Strengthening and weakening

We had

Definition: Two abstract propositions P and Q are equivalent, notation $P \stackrel{\text{\tiny val}}{=} Q$, iff

- (I) Always when P has truth value I, also Q has truth value I, and
- (2) Always when Q has truth value I, also P has truth value I.

if we relax this, we get strengthening

Strengthening

Definition: The abstract proposition P is stronger than Q, notation P Q, iff

(1) Always when P has truth value I, also Q has truth value I, and

(2) Always when Q has truth value I,

also P has truth value 1.

Q is weaker than P

Strengthening

Definition: The abstract proposition P is stronger than Q, notation $P \nvDash^{al} Q$, iff always when P has truth value I, also Q has truth value I.

always when P is true, Q is also true

Q is weaker than P

Properties

Lemma EI: $P \stackrel{val}{=} Q$ iff $P \Leftrightarrow Q$ is a tautology.

Lemma EWI:
$$P\stackrel{val}{=} Q$$
 iff $P\stackrel{val}{\models} Q$ and $Q\stackrel{val}{\models} P$.

Lemma W2:
$$P \stackrel{val}{\models} P$$

Lemma W3: If
$$P \models Q$$
 and $Q \models R$ then $P \models R$

Lemma W4:
$$P \models^{vai} Q \text{ iff } P \Rightarrow Q \text{ is a tautology.}$$

Standard Weakenings

and-or-weakening

$$P \land Q \models P$$

$$P \models P \lor Q$$

Extremes

$$F \stackrel{val}{\models} P$$

$$P \stackrel{val}{\models} T$$

Calculating with weakenings (the use of standard weakenings)

Substitution

just holds

Simple

$$\phi \models \psi$$

$$\phi\{\xi/P\} \stackrel{val}{\models} \psi\{\xi/P\}$$

Sequential

$$\phi \models \psi$$

$$\phi[\xi/P][\eta/Q] \stackrel{vai}{\models} \psi[\xi/P][\eta/Q]$$

Simultaneous

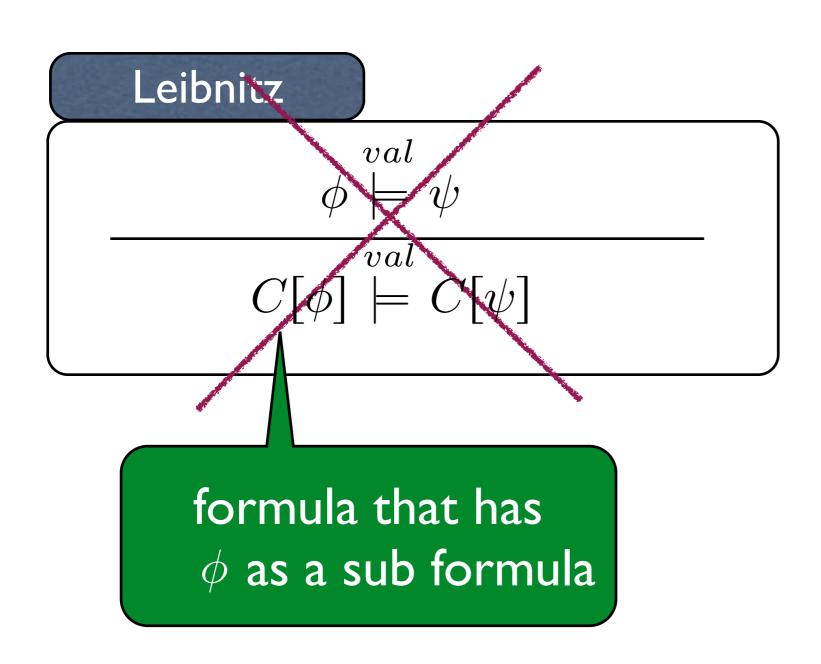
$$\phi \models^{val} \psi$$

EVERY occurrence of P

is substituted!

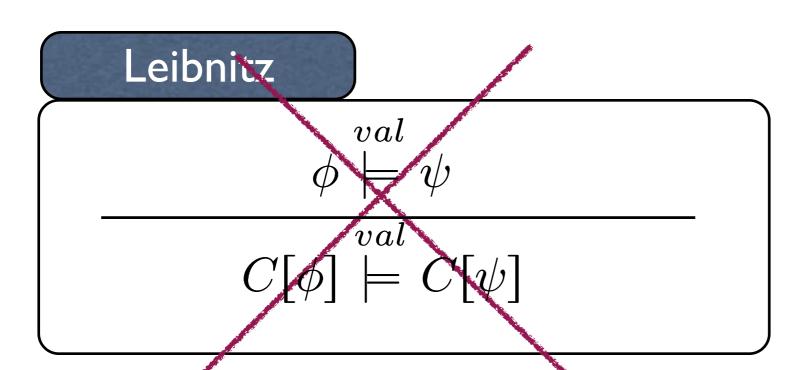
$$\phi[\xi/P, \eta/Q] \stackrel{val}{\models} \psi[\xi/P, \eta/Q]$$

The rule of Leibnitz



does not hold for weakening!

The rule of Leibnitz



does not hold for weakening!

Monotonicity

$$P \models Q$$

$$P \land R \models Q \land R$$

$$\begin{array}{c}
val \\
P \models Q \\
\hline
P \lor R \models Q \lor R
\end{array}$$