



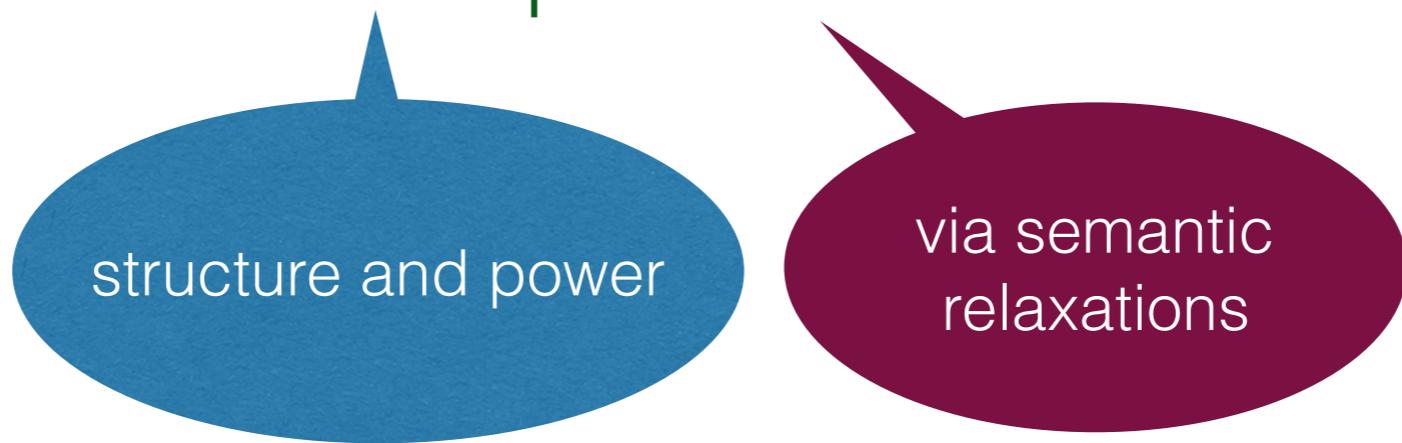


# Semantics of Concurrent Data Structures

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of SALZBURG

AVM, 25.9.2018

# Concurrent data structures correctness and performance



- \* New results enabling verifying linearizability

# Concurrent Data Structures Correctness and Relaxations



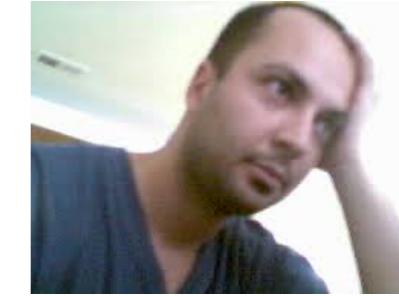
Hannes Payer  
**Google**



Tom Henzinger  
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Christoph Kirsch  
**UNIVERSITY of SALZBURG**



Ali Sezgin  
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Andreas Haas **Google**



Michael Lippautz



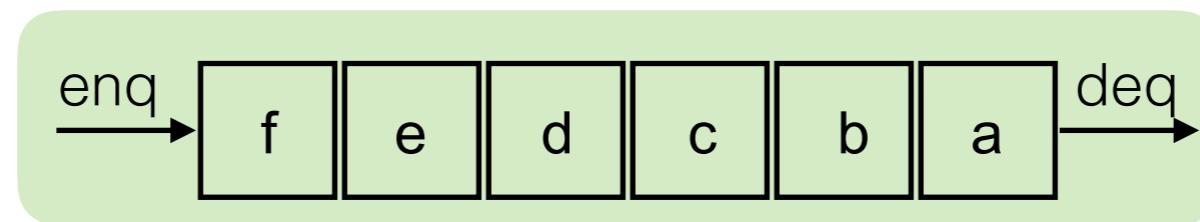
Andreas Holzer  
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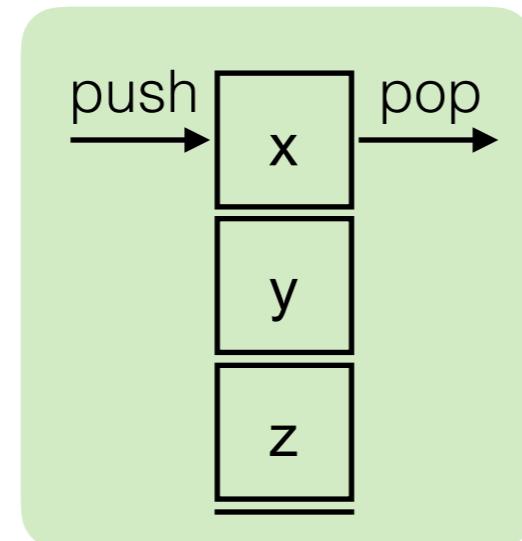
Helmut Veith  
**TU WIEN**

