Formale Systeme
Example tasks for Test 2, 2016

Task 1. (15) Write down the definitions of the following notions:

  (a) A relation R is symmetric.

  (b) A relation R is a partial order.

  (c) A function $f\colon A \to B$ is surjective.

Task 2. (20 points)   Is the following property true? If yes, prove it; if not give a counter example:

The union of any two equivalence relations is an equivalence relation.

Task 3. (20 points)   Let $f\colon A \to B$ be an injective function and $S, T \subseteq A$ be two disjoint sets. Prove that $f(S)$ and $f(T)$ are also disjoint.

Task 4. $(10 + 10)$

  (a) Let X be a set and let $1 = \{*\}$. Show that $X \sim \{f \mid f\colon 1 \to X\}$.

  (b) Prove that $\aleph_0 + \aleph_0 = \aleph_0$.

Task 5. (20) Let n be any natural number that is larger than or equal to 1. Prove (by induction) that then $3^n > 2^n$.

[Recall the inductive definition of k^n for natural numbers k, n: $k^0 = 1; k^{n+1} = k^n \cdot k$.]

Task 6. $(10 + 10)$ Construct an NFA for the language described by the regular expression:
$$(0 \cup 1)^*(11 \cup 101).$$

Determinize your automaton then to obtain a DFA for the given language.