Local Linearizability

Ana Sokolova UNIVERSITY of SALZBURG

Local Linearizability

Ana Sokolova — UNIVERSITY of SALZBURG

joint work with:

Andreas Haas — UNIVERSITY of SALZBURG
Andreas Holzer — UNIVERSITY OF TORONTO
Michael Lippautz — UNIVERSITY of SALZBURG
Ali Sezgin — UNIVERSITY OF CAMBRIDGE
Tom Henzinger — IST AUSTRIA
Christoph Kirsch — UNIVERSITY of SALZBURG
Hannes Payer — Google
Helmut Veith — TU WIEN

Concurrent Data Structures
Correctness and Performance

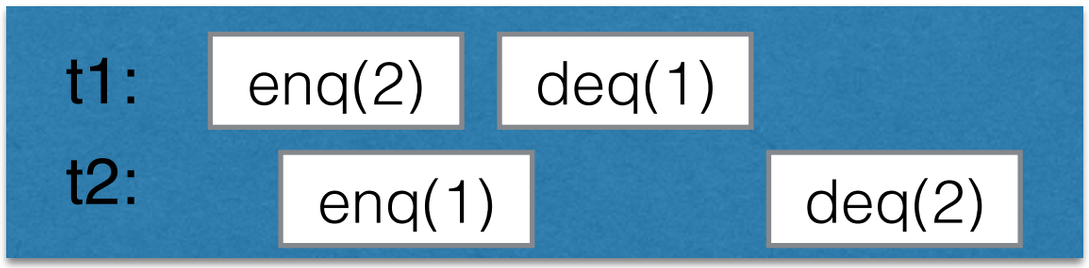
Ana Sokolova UNIVERSITY of SALZBURG

Semantics of concurrent data structures

Semantics of concurrent data structures

e.g. pools, queues, stacks

Semantics of concurrent data structures



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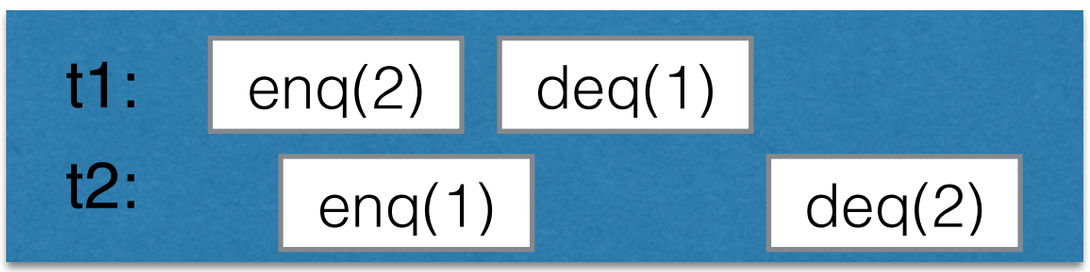

e.g. pools, queues, stacks

- Sequential specification = set of legal sequences
- Consistency condition = e.g. linearizability / sequential consistency

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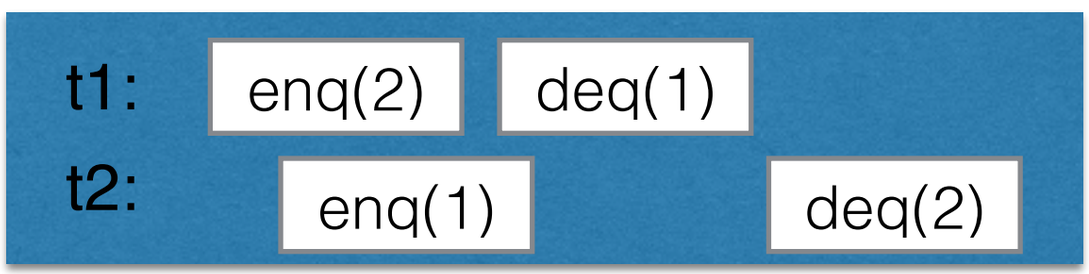

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enq(1)enq(2)deq(1)deq(2)

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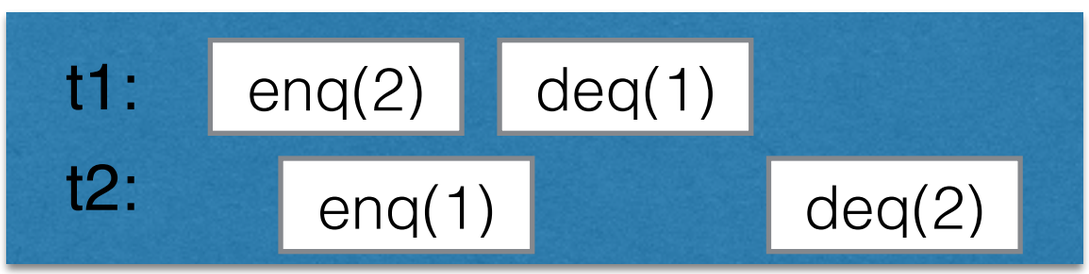

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e.g. the concurrent history above is a linearizable queue concurrent history

Consistency conditions

Linearizability [Herlihy,Wing ’90]

Sequential Consistency [Lamport’79]

Consistency conditions

there exists a sequential witness that preserves precedence

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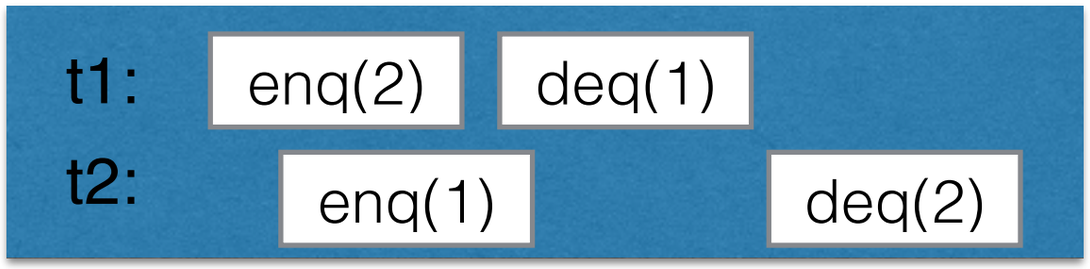

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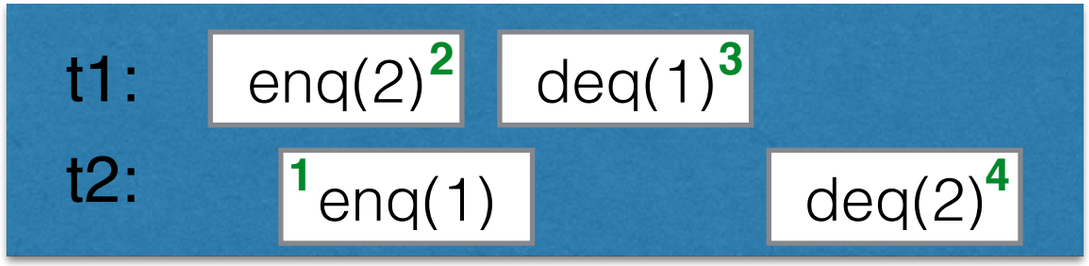

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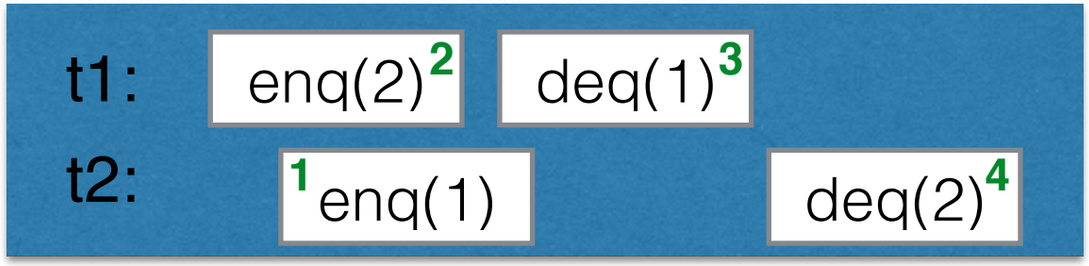

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there exists a sequential witness that preserves per-thread precedence (program order)

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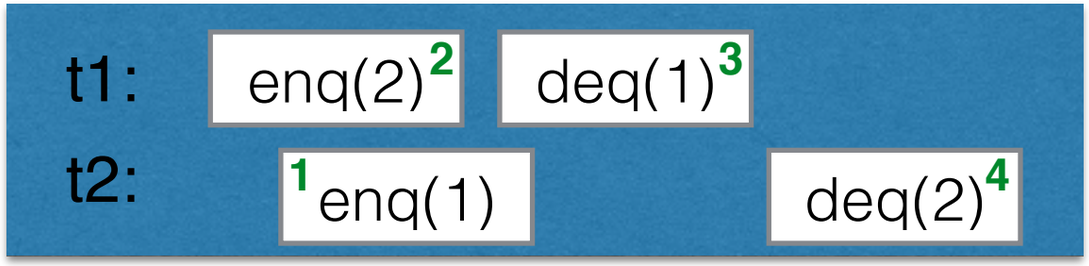

