

Algebra Linear

Anabelle Elizabeth Aratijo de Souza
Pedro Henrique Levy Fermينو Ferreira

29/10/2022

1 Exercício 1

Considere as bases do Espaço vetorial R3, A = {(4, 2, 0),(1, 1, 1),(5, 3, 3)} e B = {(1, 2, 1),(1, 5, 2),(1, 0, 1)}. Exiba as matrizes de mudança de base MB→A e MA→B. Escreva também os vetores abaixo nas bases indicadas:

- v = (0, 1, 2)A em B
- v = (1, 3, 1)B em A

Mudança B → A(b1)
x. a1 + y.a2 + z.a3 = b1

x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (1, -2, 1)

4x + y + 5z = 1
2x - y + 3z = -2
y + 3z = 1

4x + y + 5z = 1
2x - y + 3z = -2
6x + 8z = -1

2x - y + 3z = -2
y + 3z = 1
2x + 6z = -1

6x + 8z = -1
2x + 6z = -1(-3)

6x + 8z = -1
- 6x - 18z = 3

- 10z = 2
z = -2/10
z = -1/5

2x + 6z = -1
2x + 6 · (-1/5) = -1

x = 1/10

y + 3z = 1
y + 3(-1/5) = 1
- 1 + 3/5
y = 8/5

Mudança B → A(b2)

4 x+y+5 z=1
2 x=y+3 z=5
y+3 z=2

4 x+y+5 z=1
2 x-y+3 z=5

2 x-y+3 z=5
y+3=2

6 x+8 x=6
2 x+6 z=7(-3)

-10 z=-15
z=3 / 2

2 x+6 ·(3/2) = 7
x = -1

y + 3 · (3/2) = 2
y = -5/2

Mudança B → A(b3)

4x + y + 5z = 1
2x - y + 3z = 0
y + 3z = 1

4x + y = 5z = 1
2x - y + 3z = 0

2x - y + 3z = 0
y + 3z = 1
6x + 8z = 1

2x + 6z = 1
x = -1/10

z = 1/5

y + 3(1/5) = 1
y + 3/5 = 1
y = 2/5

Mudança A → B(a1)

x. b1 + y.b2 + z.b3 = a1

x.(1, -2, 1) + y.(1, 5, 2) + z.(1, 0, 1) = (4, 2, 0)

x + y + z = 4
- 2x + 5y = 2
x + 2y + z = 0

x + y + z = 4(.2)
- 2x + 5y = 2

x + y + z = 4
x + 2y + 2 = 0(-1)
- 2x + 2y + 2z = 8
- 2x + 5y = 2

7y + 2z = 10

x + y + z = 4
- x - 2y - z = 0
- y = 4
y = -4

- 2x + 5(-4) = 2
- 20
- 2x = 2
- 2x = 22
x = -11

7y + 2z = 10
7.(-4) + 2z = 10
z = 19

Mudança A → B(a2)

x + y + z = 1
-2x + 5y = -1
x + 2y + z = 1

x + y + z = 1 .(2)
- 2x + 5y = -1

x + y + z = 1
z = 2y + z = 1

2x + 2y = 2z = 2
- 2x + 5y = -1

7y + 2z = 1
y = 0

7.0 + 2z = 1
z = 1/2

- 2x + 0 = -1
- 2x + 0 = -1
- 2x = -1
x = 1/2

Mudança A → B(a3)

x + y + z = 5
-2x + 5y = 3
x + 2y + z = 3

x + y + z = 5
-2x + 5y = 3

x + y + z = 5
x + 2y + z = 3

7y + 2z = 13
y = -2

7y + 2z = 13
y = -2

7 . (-2) = 2z = 13
z = 27/2

$$\begin{aligned}
 -2x + 5(-2) &= 3 \\
 -2x - 10 &= 13 \\
 -2x &= 13 \\
 x &= -13/2
 \end{aligned}$$

