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Optimal carbon taxation and horizontal equity: A welfare-theoretic approach with application to German household data*

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ABSTRACT

We develop a model of optimal carbon taxation and redistribution taking into account horizontal equity concerns by considering heterogeneous energy efficiencies. By deriving first- and second-best rules for policy instruments including carbon taxes, transfers and energy subsidies, we then investigate analytically how horizontal equity is considered in the social welfare maximizing tax structure. We calibrate the model to German household data and a 30 percent emission reduction goal. Our results show that energy-intensive households should receive more redistributive resources than energy-efficient households if and only if social inequality aversion is sufficiently high. We further find that redistribution of carbon tax revenue via household-specific transfers is the first-best policy. Equal per-capita transfers do not suffer from informational problems, but increase mitigation costs by around 15 percent compared to the first- best for unity inequality aversion. Adding renewable energy subsidies or non-linear energy subsidies, reduces mitigation costs further without relying on observability of households' energy efficiency.

Keywords: carbon price, horizontal equity, redistribution, renewable energy subsidies, climate policy, just transition

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1 Introduction

The trade-off between equity and efficiency is one of the central topics in economics and economic policy. Analyzing this central trade-off with respect to the implementation of climate policy poses an urgent challenge: On the one hand economic theory clearly suggests that Pigouvian carbon pricing should be at the heart of economically efficient and environmentally effective climate policy (Pigou 1920, Nordhaus 2019), which is broadly supported by economists worldwide (Financial Times 2019, The Economist 2021) as well as by recent empirical findings (Andersson 2019, Gugler et al. 2021). On the other hand, there are many factors that impede the timely required implementation and political feasibility of carbon pricing (Levi 2021, Edenhofer et al. 2021). Among these factors, distributional consequences for low-income households are a key concern (Shammin and Bullard 2009, Parry 2015, Pizer and Sexton 2019) resulting in a political debate, which is often charged with emotions and thus provides breeding ground for conflicts, turmoil and deadlock. The yellow vest movement, for example, rose in France in November 2018 to protest fuel price increases due to CO_2 taxation. It is one example illustrating that climate policy analysis needs to consider appropriate redistribution measures that address equity concerns.

Due to the regressive first-order effect of carbon pricing in middle and high-income countries (Wang et al. 2016, Dorband et al. 2019, Ohlendorf et al. 2021), the existing literature has mostly focused on vertical equity between different income deciles. Empirical studies, however, have highlighted the importance of horizontal equity since distributional effects show an even larger variation within income groups (Poterba 1991, Rausch et al. 2011, Cronin et al. 2019, Pizer and Sexton 2019). The impact of carbon pricing on carbon-intensive energy consumption can vary across households with similar incomes due to household characteristics and behavior such as the climate surrounding the household, commuting distance of its members or the energy efficiency standard of a building (Rausch et al. 2011). Indeed, in the public debate negative distributional outcomes for households that are hardship cases due to their high carbon footprints, have frequently been used to argue against carbon pricing, even when the overall distributional effects are progressive, e.g., due to per-capita transfers.

Because household characteristics that determine horizontal inequality are mostly outside the scope of governmental regulation (Kaplow 1989, 1992), only very few studies have included horizontal equity concerns in a welfare-theoretic framework (e.g. Slesnick 1989, Auerbach and Hassett 2002). In this paper we put forward a normative perspective on horizontal inequality by considering heterogeneous technological abilities of households to convert energy into well-being. To achieve a certain level of utility, a long-distance commuter living in a badly insulated house, for example, is likely to require a higher amount of energy than a city dweller living in a modern apartment building. As heterogeneous technologies result in heterogeneous net incomes among households in the same decile, our welfare-theoretic model also explicitly captures the trade-off between economic efficiency and horizontal equity. Methodologically our approach therefore builds on Cremer et al. (2003) and Kaplow (2008), who suggest non-linear (energy) tax rules to take into account households' heterogeneity. In our case, however, the heterogeneity is not modelled as a 'taste' as in Cremer et al. (2003) and Kaplow (2008) but, at least in the short-run, as exogenous household-specific technology parameter capturing how efficient households can convert energy into individual well-being. It further borrows from previous works on optimal environmental policy and vertical inequality that focused, e.g., on non-constant Engel-curves in energy use (Klenert et al. 2018, Jacobs and van den Ploeg 2019, for example).