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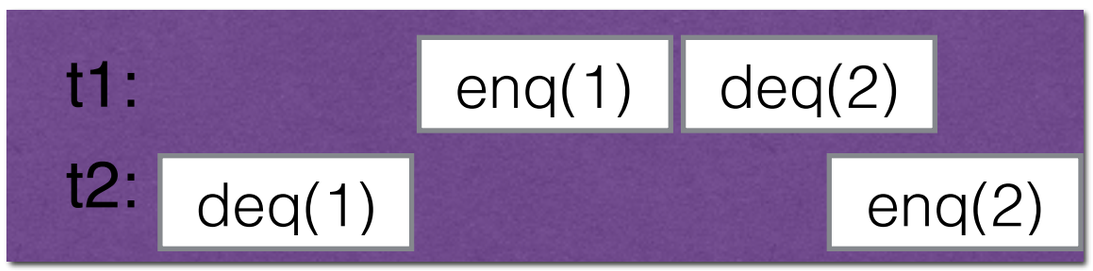

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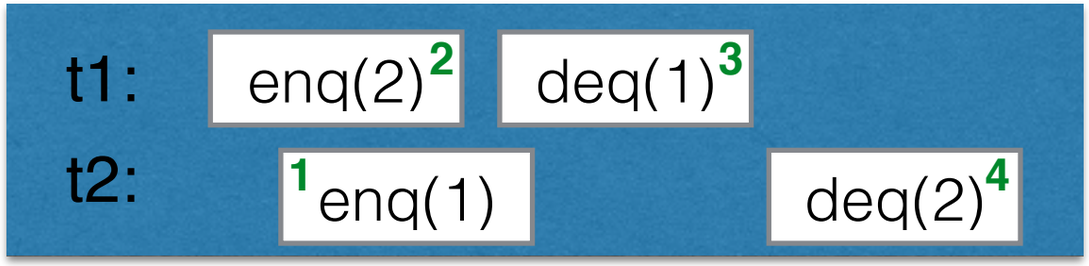

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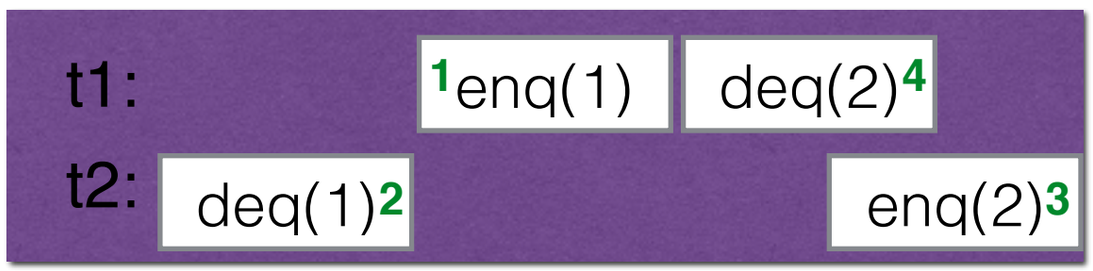
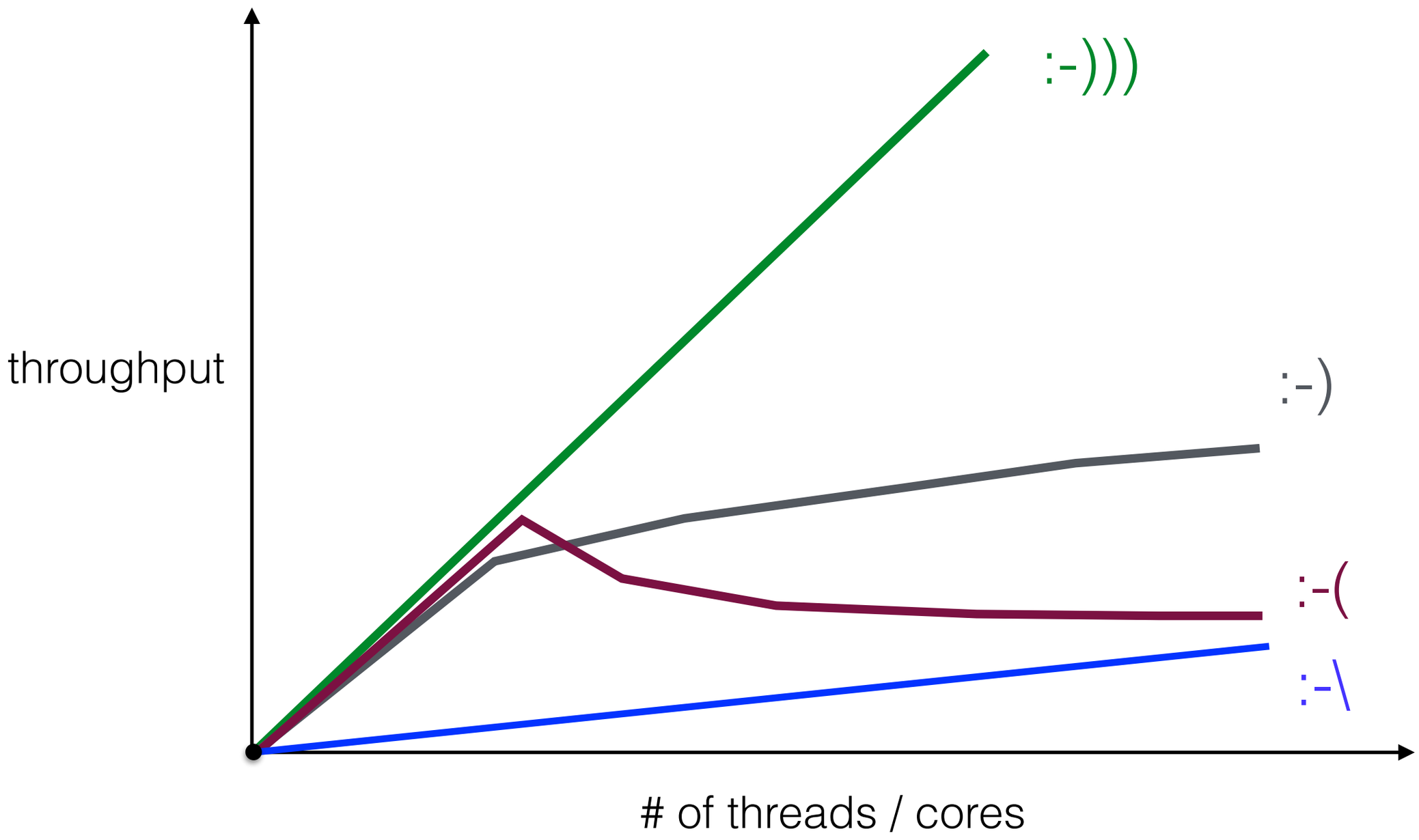


Performance and scalability



Relaxations allow trading

correctness

for

performance

Relaxations allow trading

correctness

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performance

provide the potential for better-performing implementations

Relaxing the Semantics

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Quantitative relaxations
Henzinger, Kirsch, Payer, Sezgin,S. POPL13

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not "sequentially correct"

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Local linearizability
in this talk

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for queues only
(feel free to ask for more)

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Local linearizability in this talk

for queues only
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too weak

Local Linearizability main idea

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- Partition a history into a set of local histories
- Require linearizability per local history

Local Linearizability main idea

Already present in some shared-memory consistency conditions
(not in our form of choice)

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Local sequential consistency… is also possible