$$\mathbf{M_B} \rightarrow \mathbf{A}: \left[\begin{array}{ccc} \frac{1}{10} & -1 & \frac{-1}{10} \\ \frac{8}{5} & \frac{-5}{2} & \frac{2}{5} \\ \frac{-1}{5} & \frac{3}{2} & \frac{1}{5} \end{array} \right] \cdot \left[\begin{array}{c} 1 \\ 3 \\ -1 \end{array} \right] \rightarrow \left[\begin{array}{ccc} \frac{1}{10}, 1 & -1, 3 & \frac{-1}{10}, (-1) \\ \frac{8}{5}, 1 & \frac{-5}{2}, 3 & \frac{2}{5}, (-1) \\ \frac{-1}{5}, 1 & \frac{3}{2}, 3 & \frac{1}{5}, (-1) \end{array} \right] = \left[\begin{array}{c} \frac{-25}{2} \\ -4 \\ \frac{55}{2} \end{array} \right]$$

$$\mathbf{M_A} \rightarrow \mathbf{B}: \left[\begin{array}{ccc} -11 & \frac{1}{2} & \frac{-13}{2} \\ 4 & 0 & -2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{array} \right] \cdot \left[\begin{array}{c} 0 \\ 1 \\ 2 \end{array} \right] \rightarrow \left[\begin{array}{ccc} -11, 0 & \frac{1}{2}, 1 & \frac{-13}{2}, 2 \\ 4, 0 & 0, 1 & -2, 2 \\ 19, 0 & \frac{1}{2}, 1 & \frac{27}{2}, 2 \end{array} \right] = \left[\begin{array}{c} \frac{-14}{5} \\ \frac{-63}{10} \\ \frac{41}{10} \end{array} \right]$$

2 Exercício 2

Considere o conjunto $S = \{(1, 1, 1, 1, 1), (2, 0, 1, 1, 3), (3, 1, 0, 2, 4), (2, 2, 5, 8, 1), (0, 1, 0, 2, 3)\}$.

- S é LI ou LD?

$$S = \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right] \xrightarrow{I3 \leftarrow I4 - I3} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 2 & 3 & 2 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right]$$

$$I1 \rightarrow I1 - I2 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & -1 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 2 & 3 & 2 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right] \xrightarrow{I3 \rightarrow I3 - I1} \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & -1 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right]$$

$$I2 \rightarrow I2 - I3 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & -1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right] \xrightarrow{I4 \rightarrow I4 - 3I3} \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & -1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & -1 & -1 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right]$$

$$I5 \rightarrow I5 - I1 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & -1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & -1 & -1 & 0 \\ 1 & 1 & 2 & -1 & 4 & 0 \end{array} \right] \xrightarrow{I5 \rightarrow I5 - I4} \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & -1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 0 & -5 & 5 & 0 \end{array} \right]$$

$$I5 \rightarrow I5/5 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & -1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{array} \right] \xrightarrow{I1 \rightarrow I5 + I1} \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$I4 \rightarrow I4 - I1 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{I3 \rightarrow I3 - I5} \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$I3 \leftrightarrow I4 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{I4 \rightarrow I4/3} \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$I2 \rightarrow I2 + I4 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{I3 \rightarrow I3 + I5} \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$I3 \rightarrow I3 + I4 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{I1 \rightarrow 2I2 - I1} \left[\begin{array}{ccccc|c} 2 & -2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$I3 \rightarrow -I3 \left[\begin{array}{ccccc|c} 2 & -2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right] \xrightarrow{I1 \rightarrow 2I3 - I1} \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

R : Portanto, é LD

Forma base do R-espaco vetorial R^5 ?
 R : Não forma base, pois é LD

3 Exercício 3

Considere o conjunto $W = \{(x, y, z, w, t, u) \mid x, y, z, w, t, u \in R \wedge x + y + w + z + t + u = 0 \wedge y - w - z = 0 \wedge w + t - x = 0\} \subseteq R^6$.

