Process Algebra
Uitwerkingen opgaven practicum 4

Hieronder staan de uitwerkingen van de volgende opgaven:

2.7.42: 2, 3, 13, 14 en 20;

2.8.7: 1, 2, 3 en 5.

Exercise 2.7.42.2

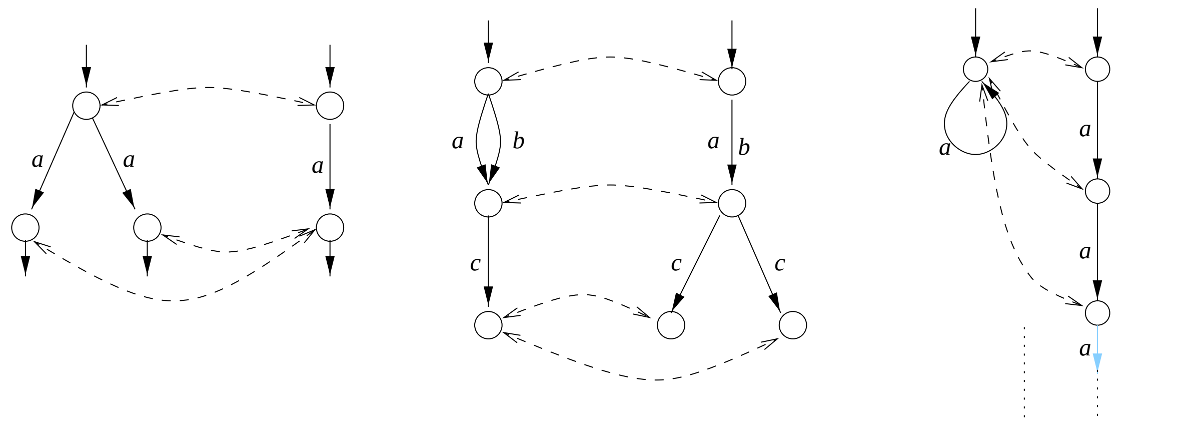
Graphs (iii) and (v) are not bisimilar. If they were, the roots should be bisimilar; however, from the root of (iii) a b step is possible and this is not possible in (v).

Graphs (iv) and (vi) are not bisimilar. In (iv), a steps can only be followed by b steps, and b steps can only be followed by a steps. In (vi), a steps may be followed by a steps as well as b steps (and vice versa).

Exercise 2.7.42.3

[**Note**: typo in exercise: 2.7.8 should be 2.7.7]

The bisimulations for 2.7.7(i), (ii), and (iii) are depicted in the following figure:

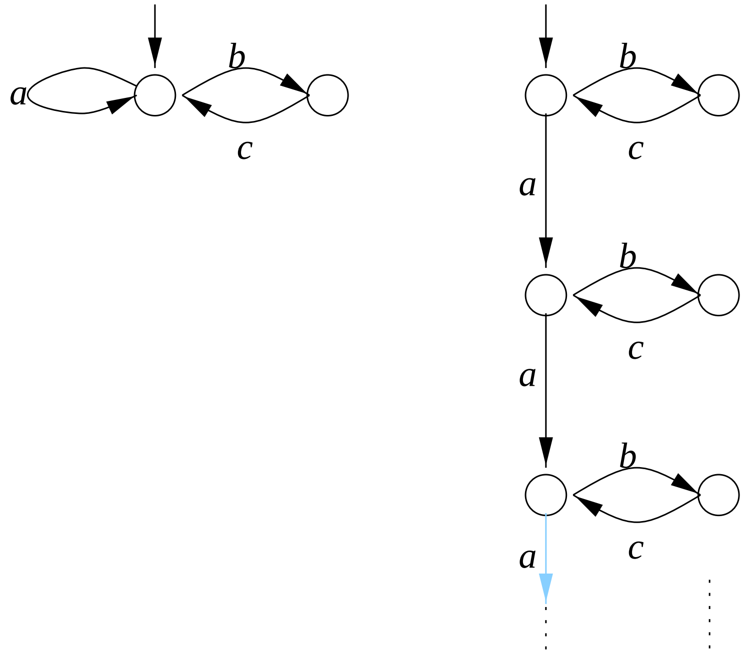


2.7.7(i) 2.7.7(ii) 2.7.7(iii)

The graphs in 2.7.7(iv) are not bisimilar: In the first graph, the root can only do an a step, whereas in the second graph, the root can do an a step or a b step. This means the roots are not bisimilar, so the graphs are not bisimilar.

