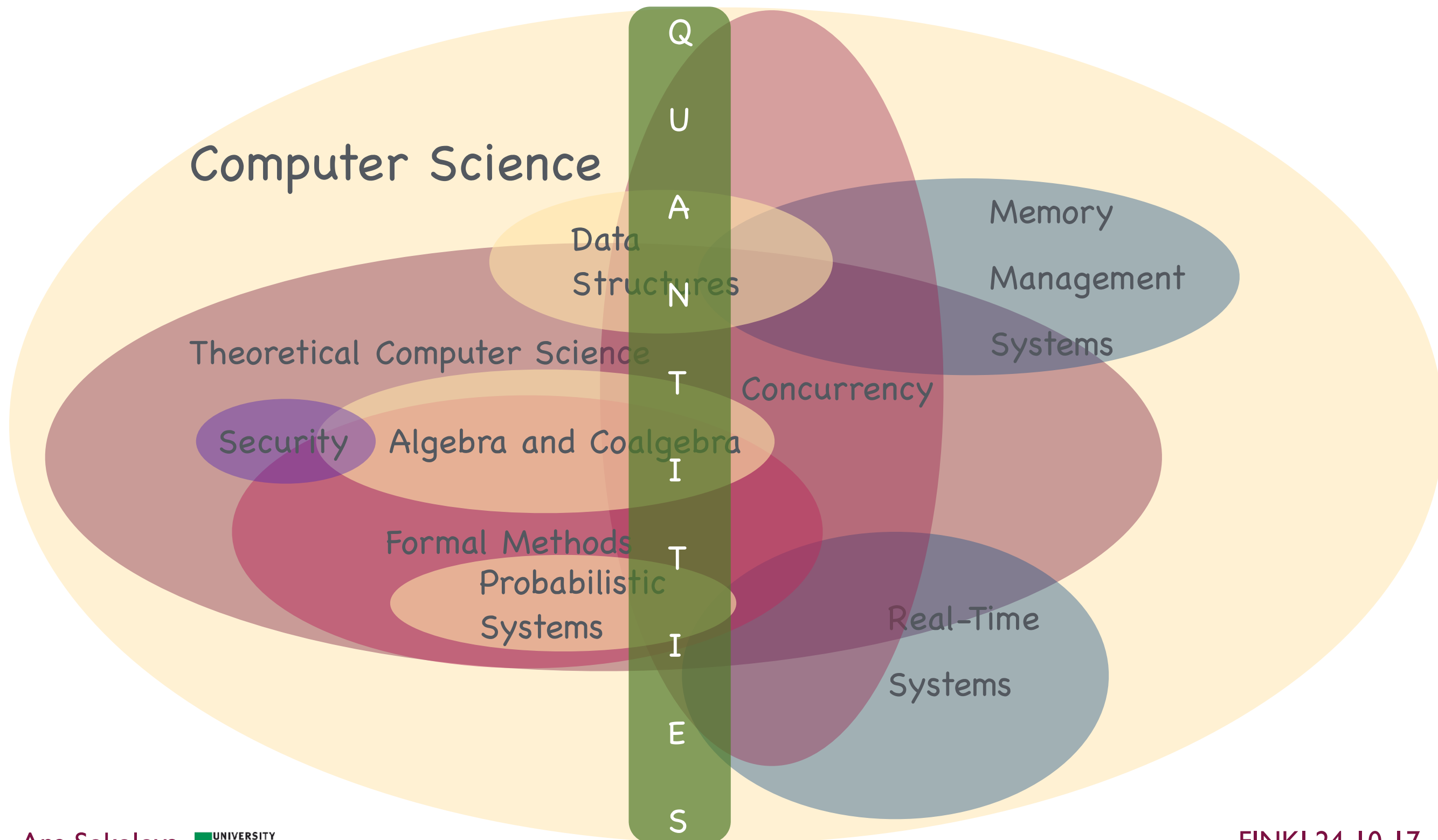


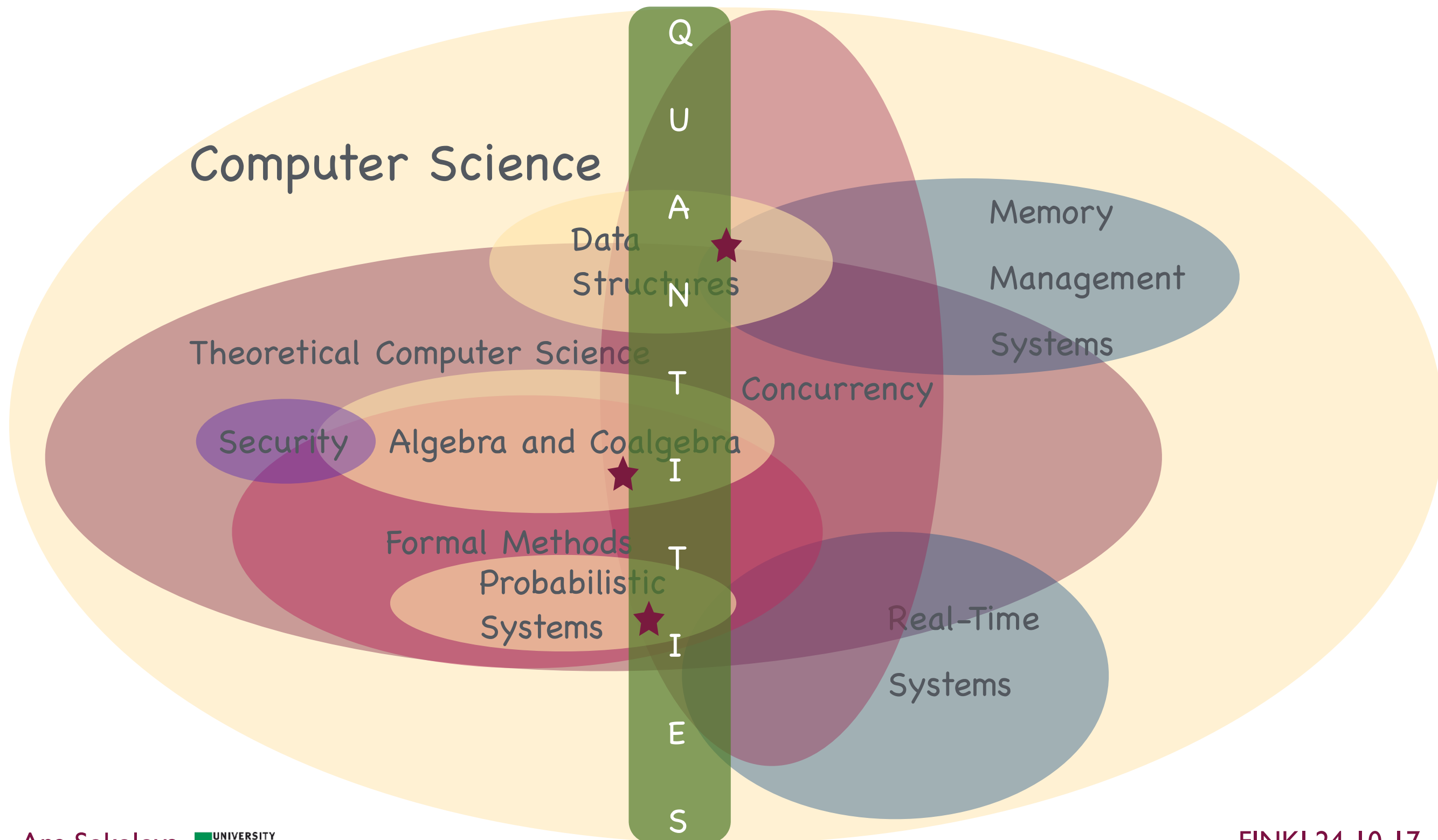
Concurrent Data Structures: Semantics and Relaxations

