Process Algebra
Uitwerkingen opgaven practicum 8

Hieronder staan de uitwerkingen van de volgende opgaven:

5.1.9: 1 en 3;

5.2.6: 1 en 2;

5.4.34: 4 en 8;

5.5.18: 9

Exercise 5.1.9.1

Show that $\text{BPA}_\delta^\tau \vdash x(\tau y + y) = xy$.

$$
\begin{aligned}
& x(\tau y + y) \\
\stackrel{\text{A3}}{=}\; & x(\tau(y + y) + y) \\
\stackrel{\text{B2}}{=}\; & x(y + y) \\
\stackrel{\text{A3}}{=}\; & xy
\end{aligned}
$$

Exercise 5.1.9.3

Give an example to show that $\pi_n \circ \tau_I(x) \neq \tau_I \circ \pi_n(x)$.
Take $x \equiv ab$, $n = 1$, and $I = \{a\}$, then we have $\pi_n \circ \tau_I(x) = \pi_1 \circ \tau_{\{a\}}(ab) = \pi_1(\tau b) = \tau\pi_1(b) = \tau b$ and $\tau_I \circ \pi_n(x) = \tau_{\{a\}} \circ \pi_1(ab) = \tau_{\{a\}}(a) = \tau$. It is clear that $\tau b \neq \tau$.
Of course, there are lots of other examples.

Exercise 5.2.6.1

i)

$$
\begin{aligned}
& a(\tau \parallel x) \\
= \;& a\tau \mathbin{|\!\lfloor} x \\
= \;& a \mathbin{|\!\lfloor} x \\
= \;& ax
\end{aligned}
$$

ii

$$
\begin{aligned}
& a(\tau x \parallel y) \\
= \; & a\tau x \mathbin{⌊\!⌊} y \\
= \; & ax \mathbin{⌊\!⌊} y \\
= \; & a(x \parallel y)
\end{aligned}
$$

iii) Show that CM6 is derivable from the other axioms of ACP$^\tau$ and **i)**. CM6: $a \mid bx = (a \mid b)x$.

There are two cases:

1. a and b communicate: $\gamma(a, b) = c$
2. a and b do not communicate: $\gamma(a, b)$ is undefined

Proof of case 1:

$$
\begin{aligned}
& a \mid bx \\
\overset{\text{B1}}{=} \; & a\tau \mid bx \\
\overset{\text{CM2}}{=} \; & (a \mid b)(\tau \parallel x) \\
\overset{\text{CF1}}{=} \; & c(\tau \parallel x) \\
\overset{\text{i}}{=} \; & cx \\
\overset{\text{CF1}}{=} \; & (a \mid b)x
\end{aligned}
$$

Proof of case 2:

$$
\begin{aligned}
& a \mid bx \\
\overset{\text{B1}}{=} \; & a\tau \mid bx \\
\overset{\text{CM2}}{=} \; & (a \mid b)(\tau \parallel x) \\
\overset{\text{CF2}}{=} \; & \delta(\tau \parallel x) \\
\overset{\text{A7}}{=} \; & \delta \\
\overset{\text{A7}}{=} \; & \delta x \\
\overset{\text{CF2}}{=} \; & (a \mid b)x
\end{aligned}
$$

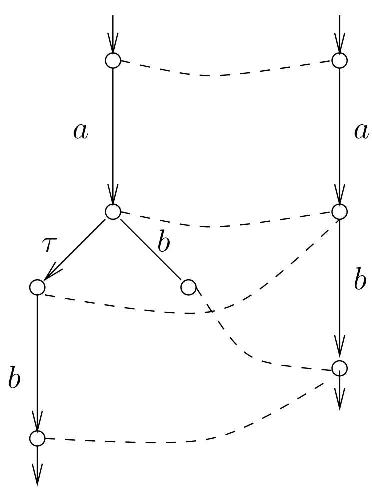
Exercise 5.2.6.2 (iv)

We will not proof i, ii, and iii (they can be proved by rewriting with the axioms of ACP$^\tau$), but we will use ii in the proof of iv. Note that i, ii, iii, and iv should be proven for arbitrary ACP$^\tau$ terms; therefore, you cannot use induction in the proofs.

