

1 Exercício 1

Considere as bases do Espaço vetorial R3, A = {(4, 2, 0),(1, 1, 1),(5, 3, 3)} e B = {(1, 2, 1),(1, 5, 2),(1, 0, 1)}. Exiba as matrizes de mudança de base MB→A e MA→B. Escreva também os vetores abaixo nas bases indicadas:

- v = (0, 1, 2)A em B
- v = (1, 3, 1)B em A

Mudança B → A(b1)
x. $a_1 + y.a_2 + z.a_3 = b_1$

x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (1, -2, 1)

$$\begin{matrix} 4x + y + 5z = 1 \\ 2x - y + 3z = -2 \\ y + 3z = 1 \end{matrix}$$

$$\begin{matrix} 4x + y + 5z = 1 \\ 2x - y + 3z = -2 \\ 6x + 8z = -1 \end{matrix}$$

$$\begin{matrix} 2x - y + 3z = -2 \\ y + 3z = 1 \\ 2x + 6z = -1 \end{matrix}$$

$$\begin{matrix} 6x + 8z = -1 \\ 2x + 6z = -1(-3) \end{matrix}$$

$$\begin{matrix} 6x + 8z = -1 \\ -6x - 18z = 3 \end{matrix}$$

$$\begin{matrix} -10z = 2 \\ z = -2/10 \\ z = -1/5 \end{matrix}$$

$$\begin{matrix} 2x + 6z = -1 \\ 2x + 6 \cdot (-1/5) = -1 \end{matrix}$$

x = 1/10

$$\begin{matrix} y + 3z = 1 \\ y + 3(-1/5) = 1 \\ -1 + 3/5 \\ y = 8/5 \end{matrix}$$

Mudança B → A(b2)

$$\begin{matrix} 4 \text{ x} + \text{y} + 5 \text{ z} = 1 \\ 2 \text{ x} = \text{y} + 3 \text{ z} = 5 \\ \text{y} + 3 \text{ z} = 2 \end{matrix}$$

$$\begin{matrix} 4 \text{ x} + \text{y} + 5 \text{ z} = 1 \\ 2 \text{ x} - \text{y} + 3 \text{ z} = 5 \end{matrix}$$

$$\begin{matrix} 2 \text{ x} - \text{y} + 3 \text{ z} = 5 \\ \text{y} + 3 = 2 \end{matrix}$$

$$\begin{matrix} 6 \text{ x} + 8 \text{ x} = 6 \\ 2 \text{ x} + 6 \text{ z} = 7(-3) \end{matrix}$$

$$\begin{matrix} -10 \text{ z} = -15 \\ \text{z} = 3 / 2 \end{matrix}$$

$$\begin{matrix} 2 \text{ x} + 6 \cdot (3/2) = 7 \\ \text{x} = -1 \end{matrix}$$

$$\begin{matrix} y + 3 \cdot (3/2) = 2 \\ y = -5/2 \end{matrix}$$

Mudança B → A(b3)

$$\begin{matrix} 4x + y + 5z = 1 \\ 2x - y + 3z = 0 \\ y + 3z = 1 \end{matrix}$$

$$\begin{matrix} 4x + y = 5z = 1 \\ 2x - y + 3z = 0 \end{matrix}$$

$$\begin{matrix} 2x - y + 3z = 0 \\ y + 3z = 1 \\ 6x + 8z = 1 \end{matrix}$$

$$\begin{matrix} 2x + 6z = 1 \\ \text{x} = -1/10 \end{matrix}$$

z = 1/5

$$\begin{matrix} y + 3(1/5) = 1 \\ y + 3/5 = 1 \\ y = 2/5 \end{matrix}$$

Mudança A → B(a1)

x. b₁ + y.b₂ + z.b₃ = a₁

x.(1, -2, 1) + y.(1, 5, 2) + z.(1, 0, 1) = (4, 2, 0)

$$\begin{matrix} x + y + z = 4 \\ -2x + 5y = 2 \\ x + 2y + z = 0 \end{matrix}$$

$$\begin{matrix} x + y + z = 4(2) \\ -2x + 5y = 2 \end{matrix}$$

$$\begin{matrix} x + y + z = 4 \\ x + 2y + 2 = 0(-1) \end{matrix}$$

$$\begin{matrix} -2x + 2y + 2z = 8 \\ -2x + 5y = 2 \end{matrix}$$

$$7y + 2z = 10$$

$$\begin{matrix} x + y + z = 4 \\ -x - 2y - z = 0 \\ -y = 4 \\ y = -4 \end{matrix}$$

$$\begin{matrix} -2x + 5(-4) = 2 \\ -20 \\ -2x = 2 \\ -2x = 22 \\ \text{x} = -11 \end{matrix}$$

$$\begin{matrix} 7y + 2z = 10 \\ 7.(-4) + 2z = 10 \\ \text{z} = 19 \end{matrix}$$

