

# Relaxing the Consistency Condition

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Local Linearizability  
(CONCUR16)

# Local Linearizability

## main idea

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- Partition a history into a set of local histories
- Require linearizability per local history

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Already present in some shared-memory consistency conditions  
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Local sequential consistency... is also possible

# Local Linearizability main idea

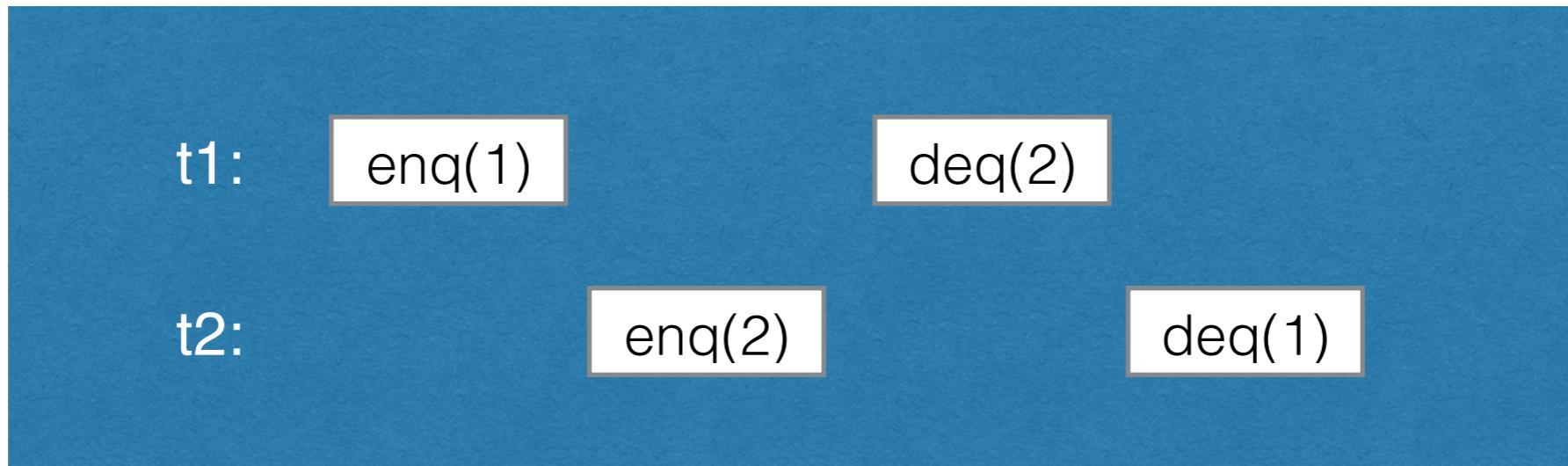
Already present in some shared-memory consistency conditions  
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- Partition a history into a set of local histories
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no global witness

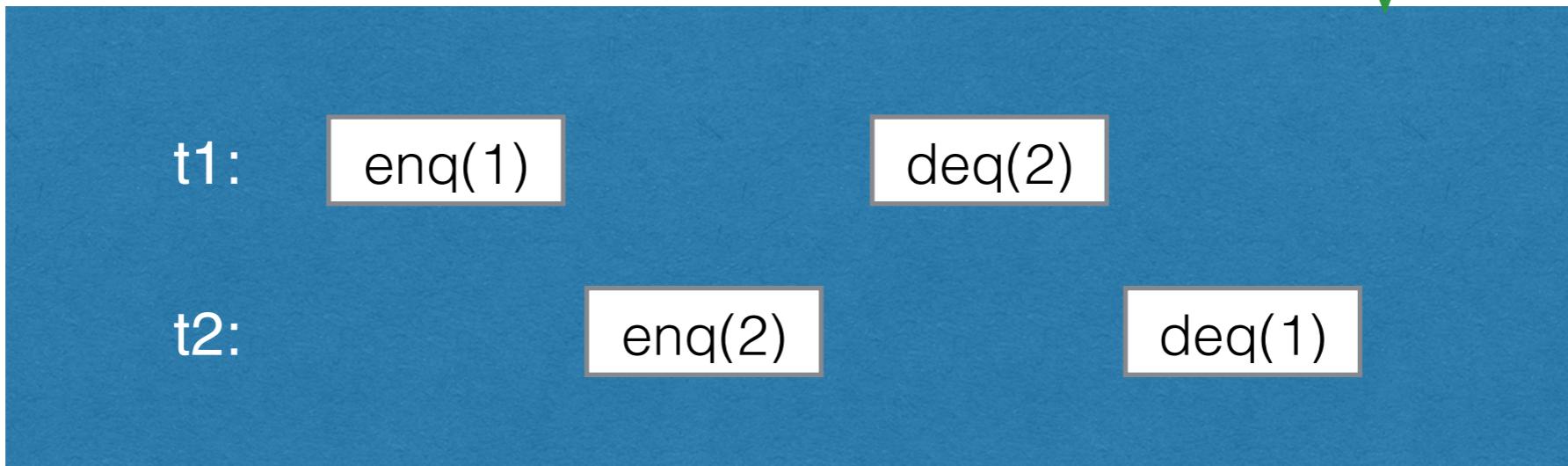
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# Local Linearizability (queue) example