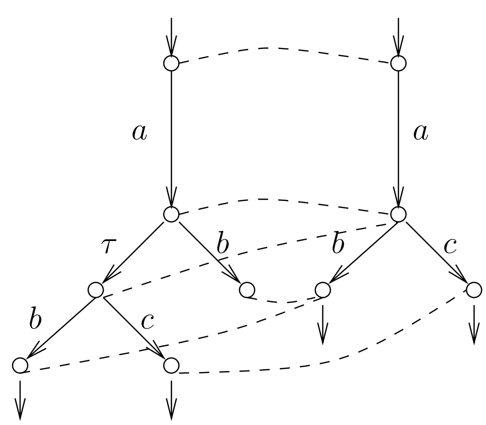
Proof $(\sum_i a_i x_i) \parallel (\sum_j b_j y_j) = (\sum_j b_j y_j) \parallel (\sum_i a_i x_i)$:

$$
\begin{aligned}
& (\textstyle\sum_i a_i x_i) \parallel (\sum_j b_j y_j) \\
\overset{\text{CM1}}{=}\ & (\textstyle\sum_i a_i x_i) \mathbin{|\!\!\lfloor} (\sum_j b_j y_j) + (\sum_j b_j y_j) \mathbin{|\!\!\lfloor} (\sum_i a_i x_i) + (\sum_i a_i x_i) \mid (\sum_j b_j y_j) \\
=\ & (\textstyle\sum_i a_i x_i) \mathbin{|\!\!\lfloor} (\sum_j b_j y_j) + (\sum_j b_j y_j) \mathbin{|\!\!\lfloor} (\sum_i a_i x_i) + (\sum_i \sum_j a_i x_i \mid b_j y_j) \\
=\ & (\textstyle\sum_i a_i x_i) \mathbin{|\!\!\lfloor} (\sum_j b_j y_j) + (\sum_j b_j y_j) \mathbin{|\!\!\lfloor} (\sum_i a_i x_i) + (\sum_j \sum_i a_i x_i \mid b_j y_j) \\
\overset{\text{ii}}{=}\ & (\textstyle\sum_i a_i x_i) \mathbin{|\!\!\lfloor} (\sum_j b_j y_j) + (\sum_j b_j y_j) \mathbin{|\!\!\lfloor} (\sum_i a_i x_i) + (\sum_j \sum_i b_j y_j \mid a_i x_i) \\
=\ & (\textstyle\sum_i a_i x_i) \mathbin{|\!\!\lfloor} (\sum_j b_j y_j) + (\sum_j b_j y_j) \mathbin{|\!\!\lfloor} (\sum_i a_i x_i) + (\sum_j b_j y_j) \mid (\sum_i a_i x_i) \\
\overset{\text{A1}}{=}\ & (\textstyle\sum_j b_j y_j) \mathbin{|\!\!\lfloor} (\sum_i a_i x_i) + (\sum_i a_i x_i) \mathbin{|\!\!\lfloor} (\sum_j b_j y_j) + (\sum_j b_j y_j) \mid (\sum_i a_i x_i) \\
\overset{\text{CM1}}{=}\ & (\textstyle\sum_j b_j y_j) \parallel (\sum_i a_i x_i)
\end{aligned}
$$

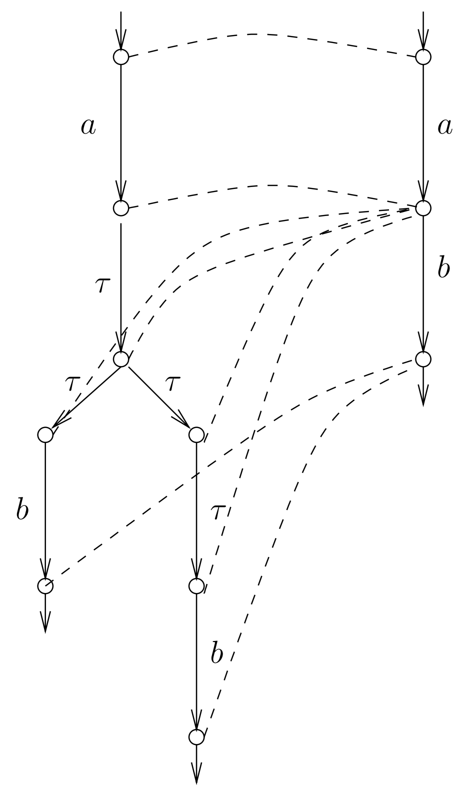
Exercise 5.4.34.4



5.4.34.4(i)



5.4.34.4(ii)



5.4.34.4(iii)

Exercise 5.4.34.8

Find two different solutions for the equation $x = \tau x$.

Axiom B1, $x\tau = x$, says that you can leave out τ's if they are not at the beginning. We can use this to find solutions for the given equation. Since the right-hand side of the equation starts with a τ, every τ following it can be left out. So, every term starting with τ is a solution for the equation. Examples: τa, $\tau(a + b)$, for some actions a and b.