Mudança A → B(a2)

$$\begin{matrix} \text{x} + \text{y} + \text{z} = 1 \\ -2\text{x} + 5\text{y} = -1 \\ \text{x} + 2\text{y} + \text{z} = 1 \end{matrix}$$

$$\begin{matrix} \text{x} + \text{y} + \text{z} = 1 \cdot (2) \\ -2x + 5y = -1 \end{matrix}$$

$$\begin{matrix} x + y + z = 1 \\ z = 2y + z = 1 \end{matrix}$$

$$\begin{matrix} 2x + 2y = 2z = 2 \\ -2x + 5y = -1 \end{matrix}$$

$$\begin{matrix} 7y + 2z = 1 \\ y = 0 \end{matrix}$$

$$\begin{matrix} 7.0 + 2z = 1 \\ \text{z} = 1/2 \end{matrix}$$

$$\begin{matrix} -2x + 0 = -1 \\ -2x + 0 = -1 \\ -2x = -1 \\ \text{x} = 1/2 \end{matrix}$$

Mudança A → B(a3)

$$\begin{matrix} \text{x} + \text{y} + \text{z} = 5 \\ -2\text{x} + 5\text{y} = 3 \\ \text{x} + 2\text{y} + \text{z} = 3 \end{matrix}$$

$$\begin{matrix} \text{x} + \text{y} + \text{z} = 5 \\ -2\text{x} + 5\text{y} = 3 \end{matrix}$$

$$\begin{matrix} \text{x} + \text{y} + \text{z} = 5 \\ \text{x} + 2\text{y} + \text{z} = 3 \end{matrix}$$

$$\begin{matrix} 7y + 2z = 13 \\ y = -2 \end{matrix}$$

$$\begin{matrix} 7y + 2z = 13 \\ y = -2 \end{matrix}$$

$$\begin{matrix} 7 \cdot (-2) = 2z = 13 \\ \text{z} = 27/2 \end{matrix}$$

-2x + 5(-2) = 3

$$l7 \rightarrow l7 - l4$$

$l6 \rightarrow -5/24.$

$$l6 \rightarrow l6 - l5$$

$l6 \rightarrow -4/5.l6$

$$l7 \rightarrow l7 - l6$$

$$30 - 10 = 20$$

$$l4 \rightarrow l4 - (10$$

12→ 12→

$11 \rightarrow l1 -$

13→ 13 -

$$l3 \rightarrow l3 - (1/3.l4) \left[\begin{array}{c|c} -1 & 0 & -2 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{c} \frac{12a+5c+30b-10g+5c}{25} \\ \frac{74e+169b-258a-76c-135d+32g-15f}{5} \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$l2 \rightarrow l2 - l4 \left[\begin{array}{c|c} -1 & 0 & -2 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{c} \frac{12a+5c+30b-10g+5c}{25} \\ \frac{16g+14c+194b-419a-86c+167d+f}{5} \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$l2 \rightarrow l2 - (4.l3) \left[\begin{array}{c|c} -1 & 0 & -2 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{c} \frac{12a+5c+30b-10g+5c}{25} \\ -\frac{480g-1070c-3070b+5883a+1330c+1565d+195f}{75} \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$l1 \rightarrow l1 + (2.l3) \left[\begin{array}{c|c} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{c} \frac{330g+655c-6048a+3080b-1295c+470d-90f}{75} \\ -\frac{480g-1070c-3070b+5883a+1330c+1565d+195f}{75} \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$l1 \rightarrow -1.l1 \left[\begin{array}{c|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{c} \frac{330g+655c-6048a+3080b-1295c+470d-90f}{75} \\ -\frac{480g-1070c-3070b+5883a+1330c+1565d+195f}{75} \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

3 Coordenadas

Portanto o conjunto forma base para o espaço vetorial R7 e as coordenadas são B = $\frac{216}{5}$; -23; 21; $-\frac{241}{5}$; $\frac{217}{10}$; 15; $\frac{19}{5}$