

AN EXPERIMENTAL TEST OF THE ANALOGY BETWEEN RADIATIVE PION ABSORPTION AND MUON CAPTURE IN ^{12}C

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Abstract: The yields of the analogous reactions $\mu^-_{\text{at rest}} + ^{12}\text{C} \rightarrow \nu + ^{12}\text{B}_{\text{stable}}$ and $\pi^-_{\text{at rest}} + ^{12}\text{C} \rightarrow \gamma + ^{12}\text{B}_{\text{stable}}$ have been determined detecting the ^{12}B activity produced in a plastic scintillator by stopping π^- and μ^- . Under certain assumptions about the strength of the π^- absorption from the mesic 2P level and about the contribution of the particle-stable excited ^{12}B states, the ratio of the 1S capture rates obtained is $R = (A_{\pi}^{\pi}/A_{\mu}^{\mu})(^{12}\text{C} \rightarrow ^{12}\text{B}_{\text{g.s.}}) = (1.99 \pm 0.53) \times 10^{12}$. The theoretically expected value is $R = (1.27 \pm 0.17) \times 10^{12}$.

E NUCLEAR REACTIONS $^{12}\text{C}(\mu^-, \nu)$, $^{12}\text{C}(\pi^-, \gamma)$, E approx 0; measured yields.

1. Introduction

The capture of negative muons in complex nuclei

$$\mu^- + (A, Z) \rightarrow \nu + (A, Z-1)$$

and the radiative pion absorption

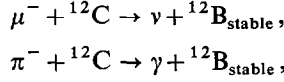
$$\pi^- + (A, Z) \rightarrow \gamma + (A, Z-1)$$

are very similar processes. It has been shown theoretically¹⁾ that the transition operator for radiative pion absorption at zero kinetic energy is essentially identical to the axial vector part of the operators describing the muon capture and the Gamow-Teller β -decay.

In order to investigate this similarity experimentally at a discrete transition one has to choose nuclei which result from pure Gamow-Teller β -decays. The known ft value then allows us to calculate the axial vector matrix element. Only light nuclei can be used since for higher Z the pion absorption from higher atomic levels increases very strongly. The analogy of the two processes has first been shown by Deutsch *et al.*²⁾ in an experiment on ^6Li . For further investigation essentially ^{12}C is left for the reasons mentioned.

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At the CERN synchrocyclotron we stopped negative muons and pions in a plastic scintillator and measured the yields of the two reactions



detecting the decay electrons of the ¹²B nuclei produced (¹²B_{stable} means ground state and particle-stable excited levels).

2. Theoretical estimate of A_γ^π/A^μ

The transition probability for muon capture in ¹²C ($T = 0, J^P = 0^+$) to the ground state of ¹²B ($T = 1, J^P = 1^+$) is given by ^{3,4)}

$$\begin{aligned}A^\mu({}^{12}\text{C} \rightarrow {}^{12}\text{B}_{\text{g.s.}}) &= \frac{v_{\text{if}}^2}{2\pi} |\bar{\varphi}_\mu|^2 \sum_{M_f} \int \frac{d\hat{\nu}}{4\pi} \left[G_A^2 \left| \int \boldsymbol{\sigma} \right|_\mu^2 + (G_P^2 - 2G_P G_A) \left| \hat{\nu} \cdot \int \boldsymbol{\sigma} \right|_\mu^2 \right] \\ &+ \text{relativistic corrections } (\hbar = c = 1),\end{aligned}$$

where

$$\int \boldsymbol{\sigma} = \langle f | \sum_{n=1}^A \tau_n^- \exp(-i\mathbf{v} \cdot \mathbf{r}_n) \boldsymbol{\sigma}_n | i \rangle,$$

v_{if} is the neutrino energy ($= m_\mu$ minus BE of muonic atom minus ΔE_{if} between nuclear states), φ_μ is the 1S orbit muonic wave function averaged over the nuclear volume [the variation in light nuclei is about $\pm 1\%$ inside the nucleus ⁴⁾], $\hat{\nu}$ is a unit vector in the direction of the neutrino momentum, G_A and G_P are the effective coupling constants commonly used in muon capture theory [see e.g. ref. ⁵⁾], the sum over M_f extends over the neutrino polarisation and the nuclear orientation.

