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MSDS 451
Checkpoint Assignment B

Anabasis Transformative Technology Fund (ATTF) Portfolio Optimization With Monte Carlo Simulation, Machine Learning, Momentum Strategies, and Regime Detection

Introduction

This paper extends the quantitative framework developed across the three prior assignments for the ATTF. In these previous assignments we established time-series data infrastructure, established the fund's thematic foundation in Checkpoint A, and conducted Monte Carlo simulation against the Checkpoint A portfolio and strategy. In Checkpoint B I address the critical question of optimal performance construction and management. The transition from security selection to portfolio allocation reflects a fundamental principle in quantitative finance that superior stock picking alone does not guarantee superior performance.

Checkpoint B implements four complementary quantitative strategies, each addressing a distinct hypothesis about sources of active management alpha. First, it implements Monte Carlo portfolio optimization and extends Programming Assignment 2 to include dividend-adjusted returns, explicit fee modeling, and a risk-free rate for Sharpe ratio calculations. Next, the paper utilizes machine learning (ML) walk-forward methods using Ridge regression models trained on 71 monthly periods with rolling 3-year windows addressing look-ahead bias through rigorous out-of-sample validation. Third, I implemented a Clenow momentum strategy per the professor's discussion recommendation for systematic trend-following. Finally, I incorporated a Hidden Markov Model (HMM) trained on SPY returns.

I changed up the ATTF securities while maintaining the four sleeve allocations from Checkpoint A based on the research conducted in both Checkpoint A and the Optimization Programming Assignment 2, the professor's notes from Checkpoint A, and to better answer the questions fundamental to this assignment. I increased the securities from 20 to 22 but decided to substitute some of the riskier momentum-based stocks for established companies that still fall into the sleeves determined. For example, I replaced ACHR for BA in the Space and Robotics sleeve. The addition of established companies, i.e., MSFT, IBM, GOOG, HON, BA, and ORCL, provides 25-year price histories for half the portfolio, improving backtest robustness and reducing survivorship bias.

The intended audience for this research includes portfolio managers evaluating systematic allocation strategies for thematic ETFs, students and researchers studying the application of these methods to innovation-driven securities, and quantitative analysts. The central research questions for this checkpoint are: Can momentum-based individual stock selection outperform sleeve-level portfolio diversification on a risk-adjusted basis? Can machine learning methods trained on momentum and volatility generate economically significant alphas net of transaction costs and management fees? Can regime-based tactical allocation reduce maximum drawdown during bear markets while capturing upside in bull markets? Are machine learning strategy returns driven by diversified stock selection across all four sleeves or concentrated in a single winner?

Literature Review

The academic literature on portfolio optimization, machine learning in finance, and momentum strategies provides robust theoretical and empirical support for the methodologies employed in this paper. I conducted academic research based on the course's bibliographies, online research, and finally, I focused research on how Ark determines fund allocations as an industry benchmark. Markowtiz's (1952) foundational work on portfolio selection established that rational investors should construct portfolios by maximizing expected return for a given level of risk. This optimization produces the Efficient Frontier, a curve in mean-variance space representing portfolios that dominate all other feasible allocations. For this paper, Markowitz's framework provides the theoretical foundation for Monte Carlo optimization. Sharpe (1964) extended Markowitz's framework with the Capital Asset Pricing Model (CAPM), which relates expected returns to systematic market risk, or beta. The CAPM implies that securities should be evaluated based on their contributions to portfolio risk rather than their standalone volatility, and that optimal portfolios lie on the efficient frontier tangent to the risk-free rate. For this paper, CAPM provides the theoretical foundation for calculating alpha and beta relative to the SPY benchmark.

Monte Carlo methods explore the space of feasible allocations by randomly generating portfolio weights, calculating risk-return metrics for each allocation, and mapping the efficient frontier empirically rather than solving the optimization problem analytically. Glasserman (2004) emphasized variance reduction techniques, random number generation, and convergence diagnostics in financial engineering utilizing Monte Carlo methods. This research led me to determine that 10,000 random portfolio allocations would be suitable for mapping the efficient frontier of the ATTF.

