Process Algebra
Uitwerkingen opgaven practicum 5

Hieronder staan de uitwerkingen van de volgende opgaven:

2.9.10: 2, 7

3.1.6: 1, 4, 8

3.2.8: 1, 5

Exercise 2.9.10.2

We use the abbreviations given in 2.9.3 (p. 64) and Figure 14 (p. 65).

Table 18:

$\pi_1(S_\lambda) = 0 + 1$

$\pi_2(S_\lambda) = 0(0 + 1 + \underline{0}) + 1(0 + 1 + \underline{1})$

$\pi_3(S_\lambda) = 0(0(0+1+\underline{0})+1(0+1+\underline{1})+\underline{0}(0+1))+1(0(0+1+\underline{0})+1(0+1+\underline{1})+\underline{1}(0+1))$

Table 19:

$\pi_1(S) = 0 + 1$

$\pi_2(S) = 0(\underline{0} + 0 + 1) + 1(\underline{1} + 0 + 1)$

$\pi_3(S) = 0(\underline{0}(0+1)+0(\underline{0}+0+1)+1(\underline{1}+0+1))+1(\underline{1}(0+1)+0(\underline{0}+0+1)+1(\underline{1}+0+1))$

We leave it to the reader to show that:

- $\pi_1(S_\lambda) = \pi_1(S)$,
- $\pi_2(S_\lambda) = \pi_2(S)$,
- $\pi_3(S_\lambda) = \pi_3(S)$.

Exercise 2.9.10.7

We have to prove that S in the specification from Table 22, satisfies S in the specification of Table 20. We relate the two specifications by the following mapping (from Table 22 to Table 20):

$$S \mapsto S$$

$$T \mapsto T$$

$$T_d \mapsto R \cdot pop(d)$$

Given this mapping, we have to show that the equations of Table 20 hold for S from Table 22. The first two equations of Table 20 hold trivially. Remains to show that:

$$R \cdot pop(d) = pop(d) + T \cdot (R \cdot pop(d))$$

Proof:

$$\begin{array}{rl} & R \cdot pop(d) \\ = & (\epsilon + T \cdot R) \cdot pop(d) \\ = & \epsilon \cdot pop(d) + (T \cdot R) \cdot pop(d) \\ = & pop(d) + T \cdot (R \cdot pop(d)) \end{array}$$

Exercise 3.1.6.1

1. $$\begin{array}{ll} aa \| bb & = \\ aa \mathbin{⌊\!⌊} bb + bb \mathbin{⌊\!⌊} aa & = \\ a(a \| bb) + b(b \| aa) & = \\ a(a \mathbin{⌊\!⌊} bb + bb \mathbin{⌊\!⌊} a) + b(b \mathbin{⌊\!⌊} aa + aa \mathbin{⌊\!⌊} b) & = \\ a(abb + b(b \| a)) + b(baa + a(a \| b)) & = \\ a(abb + b(b \mathbin{⌊\!⌊} a + a \mathbin{⌊\!⌊} b)) + b(baa + a(a \mathbin{⌊\!⌊} b + b \mathbin{⌊\!⌊} a)) & = \\ a(abb + b(ba + ab)) + b(baa + a(ab + ba)) & \end{array}$$

2. $$\begin{array}{ll} (a+b) \| (a+b) & = \\ (a+b) \mathbin{⌊\!⌊} (a+b) + (a+b) \mathbin{⌊\!⌊} (a_b) & = \\ (a+b) \mathbin{⌊\!⌊} (a+b) & = \\ a \mathbin{⌊\!⌊} (a+b) + b \mathbin{⌊\!⌊} (a+b) & = \\ a(a+b) + b(a+b) & \end{array}$$

3. $$\begin{array}{lr}
aaa\|aaa & = \\
aaa \mathbin{⌊\!⌊} aaa + aaa \mathbin{⌊\!⌊} aaa & = \\
aaa \mathbin{⌊\!⌊} aaa & = \\
a(aa\|aaa) & = \\
a(aa \mathbin{⌊\!⌊} aaa + aaa \mathbin{⌊\!⌊} aa) & = \\
a(a(a\|aaa) + a(aa\|aa)) & = \\
a(a(a \mathbin{⌊\!⌊} aaa + aaa \mathbin{⌊\!⌊} a) + a(aa \mathbin{⌊\!⌊} aa)) & = \\
a(a(aaaa + a(aa\|a)) + a(a(a\|aa))) & = \\
a(a(aaaa + a(aa \mathbin{⌊\!⌊} a + a \mathbin{⌊\!⌊} aa)) + a(a(a \mathbin{⌊\!⌊} aa + aa \mathbin{⌊\!⌊} a))) & = \\
a(a(aaaa + a(a(a\|a) + aaa)) + a(a(aaa + a(a\|a)))) & = \\
a(a(aaaa + a(a(a \mathbin{⌊\!⌊} a) + aaa)) + a(a(aaa + a(a \mathbin{⌊\!⌊} a)))) & = \\
a(a(aaaa + a(aaa + aaa)) + a(a(aaa + aaa))) & = \\
a(a(aaaa + aaaa) + aaaaa) & = \\
a(aaaaa + aaaaa) & = \\
aaaaaa &
\end{array}$$