Local Linearizability main idea

Already present in some shared-memory consistency conditions
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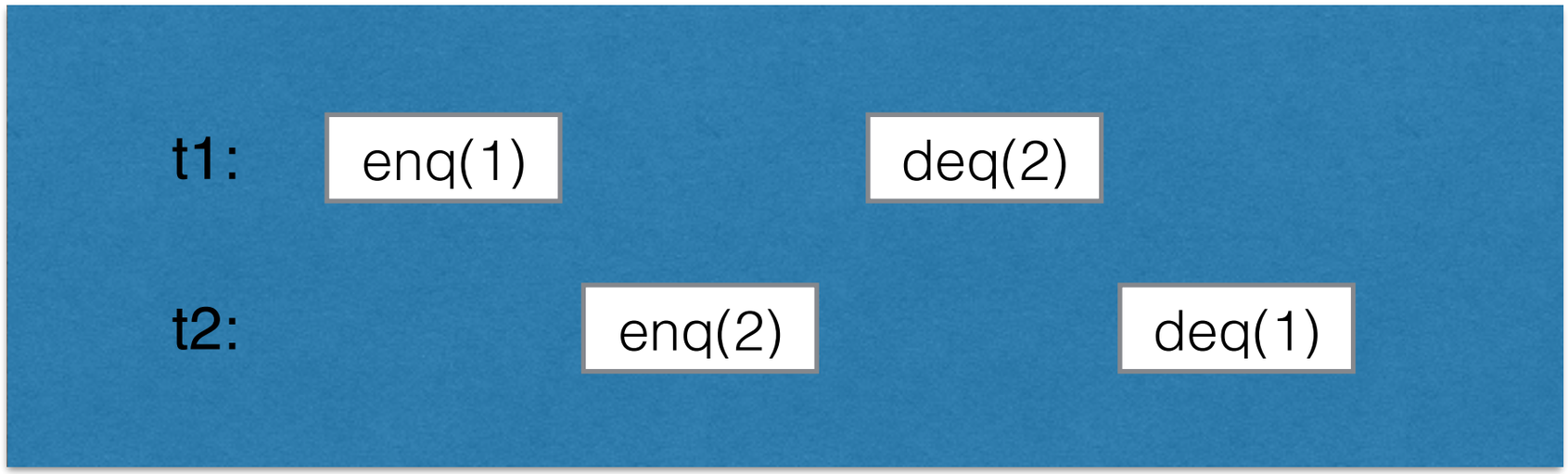
- Require linearizability per local history

no global witness

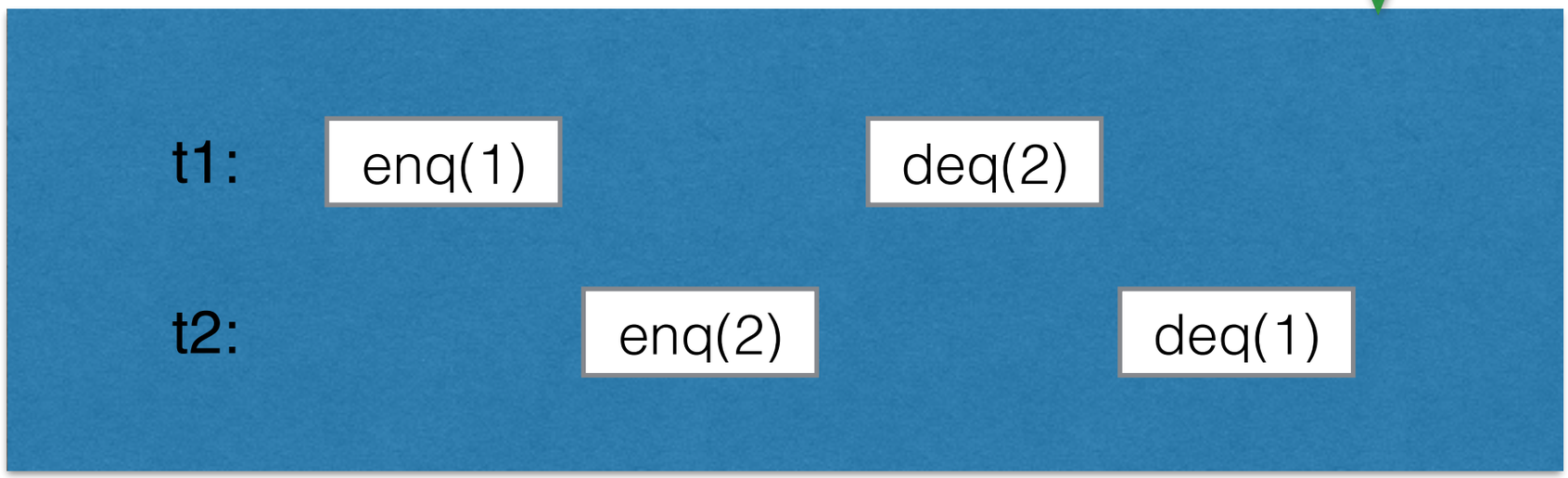
Local sequential consistency… is also possible

Local Linearizability (queue) example



Local Linearizability (queue) example

(sequential) history not linearizable



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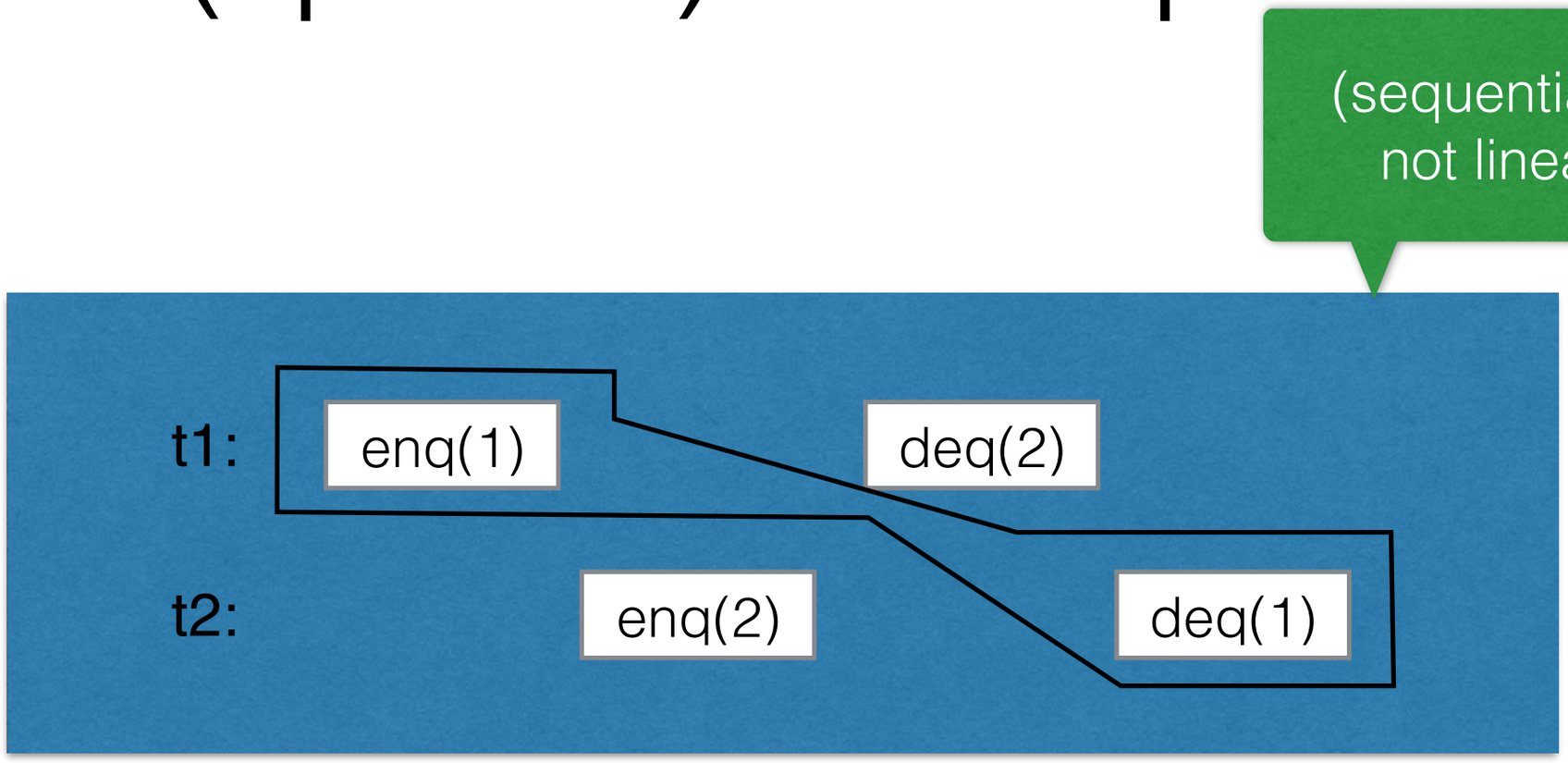