We use our model to characterize welfare-optimal first- and second-best policy instruments like carbon taxes, energy subsidies and transfer payments. Subsequently, we apply our findings to empirical data on energy consumption in Germany to quantify optimal policies. The numerical analysis considers a 30 percent emission reduction target and disregards the vertical inequality dimension (different labor productivities) to obtain a clear understanding about the trade-off between horizontal equity and efficiency in particular.

We show that the government's first best solution to address horizontal inequality is to set the carbon tax equal to the Pigouvian level and recycle the carbon tax revenue through household-specific transfer payments that account for household heterogeneity.

Thus, in our modelling framework, horizontal inequality is already considered in the standard social welfare approach. The inequality aversion derived from a general social welfare function (including the Utilitarian welfare function) also captures horizontal inequality. In general, it is not optimal to eliminate all horizontal inequality: when social inequality aversion is low, it is welfare-enhancing to redistribute resources to energy-efficient households as they can better increase social welfare due to their superior technology. Thus, an increase in horizontal inequality can be welfare enhancing. When social inequality aversion is high, the distributional motive dominates and less energy-efficient households receive larger transfers.

The fist-best policy requires that governments can perfectly observe households' energy efficiency type. We therefore consider several second-best policy instruments with lower informational requirements, like non-linear energy consumption subsidies (or taxes) and subsidies for renewable energy consumption, in combination with uniform carbon prices and transfers. These policies are, in general, welfare enhancing compared to uniform carbon prices and uniform transfers. In the numerical application for German households, we provide results for first-and second- best policy packages and calculate the increase in mitigation welfare costs ranging between 2% and 29% as compared to the first-best optimum.

The paper is structured as follows. Section 2 discusses related literature on the economics of horizontal equity in detail. In Section 3, we set up the theoretical model and provide some basic insight on first- and second-best policies to target horizontal inequality. In Section 4, we introduce functional forms and the calibration approach to German household data. Section 5 presents the numerical results for a richer set of first- and second best policy packages. Finally, Section 6 summarizes our results and discusses limitations of the approach taken in this paper.

2 Related Literature

The literature on the economics of horizontal equity can be divided into (i) empirical studies that quantify the magnitude of horizontal inequality due to some (environmental) policy reform, (ii) theoretical and applied modeling studies suggesting a welfare measure that disentangles vertical from horizontal equity and (iii) theoretical studies that consider horizontal equity within an optimal taxation framework. In the following we briefly summarize each of these literature strands.

The first literature strand focuses on empirical analyses and reports considerable within-decile variation in energy expenditure shares. Poterba (1991) analyses gasoline expenditures in the United States and finds considerable within-decile variability especially among low-income households. Pizer and Sexton (2019) confirm the high variation in energy expenditures shares also for other countries like Mexico and the United Kingdom. Rausch et al. (2011) shows that the impacts of carbon taxation in the United States puts indeed the highest burden on low-income deciles, while Cronin et al. (2019) also consider the capacity of existing transfer payments to address horizontal equity. They show that a uniform increase in all existing transfer payments increases horizontal inequality even further, which thus calls for a more targeted redistribution approach.

The second strand of the literature on the economics of horizontal equity deals with designing explicit welfare indices that incorporate horizontal equity. Slesnick (1989) proposes a welfare measure for horizontal equity that is consistent with social choice axioms and is calculated as the difference between welfare under a horizontally egalitarian distribution and the existing distribution of individual welfare. Using data on commodity taxation in the United States from 1947-1985 the study finds increasing horizontal inequality due to the heterogeneous effects of taxation on households' welfare. Auerbach and Hassett (2002) argue that horizontal equity should be justified within the context of the Atkinson inequality aversion index (Atkinson 1970). They differentiate between aversion to vertical and aversion to horizontal inequality by using a two parameter specification similar to the one that has been suggested by Epstein and Zin (1989) to disentangle preferences for risk from preferences for intertemporal substitution. When applying the suggested index to income tax data for 1994 in the United States, they find horizontal inequality to be less severe the higher the standard Atkinson inequality aversion index.

The most recent study by Pizer and Sexton (2019) proposes a welfare measure that can incorporate both vertical and horizontal equity and is based on the concept of equal sacrifice relative to a status-quo (Slesnick 1989, Kahneman and Tversky 1979). Horizontal equity concerns are seen as being related to loss aversion: If households value losses stronger than gains, heterogeneous policy costs among households of the same income group will constitute higher welfare losses than if all households of the same income group faced equal (average) policy costs. Findings show that non-Pigouvian policies like tradable performance standards lead to more horizontal inequality as compared to Pigouvian policies like a cap and trade system with equal per household rebates. Although status-quo biases might be difficult to justify from a normative perspective, policy makers would like to take them into account from a political economy perspective.