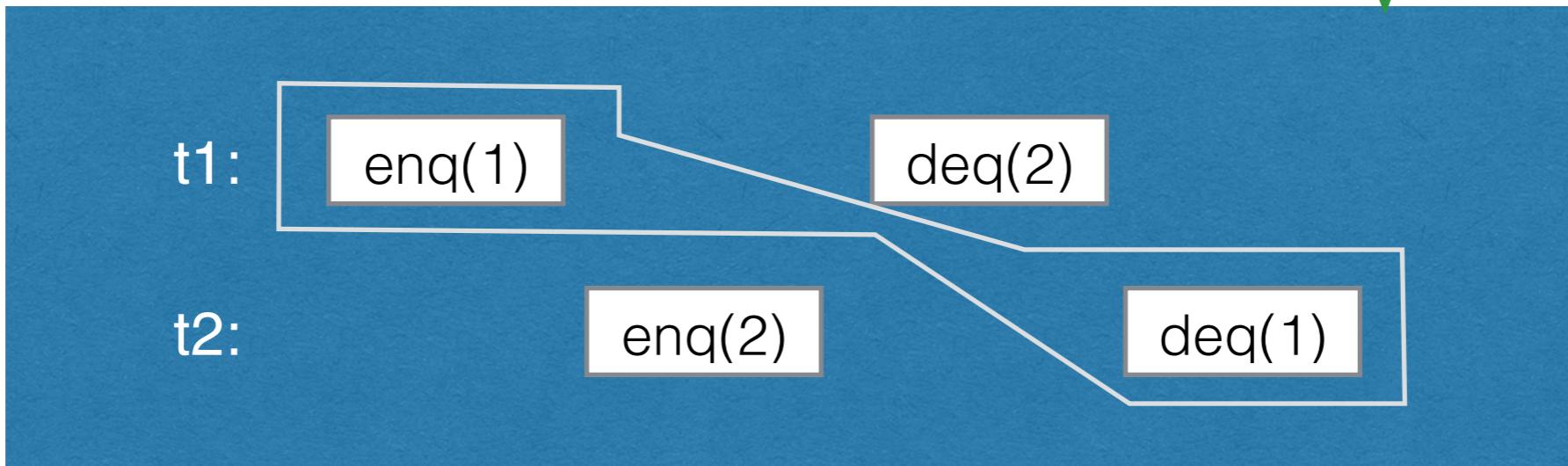
# Local Linearizability (queue) example

(sequential) history  
not linearizable



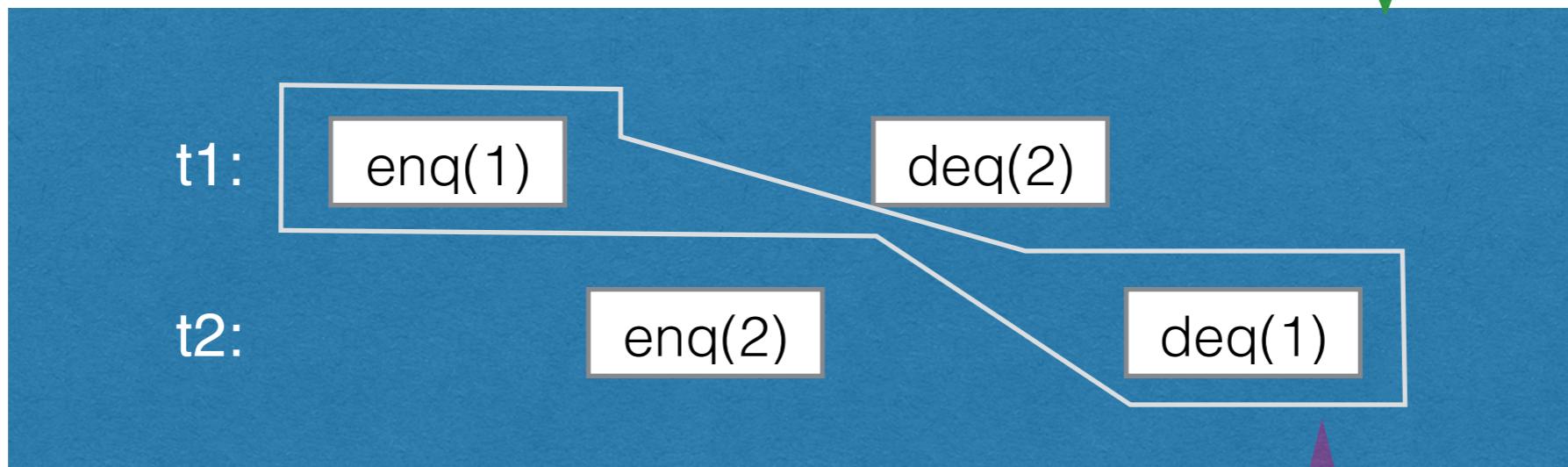
