Introducción a Redes Neuronales

Dr. Mauricio Toledo-Acosta

Diplomado Ciencia de Datos con Python

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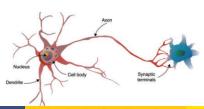
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 - Combining Perceptrons
- Gradient Based Learning
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 - Optimizers
- Fully Connected Networks
 - Architectures
- Modules



Introduction

Neural Networks

Artificial neural networks are machine learning techniques that simulate the mechanism of learning in biological organisms. The human nervous system contains cells, referred to as neurons. The neurons are connected to one another by of axons and dendrites, and the connecting regions between axons and dendrites are called synapses. The strengths of synaptic connections change in response to external stimuli. This change is how learning takes place in living organisms.



El modelo lineal de clasificación

 The Perceptrón algorithm (Rosenblatt, 1961) played an important role in Machine Learning history. It was first simulated in a computer IBM 704 at Cornell in 1957. By the early 60s, a dedicated hardware was designed to implement the learning algorithm.

El modelo lineal de clasificación

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- It was criticized by Marvin Minsky, who showed the limitations of the perceptron algorithm when dealing with a non linear separable set of points. This caused a void in the neural computation research lasting until the mid 80s.

Deep Learning

Deep Learning

Deep learning is part of a broader family of machine learning methods, which is based on artificial neural networks. Learning can be supervised, semi-supervised or unsupervised.

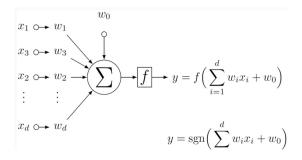
Deep learning processes machine learning by using a hierarchical level of artificial neural networks, built like the human brain, with neuron nodes connecting in a web.

The adjective *deep* in deep learning refers to the use of multiple layers in the network.

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The Rosenblatt Perceptron



- $\mathbf{w} = (w_0, w_1, ..., w_d)$ is the weights vector (matrix).
- Σ is a neuron.
- sgn is the activation function.

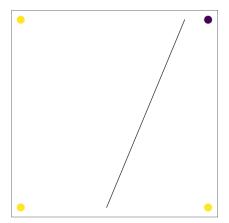
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The Algorithm

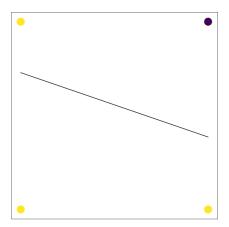
For each epoch, the algorithm is summarized as follows:

```
Data: A: training set of points, X = \{x_1, ..., x_N\} \subset \mathbb{R}^{D+1},
         Y = \{y_1, ..., y_N\}: set of labels
        \eta > 0: learning rate.
Result: w: weight vector defining the decision frontier.
Function Perceptron(X, y, \eta):
     w = 0:
     converged = False:
     while converged == False do
          for i \in \{1, ..., N\} do
               if y_i \langle \mathbf{w}, x \rangle < 0 then
                 w = w + y_i \eta x_i;
converged = False
     return w:
```

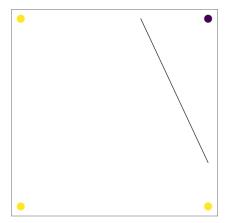
Example

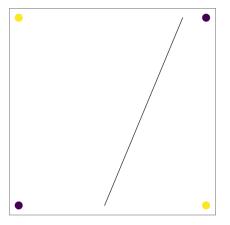


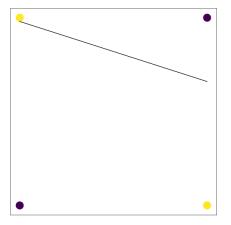
Example

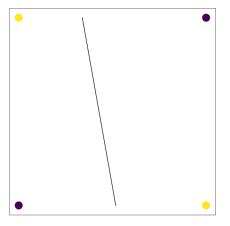


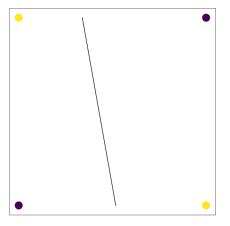
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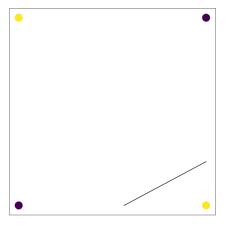


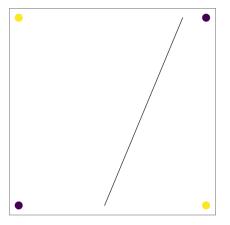










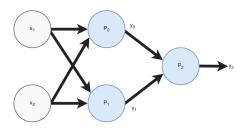


What happens if a straight line cannot separate the data points?



