# Algebra Linear

Pedro Henrique Levy Fermino Ferreira

29/10/2022

## 1 Exercício 2

Considere o conjunto S = {(1, 1, 1, 1, 1),(2, 0, 1, 1, 3),(3, 1, 0, 2, 4),(2, 2, 5, 8, 1),(0, 1, 0, 2, 3)}.

- S é LI ou LD?

$$\text{S} = \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & \text{-1} & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & \text{-1} & 3 & 0 \end{array}\right] \mathbf{l3}\leftarrow l4 - l3 \left[\begin{array}{ccccc|c} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{2} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{2} & \mathbf{2} & \mathbf{3} & \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{8} & \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{3} & \mathbf{4} & \mathbf{-1} & \mathbf{3} & \mathbf{0} \end{array}\right]$$

$$\text{l1}\rightarrow l1 - l2 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & \text{-1} & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 2 & 3 & 2 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & \text{-1} & 3 & 0 \end{array}\right] l3 \rightarrow l3 - l1 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & \text{-1} & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & \text{-1} & 3 & 0 \end{array}\right]$$

$$l2 \rightarrow l2 - l3 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & \text{-1} & 0 \\ 1 & 0 & 1 & \text{-1} & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & \text{-1} & 3 & 0 \end{array}\right] l4 \rightarrow l4 - 3l3 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & \text{-1} & 0 \\ 1 & 0 & 1 & \text{-1} & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & \text{-1} & \text{-1} & 0 \\ 1 & 3 & 4 & \text{-1} & 3 & 0 \end{array}\right]$$

$$l5 \rightarrow l5 - l1 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & \text{-1} & 0 \\ 1 & 0 & 1 & \text{-1} & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & \text{-1} & \text{-1} & 0 \\ 1 & 1 & 2 & \text{-1} & 4 & 0 \end{array}\right] l5 \rightarrow l5 - l4 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & \text{-1} & 0 \\ 1 & 0 & 1 & \text{-1} & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & \text{-1} & \text{-1} & 0 \\ 0 & 0 & 0 & \text{-0} & 5 & 0 \end{array}\right]$$

$$l5 \rightarrow l5/5 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & \text{-1} & 0 \\ 1 & 0 & 1 & \text{-1} & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & \text{-1} & \text{-1} & 0 \\ 0 & 0 & 0 & \text{-0} & 1 & 0 \end{array}\right] l1 \rightarrow l5 + l1 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & \text{-1} & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & 1 & 2 & \text{-1} & \text{-1} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$l4 \to l4 - l1 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] l3 \to l3 - l5 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$l3 \leftrightarrow 14 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] l4 \to l4/3 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$l2 \to 12 + l4 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] l3 \to l3 + l5 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$l3 \to 13 + l4 \left[\begin{array}{ccccc|c} 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] l1 \to 2l2 - l1 \left[\begin{array}{ccccc|c} 2 & -2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$l3 \to -l3 \left[\begin{array}{ccccc|c} 2 & -2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] l1 \to 2l3 - l1 \left[\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

*R: Portanto, é LD*

Forma base do R-espaço vetorial R5?
*R: Não forma base, pois é LD*

## 2 Exercício 3

Considere o conjunto W = {(x, y, z, w, t, u) | $x, y, z, w, t, u \in R \wedge x+y+w+z+t+u = 0 \wedge y-w-z = 0 \wedge w+t-x = 0\} \subseteq R^6$.

Mostre que conjunto W é um subespaço do R-espaço vetorial $R^6$.

t − x = 0
t = x
y − w − z = 0
y = w + z
x + y + w + z + t + u = 0 → $x + w + z + w + z + x + u = 0$
u = -x -y -w -z -t → $u = -2x - 2w - 2z$
W = {(x, w + z, z, w, x, - x − w − z − w − z − x)} →

W = {(x, w + z, z, w, x, - 2x − 2w − 2z)| $x, z, w \in R$}

I) $0 \in W para x = 0 z = 0 w = 0$
(w, w, w, w, w, -w) → $(x, w + z, z, w, x, -2x–2w–2z)$
= (0, 0, 0, 0, 0, -0)
= 0
Logo, $0 \in W$