Exercise 5.5.18.9

Omdat de vergelijkingen voor B_2^{13} en B_d guarded zijn, mogen we RSP gebruiken om te concluderen dat als $\tau_I \circ \partial_H(B^{12} \parallel B^{23})$ aan de vergelijkingen voor B_2^{13} voldoet, de gelijkheid geldt.

Hiertoe moeten we eerst een uitdrukking voor B_d vinden. Daar $B_2^{13} = \sum_{d \in D} r_1(d) \cdot B_d$, en $\tau_I \circ \partial_H(B^{12} \parallel B^{23}) = \tau_I \circ \partial_H(\sum_{d \in D} r_1(d) \cdot (s_2(d) \cdot B^{12} \parallel B^{23})$, dienen we $B_d = \tau_I \circ \partial_H(s_2(d) \cdot B^{12} \parallel B^{23}))$ te nemen.

De eerste vergelijking klopt dan, voor de tweede geldt:

$$\begin{array}{ll}
B_d & = \\
\tau_I \circ \partial_H(s_2(d) \cdot B^{12} \parallel B^{23}) & = \\
\tau_I \circ \partial_H(c_2(d) \cdot (B^{12} \parallel s_3(d) \cdot B^{23})) & = \\
\tau \cdot \tau_I \circ \partial_H(B^{12} \parallel s_3(d) \cdot B^{23}) & = \\
\tau \cdot \tau_I \circ \partial_H(\sum_{e \in D} r_1(e) \cdot (s_2(e) \cdot B^{12} \parallel s_3(d) \cdot B^{23}) + s_3(d) \cdot B^{12} \parallel B^{23}) & = \\
\tau \cdot \tau_I \circ \partial_H(\sum_{e \in D} r_1(e) \cdot s_3(d) \cdot \cdot_2 (e) \cdot B^{12} \parallel B^{23} + s_3(d) \cdot B^{12} \parallel B^{23}) & = \\
\tau \cdot (\sum_{e \in D} r_1(e) \cdot s_3(d) \cdot \tau_I \circ \partial_H(s_2(e) \cdot B^{12} \parallel B^{23}) + s_3(d) \cdot \tau_I \circ \partial_H(B^{12} \parallel B^{23})) & = \\
\tau \cdot (\sum_{e \in D} r_1(e) \cdot s_3(d) \cdot B_d + s_3(d) \cdot B_2^{13}) &
\end{array}$$

Hiermee zijn we bijna waar we wezen willen, er is alleen nog die vervelende τ aan het begin. Daartoe introduceren we een licht gewijzigd stelsel van specificaties:

$$B_2'^{13} = \sum_{d \in D} r_1(d) \cdot B_d'$$

$$B_d' = \tau \cdot (s_3(d) \cdot B_2^{13'} + \sum_{e \in D} r_1(e) \cdot s_2(d) \cdot B_e'$$

Hierboven is aangetoond dat $\tau_I \circ \partial_H(B^{12} \parallel B^{23})$ aan deze vergelijkingen voldoet (neem opnieuw $B_d' = \tau_I \circ \partial_H(s_2(d) \cdot B^{12} \parallel B^{23})))$, en het is ook eenvoudig in te zien dat B_2^{13} hieraan voldoet volgens $B_2^{13'} = B_2^{13}$ en $B_d' = \tau \cdot B_d$ (zie onder). Daar ook deze vergelijkingen guarded zijn, mogen we opnieuw RSP toepassen en daarom nu wel concluderen dat $\tau_I \circ \partial_H(B^{12} \parallel B^{23})$ en B_2^{13} gelijk zijn.

Het bewijs dat B_2^{13} voldoet aan de vergelijking voor $B_2^{13'}$:

Neem $B'_d = \tau \cdot B_d$.

$$\begin{array}{ll}
B_2^{13'} & = \\
B_2^{13} & = \\
\sum_{d \in D} r_1(d) \cdot B_d & = \\
\sum_{d \in D} r_1(d) \cdot \tau \cdot B_d & = \\
\sum_{d \in D} r_1(d) \cdot B'_d &
\end{array}$$

Dan:

$$\begin{array}{ll}
B'_d & = \\
\tau \cdot B_d & = \\
\tau \cdot (s_j(d) \cdot B_2^{13} + \sum_{e \in D} r_1(e) \cdot s_3(d) \cdot B_e) & = \\
\tau \cdot (s_j(d) \cdot B_2^{13} + \sum_{e \in D} r_1(e) \cdot s_3(d) \cdot \tau B_e) & = \\
\tau \cdot (s_j(d) \cdot B_2^{13'} + \sum_{e \in D} r_1(e) \cdot s_3(d) \cdot B'_e) &
\end{array}$$