### Algebra Linear

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#### 1 Exercício 1

Considere as bases do Respaço vetorial R3,  $A = \{(4, 2, 0), (1, 1, 1), (5, 3, 3)\}$  e  $B = \{(1, 2, 1), (1, 5, 2), (1, 0, 1)\}$ . Exiba as matrizes de mudança de base MB $\rightarrow$ A e MA $\rightarrow$ B. Escreva também os vetores abaixo nas bases indicadas:  $\mathbf{v} = (0, 1, 2)$ A em B

• v = (1, 3, 1)B em A

Mudança B  $\rightarrow$  A(b1)x.  $a_1 + y \cdot a_2 + z \cdot a_3 = b_1$ 

x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (1, -2, 1)

4x + y + 5z = 12x - y + 3z = -2

y + 3z = 1

4x + y + 5z = 12x - y + 3z = -2

6x + 8z = -1

2x - y + 3z = -2y + 3z = 1

2x + 6z = -1

6x + 8z = -12x + 6z = -1(-3)

6x + 8z = -1

-6x - 18z = 3

-10z = 2z = -2/10

z = -1/5

2x + 6z = -1 $2x + 6 \cdot (-1/5) = -1$ 

 $\mathbf{x} = \mathbf{1}/\mathbf{10}$ 

y + 3z = 1y + 3(-1/5) = 1

y + 3(-1/3) = -1 + 3/5y = 8/5

Mudança B ightarrow A(b2)

### Mudança B ightarrow A(b3)

4x + a + 5x = 1

4x + y + 5z = 12x - y + 3z = 0

y + 3z = 1

4x + y = 5z = 12x - y + 3z = 0

2x - y + 3z = 0

y + 3z = 16x + 8z = 1

2x + 6z = 1 $\mathbf{x} = -1/10$ 

 $\mathbf{z} = \mathbf{1/5}$ 

y + 3(1/5) = 1 y + 3/5 = 1y = 2/5

Mudança A  $\rightarrow$  B(a1)x.  $\mathbf{b}_1 + y.b_2 + z.b_3 = a_1$ 

#### x.(1, -2, 1) + y.(1, 5, 2) + z.(1, 0, 1) = (4, 2, 0)

x + y + z = 4-2x + 5y = 2

x + 2y + z = 0x + y + z = 4(.2)

x + y + z = 4x + 2y + 2 = 0(-1)

-2x + 5y = 2

-2x + 2y + 2z = 8

-2x + 2y + 2z = -2x + 5y = 2

7y + 2z = 10x + y + z = 4

-x - 2y - z = 0-y = 4

 $\mathbf{y} = -\mathbf{4}$ -2x + 5(-4) = 2

-2x + 5(-4) = -20 -2x = 2

-2x = 22 $\mathbf{x} = -11$ 

7y + 2z = 107.(-4) + 2z = 10 $\mathbf{z} = \mathbf{19}$ 

# Mudança A ightarrow B(a2)

x + y + z = 1-2x + 5y = -1

 $\mathbf{x} + 2\mathbf{y} + \mathbf{z} = 1$   $\mathbf{x} + \mathbf{y} + \mathbf{z} = 1 \cdot (2)$  -2x + 5y = -1

x + y + z = 1z = 2y + z = 1

2x + 2y = 2z = 2-2x + 5y = -1

7y + 2z = 1

 $\mathbf{y} = \mathbf{0}$ 

7.0 + 2z = 1 $\mathbf{z} = \mathbf{1/2}$ 

-2x + 0 = -1<br/>-2x + 0 = -1<br/>-2x = -1

 $\mathbf{x} = \mathbf{1/2}$ 

## Mudança A $\rightarrow$ B(a3)

x + y + z = 5 -2x + 5y = 3x + 2y + z = 3

x + y + z = 5-2x + 5y = 3

x + y + z = 5x + 2y + z = 3

7y + 2z = 13y = -2

7y + 2z = 13y = -2

 $7 \cdot (-2) = 2z = 13$ z = 27/2

z = 27/2

-2x + 5(-2) = 3 -2x - 10 = 13 -2x = 13x = -13/2

$\frac{1}{10}$ , $1  -1$ , $3  \frac{-1}{10}$ , $(-1)$ $\left[  \frac{-14}{5}  \right]$	
$\frac{8}{5} \cdot 1  \frac{-5}{2} \cdot 3  \frac{2}{5} \cdot (-1) = \begin{vmatrix} -63 \\ 10 \end{vmatrix}$	
$\frac{-1}{5}$ *! $\frac{3}{2}$ *3 $\frac{1}{5}$ * $(-1)$ $\frac{41}{10}$	
	$\frac{8}{5} \cdot 1  \frac{-5}{2} \cdot 3  \frac{2}{5} \cdot (-1) = \begin{bmatrix} \frac{-63}{10} \end{bmatrix}$