Foldy and Walecka ⁶⁾ estimated the ratio

$$\sum_{M_f} \left| \hat{\nu} \cdot \int \boldsymbol{\sigma} \right|_\mu^2 / \sum_{M_f} \left| \int \boldsymbol{\sigma} \right|_\mu^2$$

to be 0.29 ± 0.02 . The relativistic corrections should be in the order of $(10 \pm 5)\%$ [refs. ^{5,7)}].

One can derive the transition probability for radiative pion absorption to order m_π/M ($M = \text{proton mass}$) by the PCAC hypothesis with the soft-pion theorem and in impulse approximation ^{4,8)} one gets the simple result

$$(A_\gamma^\pi)^{1S}({}^{12}\text{C} \rightarrow {}^{12}\text{B}_{\text{g.s.}}) = \frac{k_{\text{if}}}{m_\pi} |\bar{\varphi}_\pi|^2 \left[\frac{1}{4\pi} \frac{e^2}{f_\pi^2} \left(1 + \frac{m_\pi}{2M} \right)^2 F_A^2(0) \right] \sum_{M_f} \int \frac{d\mathbf{k}}{4\pi} \left| \int \boldsymbol{\sigma} \right|_\pi^2.$$

The nomenclature is analogous to the one of the expression for muon capture; k_{if} is the photon energy, F_A the axial vector coupling constant and $f_\pi = 0.95 m_\pi$ the pion decay constant. The variation of φ_π is $\pm 4\%$ over the nuclear volume ⁹⁾. Apart from

cancel good events. The energy calibration of the target counter was done by means of the passing beam.

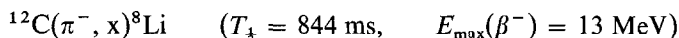
The synchrocyclotron produced a pulse of $350\ \mu\text{s}$ length every 74 ms. In the period between the pulses the time distribution of the β -decay events ($4\ 135$) was recorded in a multichannel analyzer. By means of a copper moderator of variable thickness put into the iron collimator a range curve of the ^{12}B activity was taken.

The target counter was operated at a threshold of 1.8 MeV. 97 % of the electrons from ^{12}B decay have energies > 1.8 MeV. The detection efficiency for ^{12}B decays is slightly smaller than 97 % due to the ^{12}B nuclei produced near the surface of the target. A Monte Carlo calculation has shown that for ^{12}B nuclei distributed like the π^- and μ^- stops in the respective peaks of the range curve, the detection efficiency for ^{12}B electrons at a threshold of 1.8 MeV is $(94 \pm 3)\%$.

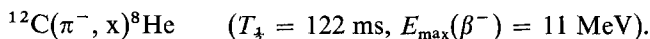
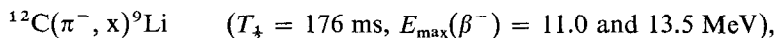
The (1, 2) coincidence rate served as a monitor. In order to derive from this rate the number of μ^- decays and π^- stops, it was calibrated in the following way: The number of μ -decays occurring in the target in the μ^- peak per (1, 2) coincidence was counted and corrected for the detection efficiency for μ -decay electrons at the threshold of 1.8 MeV, known from a Monte Carlo calculation to be $(97 \pm 3)\%$. For the π^- stop calibration, the target counter was replaced by a graphite target having the same stopping power, behind which an anticoincidence counter was placed. The number of π^- stops per monitor coincidence in the π^- peak was then determined from a range curve. The detection efficiency for stopped pions was assumed to be the same as for ^{12}B electrons within an error of 5 %.

4. Evaluation of the data

In a χ^2 analysis the time spectra have been separated into real and background events. Part of the background was proved not to depend on the moderator thickness and was attributed to neutrons associated with the cyclotron pulse. This part was subtracted in all time spectra. The remaining background was found to be mainly due to the reaction



and partly due to



These reactions cannot occur in the μ -peak of the activity range curve and similar reactions induced by muons are very unlikely. A typical time spectrum is shown in fig. 2.

The contribution of charge exchange from pions entering the target with an energy above threshold should be negligible. For the cross section of the mirror reaction $^{12}\text{C}(\pi^+, \pi^0)^{12}\text{N}_{\text{g.s.}}$ an upper limit of $120\ \mu\text{b}$ was obtained at 70 MeV pion energy ¹¹⁾.

