Formale Systeme
Example tasks for Test 1, 2015

Task 1. (15) Write down the following statement as a predicate formula:

There exists a 3-element subset of natural numbers that has a 2-element subset whose sum of elements is not divisible by 2.

Is this statement true?

Task 2. (15) Check if the following propositional formula is a tautology. Prove your answer.
$$(P \Rightarrow Q) \vee P \vee (Q \wedge \neg R).$$

Task 3. (15) Prove with a calculation that the following abstract propositions are equivalent

$$((a \Rightarrow b) \Rightarrow \neg a) \qquad \text{and} \qquad (\neg b \vee \neg a) \wedge (\neg b \vee b).$$

Task 4. (15 + 5) Prove with a derivation that the following formula is a tautology:

$$(\forall_x[D(x) : P(x)] \wedge \neg \exists_y[D(y) : P(y)]) \Rightarrow \neg \exists_z[D(z) : T].$$

Using that, show that the following statement is true: If all sheep are white and there is no white sheep, then there is no sheep.

Task 5. (15) Prove with a calculation that for any two sets A and B we have

$$A \subseteq B \quad \text{if and only if} \quad B^c \subseteq A^c.$$

Task 6. (20) Prove that for any two sets A and B

$$A \cup B \subseteq A \quad \text{if and only if} \quad A^c \subseteq B^c.$$