Compared to the literature on welfare measures, our paper specifically introduces income heterogeneity in households' energy expenditure shares in the modelling structure, but otherwise applies a standard utilitarian social welfare approach to evaluate policies. Thus, our model is capable of capturing horizontal inequality with the standard Atkinson inequality aversion index as suggested by Auerbach and Hassett (2002). While in Pizer and Sexton (2019) more horizontal equity is always welfare-increasing, this paper not only considers the positive effect on welfare due to a more egalitarian within-decile income distribution, but also takes into account that society sacrifices efficiency gains when compensating hardship cases with higher transfer payments. Our approach can thus be considered more general as we seek to understand under which conditions (i.e. social preferences) a benevolent government would care about horizontal equity without prescribing a distinct welfare measure in the first place.

The third strand of the literature on the economics of horizontal equity deals with optimal taxation (Ramsey 1928, Diamond and Mirrlees 1971) with the aim of implementing a tax system that maximizes a social welfare function subject to economic constraints (Mankiw et al. 2009). Traditional utilitarian welfare theory (Bentham 1789) is based on the principle of diminishing marginal utility of income, which motivates the dominating interest in vertical equity in optimal taxation models. Horizontal equity, in turn, is in this context typically interpreted as treating tax payers at equal positions equally, which is the more fundamental and widely accepted principle of fairness as an acceptable pattern of differentiation between income groups must be chosen (Musgrave 1990). Nevertheless, the literature on optimal taxation and horizontal equity is relatively scarce (Atkinson and Stiglitz 1976, Fischer and Pizer 2019).

Stiglitz (1982) shows that horizontal equity cannot be derived from a utilitarian social welfare function and can be inconsistent with Pareto efficiency. This is because randomization of the tax system enables the government to differentiate between high and low ability types at lower cost by taxing individuals with equal circumstances such that the high ability type is even more productive thereby raising average productivity and economic output. Jordahl and Micheletto (2005) incorporate a horizontal equity constraint in the problem of finding an optimal utilitarian tax structure, which has already been suggested by Atkinson and Stiglitz (1976) in order to circumvent the equity-efficiency trade-off when horizontal equity is built into the measurement of social welfare itself. The horizontal equity constraint in Jordahl and Micheletto (2005) is based on the interpretation of Bossert (1995) in terms of 'equal transfers for equal circumstances' and requires that heterogeneous households with the same abilities should pay the same taxes. Kaplow (2008) reconsiders central results of optimal income and commodity taxation when preferences are heterogeneous and are either observable or not. Based on a utility function that can embody different types of heterogeneity, results reveal that preference heterogeneity can lead to both higher and lower levels of income taxation depending on the type of heterogeneity, it's strengths, and the concavity of private utility and the social welfare function. However, both Jordahl and Micheletto (2005) and Kaplow (2008) do not make any explicit connection to environmental policy and carbon taxation specifically.

Within the optimal taxation literature there is an established sub-field on optimal taxation and environmental externalties[1] that goes back to Pigou (1920). Later Sandmo (1975) contributed the seminal paper based on a model of optimal linear taxation of a commodity that generates a negative atmospheric externality. The optimal commodity tax rule that results from this modeling setup includes one additive term that corrects

[1]See Aronsson and Sjögren (2018) for a very good overview of this literature.

for the externality thereby fulfilling the so-called ‘additivity property’. The resulting optimal tax system has two main objectives including (i) the correction of the environmental externality and (ii) achieving a distribution among heterogeneous individuals that is optimal according to a particular social welfare function.

This literature has been extended to also consider heterogeneous households (Cremer et al. 2003) which could in principle also cover horizontal inequality as a source of heterogeneity. Cremer et al. (2003) analyses how taste heterogeneity in households' preferences is captured in different systems of optimal environmental taxation and applies the model to energy consumption in France. The authors argue that type-specific non-linear environmental taxes should be implemented when individual consumption levels are observable as in the case of electricity consumption.