Mostre que conjunto W é um subespaço do R-espaco vetorial R^6 .

$$\begin{aligned} t - x &= 0 \\ t &= x \\ y - w - z &= 0 \\ y &= w + z \\ x + y + w + z + t + u &= 0 \rightarrow x + w + z + w + z + x + u = 0 \\ u &= -x - y - w - z - t \rightarrow u = -2x - 2w - 2z \\ W &= \{(x, w + z, z, w, x, -x - w - z - w - z - x)\} \rightarrow \\ W &= \{(x, w + z, z, w, x, -2x - 2w - 2z) \mid x, z, w \in R\} \end{aligned}$$

$$\begin{aligned} \text{I) } 0 &\in W, \text{ para } x = 0, z = 0, w = 0 \\ (w, w, w, w, w, -w) &\rightarrow (x, w + z, z, w, x, -2x - 2w - 2z) \\ &= (0, 0, 0, 0, 0, -0) \\ &= 0 \\ \text{Logo, } 0 &\in W \end{aligned}$$

$$\begin{aligned} \text{II) } u, v &\in W \rightarrow u + v \in W, \text{ sendo:} \\ u &= (u1, u2, u3, u4, u5, -u6) \rightarrow (x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1) \\ v &= (v1, v2, v3, v4, v5, -v6) \rightarrow (x_2, w_2 + z_2, z_2, w_2, x_2, -2x_2 - 2w_2 - 2z_2) \\ u + v &= (x_1 + x_2, (w_1 + z_1) + (w_2 + z_2), z_1 + z_2, w_1 + w_2, x_1 + x_2, (-2x_1 - 2w_1 - 2z_2) + (-2x_2 - 2w_2 - 2z_2)) \\ u + v &= (x_1 + x_2, w_1 + z_1 + w_2 + z_2, z_1 + z_2, w_1 + w_2, x_1 + x_2, -2x_1 - 2x_2 - 2w_1 - 2w_2 - 2z_1 - 2z_2) \\ \text{Logo, } u + v &\in W \end{aligned}$$

$$\begin{aligned} \text{III) } a &\in R, v \in W \rightarrow av \in W, \text{ sendo:} \\ v &= (v1, v2, v3, v4, v5, -v6) \rightarrow (x, w + z, z, w, x, -2x - 2w - 2z) \\ av &= a \cdot (x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1) \\ av &= (a \cdot x_1, a \cdot w_1 + z_1, a \cdot z_1, a \cdot w_1, a \cdot x_1, a \cdot (-2x_1 - 2w_1 - 2z_1)) \\ av &= (ax_1, aw_1 + az_1, aw_1 + az_1, a \cdot (-2x_1 - 2w_1 - 2z_1)) \\ \text{Logo, } av &\in W. \end{aligned}$$

Logo W é subespaço vetorial de R^6 .

- O conjunto $W = \{(x, y, z) \mid x, y, z \in R \wedge x - z = 1 \wedge y + x = 0\}$ é um subespaço vetorial de R^3 ? Esboce graficamente W .

$$\begin{aligned} x - z &= 1 \text{ à } x = 1 + z \\ y + x &= 0 \text{ à } y + 1 + z = 0 \text{ à } y = -1 - z. \\ W &= \{(1 + z, -1 - z, z)\} \end{aligned}$$

$$\begin{aligned} \text{I) } 0 &\in W, \text{ para } z = 0 \\ (1 + z, -1 - z, z) &\rightarrow (1 + 0, -1 - 0, 0) = (1, -1, 0). \\ \text{Logo } 0 &\text{ NÃO pertence a } W \text{ para } z = 0. \text{ Portanto, } W \text{ NÃO é subespaço vetorial.} \end{aligned}$$