For isospin reasons, the cross section of ^{12}B production by charge exchange should be of the same order, since among the particle-stable states of ^{12}B mainly the ground state (analogue state to $^{12}\text{N}_{\text{g.s.}}$) is expected to be populated ¹¹).

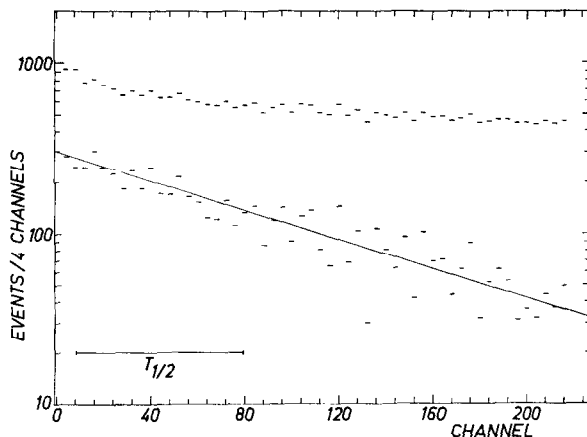


Fig. 2. Time distribution of 4135 events in the pion peak of the ^{12}B activity range curve before and after background subtraction. The solid line corresponds to $T_{\frac{1}{2}}(^{12}\text{B}) = 20.41$ ms.

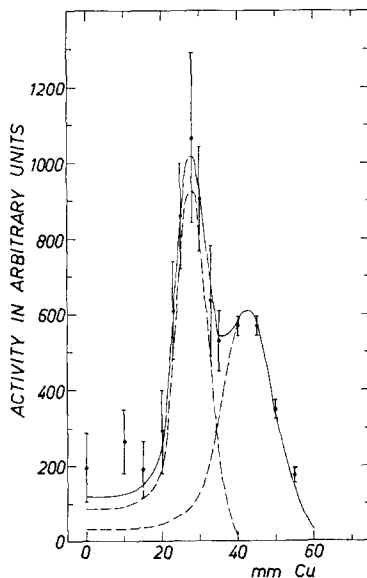


Fig. 3. ^{12}B activity range curve fitted with π - and μ -range curves. The points are the background corrected ^{12}B activities. The bars indicate statistical errors.

The resulting ^{12}B activity range curve is shown in fig. 3. The line shapes of the π - and μ -range curves indicated in the figure have been measured in the positive beam using the decay chains $\pi \rightarrow \mu \rightarrow e$ and $\mu \rightarrow e$ for identification. The relative peak half-widths for positive and negative pions as well as for π and μ have been found experimentally to be the same within the errors.

5. Results

From the activity in the π^- peak of the activity range curve (fig. 3) we obtain for A^π , the number of ^{12}B nuclei produced per π^- stopping in the target, the value

$$A^\pi = (5.8 \pm 1.3) \cdot 10^{-3}.$$

In the μ^- peak the corresponding value

$$A^\mu = (1.36 \pm 0.13) \cdot 10^{-2},$$

is derived for the number of ^{12}B nuclei produced per decaying μ^- . The error in A^π is larger than the one in A^μ in spite of the better statistics. This is due to the uncertainty in the background, which has more components for the π^- stops as pointed out in sect. 4. From A^μ , the rate of μ^- capture from the 1S state (all muons reach the 1S state) leading to particle-stable ^{12}B states, will then be

$$A^\mu(^{12}\text{C} \rightarrow ^{12}\text{B}_{\text{stable}}) = A^\mu A_D^\mu = (6.95 \pm 0.71) \cdot 10^3 \text{ sec}^{-1},$$

where $A_D^\mu = 0.455 \cdot 10^6 \text{ sec}^{-1}$ is the decay rate for the free muon which is the same as in carbon ¹²).

This value agrees with the one of Maier *et al.* ¹³),

$$A^\mu = (7.05 \pm 0.28) \cdot 10^3 \text{ sec}^{-1}.$$

As for the corresponding rate of the π^- absorption, two corrections have to be applied to the experimental number:

(i) Only $(45.5 \pm 3.0)\%$ of the pions reach the 1S state ¹⁴), the others being absorbed from higher levels, essentially the 2P level.