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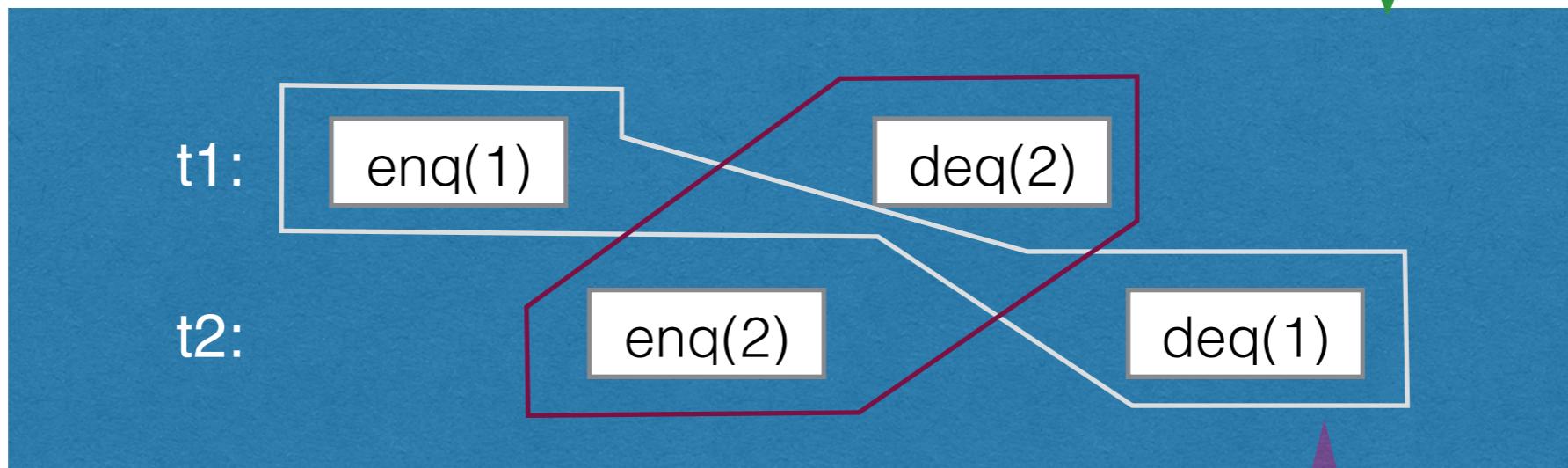
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t1-induced history,  