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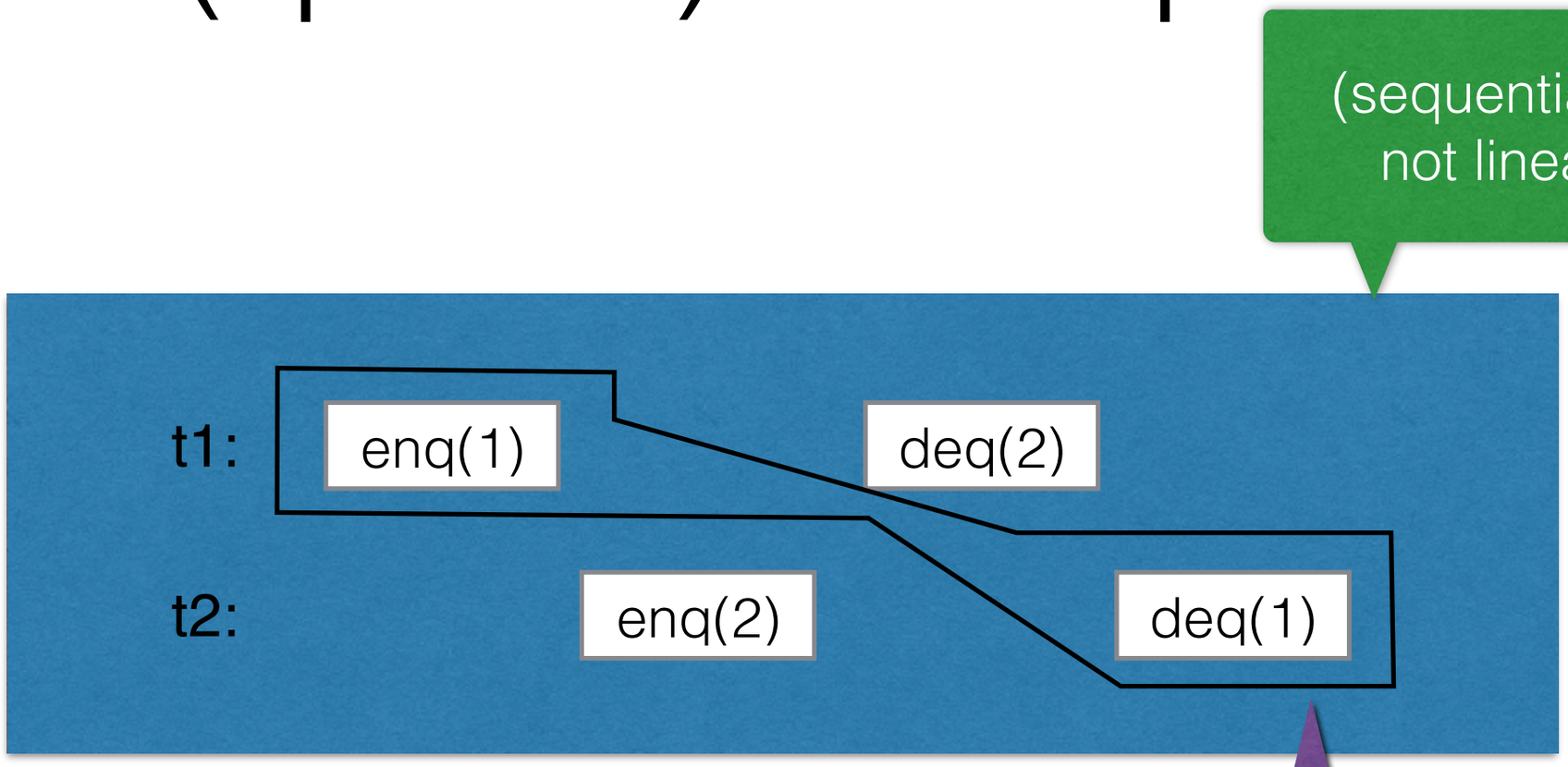

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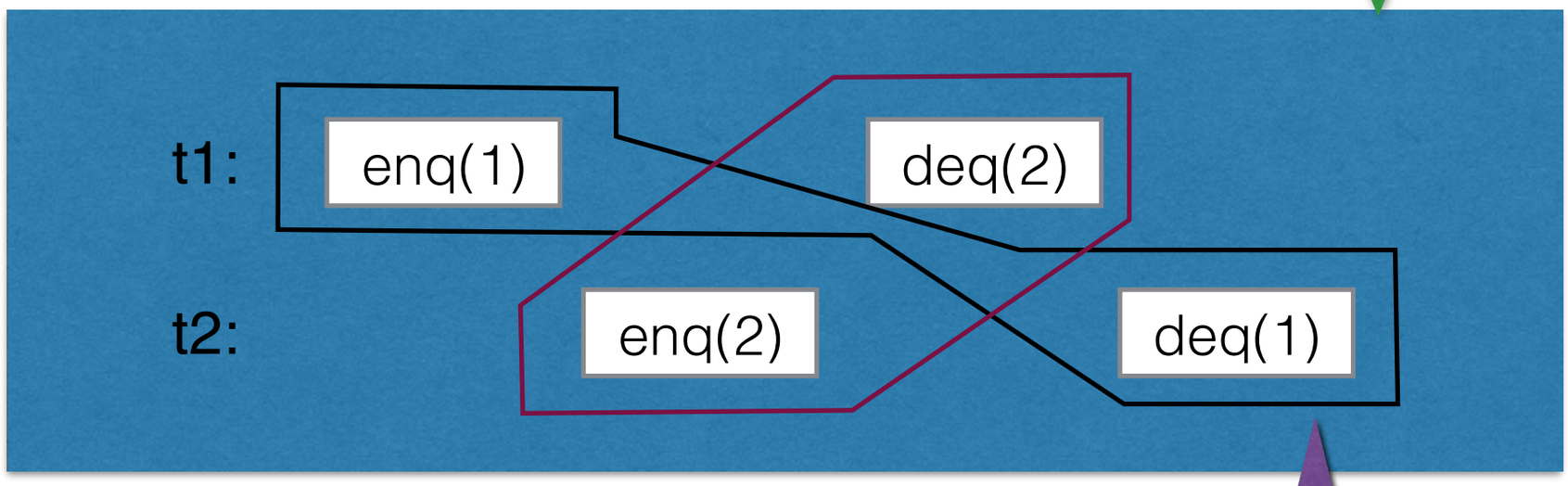
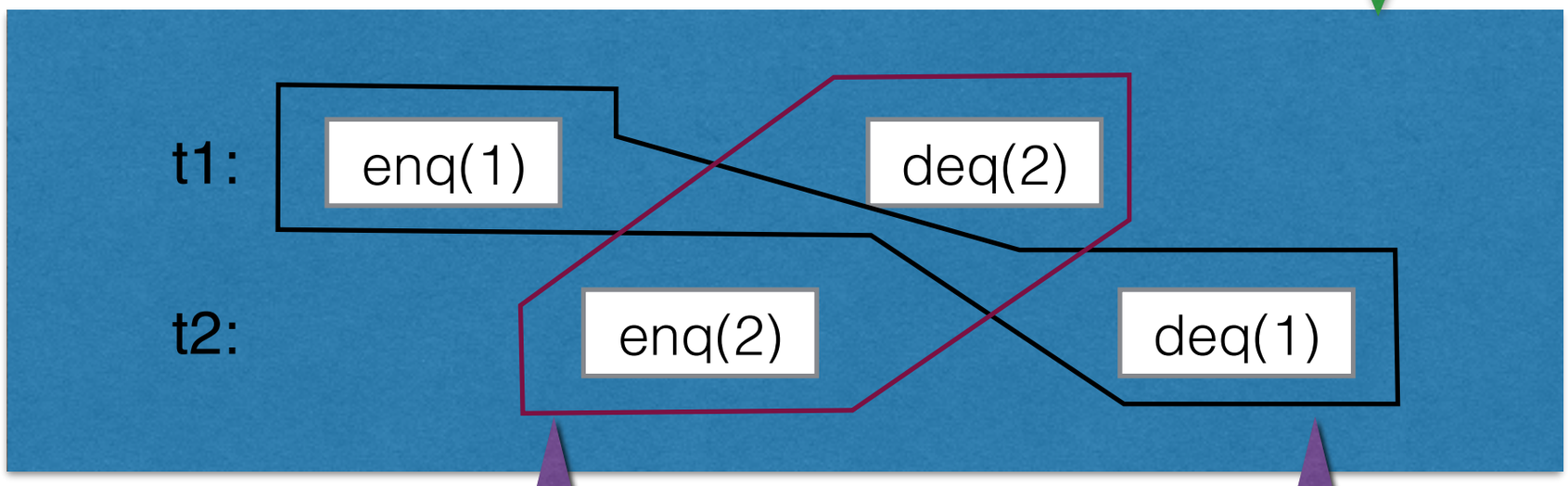