		$\frac{1}{2}$	$\frac{-13}{2}$		0			$\frac{1}{2} * 1$	$\frac{-13}{2}$ *2		$\frac{-25}{2}$	
$\mathbf{M}_A  o_B$ :	4	0	-2		1	$\rightarrow$	4*0	0*1	$-2_{*}2$	=	-4	
	19	$\frac{1}{2}$	$\frac{27}{2}$		2		19*0	$\frac{1}{2} * 1$	$\frac{27}{2} * 2$		$\frac{55}{2}$	
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#### 2 Exercício 4

$\text{Início} \left[ \begin{array}{cccccccccccccccccccccccccccccccccccc$
$l2 \rightarrow l2 + l1 \begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 & a \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 1 & -1 & 1 & 0 & 2 & -1 & 2 & c \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 & d \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & f \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & g \end{bmatrix}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$16 \rightarrow l6 + l1 \begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 & a \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & d+a \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & g \end{bmatrix} l7 \rightarrow l7 + l1 \begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 & a \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & d+a \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{bmatrix}$
$l1 \rightarrow l1 - l2 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & d+a \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{bmatrix} l4 \rightarrow l4/2 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 1 & 3/2 & 0 & 0 & 0 & 3/2 & d+a/2 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{bmatrix}$
$l4 \rightarrow l4 - l2 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 0 & -5/2 & 0 & -1 & 0 & -3/2 & d+3a-2b/2 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{bmatrix} l5 \rightarrow l5 - l2 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 0 & -5/2 & 0 & -1 & 0 & -3/2 & d+3a-2b/2 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & e-b+a \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{bmatrix}$
$l7 \rightarrow l7 - l2 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 0 & -5/2 & 0 & -1 & 0 & -3/2 & d+3a-2b/2 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & e-b+a \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{bmatrix} \\ l3 \rightarrow l3/3 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & -5/2 & 0 & -1 & 0 & -3/2 & d+3a-2b/5 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & e-b+a \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & e-b+a \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{bmatrix}$
$l4 \rightarrow -2.5.l4 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 1 & 0 & 2/5 & 0 & 3/5 & -d+3a-2b/5 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & e-b+a/5 \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{bmatrix} \\ l5 \rightarrow l5/5 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 1 & 0 & 2/5 & 0 & 3/5 & -d+3a-2b/5 \\ 0 & 0 & -1 & 1/5 & 0 & -1/5 & -1/5 & e-b+a/5 \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & b+a/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{bmatrix}$
$l6 \rightarrow l6/3 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 \\ 0 & 0 & 1 & 0 & 2/5 & 0 & 3/5 \\ 0 & 0 & -1 & 1/5 & 0 & -1/5 & -1/5 \\ 0 & 0 & 1 & 2/3 & 2/3 & 0 & 2/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 \\ \end{bmatrix} \begin{array}{c} 2a+b \\ b+a \\ c+a/3 \\ e-b+a/5 \\ b+a/3 \\ 0 & 0 & -1 & 1/5 & 0 & -1/5 & -1/5 \\ 0 & 0 & 1 & 2/3 & 2/3 & 0 & 2/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 \\ \end{bmatrix} \\ l4 \rightarrow l4 - l3 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 & -3d-14a+6b-3c \\ 0 & 0 & -1 & 1/5 & 0 & -1/5 & -1/5 & e-b+a/5 \\ 0 & 0 & 1 & 2/3 & 2/3 & 0 & 2/3 & f+a/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \\ \end{bmatrix}$