Ana Sokolova  UNIVERSITY
of SALZBURG

Background big picture



Favourites



Concurrent Data Structures: Semantics and Relaxations

Ana Sokolova  UNIVERSITY
of SALZBURG

Concurrent Data Structures: Correctness and Performance

Semantics of concurrent data structures

t1: enq(2) deq(1)
t2: enq(1) deq(2)

e.g. pools, queues, stacks

- Sequential specification = set of legal sequences

e.g. queue legal sequence
enq(1)enq(2)deq(1)deq(2)

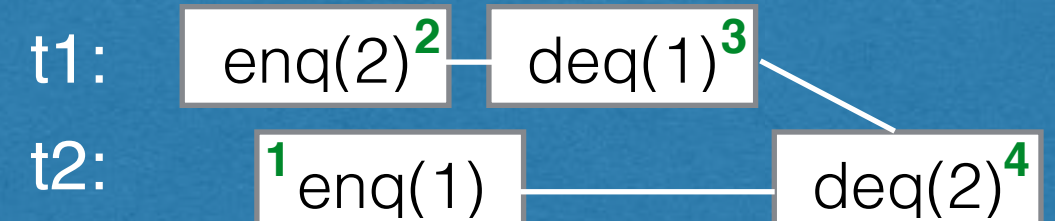
- Consistency condition = e.g. linearizability / sequential consistency

e.g. the concurrent history above is a linearizable queue concurrent history

Consistency conditions

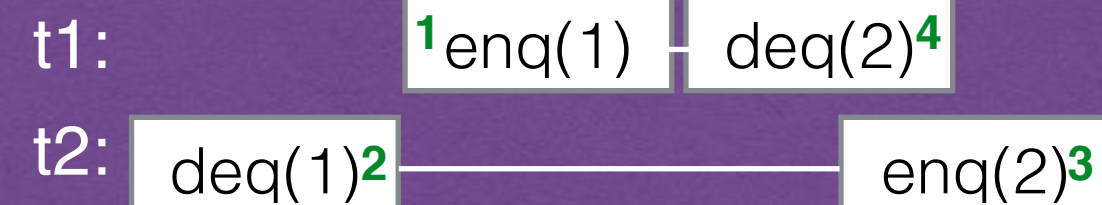
there exists a legal sequence that preserves precedence

Linearizability [Herlihy, Wing '90]

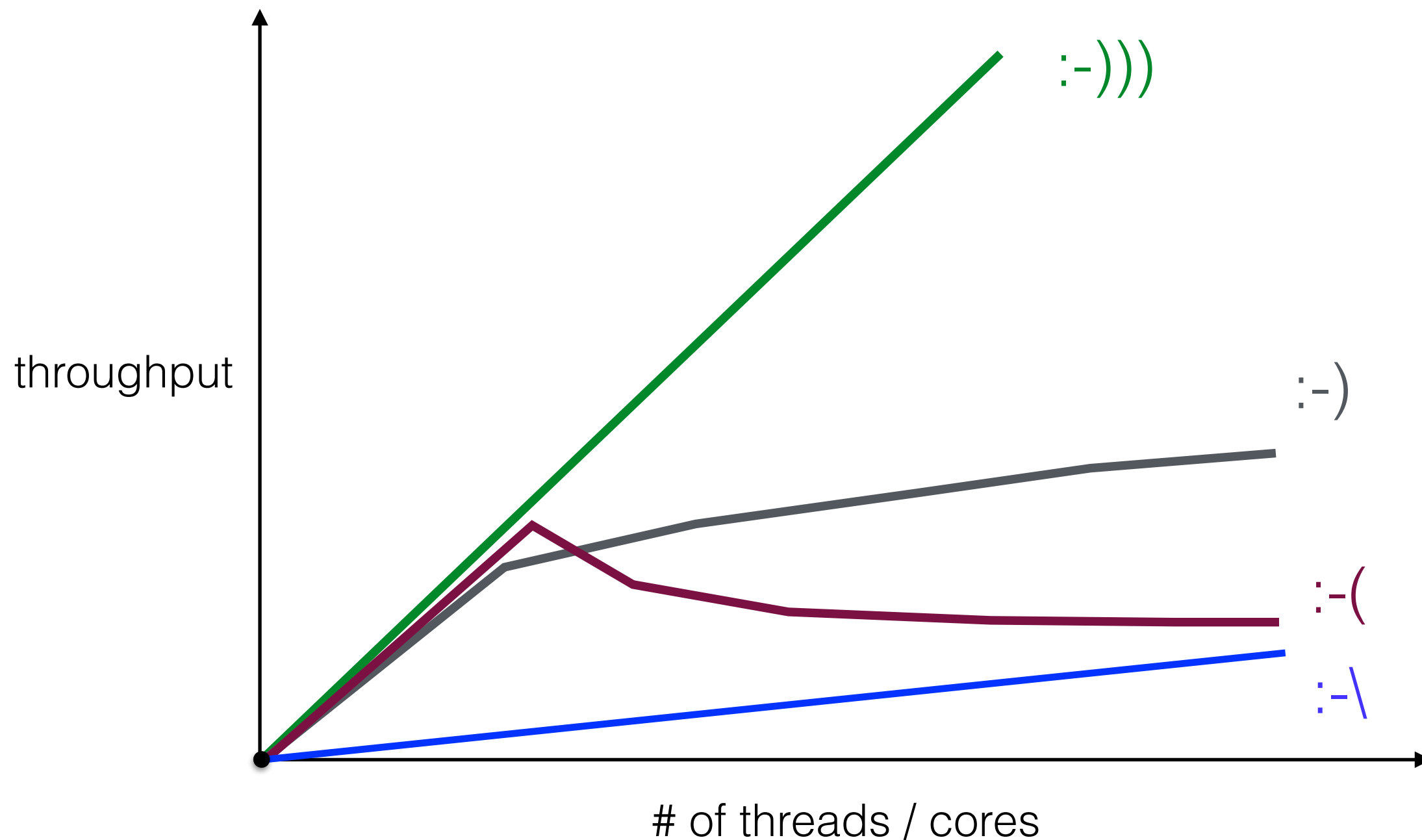


Sequential Consistency [Lamport'79]

there exists a legal sequence that preserves per-thread precedence (program order)