Compared to the previous literature on optimal (environmental) taxation and horizontal equity, our paper explicitly considers horizontal inequality in the modelling structure by implementing an additional horizontal dimension of income inequality via heterogeneous energy efficiency technologies within the same income group. Thus, welfare-optimal redistributive policies explicitly take into account this second dimension of income inequality by implementing targeted transfers or subsidies for heterogeneous energy efficiency types. In contrast to previous studies our modelling framework avoids conflicts of horizontal equity with Pareto efficiency (Stiglitz 1982, Elkins 2006) and exhibits the trade-off between equity and efficiency in the context of climate policy in a transparent way.

3 Theoretical Model

In the following, we introduce a parsimonious model in order to convey a few basic intuitions about optimal policies. In Section 3.2, we characterize the first-best optimal allocation that a social planner would implement. Then we compare the first-best with the outcomes that a government can achieve by using different sets of first-best (Section 3.3) and second-best policy instruments (Section 3.4). Finally, in Section 3.5, we discuss two possible extensions of the model: efficiency enhancing investments and the possibility to substitute carbon-free for fossil-based energy.

3.1 Basic model

We assume that a benevolent government seeks to maximize the welfare of $j = 1, ..., n$ heterogeneous households, which derive utility u^j from a numeraire consumption good c^j and carbon-intensive energy services E^j. We assume that households have exogenously given income y^j, that they demand raw energy $\widetilde{E}^j$ and use a technology f to convert it to energy services E^j. Thus, we assume that $E^j = f(x_0^j)\tilde{E}^j = \alpha_j \tilde{E}^j$ where we define $f(x_0^j) =: \alpha_j$. All households use the same technology, but are heterogeneous with respect to their capital endowments x_0^j. For example, energy services constitute an optimal room temperature which can require more or less raw energy depending on the type of heating and building insulation; mobility – reaching certain places for working, shopping etc. – constitutes another energy service, which requires different amounts of raw energy depending on distance, availability of public transport infrastructure or fuel efficiency of the household's car.

The heterogeneity in income is based on differences in labor productivity and reflects the vertical heterogeneity of households. Households' capital endowments x_0^j reflect their horizontal heterogeneity. For now, we will assume that they cannot make additional investments in efficiency enhancing capital. This is reasonable for time periods of five to ten years over which housing location decisions or energy investments of buildings hardly change. Later, we will discuss how relaxing this assumption affects our results. Thus, we have

$$
\begin{aligned}
u^j =& u(c^j, E^j) \\
E^j =& f(x_0^j)\tilde{E}^j = \alpha_j \widetilde{E}^j
\end{aligned} \tag{1}
$$

where $x_0^j > 0$ and $f' > 0 > f''$. The households' budget equation is

$$
b^j = y^j + R^j = c^j + \underbrace{(p_E + t_E^j)}_{=:q^j} \widetilde{E}^j \tag{2}
$$

The producer price of energy p_E is assumed to be fixed and given. Possible policy instruments that the government could implement include carbon taxes t_E^j on CO_2-intensive energy consumption and transfers R^j – which may be uniform or household-specific.

The Lagrangian of households is given by

$$L^H = u(c^j, E^j) + \lambda^j(b^j - c^j - \frac{q^j E^j}{\alpha_j}), \tag{3}$$

where we have used (1) to eliminate raw energy $\widetilde{E}^j$. The first order conditions are as follows:

$$\frac{\partial u^j}{\partial c^j} = \lambda^j \tag{4}$$

$$\frac{\partial u^j}{\partial E^j}\frac{\alpha_j}{q^j} = \lambda^j \tag{5}$$

Combining equations (4) and (5) shows that in the optimum the marginal rate of substitution between consuming an additional unit of energy services and consuming an additional unit of the numeraire consumption good must be equal to the market price of energy scaled by the energy efficiency parameter α_j. Moreover, (5) reveals that the marginal utility of the individual household's income λ^j depends on the energy efficiency parameter α_j, i.e. we have $\lambda^j(\alpha_j)$ with $\lambda^{j\prime}(\alpha_j) = \frac{\partial u^j}{\partial E}\frac{1}{q^j} > 0$.

An individual household's maximization results in the conditional demand functions $c^j = c(b^j, q^j)$ and $E^j = E(b^j, q^j)$, which together determine the household's indirect utility function $v^j = v(b^j, q^j)$.