$l5 \rightarrow l5 + l3 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 \\ 0 & 0 & 1 & 2/3 & 2/3 & 0 & 2/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 \\ 0 & 0 & 0 & 1 & 2/3 & 2/3 & 0 & 2/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1/3 & -1/5 & 22/15 \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 \\ 0 & 0 & 0 & -1/3 & -1/5 & 22/15 \\ 0 & 0 & 0 & -1/3 & -1/5 & 22/15 \\ 0 & 0$
$l7 \rightarrow l7 + l3 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 & \frac{-3d-14a+6b-3c}{15} \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 & \frac{8a+3e-3b+3c}{15} \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 & \frac{5a+3e-3b+3c}{15} \\ 0 & 0 & 0 & 4/3 & 10/3 & 2 & 11/3 & \frac{3g+7a-3b+c}{3} \end{bmatrix} \\ l4 \rightarrow 3.l4 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & 1 & 1/45 & 0 & 16/5 & \frac{-3d-14a+6b-3c}{3} \\ 0 & 0 & 0 & 1 & 1/45 & 0 & 16/5 & \frac{-3d-14a+6b-3c}{3} \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 & \frac{5a+3e-3b+3c}{15} \\ 0 & 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 & \frac{5c-3}{3} \\ 0 & 0 & 0 & 4/3 & 10/3 & 2 & 11/3 & \frac{3g+7a-3b+c}{3} \end{bmatrix}$
$l5 \rightarrow 15/8.l5 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-3c}{8} \\ 0 & 0 & 0 & 1 & 5/2 & -3/8 & 11/4 & \frac{8a+3e-3b+3c}{8} \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 & f-c/3 \\ 0 & 0 & 0 & 4/3 & 10/3 & 2 & 11/3 & \frac{3g+7a-3b+c}{3} \end{bmatrix} \\ l6 \rightarrow 3.l6 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-3c}{5} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-3c}{5} \\ 0 & 0 & 0 & 1 & 5/2 & -3/8 & 11/4 & \frac{8a+3e-3b+3c}{8} \\ 0 & 0 & 0 & 1 & -2 & 0 & -3 & f-c \\ 0 & 0 & 0 & 4/3 & 10/3 & 2 & 11/3 & \frac{3g+7a-3b+c}{3} \end{bmatrix}$
$l5 \rightarrow l5 - l4 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-}{5} \\ 0 & 0 & 0 & 0 & -3/10 & -3/8 & -9/20 \\ 0 & 0 & 0 & 1 & -2 & 0 & -3 & f-c \\ 0 & 0 & 0 & 1 & 5/2 & 3/2 & 11/4 \end{bmatrix} l6 \rightarrow l6 - l4 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-}{5} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-}{5} \\ 0 & 0 & 0 & 0 & -3/10 & -3/8 & -9/20 & \frac{-72a+13c-+55b+15c-24d}{40} \\ 0 & 0 & 0 & 0 & -24/5 & 0 & -31/5 & f-c \\ 0 & 0 & 0 & 1 & 5/2 & 3/2 & 11/4 & \frac{3g+7a-3b+c}{4} \end{bmatrix}$
$l7 \rightarrow l7 - l4 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 &   & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 &   & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{3d-14a+6b-5c}{5} \\ 0 & 0 & 0 & 0 & -3/10 & -3/8 & -9/20 &   & \frac{-72a+15c+33b-15c-24d}{40} \\ 0 & 0 & 0 & 0 & -24/5 & 0 & -31/5 &   & \frac{5f-10c-3d-14a+6b}{5} \\ 0 & 0 & 0 & 0 & -3/10 & 3/2 & -9/20 &   & \frac{15g-21a+9b-15c-12d}{20} \end{bmatrix} \right] \\ = -10/3.l4 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 &   & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 &   & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{3d-14a+6b-5c}{5} \\ 0 & 0 & 0 & 1 & 5/4 & 3/2 &   & -\frac{24a+5c+11b-5c-8d}{4} \\ 0 & 0 & 0 & 0 & -24/5 & 0 & -31/5 &   & \frac{5f-10c-3d-14a+6b}{5} \\ 0 & 0 & 0 & 0 & -24/5 & 0 & -31/5 &   & \frac{5f-10c-3d-14a+6b}{5} \\ 0 & 0 & 0 & 0 & -3/10 & 3/2 & -9/20 &   & \frac{15g-21a+9b-15c-12d}{20} \\ \end{bmatrix}$