Performance and scalability



Relaxations allow trading

correctness
for
performance

provide the **potential**
for better-performing
implementations

Relaxing the semantics

not
“sequentially
correct”

Quantitative relaxations
POPL13

- Sequential specification = set of legal sequences
- Consistency condition = e.g. linearizability / sequential consistency

for queues/stacks only
(feel free to ask for more)

Local linearizability
CONCUR16

too weak

Relaxing the sequential specification

Quantitative
relaxations
(POPL13)

Goal

Stack - incorrect behavior

`push(a)push(b)push(c)pop(a)pop(b)`

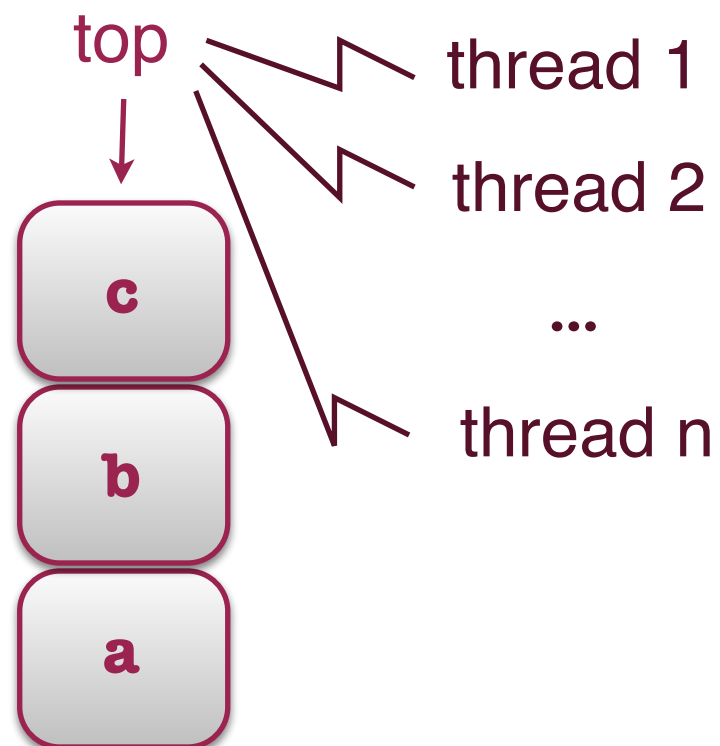
- trade correctness for performance
- in a controlled way with quantitative bounds

correct in a relaxed stack
... 2-relaxed? 3-relaxed?

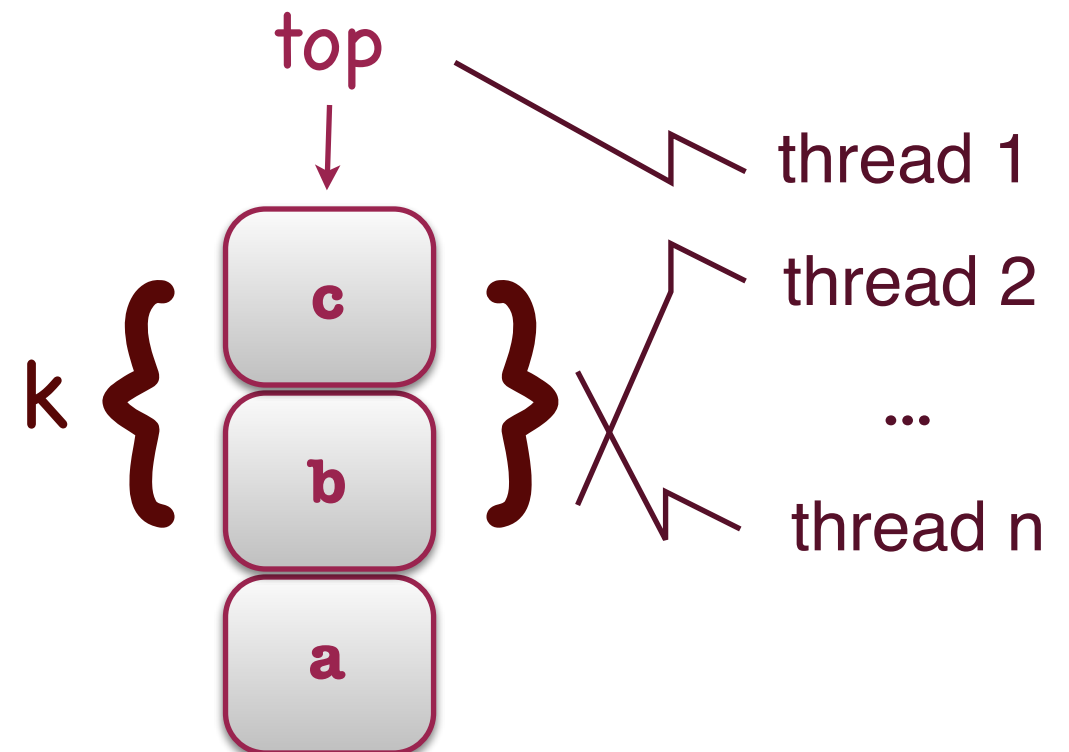
measure the
error from correct
behaviour

How can relaxing help?