# Data structures

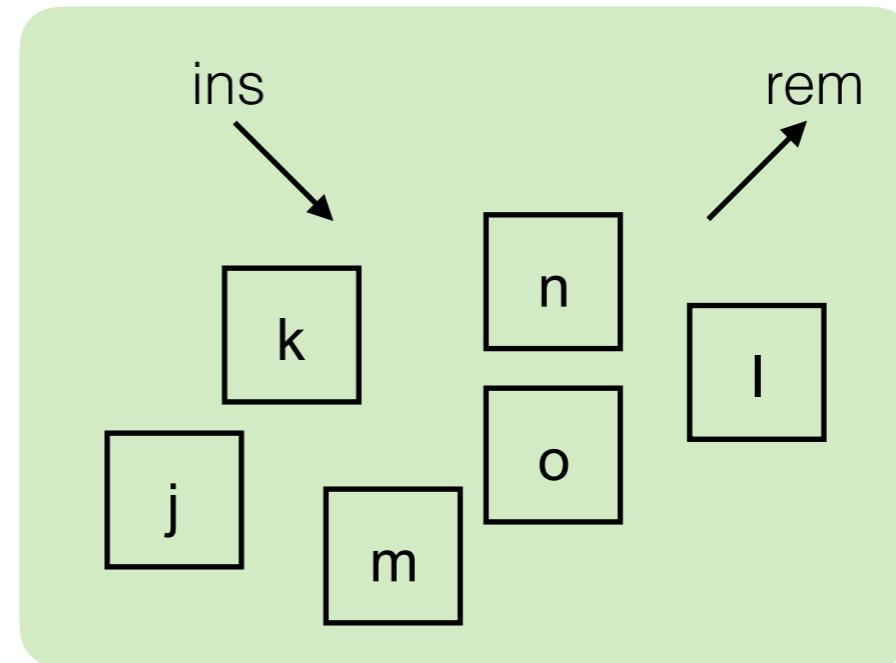
- Queue FIFO



- Stack LIFO

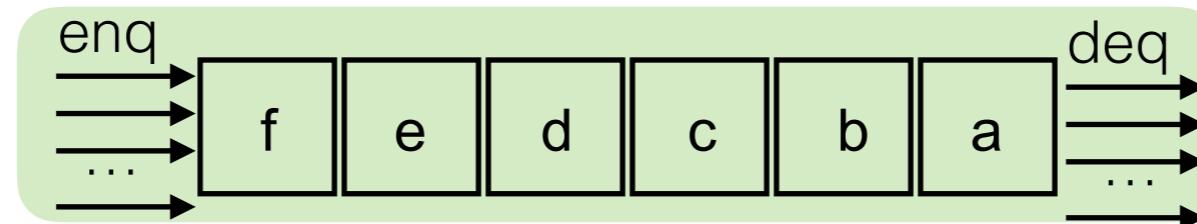


- Pool unordered

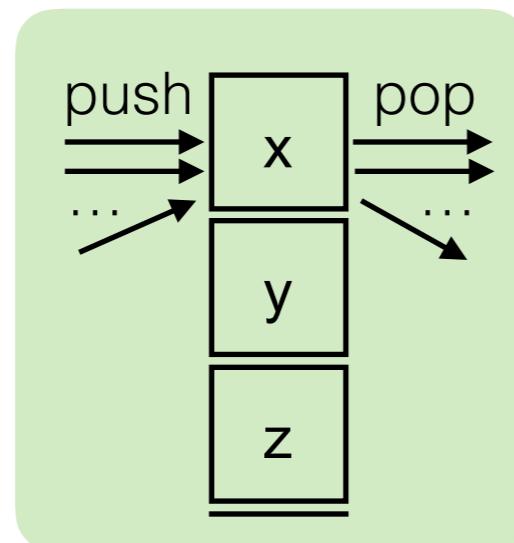


# Concurrent data structures

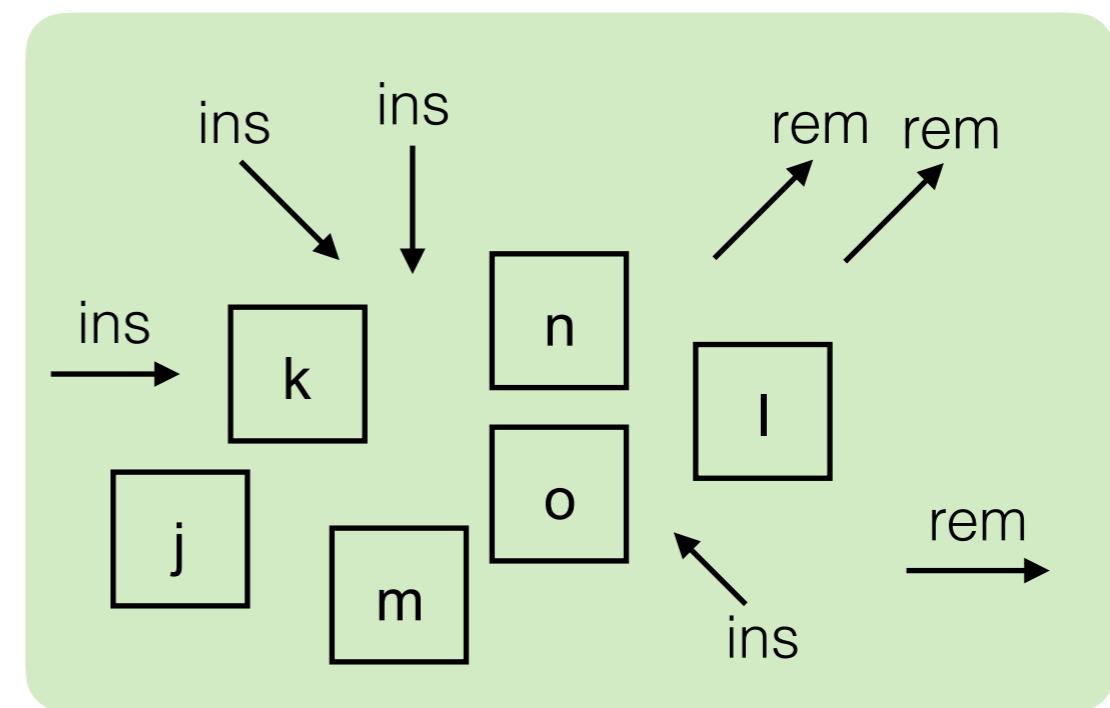
- Queue FIFO



- Stack LIFO



- Pool unordered



# Semantics of concurrent data structures

t1:    enq(2)    deq(1)  
t2:    enq(1)    deq(2)

e.g. queues

- Sequential specification = set of legal sequences

e.g. queue legal sequence  
enq(1)enq(2)deq(1)deq(2)

- Consistency condition = e.g. linearizability / sequential consistency

e.g. the concurrent history above is a linearizable queue concurrent history

# Consistency conditions

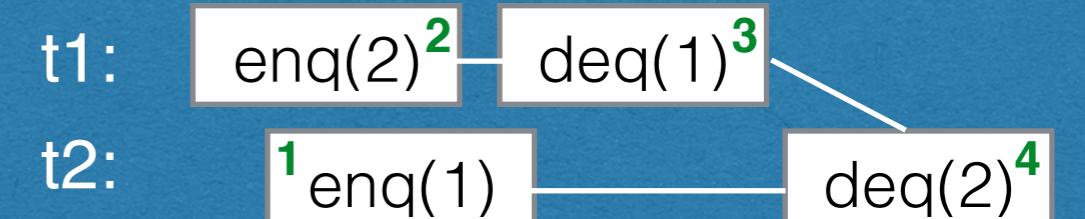
there exists a legal sequence that preserves precedence order

consistency is about extending partial orders to total orders

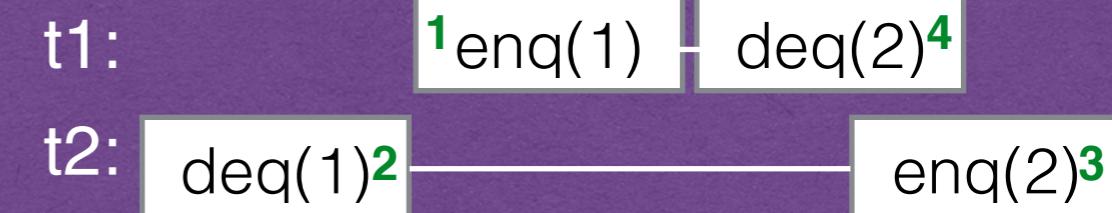
there exists a legal sequence that preserves per-thread precedence (program order)

