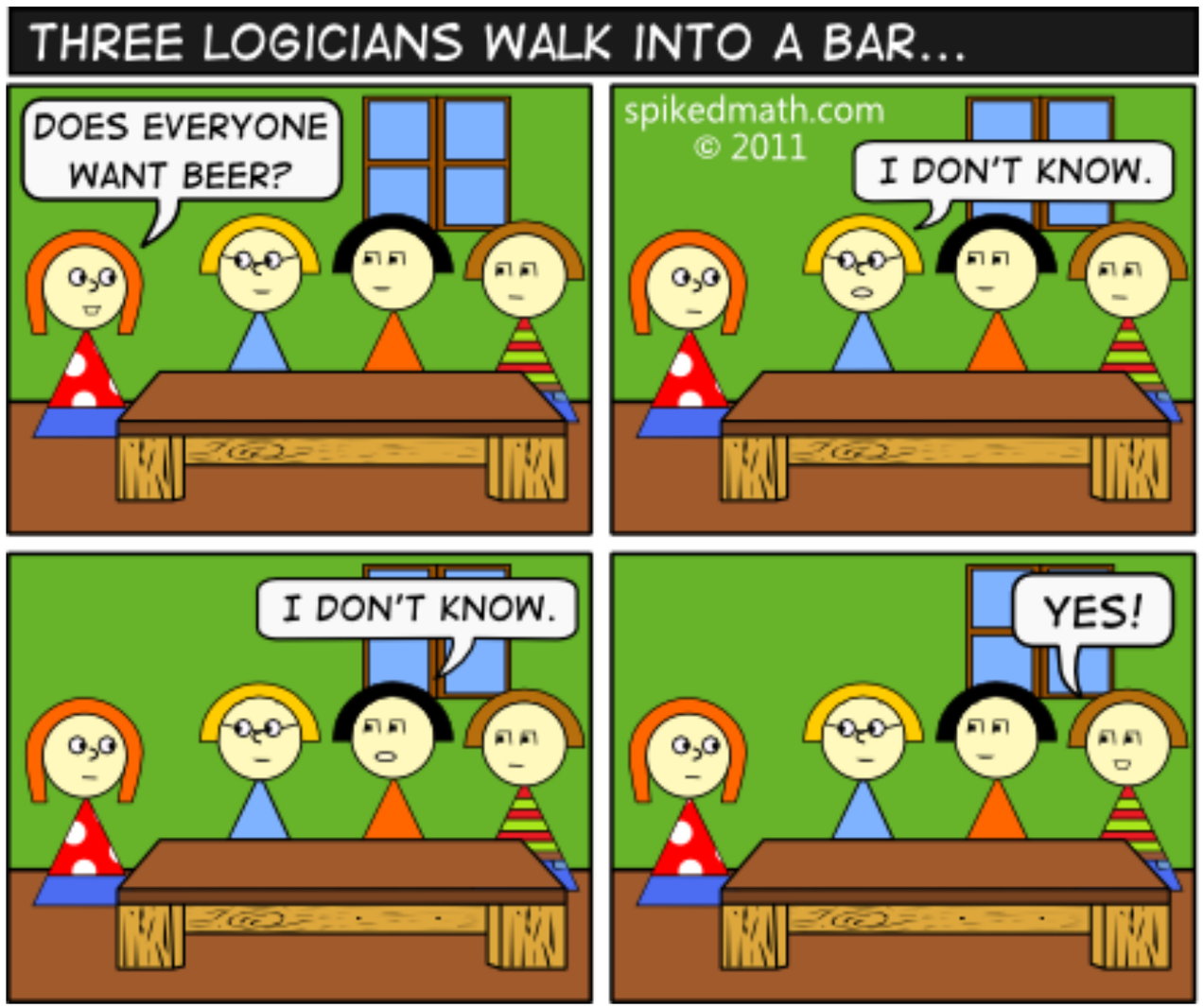
What is logic ?