Stack



k-Relaxed stack



What we have

- Framework

for semantic
relaxations

- Generic examples

out-of-order /
stuttering

- Concrete relaxation examples

stacks, queues,
priority queues,.. /
CAS, shared counter

- Efficient concurrent implementations

of relaxation
instances

The big picture



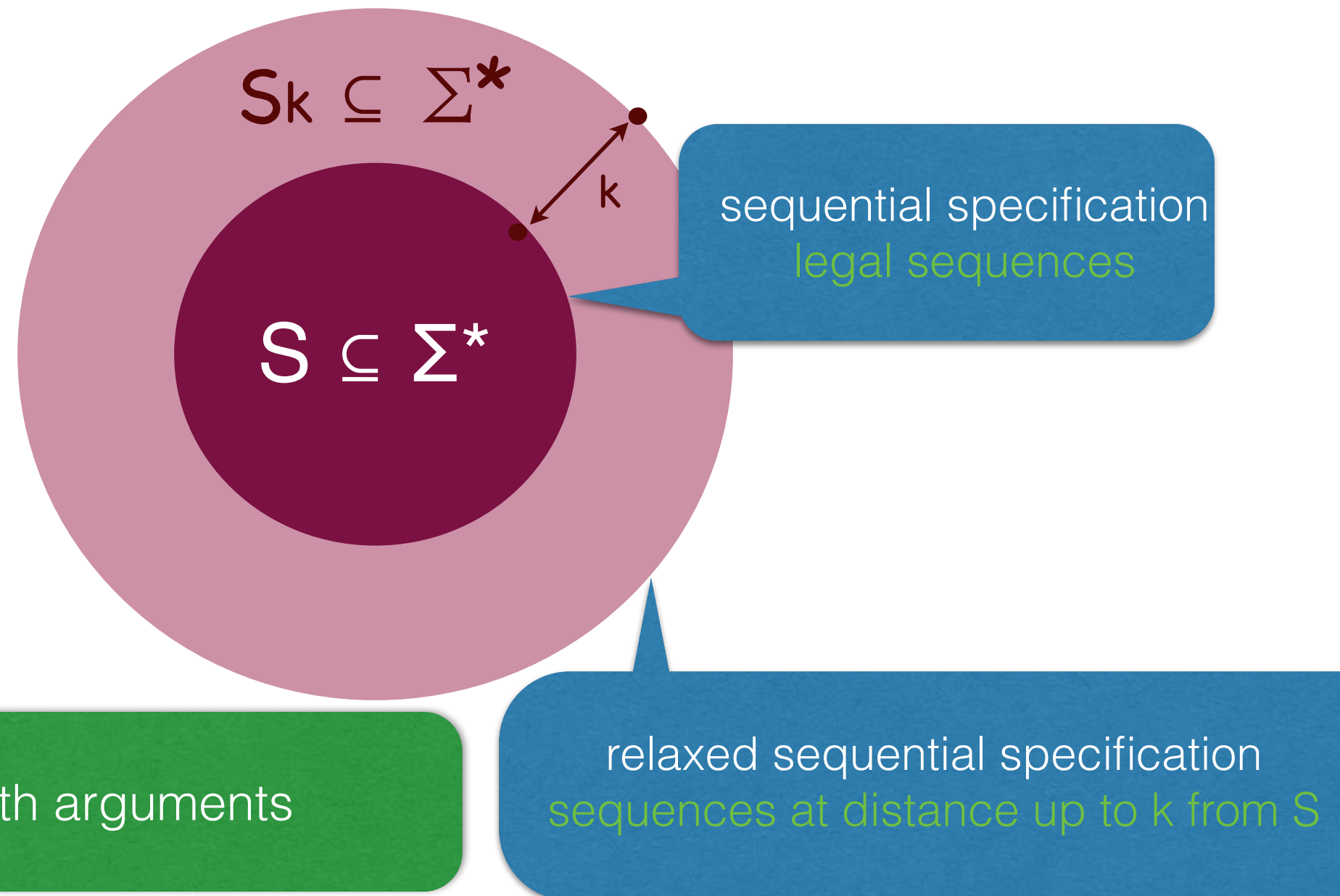
A diagram consisting of a large maroon circle containing the mathematical expression $S \subseteq \Sigma^*$. A blue speech bubble points from the right side of the circle to the text "sequential specification" and "legal sequences".

$$S \subseteq \Sigma^*$$

sequential specification
legal sequences

Σ - methods with arguments

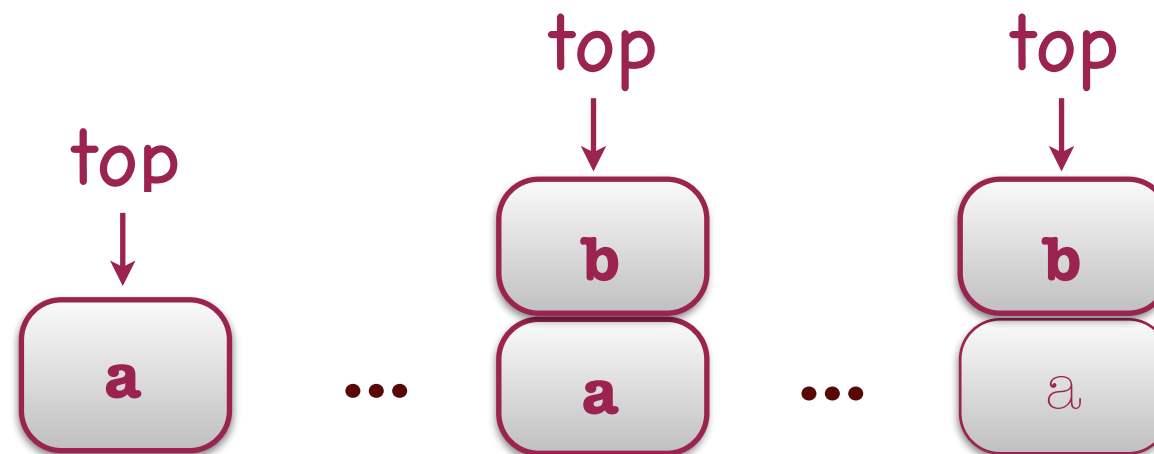
The big picture



Syntactic distances do not help

$\text{push}(a)[\text{push}(i)\text{pop}(i)]^n\text{push}(b)[\text{push}(j)\text{pop}(j)]^m\text{pop}(a)$

is a 1-out-of-order stack sequence



its permutation distance is $\min(2n, 2m)$

Semantic distances need a notion of state

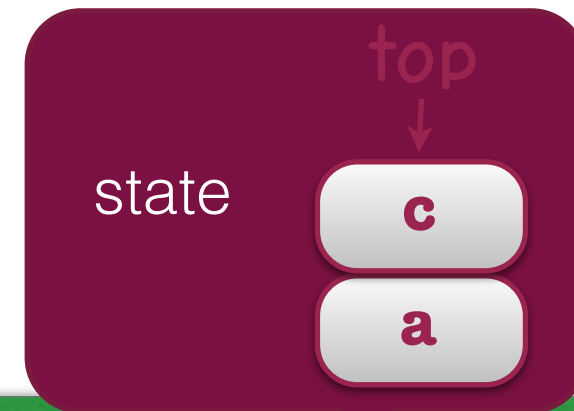
- States are equivalence classes of sequences in S