3.2 Social planner optimum

The first-best optimal allocation that a social planner would choose is determined by maximizing a Bergson-Samuelson social welfare function $W(u^1, ..., u^n)$ with $\frac{\partial W}{\partial u^j} \geq 0$ and $\frac{\partial^2 W}{\partial {u^j}^2} \leq 0$ for all j, subject to an exogenous aggregate environmental target $E^* = \sum_j \widetilde{E}^j$ and a resource constraint $\sum_j y^j - c^j - p_E \widetilde{E}^j = 0$. We abstract from an explicit representation of environmental damages to keep the analysis as simple as possible. In the context of climate change, this is also reasonable as damages occur in the very-long run and are globally distributed. The social planner's Lagrangian is

$$L^{SP} = W(u^1, ..., u^n) + \mu(E^* - \sum_j \widetilde{E}^j) + \gamma(\sum_j y^j - c^j - p_E \widetilde{E}^j) \tag{6}$$

and the first order conditions along with their interpretation are given in the following proposition.

Proposition 1. *The social optimum in the basic model can be achieved under the condition that*

$$0 = W_u^j u_E^j \alpha_j - \mu - \gamma p_E \quad \forall j \tag{7}$$

$$0 = W_u^j u_c^j - \gamma \quad \forall j \tag{8}$$

The social planner chooses an allocation that balances households' welfare weights W_u^j, their marginal utilities u_E^j and u_c^j and their energy efficiencies α_j. Thus, normative distributional social preferences are balanced with efficiency in consumption.

From the social planner's first-order conditions it follows that

$$\frac{W_u^i}{W_u^j} = \frac{u_E^j}{u_E^i}\frac{\alpha_j}{\alpha_i}$$
$$\text{and} \quad \frac{W_u^i}{W_u^j} = \frac{u_c^j}{u_c^i} \quad \forall i, j.$$

To interpret these equations, assume that i has a higher normative welfare weight than j ($W_u^i > W_u^j$). Then, i must also have a higher level of numeraire consumption. Moreover, if $\alpha_i = \alpha_j$, household i must also have higher level of energy service consumption. However, if j is more energy efficient than i ($\alpha_i < \alpha_j$), then the difference between normative welfare weights could be offset by energy efficiency considerations. Social optimality requires the social planner to allocate relatively more energy service consumption to households that are more efficient in transforming energy services to utility and hence social welfare.

3.3 First-best optimal governmental policy for the decentralized economy

The government maximizes the same social welfare function $W(u^1, ..., u^n)$ as the social planner does, subject to the same aggregate environmental target $E^* = \sum_j \widetilde{E}^j$ and in addition to its budget constraint $\sum_j (t_E \widetilde{E}^j - R^j) = 0$. Using the indirect utility function implies that the government maximizes social welfare taking into account the individual household's optimization behavior.

We abstract for now from problems of self-selection to clarify the fundamental mechanisms that determine the optimal tax system when heterogeneity is observable. While this case also serves as an important first-best policy benchmark we relax the assumption later when discussing second-best policies. The optimal policy is characterized by

Proposition 2. *If the government uses a uniform carbon tax $t_E^j = t_E \, \forall j$ and household-specific transfers R^j, it can achieve the first-best optimum by setting*

$$t_E^* = \frac{\mu}{\gamma} \ \forall j, \tag{9}$$

The optimal transfers ${R^j}^*$ must ensure that the households' marginal rates of substitution, weighted by energy efficiency α_j, are equal for all households,

$$\frac{\mu}{\gamma} + p_E = \alpha_j \frac{{u_E^j}^*}{{u_c^j}^*} = \alpha_j {MRS_{c,E}^j}^* = {MRS_{c,\widetilde{E}}^j}^* \ \forall j \tag{10}$$

and the governments budget is balanced, that is, $t_E^* E^* = \sum_j {R^j}^*$. Asterisks denote the values obtained from evaluation at the optimal allocation.

Proof. See appendix. □

Proposition 2 emphasizes that the carbon tax should be complemented by type-specific transfers that account for heterogeneity in energy efficiency technology as well as labor productivity.

3.4 Second-best policies

To achieve the first-best, it is sufficient to differentiate transfers to households and implement a uniform carbon tax. In this section, we briefly discuss alternative sets of policy instruments that may be less restrictive on the assumption that households' energy efficiency type is public information. We begin with a policy where the government sets uniform transfers to households but specific carbon taxes ${t_E^j}^*$ (see Proposition 3). While this policy requires the government to observe energy technologies of households, it serves as a benchmark for non-linear or linear climate policies that do not require this observability.

