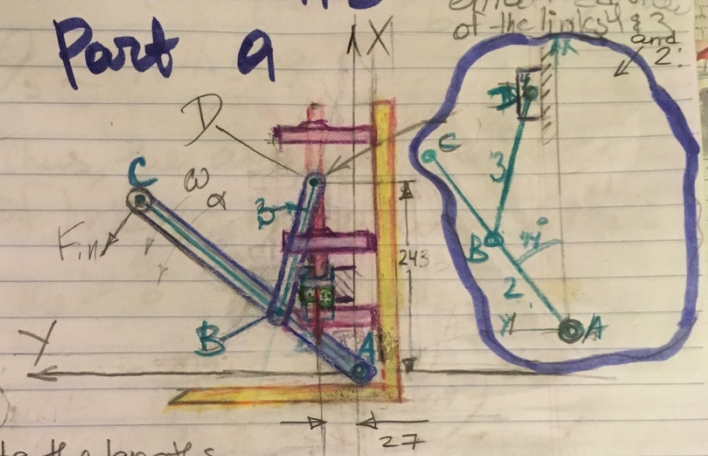
Anastasiya Travina

Problem #1
Part a



1

Let us denote the lengths of links 2, 3, and 4, as a, b, and c, respectively:
length of link 2, AB = 105 mm @ ; i.e. a := 105 mm, while the offset is c := 27 mm
length of link 3, BD = 172 mm ; i.e. b := 172 mm
length of link 2, AC = 301 mm @ 44°, i.e. r_{in} := 301 mm because this is the distance to force

Since the position of the 2nd link is defined by $\Theta_2 = 44°$, hence our $\omega_2 = 1$ rad/sec

To find Θ_3 we would use the formula (4.7) (*)

$$\theta_3 := a\sin\left(\frac{a\cdot\sin(\theta_2)-c}{b}\right)+\pi$$

$$\theta_3 = 105mm\cdot\sin\left(\frac{105mm\sin(44^\circ)-27mm}{b}\right)+\pi$$

$$\theta_3 \approx 164.509^\circ$$

Thus, the angular velocity of link 3 by equation 6.22a:

$$\omega_3 := \frac{a}{b}\cdot\frac{\cos(\theta_2)}{\cos(\theta_3)}\cdot\omega_2$$

$$= \boxed{\frac{105}{172}\cdot\frac{\cos(44^\circ)}{\cos(164.509^\circ)}\cdot 1\,rad\cdot sec^{-1}}$$

The velocity of join D with the same 6.22c equation but its b part:

$$V_D = -a\cdot\omega_2\sin(\theta_2)+b\cdot\omega_3\sin(\theta_3)$$

$$= -105mm\cdot 1\,rad\cdot sec^{-1}+172mm\cdot(\omega)_3\cdot\sin(164.509^\circ)$$

the fact that we obtain a positive result indicates this is a positive upward velocity.

Now, the velocity of point C

$$\mathbf{V_C} := r_{in} \cdot \omega_2 \left(-\sin(\theta_2) + j\cos(\theta_2)\right) = ?$$

hence,

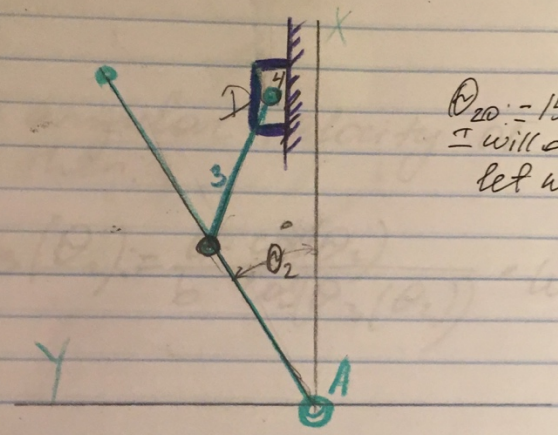
$$V_C := |\mathbf{V_C}|$$

Thus, the mechanical advantage is:

$$m_A := \frac{V_C}{|V_D|} \qquad m_A = 3.206$$

6.55 b:

Now let me redraw the enlarged representation of the linkage from my selection of part a of 6.55 with some modifications:



$\theta_{20} := 15^\circ$, $\theta_{21} := 60^\circ$
I will again let $\omega_2 := 1 \cdot rad \cdot sec^{-1}$

Since here we got a slider-crank linkage let us find the motion range:

$$\theta_2 := \theta_{20}, \theta_{20} + 1^\circ \ldots etc\ \theta_{21}$$

it's θ_{20}, since the initial position of the link 2

the angle of the final position of link 2

Hence, θ_3:

$$\theta_3(\theta_2) := \operatorname{asin}\left(-\frac{a \cdot \sin(\theta_2) - c}{b}\right) + \pi$$

the angular velocity of the 3rd link is then:

$$\omega_3(\theta_2) := \frac{a \cdot \cos(\theta_2)}{b \cdot \cos(\theta_3(\theta_2))} \cdot \omega_2$$

The velocity of pin D is:

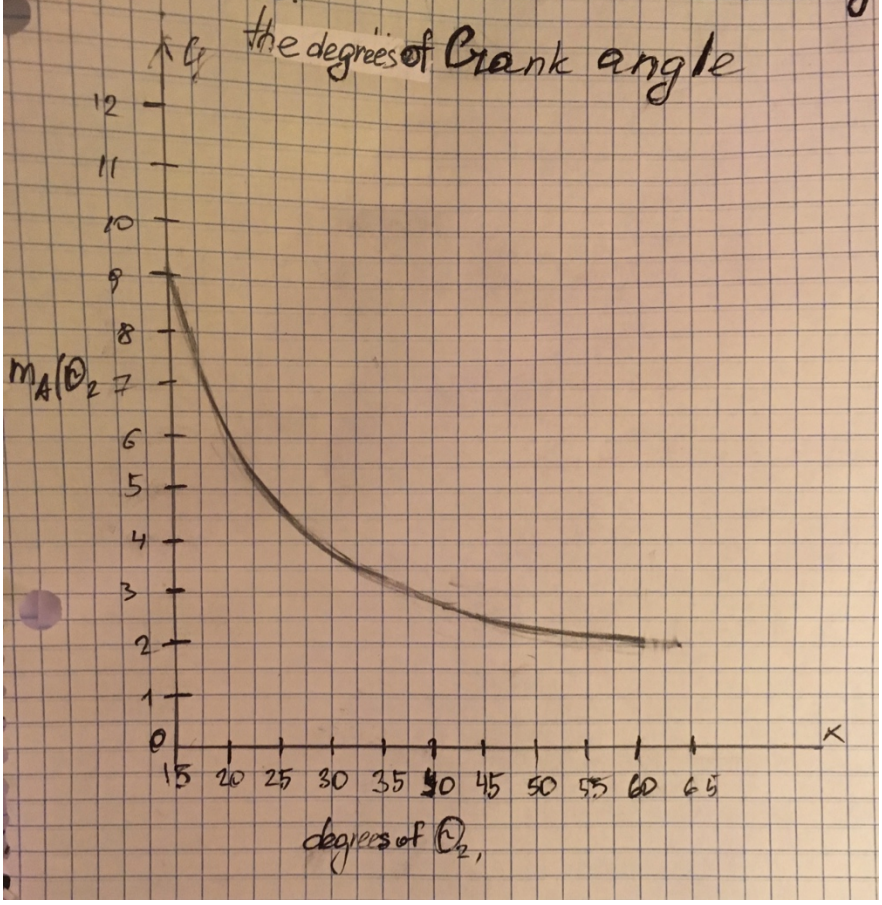
$$v_D(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b\,\omega_3(\theta_2) \cdot \sin(\theta_3(\theta_2))$$

$=$

so the result is positive upward

Problem 1, 6.55 Part b

The plot of the Mechanical advantage visualization vs. the degrees of Crank angle



The velocity of point C:

$$\mathbf{V_C}(\theta_2) = r_{in} \cdot \omega_2 \cdot \left(-\sin(\theta_2) + j\cos(\theta_2)\right)$$

Hence,

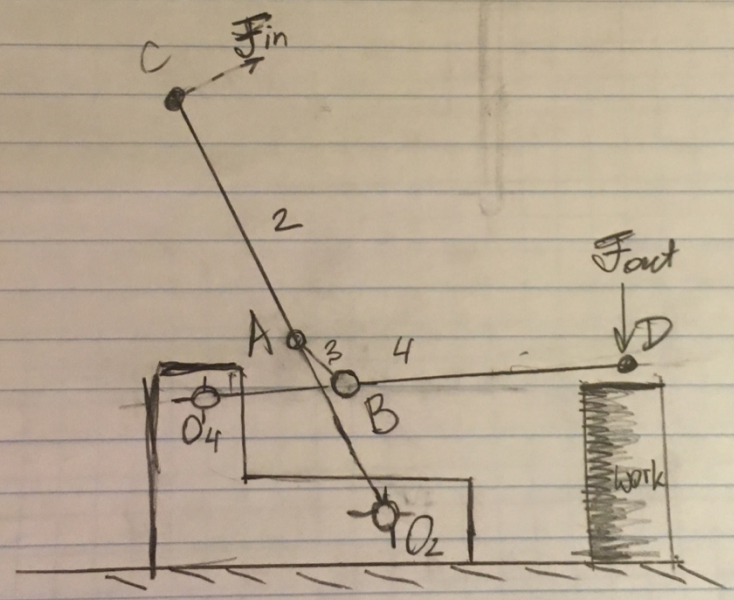
$$V_C(\theta_2) := |\mathbf{V_C}(\theta_2)|$$

Then $m_A(\theta_2) := \dfrac{V_C(\theta_2)}{|V_D(\theta_2)|}$

Question #2

6.59a

The linkage toggles as the 2nd link approaches 90°. For the dimensions



link2 (O_2A)

$a := 70 \cdot mm$

link3 (AB) $b := 35 \cdot mm$

link 4 (O_4B) $c := 34 \cdot mm$

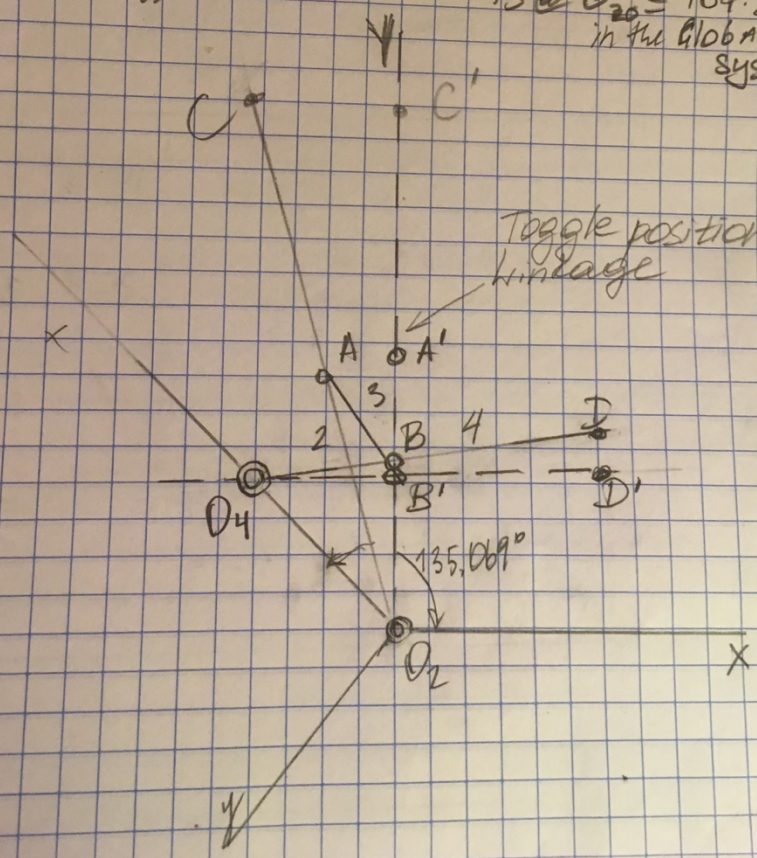
link 1 (O_2O_4) $d := 48 \cdot mm$

Distance to force app:

link2 (O_2C): $r_{in} := 138 \cdot mm$

link4 (O_4D), $r_{out} := 82 \cdot mm$

the initial position of the second link is @ $\theta_{20} := 104 \cdot deg$ in the Global XY system



length of link 1, i.e. O_2O_4, let us denote it as $d := 48$ mm

length of link 2, i.e. O_2A, $a := 70$ mm

length of link 3, i.e. AB, $b := 35$ mm

length of link 4, i.e. O_4B, $c := 34$ mm

In terms of distance to force application,

link 2, i.e. O_2C, $r_{in} = 138$ mm,

link 4, i.e. O_4D; $r_{out} = 82$ mm

Initial position of link 2 in the global XY plane:

$\theta_{20} := 104°$

As the toggle position of $\theta_2 = 90°$,

in the global coordinate system, the position of O_4 with respect to O_2 will look the following way:

from the picture, we can then conclude that the pivot O_4 is 48 mm from O_2;

the pivot O_4 is also 34 mm from B"

Now, in the local coordinate system, rotation angle α of the local xy-coordinates to the global coord. system previously denoted as XY is $\alpha := 135.069°$

$$\Rightarrow \theta_2 := \theta_{20} - \alpha$$

$$\theta_2 = -31.069°$$

To find θ_4, we need a constant, let us denote it by C (for a "constant")

$\Rightarrow C_1 := \frac{d}{a}$, $C_1 \approx 0.6857$

$C_2 := \frac{d}{c}$, $C_2 \approx 1.4118$

$$\Rightarrow C_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}, \quad C_3 = 1.4989$$

* all my $\boxed{\text{capital}}$ C's here stand for a Constant.

However, here C means the point C or "position C":

$$A(\theta_2) := \cos(\theta_2) \cdot C_1 - C_2 \cdot \cos(\theta_2) + C_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := C_1 - (C_2 + 1) \cdot \cos(\theta_2) + C_3$$

Let me find the value of θ_4 for this open circuit using 4.10b formula:

$$\theta_4(\theta_2) := 2 \left[\operatorname{atan2}\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right)\right]$$

(from the 4.10b formula)

Problem #2 6.59 b.

From part a, we continue to find the mechanical advantage.

The constants C_4 & C_5 are:

$$C_4 := \frac{d}{b} = 1.3714, \quad C_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} = -1.4843$$

$$D(\theta_2) := \cos(\theta_2) - C_1 + C_4 \cdot \cos(\theta_2) + C_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := C_1 + (C_4 - 1) \cdot \cos(\theta_2) + C_5$$

$$\theta_3(\theta_2) := 2 \cdot \left[\operatorname{atan2}\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right)\right]$$

$$\nu(\theta_2) := \theta_2 - \theta_3(\theta_2)$$

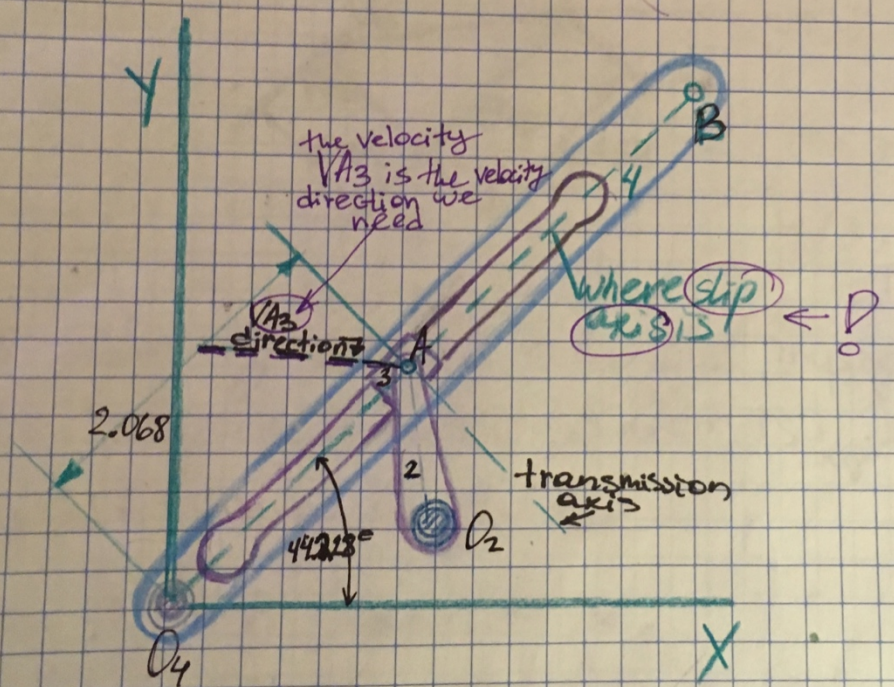
$$\mu(\theta_2) := \theta_4(\theta_2) - \theta_3(\theta_2)$$

The mechanical advantage is

$$M_A(\theta_2) := \frac{c \cdot \sin(\mu(\theta_2))}{a \cdot \sin(\nu(\theta_2))} \cdot \frac{r_{in}}{r_{out}}$$

Problem #3, 6.66:

To calculate the velocities of A, B, and C, as well as the slip velocity for the position; link 1 ($O_2 O_4$), $d := 1.69 \cdot in$, $\angle O_2O_4$ & X-axis, $\theta_1 := 155 \cdot deg$
link 2 (L_2), $a := 1.00 \cdot in$, where $\angle$ link 2 & X-axis, $\theta_2 := 99 \cdot deg$
link 4 (L_4), $e := 4.76 \cdot in$
link 5 (L_5), $f := 4.55 \cdot in$
while the offset (O_2 C), $g := 2.86 \cdot in$
finally, the angular velocity of link 2 is $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ ccw



$V_{A2} := a \cdot \omega_2$ $\quad \theta_{VA2} := \theta_2 + 90 \cdot deg$

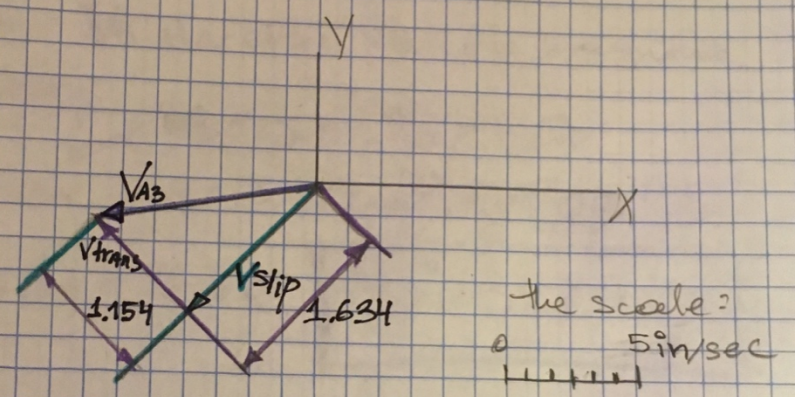
$V_{A2} = 10 \frac{in}{sec}$ $\quad \theta_{VA2} := 189 \cdot deg$

these give us the magnitude of the velocity at point A on the link 2 & the link 3

Then, we have the equation

$V_{A3} = V_{trasn} + V_{slip}$

I will use the graphic approach and it allows me to solve the above equation and get this picture



The velocity triangle implies that the velocity scale factor is $k_v := \frac{5 \cdot in \cdot sec^{-1}}{in}$

$V_{slip} := 1.634 \cdot in \cdot k_v$, $V_{slip} = 8.17\ in/sec$

$V_{trans} := 1.154 \cdot in \cdot k_v$, $V_{trans} = 5.77\ in/sec$

The true velocity of point A is therefore (of A on link 4)

$V_{A4} := V_{trans}$

$\Rightarrow V_{A4} = 5.77\ in/sec$

The Therefore angular velocity of link 4 is, usi the sketched solution to the equation $V_{A3} = V_{trans} + V_{slip}$ is:

$c := 2.068 \cdot in$ and $\theta_{4i} := 44.228 \cdot deg$

$\omega_{4i} := \frac{V_{A4}}{c}$, $\omega_4 = 2.79 \frac{rad}{sec}$ CCW

the vector V_B therefore has the magnitude:

$V_{Bi} := e \cdot \omega_4$, $V_B = 13.281$ in/sec

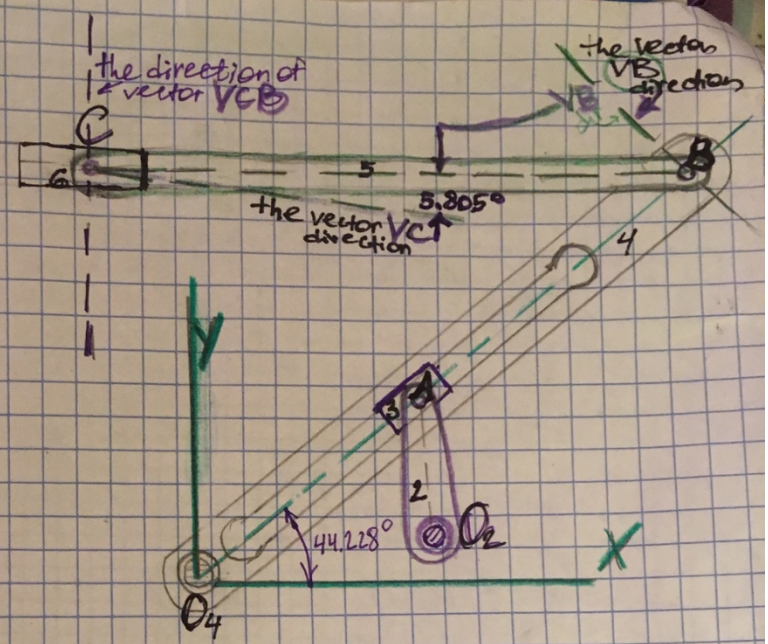
$\theta_{VBi} := \theta_4 + 90 \cdot deg$, $\boxed{\theta_{VB} = 134.228 deg}$

With the following scale, we can draw the directions of the velocity vectors VC, VB, VCB:

Hence, the graphic solution of the equation

$V_C = V_B + V_{CB}$

is this picture solution →



Thus, the velocity triangle helps us determine the velocity scale factor for the following eventual graphic solution →

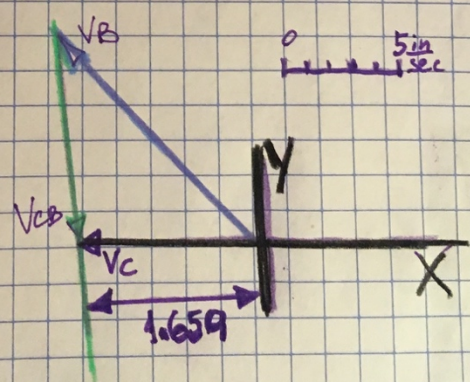
$K_V := 5 \cdot \frac{in \cdot sec^{-1}}{in}$

the scale factor

$V_C := 1.659 \cdot in \cdot K_V$

$V_C = 8.3\ in/sec$

$\theta_{VC} := 180 \cdot deg$



Problem 4, 6.71

Link lengths:

Fourbar ground link → $L_1 := 1$; i.e. $d := 1$

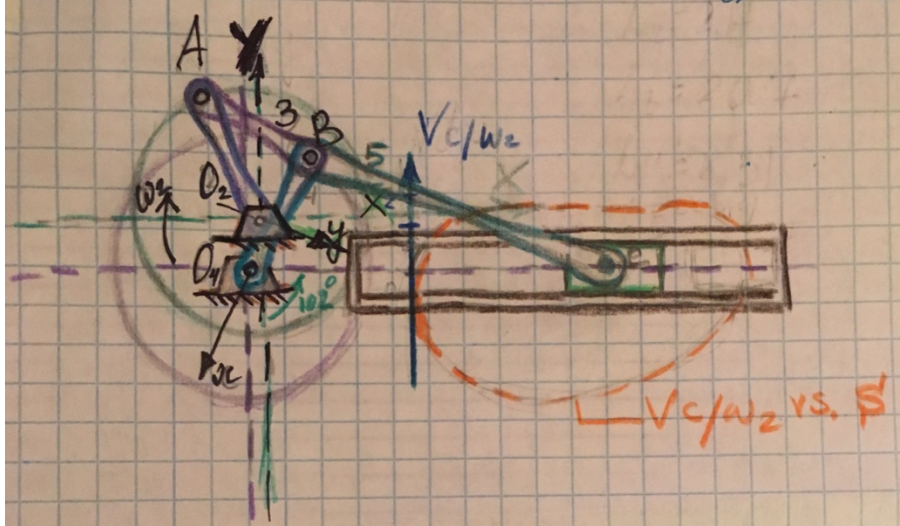
input crank → $L_2 := 2.17$, i.e. $a := 2.17$

fourbar coupler → $L_3 := 2.067$; $b := 2.067$

output crank → $L_4 := 2.31$; $c := 2.31$

Slider coupler → $L_5 := 5.4$; $e := 5.4$

Crank velocity, $\omega_2 := -1 \cdot \frac{rad}{sec}$



Using Grashof (fourbar) double crank in series with a crank-slider mechanism, as well as the output of the fourbar, link and link 4 being the input to the crank-slider, we can analyze this mechanism. Thus, let the revolution of the input crank be denoted by θ_2 such that $\theta_2 := 0 \cdot deg, 1 \cdot deg, \ldots 360 \cdot deg$

Problem #4, 6.71 continued

In the global XY coord. system, 2

$K_1 := d/a \Rightarrow K_1 = 0.4608$

$K_2 := d/c, \quad K_2 = 0.4329$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}, \quad K_3 = 0.6755$$

$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$

$B(\theta_2) := -2 \cdot \sin(\theta_2), \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$

$\theta_{41}(\theta_2) := 2 \cdot \left[\operatorname{atan2}\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right) \right] - 10$

$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$

$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$

Inserting the angle θ_2 in the equations, the slider-crank motion will therefore be:

$$\theta_5(\theta_2) := \operatorname{asin}\left(\frac{-c \cdot \sin(\theta_4(\theta_2))}{e}\right) + \pi$$

$f(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) - e \cdot \cos(\theta_5(\theta_2))$

To find θ_3, we need to obtain the K's, i.e. the constants first:

Problem #4 6.71 continued

In the global XY coord. system, 2

$K_1 := d/a \Rightarrow K_1 = 0.4608$

$K_2 := d/c, \quad K_2 = 0.4329$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}, \quad K_3 = 0.6755$$

$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$

$B(\theta_2) := -2 \cdot \sin(\theta_2), \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$

$\theta_{41}(\theta_2) := 2 \cdot \left[\operatorname{atan2}\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right) \right] - 102 \cdot \frac{\pi}{180}$

$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$

$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$

Inserting the angle θ_4 in the equations, the slider-crank motion will therefore be:

$$\theta_5(\theta_2) := \operatorname{asin}\left(\frac{-c \cdot \sin(\theta_4(\theta_2))}{e}\right) + \pi$$

$f(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) - e \cdot \cos(\theta_5(\theta_2))$

To find θ_3, we need to obtain the K's, i.e. the constants first:

$K_4 := d/b$, $K_4 = 0.4838$

$K_5 := \dfrac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$, $K_5 = -0.5178$

$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$

$E(\theta_2) := -2 \sin(\theta_2)$

$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$

Finally, to find θ_3 I will use the above derivations & obtain:

$$\theta_3(\theta_2) := 2 \cdot \left[\operatorname{atan2}\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right) \right]$$

Hence, for the open circuit, the angular velocity of link 4 is:

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin[\theta_2 - \theta_3(\theta_2)]}{\sin[\theta_4(\theta_2) + 102 \cdot deg - \theta_3(\theta_2)]}$$

Hence, the angular velocity of link 5 is:

$$\omega_5(\theta_2) := \frac{c}{e} \frac{\cos(\theta_4(\theta_2))}{\cos(\theta_5(\theta_2))} \cdot \omega_4(\theta_2)$$

Finally, the pin C has the velocity:

$$V_c(\theta_2) := -c\,\omega_4(\theta_2) \cdot \sin[\theta_4(\theta_2)] + e \cdot \omega_5(\theta_2) \cdot \sin[\theta_5(\theta_2)]$$

minus!!