example: for stack

$\text{push(a)push(b)pop(b)push(c)} \equiv \text{push(a)push(c)}$

- Two sequences in S are equivalent iff they have an indistinguishable future

$$x \equiv y \iff \forall u \in \Sigma^*. (xu \in S \iff yu \in S)$$



Semantics goes operational

$S \subseteq \Sigma^*$ is the sequential specification

states

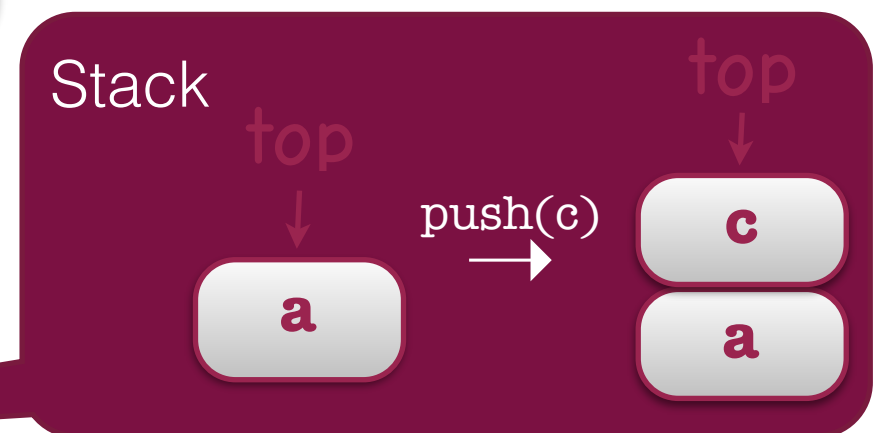
labels

initial state

$\text{LTS}(S) = (S/\equiv, \Sigma, \rightarrow, [\varepsilon]_\equiv)$ with

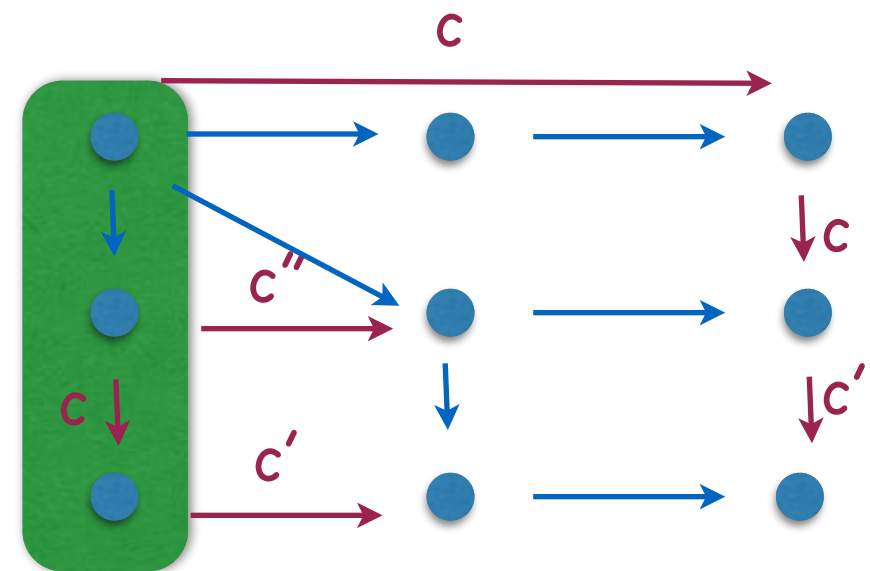
transition relation

$$[s]_\equiv \xrightarrow{m} [sm]_\equiv \iff sm \in S$$



The relaxation framework

- Start from LTS(S)
- Add transitions with transition costs
- Fix a path cost function



distance - minimal cost on all paths labelled by the sequence

Generic out-of-order

$$\text{segment_cost}(q \xrightarrow{m} q') = |\mathbf{v}|$$

transition cost

Where \mathbf{v} is a sequence of minimal length s.t.

removing \mathbf{v} enables a transition

or

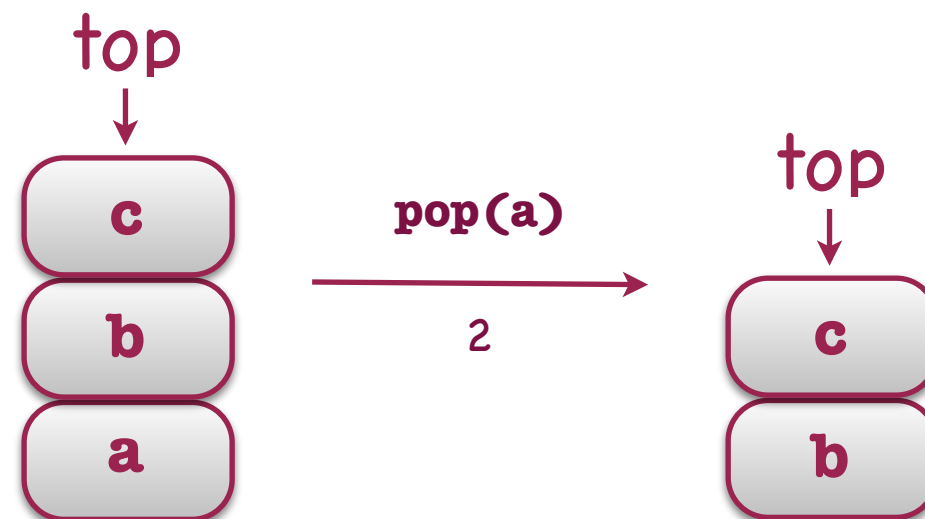
inserting \mathbf{v} enables a transition

goes with different path costs

Out-of-order stack

Sequence of push's with no matching pop

- Canonical representative of a state
- Add incorrect transitions with **segment-costs**



- Possible path cost functions **max**, **sum**,...

also more advanced

Relaxing the Consistency Condition

Local Linearizability
(CONCUR16)

