the exogenous inputs, while the quantity to buy or sell, represented by the hedge ratio ( $\beta_1$ ), is the variable estimated by the model.

The resulting parameters form the initial state vector  $S = (R, I, B)$ , containing the residual variance ( $R$ ), the intercept ( $I$ ), and the hedge ratio ( $B$ ). This provides a baseline equilibrium relationship that the Kalman filter will update dynamically as new market data arrive.

### Parameter estimation methodology

Once initialized, the Kalman Filter begins to recursively estimate and update the hedge ratio as new observations of the asset prices arrive. At each time step, it minimizes the prediction error between the observed and estimated spread, refining the parameters  $\beta_t = (\beta_0, \beta_1)$ .

This stage adapts the regression in real time, allowing the model to capture time-varying relationships between the assets rather than assuming a fixed coefficient as in traditional OLS.

Mathematically, the filter minimizes:

$$\min C(S_t, X^T(S_t)) \mid S_0$$

Where  $C(\cdot)$  represents the deviation between the observed and predicted spread. This ensures that the dynamic hedge ratio continuously reflects the most recent market information.

### Reestimation schedule and validation approach

The second stage (Kalman 2) incorporates information from the cointegration test to ensure that the dynamic relationship between assets remains valid over time.

Here, the eigenvectors derived from the Johansen cointegration test are exogenous inputs, while the idea or direction of the eigenvector becomes the variable used by the Kalman Filter to adjust its estimates.

The z-score ( $\theta$ ) acts as a decision threshold, signaling when deviations from equilibrium justify trading the assets and closing or holding the positions.

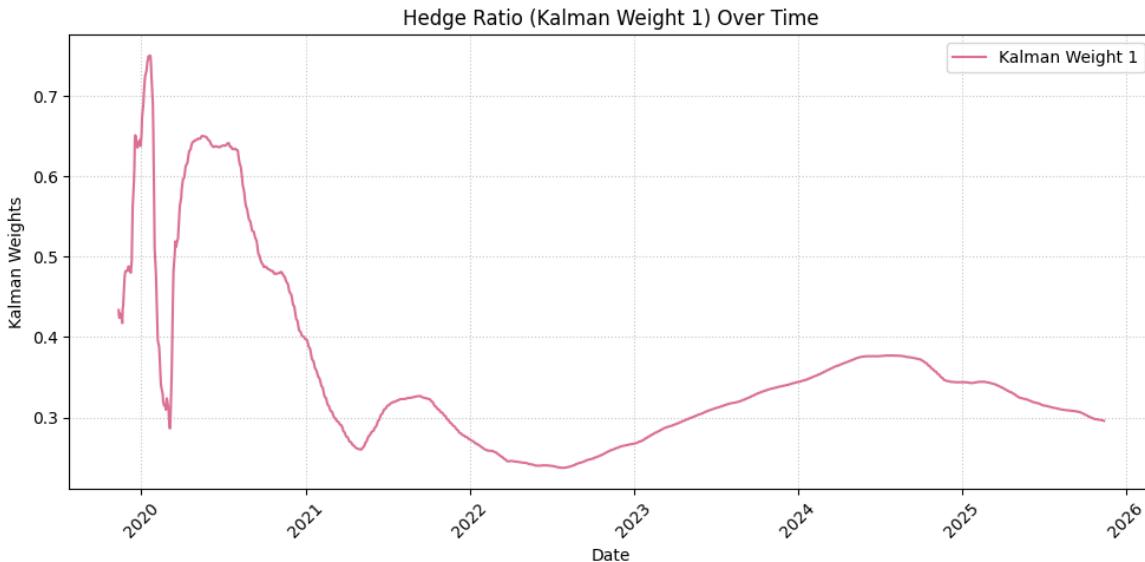
The decision logic follows:

$$X^\pi(S_t) = \begin{cases} \varphi(\text{VECM}) > \theta & \rightarrow \text{long asset } y \text{ and short asset } x \\ \varphi(\text{VECM}) < -\theta & \rightarrow \text{short asset } y \text{ and long asset } x \\ |\varphi(\text{VECM})| < 0.05 & \rightarrow \text{close positions} \\ \text{otherwise} & \rightarrow \text{hold} \end{cases}$$

The transition function is provided by the Kalman filter itself, which determines how the system moves from one state to the next. The objective function in this stage is to maximize the Kalman performance, to achieve the best trade-off between profit (signal strength) and stability (low noise).

