# FMAT3888 - Projects in Financial Mathematics

## Portfolio Optimisation

## 2024S2 - Group 6


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## 28th Oct 2024


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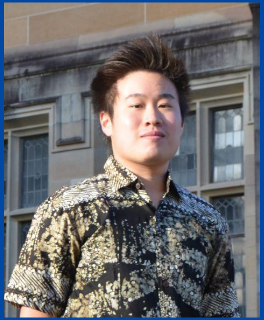

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# Q1. Parameters α and B

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# Parameters α and B

### Mean Values for Periods A and B

| | Mean A | Mean B |
|---|---|---|
| AEQ | 0.009147 | 0.006502 |
| ILE_H | 0.011787 | 0.009373 |
| ALP | 0.015114 | 0.004852 |
| ILP | 0.011509 | 0.002054 |
| ILI | 0.010477 | 0.006713 |
| AFI | 0.004437 | 0.004045 |
| IFI | 0.005130 | 0.003575 |
| CASH | 0.002433 | 0.001144 |

### Covariance Matrix - Period A

| | AEQ | ILE_H | ALP | ILP | ILI | AFI | IFI | CASH |
|---|---|---|---|---|---|---|---|---|
| AEQ | 0.001203 | 0.000659 | 0.000872 | 0.000691 | 0.000491 | 0.000016 | 0.000073 | 0.000003 |
| ILE_H | 0.000659 | 0.000870 | 0.000329 | 0.000458 | 0.000467 | -0.000043 | 0.000016 | 0.000001 |
| ALP | 0.000872 | 0.000329 | 0.001129 | 0.000782 | 0.000503 | 0.000091 | 0.000119 | 0.000004 |
| ILP | 0.000691 | 0.000458 | 0.000782 | 0.001139 | 0.000712 | 0.000098 | 0.000161 | 0.000002 |
| ILI | 0.000491 | 0.000467 | 0.000503 | 0.000712 | 0.000688 | 0.000050 | 0.000091 | 0.000002 |
| AFI | 0.000016 | -0.000043 | 0.000091 | 0.000098 | 0.000050 | 0.000063 | 0.000039 | 0.000002 |
| IFI | 0.000073 | 0.000016 | 0.000119 | 0.000161 | 0.000091 | 0.000039 | 0.000055 | 0.000001 |
| CASH | 0.000003 | 0.000001 | 0.000004 | 0.000002 | 0.000002 | 0.000002 | 0.000001 | 0.000000 |

Period A = January 2012 to December 2015

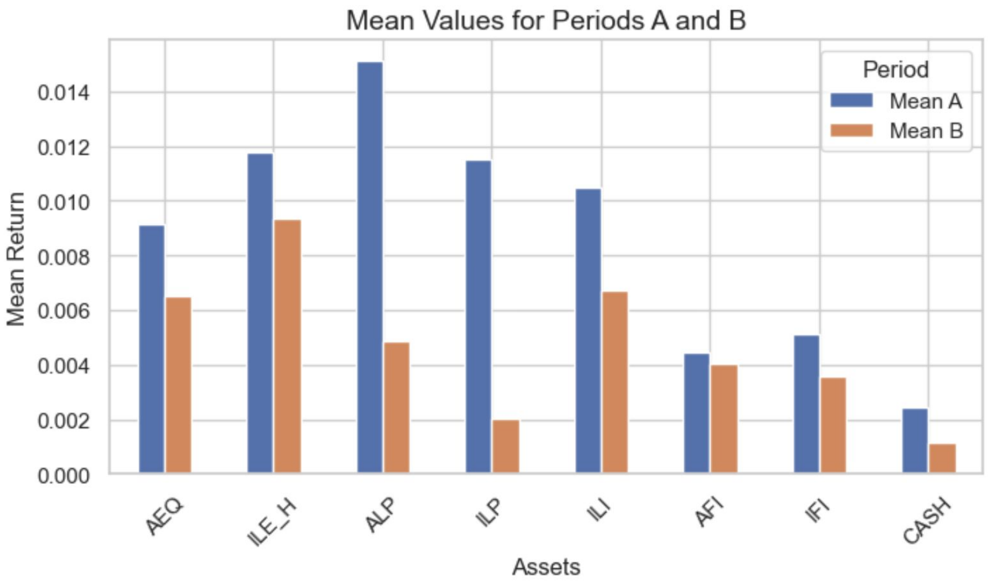
Period B = January 2016 to December 2019

# Parameters α and B

### Covariance Matrix - Period B

| | AEQ | ILE_H | ALP | ILP | ILI | AFI | IFI | CASH |
|---|---|---|---|---|---|---|---|---|
| **AEQ** | 0.002288 | 0.001634 | 0.003199 | 0.001970 | 0.001194 | 0.000026 | 0.000120 | -0.000002 |
| **ILE_H** | 0.001634 | 0.001927 | 0.002226 | 0.001777 | 0.001176 | -0.000083 | 0.000029 | -0.000004 |
| **ALP** | 0.003199 | 0.002226 | 0.005668 | 0.003333 | 0.001910 | 0.000122 | 0.000302 | -0.000003 |
| **ILP** | 0.001970 | 0.001777 | 0.003333 | 0.002667 | 0.001508 | 0.000012 | 0.000163 | -0.000001 |
| **ILI** | 0.001194 | 0.001176 | 0.001910 | 0.001508 | 0.001196 | 0.000020 | 0.000091 | 0.000001 |
| **AFI** | 0.000026 | -0.000083 | 0.000122 | 0.000012 | 0.000020 | 0.000056 | 0.000038 | 0.000001 |
| **IFI** | 0.000120 | 0.000029 | 0.000302 | 0.000163 | 0.000091 | 0.000038 | 0.000057 | -0.000001 |
| **CASH** | -0.000002 | -0.000004 | -0.000003 | -0.000001 | 0.000001 | 0.000001 | -0.000001 | 0.000000 |

# Parameters α and B



# Mean, covariance, correlation

# Mean, covariance, correlation

$$R_i^{(j)} = e^{Y_i} - 1 \quad \text{with} \quad Y_i \sim \mathcal{N}(12ja_i, 12jb_{ii}) \quad \text{for values of} \quad j = 1, 2.$$