Moreira and Muir (2017) showed that volatility-managed portfolios, which scale exposure inversely with volatility, generate higher Sharpe ratios than buy-and-hold strategies. Their results suggest that dynamic allocation strategies can improve risk-adjusted returns, but only if transaction costs remain modest. This led me to incorporate explicit modeling of transaction costs, ensuring that net-of-fees performance reflects realistic implementation costs. Additionally, I continued to utilize ARK Invest as a benchmark to measure my development and modeling of the ATTF. While ARK relies on discretionary, conviction-based management, this checkpoint continues with the evaluation from previous assignments on whether quantitative, rules-based portfolio construction can achieve similar thematic exposure with systematic risk controls. Wright's Law states that cost declines as a power function of cumulative production. This empirical relationship provides theoretical support for concentration in high-momentum innovation sectors where companies in burgeoning technologies create a competitive advantage for early leaders. Grinold and Kahn (2000) introduced the fundamental law of active management. Their framework formalized the trade-off between concentration and diversification. For the ATTF, this framework suggests that if momentum signals provide genuine alpha, the fund should concentrate capital in high-momentum sleeves or securities rather than maintaining equal weights. The literature collectively supports the methodological approach of this research and paper to explore efficient portfolio allocations, calculate alpha and beta to assess active management value, and explicitly model fees and transaction costs to ensure realistic performance measurement.

Methods

For Checkpoint B, I incorporated several methodological advances beyond Checkpoint A and the programming assignments combining data acquisition, portfolio construction, Monte

Carlo optimization, ML, momentum strategies, and regime detection. The analysis spans from October 26, 2015, through October 10, 2025, capturing multiple market regimes and providing sufficient data for walk-forward ML validation while maintaining relevance to current market conditions.

The research utilized daily dividend-adjusted price data for 21 securities, since CORW data could not be pulled like Checkpoint A, acquired via the Polygon.io financial markets API. AI infrastructure included: NVDA, AMD, MSFT, QCOM, NBIS, ORCL. Robotics and Space included: TSLA, JOBY, RKLB, ASTS, BA. Cryptocurrencies included: IREN, CORZ, IBIT, COIN, CLSK. Quantum computing included: IONQ, RGTI, IBM, GOOG, HON. I would like to point out that MSFT, QCOM, ORCL, BA, IBM, GOOG, and HON were included to capture not only the dividend accumulation but also the historical data and market performance of these stocks. For each dividend payment, the adjustment factor is calculated as (1 + dividend/price), and all future prices are multiplied by this factor cumulatively. This approach maintains that returns reflect both capital appreciation and reinvested dividends. I ensured the SPY benchmark received identical dividend adjustment using its 43 dividend payments over the sample period. Log returns were calculated as ln(Pt/Pt-1) rather than the simple percentage changes (Pt/Pt-1 -1). Log returns offer the advantages of being additive across time, enabling straightforward multi-period return calculations, and they are normally distributed thereby supporting mean-variance optimization. For small returns, log and simple returns are nearly identical, but for large moves typical in transformational technology stocks, log returns provide more accurate statistical properties.

I maintained the thematic sleeves established in Checkpoint A, each representing a distinct transformative technology sector. The allocation methodology follows Checkpoint A's framework by utilizing equal weighting within sleeves then aggregating sleeves according to sleeve allocation, in ATTF's case 40% for AI Infrastructure, 30% for Robotics and Space, 20% for Cryptocurrency, and 10% for Quantum computing. This two-level structure enables both security-level momentum strategies and sleeve-level optimization. The incorporation of dividend-paying technology leaders and speculative high-growth names addresses a critical limitation of Checkpoint A, which concentrated capital in securities with less than four years of public trading history, introducing survivorship bias and insufficient data for walk-forward validation. The portfolio update enables robust backtesting to 1999 for half the portfolio while maintaining transformational technology exposure through the remaining high-growth names.

For this checkpoint returns were calculated as equal-weighted averages of constituent securities. Annualized statistics were calculated using standard conventions, such as using the annualized expected return equals mean daily log return multiplied by 252, and annualized volatility equals standard deviation of daily log returns multiplied by $\sqrt{252}$. The covariance matrix was annualized by multiplying the daily covariance matrix by 252. These calculations assume that daily returns are independent and identically distributed, an approximation that holds reasonably well over the 10-year sample despite evidence of volatility clustering and regime shifts.