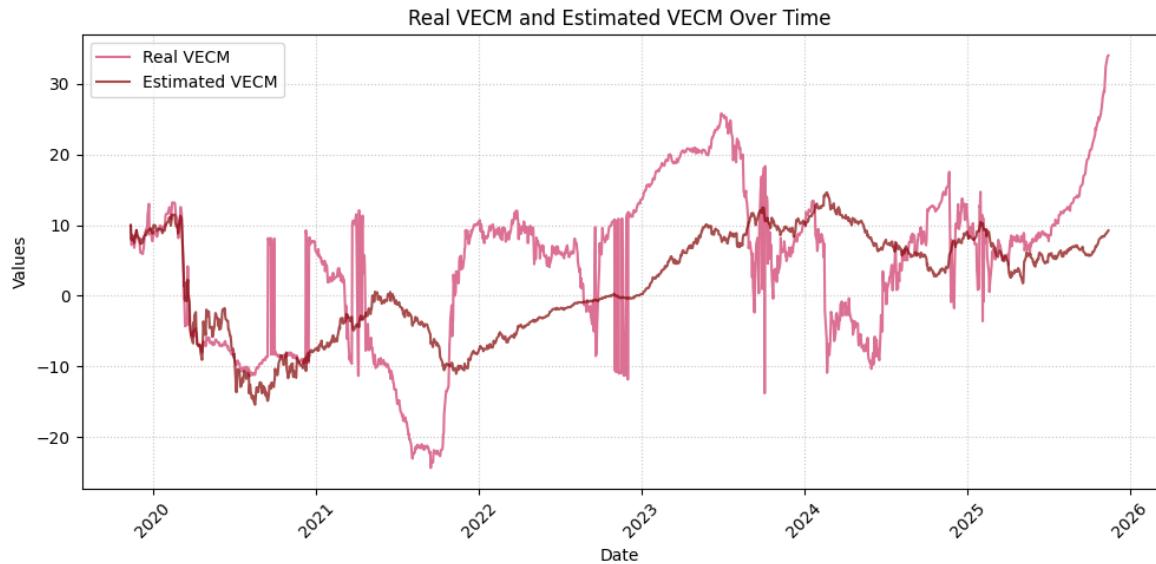
## Convergence analysis and filter stability

For first Kalman Filter:



As can be seen in the graph above, at the beginning of the sample, the hedge ratio exhibits sharp movements. This early volatility is expected: the filter is still learning the relationship between the two assets and adjusting to initial uncertainty. As time progresses, the hedge ratio becomes increasingly smooth and begins to move within a much narrower range. This behavior signals that the filter has successfully converged. In other words, it has gained confidence in its estimates and now reacts only moderately to new information. The long period of stability from 2022 onward indicates a well-calibrated and reliable hedge ratio estimate, which is essential for consistent risk management and trading decisions.

For second Kalman Filter:



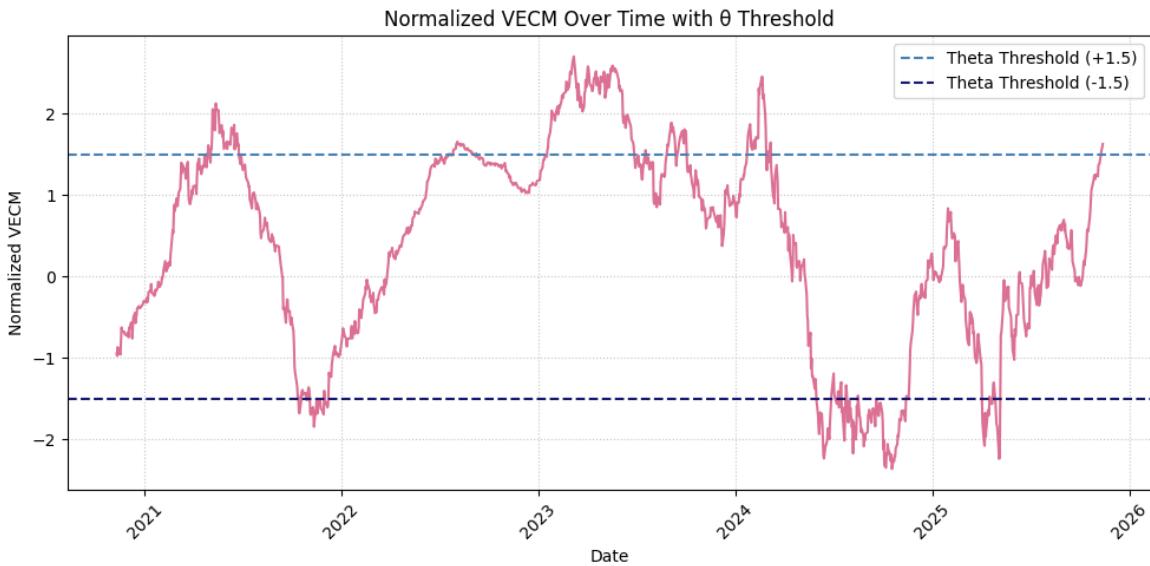
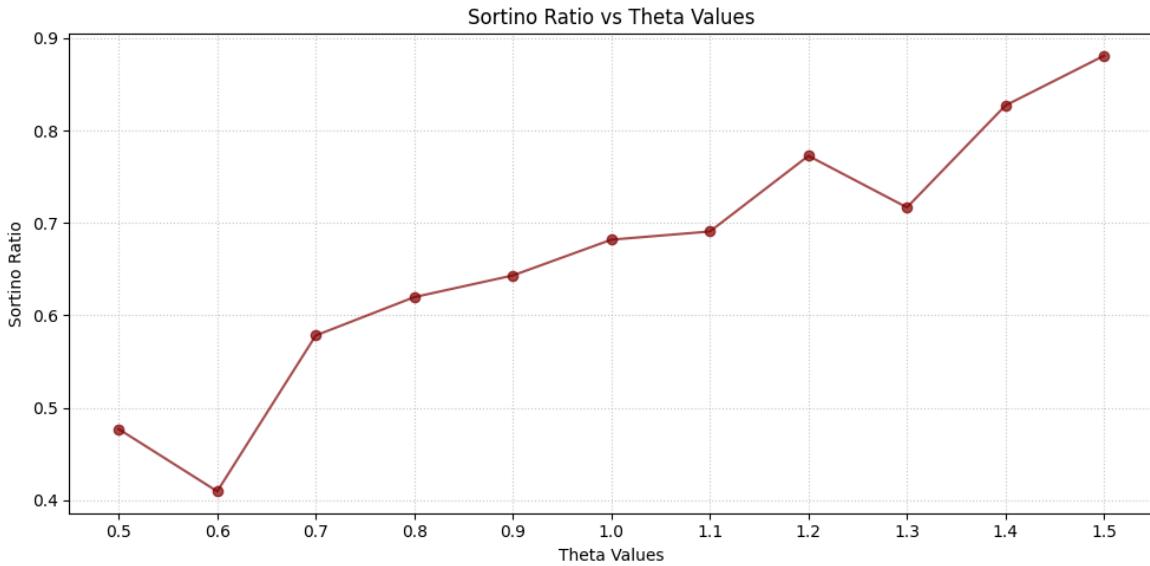
The graph above compares the true underlying VECM signal with the Kalman Filter's estimate. Early in the sample, the estimated values fluctuate widely and show large deviations from the real signal. This reflects the filter's initial uncertainty and the complexity of modelling the error-correction dynamics. Over time, however, the filter begins to align more closely with the overall direction and turning points of the real VECM. Although it continues to exhibit higher short-term volatility, the long-term trend tracking improves noticeably. This pattern indicates partial convergence: the filter is learning the broad structure of the system but still responds sensitively to short-term noise. Such behavior is typical when the underlying process is more volatile or structurally complex than a simple hedge-ratio model.