A history is ... wrt a sequential specification iff

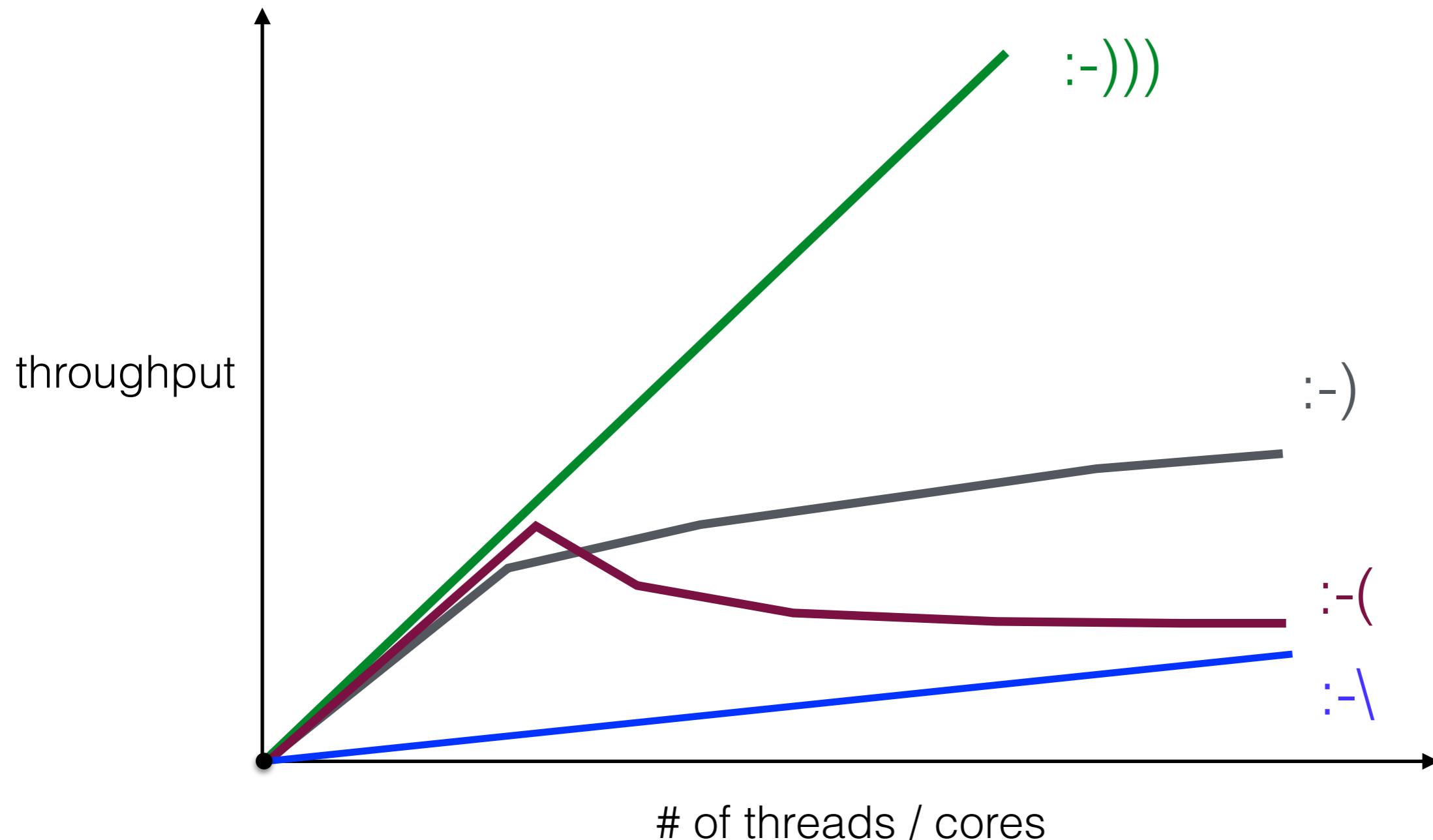
Linearizability [Herlihy,Wing '90]



Sequential Consistency [Lamport'79]



# Performance and scalability



# Relaxations allow trading

correctness  
for  
performance

provide the potential  
for better-performing  
implementations

# Relaxing the Semantics

Quantitative relaxations

Henzinger, Kirsch, Payer, Sezgin, S. POPL13

- Sequential specification = set of legal sequences
- Consistency condition = e.g. linearizability / sequential consistency

Local linearizability

Haas, Henzinger, Holzer, ..., S, Veith CONCUR16

# Relaxing the Sequential Specification

Quantitative  
Relaxations  
(POPL13)

# Goal

Stack - incorrect behavior

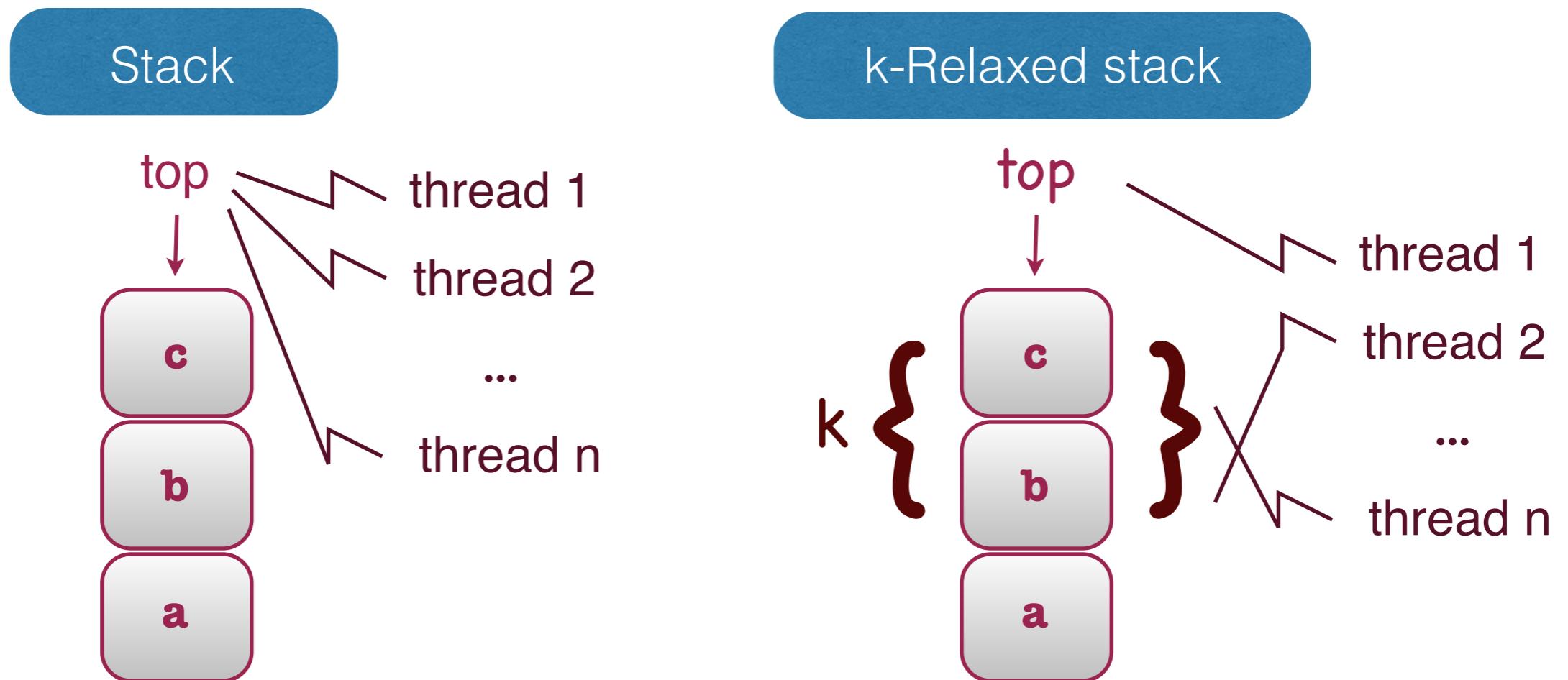
push(a)push(b)push(c)pop(a)pop(b)

