

Algebra Linear

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1 Exercício 1

Considere as bases do Espaço vetorial R3, A = {(4, 2, 0),(1, 1, 1),(5, 3, 3)} e B = {(1, 2, 1),(1, 5, 2),(1, 0, 1)}. Exiba as matrizes de mudança de base MB→A e MA→B. Escreva também os vetores abaixo nas bases indicadas:

- v = (0, 1, 2)A em B
- v = (1, 3, 1)B em A

Mudança B → A(b1)  
x. a1 + y.a2 + z.a3 = b1

x.(4, 2, 0) + y.(1, -1, 1) + z.( 5, 3, 3) = (1, -2, 1)

4x + y + 5z = 1  
2x - y + 3z = -2  
y + 3z = 1

4x + y + 5z = 1  
2x - y + 3z = -2  
6x + 8z = -1

2x - y + 3z = -2  
y + 3z = 1  
2x + 6z = -1

6x + 8z = -1  
2x + 6z = -1(-3)

6x + 8z = -1  
- 6x - 18z = 3

- 10z = 2  
z = -2/10  
z = -1/5

2x + 6z = -1  
2x + 6 . (-1/5) = -1

x = 1/10

y + 3z = 1  
y + 3(-1/5) = 1  
- 1 + 3/5  
y = 8/5

Mudança B → A(b2)

Mudança B → A(b3)

4x + y + 5z = 1  
2x - y + 3z = 0  
y + 3z = 1

4x + y = 5z = 1  
2x - y + 3z = 0

2x - y + 3z = 0  
y + 3z = 1  
6x + 8z = 1

2x + 6z = 1  
x = -1/10

z = 1/5

y + 3(1/5) = 1  
y + 3/5 = 1  
y = 2/5

Mudança A → B(a1)

x. b1 + y.b2 + z.b3 = a1

x.(1, -2, 1) + y.(1, 5, 2) + z.( 1, 0, 1) = (4, 2, 0)

x + y + z = 4  
- 2x + 5y = 2  
x + 2y + z = 0

x + y + z = 4(.2)  
- 2x + 5y = 2

x + y + z = 4  
x + 2y + 2 = 0(-1)

- 2x + 2y + 2z = 8  
- 2x + 5y = 2

7y + 2z = 10

x + y + z = 4  
- x - 2y - z = 0  
- y = 4  
y = -4

- 2x + 5(-4) = 2  
- 20  
- 2x = 2  
- 2x = 22  
x = -11

7y + 2z = 10  
7.(-4) + 2z = 10  
z = 19

Mudança A → B(a2)

x + y + z = 1  
-2x + 5y = -1  
x + 2y + z = 1

x + y + z = 1 .(2)  
- 2x + 5y = -1

x + y + z = 1  
z = 2y + z = 1

2x + 2y = 2z = 2  
- 2x + 5y = -1

7y + 2z = 1  
y = 0

7.0 + 2z = 1  
z = 1/2

- 2x + 0 = -1  
- 2x + 0 = -1  
- 2x = -1  
x = 1/2

Mudança A → B(a3)

x + y + z = 5  
-2x + 5y = 3  
x + 2y + z = 3

x + y + z = 5  
-2x + 5y = 3

x + y + z = 5  
x + 2y + z = 3

7y + 2z = 13  
y = -2

7y + 2z = 13  
y = -2

7 . (-2) = 2z = 13  
z = 27/2

-2x + 5(-2) = 3  
-2x - 10 = 13  
-2x = 13  
x = -13/2

MB →A: 
$$\begin{bmatrix} \frac{1}{10} & -1 & \frac{-1}{10} \\ \frac{8}{5} & \frac{-5}{2} & \frac{2}{5} \\ \frac{-1}{5} & \frac{3}{2} & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{10} \cdot 1 & -1 \cdot 3 & \frac{-1}{10} \cdot (-1) \\ \frac{8}{5} \cdot 1 & \frac{-5}{2} \cdot 3 & \frac{2}{5} \cdot (-1) \\ \frac{-1}{5} \cdot 1 & \frac{3}{2} \cdot 3 & \frac{1}{5} \cdot (-1) \end{bmatrix} = \begin{bmatrix} \frac{-14}{5} \\ \frac{-63}{10} \\ \frac{41}{10} \end{bmatrix}$$



$$l6 \rightarrow -5/24.l6$$

$$l6 \rightarrow l6 - l5$$

$$l6 \rightarrow -4/5.l6$$

$$l7 \rightarrow l7 - l6$$

$$l6 \rightarrow l6 - (1/6.l7)$$

$$l4 \rightarrow l4 - (16/5.l7$$

$$12 \rightarrow l2 - (3.l7)$$

$$11 \rightarrow 11 \rightarrow 16$$

10 10 44/2

$$10^{-10} = 10^{-10} \quad (1.10)$$

$$I2 \rightarrow I2 - (4.I3) \left[ \begin{array}{cccccc|c} -1 & 0 & -2 & 0 & 0 & 0 & 0 & \frac{12a+5e+30b-10g+5c}{25} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -480g-1070e-3070b+5883a+1330c+1565d+195f \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$I1 \rightarrow I1 + (2.I3) \left[ \begin{array}{cccccc|c} -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{330g+655e-6048a+3080b-1295c+470d-90f}{75} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -480g-1070e-3070b+5883a+1330c+1565d+195f \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

### 3 Coordenadas

Portanto o conjunto forma base para o espaço vetorial R7 e as coordenadas são B =  $\frac{216}{5}$ ; -23; 21;  $-\frac{241}{5}$ ;  $\frac{217}{10}$ ; 15;  $\frac{19}{5}$

$$I1 \rightarrow -1.I1 \left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{330g+655e-6048a+3080b-1295c+470d-90f}{75} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -480g-1070e-3070b+5883a+1330c+1565d+195f \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$