## Trading Strategy Logic

### Z-score definition using VECM

The Z-score was computed from the residuals of the Vector Error Correction Model (VECM), representing deviations of the current spread from its long-term equilibrium. After estimating the cointegrating relationship, the spread series was standardized by subtracting its mean and dividing by its standard deviation, both calculated over a rolling window to capture changes in market conditions. This transformation makes it easier to compare deviations over time, identifying overbought and oversold conditions in the pair relationship.

## Optimal entry and exit Z-score policy found



The optimal  $\theta$  threshold of  $\pm 1.5$  was determined by manually testing different threshold values ranging from 0.5 to 1.5 in increments of 0.1 and selecting the one that maximized the Sortino ratio.

A systematic trading policy that determines when to enter and exit positions based on the deviation of the VECM (Vector Error Correction Model) spread from its long-run equilibrium was created. This deviation is converted into a normalized Z-score, which provides a consistent measure of how far markets have moved from their expected relationship.

$$X^\pi(S_t) = \begin{cases} \varphi(\text{VECM}) > 1.5 & \rightarrow \text{long asset } y \text{ and short asset } x \\ \varphi(\text{VECM}) < -1.5 & \rightarrow \text{short asset } y \text{ and long asset } x \\ |\varphi(\text{VECM})| < 0.05 & \rightarrow \text{close positions} \\ \text{otherwise} & \rightarrow \text{hold} \end{cases}$$

## Cost treatment

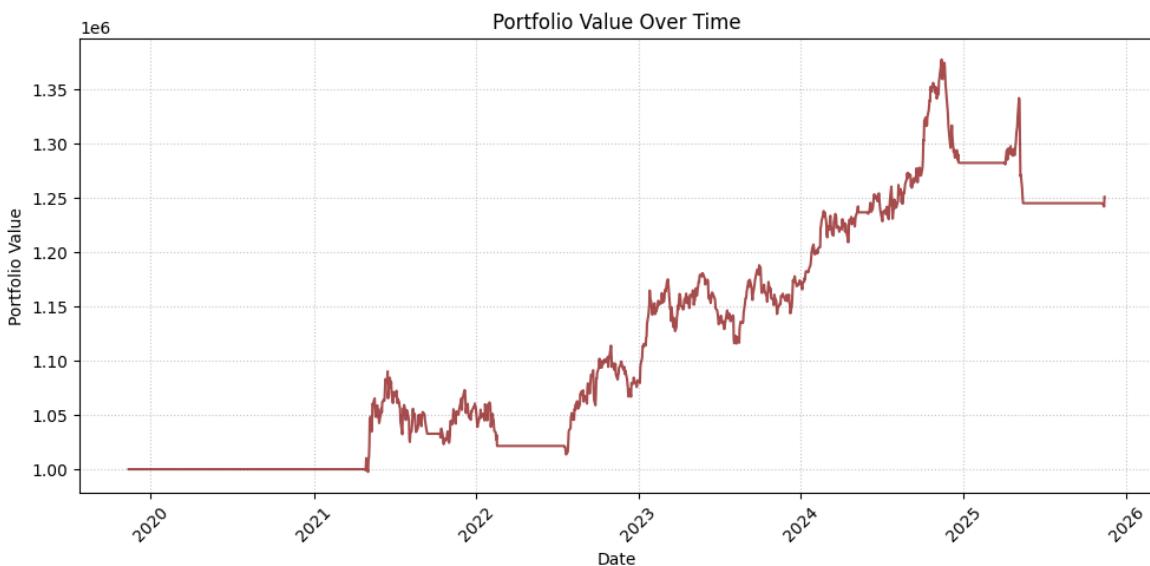
Transaction and financing costs were included to reflect realistic trading conditions. A commission rate of 0.125% was applied to each transaction, both when opening and closing positions. For short positions, a borrow rate of 0.25% annualized was charged and accrued daily on the borrowed amount. The strategy invested 80% of the available cash (\$1,000,000) equally across both assets, maintaining approximately 40% exposure per asset. Positions were rebalanced daily in line with updated hedge ratios to ensure consistency with the model's dynamic adjustments.