The expected value can be computed by:

$$\mathbb{E}[e^{Y_i} - 1] = \mathbb{E}[e^{Y_i}] - 1 = e^{12ja_i - \frac{1}{2}12jb_{ii}} - 1.$$

The covariance can be computed by:

$$\mathrm{Cov}(e^{Y_i} - 1, e^{Y_l} - 1) = \mathrm{Cov}(e^{Y_i}, e^{Y_l}) = \mathbb{E}[e^{Y_i + Y_l}] - \mathbb{E}[e^{Y_i}]\mathbb{E}[e^{Y_l}].$$

This gives us:

$$\mathbb{E}[e^{Y_i + Y_l}] = e^{12j\left(a_i + a_l - \frac{1}{2}(b_{ii} + b_{ll})\right)}.$$

Thus, the covariance is equal to:

$$\mathrm{Cov}(e^{Y_i} - 1, e^{Y_l} - 1) = e^{12j\left(a_i + a_l - \frac{1}{2}(b_{ii} + b_{ll})\right)} \left(e^{-12jb_{il}} - 1\right).$$

# Mean

Mean Values for Each Data Set

| | Mean R1 for A | Mean R2 for A | Mean R1 for B | Mean R2 for B |
|---|---|---|---|---|
| AEQ | 0.107991 | 0.227643 | 0.066410 | 0.137231 |
| ILE_H | 0.145933 | 0.313162 | 0.106173 | 0.223620 |
| ALP | 0.190769 | 0.417931 | 0.024513 | 0.049627 |
| ILP | 0.140273 | 0.300222 | 0.008683 | 0.017442 |
| ILI | 0.129300 | 0.275319 | 0.076146 | 0.158091 |
| AFI | 0.054294 | 0.111537 | 0.049389 | 0.101217 |
| IFI | 0.063146 | 0.130278 | 0.043481 | 0.088852 |
| CASH | 0.029625 | 0.060129 | 0.013825 | 0.027840 |

# Covariance

### Covariance Matrix - R1 for dataset A

| | AEQ | ILE_H | ALP | ILP | ILI | AFI | IFI | CASH |
|---|---|---|---|---|---|---|---|---|
| **AEQ** | -0.017602 | -0.010000 | -0.013727 | -0.010437 | -0.007357 | -0.000228 | -0.001034 | -0.000039 |
| **ILE_H** | -0.010000 | -0.013643 | -0.005369 | -0.007157 | -0.007228 | 0.000625 | -0.000227 | -0.000017 |
| **ALP** | -0.013727 | -0.005369 | -0.019077 | -0.012690 | -0.008096 | -0.001367 | -0.001806 | -0.000062 |
| **ILP** | -0.010437 | -0.007157 | -0.012690 | -0.017651 | -0.010953 | -0.001412 | -0.002340 | -0.000031 |
| **ILI** | -0.007357 | -0.007228 | -0.008096 | -0.010953 | -0.010482 | -0.000711 | -0.001311 | -0.000023 |
| **AFI** | -0.000228 | 0.000625 | -0.001367 | -0.001412 | -0.000711 | -0.000836 | -0.000530 | -0.000020 |
| **IFI** | -0.001034 | -0.000227 | -0.001806 | -0.002340 | -0.001311 | -0.000530 | -0.000748 | -0.000015 |
| **CASH** | -0.000039 | -0.000017 | -0.000062 | -0.000031 | -0.000023 | -0.000020 | -0.000015 | -0.000004 |

### Covariance Matrix - R2 for dataset A

| | AEQ | ILE_H | ALP | ILP | ILI | AFI | IFI | CASH |
|---|---|---|---|---|---|---|---|---|
| **AEQ** | -0.042908 | -0.025293 | -0.036032 | -0.026264 | -0.018356 | -0.000533 | -0.002435 | -0.000088 |
| **ILE_H** | -0.025293 | -0.035645 | -0.014625 | -0.018652 | -0.018654 | 0.001511 | -0.000554 | -0.000040 |
| **ALP** | -0.036032 | -0.014625 | -0.053735 | -0.034300 | -0.021708 | -0.003430 | -0.004570 | -0.000153 |
| **ILP** | -0.026264 | -0.018652 | -0.034300 | -0.045590 | -0.028087 | -0.003392 | -0.005667 | -0.000073 |
| **ILI** | -0.018356 | -0.018654 | -0.021708 | -0.028087 | -0.026625 | -0.001692 | -0.003147 | -0.000054 |
| **AFI** | -0.000533 | 0.001511 | -0.003430 | -0.003392 | -0.001692 | -0.001857 | -0.001187 | -0.000044 |
| **IFI** | -0.002435 | -0.000554 | -0.004570 | -0.005667 | -0.003147 | -0.001187 | -0.001690 | -0.000033 |
| **CASH** | -0.000088 | -0.000040 | -0.000153 | -0.000073 | -0.000054 | -0.000044 | -0.000033 | -0.000009 |

### Covariance Matrix - R1 for dataset B

| | AEQ | ILE_H | ALP | ILP | ILI | AFI | IFI | CASH |
|---|---|---|---|---|---|---|---|---|
| **AEQ** | -0.030805 | -0.022899 | -0.041150 | -0.025135 | -0.016321 | -0.000353 | -0.001599 | 0.000028 |
| **ILE_H** | -0.022899 | -0.027975 | -0.029870 | -0.023540 | -0.016681 | 0.001159 | -0.000408 | 0.000051 |
| **ALP** | -0.041150 | -0.029870 | -0.069019 | -0.040513 | -0.024982 | -0.001569 | -0.003872 | 0.000043 |
| **ILP** | -0.025135 | -0.023540 | -0.040513 | -0.032048 | -0.019472 | -0.000151 | -0.002053 | 0.000011 |
| **ILI** | -0.016321 | -0.016681 | -0.024982 | -0.019472 | -0.016498 | -0.000266 | -0.001231 | -0.000010 |
| **AFI** | -0.000353 | 0.001159 | -0.001569 | -0.000151 | -0.000266 | -0.000740 | -0.000497 | -0.000010 |
| **IFI** | -0.001599 | -0.000408 | -0.003872 | -0.002053 | -0.001231 | -0.000497 | -0.000743 | 0.000001 |
| **CASH** | 0.000028 | 0.000051 | 0.000043 | 0.000011 | -0.000010 | -0.000010 | 0.000001 | -0.000004 |