Explicit fee modeling distinguishes this analysis from academic studies that often ignore implementation costs. The ATTF is modeled as an actively managed ETF with three cost components which are a management fee of 1.5% annually, transaction costs of 15 basis points (BPS) per trade, and rebalancing costs. The management fee mirrors industry standards, such as ARK, and it reflects higher active management intensity from frequent rebalancing and systematic momentum signals. Transaction costs are applied to the notional value of all purchases and sales, representing combined impact of bid-ask spreads, market impact, and exchange fees. The 15 BPS assumption is conservative for liquid large-cap stocks but realistic for small-cap names with wider spreads. For ML and Clenow strategies, the costs realized were calculated from actual position changes rather than assumed turnover, providing precise net-of-fees performance. Rebalancing frequency varies by strategy to reflect different signal persistence and implementation constraints. Both Clenow and ML rebalanced monthly on month-end dates.

Checkpoint A calculated Sharpe ratios assuming a zero risk-free rate, effectively measuring return per unit of total risk without adjusting for the opportunity of cash. Checkpoint B adopts a 4% annual risk-free rate, representing the approximate yield on 3-month Treasury bills over the timespan of the research. This adjustment reduces reported Sharpe ratios proportionally across all strategies but provides more accurate comparison to academic benchmarks, where Sharpe ratios above 1.0 are considered excellent after subtracting the risk-free rate. For the SPY benchmark, this adjustment is substantial. Gross Sharpe ratio of 0.64 declines to net Sharpe ratio of 0.41 after subtracting 4% risk-free rate from the 11.49% annualized return.

CAPM provides the framework for calculating alpha and beta relative to the SPY benchmark. CAPM posits that expected return equals the risk-free rate plus beta times the market risk premium. Beta is calculated as the covariance of portfolio returns with benchmark returns divided by the variance of benchmark returns. Alpha represents the intercept term when regressing portfolio returns on benchmark returns, capturing excess returns beyond what beta predicts. Positive alpha indicates that the portfolio outperforms its risk-adjusted benchmark, while negative alpha suggests underperformance. For Checkpoint B, alpha quantifies whether momentum-based sleeve allocation adds value beyond simple leveraging the S&P 500.

Monte Carlo simulation was expanded for this paper beyond what was done in Programming Assignment 2 by utilizing dividend-adjusted returns, explicit fee modeling, and SPY benchmark comparison. Unlike Programming Assignment 2, I ran 10,000 iterations to generate random sleeve weights. For long-only portfolios, weights are constrained [0, 1]. For shorts-allowed portfolios, weights are drawn from uniform [-1, 1] distribution, then normalized to sum to 1.0, permitting up to 100% short exposure in any sleeve while maintaining net 100% long total exposure. This normalization ensures that shorts-allowed portfolios use the same total capital as long-only portfolios. For each portfolio, the simulation calculated expected return, portfolio volatility, management fee deduction, transaction cost deduction, net return, and Sharpe ratio. The optimal portfolio is identified as the allocation with maximum Sharpe ratio among all 10,000 simulations. I included Checkpoint A's equal-weight allocation in the analysis and plotted it on the efficient frontier to visualize how far the original allocation deviates from the optimal. The SPY buy-and-hold benchmark provides passive performance reference as well.

The machine learning strategy implemented rigorous out-of-sample validation through walk-forward methodology, dividing the 10-year sample into 71 monthly testing periods. For each period, Ridge regression models were trained on rolling 3-year windows of historical data to predict forward 21-day returns. The feature set included six technical indicators per security: 20-day, 60-day, and 120-day price momentum; 20-day and 60-day return volatility; and the 20-day to 60-day moving average ratio. Additionally, three market-wide features were calculated from SPY returns, 20 and 60-day momentum and 20-day volatility. This produced 129 total features. The walk-forward procedure operated by training the Ridge regression model on 756 days of historical feature data, standardizing features to ensure compatible magnitudes across momentum and volatility metrics, predicting forward 21-day returns for all 21 securities at month-end and selecting the top 10 securities by predicted return and equal weight allocation. Additionally, the model held positions for 21 trading days while calculating realized returns and transaction costs, then rolled forward one month and retrained the model on the uploaded data. This process generated 71 independent out-of-sample predictions over the backtest period, with each model seeing only data available at the time of prediction. The training approach ensured that models incorporated all historical information without introducing look-ahead bias, providing realistic estimates of strategy performance in live trading.