## Results and Performance Analysis

### Equity curve plots

#### Test

The test set represents 40% of the complete data, and it includes the 20% that is normally used for testing and the 20% that is normally used for validation, for practical reasons they were combined into one.



As we can see, in 2020 and early 2021, the portfolio value remained unchanged, staying at the initial cash level, and shortly after it began to experience some gains and losses. The portfolio value increased steadily from mid 2022 to late 2024,

reaching nearly \$1.4 millions. However, after that major peak, there was a sharp decline and a period of slight inactivity, bringing the portfolio value to a final level of \$1.25 millions.

## Performance metrics

To evaluate the effectiveness of the strategy, several metrics were used to provide a more comprehensive view of its performance over several periods. The implemented metrics were as follows:

### Sharpe Ratio

This metric measures the risk adjusted return relative to its volatility. The higher the value, the better the relationship between return and risk. It is calculated as follows:

$$\text{Sharpe Ratio} = \frac{\text{Average return}}{\text{Volatility}}$$

### Sortino Ratio

This metric is similar to Sharpe Ratio but focuses on downside volatility by considering only negative returns. The higher the value, the better protection against losses. It is calculated as follows:

$$\text{Sortino Ratio} = \frac{\text{Average return}}{\text{Downside volatility}}$$

### Calmar Ratio

This metric measures the return relative to the maximum loss risk. The higher the value, the better the risk-return efficiency. It is calculated as follows:

$$\text{Calmar Ratio} = \frac{\text{Average return}}{\text{Max Drawdown}}$$

### Max Drawdown

This metric indicates the largest drop that can occur from a peak to a trough. The lowest the value, the lower the exposure to risk. It is calculated as follows:

$$\text{Max Drawdown} = \frac{\text{Minimum value} - \text{Maximum value}}{\text{Maximum value}}$$

### Test

The test set represents 40%, and it includes the 20% that is normally used for testing and the 20% that is normally used for validation, but for practical reasons they were combined into one.

| Sharpe Ratio | Sortino Ratio | Max Drawdown | Calmar Ratio |
|--------------|---------------|--------------|--------------|
| 0.5730       | 0.8809        | 0.0984       | 0.3436       |

As we can see, the strategy obtained a Sharpe ratio of 0.57, which being greater than 0.5 indicates that the strategy delivers positive returns relative to the risk taken. The Sortino ratio was 0.88, showing that downside risk is somewhat controlled but losses are still relatively high compared to gains. The maximum drawdown was 0.10, indicating that the largest peak-to-trough loss was relatively small, and that the strategy preserves capital reasonably well. Finally, the Calmar ratio was 0.34, which, being below 1, suggests that the returns do not sufficiently exceed the maximum loss. However, this is not concerning given that the maximum loss itself is pretty low.

## Trade statistics

### Total trades

This statistic represents the total number of trades executed by the strategy, including both sells and buys. A higher number of trades indicate more activity or a shorter holding period. It is calculated as follows:

$$\text{Total trades} = \text{Sells} + \text{Buys}$$

### Test

The test set represents 40%, and it includes the 20% that is normally used for testing and the 20% that is normally used for validation, but for practical reasons they were combined into one.

| Total Trades |
|--------------|
| 12           |

As we can see, the total number of trades executed was 12, which is a relatively small value considering the number of years over which the strategy was applied.

