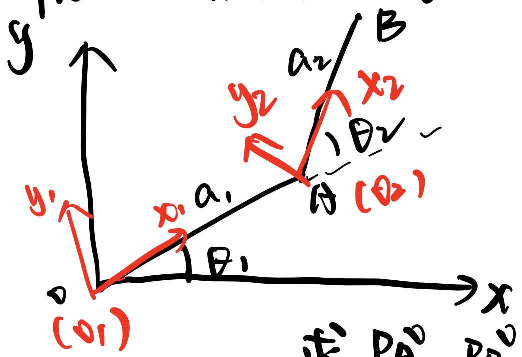
1) Kinematics

Example: 2-link robot arm



已知 $A(a_1, 0)$ in $\{1\}$ $P_A^1(a_1, 0)$

B: $P_B^2 = (a_2, 0)$

P_A^1 P_B^2: local coordinates

求 P_A^0 P_B^0

$\{0\} \rightarrow \{1\} \rightarrow \{2\}$

$P_A^0 \Leftarrow P_A^1$

$P_B^0 \Leftarrow P_B^1 \Leftarrow P_B^2$

$$\underset{2\times1}{P_B^1} = \underset{2\times1}{P_A^1} + \underset{2\times2}{R_2^1} \underset{2\times1}{P_B^2} \qquad P_A^1 = \begin{pmatrix} a_1 \\ 0 \end{pmatrix} \quad R_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \quad P_B^2 = \begin{pmatrix} a_2 \\ 0 \end{pmatrix}$$

$$P_B^1 = \begin{pmatrix} a_1 + a_2\cos\theta_2 \\ a_2\sin\theta_2 \end{pmatrix}$$

Homogeneous Transformation

齐次

$$P_B^1 = \underset{3\times3}{H} P_B^2$$

$$H = \begin{bmatrix} \cos\theta & -\sin\theta & x_1 \\ \sin\theta & \cos\theta & y_1 \\ 0 & 0 & 1 \end{bmatrix} \qquad P_B^2 = \begin{pmatrix} a_2 \\ 0 \\ 1 \end{pmatrix} \quad P_A^1 = \begin{pmatrix} a_1 \\ 0 \\ 1 \end{pmatrix}$$

$$P_B^1 = H P_B^2$$

$$= \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & a_1 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_1 + a_2\cos\theta_2 \\ a_2\sin\theta_2 \\ 1 \end{bmatrix}$$

$H_{4\times4}$

Robot: 3D space

$$H_{4\times4} = \begin{bmatrix} R_{3\times3} & d_{3\times1} \\ 0_{1\times3} & 1 \end{bmatrix}$$

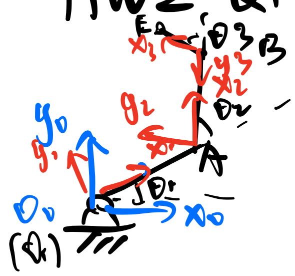
$$P_B^1 = \underset{4\times4}{H} P_B^2$$

$R_{3\times3}$

$$\underset{4\times4}{H} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_1 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} x \\ y \\ z. \\ \end{array}$$

$$p_B^1 = H \cdot \begin{bmatrix} a_2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 + a_2\cos\theta_2 \\ a_2 \sin\theta_2 \\ 0 \\ 1 \end{bmatrix}$$

HW2 Q10: 3-link robot arm



已知: $L_1 = 3 \quad L_2 = 4 \quad L_3 = 2$

$\theta_1 = \frac{\pi}{4} \quad \theta_2 = \frac{\pi}{3} \quad \theta_3 = \frac{\pi}{6}$

$P_A^1 = (3, 0) \quad P_B^2 = (4, 0) \quad P_E^3 = (2, 0)$

求 P_E^0

$\{0\} \to \{1\} \to \{2\} \to \{3\}$

$P_E^0 \xleftarrow{R} P_E^1 \xleftarrow{R+d} P_E^2 \xleftarrow{R+d} P_E^3$

$H_{3\times3}$

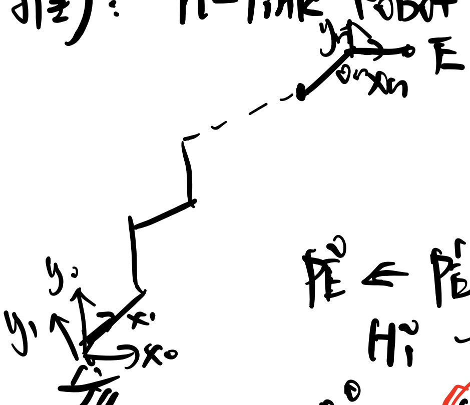
$P_E^2 = H_3^2\, P_E^3 \qquad P_E^1 = H_2^1 \cdot P_E^2 \qquad P_E^0 = H_1^0 P_E^1$

$$\Rightarrow P_E^0 = \overset{H_3^0}{H_1^0 H_2^1 H_3^2}\, P_E^3$$

$$= \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & a_1 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & a_2 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_3 \\ 0 \\ 1 \end{pmatrix}$$