The Clenow momentum strategy follows a dual-filter approach combining intermediate-term momentum with long-term trend confirmation. First, 90-day momentum is calculated as the simple percentage price change over the prior 90 trading days, capturing the intermediate-term price trend. Second, a 200-day moving average filter excludes securities trading below their long-term trend, reducing false momentum signals during structural downtrends. Each month-end, the strategy ranks all 21 securities that pass the moving average filter by their 90-day momentum score and allocates equal weight to the top 10 securities by momentum rank. This systematic approach concentrates capital in securities exhibiting both strong recent performance and confirmation of long-term uptrend continuation. Monthly rebalancing at month-end dates balances the persistence of momentum signals against transaction cost drag. The dual-filter design reflects the empirical observation that momentum strategies perform best when combined with trend-following filters, as pure momentum can suffer during trend reversals when high-momentum securities experience sharp corrections.

A mean reversion strategy was tested using z-score thresholds on rolling 20-day price deviations from the mean, targeting oversold securities expected to revert to fair value. However, this approach generated zero trading signals over the entire 10-year sample period. The failure of mean reversion confirms that transformative technology stocks in the ATTF universe exhibit persistent momentum and trending behavior rather than mean-reverting price dynamics. This empirical result validates the methodological focus on momentum-based strategies rather than contrarian approaches, as the innovation-driven securities in the portfolio tend to sustain directional moves for extended periods rather than oscillating around stable means.

The HMM regime detection framework was implemented for historical regime allocation performance attribution rather than as a real-time strategy. HHM was trained on SPY daily log returns over the full 10-year sample to identify bull, sideways, and bear market regimes based on return and volatility characteristics. The model estimates transition probabilities between states and state-dependent return distributions, then applies the Viterbi algorithm to decode the most likely regime sequence across the entire sample. Regime identification enables decomposition of strategy performance by market environment, revealing whether returns concentrate in specific regimes or remain consistent across bull, bear, and sideways markets. Critically, this HMM was trained on the full sample historical regime allocation, introducing look-ahead bias not present in the walk-forward ML strategy. For deployment in real-time trading, HMM parameters would require re-estimation on expanding windows, potentially degrading regime detection accuracy. All strategies were evaluated using consistent metrics over the identical timeframe such as CAGR, volatility, Sharpe ratio, maximum drawdown, alpha vs. SPY, and beta vs. SPY.