Alternatives:

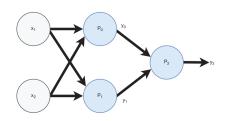
- We change the model of a neuron or
- We combine multiple of them to solve this limitation.



Now, we can solve the classification problem

$$X = \begin{pmatrix} -1 & 1 \\ 1 & -1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}, \ y = \begin{pmatrix} -1 \\ +1 \\ +1 \\ -1 \end{pmatrix}$$



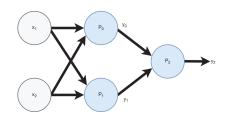


$$y_{0} = f_{0} \left(\left(\begin{array}{ccc} w_{0}^{(0)} & w_{1}^{(0)} & w_{2}^{(0)} \end{array} \right) \left(\begin{array}{c} 1 \\ x_{1} \\ x_{2} \end{array} \right) \right)$$

$$y_{1} = f_{1} \left(\left(\begin{array}{ccc} w_{0}^{(1)} & w_{1}^{(1)} & w_{2}^{(1)} \end{array} \right) \left(\begin{array}{c} 1 \\ x_{1} \\ x_{2} \end{array} \right) \right)$$

$$y_{2} = f_{2} \left(\left(\begin{array}{ccc} w_{0}^{(2)} & w_{1}^{(2)} & w_{2}^{(2)} \end{array} \right) \left(\begin{array}{c} 1 \\ y_{0} \\ y_{1} \end{array} \right) \right)$$

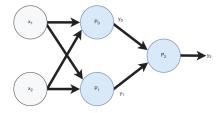
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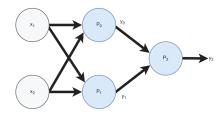
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$$y_{2} = \operatorname{sgn} \begin{pmatrix} \begin{pmatrix} w_{0}^{(2)} & w_{1}^{(2)} & w_{2}^{(2)} \end{pmatrix} \begin{pmatrix} 1 \\ y_{0} \\ y_{1} \end{pmatrix}$$



This neural network is one of the simplest examples of a fully connected feedforward network. Fully connected means that the output of each neuron in one layer is connected to all neurons in the next layer. Feedforward means that there are no backward connections. A multilevel neural network has an input layer, one or more hidden layers, and an output layer. The input layer does not contain neurons but contains only the inputs themselves.



- We have two features (white).
- We have two layers of neurons (blue).
- The hidden layer has two neurons, with no activation function.
- The output layer has one neuron with activation function sgn.

To train this network of perceptrons means to find the best parameters $w_i^{(k)}$.

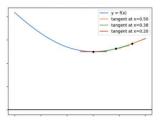
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$$x_{n+1} = x_n - \eta f'(x_n).$$

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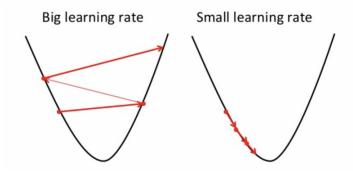
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- The algorithm is not guaranteed to find the global minimum because it can get stuck in a local minimum.

Learning Rate

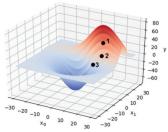


The high-dimensional case

• Now, consider the function $f(x_0, x_1)$.