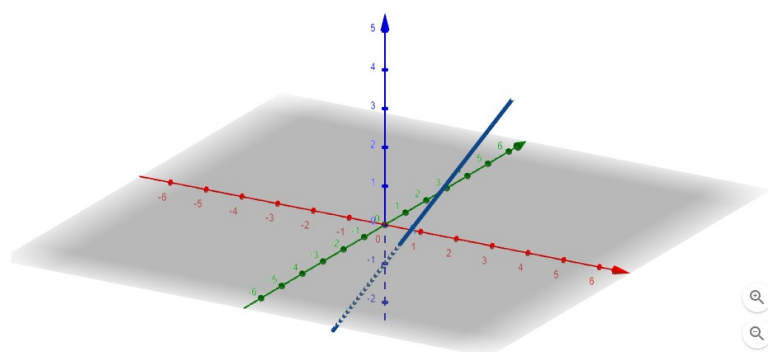


Figure 1: Representação gráfica.

- Invente seu subespaço vetorial em qualquer R^n com n maior igual a 2. Mostre que o conjunto apresentado é de fato um subespaço vetorial. Não vale usar nenhum exemplo da aula ou da prova

$$\begin{aligned} Z &= \{(x, y, z) \mid 2y + z = 0 \wedge x + y = 0\} \\ 2y + z &= 0 \\ x + y &= 0 \\ z &= -2y \\ x &= -y \\ Z &= \{-y, y, -2y\} \\ Z &= \{(-z, z, -z) \mid Z \in R\} \end{aligned}$$

$$\begin{aligned} \text{I) } 0 &\in Z, \text{ para } z = 0 \\ (z, z, z) &\rightarrow (-y, y, -2y) \\ y &\rightarrow 0 \\ (-y, y, -2y) &= (-0, 0, -0) = (0, 0, 0) \\ \text{Logo, } 0 &\in Z \end{aligned}$$

$$\begin{aligned} \text{II) } u, v &\in Z \rightarrow u + v \in Z, \text{ sendo:} \\ u &= (-y1, y1, -2y1) \\ v &= (-y2, y2, -2y2) \\ u + v &= (-y, y, -2y) + (-y2, y2, -2y2) \\ u + v &= (-y1 - y2, y1 + y2, -2y - 2y2) \\ \text{Logo, } u + v &\in Z \end{aligned}$$

$$\begin{aligned} \text{III) } a &\in R, v \in Z \rightarrow av \in Z. \text{ Sendo:} \\ v &= (v1, v2, v3) \rightarrow (-y, y, -2y) \\ av &= a \cdot (-y, y, -2y) \\ av &= (a \cdot (-y), a \cdot y, a \cdot (-2y)) \end{aligned}$$

$$l6 \rightarrow -5/24.l6$$

$$l6 \rightarrow l6 - l5$$

$$l6 \rightarrow -4/5.l$$

$$l7 \rightarrow l7 - l6$$

$$l6 \rightarrow l6 - (1/6.l7)$$

$$l4 \rightarrow l4 - (16/5.l7)$$

$$12 \rightarrow l2 - (3.17)$$

$$11 \rightarrow l1 - l6$$

$$13 \rightarrow l3 - (4/3.l$$

$$l3 \rightarrow l3 - (1/3.l$$

$$I2 \rightarrow I2 - (4.I3) \left[\begin{array}{cccccc|c} -1 & 0 & -2 & 0 & 0 & 0 & 0 & \frac{12a+5e+30b-10g+5c}{25} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -480g-1070e-3070b+5883a+1330c+1565d+195f \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$I1 \rightarrow I1 + (2.I3) \left[\begin{array}{cccccc|c} -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{330g+655e-6048a+3080b-1295c+470d-90f}{75} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -480g-1070e-3070b+5883a+1330c+1565d+195f \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

5 Coordenadas

Portanto o conjunto forma base para o espaço vetorial R7 e as coordenadas são B = $\frac{216}{5}$; -23; 21; $-\frac{241}{5}$; $\frac{217}{10}$; 15; $\frac{19}{5}$

$$I1 \rightarrow -1.I1 \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{330g+655e-6048a+3080b-1295c+470d-90f}{75} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -480g-1070e-3070b+5883a+1330c+1565d+195f \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$