### Win rate

This statistic measures the number of profitable trades relative to the total number of trades. The higher the value, the more consistent the strategy. It is calculated as follows:

$$\text{Win Rate} = \frac{\text{Positive trades}}{\text{Positive trades} + \text{Negative trades}}$$

## Test

The test set represents 40%, and it includes the 20% that is normally used for testing and the 20% that is normally used for validation, but for practical reasons they were combined into one.

| Wins | Losses | Win Rate |
|------|--------|----------|
| 6    | 6      | 50.00%   |

As we can see, from the 12 trades executed, 6 of them were wins while also 6 of them were losses, which gives a win rate of 50%. This indicates that the strategy's performance was driven not by a high frequency of winning trades, but by the magnitude of returns generated during profitable trades compared with the losses incurred.

## Average win/loss

This statistic compares the average profit per winning trade to the average loss per losing trade. It tells you how much you can make when you win compared with how much you lose when you lose. The higher the value, the better, because it means your average profit per winning trade is larger compared to your average loss per losing trade. It is calculated as follows:

$$\text{Average win/loss} = \frac{\text{Average profit on wins}}{\text{Average loss on losses}}$$

## Test

The test set represents 40%, and it includes the 20% that is normally used for testing and the 20% that is normally used for validation, but for practical reasons they were combined into one.

| Avg Win/Loss |
|--------------|
| 1.66         |

As we can see, the average win/loss obtained was 1.66, indicating that, on average, each winning trade was 1.66 times larger than each losing trade. Even with a win rate of 50%, this ratio shows that the strategy generated larger gains on profitable trades than the losses incurred on unprofitable ones, which is a positive sign of risk-adjusted performance. A win/loss ratio above 1 suggests that the strategy can remain profitable even with a relatively modest hit rate, as long as the average win continues to outweigh the average loss.

## Profit factor

This statistic measures the relationship between total profits and total losses in a trading strategy, measuring the amount of profit earned for every dollar lost. The

higher the value, the more profitable and efficient the strategy is overall. It is calculated as follows:

$$\text{Profit factor} = \frac{\text{Gross profit}}{\text{Gross loss}}$$

#### Test

The test set represents 40%, and it includes the 20% that is normally used for testing and the 20% that is normally used for validation, but for practical reasons they were combined into one.

| Profit Factor |
|---------------|
| 1.66          |

As we can see, the profit factor obtained was 1.66, which means that for every dollar lost, the strategy earned 1.66 dollars in profit. This shows that the strategy was overall profitable and managed to generate gains that clearly outweighed its losses.

#### Cash and portfolio value

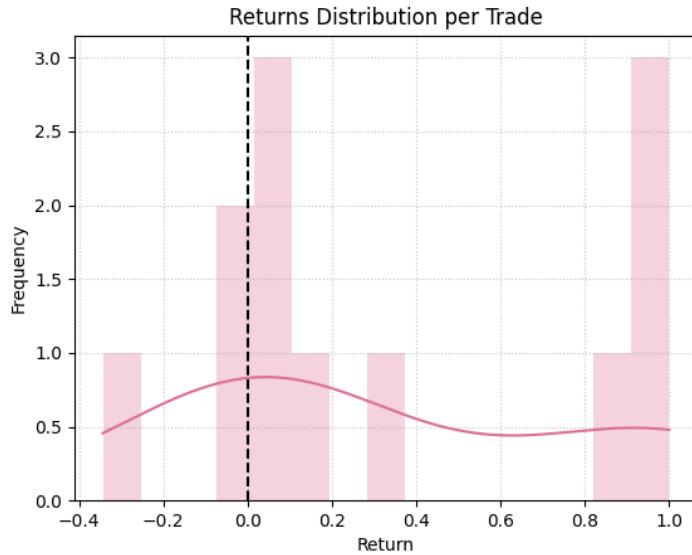
#### Test

The test set represents 40%, and it includes the 20% that is normally used for testing and the 20% that is normally used for validation, but for practical reasons they were combined into one.

| Cash           | Portfolio Value |
|----------------|-----------------|
| \$1,249,956.78 | \$1,251,234.48  |

As we can see, the strategy ended with a cash balance of \$1,249,956.78 and a total portfolio value of \$1,251,234.48. This reflects consistent growth during the testing period, suggesting that the model managed trades effectively and maintained solid capital preservation.