Problem #5

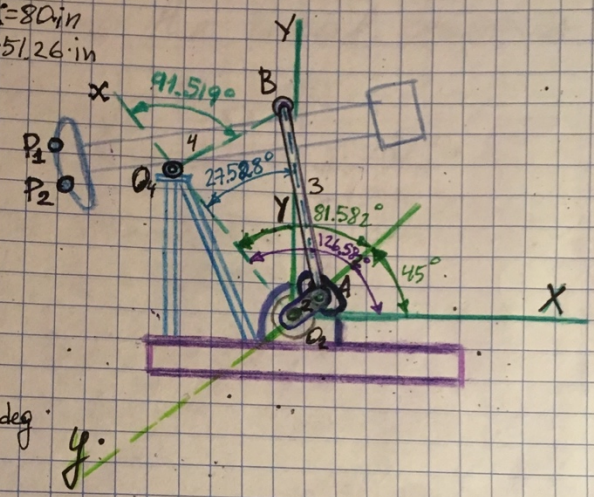
length of link 1 (given the X-offset) $d_x := 47.5 \cdot in$
length of link 1 (given the Y-offset), $d_y := 76 \cdot in - 12 \cdot in$
length of link 2 $(O_2 \rightarrow A)$ $a := 14 \cdot in$
length of link 3 $(A \rightarrow B)$, $b := 80 \cdot in$
length of link 4 $(O_4 \rightarrow B)$, $c := 51.26 \cdot in$

length of coupler point x-offset:
$p_x := 114.68 \cdot in$

coupler point y-offset:
$p_y := 33.19 \cdot in$

Crank angle:

$\theta_{2xy} := 0 \cdot deg, 1 \cdot deg ... 360 \cdot deg$
in the XY coordinate system



Therefore, the transformation angle that allows me the transformation into the local coordinate system from global:

$\delta := 126.582 \cdot deg$

the angular velocity for the input crank:

$\omega_2 := 10 \cdot rad \cdot sec^{-1}$ ccw

On the following page, I will illustrate the transformation from one coordinate system into another algebraically:

Problem #5, 6.85

First of all, in this problem, we have the global coordinate system crank angle and I will change it to the local coordinate system: $\theta_2(\theta_{2xy}) := \theta_{2xy} - 8$

Hence, the O_2O_4 distance is $d := \sqrt{d_x^2 + d_y^2}$, $d = 79.701$ in

Let the letter K stand for a constant

hence, $K_1 := d/a$, $K_1 = 5.6929$

$K_2 := d/c$, $K_2 = 1.5548$

$K_3 := \dfrac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$, $K_3 = 1.934$

$A(\theta_{2xy}) := \cos[\theta_2(\theta_{2xy})] - K_1 - K_2 \cdot \cos[\theta_2(\theta_{2xy})] + K_3$

$B(\theta_{2xy}) := -2 \cdot \sin(\theta_2(\theta_{2xy}))$

$C(\theta_{2xy}) := K_1 - (K_2 + 1) \cdot \cos[\theta_2(\theta_{2xy})] + K_3$

Hence, I can now proceed to find θ_4 values:

$$\theta_4(\theta_{2xy}) := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot A(\theta_{2xy}), -B(\theta_{2xy}) + \sqrt{B(\theta_{2xy})^2 - 4A(\theta_{2xy}) \cdot C(\theta_{2xy})}\right)\right)$$

$C(\theta_{2xy})$

Now, the constants w.r.t. θ_3 are:

$K_4 := d/b$ $\quad K_4 = 0.9963$

$K_5 := \dfrac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$, $K_5 = -4.6074$

$$(\theta_{2xy}) := \cos[\theta_2(\theta_{2xy})] - K_1 + K_4 \cdot \cos[\theta_2(\theta_{2xy})] + K_5$$

$$E(\theta_{2xy}) := -2 \cdot \sin(\theta_2(\theta_{2xy}))$$

$$F(\theta_{2xy}) := K_1 + (K_4 - 1) \cdot \cos[\theta_2(\theta_{2xy})] + K_5$$

Now, the crossed circuit θ_3 values are:

$$\theta_3(\theta_{2xy}) := 2 \cdot \left[\operatorname{atan2}\left(2 \cdot D(\theta_{2xy}), -E(\theta_{2xy}) + \sqrt{E(\theta_{2xy})^2 - 4 \cdot D(\theta_{2xy}) \cdot F(\theta_{2xy})}\right)\right]$$

$\Rightarrow$ the angular velocity for the crossed circuit for link 4 is:

$$\omega_4(\theta_{2xy}) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2(\theta_{2xy}) - \theta_3(\theta_{2xy}))}{\sin(\theta_4(\theta_{2xy}) - \theta_3(\theta_{2xy}))}$$

Finally, the distance from O_4 to P_1 is:

$$e := \sqrt{(p_x - d)^2 + p_y^2} \qquad e = 48.219\,in$$

Hence, the distance to the angle BO_4P_1

$$\delta_4 := 180 \cdot deg - \operatorname{atan}\left(\frac{p_y}{p_x - d}\right), \quad \delta_4 = 136.503°$$

$$V_{P1}(\theta_{2xy}) := e \cdot \omega_4(\theta_{2xy}) \cdot (-\sin(\theta_4(\theta_{2xy}) + \delta_4) + j \cdot \cos(\theta_4(\theta_{2xy}) + \delta_4))$$

$$V_{P1}(\theta_{2xy}) := |V_{P1}(\theta_{2xy})|$$

$$\theta_{VPXY}(\theta_{2xy}) := \arg(V_{P1}(\theta_{2xy})) + \delta$$

Hence, the velocity magnitude plot can be visualized as follows:

Problem #6, 6.87a

lengths of: L_2, O_2 to A, $a := 9.17\,in$

L_3, A to B, $b := 12.97\,in$

L_4, O_4 to B, $c := 9.57\,in$

Link 1 with X-offset:

$dx := 2.79\,in$

with Y-offset:

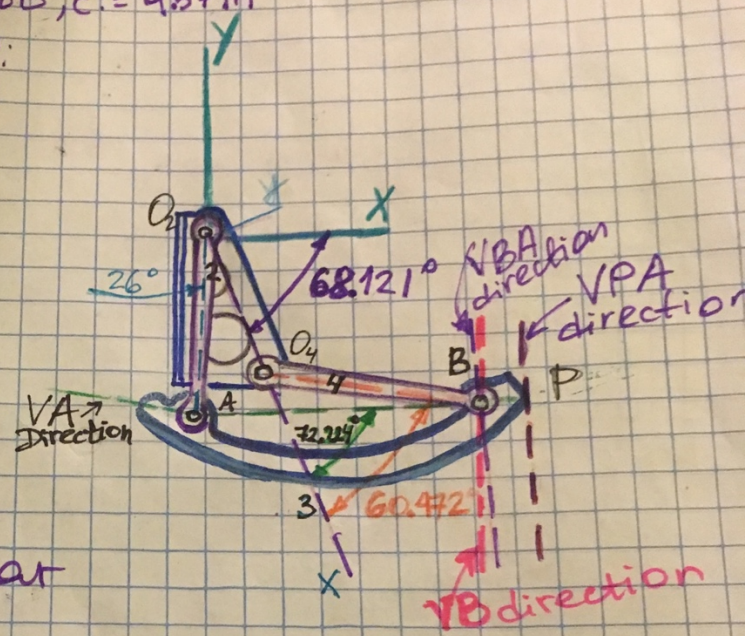
$dy := -6.95\,in$

Coupler point:

$p := 15\,in$, $\theta := 0\,deg$

Crank angle:

$\theta_2 := -94.121\,deg$



Input of the crank angular velocity

$\omega_2 := 1\,rad\cdot sec^{-1}$, CW

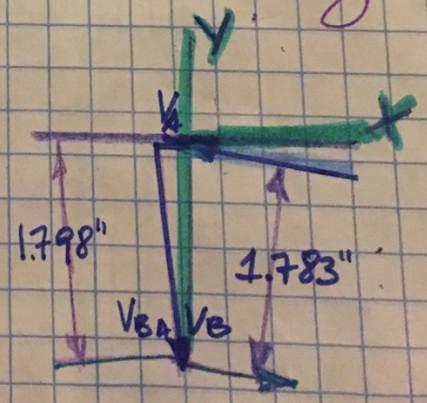
Thus, the @point A velocity:

$V_A := a\cdot\omega_2$, $V_A = 9.17\,\frac{in}{sec}$

$\theta_{VA} := \theta_2 - 90\,deg$, $\theta_{VA} = -184.121\,deg$

Graphically, 0 25 in/sec

this equation $V_B = V_A + V_{BA}$ solution is the following picture →



Hence, the velocity scale factor:

$k_v := \frac{25 \text{ in sec}^{-1}}{\text{in}}$

$V_B := 1.783 \cdot in \cdot k_v$, $V_B = 44.575 \frac{in}{sec}$,

$\theta_{VB} := 262.352 \cdot deg$

$V_{BA} := 1.798 \cdot in \cdot k_v$, $V_{BA} = 44.95 \frac{in}{sec}$

$\theta_{VBA} := 274.103 deg$

The angular velocity of link 3:

$\omega_3 := \frac{-V_{BA}}{b}$, $\omega_3 = -3.466 \frac{rad}{sec}$

$\omega_4 := -\frac{V_B}{c}$, $\omega_4 = -4.658 \frac{rad}{sec}$

The magnitude of $\vec{V}_{PA}$ =

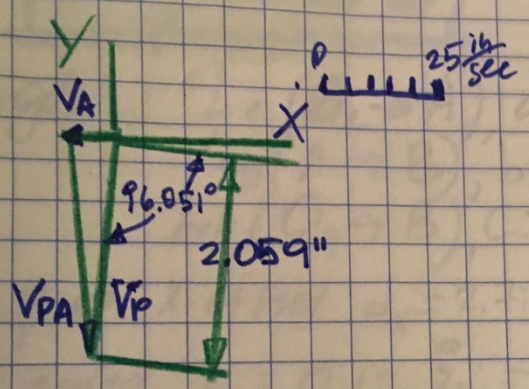
$\vec{V}_{PA} := |p \cdot \omega_3|$, $\vec{V}_{PA} = 51.985 \frac{in}{sec}$

$\theta_{VPA} := \theta_{VBA}$, $\theta_{VPA} = 274.103°$

In order to find the magnitude of velocity at point P, I will solve this equation

$$\mathbf{V_P = V_A + V_{PA}}$$

with this picture:



This velocity scale factor obtained using the velocity triangle is hence:

$k_v := 25 \frac{in \cdot sec^{-1}}{in}$

$V_P := 2.059 \cdot in \cdot k_v$

$V_P = 51.475 \, in/sec$

$\theta_P := -96.051 \cdot deg$

67, Part b

lengths: $L_2 (O_2 \to A)$, $a := 9.174 \cdot in$

$L_3 (A \to B)$, $b := 12.971 \cdot in$

$L_4 (O_4 \to B)$, $c := 9.573 \cdot in$

link 1, given X-offset: $d_x := 2.7 in$, link 1, Y-offset: $d_y := -6.948 \cdot in$

Coupler point: $p := 15 \cdot in$, $\delta_3 := 0 \cdot deg$,

Crank angle: $\theta_2 := -94.121 \cdot deg$

Input crank

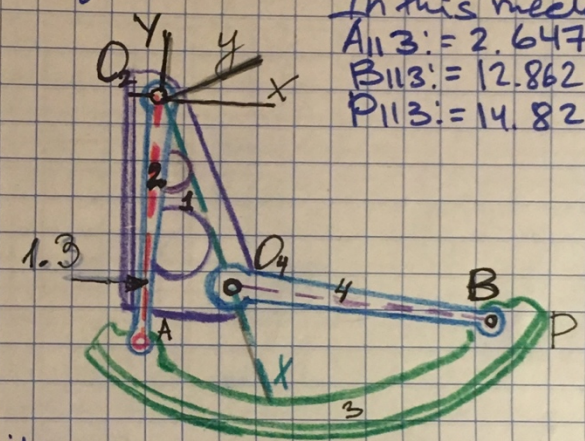
$\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CW