(sequential) history not linearizable

t1-induced history, linearizable

Local Linearizability (queue) example



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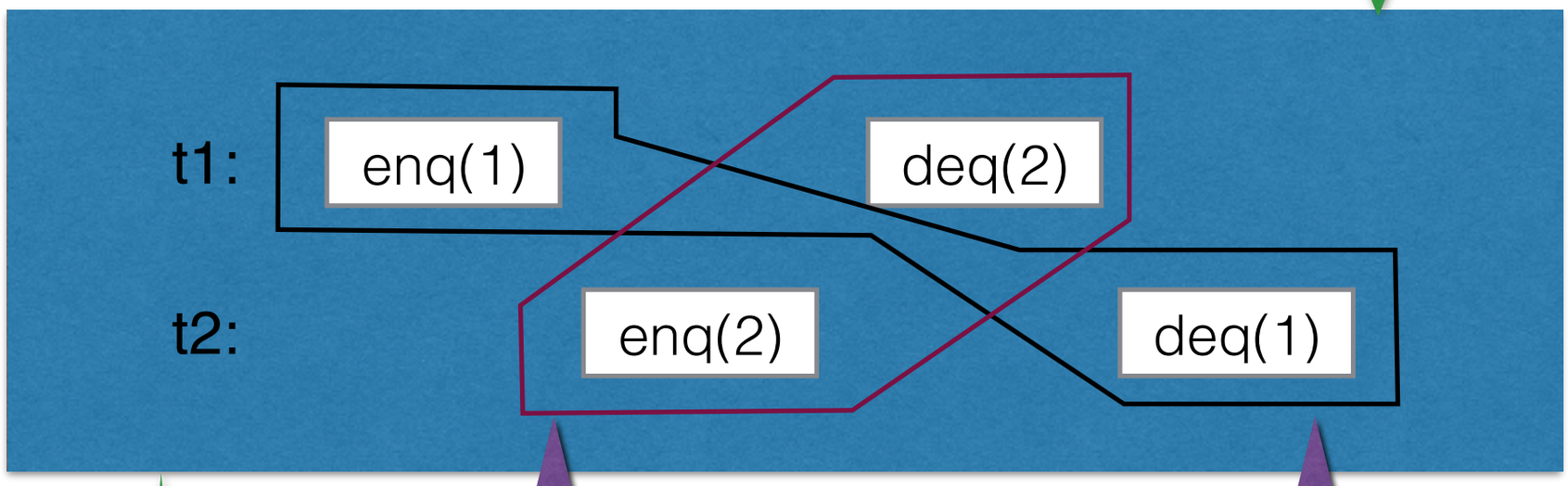

Local Linearizability (queue) definition

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Queue signature $\sum = \{enq(x) \mid x \in V\} \cup \{deq(x) \mid x \in V\} \cup \{deq(empty)\}$

Local Linearizability (queue) definition

Queue signature $\Sigma = \{enq(x) \mid x \in V\} \cup \{deq(x) \mid x \in V\} \cup \{deq(empty)\}$

For a history **h** with n threads, we put

$$In_{\mathbf{h}}(i) = \{enq(x)^i \in \mathbf{h} \mid x \in V\}$$

$$Out_{\mathbf{h}}(i) = \{deq(x)^j \in \mathbf{h} \mid enq(x)^i \in In_{\mathbf{h}}(i)\} \cup \{deq(empty)\}$$

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h is locally linearizable iff every thread-induced history

$\mathbf{h}_i = \mathbf{h} \mid (In_{\mathbf{h}}(i) \cup Out_{\mathbf{h}}(i))$

is linearizable.

Generalizations of Local Linearizability

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Signature ∑

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For a history **h** with n threads, choose

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Generalizations of Local Linearizability

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For a history **h** with n threads, choose

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by increasing the in-methods, LL gradually moves to linearizability

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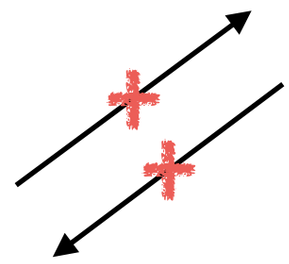
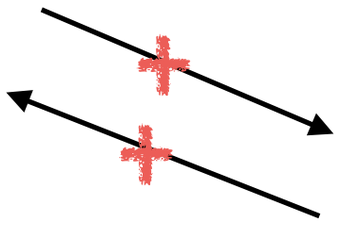
Where do we stand?

In general

Linearizability



Local Linearizability



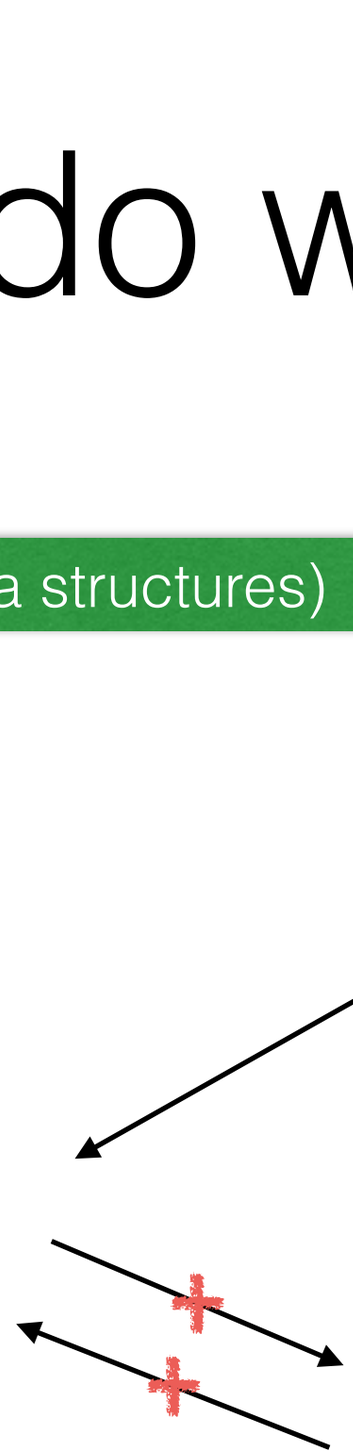
Sequential Consistency

Where do we stand?

For queues (and all pool-like data structures)

Linearizability

Local Linearizability



Sequential Consistency

Where do we stand?

C: For queues

Linearizability

Local Linearizability & Pool-seq.cons.

Sequential Consistency

Properties

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Local linearizability is compositional

Properties

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like linearizability
unlike sequential consistency

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h (over multiple objects) is locally linearizable
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Local linearizability is modular / "decompositional"

uses decomposition into smaller histories, by definition

may allow for modular verification

Verification (queue)

Queue sequential specification (axiomatic)

s is a legal queue sequence

iff

1. **s** is a legal pool sequence, and
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precedence order

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thread-local precedence order

Generic Implementations

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Your favorite linearizable data structure implementation

Generic Implementations

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Φ

turns into a locally linearizable implementation by:

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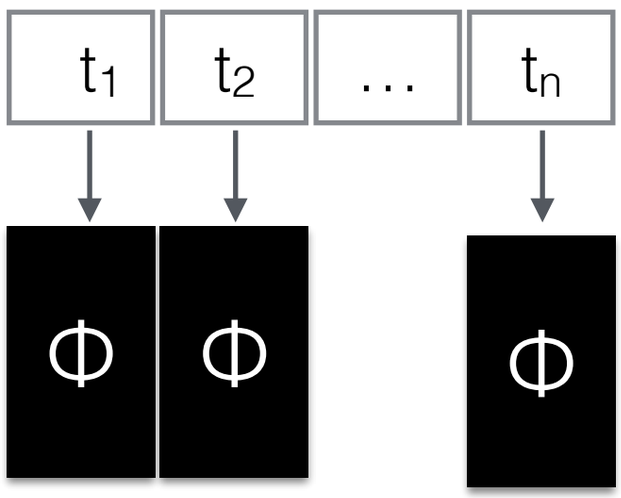
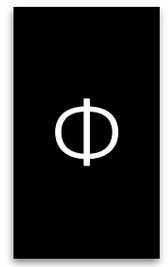
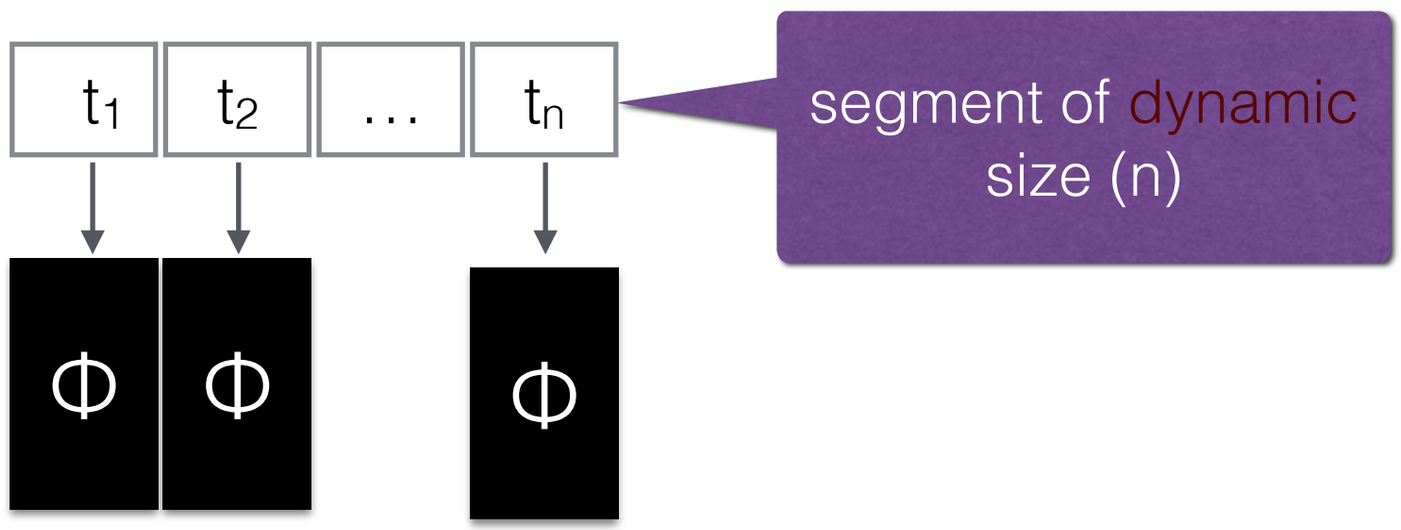


