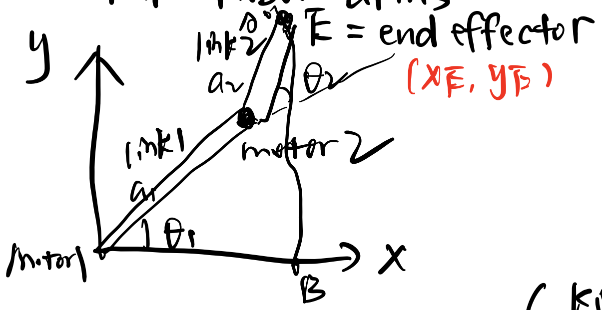
Basic concepts: Kinematics, Dynamics, Control

eg. 2-link Robot arms



actuator — arms
sensor — sense
controller — brain

Robot ⇐⇒ human

Goal: A → B
- Kinematics: compute current location E
- dynamics: apply desired force/torque: τ
- control: control methods (PID, MPC etc.)

① Kinematics

Forward Kinematics: given θ_1, θ_2, ⇒ compute x_E, y_E

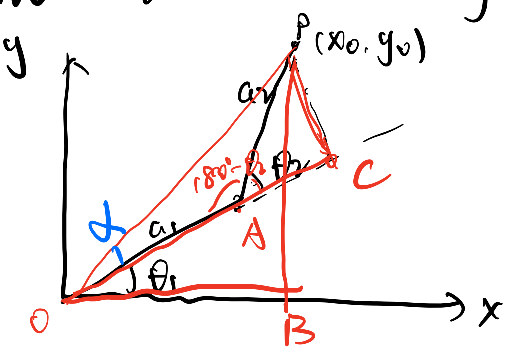
$$x_E = a_1 \cos\theta_1 + a_2 \cdot \cos(\theta_1+\theta_2) \qquad y_E = a_1 \sin\theta_1 + a_2 \sin(\theta_1+\theta_2)$$

(HW1 Q3) given $a_1 = a_2 = 1m$, $\theta_1 = 30°$ $\theta_2 = 45°$

$$\Rightarrow x_E = \cos 30° + \cos(30°+45°)$$

$$y_E = \sin 30° + \sin(30°+45°)$$

Inverse Kinematics: given x_E, y_E ⇒ compute θ_1, θ_2



余弦定理: ($\triangle OAP$)

$$OP^2 = x_0^2 + y_0^2 = a_1^2 + a_2^2 - 2a_1a_2\cos(180° - \theta_2) \quad \cos(180°-\theta_2) = -\cos\theta_2$$

$$\Rightarrow \theta_2 = \arccos\frac{x_0^2+y_0^2-a_1^2-a_2^2}{2a_1a_2}$$

$$\theta_1 = \arctan\frac{y_0}{x_0} - \alpha.$$

$\triangle POC$: $$\alpha = \arctan\frac{a_2\sin\theta_2 \;\; PC}{a_1+a_2\cos\theta_2 \;\; OC}$$

$$\Rightarrow \theta_1 = \arctan\frac{y_0}{x_0} - \arctan\frac{a_2\sin\theta_2}{a_1+a_2\cos\theta_2}$$

(HW1 Q4) Given: $P_x = 2.232$ $P_y = 1.866$ $L_1 = 2$ $L_2 = 1$

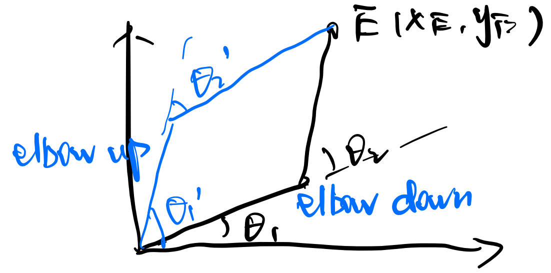
$$\theta_2 = \arccos\frac{2.232^2+1.866^2-2^2-1^2}{2\times2\times1} = ?$$

$\theta_1 = - - - \quad — = ?$

有解条件: $OP \le a_1 + a_2 \Rightarrow x_E^2 + y_E^2 \le (a_1+a_2)^2$.

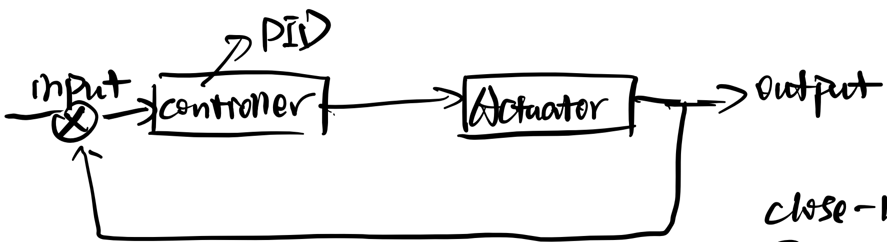
if $OE > a_1+a_2 \Rightarrow E$ is outside the workspace $\Rightarrow$ No feasible solution