### Covariance Matrix - R2 for dataset B

| | AEQ | ILE_H | ALP | ILP | ILI | AFI | IFI | CASH |
|---|---|---|---|---|---|---|---|---|
| **AEQ** | -0.069115 | -0.053500 | -0.088223 | -0.053442 | -0.037195 | -0.000790 | -0.003557 | 0.000060 |
| **ILE_H** | -0.053500 | -0.067678 | -0.066809 | -0.051978 | -0.039436 | 0.002692 | -0.000941 | 0.000115 |
| **ALP** | -0.088223 | -0.066809 | -0.140125 | -0.082092 | -0.054461 | -0.003371 | -0.008264 | 0.000089 |
| **ILP** | -0.053442 | -0.051978 | -0.082092 | -0.064186 | -0.041895 | -0.000319 | -0.004318 | 0.000023 |
| **ILI** | -0.037195 | -0.039436 | -0.054461 | -0.041895 | -0.037940 | -0.000600 | -0.002764 | -0.000022 |
| **AFI** | -0.000790 | 0.002692 | -0.003371 | -0.000319 | -0.000600 | -0.001629 | -0.001089 | -0.000021 |
| **IFI** | -0.003557 | -0.000941 | -0.008264 | -0.004318 | -0.002764 | -0.001089 | -0.001618 | 0.000003 |
| **CASH** | 0.000060 | 0.000115 | 0.000089 | 0.000023 | -0.000022 | -0.000021 | 0.000003 | -0.000008 |

# Utility maximisation

# Utility maximisation

The main objective is to maximize:

$$\mathbb{E}[U(R^{(2)}w)] = -\mathbb{E}\left[e^{-aR^{(2)}w}\right].$$

As mentioned previously, this is equivalent to:

$$-\mathbb{E}\left[e^{-aR^{(2)}w}\right] = -e^{-aR^{(2)}w+\frac{1}{2}w^T BR^{(2)}w}.$$

Solving the equation can be done by minimizing the following equation:

$$-aR^{(2)}w + \frac{1}{2}w^T BR^{(2)}w.$$

# Utility maximisation

## By setting the boundaries to 0 and 1 with gamma = 1, we got:

```
Optimal Weights for Period A:
AEQ: 0.0000
ILE_H: 0.0000
ALP: 1.0000
ILP: 0.0000
ILI: 0.0000
AFI: 0.0000
IFI: 0.0000

Optimal Weights for Period B:
AEQ: 0.0000
ILE_H: 1.0000
ALP: 0.0000
ILP: 0.0000
ILI: 0.0000
AFI: 0.0000
IFI: 0.0000
```

**Utility Function Dynamics:** The risk-averse utility function, constrained to keep asset weights between 0 and 1, often results in concentrated investments in select assets rather than diversified portfolios.

**Optimization Goals:** The ideal strategy involves finding an equilibrium between risk mitigation and return maximization in fluctuating market conditions.

**To Adjust:** Strategies should include broadening investment diversification, adjusting the risk aversion parameters to suit different market scenarios, and evaluating various utility functions to cater to diverse investor needs.

# Utility maximisation

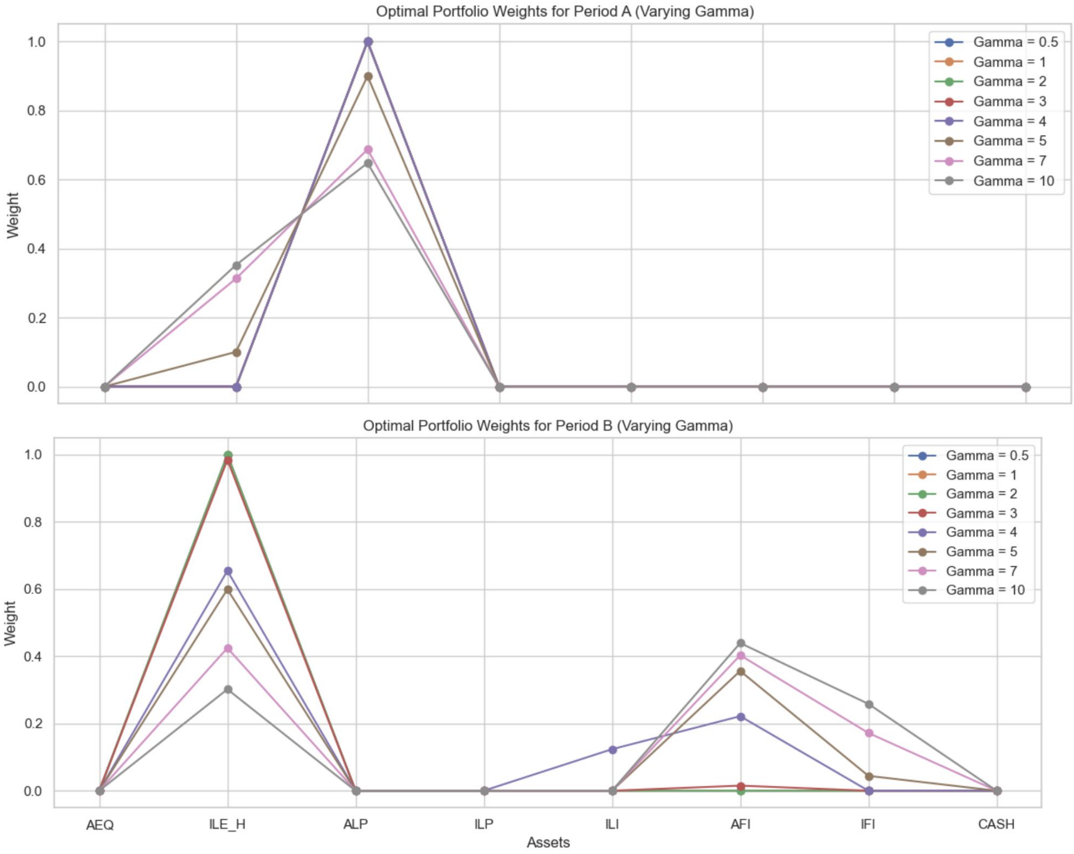
## For period C (January 2020 to December 2021)

```
Maximum utililty Period C:
2.77555756e-17
1.0
2.77555756e-17
0.0
0.0
5.55111512e-17
0.0

Maximum utility returns Period C:
0.300563543
```