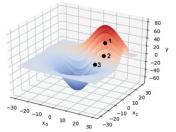
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• If we are at the point $\mathbf{x_n} = (x_0^{(n)}, x_1^{(n)})$ and want to minimize y, then we choose our next point as

$$\mathbf{x}_{\mathsf{n}+1} = \mathbf{x}_{\mathsf{n}} - \eta \nabla f(\mathbf{x}_{\mathsf{n}})$$

Gradient-based Learning

When applying gradient descent to our neural network, we consider input values \mathbf{x} to be constants, with our goal being to adjust the weights \mathbf{w} .

Which function we want to minimize? A loss function.

When applying gradient descent to our neural network, we consider input values x to be constants, with our goal being to adjust the weights w.

- Which function we want to minimize? A loss function.
- In the context of an optimization algorithm, the function used to evaluate a candidate solution is referred to as the objective function. With neural networks, we seek to minimize the error. As such, the objective function is often referred to as a cost function or a loss function.

In the case of the Perceptron, the loss function is given by

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$$L^{(0/1)}(\mathbf{w}) = \frac{1}{2}(y_i - \operatorname{sign}\langle \mathbf{w}, \mathbf{x}_i \rangle)^2 = 1 - y_i \operatorname{sign}\langle \mathbf{w}, \mathbf{x}_i \rangle$$

This function is not smooth, we use the smooth surrogate loss function

$$L(\mathbf{w}) = \max\{-y_i \operatorname{sign}\langle \mathbf{w}, \mathbf{x}_i \rangle, 0\}.$$

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Applying gradient descent

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \eta \nabla L(\mathbf{w})$$

$$= \begin{cases} \mathbf{w}_n + \eta y_i \mathbf{x}_i, & \text{well classified} \\ \mathbf{w}_n, & \text{misclassified} \end{cases}$$

Loss Functions

Other examples of loss functions

▶ Keras Losses

Loss Functions: MSE

Mean Square Error (MSE)

$$L(y,t)=(t-y)^2.$$

• L2 loss.

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- L2 loss.
- Good for regression tasks.
- Trivial derivative for gradient descent.

Loss Functions: MAE

Mean Absolute Error (MAE)

$$L=|t-y|.$$

- I 1 loss.
- More robust to outliers than mse.
- Good for regression tasks.
- Discontinuity in its derivative.

Loss Functions: Hinge

Hinge Loss

$$L = \max\{-y_i\hat{y}_i, 0\}.$$

- Used in SVMs and Perceptron.
- Penalizes errors, but also correct predictions of low confidence (probabilities).
- Good for binary classification tasks.

Loss Functions: Categorical cross entropy

Categorical cross entropy

$$L = \sum_{i}^{n} y_{i} \log \left(\hat{y}_{i}\right).$$

- Good for multi-class classification problems.
- Considers y to be a one-hot encoding vector in *n* classes.

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Advantages:

- Easy computation.
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- Easy to understand.

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Disadvantages:

- May trap at local minima.
- Weights are changed after calculating gradient on the whole dataset.
 May take a long time to converge.
- Requires large memory to calculate gradient on the whole dataset.

Optimizers

We can use other optimizing strategies:

- Gradient Descent
- Stochastic Gradient Descent
- Stochastic Gradient Descent with momentum
- Mini-Batch Gradient Descent
- Adagrad
- RMSProp
- AdaDelta
- Adam

▶ Keras Optimizers



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- Due to an increase in the number of iterations, the overall computation time increases. Still, the computation cost is still less than that of the gradient descent optimizer.
- If the data is enormous and computational time is an essential factor, stochastic gradient descent should be preferred over batch gradient descent algorithm.

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- The benefit of using Adagrad is that it abolishes the need to modify the learning rate manually. It is more reliable than gradient descent algorithmss, as it reaches convergence at a higher speed.
- One downside of AdaGrad optimizer is that it decreases the learning rate aggressively and monotonically. There might be a point when the learning rate becomes extremely small.

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- Adam optimizer updates the learning rate for each network weight individually.

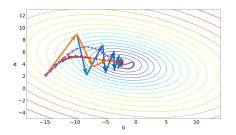
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- It tends to focus on faster computation time, it might not generalize the data well enough.