- trade correctness for performance
- in a controlled way with quantitative bounds

correct in a relaxed stack  
... 2-relaxed? 3-relaxed?

measure the  
error from correct  
behaviour

# How can relaxing help?



# We have got

- Framework
- Generic examples
  - out-of-order /  
stuttering
- Concrete relaxation examples
  - stacks, queues,  
priority queues,.. /  
CAS, shared counter
- Efficient concurrent implementations
  - of relaxation  
instances

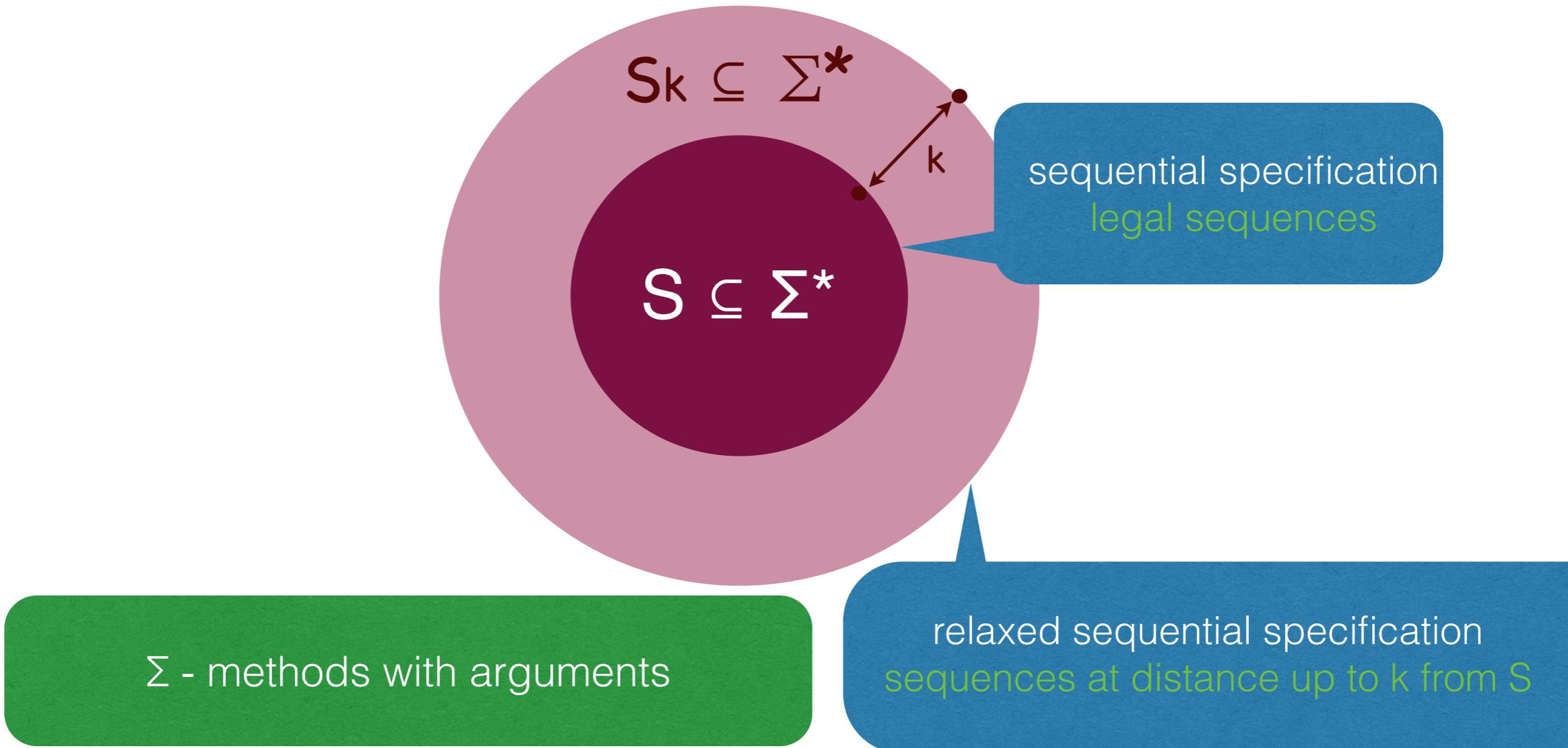
# The big picture

$$S \subseteq \Sigma^*$$

sequential specification  
legal sequences

$\Sigma$  - methods with arguments

# The big picture



# Relaxing the Consistency Condition

Local Linearizability  
(CONCUR16)

# Local Linearizability main idea

Already present in some shared-memory consistency conditions  
(not in our form of choice)

