PSAT Week 5: Sorting Algorithms I

Objectives: algorithms, complexity, Selection Sort

Introduction:

- An **algorithm** is a finite sequence of well-defined instructions to solve a problem or to perform a computation
- Typical "building blocks": Repetitions, case distinctions, elementary instructions

Task 1 (10 min):

- Add the two numbers 532 and 486
- Write down instructions for adding two numbers as precisely as possible.

Task 2 (10 min):

- Exchange instructions with your neighbor.
- Execute your neighbor's instruction literally to add the two numbers 718 and 611.
- Discuss: How do the instructions need to be modified to be "well-defined"?

Addition Algorithm:

```
Write down the two given numbers below each other in a right-adjusted
manner such that — in case of unequal length — the longer number is at the
bottom
Iterate over the digits of the bottom number from right to left and for
each digit execute the following instructions:
      Add to the current digit of the bottom number the matching digit of
      the top number (if existent) and a potentially memorized carry
      If the result of this addition is less than 10, then write this
      result below the current digit of the bottom number
      If the result of this addition is at least 10, then write the units
      digit of this result below the current digit of the bottom number
      and memorize its tens digit as the carry
If you memorized a carry after the last addition of digits, then write it
to the left of the overall result at the bottom
```

Complexity

- Algorithms might differ in their efficiency
- Typical complexity measure: number of elementary instructions
- Addition: Lookup in addition table (range 0 to 20)
- Number n has length $\leq 1 + \log_{10} n$
- To add up two numbers $\leq n$ we need $\leq 1 + \log_{10} n$ lookups in addition table

Task 3 (10 min):

- Take a pile of cards and shuffle it.
- Find the smallest card in the pile.
- Which method did you use for finding the smallest card? Write down instructions for finding the minimum as precisely as possible.
- Assumption: We have fixed a unique order on the cards – comparing two cards, we can immediately tell which one has higher value
- Discuss: Which "elementary" instructions can be performed for pile of cards?

Minimum Algorithm:

```
Turn the given pile such that the front side of the cards faces upwards
Pick up the first card of the pile to your hand
Scan through pile from top to bottom and for each scanned card execute the
following instruction:
      If the currently scanned card is smaller than the card in your hand,
      then exchange these two cards
Return the card in your hand as the result
```

Discussion: Basic instructions on cards:

- Compare two cards
- Pick up a card
- Move a card from one pile to another one
- Memorize the value of a card
- ...

Selection Sort algorithm:

```
Turn the given pile such that the front side of the cards faces upwards
Put the pile to the left and start an empty pile to the right
Repeat the following instruction until the left pile is empty:
      Find smallest card in left pile using the Minimum algorithm and move
      it to bottom of right pile
Return the right pile as the result
```