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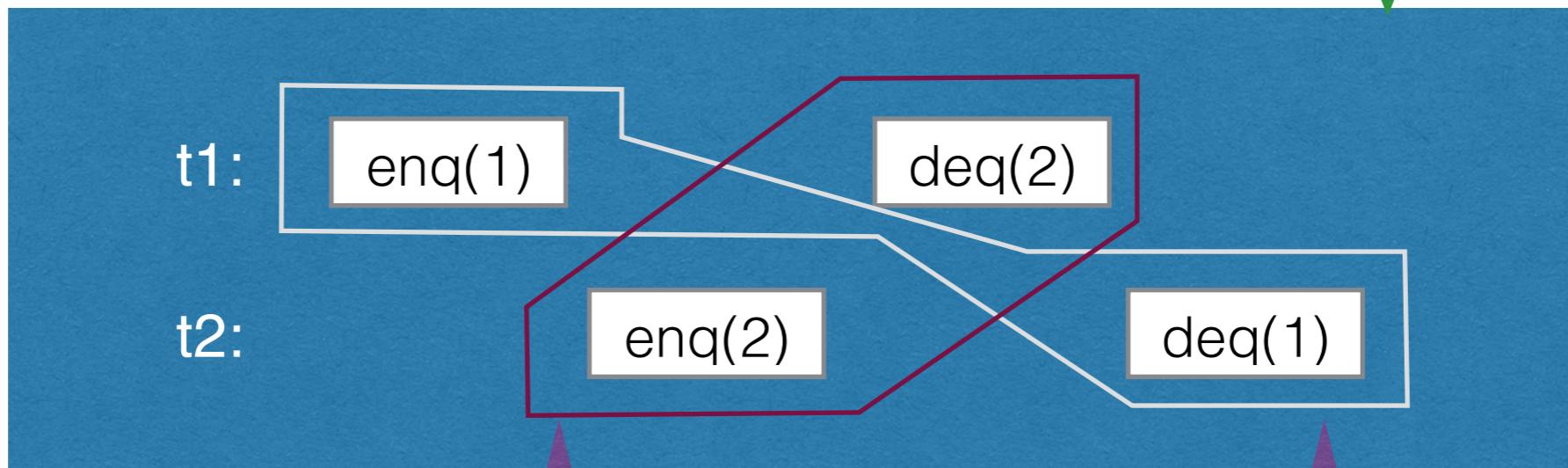
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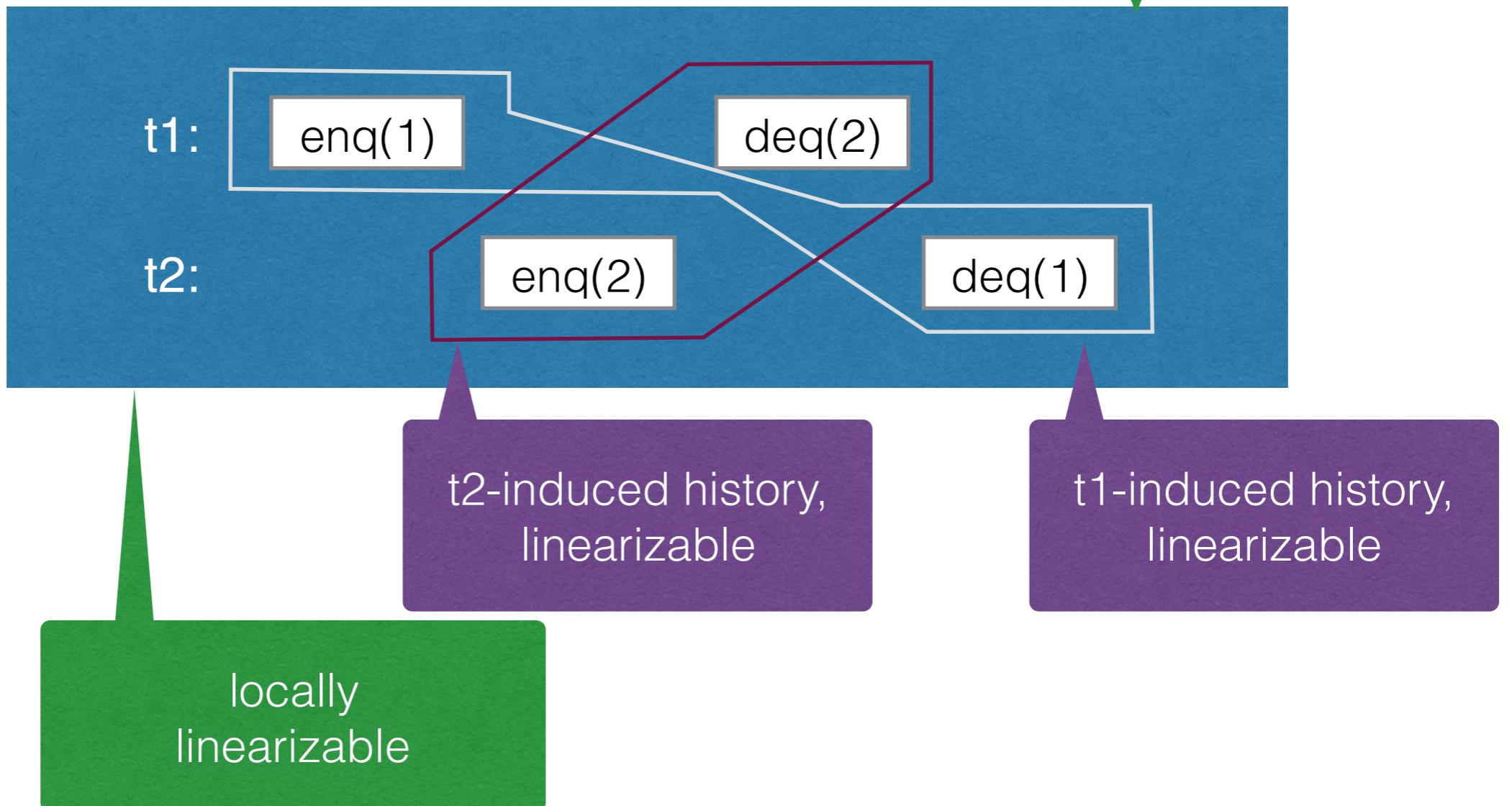