$x_E^2+y_E^2 < (a_1+a_2)^2$



② Control

经典控制:



close-loop
Feedback control

eg. PID

Mathematical models for control system:

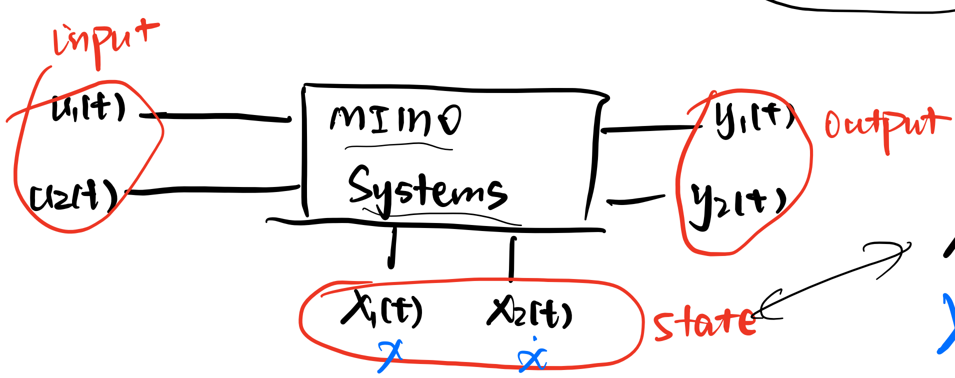
经典

① Differential Equation Model

eg. $C_{pid}(t) = K_p \cdot e(t) + K_i \int e(t)\,dt + K_d \frac{de(t)}{dt}$

② Transfer Function Model

③ State space model $\quad \dot{x} = f(x,u)$



$X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

$X = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}$

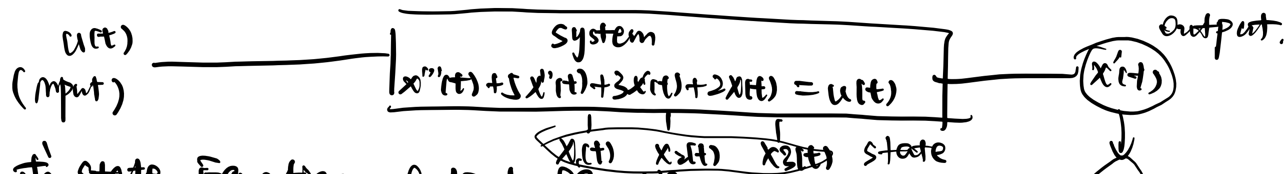
State Equation: $x'(t) = Ax(t) + Bu(t)$

多 state: $\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$

Out Equation: $y(t) = Cx(t) + Du(t)$

多 output $\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$.

[例]



求 state Equation, Output Equation:

Ans: $X = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$

$u(t) = x'''(t) + 5x''(t) + 3x'(t) + 2x(t)$

$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dddot{x} \end{bmatrix}$

$x_1' = \dot{x} = X(2)$

$x_2' = \ddot{x} = X(3)$

$x_3' = \dddot{x} = -5x_3 - 3x_2 - 2x_1 + u(t)$

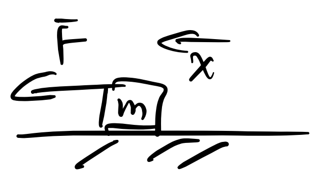
$\Rightarrow \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$

Output Equation: $y = Cx + Du = x'(t)$.

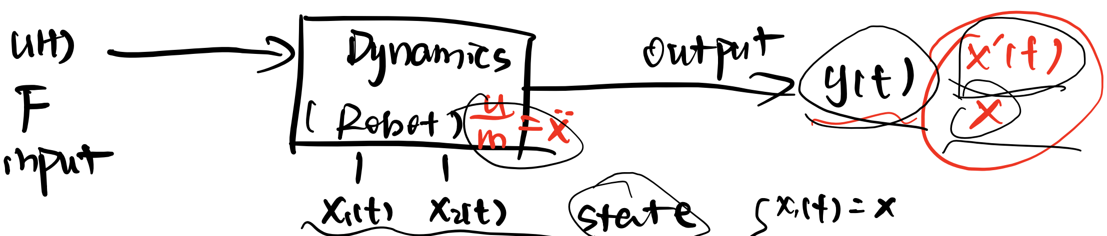
$y = x_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$

③ Dynamics

1-D single mass



$F = ma \quad a = \ddot{x} \Rightarrow F = m\ddot{x} \Rightarrow \ddot{x} = \frac{F}{m} = \frac{u}{m}$



$\begin{cases} x_1(t) = x \\ x_2(t) = \dot{x} \end{cases}$

$X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad u = F$

$x' = X(2)$

$x'' = \frac{u}{m}$

State Equation: $\dot{x} = AX + Bu$

$$\dot{x} = \begin{bmatrix} x' \\ x'' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u \qquad \dot{x} = f(x, u)$$

$\frac{u}{m}$

Given control input u } ⇒ solve for $x(t)$, $x'(t)$

⇒ simulate the robot

⇒ prediction of the state