Finite Automata

Finite automata provide one of the simplest, and yet very well known and useful, model of computation. They also provide a model of computation with seriously limited resources: finite memory. With help of finite automata we can solve problems that require only finite memory for their solution.

A *(deterministic) finite automaton* consists of:

- A finite set of *states*, usually denoted by Q;
- A finite (input) *alphabet*, often denoted by A;
- A designated *initial state* q_0;
- A set of *final, or accepting, states* F;
- A *transition function* that for every state q and every letter of the alphabet a, determines the next state q' that the automaton reaches if it reads the letter a in state q. We write $q \xrightarrow{a} q'$ in such a case.

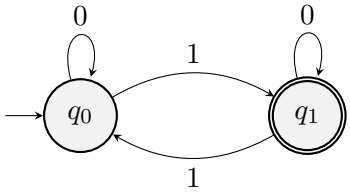
Finite automata recognise words over alphabet A as follows. Given a word $w = a_1 a_2 \dots a_n$, the execution of the automaton on this word is the sequence of transitions:

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \dots q_{n-1} \xrightarrow{a_n} q_n.$$

The word w is accepted if and only if the state q_n reached after reading the word is an accepting (final) state. The language of the automaton consists of all the words that are accepted by the automaton. A language recognised by a finite automaton is called *regular language*.

Finite automata have a very appealing graphical presentation: We draw a circle for each state; We denote the initial state by an incoming arrow (out of nowhere); We denote the final states by a double circle; For every state q and letter a, we draw the transition from q by a to q', $q \xrightarrow{a} q'$, by an arrow from state q to state q' labelled by a.

Example 1 Here is an example of a finite automaton with 2 states. Does the automaton accept the following words: (a) 00011, (b) 10010, (c) 111110? What is the language of this automaton?



Example 2 Together, we will construct a finite automaton that accepts all words over alphabet $\{0, 1\}$ that start with a 0.

Task 1 Construct a finite automaton that accepts all words over alphabet $\{0, 1\}$ that end with a 0.

Task 2 Construct a finite automaton that accepts all words over alphabet $\{0, 1\}$ that start with a 0 and end with a 0.

Task 3 Construct a finite automaton over the Latin alphabet that accepts your first name.

Task 4 Construct a finite automaton that accepts all words over alphabet $\{a, b\}$ that end with abb.

Task 5 Do you think that one can construct a finite automaton that accepts the words of shape 0^n1^n for some natural number n ?