Exercise 2.7.42.13

Find a process graph for $\{X = aX + bcX\}$. The following graphs are two possible solutions:



Exercise 2.7.42.14

Find two non-bisimilar graphs that are solutions of the unguarded specification $\{X = Xa + Xb\}$.

What would be a solution of $\{X = Xa + Xb\}$? Suppose p is a solution and p is a finite process that terminates successfully. This means (according to definition 2.4.5) that there is a closed term x without recursion constructs such that p is the interpretation of x and p does not lead to deadlock.

Since x has no recursion, every trace of x is finite. Since the maximal trace is a trace of X, it should also be a trace of $Xa + Xb$. However, it is not a maximal trace of this process. Therefore, the finite process p cannot be a solution for $\{X = Xa + Xb\}$.

It follows that solutions of the process have to be infinite or they should always lead to deadlock. For example, δ is a solution, since $\delta = \delta a + \delta b$. Another solution is $a\delta$. It is clear that δ and $a\delta$ are not bisimilar; not even at the root.

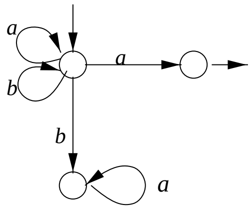
Exercise 2.7.42.20

Show that $\{X = aX\}$ has no solution in $\mathbb{F}/\underline{\leftrightarrow}$. A solution in $\mathbb{G}/\underline{\leftrightarrow}$ is a^ω. Processgraphs of this solution are given in example 2.7.5 (p47). Some of the processgraphs given there, have a finite number of nodes, so they are in $\mathbb{R}/\underline{\leftrightarrow}$. Because of RSP, any solution in $\mathbb{R}/\underline{\leftrightarrow}$ must be bisimilar to this one.

In $\mathbb{F}/\underline{\leftrightarrow}$, you have no cycles and a finite number of nodes and edges. Therefore, every trace of a process in $\mathbb{F}/\underline{\leftrightarrow}$ is finite. So, you cannot find a processgraph in $\mathbb{F}/\underline{\leftrightarrow}$ for a^ω, since this process has infinite traces. As any processgraph in $\mathbb{F}/\underline{\leftrightarrow}$ is also in $\mathbb{R}/\underline{\leftrightarrow}$, there cannot be another solution.

Exercise 2.8.7.1

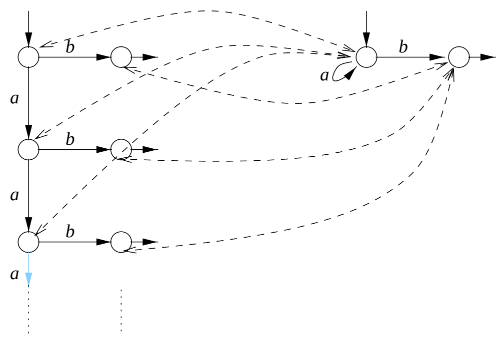
Determine $G(X)$ for $\{X = (a+b)X + a + bY, Y = aY\}$.



Exercise 2.8.7.2

Let X_0 be given by $E = \{X_n = aX_{n+1} + b \,:\, n \geq 0\}$ in one of the models of 2.5–2.7. Is X_0 a regular process ($X_0 \in \mathbb{R}/\underline{\leftrightarrow}$)?

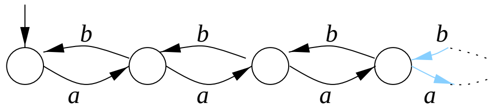
Below $G(X_0)$ is given together with a graph that is bisimilar to $G(X_0)$.



It is clear that second graph is in $\mathbb{R}/\underline{\leftrightarrow}$, so X_0 is a regular process.

Exercise 2.8.7.3

Y_0 is given by $\{Y_0 = aY_1\} \cup \{Y_n = aY_{n+1} + bY_{n-1} \,:\, n \geq 1\}$. $G(Y_0)$ is given by the following graph:



It is clear that this graph is in $\mathbb{G}/\leftrightarrows$. The question is if it is in $\mathbb{R}/\leftrightarrows$, as well.

Note that each Y_i ($i \geq 0$) is different from every other Y_j ($i \neq j$ and $j \geq 0$), since the maximal number of successive b steps Y_i can do is determined by the index i. There are infinitely many Y_i's, so there is no solution in $\mathbb{R}/\leftrightarrows$.

Exercise 2.8.7.5