(ii) The stable ^{12}B states are populated by radiative absorption from the 2P state. We call K_A the fraction of the ^{12}B nuclei produced by radiative absorption from the 1S state, and give an estimate of K_A in the following discussion of the results.

The rate A_{tot}^π of π^- absorption from the 1S state of ^{12}C has been measured to be $(45.0 \pm 3.8) \cdot 10^{17} \text{ sec}^{-1}$ by the determination of the line width of the pionic (2P–1S) transition ¹⁵). With this value, we get for the absorption rate of π^- from the 1S state, leading to particle-stable ^{12}B states, the value

$$(A_\gamma^\pi)^{1\text{S}}(^{12}\text{C} \rightarrow ^{12}\text{B}_{\text{stable}}) = A^\pi \frac{1}{0.455} (A_{\text{tot}}^\pi)^{1\text{S}} = (5.8 \pm 1.4) \cdot 10^{16} K_A \text{ sec}^{-1}.$$

The ratio of the capture rates is

$$(A_\gamma^\pi/A^\mu)^{1\text{S}}(^{12}\text{C} \rightarrow ^{12}\text{B}_{\text{stable}}) = (8.3 \pm 2.2) \cdot 10^{12} K_A.$$

6. Discussion

Comparing the experimental result with the theory we face two difficulties: The reduction of the activity from π^- stops to the part coming from 1S absorption, and the estimation of the population of particle-stable excited states in ^{12}B which contribute to the measured activity.

For the muon capture table 1 shows that the excited states can only be reached by forbidden transitions. In fact Maier *et al.* ¹³⁾ have measured only a $(11.8 \pm 4.3)\%$ contribution of the excited ^{12}B states in the muon capture process. In the case of π^- absorption the situation is more complex due to the contribution of the 2P shell. Table 1 shows the multipole order, through which the different levels of ^{12}B can be reached by π^- radiative absorption from the 1S and the 2P state. In the first approximation we will consider only E1 transitions. This seems to be justified by regarding the inverse process of π -photoproduction. Chew *et al.* ¹⁷⁾ have shown that the M1 and E2 amplitudes are connected with operators proportional to the pion momentum.

TABLE 1
Particle-stable states in ^{12}B and the corresponding transitions ^{a)}

J^P (^{12}C ground state) = 0^+					
nuclear state ^{12}B		μ -capture 1S		π -absorption 1S 2P	
1^+	ground state	GT	allowed	E1	M1
2^+	0.95 MeV	GT+F	$2 \times$ forbidden	M2	M1
2^-	1.67 MeV	GT	$1 \times$ forbidden	E2	E1
1^-	2.62 MeV	GT+F	$1 \times$ forbidden	M1	E1

^{a)} The 2.72 MeV level is not mentioned because of lack of information on its spin and parity ($J^P \leq 3^+$) [ref. ¹⁶⁾].

To clarify the relative strength of E1 transitions from 1S and 2P atomic states a simple plausibility argument may be helpful: We interpret the measured ratio of the total absorption yields $Y(2\text{P})/Y(1\text{S}) = 1.20 \pm 0.08$ [ref. ¹⁴⁾] as a measure of the relative interaction strength $P(2\text{P})/P(1\text{S})$ of the pions with the ^{12}C nucleus and discard differences in the individual structure of the final nuclear states. Then, equivalent multipole radiations following absorption from both pionic shells should have approximately the same strength if they lead to nuclear states of the same multiplicity i.e.

$$P^{\text{E1}}(2\text{P}) = P^{\text{E1}}(1\text{S})P(2\text{P})/P(1\text{S}).$$

However this gives rather an upper limit of the 2P contribution. If one describes the absorption process as the interaction with one single nucleon, the pion must be captured in a relative S-state because of spin and parity conservation. The probability for a relative S-state is different for the 1S and 2P mesic shells. Only if these probabilities are equal is the above formula satisfied. We should write therefore: $P^{\text{E1}}(2\text{P}) \leq P^{\text{E1}}(1\text{S})P(2\text{P})/P(1\text{S})$.

The correction factor introduced in the last section then becomes for the extreme case above

$$K_A = \frac{\sum_{1S} M_i P_i^{E1}}{\sum_{1S} M_i P_i^{E1} + \sum_{2P} M_K P_K^{E1}} = 0.24,$$

where the multiplicities M_i of the nuclear states involved are taken into account.