Local Linearizability

main idea

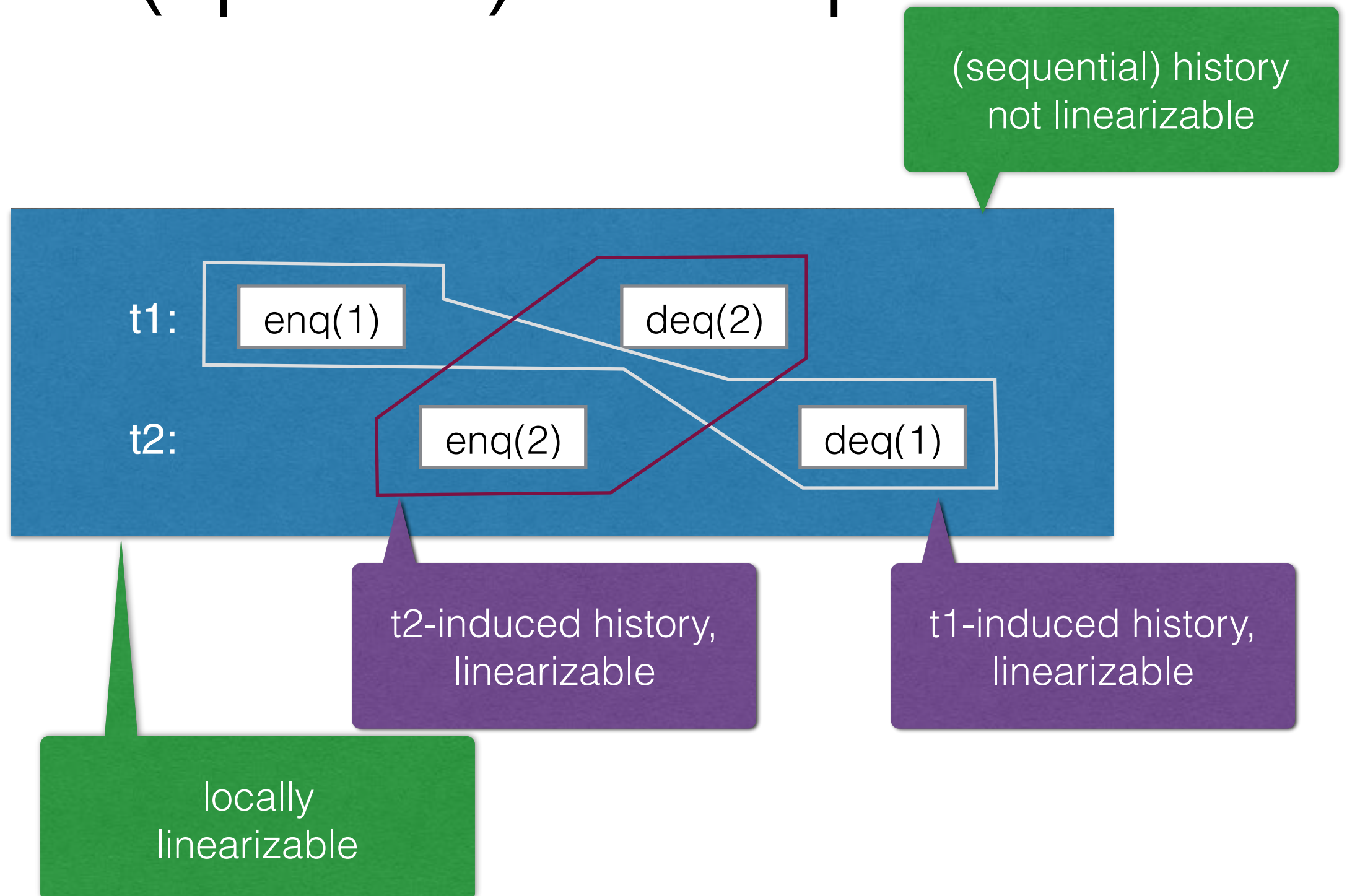
Already present in some shared-memory consistency conditions
(not in our form of choice)

- Partition a history into a set of local histories
- Require linearizability per local history

Local sequential consistency... is also possible

no global witness

Local Linearizability (queue) example



Local Linearizability (queue) definition

Queue signature $\Sigma = \{\text{enq}(x) \mid x \in V\} \cup \{\text{deq}(x) \mid x \in V\} \cup \{\text{deq}(\text{empty})\}$

For a history **h** with a thread T, we put

$$I_T = \{\text{enq}(x)^T \in \mathbf{h} \mid x \in V\}$$

in-methods of thread T
are
enqueuees performed
by thread T

$$O_T = \{\text{deq}(x)^{T'} \in \mathbf{h} \mid \text{enq}(x)^T \in I_T\} \cup \{\text{deq}(\text{empty})\}$$

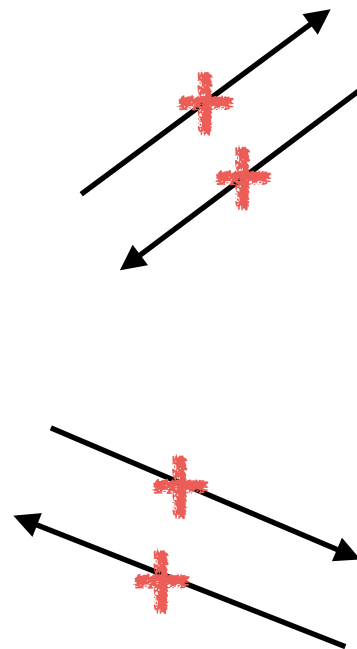
out-methods of thread T
are dequeuees
(performed by any thread)
corresponding to enqueuees that
are in-methods

h is locally linearizable iff every thread-induced history
 $\mathbf{h}_T = \mathbf{h} \mid (I_T \cup O_T)$
is linearizable.

Where do we stand?

In general

Local Linearizability



Linearizability



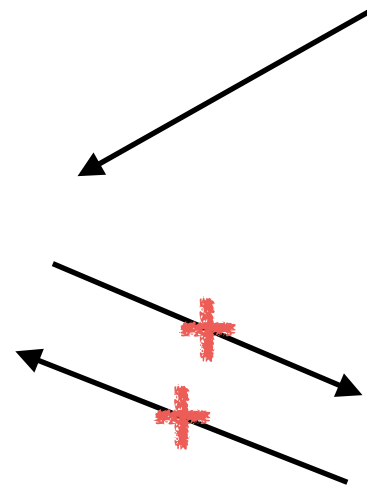
Sequential Consistency

Where do we stand?