# Extension

# Extension – varying γ



# Extension – adding ILE_UH

Mean Values for Periods A and B - with IEH_UH

| | Mean A | Mean B |
|---|---|---|
| AEQ | 0.009147 | 0.006502 |
| ILE_H | 0.011787 | 0.009373 |
| ILE_UH | 0.015248 | 0.009411 |
| ALP | 0.015114 | 0.004852 |
| ILP | 0.011509 | 0.002054 |
| ILI | 0.010477 | 0.006713 |
| AFI | 0.004437 | 0.004045 |
| IFI | 0.005130 | 0.003575 |
| CASH | 0.002433 | 0.001144 |

Covariance Matrix - Period A (New Data)

| | AEQ | ILE_H | ILE_UH | ALP | ILP | ILI | AFI | IFI | CASH |
|---|---|---|---|---|---|---|---|---|---|
| AEQ | 0.001203 | 0.000659 | 0.000316 | 0.000872 | 0.000691 | 0.000491 | 0.000016 | 0.000073 | 0.000003 |
| ILE_H | 0.000659 | 0.000870 | 0.000524 | 0.000329 | 0.000458 | 0.000467 | -0.000043 | 0.000016 | 0.000001 |
| ILE_UH | 0.000316 | 0.000524 | 0.000807 | 0.000084 | 0.000142 | 0.000216 | 0.000018 | -0.000020 | 0.000002 |
| ALP | 0.000872 | 0.000329 | 0.000084 | 0.001129 | 0.000782 | 0.000503 | 0.000091 | 0.000119 | 0.000004 |
| ILP | 0.000691 | 0.000458 | 0.000142 | 0.000782 | 0.001139 | 0.000712 | 0.000098 | 0.000161 | 0.000002 |
| ILI | 0.000491 | 0.000467 | 0.000216 | 0.000503 | 0.000712 | 0.000688 | 0.000050 | 0.000091 | 0.000002 |
| AFI | 0.000016 | -0.000043 | 0.000018 | 0.000091 | 0.000098 | 0.000050 | 0.000063 | 0.000039 | 0.000002 |
| IFI | 0.000073 | 0.000016 | -0.000020 | 0.000119 | 0.000161 | 0.000091 | 0.000039 | 0.000055 | 0.000001 |
| CASH | 0.000003 | 0.000001 | 0.000002 | 0.000004 | 0.000002 | 0.000002 | 0.000002 | 0.000001 | 0.000000 |

Covariance Matrix - Period B (New Data)

| | AEQ | ILE_H | ILE_UH | ALP | ILP | ILI | AFI | IFI | CASH |
|---|---|---|---|---|---|---|---|---|---|
| AEQ | 0.002288 | 0.001634 | 0.001134 | 0.003199 | 0.001970 | 0.001194 | 0.000026 | 0.000120 | -0.000002 |
| ILE_H | 0.001634 | 0.001927 | 0.001112 | 0.002226 | 0.001777 | 0.001176 | -0.000083 | 0.000029 | -0.000004 |
| ILE_UH | 0.001134 | 0.001112 | 0.000959 | 0.001555 | 0.001015 | 0.000783 | 0.000019 | 0.000037 | -0.000001 |
| ALP | 0.003199 | 0.002226 | 0.001555 | 0.005668 | 0.003333 | 0.001910 | 0.000122 | 0.000302 | -0.000003 |
| ILP | 0.001970 | 0.001777 | 0.001015 | 0.003333 | 0.002667 | 0.001508 | 0.000012 | 0.000163 | -0.000001 |
| ILI | 0.001194 | 0.001176 | 0.000783 | 0.001910 | 0.001508 | 0.001196 | 0.000020 | 0.000091 | 0.000001 |
| AFI | 0.000026 | -0.000083 | 0.000019 | 0.000122 | 0.000012 | 0.000020 | 0.000056 | 0.000038 | 0.000001 |
| IFI | 0.000120 | 0.000029 | 0.000037 | 0.000302 | 0.000163 | 0.000091 | 0.000038 | 0.000057 | -0.000001 |
| CASH | -0.000002 | -0.000004 | -0.000001 | -0.000003 | -0.000001 | 0.000001 | 0.000001 | -0.000001 | 0.000000 |