2

$l6 \rightarrow -5/24.l6 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 &   & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 &   & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{-3d-14a+6b-5c}{5} \\ 0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 &   & -\frac{-24a+5e+11b-5c-8d}{4} \\ 0 & 0 & 0 & 0 & 1 & 0 & 31/24 &   & -\frac{5f-10c-3d-14a+6b}{24} \\ 0 & 0 & 0 & 0 & -3/10 & 3/2 & -9/20 &   & \frac{15g-21a+9b-15c-12d}{20} \end{bmatrix} \end{bmatrix} \\ I7 \rightarrow -10/3.l7 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 &   & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 &   & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{3d-14a+6b-5c}{3} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{3d-14a+6b-5c}{4} \\ 0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 &   & -\frac{24a+5e+11b-5c-8d}{4} \\ 0 & 0 & 0 & 0 & 1 & 0 & 31/24 &   & -\frac{5f-10c-3d-14a+6b}{24} \\ 0 & 0 & 0 & 0 & 1 & -5 & 3/2 &   & -\frac{5g+7a+3b-5c-4d}{2} \end{bmatrix}$
$l6 \rightarrow l6 - l5 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 &   & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 &   & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{-3d-14a+6b-5c}{5} \\ 0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 &   & -\frac{-24a+5c+11b-5c-8d}{4} \\ 0 & 0 & 0 & 0 & 0 & -5/4 & -5/24 &   & \frac{-20c+30c-5f-45d-130a+60b}{24} \\ 0 & 0 & 0 & 0 & 1 & -5 & 3/2 &   & -\frac{5g+7a+3b-5c-4d}{2} \end{bmatrix} \right] \\ = \begin{cases} -1 & 0 & -2 & 0 & 0 & 1 & 0 &   & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 &   & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} \\ 0 & 0 & 0 & 1 & 1/4/5 & 0 & 16/5 &   & -\frac{3d-14a+6b-5c}{5} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{-3d-14a+6b-5c}{5} \\ 0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 &   & -\frac{-24a+5c+11b-5c-8d}{4} \\ 0 & 0 & 0 & 0 & 0 & -5/4 & -5/24 &   & \frac{-20c+30c-5f-45d-130a+60b}{24} \\ 0 & 0 & 0 & 0 & 0 & 0 & -5/4 & -5/24 &   & \frac{-20c+30c-5f-45d-130a+60b}{24} \\ 0 & 0 & 0 & 0 & 0 & 0 & -25/4 & 0 &   & & \frac{-38a+5c-10g+5b+5c}{4} \\ \end{cases}$
$l6 \rightarrow -4/5.l6 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 &   & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 &   & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{-3d-14a+6b-5c}{5} \\ 0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 &   & -\frac{-24a+5e+11b-5c-8d}{4} \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/6 &   & -\frac{4c+6e-f-9d-26a+12b}{6} \\ 0 & 0 & 0 & 0 & 0 & -25/4 & 0 &   & \frac{-38a+5e-10g+5b+5e}{4} \end{bmatrix}$
$l7 \rightarrow l7 - l6 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 &   & 2a+b & \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 &   & b+a & \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} & \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{-3d-14a+6b-5c}{5} & \\ 0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 &   & -\frac{-24a+5e+11b-5c-8d}{6} & \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/6 &   & -\frac{-4c+6e-f-9d-26a+12b}{6} & \\ 0 & 0 & 0 & 0 & 0 & 0 & -1/6 &   & \frac{120c-422a+270b-130c-225d+60g-25f}{150} \end{bmatrix} \\ l7 \rightarrow -6.l7 \\ \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 1 & 0 &   & 2a+b & \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 &   & b+a & \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} & \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 &   & \frac{c+a}{3} & \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 &   & -\frac{3d-14a+6b-5c}{5} & \\ 0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 &   & -\frac{-24a+5e+11b-5c-8d}{4} & \\ 0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 &   & -\frac{-24a+5e+11b-5c-8d}{4} & \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/6 &   & -\frac{-4c+6e-f-9d-26a+12b}{6} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 &   & -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{bmatrix}$
$l6 \rightarrow l6 - (1/6.l7) \left[ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$l4 \rightarrow l4 - (16/5.l7) \left[ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$12 \rightarrow 12 - (3.17) \left[ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$11 \rightarrow l1 - l6 \begin{bmatrix} -1 & 0 & -2 & 0 & 0 & 0 & 0 &   & \frac{12a+5e+30b-10g+5c}{25} \\ 0 & 1 & 4 & 1 & 2 & 0 & 0 &   & \frac{360e+835b-1214a-390c-675d+180g-75f}{25} \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 0 &   & \frac{120e-125c-417a+270b-255d+60g-25f}{15} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 0 &   & \frac{1920e-3525d-3402a+4170b-1955c+960g-400f}{125} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 &   & \frac{60e-1061a+685b-315e+575d+155g-75f}{50} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 &   & \frac{10g-5e-5e+38a-5b}{25} \\ 0 & 0 & 0 & 0 & 0 & 1 &   & -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{bmatrix} $
$13 \rightarrow l3 - (1/3.l4) \left[ \begin{array}{cccccccccccccccccccccccccccccccccccc$