II) u, v $\in W \rightarrow u + v \in W, sendo:$
u = (u1, u2, u3, u4, u5, -u6) → $(x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1–2w_1–2z_1)$
v = (v1, v2, v3, v4, v5, -v6) → $(x_2, w_2 + z_2, z_2, w_2, x_2, -2x_2–2w_2–2z_2)$
u + v = (x$_1 + x_2, (w_1 + z_1) + (w_2 + z2), z_1 + z_2, w_1 + w_2, x_1 + x_2, (-2x_1 - 2w_1 - 2z_2) + (-2x_2 - 2w_2 - 2z_2))$
u + v = (x$_1 + x_2, w_1 + z_1 + w_2 + z_2, z_1 + z_2, w_1 + w_2, x_1 + x_2, -2x_1 - 2x_2, -2w_1 - 2w_2, -2z_1 - 2z_2)$
Logo, u + v $\in W$

III) a $\in R, v \in W \rightarrow av \in W, sendo:$
v = (v1, v2, v3, v4, v5, -v6) → $(x, w + z, z, w, x, -2x–2w–2z)$
av = a . (x$_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1)$
av = (a. x$_1, a.w_1 + z_1, a.z_1, a.w_1, a.x_1, a. - 2x_1 - 2w_1 - 2z_1)$
av = (ax$_1, aw_1w_2, az_1, aw_1, ax_1, a - 2x_1 - 2w_1 - 2z_1)$
Logo, av $\in W$.

Logo W é subespaço vetorial de R6.

- O conjunto W = {(x, y, z) | $x, y, z \in R \wedge x - z = 1 \wedge y + x = 0$}
  é um subsespaço vetorial de R3? Esboce graficamente W.

x − z = 1 à x = 1 + z
y + x = 0 à y + 1 + z = 0 à y = -1 − z.
W = {(1 + z, -1 - z, z)}

I) $0 \in W, para z = 0$
(1 + z, - 1 − z, z) → $(1 + 0, -1–0, 0) = (1, -1, 0)$.
Logo 0 NÃO pertence a W para z = 0. Portanto, W NÃO é subespaço vetorial.

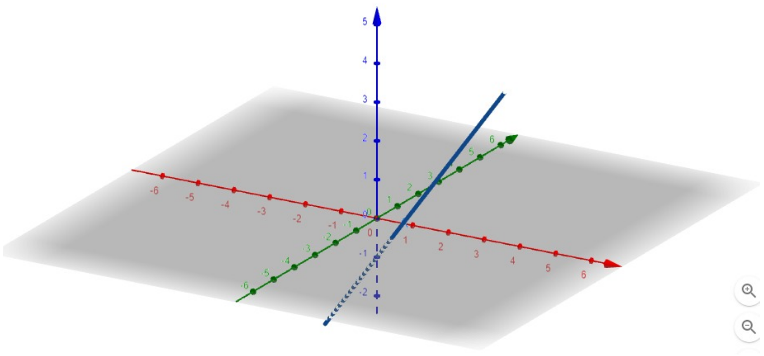


Figure 1: Representação gráfica.

- Invente seu subespaço vetorial em qualquer R n com n maior igual a 2. Mostre que o conjunto apresentado é de fato um subespaço vetorial. Não vale usar nenhum exemplo da aula ou da prova

Z = {(x, y, z) | $2y + z = 0 \wedge x + y = 0$}
2y + z = 0
x + y = 0
z = -2y
x = -y

Z= (-y, y, -2y)
Z = {(-z, z, -z) | $Z \in R$}

I)0 $\in Z, para z = 0$
(z, z, z) $\rightarrow (-y, y, -2y)$
y $\rightarrow$ 0
(-y, y, -2y) = (-0, 0, -0) = (0, 0, 0)
Logo, 0 $\in Z$

II) u, v $\in Z \rightarrow u + v \in Z, sendo:$
u = (-y1, y1, -2y1)
v = (-y2, y2, -2y2)
u + v = (-y, y, -2y) + (-y2, y2, -2y2)
u + v = (-y1-y2, y1+y2, -2y-2y2)
Logo, u + v $\in Z$

III) a $\in R, v \in Z \rightarrow av \in Z. Sendo:$
v = (v1,v2,v3) $\rightarrow (-y, y, -2y)$
a.v = a . (-y, y, -2y)
a.v = (a . (-y), a . y, a . (-2y))
a.v = (-ay, ay, -a2y)
Logo, av $\in Z$

Logo Z é subespaço vetorial de R3