$= \ ?$

推广: n-link robot arm



已知 $P_E^n = (a_n, 0)$ in $\{n\}$

求 P_E^0

$P_E^0 \leftarrow P_E^1 \leftarrow \cdots \leftarrow P_E^{n-1} \leftarrow P_E^n$

$H_1^0 \quad \text{-----} \quad H_{n-1}^{n-2} \quad H_n^{n-1}$

$P_E^0 = H_n^0\, P_E^n$

$H_1^1 H_2^1 --- H_n^{n-1}$

HW3 Q2:

① 逆90° $H_1 = \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

② 平移 $4\vec{i} - 3\vec{j} + 7\vec{k}$ $H_2 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\theta = 0^\circ$

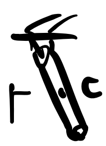
$H_0 = H_1 \times H_2$

Q3: $X_0 = H_2 \times \underline{H_1 \times \begin{bmatrix} 7 \\ 3 \\ 2 \end{bmatrix}}$

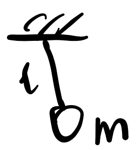
2) Dynamics:

基础: Newton Law: $F = ma$ $\quad M = I\alpha$

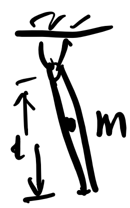
I: 转动惯量.



杆长m m均匀分布 $\underline{I = \int r^2 \, dm}$



$\underline{I = ml^2}$



$I = I_c + m\left(\frac{l}{2}\right)^2$

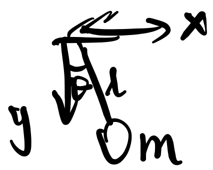
$= \frac{1}{12}ml^2 + \frac{1}{4}ml^2$

$= \underline{\frac{1}{3}ml^2}$

动能 $E_k = \frac{1}{2}mv^2$ $\quad E_k = \frac{1}{2} I \cdot w^2$

势能: $E_p = mgy$

广义坐标q：描述一个系统的最少独立变量个数.



$$\begin{cases} x = l\cos\theta \\ y = l\cdot\sin\theta \end{cases}$$

θ.

$q=1 : \theta$

x, y

$$\begin{cases} m\ddot{x} = -c\dot{x}\sqrt{\dot{x}^2+\dot{y}^2} \\ m\ddot{y} = -c\dot{y}\sqrt{\dot{x}^2+\dot{y}^2} - mg \end{cases}$$

$q=2 : x, y$

n个q，n个动力学方程.

Euler-Lagrange Equation：

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau$$

Lagrangian $L = E_k - E_p$

注：① q：广义坐标. n个q，n个 equation

② 若q为线位移，τ为力(N)

若q为角位移，τ为力矩(N·m)

[例] falling ball system

法1：Newton Law

$u - mg = ma$，$a = \ddot{y}$

$\Rightarrow \ddot{y} = \frac{u}{m} - g$

法2： $L = k - p$ $k = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{y}^2$ $p = mgy$

$L = \frac{1}{2}m\dot{y}^2 - mgy$

代入 Equation $q = y$. $\dot{q} = \dot{y}$

$$\frac{\partial L}{\partial \dot{y}} = \frac{\partial}{\partial \dot{y}}\left(\frac{1}{2}m\dot{y}^2\right) = \frac{1}{2}m\cdot 2\dot{y} = \underline{m\dot{y}}$$

$$\frac{\partial L}{\partial y} = \frac{\partial}{\partial y}(-mgy) = -mg$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = \frac{d}{dt}(m\dot{y}) = m\cdot\frac{d\dot{y}}{dt} = m a_y = m\ddot{y}$$

$\Rightarrow$ 代入 $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = \tau$

$\Rightarrow m\ddot{y} = u - mg$

总结:

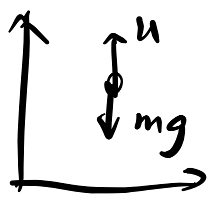
① 选取合适的广义坐标 $q_1, q_2, \cdots q_n$

② 求各广义坐标上输入力/力矩 τ

③ 求 $L = E_k - E_p$

④ 代入 Equation. 得到动力学方程.

HW3 Q9: falling ball system



已知: $\underline{\ddot{y} = \frac{u}{m} - g}$ = 二阶微分方程

求 $y(t)$ $\dot{y}(t)$ 在 $[0, 2]$ s

$$[t, \vec{y}] = \begin{bmatrix} t_1 & y_1(t=1) & y_2(t=1) \\ t_2 & y_1(t=2) & \vdots \\ t_3 & \vdots & \\ \vdots & & \\ t_n & y_1(t=t_n) & y_2(t=t_n) \end{bmatrix}$$