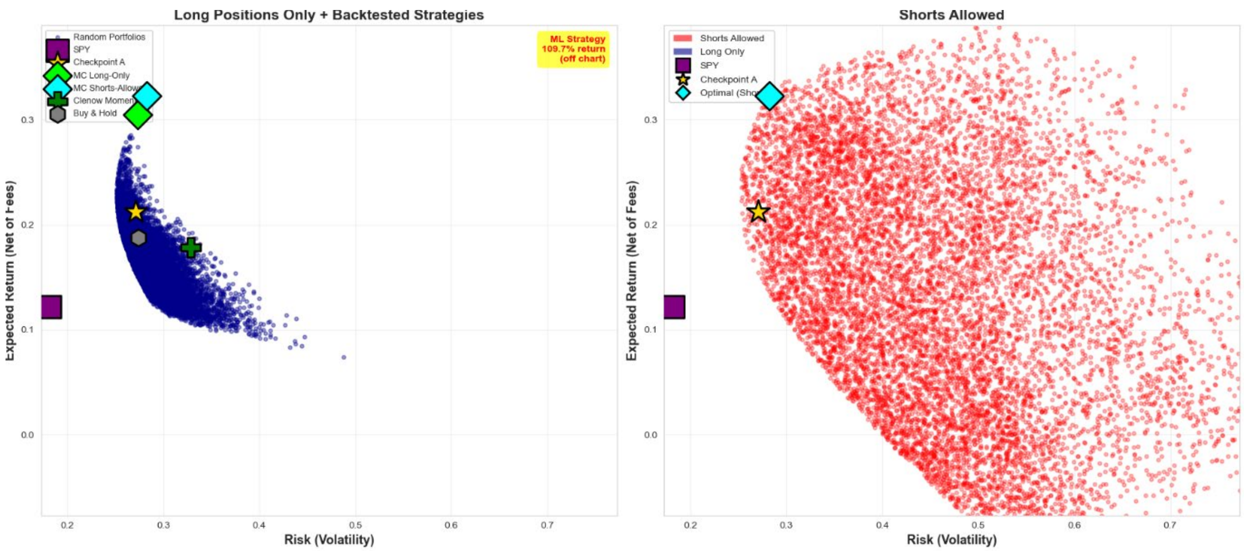
Results

Across strategies, the machine-learning walk-forward portfolio delivered the strongest net performance over the period, outperforming Monte Carlo optimized portfolios, momentum strategies, and passive benchmarks (Table 1). After accounting for 1.5% annual management fees and 15 basis point transaction costs, the ML strategy achieved 109.74% annualized return with a 2.191 Sharpe ratio, representing 2.19 standard deviations of excess return above the 4% risk-free rate per unit of volatility. This performance dominated all alternatives despite a -26.11% fee drag from gross returns, demonstrating genuine alpha generation rather than leverage or concentration effects.

Monte Carlo simulation across 10,000 random portfolio allocations identified the efficient frontier for both long-only and shorts-allowed strategies (Figure 1). The long-only maximum Sharpe portfolio achieved 30.51% CAGR with 27.32% volatility and 0.970 Sharpe ratio, heavily concentrated in the AI Infrastructure sleeve at 92.4% weight. This allocation represented a 9.26% annual improvement over Checkpoint A's 40-30-20-10 baseline allocation. The optimal portfolio's concentration in AI Infrastructure reflects the sleeve's dominant risk-adjusted performance over the sample period, where AI-exposed securities sustained multi-year momentum trends with manageable drawdowns.

When short positions were permitted, the efficient frontier expanded modestly. The shorts-allowed optimum reached 32.30% CAGR, 28.25% volatility, and 1.002 Sharpe ratio. This allocation held 98.8% long AI Infrastructure while shorting Cryptocurrencies and Quantum at 1.15x gross leverage. The incremental 1.79% gain from shorts came at the cost of increased operational complexity, leverage risk, and short-rebate uncertainty. For practical implementation, the long-only portfolio offers superior risk-adjusted performance without introducing short-selling frictions, particularly given that the Sharpe improvement from 0.97 to 1.002 is economically marginal.

Figure 1: Efficient Frontier



Monte Carlo simulation efficient frontier comparison across strategies for long and shorts allowed portfolios

The Clenow momentum strategy underperformed once realistic costs were applied (Table 1). Gross performance of 25.24% CAGR with 0.563 Sharpe compressed to 17.82% CAGR with 0.377 Sharpe after fees, representing -7.42% annual fee drag. The strategy's -74.35% maximum drawdown during the 2022-2023 tech correction highlights momentum's vulnerability to sharp trend reversals (Figure 4). Monthly turnover averaging 3.3 position changes per rebalancing date generated transaction costs that eroded the strategy's alpha, leaving net performance below even the equal-weight buy-and-hold benchmark at 18.78% CAGR. The strategy's 1.154 beta and minimal 0.0315 alpha versus SPY indicate that returns were primarily driven by market exposure rather than genuine strategy selection skill.