In this mechanism,

$A_{113} := 2.647 \cdot in$

$B_{113} := 12.862 \cdot in$

$P_{113} := 14.825 \cdot in$



Using this mechanism layout →

$V_A := a \cdot \omega_2$, $V_A = 9.174 \frac{in}{sec}$

$\theta_{VA} := \theta_2 - 90 \cdot deg$, $\theta_{VA} = -184.121 deg$

$\omega_3 := \frac{V_A}{A_{113}}$ $\omega_3 = 3.466 \frac{rad}{sec}$ CW

$V_B := B_{113} \cdot \omega_3$, $V_B = 44.577 \frac{in}{sec}$

Hence, link 4 angular velocity: $\omega_4 := \frac{V_B}{c}$, $\omega_4 = 4.657 \frac{rad}{sec}$, C

Hence, at point P.

$V_P := P_{113} \cdot \omega_3$, $V_P = 51.381 \frac{in}{sec}$

Problem 67-C

length of link 1, X-offset $d_x := 2.79 \cdot in$

length of link 1, Y-offset $d_y := -6.948 \cdot in$

length of link 2 $(O_2 \to A)$, $a := 9.174 \cdot in$

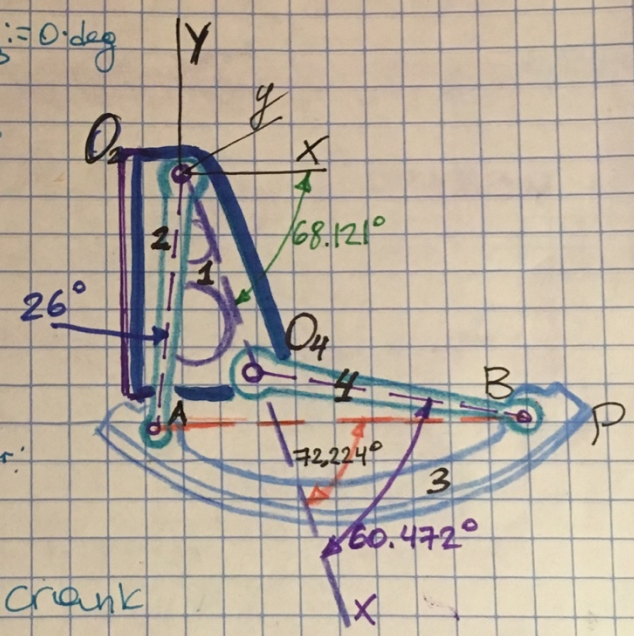
length of link 3 $(A \to B)$, $b := 12.971 \cdot in$

length of link 4 $(O_4 \to B)$, $c := 9.573 \cdot in$

Coupler point: $p := 15 \cdot in$, $\delta_3 := 0 \cdot deg$

Crank angle in the Global XY coordinate system:

$\theta_{2XY} := -94.121 \cdot deg$



The transformation angle to change from one coordinate system into another:

$\delta := -68.121 \cdot deg$

Input of crank of angular velocity:

$\omega_2 := -1 \cdot rad \cdot sec^{-1}$ CW

The transformation from Global into the local coordinate system of xy:

$\theta_2 := \theta_{2XY} - \delta$ the O_2O_4 distance: $d := \sqrt{d_x^2 + d_y^2}$

$d = 7.487 \, in$

$\theta_2 := -26 deg$

Let us find the constants k's:

$K_1 := d/a$, $K_1 = 0.8161$

$K_2 := d/c$, $K_2 = 0.7821$

$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$, $K_3 = 0.3622$

$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$, $A = -.2581$

$B := -2 \cdot \sin(\theta_2)$, $B = .8767$

$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$, $C = -.4234$

Hence, θ_4 (the crossed unit) is:

$\theta_4 := 2\left(\operatorname{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) + 2\pi$, $\theta_4 = 60.488 \cdot deg$

The constants:

$K_4 := d/b$ $K_4 = .5772$

$K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$ $K_5 = -.9111$

$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$, $D = -.3096$

$E := -2 \sin(\theta_2)$, $E = .8767$

$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$, $F = -.4749$

$\theta_3 := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) + 2\pi$, $\theta_3 = 72.236 deg$

Now, the angular velocity of links.

$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)}$, $\omega_3 = -3.467$ rad/sec

the angular velocity of link 4

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)},$$

$$\omega_4 = -4.658 \, rad/sec$$

Then, the velocity of point A:

$$V_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_A = (-4.022 - 8.246i) \frac{in}{sec}, \quad |V_A| = 9.174 \frac{in}{sec}$$

$$\theta_{VAXY} := \arg(V_A) + \delta, \quad \theta_{VAXY} = -184.121 deg$$

the velocity of point B:

$$V_B := c \cdot \omega_4 (-\sin(\theta_4) + j \cos(\theta_4))$$

$$V_B = (38.807 - 21.967i) \frac{in}{sec}, \quad |V_B| = 44.593 \frac{in}{sec}$$

$$\theta_{VBXY} := \arg(V_B) + \delta, \quad \theta_{VBXY} = -97.633 deg$$

Thus, for the crossed circuit, point P's velocity:

$$V_{PA} := p \cdot \omega_3 \cdot (-\sin(\theta_2 + \delta) + j \cdot \cos(\theta_3 + \delta_3))$$

$$V_{PA} = (49.529 - 15.867i) \frac{in}{s}$$

$$V_P := V_A + V_{PA}, \quad V_P = (45.507 - 24.113i) \frac{in}{sec},$$

$$|V_P| = 51.50 \frac{in}{sec}, \quad \theta_{VPXY} := \arg(V_P) + \delta$$

$$\theta_{VPXY} = -96.039 deg$$