4. $$\begin{array}{lr}
abc\|(d+e) & = \\
abc \mathbin{⌊\!⌊} (d+e) + (d+e) \mathbin{⌊\!⌊} abc & = \\
a(bc\|(d+e)) + d \mathbin{⌊\!⌊} abc + e \mathbin{⌊\!⌊} abc & = \\
a(bc \mathbin{⌊\!⌊} (d+e) + (d+e) \mathbin{⌊\!⌊} bc) + dabc + eabc & = \\
a(b(c\|(d+e)) + d \mathbin{⌊\!⌊} bc + e \mathbin{⌊\!⌊} bc) + dabc + eabc & = \\
a(b(c \mathbin{⌊\!⌊} (d+e) + (d+e) \mathbin{⌊\!⌊} c) + dbc + ebc) + dabc + eabc & = \\
a(b(c(d+e) + d \mathbin{⌊\!⌊} c + e \mathbin{⌊\!⌊} c) + dbc + ebc) + dabc + eabc & = \\
a(b(c(d+e) + dc + ec) + dbc + ebc) + dabc + eabc &
\end{array}$$

Exercise 3.1.6.4

$K' = ((q\|d)\|n)cK'$

Exercise 3.1.6.8

Neem bijvoorbeeld $x = a, y = b, z = c$. Dan:

$$(a+b)\|c = ac + bc + c(a+b)$$

$$a\|c + b\|c = ac + ca + bc + cb$$

en deze twee zijn ongelijk.

Exercise 3.2.8.1

(i)
$$
\begin{array}{ll}
(a\|b)\|c & = \\
(a\|b) \mathbin{\lfloor\!\lfloor} c + c \mathbin{\lfloor\!\lfloor} (a\|b) & = \\
(ab+ba) \mathbin{\lfloor\!\lfloor} c + c(ab+ba) & = \\
ab \mathbin{\lfloor\!\lfloor} c + ba \mathbin{\lfloor\!\lfloor} c + c(ab+ba) & = \\
a(b\|c) + b(a\|c) + c(ab+ba) & = \\
a(bc+cb) + b(ac+ca) + c(ab+ba) &
\end{array}
$$

(ii) $a(bc+cb)+b(ca+ac)+c(ba+ab)$

$$
\begin{array}{ll}
a\|(b\|c) & = \\
a\|(bc+cb) & = \\
a \mathbin{\lfloor\!\lfloor} (bc+cb) + (bc+cb) \mathbin{\lfloor\!\lfloor} a & = \\
a(bc+cb) + bc \mathbin{\lfloor\!\lfloor} a + cb \mathbin{\lfloor\!\lfloor} a & = \\
a(bc+cb) + b(c\|a) + c(b\|a) & = \\
a(bc+cb) + b(ca+ac) + c(ba+ab) &
\end{array}
$$

(iii)
$$
\begin{array}{ll}
(a \mathbin{\lfloor\!\lfloor} b) \mathbin{\lfloor\!\lfloor} c & = \\
ab \mathbin{\lfloor\!\lfloor} c & = \\
a(b\|c) & = \\
a(bc+cb) &
\end{array}
$$

(iv)
$$
\begin{array}{ll}
a \mathbin{\lfloor\!\lfloor} (b\|c) & = \\
a(b\|c) & = \\
a(bc+cb) &
\end{array}
$$

Exercise 3.2.8.5

Te bewijzen: $a^n = a^{\underline{\underline{n}}}$. We bewijzen eerst een lemma:

$$a^n\|a = a^{n+1}$$

Bewijs lemma: Inductie naar n.

Basis: $n=1$: $a\|a = aa = a^2$.

Stap: $n=k+1$: $a^{k+1}\|a = a(a^k\|a) + aa^{k+1} \overset{\text{IH}}{=} aa^{k+1} + aa^{k+1} = a^{k+2}$.

Nu bewijzen we de gevraagde stelling (opnieuw met inductie naar n):

Basis: $n=1$: $a^{\underline{\underline{1}}} = a = a^1$.

Stap: $n=k+1$: $a^{\underline{\underline{k+1}}} = a^{\underline{\underline{k}}}\|a \overset{\text{IH}}{=} a^k\| = a^{k+1}$.

Gezocht: een t zodanig dat $t^{\underline{\underline{n}}} \neq t^n$. Neem bijvoorbeeld $t = ab$, dan:

$$t^{\underline{\underline{2}}} = ab\|ab = a(bab+abb) \neq abab = t^2$$