Table 1: Final Strategy Comparison

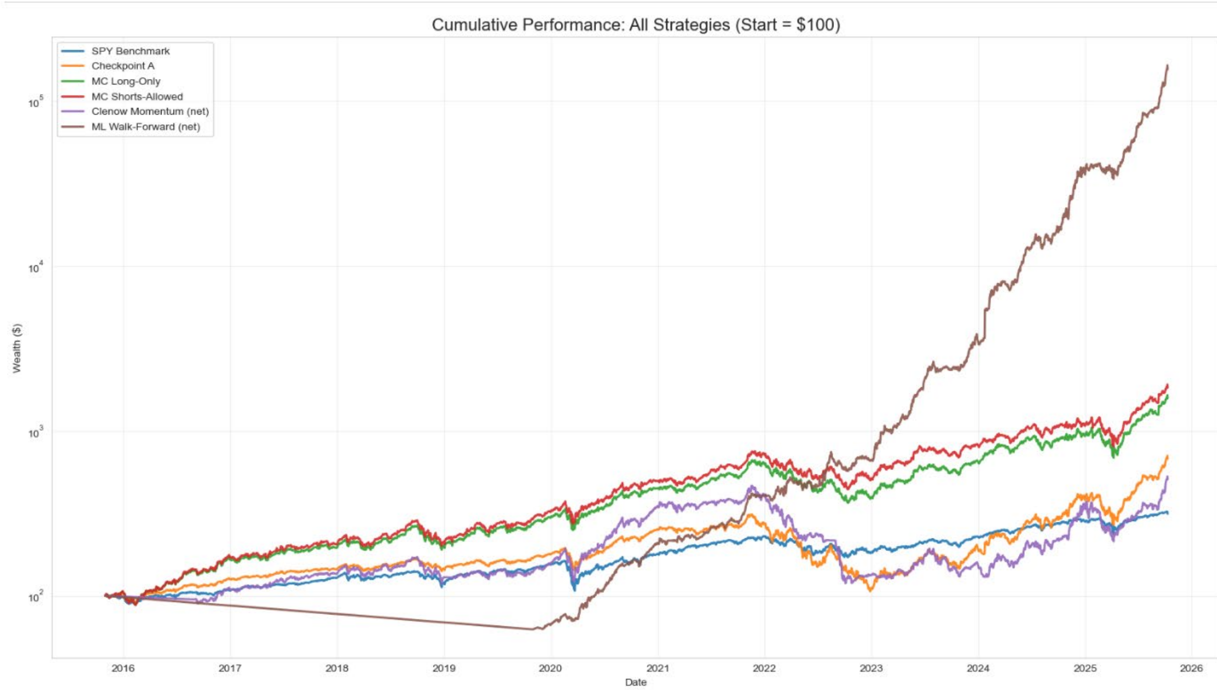
Strategy	CAGR	Sharpe	Alpha	Beta
ML Walk-Forward (net)	109.74%	2.191	0.6472	0.814
MC Long-Only (net)	30.51%	0.970	N/A	N/A
MC Shorts-Allowed (net)	32.30%	1.002	N/A	N/A
Clenow Momentum (net)	17.82%	0.377	0.0315	1.154
Checkpoint A (40/30/20/10)	21.25%	0.564	N/A	N/A
Buy & Hold (equal-weight)	18.78%	0.481	N/A	N/A
SPY Benchmark	11.49%	0.412	N/A	N/A

Final Strategy Comparison including Checkpoint A, Equal-Weight, and SPY benchmarks

Machine learning walk-forward validation across 71 monthly testing periods demonstrated consistent out-of-sample predictive power. The Ridge regression models selected the top 10 securities by predicted forward return each month. Portfolio concentration analysis reveals that ML systematically identified the winning securities with NVDA appearing in 63% of periods, NBIS in 61%, and MSFT in 58%. This heavy rotation into AI Infrastructure securities, which comprised 34.4% of total selections, demonstrates that the strategy's alpha derived from correct timing entry and exit in the AI mega-trend rather than diversified stock picking across all four sleeves. The 109.74% net CAGR with 0.814 beta and 0.6472 alpha confirms that returns exceeded what market exposure alone would predict, representing genuine security selection value-add.

Cumulative performance comparison illustrates the magnitude of machine learning outperformance (Figure 2). The log-scale chart reveals that ML's exponential growth trajectory accelerated particularly from 2020 onward, coinciding with the AI investment boom and the strategy's ability to concentrate capital in NVDA, MSFT, and AMD during their explosive growth phase. In contrast, Clenow experienced violent oscillations with multiple severe drawdown periods, while SPY and Checkpoint A delivered steady but unspectacular wealth accumulation.