For queues (and most container-type data structures)

Local Linearizability

Linearizability



Sequential Consistency

Properties

Local linearizability is compositional

like linearizability
unlike sequential consistency

h (over multiple objects) is locally linearizable
iff

each per-object subhistory of **h** is locally linearizable

Local linearizability is modular /
“decompositional”

uses decomposition into smaller
histories, by definition



may allow for modular verification

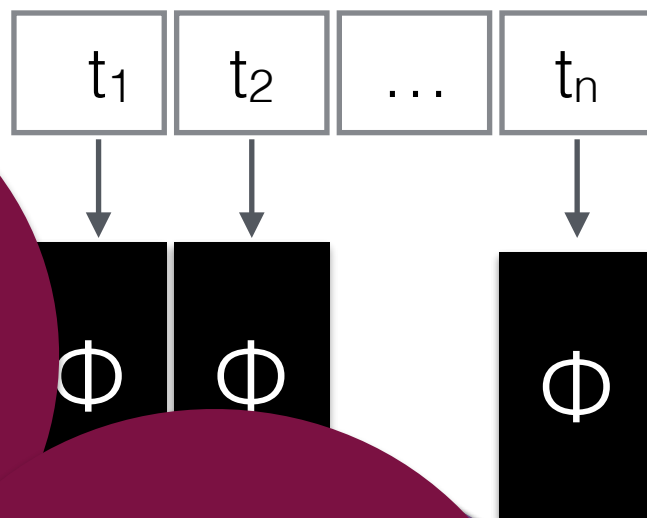
Generic Implementations

Your favorite linearizable data structure implementation

Φ

turns into a locally linearizable implementation by:

LLD Φ
(locally
linearizable)

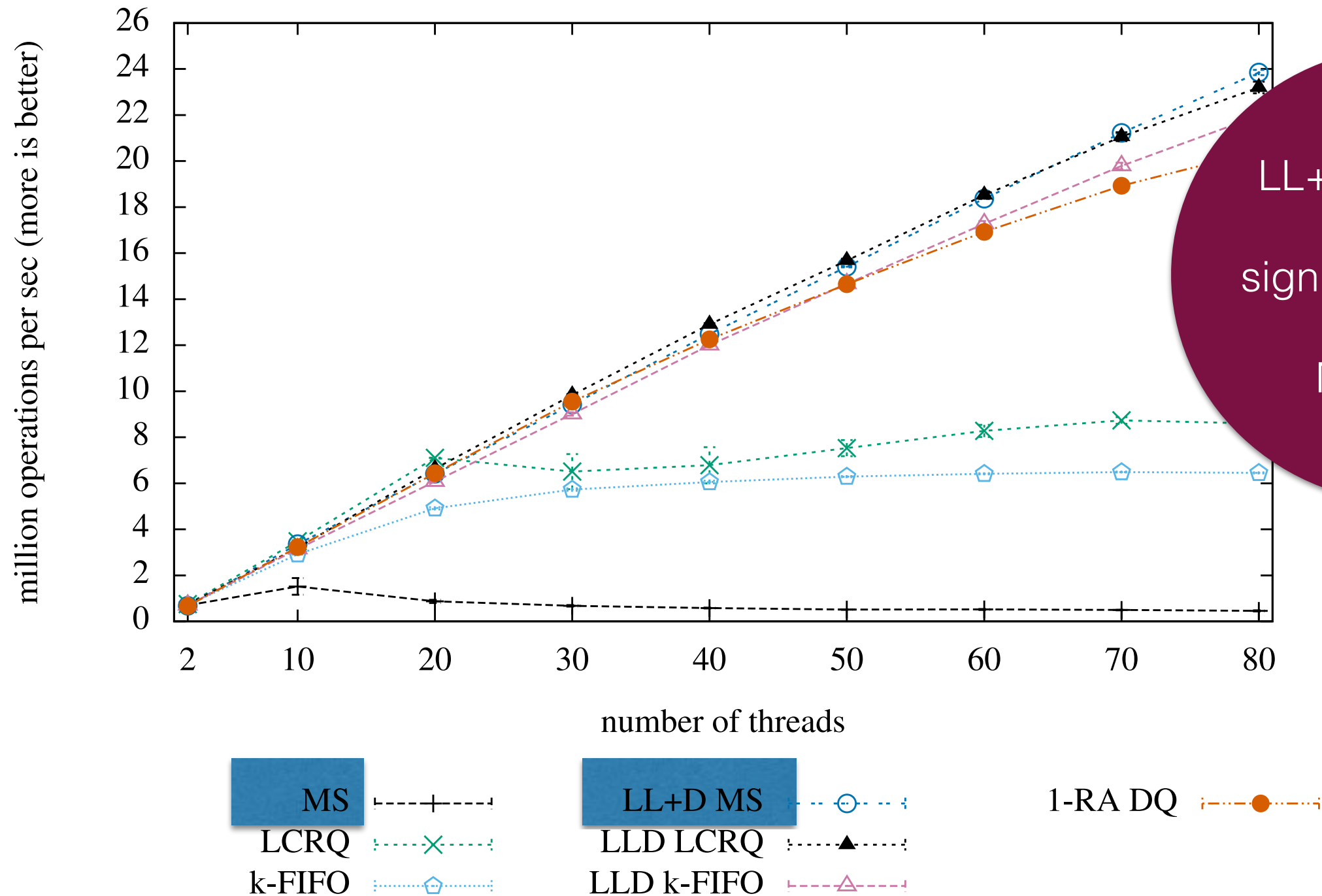


segment of possibly
dynamic size (n)

LL+D Φ
(also pool
linearizable)

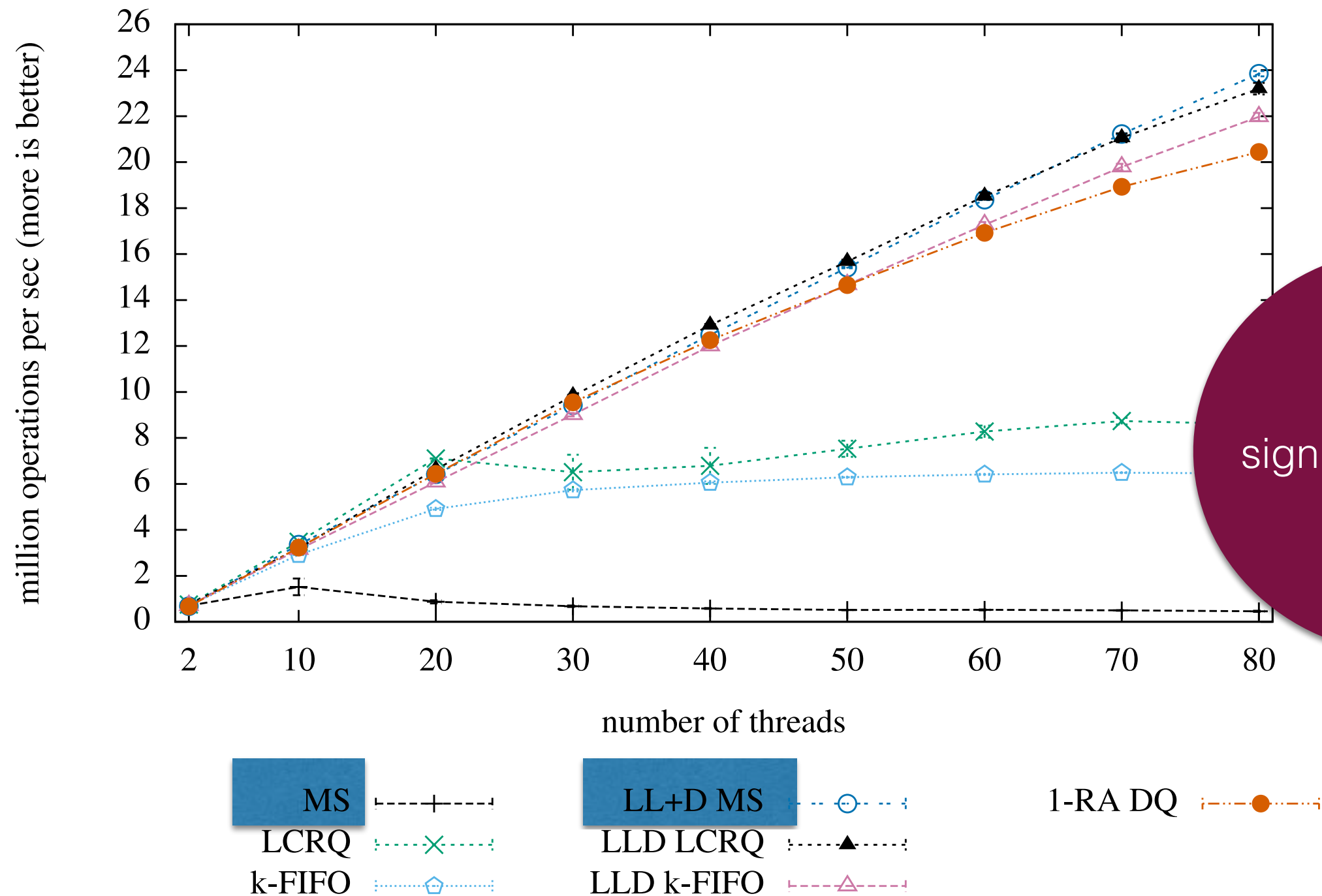
local inserts / global (randomly distributed) removes