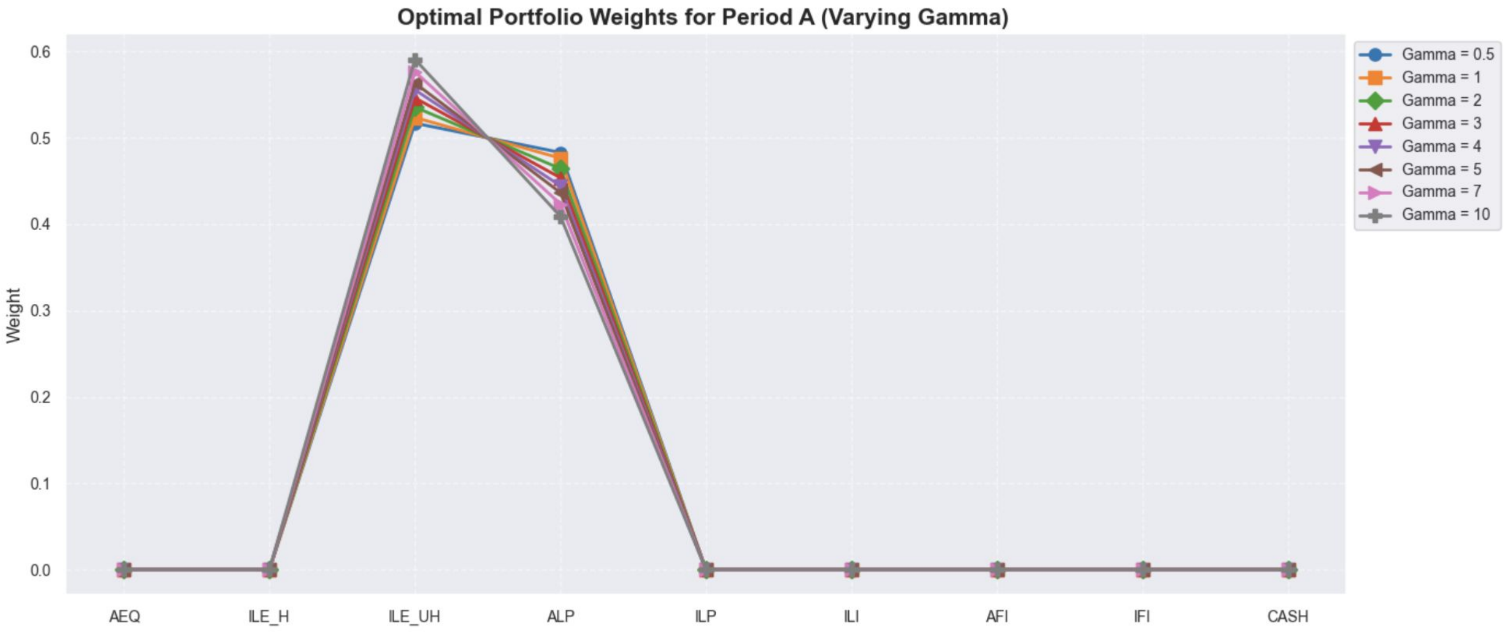
# Extension - adding ILE_UH

**Using gamma = 1,**

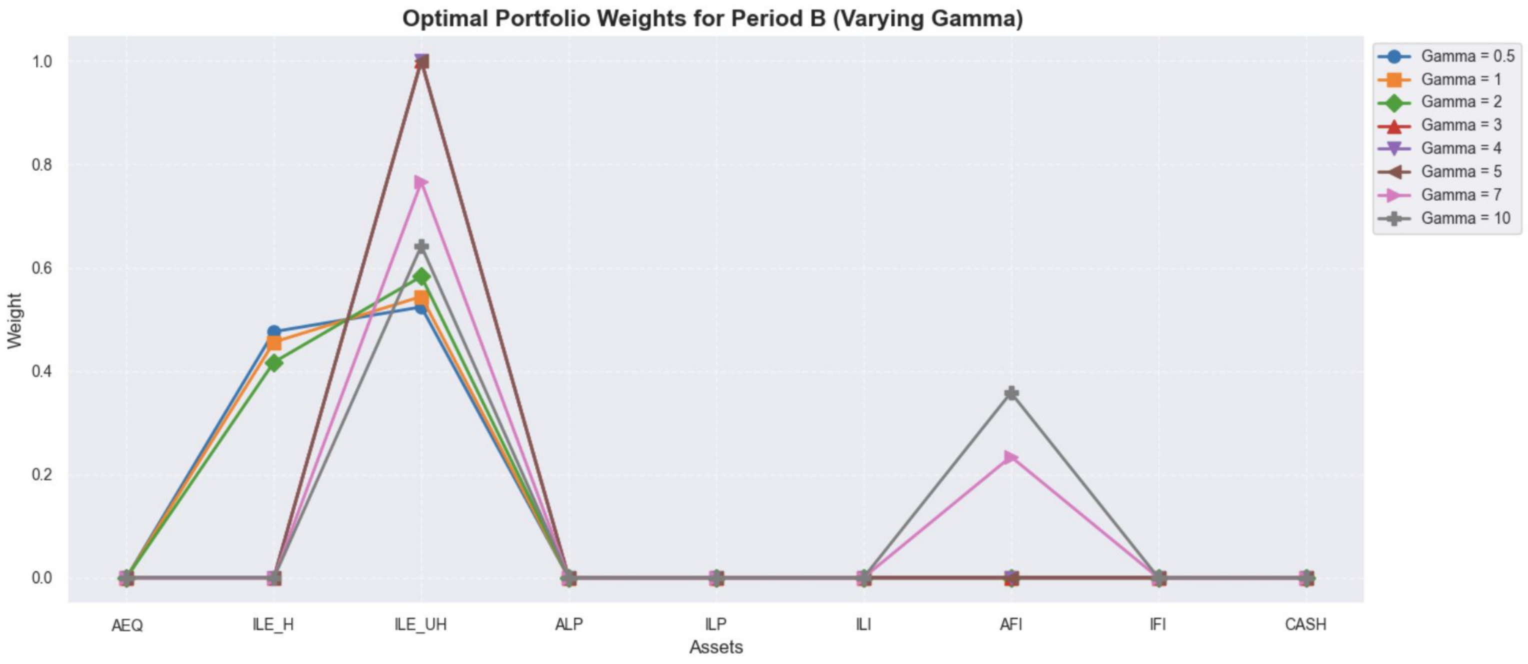
```
Optimal Weights for Period A:
AEQ: 0.0000
ILE_H: 0.0000
ILE_UH: 0.5235
ALP: 0.4765
ILP: 0.0000
ILI: 0.0000
AFI: 0.0000
IFI: 0.0000
CASH: 0.0000

Optimal Weights for Period B:
AEQ: 0.0000
ILE_H: 0.4559
ILE_UH: 0.5441
ALP: 0.0000
ILP: 0.0000
ILI: 0.0000
AFI: 0.0000
IFI: 0.0000
CASH: 0.0000
```