I

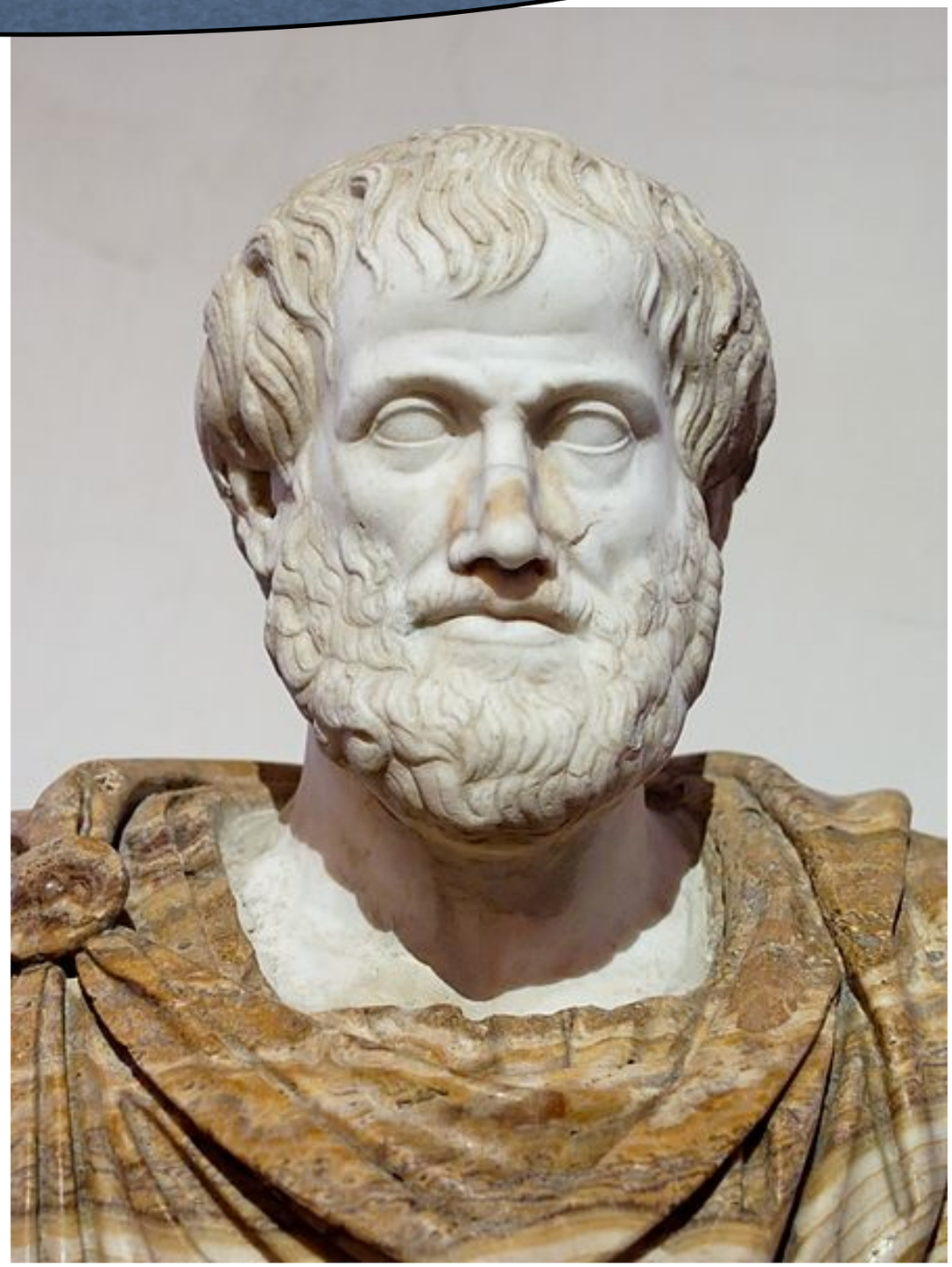
Logic = study of correct reasoning

In the beginning

Aristotle +/- 350 B.C.

