Base Conversion

Between Powers of 2: 2, 8, 16 Examples: Rechunk

1. Convert each digit to binary: Lookup Table or from memory!
2. Merge then rechunk the bits
3. Convert each chunk to the appropriate digit

Base *N* to 10

- Use Expanded Notation, or
- For each digit: multiply by *N*, and then add the value of the digit

Base 10 to Base *N*

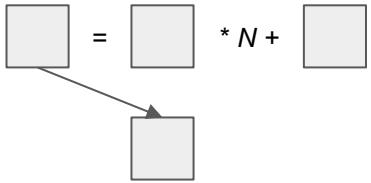
- Successively divide the number by *N*
- The concatenation of the remainders produce the final value
- Consider the examples via the spreadsheet: Base Conversion

Base *N* to Base 10

Algorithm: multiply, add, and shift

- set v = 0
- For each digit (from left to right)
  - v = v * base; # Multiple by the base
  - v = v + $digit_{10}$; # Add the next digit
- emit v

Consider: 16# 5a2 == 1442



16# 5a2

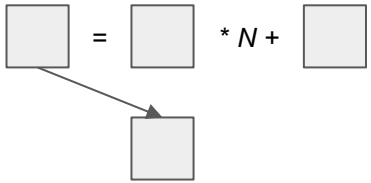
v:	5	=	0	* 16 +	5	5
v:	90	=	5	* 16 +	10	a
v:	1442	=	90	* 16 +	2	2

Base *N* to Base 10

Algorithm: multiply, add, and shift

- set v = 0
- For each digit (from left to right)
  - v = v * base; # Multiple by the base
  - v = v + $digit_{10}$; # Add the next digit
- print v

Consider: 2# 10110101 == 181



2# 10110101

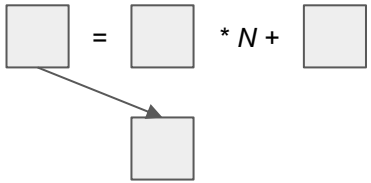
v:	1	=	0	* 2 +	1	1
v:	2	=	1	* 2 +	0	0
v:	5	=	2	* 2 +	1	1
	11	=	5	* 2 +	1	1
	22	=	11	* 2 +	0	0
	45	=	22	* 2 +	1	1
	90	=	45	* 2 +	0	0
	181	=	90	* 2 +	1	1

Base *N* to Base 10

Algorithm: multiply, add, and shift

- set v = 0
- For each digit (from left to right)
  - v = v * base; # Multiple by the base
  - v = v + $digit_{10}$; # Add the next digit
- print v

Consider: 8# == ?



8# 453

v:	4	=	0	* 8 +	4	4
v:	37	=	4	* 8 +	5	5
v:	299	=	37	* 8 +	3	3

Base Conversion of Real Numbers

Base 10 to Base *N*

- The whole portion is divided by the new base, repeatedly
  - Dividend / Divisor = (Quotient, Remainder)
  - The concatenation of the Remainders provide you with the final digits
- The fraction portion is multiplied by the new base, repeatedly
  - Multiplier * Multiplicant = (Overflow, Product)
  - The concatenation of the Overflows provide you with the final digits
- Consider the examples via the spreadsheet: Base Conversion

Decimal Real to Binary Real

1. Split the number at the radix point: *whole . fractional*

2. With the whole part,

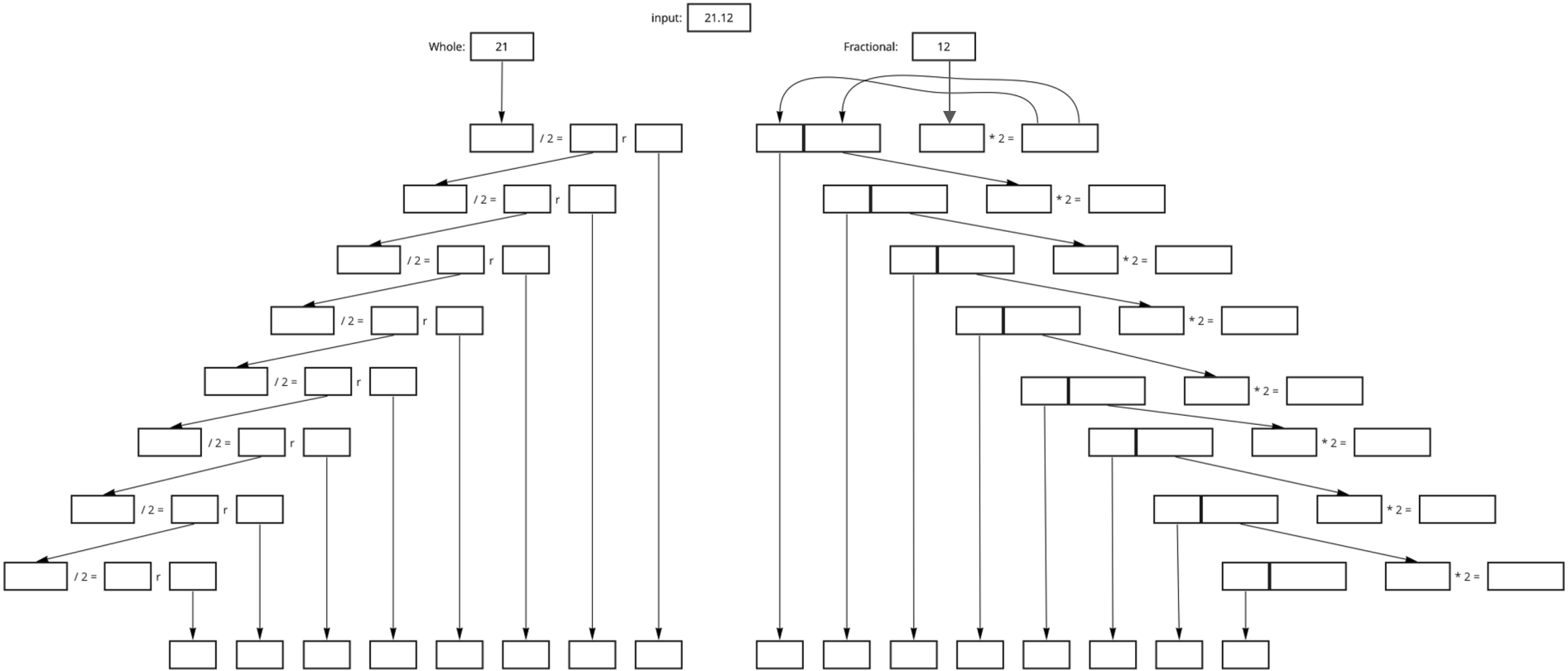
```
number = whole;
while (number != 0 ){
    push ( number % 2 )
    number = number / 2
}
pop_all();
```