# Extension – adding ILE_UH and varying γ

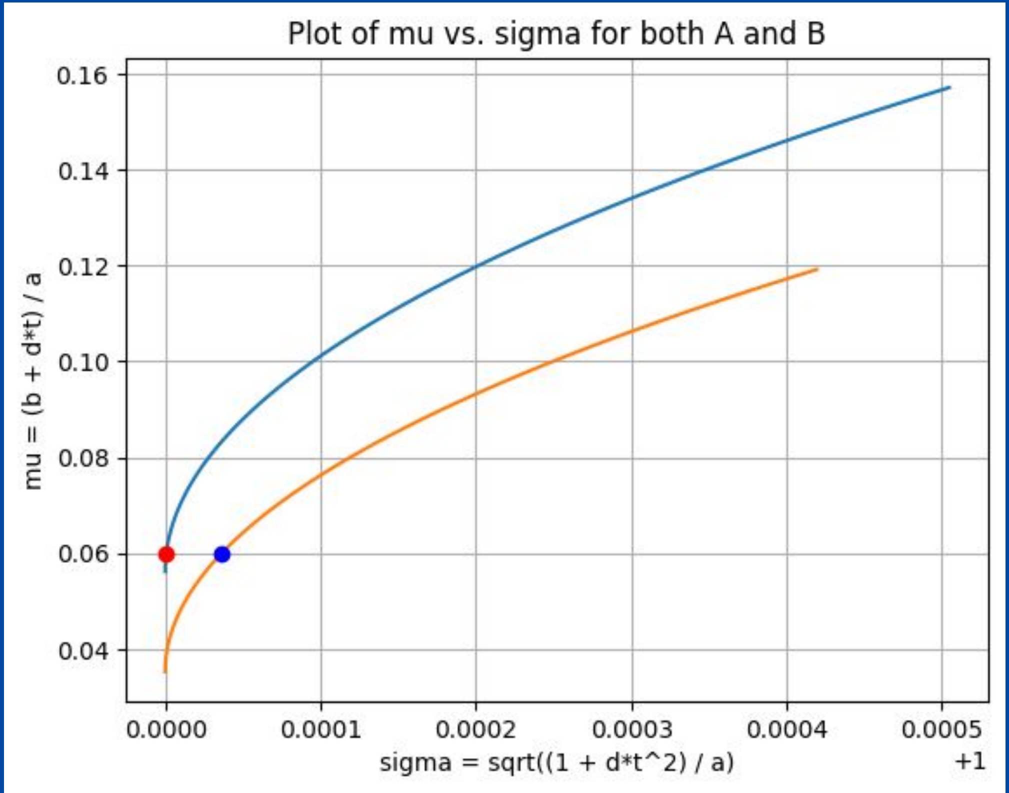


# Extension - adding ILE_UH and varying γ



# Efficient Frontier and Minimum Variance

1. Creating an EF and finding the portfolio with minimum variance that yields a set level of return.
2. Adjusting the portfolio to still yield set returns without short selling.

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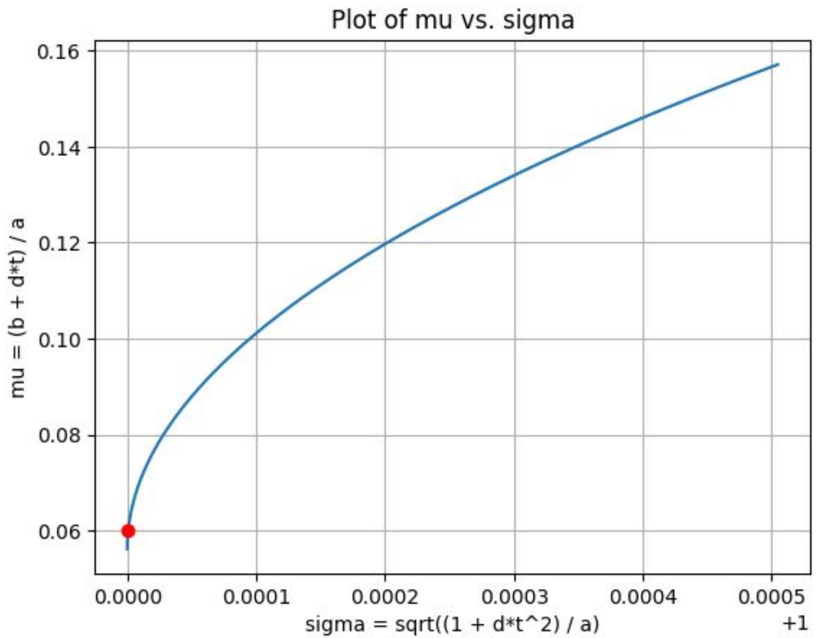
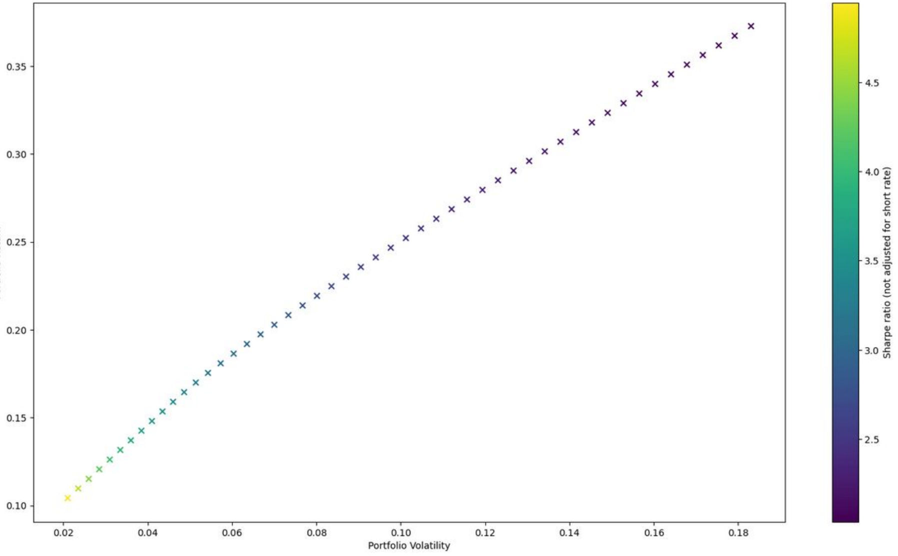
# Efficient Frontier and Minimum Variance

## Period A using both portfolio theory and simulated portfolios:



# Efficient Frontier and Minimum Variance

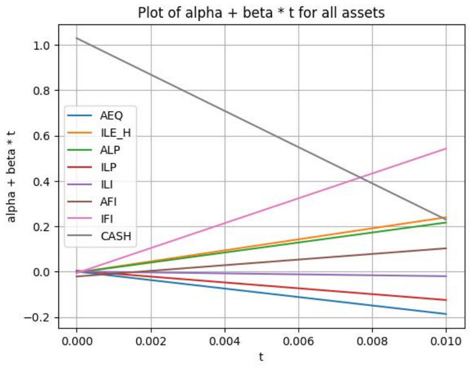
## Period A - Minimized variance for 6% returns: (SD = 0.002891)

| Assets | Optimal Weights |
|---|---|
| Australian Equities (AEQ) | -0.007 |
| International Listed Equity - Hedged (ILE) | 0.006 |
| Australian Listed Property (ALP) | 0.004 |
| International Listed Property (ILP) | -0.001 |
| International Listed Infrastructure (ILP) | 0 |
| Australian Fixed Income | -0.016 |
| International Fixed Income | 0.015 |
| CASH | 0.999 |