Organon

19 syllogisms



Formal Logic

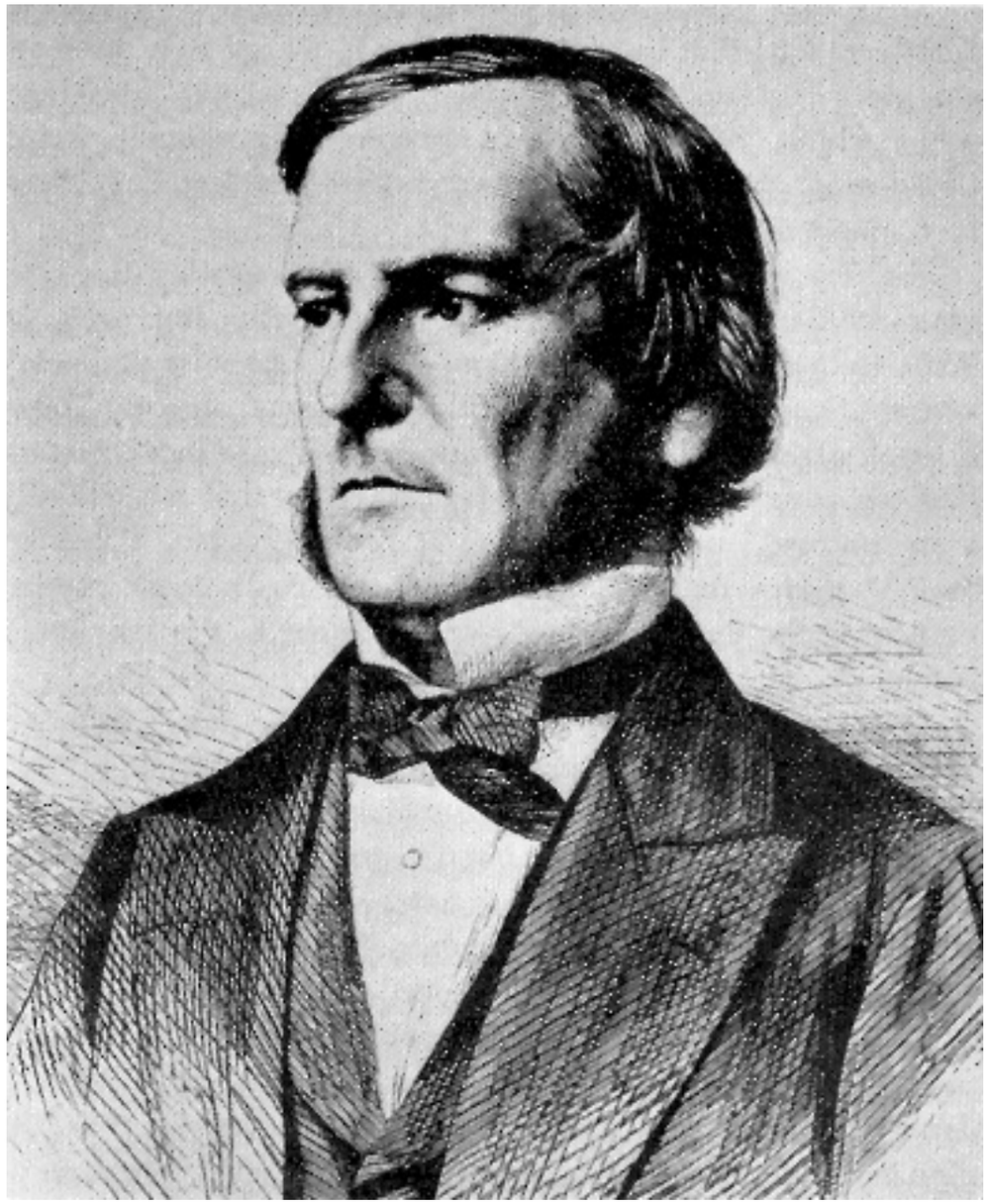
Gottfried Wilhelm Leibnitz
(1646 - 1716)

Beginnings of symbolic logic

Boolean Logic

George Boole
(1815 - 1864)

Boolean logic,
the foundations of the
information age



Modern Logic

Gottlob Frege
(1848 - 1925)

Predicate logic,
foundations of mathematics

What is logic ?

Thanks to Helle Hvid Hansen

- Reasoning is typical human activity
- There are inference rules that determine whether an argument is correct of not

- Logic is the study of these inference rules

From Language, Proof and Logic [LPL], p.1:

. . . all rational inquiry depends on logic, on the ability of people to reason correctly most of the time, and, when they fail to reason correctly, on the ability of others to point out the gaps in their reasoning.

Logic focuses on Validity

Thanks to Helle Hvid Hansen

From [LPL], p.44:

An argument is valid if the conclusion must be true in any circumstance in which the premises are true.

We say that the conclusion of a logically valid argument is a logical consequence of its premises.

If I miss the train, I will be late. — premise
I miss the train. — premise

I will be late. — conclusion

And on abstract reasoning

Thanks to Helle Hvid Hansen

Logic does not study concrete statements, but rather abstract reasoning schemes:

$$\frac{\text{If P, then Q.} \quad \text{P}}{\text{Q}}$$

premise
premise
conclusion

And on abstract reasoning

Thanks to Helle Hvid Hansen

Logic does not study concrete statements, but rather abstract reasoning schemes:

Symbolically

$$\frac{P \Rightarrow Q \qquad P}{Q}$$

premise
premise
conclusion

How to show that an argument is valid ?

Thanks to Helle Hvid Hansen

Give a proof !

From [LPL], p.46-47:

A proof is a step-by-step demonstration that a conclusion (say S) follows from some premises (say P, Q, R). The way a proof works is by establishing a series of intermediate conclusions, each of which is an obvious consequence of the original premises and the intermediate conclusions previously established. The proof ends when we finally establish S as an obvious consequence of the original premises and the intermediate conclusions.

How to show that an argument is not valid ?

Thanks to Helle Hvid Hansen

Give a counterexample !

From [LPL], p.63:

To show that a sentence Q is not a consequence of premises P1, .., Pn, we must show that the argument with premises P1, .., Pn and conclusion Q is invalid. This requires us to demonstrate that it is possible for P1, …, Pn to be true while Q is simultaneously false. That is, we must show that there is a possible situation or circumstance in which the premises are all true while the conclusion is false. Such a circumstance is said to be a counterexample to the argument.