Generic Implementations

Your favorite linearizable data structure implementation



turns into a locally linearizable implementation by:



Generic Implementations

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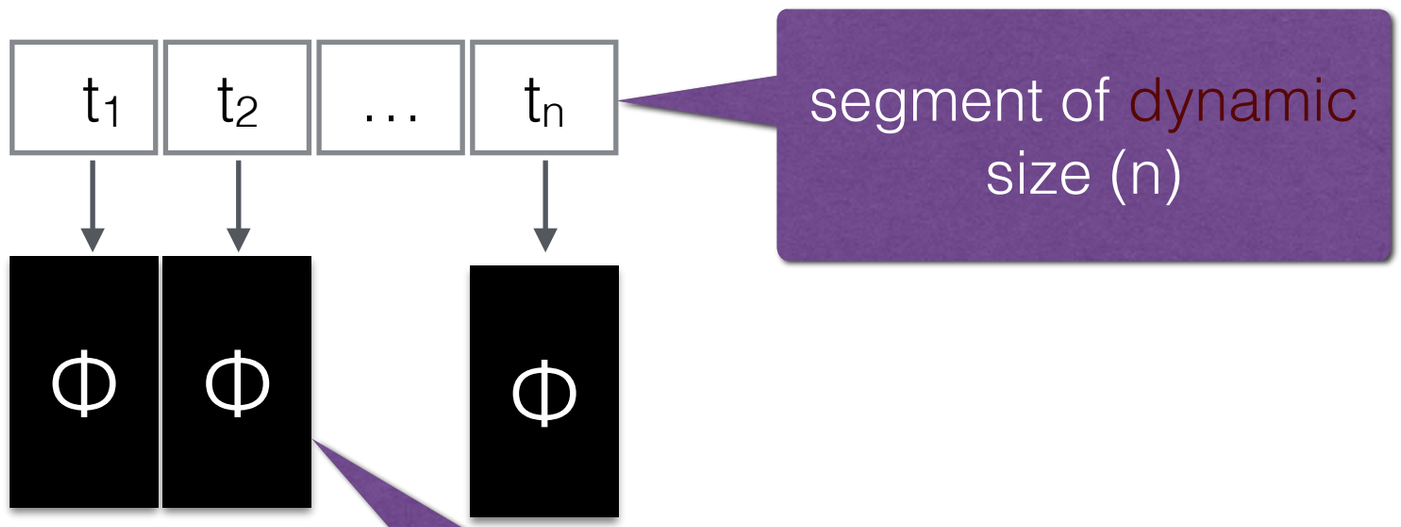

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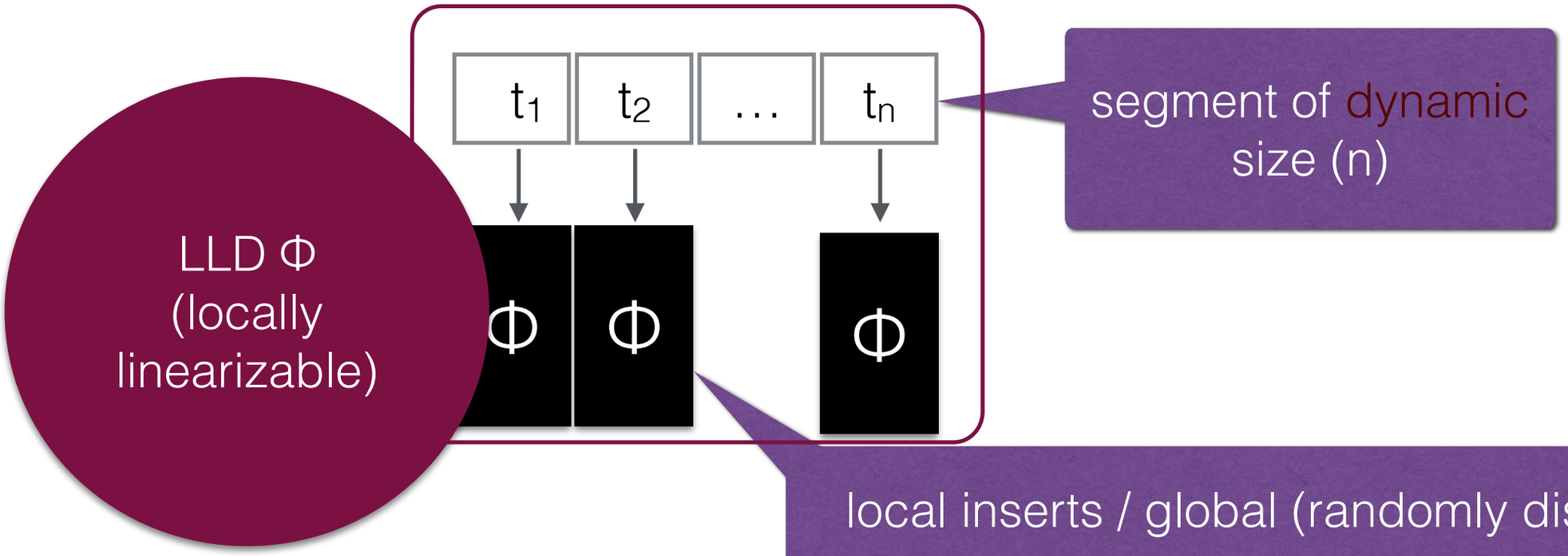

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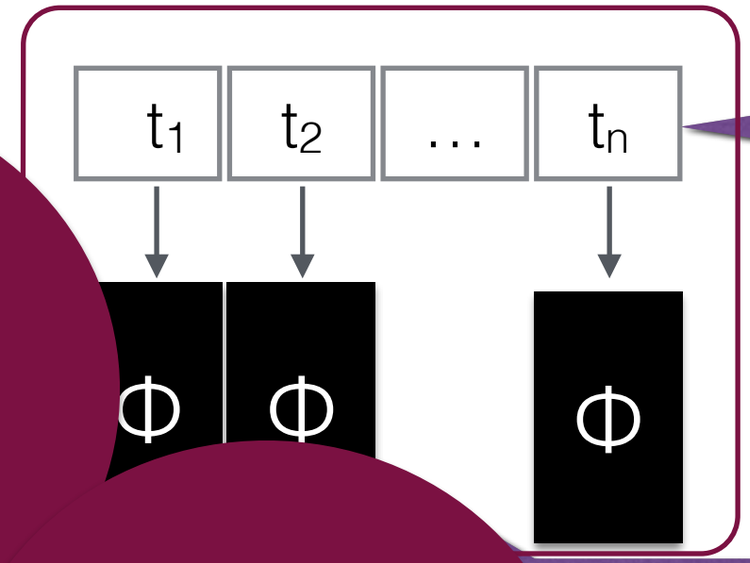