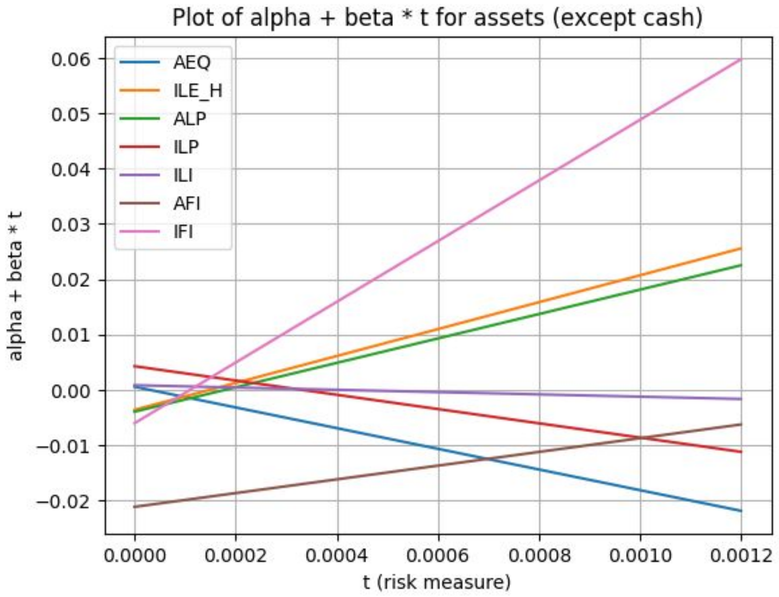
# These are the expected returns for all assets under A

These are the expected returns for every asset when using possible portfolios along the efficient frontier.

There's no scenario without short selling on the EF.

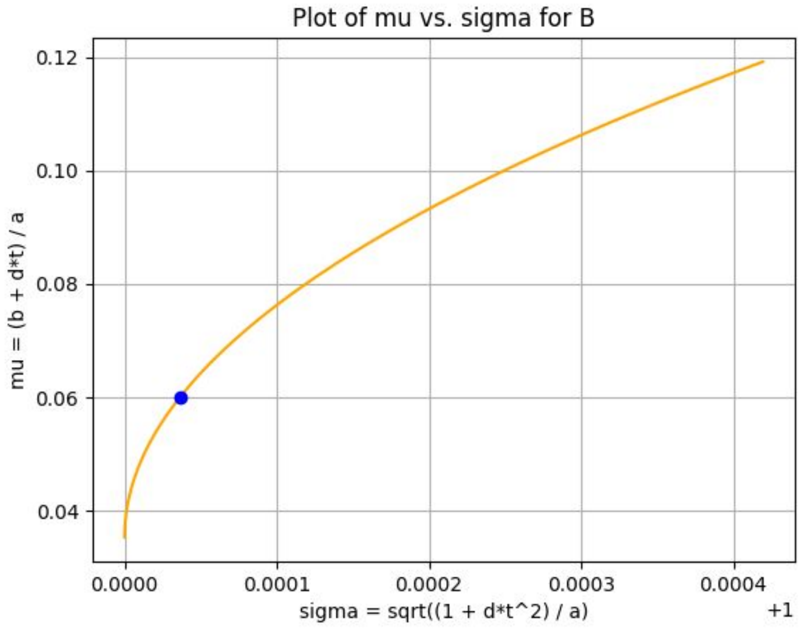


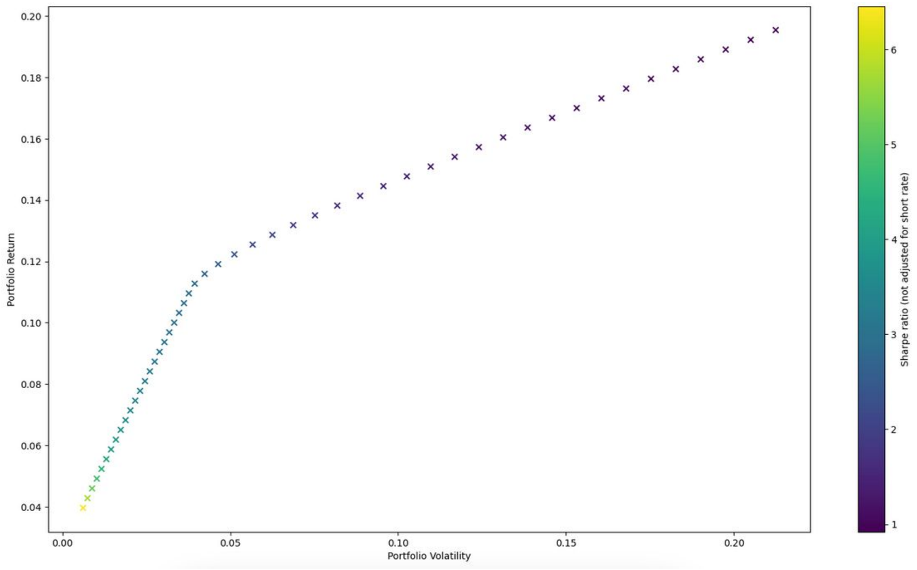

Note: Cash is what you want to get rid of the most when choosing a riskier portfolio.