Performance



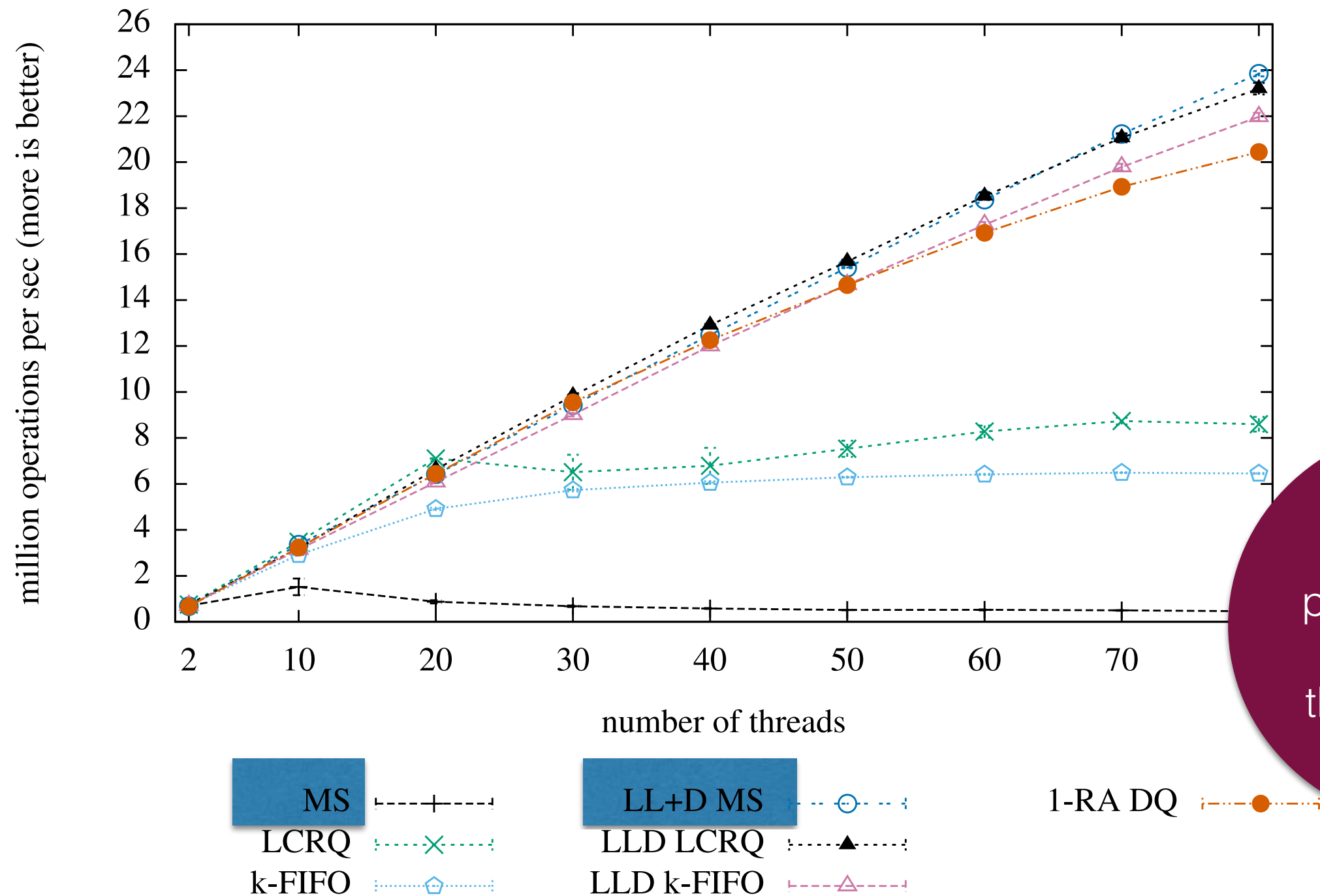
(a) Queues, LL queues, and “queue-like” pools

Performance



(a) Queues, LL queues, and “queue-like” pools

Performance



LL+D MS
queue
performs better
than
the best known
pools

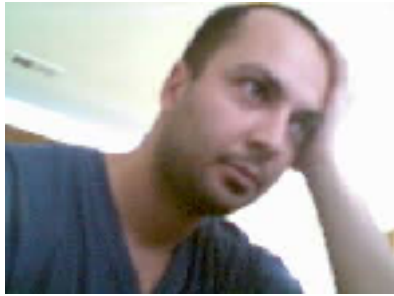
(a) Queues, LL queues, and “queue-like” pools


A large blue oval with a slight drop shadow, containing the text "Thank You!".

Thank You!

and many thanks to
my dear coauthors





Ali Sezgin 



Hannes Payer





Andreas Holzer 




Michael Lippautz



Andreas Haas 



Tom Henzinger




Helmut Veith 



Christoph Kirsch