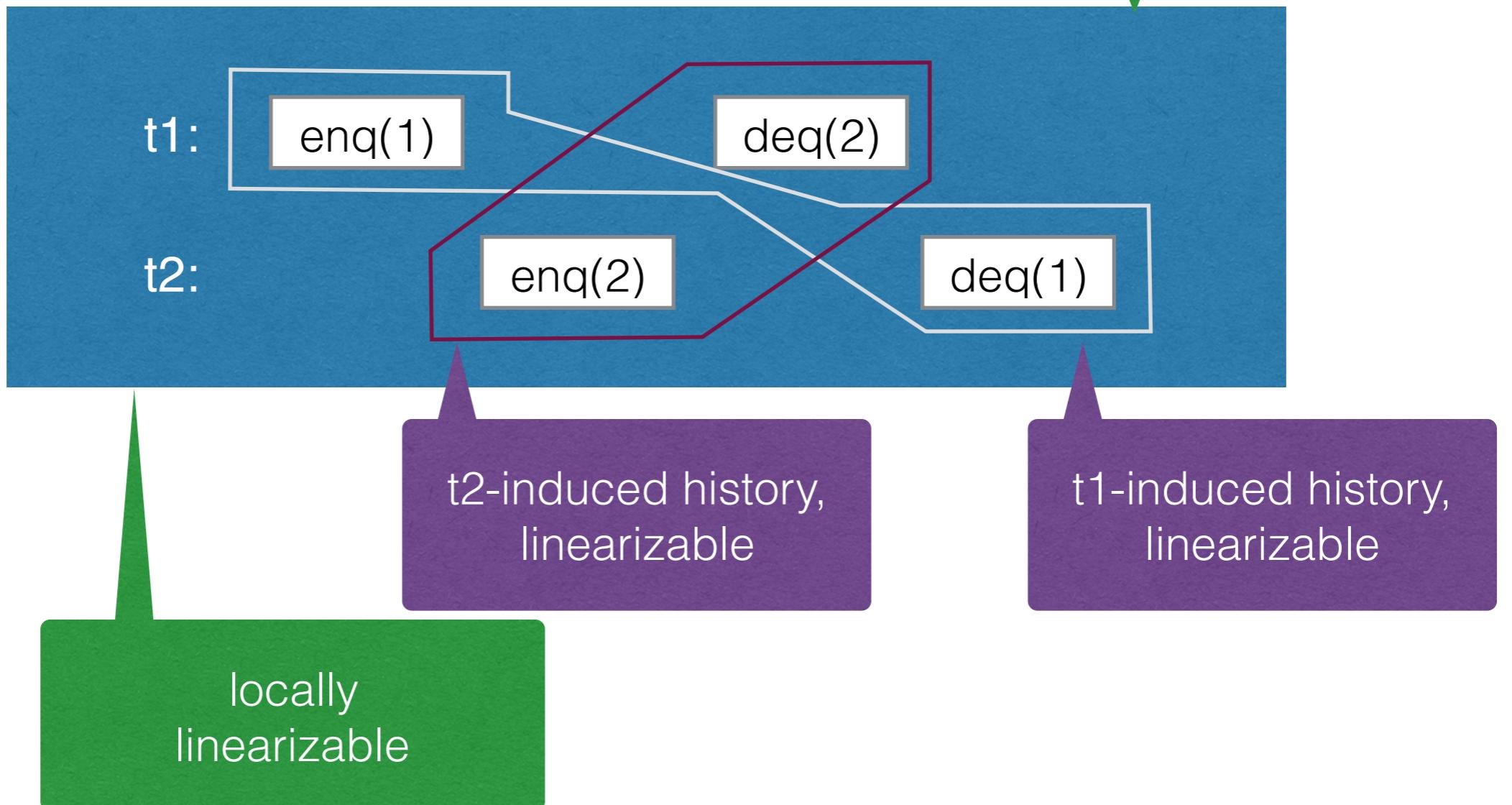
- Partition a history into a set of local histories
- Require linearizability per local history

no global witness

Local sequential consistency... is also possible

# Local Linearizability (queue) example

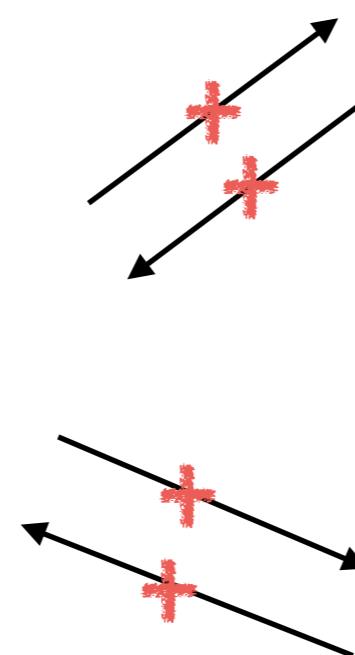
(sequential) history  
not linearizable



# Where do we stand?

In general

Local Linearizability



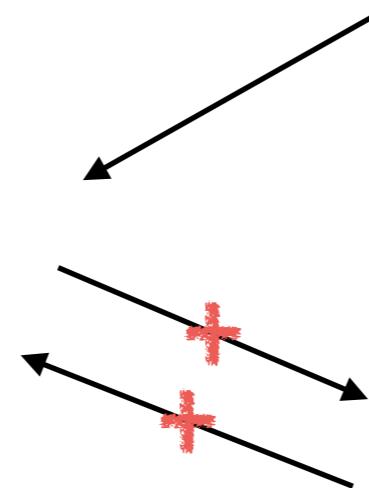
Linearizability

Sequential Consistency

# Where do we stand?

For queues (and most container-type data structures)