# Efficient Frontier and Minimum Variance

## Period B using both portfolio theory and simulated portfolios:



# Efficient Frontier and Minimum Variance

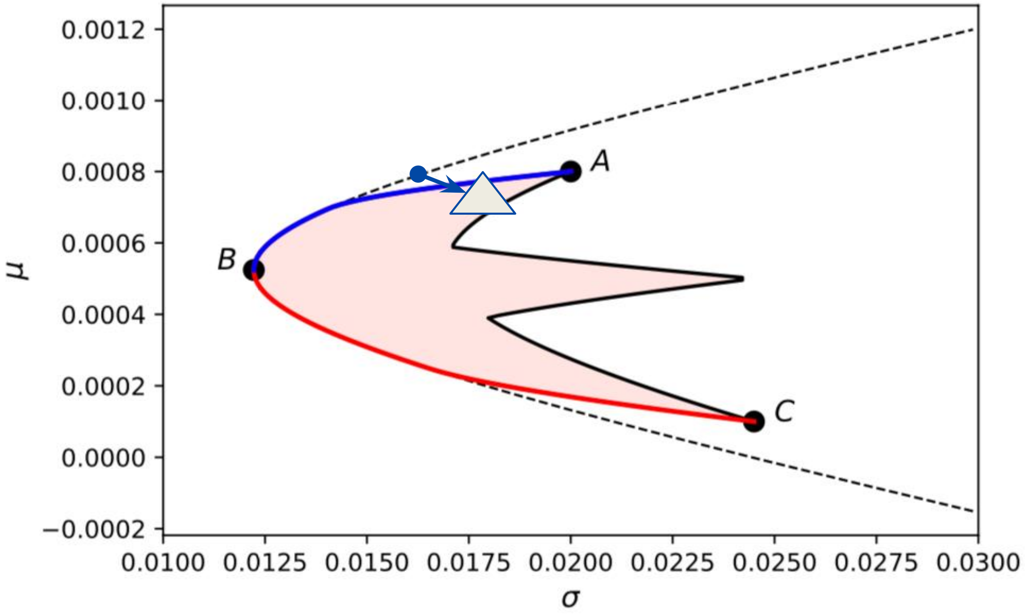
## Period B - Minimized variance for 6% returns: (SD = 0.008661)

| Assets | Optimal Weights |
| --- | --- |
| Australian Equities (AEQ) | 0.029 |
| International Listed Equity - Hedged (ILE) | 0.027 |
| Australian Listed Property (ALP) | -0.02 |
| International Listed Property (ILP) | -0.044 |
| International Listed Infrastructure (ILI) | 0.058 |
| Australian Fixed Income (AFI) | 0.083 |
| International Fixed Income (IFI) | 0.123 |
| CASH | 0.744 |

# Approximating a portfolio:

**For these assets, the efficient frontier always has short selling.**

**We can adjust the EF portfolio to have no short selling, giving us a feasible portfolio close to the EF.**



# Adjusting for no short selling (moving weight from cash)

| Assets | Optimal Weights (A) | Optimal Weights (B) |
|---|---|---|
| Australian Equities (AEQ) | 0 | 0.029 |
| International Listed Equity - Hedged (ILE) | 0.006 | 0.027 |
| Australian Listed Property (ALP) | 0.004 | 0 |
| International Listed Property (ILP) | 0 | 0 |
| International Listed Infrastructure (ILI) | 0 | 0.058 |
| Australian Fixed Income (AFI) | 0 | 0.083 |
| International Fixed Income (IFI) | 0.015 | 0.123 |
| CASH | 0.975 | 0.68 |

**Whilst it wouldn't be the exact minimum variance (but still close), moving the weight from cash ensures the portfolio yields 6% returns.**

**This strategy gives returns of 6.19% and 6.84% respectively.**

# Dynamic portfolio optimization

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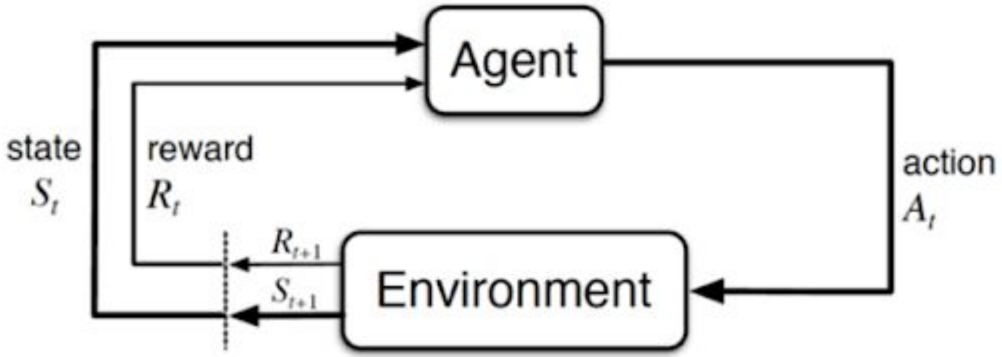
# Finite Markov Decision Process




- An MDP is a 4-tuple (S, A, P, R)
- Issue is the model environment is not known

# Setup for the Constant-α MC

| **Variable** | **Value** |
|---|---|
| α | 0.0002 |
| ε | 0.1 |
| M | 10000 |

- Results are for the period Jan 2016 to Dec 2017
- 50 Portfolios generated at time 1 and time 2

# Results (Max Utility) [risk-aversion = 1]

Optimal dynamic portfolio generated using **Constant-α MC** was:

- *Year 1*: [0.52, 0.08, 0.02, 0.06, 0.11, 0.07, 0.11, 0.02]
- *Year 2*: [0.08, 0.19, 0.13, 0.07, 0.01, 0.02, 0.24, 0.24]
- *Expected returns* = 0.10924

(weights rounded to 2 d.p)

Conclusion: the optimal static portfolio for this time period yielded greater returns (0. 2236).

# Results (Min Variance)

Optimal dynamic portfolio generated using **Constant-α MC** was:

- *Year 1*: [0.02, 0.03, 0.09, 0.01, 0.09, 0.21, 0.43, 0.13]
- *Year 2*: [0.03, 0.11, 0.15, 0.05, 0.21, 0.42, 0.03, 0.01]
- *Expected returns*: 0.10187

(weights rounded to 2 d.p)

Conclusion: yielded greater returns than the static portfolio optimisation.

# Conclusion

- **We constructed static portfolios with maximum utility.**
  - Ext: We demonstrated with differing values of gamma for the utility function
  - Ext: We also included the unhedged International Listed Equity
- **We constructed static portfolios on the EF at 6% returns and then adjusted for no short selling.**
- **Our dynamic portfolio gave the greatest returns with the lowest volatility, therefore giving us our best portfolio.**

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# Thank You!

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