## Out-of-sample performance on validation period



As we can see in the Portfolio Value Over Time plot and the Returns Distribution per Trade, the strategy performed well on unseen data, as the returns are mostly concentrated on the right side of the distribution, indicating positive results. The presence of several trades with returns close to zero suggests periods of stability, while the positive tail shows that some trades generated significant profits. Additionally, the relatively low frequency of negative returns implies that losses were limited compared to the profitable trades, reflecting a favorable risk-reward balance.

## Cost analysis

### Total commissions paid

| Total Commissions |
|-------------------|
| \$8,154.32        |

As we can see, the total commissions paid amounted to \$8,154.32, resulting from the 0.125% commission rate applied to both entry and exit trades. This reflects the transaction costs associated with actively managing positions across both assets, since the strategy invested 80% of the available capital (40% per asset) and executed a limited number of trades.

### Total borrow costs

| Total Borrow Costs |
|--------------------|
| \$24.68            |

As we can see, the total borrow costs amounted to \$24.68. With a borrow rate of 0.25% annualized and daily interest accrual on borrowed amounts, this low value suggests that short exposure was minimal, leading to very low financing costs.

## Conclusions

### Key findings and strategy viability

The results of this project show that the pairs trading strategy built on the ET–WMB cointegrated relationship is statistically validated through the tests performed and operationally viable. The pair passed all three selection filters: correlation, Engle-Granger, and Johansen tests, demonstrating both short-term co-movement and a robust long-term equilibrium relationship. The use of the Kalman Filter for dynamic hedge ratio estimation and VECM-based Z-score thresholds allowed the strategy to adapt to evolving market conditions and capture short-term mean-reversion opportunities effectively.

The equity curve and trade distribution indicate that the strategy generated consistent gains during the test period, reaching a peak near \$1.4 millions and ending at approximately \$1.25 millions. Performance metrics such as Sharpe, Sortino, and Calmar ratios suggest a favorable risk-adjusted return profile, with limited drawdowns and a positive skew in trade outcomes. Overall, the model behaved as expected for a market-neutral, mean-reversion-based approach.

## Whether strategy would be profitable after costs

The cost analysis confirms that the strategy would remain profitable once realistic trading expenses are incorporated. The total commissions paid were \$8,154.32 and the total borrow costs were \$24.68, which represent a small fraction of portfolio value, especially given the limited number of trades and moderate turnover. The final portfolio value of \$1,251,234.48 and the consistently positive distribution of returns in the out-of-sample validation confirm that profits comfortably exceed transactional and financing costs.

## Potential improvements or extensions

Although the strategy demonstrated positive performance, several improvements could reinforce its robustness and adaptability. One important extension involves incorporating mechanisms to detect changes in market behavior, this way is easier to identify periods when the cointegration relationship weakens or temporarily breaks down. As observed around 2016, structural changes in the energy sector can alter the spread dynamics, reducing mean-reversion strength. Introducing filters that detect such market behavior, using rolling cointegration tests, spread volatility shifts, or trend indicators, would allow the strategy to pause trading or adjust its parameters during unfavorable periods.

Another potential improvement involves expanding the strategy structure into a multi-pair or portfolio-based approach. Instead of trading only one cointegrated pair, selecting several uncorrelated or weakly correlated cointegrated pairs would improve diversification and smooth the equity curve. A portfolio of pairs reduces risk concentration, increases the number of trading opportunities, and often delivers superior risk-adjusted returns compared to a single-pair strategy.

Finally, the strategy could explore data-driven optimization techniques to refine entry and exit thresholds. While the  $\pm 1.5$  Z-score level was selected through manual testing, more advanced methods, such as Bayesian optimization, walk-forward validation, or reinforcement learning, could identify adaptive thresholds that respond to changing market behavior. These techniques may uncover nonlinear patterns or dynamic threshold rules that improve profitability and reduce downside risk.

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