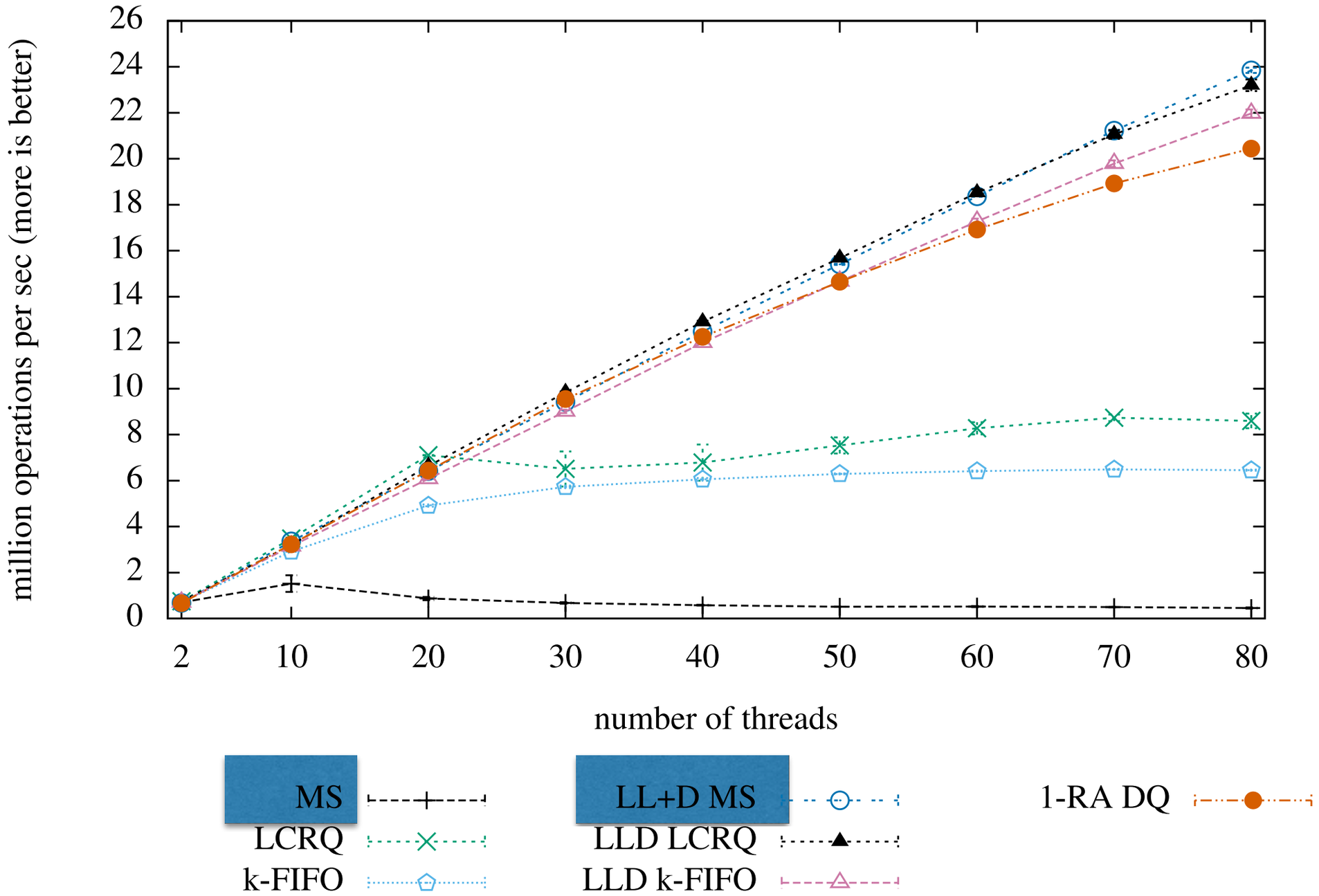
segment of dynamic size (n)

LLD Φ (locally linearizable)

LL+D Φ (also pool linearizable)

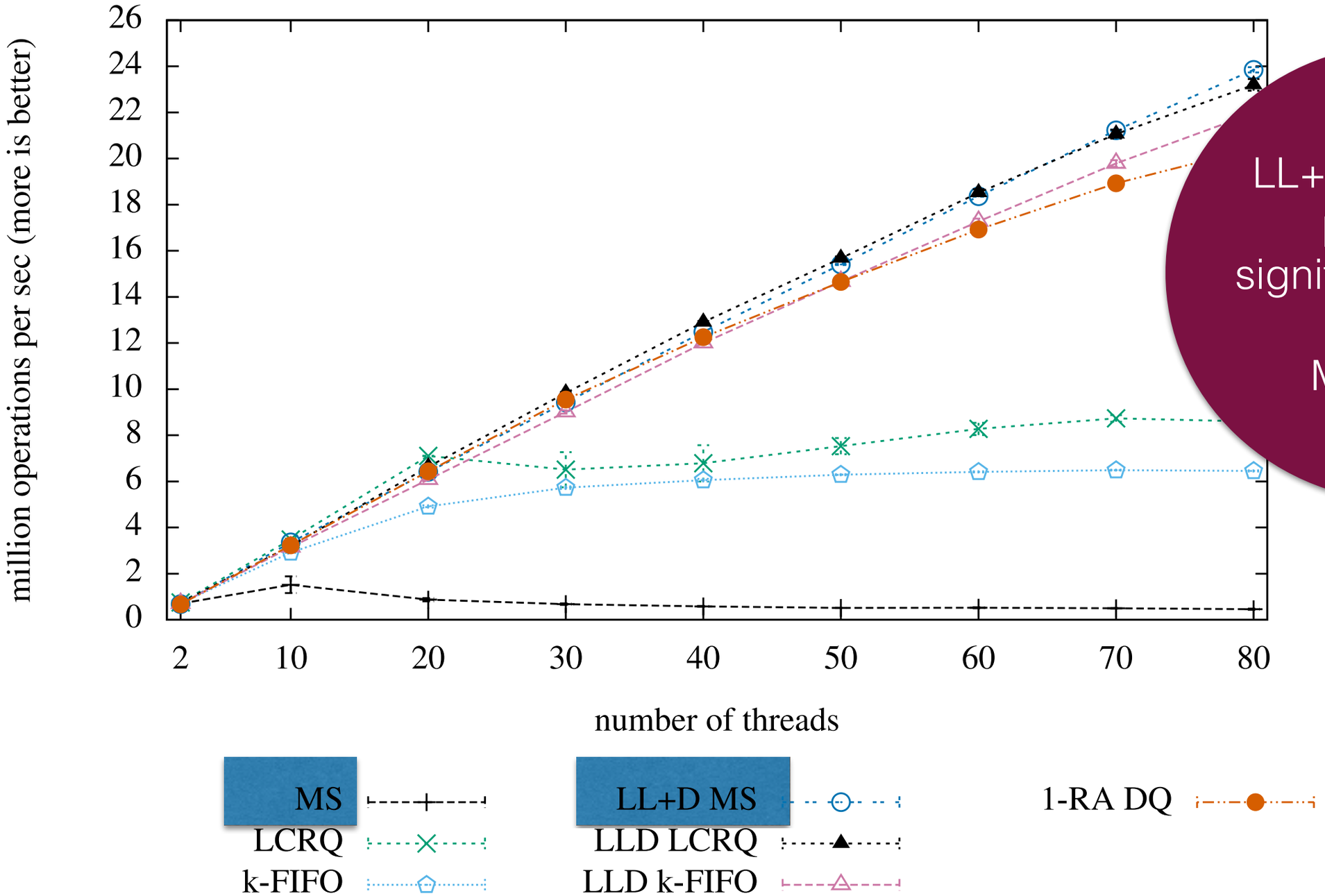
local inserts / global (randomly distributed) removes

