Formale Systeme Proseminar

Tasks for Week 11, 17.12.2015

Task 1 Which of the following relations between $A = \{a, b, c\}$ and $B = \{1, 2\}$ are graphs of functions from A to B?

   (a) $R_1 = \{(a, 1), (b, 2)\}$.

   (b) $R_2 = \{(a, 1), (b, 1), (b, 2), (c, 1)\}$.

   (c) $R_3 = \{(a, 1), (b, 2), (a, 2)\}$.

   (d) $R_4 = \{(a, 1), (b, 2), (c, 1)\}$.

Why?

Task 2 Let $A = \{a, b, c\}$ and $B = \{1, 2\}$. Give an example of a surjective function $f \colon A \to B$.

Task 3 Give an example of an injective function $f \colon \mathbb{N} \to \mathbb{N}$.

Task 4 Let $X = \{1, 2, 3, 4, 5\}$ and consider the function $c \colon \mathcal{P}(X) \setminus \{\emptyset\} \to X$ defined by $c(Y) = |Y|$ for any $Y \subseteq X$, $Y \neq \emptyset$. Show that c is surjective but not injective.

Task 5 Prove Lemma I1 from the lectures, that is, show that $f \colon A \to B$ is an injective function if and only if for all $b \in B$, $|\{f^{-1}(b)\}| \leq 1$.

Task 6 Let X and Y be finite sets with $|X| = |Y|$. Prove that every injective function $f \colon X \to Y$ must also be surjective (and hence bijective).

Task 7 Prove Proposition S3 from the lectures, that is, show that if $f \colon A \to B$ is a surjective function and $B' \subseteq B$ then $f(f^{-1}(B')) = B'$.