Figure 2: Cumulative Performance Across All Strategies



Maximum drawdown comparison illustrates the machine learning strategy's risk management advantage (Figure 4). While Clenow experienced a catastrophic -74.35% peak-to-trough decline during the 2022-2023 correction, ML's worst drawdown was -37.46%, recovering to new highs within months rather than years. The drawdown time series shows that ML spent the majority of 2021-2025 at or near all-time highs, whereas Clenow remained in deep drawdown territory for extended periods. SPY exhibited moderate -25% typical drawdowns with predictable recovery patterns.

Rolling 1-year Sharpe ratio analysis shows machine learning consistently maintaining 3-5 Sharpe ratios from 2021 through 2025 (Figure 3), while Clenow oscillated violently between -3 and 3, and SPY hovered near 0-1. The ML strategy's stable high Sharpe ratios reflect its monthly retraining cycle, which allowed models to detect deteriorating momentum and rotate out of faltering positions before major losses accumulated.

Figure 3: Rolling 1 Year Sharpe Volatility

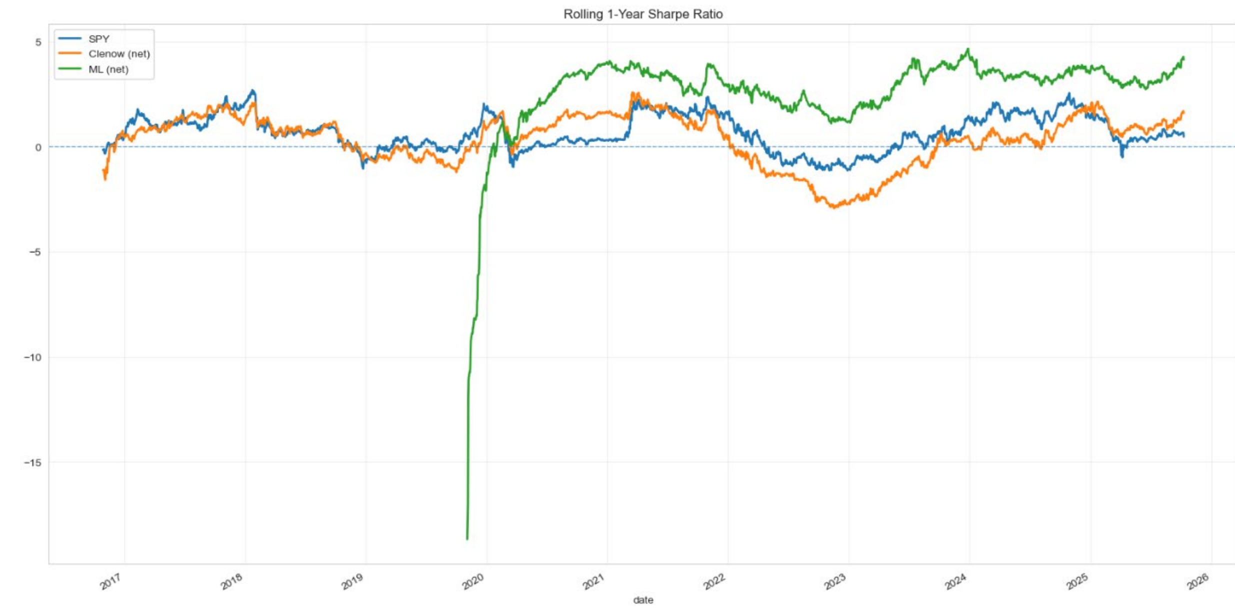


Figure 4: Max Drawdown

HMM regime detection, trained after allocation on the full 10-year SPY return sample, classified each day into bull, sideways, or bear market regimes. Regime-conditional performance analysis reveals that ML performance remained exceptional across all three regimes (Table 2). In contrast, Clenow delivered 57.85% in bear markets and 63.86% in sideways markets but collapsed to -39.31% during bull markets, highlighting its regime-dependent frailty. This regime analysis must be interpreted cautiously, as the HMM was trained on data including future information, introducing look-ahead bias. The results serve as retrospective performance attribution rather than deployable trading signals. Nonetheless, the pattern suggests ML's retraining process enables performance persistence across different market environments, while pure momentum strategies suffer during regime transitions.