t2-induced history,  
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t1-induced history,  
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For a history  $\mathbf{h}$  with a thread  $T$ , we put

$$I_T = \{\text{enq}(x)^T \in \mathbf{h} \mid x \in V\}$$

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by increasing the  
in-methods,  
LL gradually moves to  
linearizability

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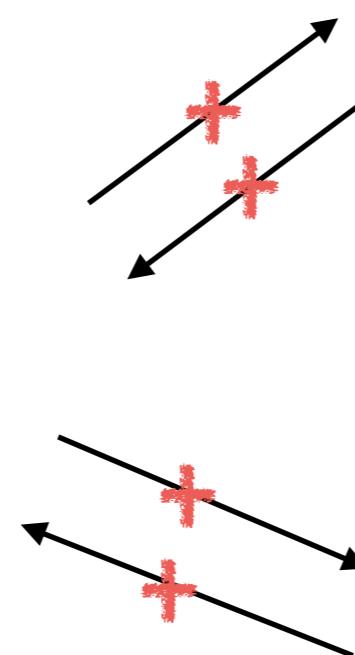
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# Where do we stand?

In general

Local Linearizability



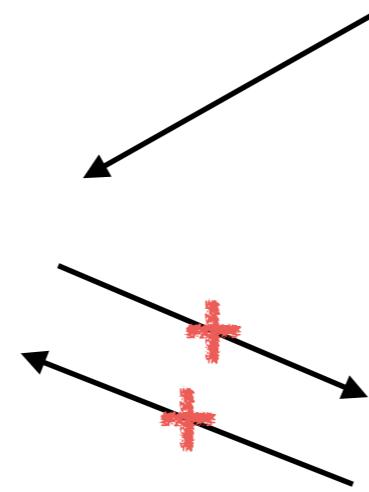
Linearizability

Sequential Consistency

# Where do we stand?

For queues (and most container-type data structures)

Local Linearizability



Linearizability

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may allow for modular verification