Different Optimizers

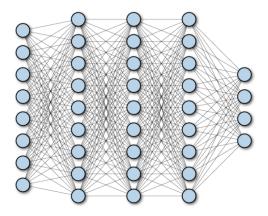


Visualizaciones, Several optimizers

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Fully Connected Networks



A fully connected neural network consists of a series of fully connected layers that connect every neuron in one layer to every neuron in the other layer.

June 2, 2023

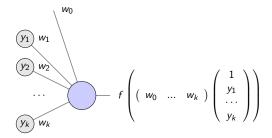
Advantages and Disadvantages

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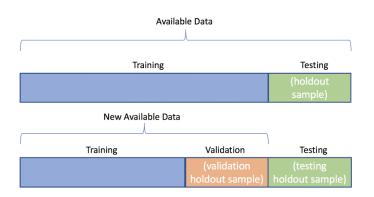
- The major advantage of fully connected networks is that they are no special assumptions needed to be made about the input.
- While being structure agnostic makes fully connected networks very broadly applicable, such networks do tend to have weaker performance than special-purpose networks tuned to the structure of a problem space.

How each neuron works?



The previous layer has k neurons, the activation function is f.

Division of the dataset



The Algorithm

The algorithm consists of three steps:

• First, present one or more training examples to the neural network: Feed-Forward.

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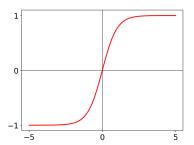
The Algorithm

The algorithm consists of three steps:

- First, present one or more training examples to the neural network: Feed-Forward.
- Second, compare the output of the neural network to the desired value: Loss function.
- Finally, adjust the weights to make the output get closer to the desired value using gradient descent: Back propagation.

Other Activation Functions: Hyperbolic Tangent

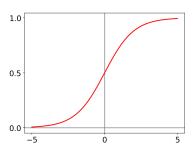
$$tanh(x) = \frac{e^{2x} - 1}{e^{2x} + 1} \in (-1, 1)$$
 $tanh'(x) = 1 - tanh^2(x)$



It is often used in the hidden layer. It can also be used if the predicted value of a regression task has values in (-1,1).

Other Activation Functions: The Sigmoid

$$S(x) = \frac{1}{1 + e^{-x}} \in (0, 1)$$
$$S'(x) = S(x)(1 - S(x))$$



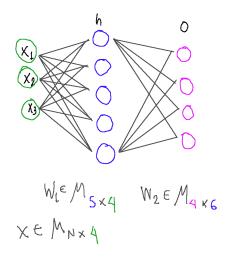
It is used when we need the output to be in (0,1). For example, in binary classification.

Choice of Activation Functions

There exist a large number of activation functions. Two popular choices are tanh and the logistic sigmoid function. When picking between the two, choose tanh for hidden layers and logistic sigmoid for the output layer.

► Keras Activation Functions

Feed-Forward: An Example



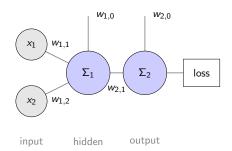
$$X = \begin{pmatrix} \downarrow & \chi_{1}^{(1)} & \chi_{2}^{(1)} & \chi_{3}^{(1)} \\ \downarrow & \chi_{1}^{(N)} & \chi_{2}^{(N)} & \chi_{3}^{(N)} \end{pmatrix}$$

$$h = \oint_{I} \left(X \cdot W_{I}^{T} \right)$$

$$0 = \oint_{Z} \left(\overline{h} \cdot W_{Z}^{T} \right)$$

$$h \in \mathcal{M}_{N \times 5}$$

$$\overline{h} \in \mathcal{M}_{N \times 6}$$



This neural network implements

$$f = S(w_{2,0} + w_{2,1} \tanh(w_{1,0} + w_{1,1}x_1 + w_{1,2}x_2))$$

We use the MSE loss:

$$e(f) = \frac{1}{2}(y - f)^2.$$



The error is given by

$$e(\mathbf{w}) = \frac{1}{2} \left(y - S \left(w_{2,0} + w_{2,1} \mathsf{tanh} (w_{1,0} + w_{1,1} x_1 + w_{1,2} x_2) \right) \right)^2$$
 where $\mathbf{w} = \left(w_{1,0}, w_{1,1}, w_{1,2}, w_{2,0}, w_{2,1} \right)$