Performance



(a) Queues, LL queues, and “queue-like” pools

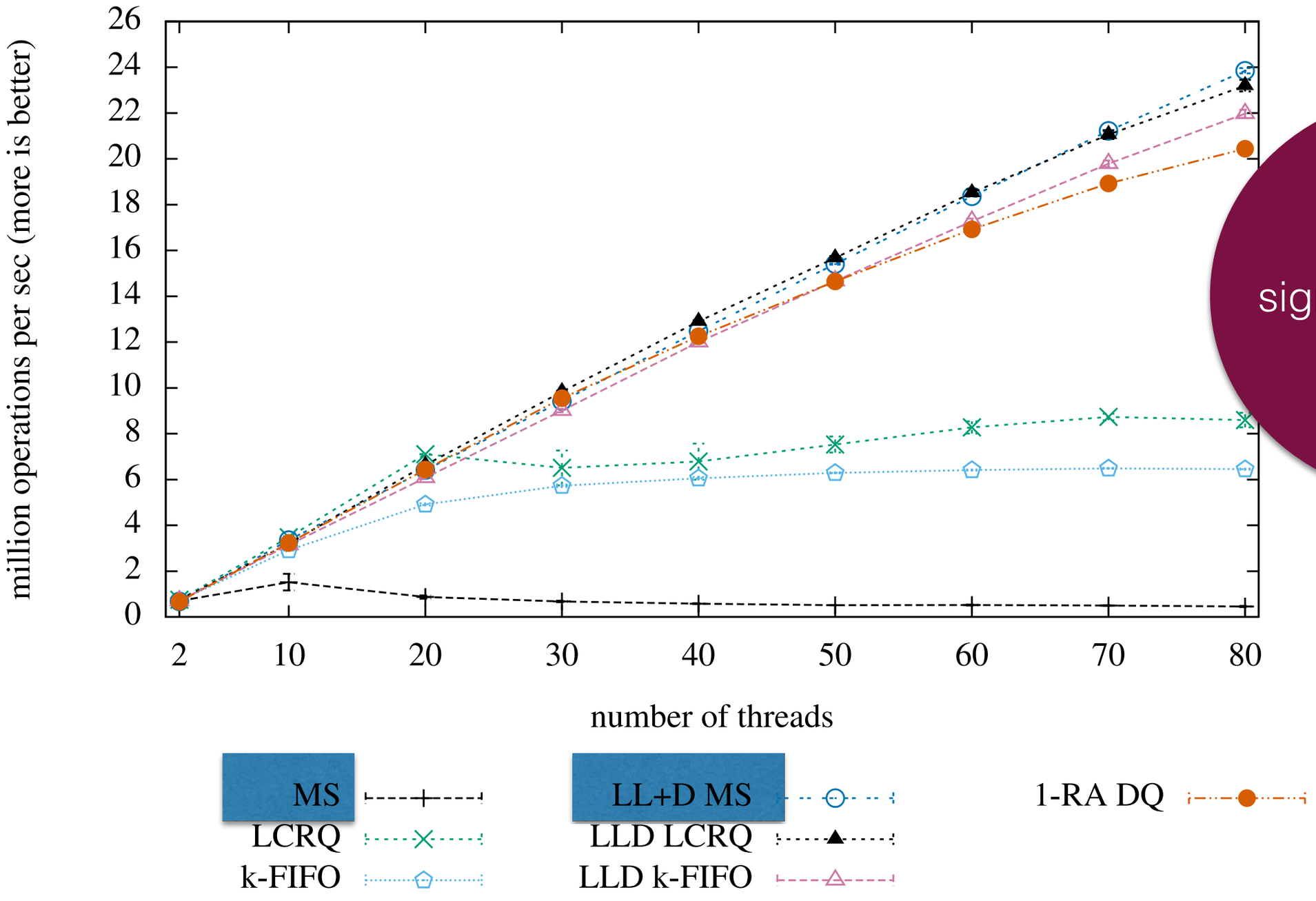
Performance



LL+D MS queue performs significantly better than MS queue

(a) Queues, LL queues, and "queue-like" pools

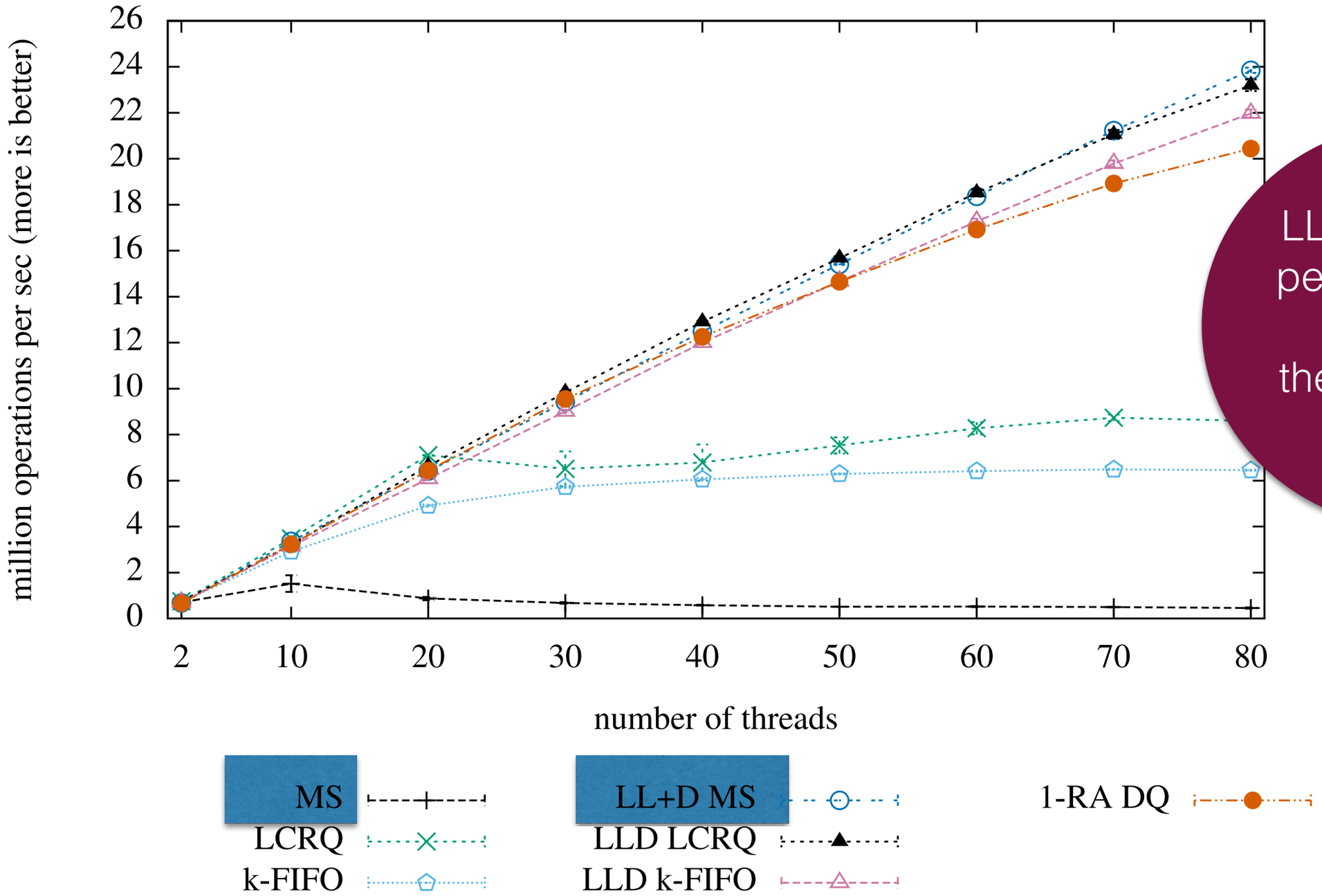
Performance



LLD Φ performs significantly better than Φ

(a) Queues, LL queues, and “queue-like” pools

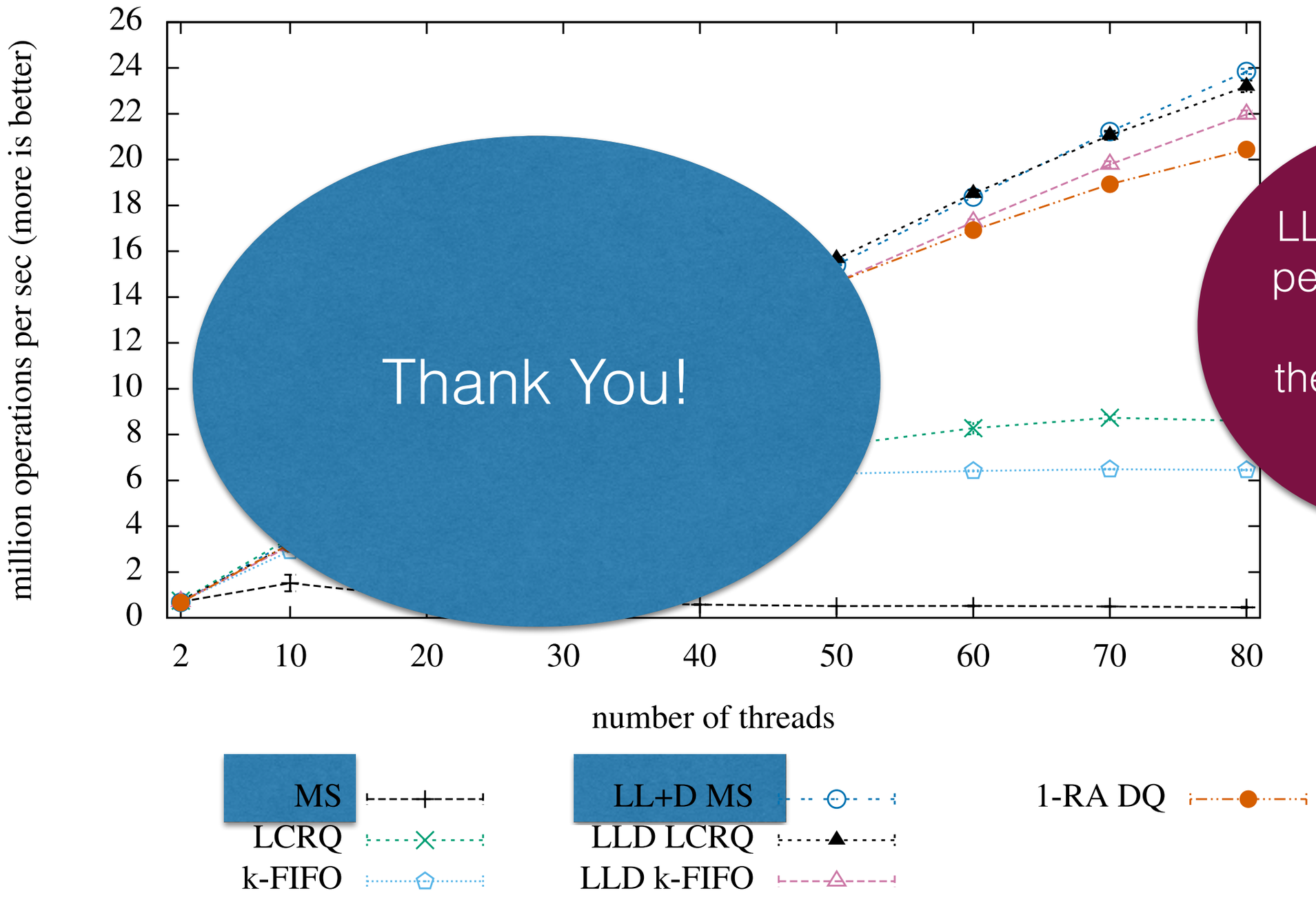
Performance



LLD MS queue performs better than the best known pools

(a) Queues, LL queues, and “queue-like” pools

Performance



(a) Queues, LL queues, and “queue-like” pools