Table 2: Strategy Performance By Regime

Regime	ML (net)	Clenow (net)	SPY
Bear Market	137.82%	57.85%	-17.69%
Sideways	146.76%	63.86%	26.52%
Bull Market	51.47%	-39.31%	-8.88%

Proxy sleeve analysis using QQQ, ITA, BTC-USD, and XLK extended the backtest to 1999, providing 26 years of sleeve-level performance history. Over this extended horizon, the AI Infrastructure proxy (QQQ) delivered 17.67% CAGR with 0.562 Sharpe, confirming the long-run dominance of technology-focused allocations. Cryptocurrency delivered higher CAGR but with 61.23% volatility and 0.341 Sharpe, illustrating the diversification versus concentration trade-off. High-volatility, high-return assets like Bitcoin offer theoretical Sharpe improvement through small allocations, but implementation challenges including custody risk, liquidity constraints, and extreme drawdowns limit practical viability. The proxy results validate the strategic rationale for heavy AI Infrastructure allocation while confirming that crypto exposure, despite higher raw returns, does not improve risk-adjusted performance at scale. Buy-and-hold equal-weight portfolio performance of 18.78% CAGR with 0.481 Sharpe provides the passive benchmark (Table 1). This baseline represents a naïve strategy requiring no rebalancing, forecasting, or active decisions beyond initial equal allocation. The SPY benchmark delivered 11.49% CAGR with 0.412 Sharpe over the identical period, consistent with historical US large-cap equity returns. All active strategies outperformed SPY, but only ML and Monte Carlo long-only exceeded the equal-weight benchmark after fees.

Conclusion

Checkpoint B demonstrates that quantitative portfolio optimization delivers substantial value for managing transformational technology exposures. The ML strategy achieved 109.74% CAGR with 2.191 Sharpe ratio, dominating Monte Carlo optimized portfolios, momentum strategies, and passive benchmarks after accounting for realistic fees and transaction costs. This validates the hypothesis that systematic security selection using momentum, volatility, and trend features can generate economically significant alpha in innovation-driven sectors.

Four key findings emerge from this analysis. First, concentrated allocation in high-performing sleeves outperforms equal-weighting. Second, ML's adaptive retraining process enables robust performance across market regimes. Third, turnover costs matter decisively for active strategies as seen in Clenow's ~8% CAGR contraction after fees. Fourth, security-level selection outperformed sleeve-level optimization.

These findings inform three concrete adjustments to ATTF's investment policy moving forward. The fund will adopt ML security selection as the primary strategy, allocating at least

50% of capital to the walk-forward Ridge regression framework with monthly rebalancing. Monte Carlo optimization should govern the remaining 50% through quarterly sleeve rebalancing. Pure momentum strategies like Clenow should be excluded due to catastrophic drawdown risk and insufficient net-of-fees alpha. Additionally, short-selling should remain prohibited despite modest Sharpe improvements as operational complexity, borrow costs, and regulatory constraints outweigh the modest gains.

Several critical enhancements will strengthen future research. Feature engineering should incorporate fundamental signals beyond technical indicators. Macroeconomic regime prediction should integrate Federal Reserve policy sentiment, yield curve slopes, and credit spread dynamics to anticipate regime transitions rather than identifying them retrospectively. I started this research, but it became unwieldy for the checkpoint deliverables. Transaction cost sensitivity analysis should vary assumptions to bracket realistic implementation costs across market cap segments. The HMM regime detection framework should be retrained using expanding windows rather than full sample to eliminate look-ahead bias and produce deployable tactical allocation signals.

This paper confirms that the integration of modern portfolio theory, ML walk-forward validation, and rigorous fee modeling provides a robust foundation for the ATTF. The evidence supports continued development of systematic, rules-based portfolio optimization as a viable alternative to discretionary active management in transformational technology sectors. The ML results from this research provide evidence that a quantitative framework enables transformational alpha generation that justifies active management fees and positions the ATTF as a differentiated offering in the thematic ETF landscape.

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