• We write it as:

$$e(f) = \frac{1}{2}(y - f)^{2}$$

$$f(z_{f}) = S(z_{f})$$

$$z_{f}(w_{2,0}, w_{2,1}, g) = w_{2,0} + w_{2,1}g$$

$$g(z_{g}) = \tanh(z_{g})$$

$$z_{g}(w_{1,0}, w_{1,1}, w_{1,2}) = w_{1,0} + w_{1,1}x_{1} + w_{1,2}x_{2}$$

Compute the partial derivatives

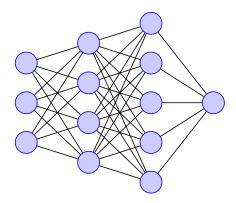
$$\begin{split} &\frac{\partial}{\partial w_{2,0}}e = -(y-f)S'(z_f)\\ &\frac{\partial}{\partial w_{2,1}}e = -(y-f)S'(z_f)g\\ &\frac{\partial}{\partial w_{1,0}}e = -(y-f)S'(z_f)w_{2,1}\tanh'(z_g)\\ &\frac{\partial}{\partial w_{1,1}}e = -(y-f)S'(z_f)w_{2,1}\tanh'(z_g)x_1\\ &\frac{\partial}{\partial w_{1,2}}e = -(y-f)S'(z_f)w_{2,1}\tanh'(z_g)x_2 \end{split}$$

Finally, we update the weights, via gradient descent

$$\begin{aligned} w_{2,0} &\leftarrow w_{2,0} - \eta \frac{\partial e}{\partial w_{2,0}} \\ w_{2,1} &\leftarrow w_{2,1} - \eta \frac{\partial e}{\partial w_{2,1}} \\ w_{1,0} &\leftarrow w_{1,0} - \eta \frac{\partial e}{\partial w_{1,0}} \\ w_{1,1} &\leftarrow w_{1,1} - \eta \frac{\partial e}{\partial w_{1,1}} \\ w_{1,2} &\leftarrow w_{1,2} - \eta \frac{\partial e}{\partial w_{1,2}} \end{aligned}$$

Regression

If we have three independent variables and one dependent variables

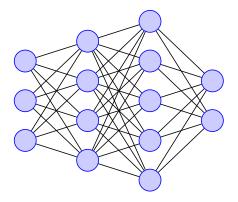


The activation in the output layer is usually None (the identity).

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Multivariate Regression

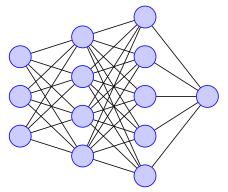
If we have three independent variables and two dependent variable



The activation in the output layer is usually None (the identity).

Binary Classification

If we have three independent variables and two classes

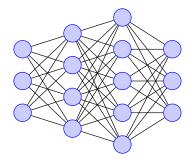


The activation function in the output layer is the sigmoid

$$\sigma(x) = \frac{1}{1 + e^{-x}} \in (0, 1)$$

Multi-class Classification

If we have three classes



The activation function in the output layer is softmax

$$\sigma(j;(x_1,...,x_n)) = \frac{e^{x_j}}{\sum_{i=1}^n e^{x_i}} \in (0,1)$$

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Deep Learning Libraries





Tensorflow

Pytorch

Deep Learning Libraries

	Keras	Tensorflow	Pytorch
API Level	High	High and low	Low
Architecture	Simple, read- able	Not easy to use	Complex, less readable
Datasets	Smaller datasets	Large datasets	Large datasets
Debugging	Debugging is not often needed	Good	Difficult
Written in	Python	C++, CUDA, Python	Lua
Trained models	Yes	Yes	Yes
Speed	Slow, low per- formance	Fast, high- performance	Fast, high- performance