With this estimation and a 10 % correction due to the particle-stable excited states populated by 1S absorption we get

$$\begin{aligned} (A_\gamma^\pi)^{1S}(^{12}\text{C} \rightarrow ^{12}\text{B}_{\text{g.s.}}) &= (1.24 \pm 0.30) \cdot 10^{16} \text{ sec}^{-1}, \\ (A_\gamma^\pi/A^\mu)^{1S}(^{12}\text{C} \rightarrow ^{12}\text{B}_{\text{g.s.}}) &= (1.99 \pm 0.53) \cdot 10^{12}. \end{aligned}$$

The errors quoted do not contain the uncertainty of the correction factor K_A and of the contribution of the excited states which can only give the order of magnitude. These results have to be compared with

$$\begin{aligned} (A_\gamma^\pi)^{1S}(^{12}\text{C} \rightarrow ^{12}\text{B}_{\text{g.s.}}) &= (0.89 \pm 0.14) \cdot 10^{16} \text{ sec}^{-1}, \\ (A_\gamma^\pi/A^\mu)^{1S}(^{12}\text{C} \rightarrow ^{12}\text{B}_{\text{g.s.}}) &= (1.27 \pm 0.17) \cdot 10^{12}, \end{aligned}$$

where the first value has been calculated using the known ft value of the β -decay $^{12}\text{B} \rightarrow ^{12}\text{C}_{\text{g.s.}}$: $ft = (1.180 \pm 0.007) \cdot 10^4 \text{ sec}$ [ref. ¹⁸].

If we also take into account the uncertainties in the order of magnitude estimation of the 2P absorption and in the contribution of the excited ^{12}B states as the approximations of the calculation of the transition probabilities, the agreement is satisfying. The experimental results can significantly be improved only by separating the 1S capture in a coincidence experiment with the pionic X-rays.

The result for the ratio of the capture rates of ^{12}C is to be compared with the value for ^6Li [ref. ²)] which is reported to be $(1.25_{-0.13}^{+0.25}) \cdot 10^{12}$ under the assumption of 1S absorption only. The agreement is remarkable in view of the fact that the corresponding ft values of the ^6He and ^{12}B β -decays differ by a factor of about 15. A further value to compare with is the ratio of the total radiative pion absorption rate from the 1S shell to the total muon capture rate for ^{12}C . According to the yield measurements of radiative pion absorption in carbon, 1.6 % of all pions are absorbed under gamma emission ^{19,20}). If one assumes the relative radiative yields $Y_\gamma^\pi/Y_{\text{tot}}^\pi$ for 1S and 2P absorption to be the same, the mentioned ratio can be calculated taking into account the muon capture rate ⁵). One gets $(A_\gamma^\pi/A^\mu)^{1S}(^{12}\text{C} \rightarrow \text{all final states}) = (1.93 \pm 0.29) \cdot 10^{12}$.

We thank the operation group of the CERN synchrocyclotron for the specially pulsed beam needed for this experiment, and Prof. P. Preiswerk for his continuous interest and support.

Note added in proof. In a recent paper Bistirlich *et al.* ²¹⁾ give $(0.97 \pm 0.09) \cdot 10^{-3}$ for the ratio $A_{\pi \text{ rad}}/A_{\pi \text{ tot}}$ which corresponds to our A_{π} . This is not in agreement with our value of $A_{\pi} = (5.8 \pm 1.3) \cdot 10^{-3}$. Our result is supported by the fact that our pion data were measured with the same experimental arrangement and evaluated in the same way as our muon data which are in good agreement with the results of the precision experiment of Maier *et al.* ¹³⁾. It should also be noted that Bistirlich *et al.* used values for the relative probabilities for absorption from the 1S and 2P orbit ($Y(1S) = 12.3\%$ and $Y(2P) = 88.7\%$) which differ from the ones we used ($Y(1S) = 45.5\%$ and $Y(2P) = 54.5\%$). We believe that their values are due to a misinterpretation of the pionic atom data of M. Koch *et al.* ¹⁴⁾. This same interpretation can be found in the paper of Cheswire ²²⁾ where the agreement between experiment and theory is significantly improved using our values for $Y(1S)$ and $Y(2P)$.

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