Logic is the foundation of computing

and the foundation of set theory, which enables formal modelling and reasoning

- Bits and Boolean Logic
- Formal verification of programs and systems
- Reasoning about computability

Let's start with a puzzle…

The river-crossing puzzle

[Hopcroft et al, Kastens et al]

The river-crossing puzzle

- A man stands with a wolf, a goat, and a cabbage at the left bank of a river, that he wants to cross.
- The man has a boat that is large enough to carry him and another object to the other side.
- If the man leaves the wolf and the goat, or the goat and the cabbage on one side without supervision, one of them will get eaten :-(
- Is it possible to cross the river so that neither the goat nor the cabbage is eaten?

[Hopcroft et al, Kastens et al]

The river-crossing puzzle

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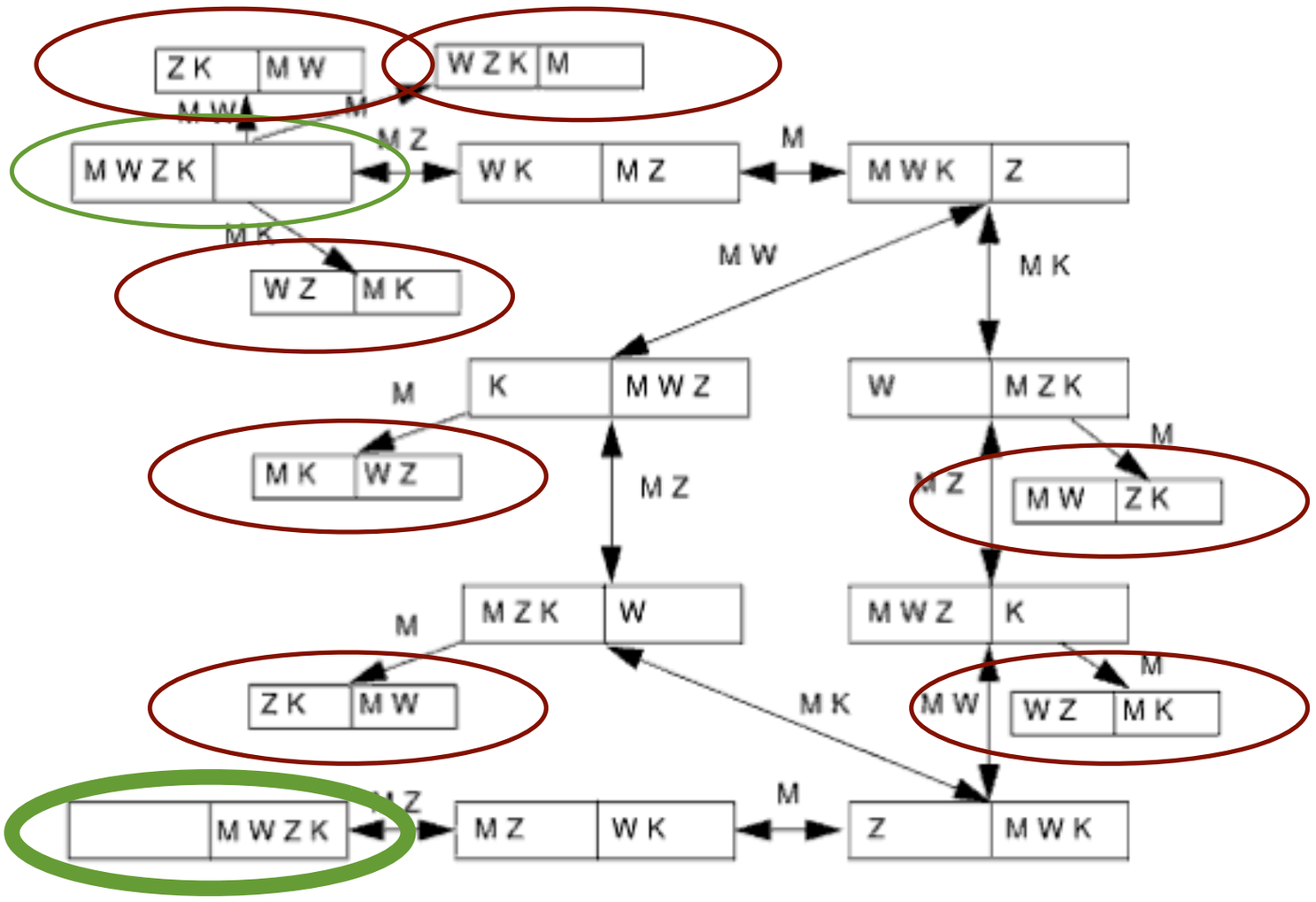
The river-crossing puzzle

Formalization with a finite automaton [Kastens et al.] :



states and transitions

Another model example