Local Linearizability



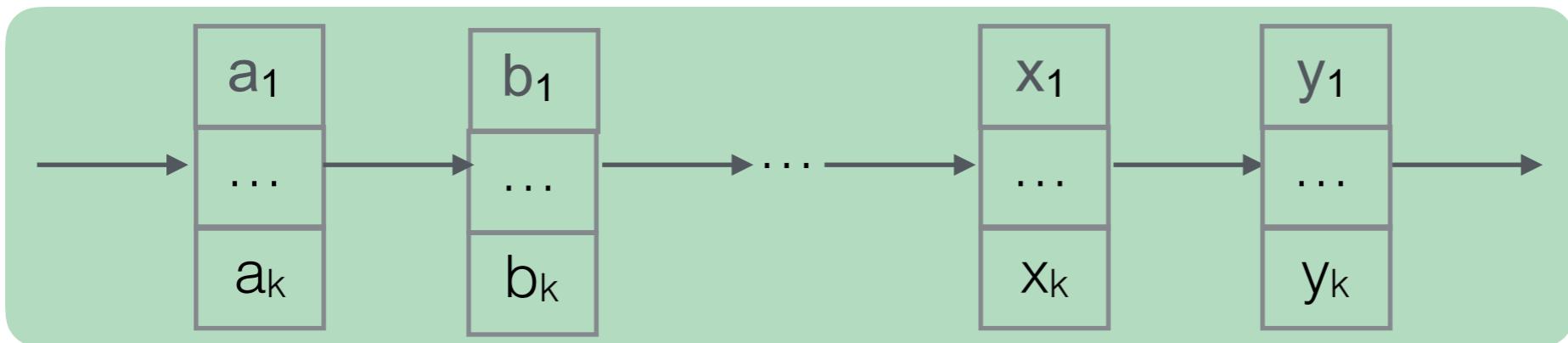
Linearizability



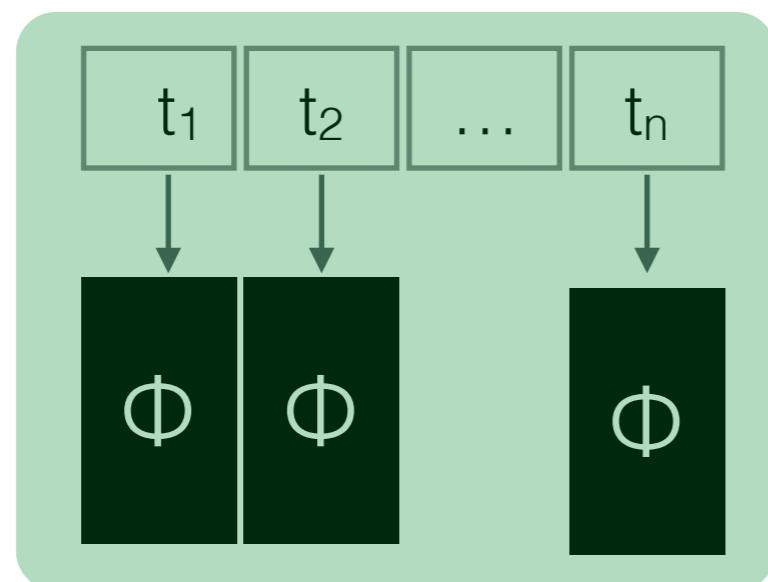
Sequential Consistency

# Lead to scalable implementations

e.g. k-FIFO, k-Stack

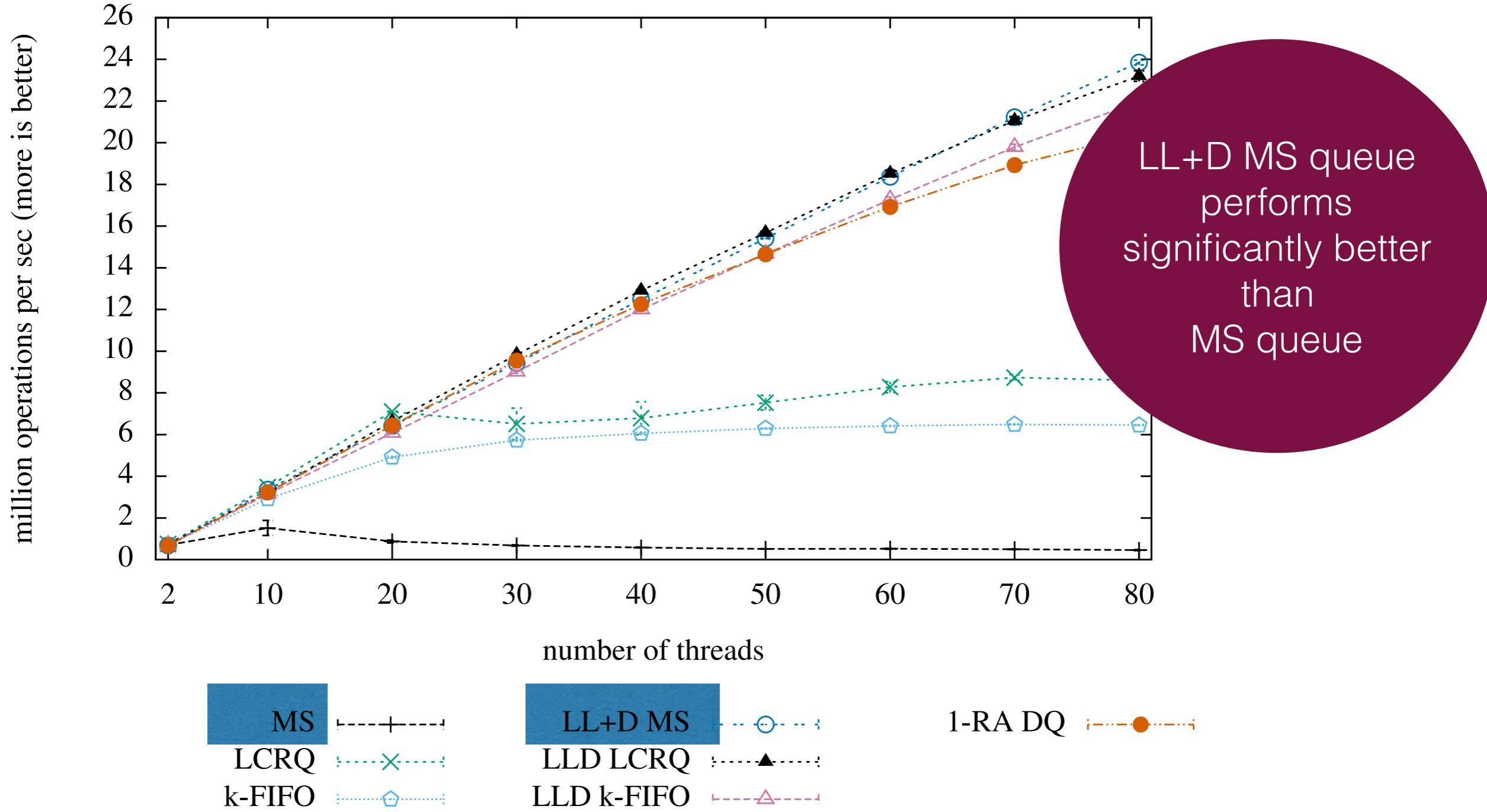


locally linearizable distributed implementation



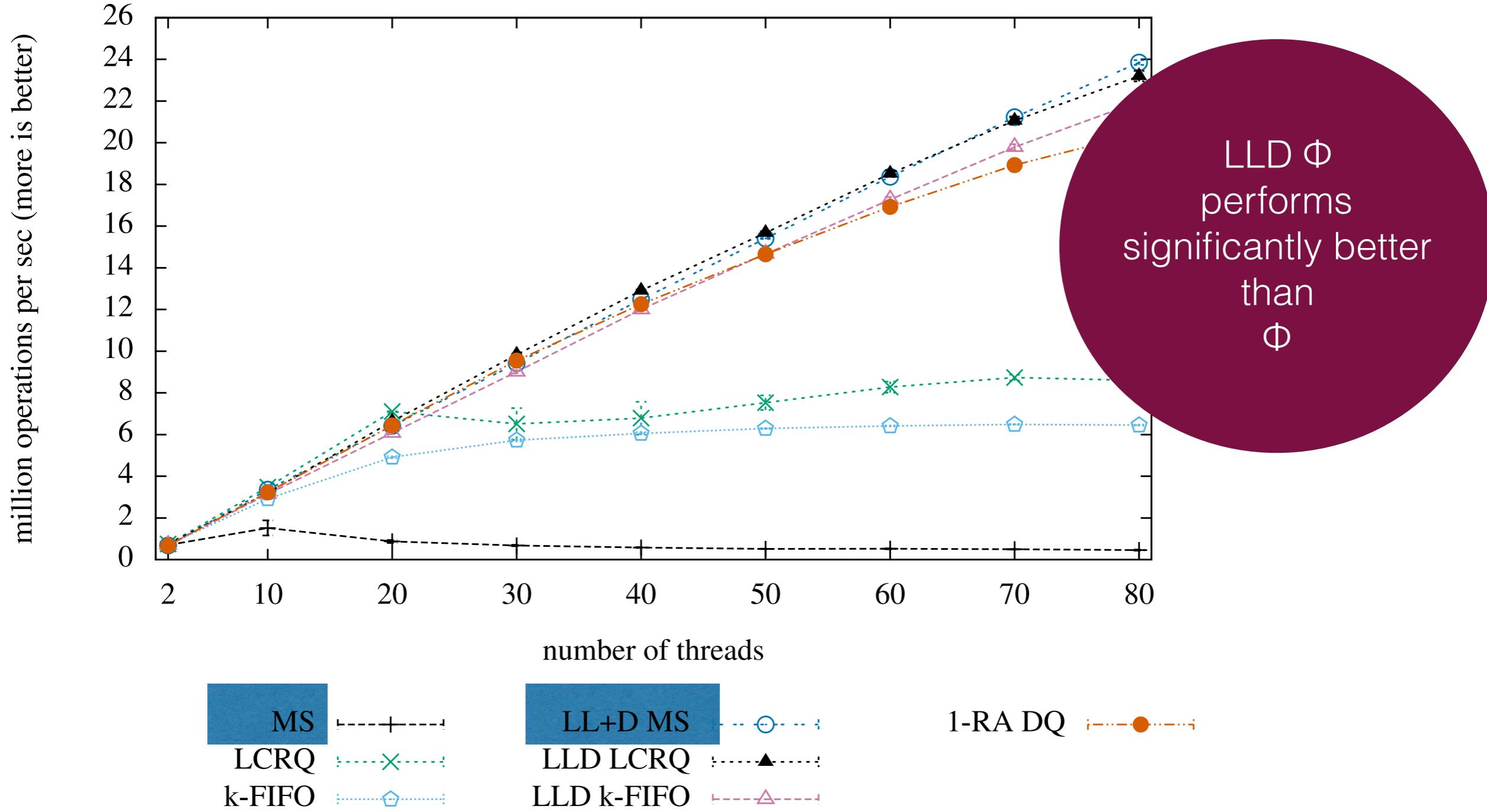
local inserts / global removes

# Performance



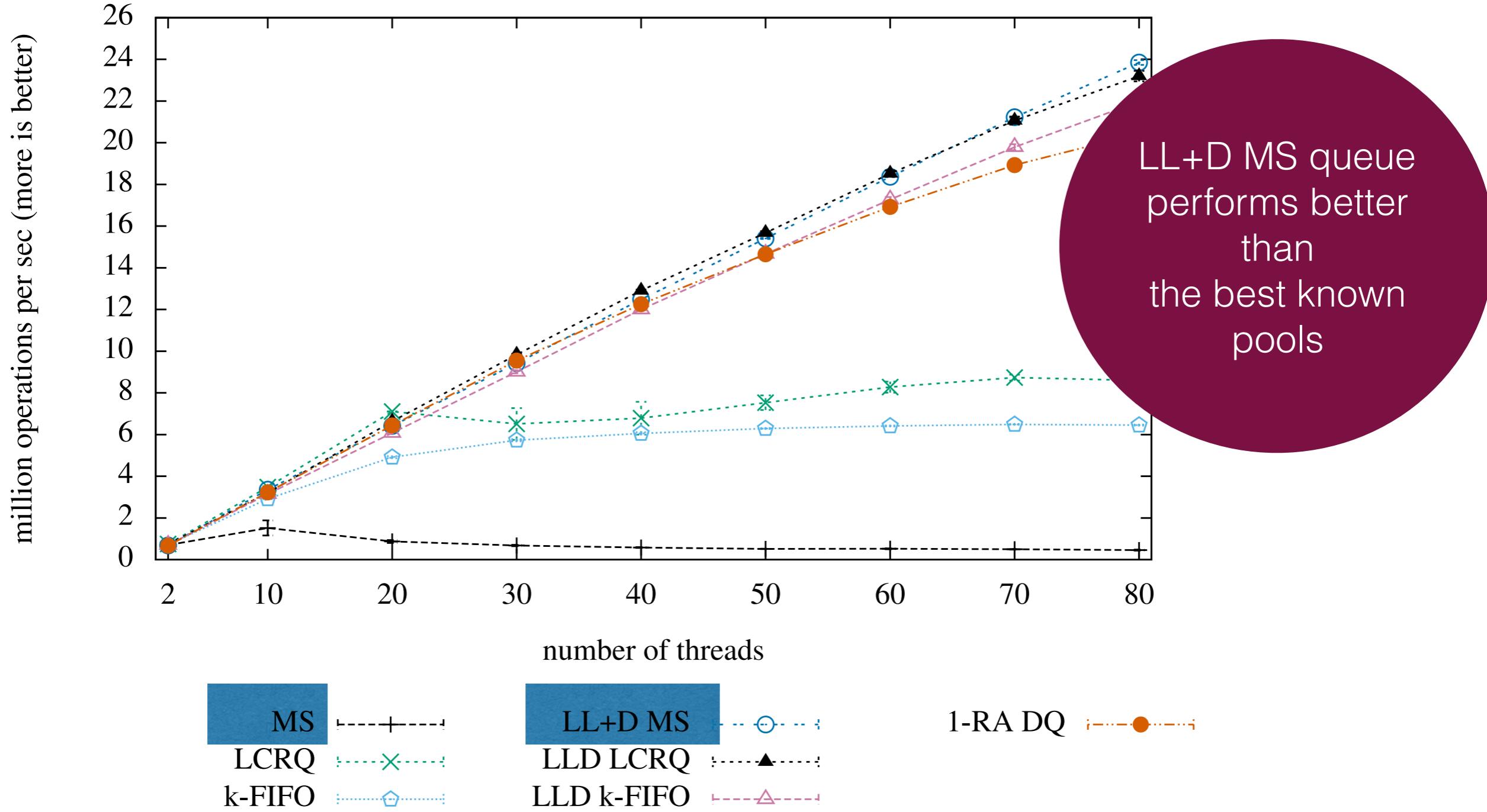
(a) Queues, LL queues, and “queue-like” pools

# Performance



(a) Queues, LL queues, and “queue-like” pools

# Performance



(a) Queues, LL queues, and “queue-like” pools

[scal.cs.uni-salzburg.at](http://scal.cs.uni-salzburg.at)

Scal



High-Performance Multicore-Scalable Computing

We study the design, implementation, performance, and scalability of concurrent objects on multicore systems by analyzing the apparent trade-off between adherence to concurrent data structure semantics and scalability.



Thank You !

[scal.cs.uni-salzburg.at](http://scal.cs.uni-salzburg.at)

Scal



High-Performance Multicore-Scalable Computing



We study the design, implementation, performance, and scalability of concurrent programs. There aren't trade-off between performance and scalability.