3. With the fractional part

```
max = 10 ** stringlength(fractional);
number = fractional
while (number != 0 ) {
    number = number * 2
    if ( number > max ) {
       emit 1
       number = number - max
    } else {
       emit 0
    }
}
```

4. Put the two pieces together

Real: Decimal to Binary



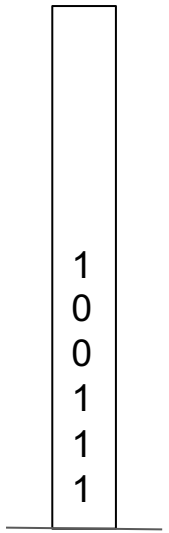
Example: - 39.234

Whole Part

1. Convert the base 10 number into base 2
2. Split the number at the radix point: - *39 . fractional*
2. With the whole part,

```
number = whole
while (number != 0 ){
     push ( number % 2 )
     number = number / 2
}
pop_all();
```

```
number: 39
    number = 39 / 2      → 19, 1
    push ( 39 % 2 )      → 1

    number = 19 / 2      → 9, 1
    push( 19 % 2)        → 1

    number = 9 / 2       → 4, 1
    push( 9 % 2)         → 1

    number = 4 /2        → 2, 0
    push( 4 % 2)         → 0

    number = 2 / 2       → 1, 0
    push ( 2 % 2 )       → 0

    number = 1 / 2       → 0, 1
    push ( 1 % 2 )       → 1
```



4. Put the two pieces together

-	100111	.

Example: - 39.234

Fractional Part

1. Convert the base 10 number into base 2
2. Split the number at the radix point: *whole . 234*

   max = 10 ** |234| == 1,000

   number = 234

   number = 234 * 2 = 0,468

   number = 468 * 2 = 0,936

   number = 936 * 2 = 1,872 - 1000 = 872

   number = 872 * 2 = 1,744 - 1000 = 744

   number = 744 * 2 = 1,488 - 1000 = 488

   number = 488 * 2 = 0,976

   number = 976 * 2 = 1,952 - 1000 = 952

3. With the fractional part

```
max = 10 ** stringlength(fractional);
number = fractional
while (number != 0 ) {
    number = number * 2
    if ( number > max ) {
        emit 1
        number = number - max
    } else {
        emit 0
    }
}
```

4. Put the two pieces together

.	0011101

Example: - 39.234

Both Parts

1. Split the number at the radix point: *whole . fractional*

2. With the whole part,

```
number = whole
while (number != 0 ){
     push ( number % 2 )
     number = number / 2
}
pop_all();
```

3. With the fractional part

```
max = 10 ** stringlength(fractional);
number = fractional
while (number != 0 ) {
    number = number * 2
    if ( number > max ) {
       emit 1
       number = number - max
    } else {
       emit 0
    }
}
```

4. Put the two pieces together

-	100111	.	0011101

Example: 45.45

Whole Part

1. Split the number at the radix point: *whole . fractional*

2. With the whole part,

```
number = whole;
while (number != 0 ){
     push ( number % 2 )
     number = number / 2
}
pop_all();
```

number: 45

number = 45 / 2 → 22, 1
push (1)
number = 22 / 2 → 11, 0
push (0)
number = 11 / 2 → 5, 1
push (1)
number = 5 / 2 → 2, 1
push (1)
number = 2 / 2 → 1, 0
push (1)
number = 1 / 2 → 0, 1
push (1)

---

number = 0 / 0

1
0
1
1
0
1

4. Put the two pieces together

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Example: 45.45

Fractional Part

1. Split the number at the radix point: *whole . 45*

```
max = 10 ** |45| == 100
number = 45
   number = number * 2 = 90
   number = number * 2 = 180 - 100 = 80
   number = 80 * 2 = 160 - 100 = 60
   number = 60 * 2 = 120 - 100 = 20
   number = 20 * 2 = 40
   number = 40 * 2 = 80
   number = 80 * 2 = 160 = 100 = 60
```

3. With the fractional part

```
max = 10 ** stringlength(fractional);
number = fractional
while (number != 0 ) {
    number = number * 2
    if ( number > max ) {
       emit 1
       number = number - max
    } else {
       emit 0
    }
}
```

4. Put the two pieces together

.	0111001

Example: 45.45

1. Split the number at the radix point: *whole . fractional*

2. With the whole part,

```
number = whole;
while (number != 0 ){
     push ( number % 2 )
     number = number / 2
}
pop_all();
```

3. With the fractional part

```
max = 10 ** stringlength(fractional);
number = fractional
while (number != 0 ) {
    number = number * 2
    if ( number > max ) {
       emit 1
       number = number - max
    } else {
       emit 0
    }
}
```

4. Put the two pieces together

101101	.	0111001