Formale Systeme Proseminar

Tasks for Week 4

Task 1 Prove that:

    (a) $P \Rightarrow Q$ is not equivalent to $Q \Rightarrow P$

    (b) $P \Rightarrow Q$ is not equivalent to $\neg P \Rightarrow \neg Q$

    (c) $P \Leftrightarrow Q \Leftrightarrow R$ is not equivalent to $(P \Leftrightarrow Q) \wedge (Q \Leftrightarrow R)$

Remember this!

Task 2 Show the following equivalences by calculating with propositions. Always state precisely: (1) which standard equivalence(s) you use, (2) whether you apply Substitution or Leibnitz, or both, and (3) how you do this.

    (a) $P \vee (\neg P \wedge Q) \stackrel{val}{=} P \vee Q$

    (b) $P \wedge (P \Rightarrow Q) \stackrel{val}{=} P \wedge Q$

    (c) $P \vee (P \wedge Q) \stackrel{val}{=} P$

    (d) $P \wedge (P \vee Q) \stackrel{val}{=} P$

    (e) $P \Rightarrow \neg Q \stackrel{val}{=} \neg(P \wedge Q)$

Task 3 Show with a calculation that the following formulas are tautologies

    (a) $\neg(P \Rightarrow Q) \Leftrightarrow (P \wedge \neg Q)$

    (b) $P \vee \neg((P \Rightarrow Q) \Rightarrow P)$

Task 4 Show with calculations that for arbitrary sets A and B, we have $A \subseteq B$ if and only if $B^c \subseteq A^c$.

Task 5 Check with a calculation whether the following abstract propositions are equivalent:

    (a) $((a \Rightarrow b) \Rightarrow \neg a)$ and $(\neg b \vee \neg a) \wedge (\neg b \vee b)$

    (b) $a \wedge b$ and $(\neg a \vee b) \Leftrightarrow a$

Task 6 Prove with a calculation that the following propositions are equivalent:

    (a) $x \in A \cup (A \cap B)$ and $x \in A \cup (B \cap B^c)$

    (b) $x \in A \cap (B \cup A^c)^c$ and $x \in B^c \cap A \cap (A \cup A^c)$.

Task 7 Prove with a calculation that

    (a) $(A^c)^c = A$ for any set A

    (b) $A \cup (A \cap B) = A$ for any two sets A and B.