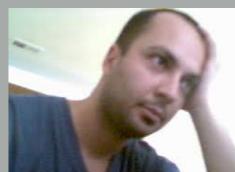
Henning  
Google



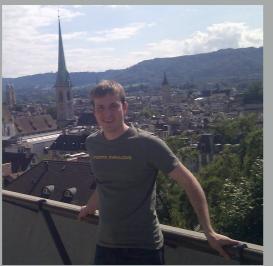
Tom  
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Christian  
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Alin  
UNIVERSITY OF CAMBRIDGE



Andrasz  
Google



Michael



Andrasz  
Google



Thomas  
TU WIEN



Thank You !

# Concurrent Data Structures

## Correctness and Performance



Hannes Payer



Tom Henzinger



Christoph Kirsch



Ali Sezgin



Andreas Haas



Michael Lippautz



Andreas Holzer



Helmut Veith



Thank You !

# Linearizability via Order Extension Theorems

joint work with



foundational results  
for  
verifying linearizability

# Inspiration

Queue sequential specification (axiomatic)

**s** is a legal queue sequence  
iff

1. **s** is a legal pool sequence, and
2.  $\text{enq}(x) <_s \text{enq}(y) \wedge \text{deq}(y) \in s \Rightarrow \text{deq}(x) \in s \wedge \text{deq}(x) <_s \text{deq}(y)$

Queue linearizability (axiomatic)

Henzinger, Sezgin, Vafeiadis CONCUR13

**h** is queue linearizable  
iff

1. **h** is pool linearizable, and
2.  $\text{enq}(x) <_h \text{enq}(y) \wedge \text{deq}(y) \in h \Rightarrow \text{deq}(x) \in h \wedge \text{deq}(y) <_h \text{deq}(x)$

precedence order

As well as  
Reducing Linearizability to  
State Reachability  
[Bouajjani, Emmi, Enea, Hamza]  
ICALP15 + ...

# Concurrent Queues

Data independence => verifying executions where each value is enqueued at most once is sound

Reduction to assertion checking = exclusion of "bad patterns"

Value v dequeued without being enqueued

$\text{deq} \Rightarrow v$



Value v dequeued before being enqueued

$\text{deq} \Rightarrow v$      $\text{enq}(v)$



Value v dequeued twice

$\text{deq} \Rightarrow v$      $\text{deq} \Rightarrow v$



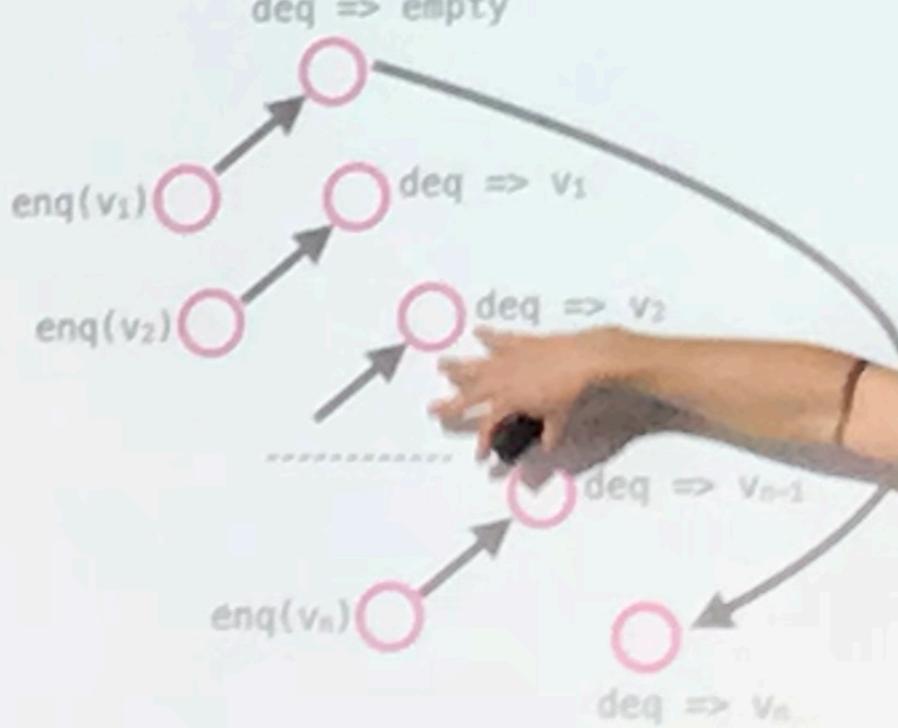
Value  $v_1$  and  $v_2$  dequeued in the wrong order

$\text{enq}(v_1)$      $\text{enq}(v_2)$      $\text{deq} \Rightarrow v_2$      $\text{deq} \Rightarrow v_1$



Dequeue wrongfully returns empty

$\text{deq} \Rightarrow \text{empty}$



# Problems (stack)

## Stack sequential specification (axiomatic)

**s** is a legal stack sequence  
iff

1. **s** is a legal pool sequence, and
2.  $\text{push}(x) <_s \text{push}(y) <_s \text{pop}(x) \Rightarrow \text{pop}(y) \in \mathbf{s} \wedge \text{pop}(y) <_s \text{pop}(x)$

## Stack linearizability (axiomatic)

**h** is stack linearizable  
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???

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# Problems (stack)

t1: push(1)

pop(1)

t2: push(2)

pop(2)

t3: push(3)

pop(3)

not stack  
linearizable

## Stack linearizability (axiomatic)

~~**h** is stack linearizable  
iff~~

1. **h** is pool linearizable, and
2.  $\text{push}(x) <_{\mathbf{h}} \text{push}(y) <_{\mathbf{h}} \text{pop}(x) \Rightarrow \text{pop}(y) \in \mathbf{h} \wedge \text{pop}(x) \not<_{\mathbf{h}} \text{pop}(y)$

# Linearizability verification

## Data structure

- signature  $\Sigma$  - set of method calls including data values
- sequential specification  $S \subseteq \Sigma^*$ , prefix closed

identify sequences with total orders

## Sequential specification via violations

Extract a set of violations  $V$ . relations on  $\Sigma$ . such that  $\mathbf{s} \in S$  iff  $\mathbf{s}$  has no violations

it is easy to find a large  $CV$ ,  
but difficult to find a small representative

$$\mathcal{P}(\mathbf{s}) \cap V = \emptyset$$

## Linearizability verification

Find a set of violations  $CV$  such that: every interval order with no  $CV$  violations  
extends to a total order with no  $V$  violations.

we build  
CV iteratively  
from  $V$

legal sequence

concurrent history

# It works for

- Pool without empty removals
- Queue without empty removals
- Priority queue without empty removals
- Pool
- Queue
- Priority queue

Thank You !

But not yet for Stack:  
infinite CV violations  
without clear  
inductive structure

Exploring the space of  
data structures  
as well as new ideas  
for problematic cases