	「 -1	0	-2	0	١	0	0	0	$\frac{12a + 5e + 30b - 10g + 5c}{25}$	]	-1	0	0	0	0	0	0	$\frac{330g + 655e - 6048a + 3080b - 1295c + 470d - 90f}{75}$
	0	1	0	0	)	0	0	0	$\frac{-480g - 1070e - 3070b + 5883a + 1330c + 1565d + 195f}{75}$		0	1	0	0	0	0	0	$\frac{-480g - 1070e - 3070b + 5883a + 1330c + 1565d + 195f}{75}$
	0	0	1	0	)	0	0	0	$\frac{180g + 320e - 655c - 3042a + 1495b + 235d - 45f}{75}$	$l1 \rightarrow l1 + (2.l3)$	0	0	1	0	0	0	0	$\frac{180g + 320e - 655c - 3042a + 1495b + 235d - 45f}{75}$
$l2 \rightarrow l2 - (4.l3)$	0	0	0	1		0	0	0	$\frac{16g\!+\!60e\!-\!302d\!+\!161a\!-\!25b\!+\!10c\!-\!16f}{5}$		0	0	0	1	0	0	0	$\frac{16g + 60e - 302d + 161a - 25b + 10c - 16f}{5}$
	0	0	0	0	)	1	0	0	$\frac{60e - 1061a + 685b - 315c + 575d + 155g - 75f}{50}$		0	0	0	0	1	0	0	$\frac{60e - 1061a + 685b - 315c + 575d + 155g - 75f}{50}$
	0	0	0	0	)	0	1	0	$\frac{10g - 5e - 5c + 38a - 5b}{25}$		0	0	0	0	0	1	0	$\frac{10g - 5e - 5c + 38a - 5b}{25}$
	0	0	0	0	)	0	0	1	$-\frac{120e - 422a + 270b - 130c - 225d + 60g - 25f}{25}$		0	0	0	0	0	0	1	$-\frac{120e - 422a + 270b - 130c - 225d + 60g - 25f}{25}$

	1	0	0	0	0	0	0	$-\frac{330g+655e-6048a+3080b-1295c+470d-90f}{75}$
	0	1	0	0	0	0	0	$\frac{-480g - 1070e - 3070b + 5883a + 1330c + 1565d + 195f}{75}$
	0	0	1	0	0	0	0	$-\frac{330g+655e-6048a+3080b-1295c+470d-90f}{75}$ $-480g-1070e-3070b+5883a+1330c+1565d+195f}{75}$ $-\frac{180g+320e-655c-3042a+1495b+235d-45f}{75}$ $-\frac{16g+60e-302d+161a-25b+10c-16f}{5}$ $-\frac{60e-1061a+685b-315c+575d+155g-75f}{50}$ $-\frac{10g-5e-5c+38a-5b}{25}$ $-\frac{120e-422a+270b-130c-225d+60g-25f}{25}$
$l1 \rightarrow -1.l1$	0	0	0	1	0	0	0	$\frac{16g + 60e - 302d + 161a - 25b + 10c - 16f}{5}$
	0	0	0	0	1	0	0	$\frac{60e - 1061a + 685b - 315c + 575d + 155g - 75f}{50}$
	0	0	0	0	0	1	0	$\frac{10g - 5e - 5c + 38a - 5b}{25}$
	0	0	0	0	0	0	1	$-\frac{120e-422a+270b-130c-225d+60g-25f}{25}$
	l							

#### 3 Coordenadas

Portanto o conjunto forma base para o espaço vetorial R7 e as coordenadas são B =  $\frac{216}{5}$ ; -23; 21;  $\frac{-241}{5}$ ;  $\frac{217}{10}$ ; 15;  $\frac{19}{5}$