Let us assume that the government can target individual households by setting individual carbon tax rates, but must use the same lump-sum transfer for all households. Then the Lagrangian of the government is $L^G = W(v^1, ..., v^n) + \mu(E^* - \sum_j \widetilde{E}^j) + \gamma \sum_j \left(t_E^j \widetilde{E}^j - R\right)$, which yields first order conditions

$$L_R^G = \sum_j W_{v^j} v_{b^j}^j - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial b^j} + \gamma \sum_j \left(t_E^j \frac{\partial \widetilde{E}^j}{\partial b^j} - 1 \right) = 0 \tag{11}$$

$$L_{t_{\widetilde{E}}^j}^G = W_{v^j} v_{q^j}^j - \mu \frac{\partial \widetilde{E}^j}{\partial q^j} + \gamma \left(t_E^j \frac{\partial \widetilde{E}^j}{\partial q^j} + \widetilde{E}^j \right) = 0 \tag{12}$$

Definition 1. *We define the social shadow price of achieving the environmental target E^* measured in terms of public funds, μ/γ, as the* Pigouvian *component of environmental taxation.*[2]

By using the private households' marginal utility of income $\lambda^j(\alpha_j)$ and Roy's identity[3] we can derive the rule for individual carbon taxes from (12) as given in Proposition 3.

[2]We are aware of the fact that the Pigouvian component is typically associated with capturing the sum of marginal environmental damages. We abstract from environmental damages here and instead introduce a fixed upper bound on total energy use. However, our shadow price μ/γ plays a similar role as the corresponding shadow price in the literature on optimal taxation and environmental externalities.

[3]Specifically we use $\frac{\partial v^j}{\partial q_E} = -\underbrace{\frac{\partial v^j}{\partial I^j}}_{\lambda^j} \widetilde{E}^j$ and $\frac{\partial v^j}{\partial b} = \underbrace{\frac{\partial v^j}{\partial I^j}}_{\lambda^j}$

Proposition 3. *When the government can perfectly observe the household-specific energy efficiencies α_j but has to use uniform transfers, the optimal household-specific carbon tax can be written as*

$$
{t_E^j}^* = \frac{\mu}{\gamma} + \underbrace{\frac{\partial W}{\partial v^j}\frac{\lambda^j}{\gamma}\frac{\widetilde{E}^j}{\frac{\partial \widetilde{E}^j}{\partial q^j}} - \frac{\widetilde{E}^j}{\frac{\partial \widetilde{E}^j}{\partial q^j}}}_{\Phi_j}.
$$

The optimal individual carbon tax is the sum of two components:

(i) a Pigouvian component μ/γ, which is the same for each household.

(ii) a distributional, household-specific component Φ_j, which takes into account differences in disposable income resulting from heterogeneous α_j.

Hence, if energy expenditures only make up a negligible part of the household's total expenditure, this term is very small and the tax is close to the Pigouvian level. The greater the share of energy expenditure, the more the tax deviates from the shadow price, i.e distributional aspects have a larger impact on the optimal carbon tax rule. In the following corollary, we summarize the considerations taken into account by the government when it trades off equity and efficiency.

Corollary 1. *In setting the optimal energy tax, the government faces an equity-efficiency trade-off. The equity motive is determined by the welfare function, while the efficiency motive is determined by the curvature of households' indirect utility function with respect to disposable income (measured by λ^j), their energy-efficiency (measured by α_j) and the marginal social value of public funds γ.*

Proof. If $\Phi_j = 0$, household j is taxed at the Pigouvian level. In general, however, Φ_j could be greater or less than zero, implying that the optimal household-specific carbon tax lies above or below the Pigouvian level. Without loss of generality we focus in the following on the case in which $\Phi_j < 0$ and, hence, $t_j^{E*} < \frac{\mu}{\gamma}$. The discussion of the case $\Phi_j > 0$ would be analogous. Assuming that energy is a normal good in the sense that

$\frac{\partial \widetilde{E}^j}{\partial q^j} < 0$, we have

$$t_E^{j\,*} < \frac{\mu}{\gamma} \quad \Longleftrightarrow \quad \Phi_j < 0 \quad \Longleftrightarrow \quad \frac{\gamma}{\lambda^j} < \frac{\partial W}{\partial v^j}. \tag{13}$$

Above inequality (13) allows us to infer four reasons for the government to tax a household below the Pigouvian rate. We now discuss each of these four reasons assuming all else is equal. First, the optimal individual carbon tax is more likely to be set below the Pigouvian level if the government puts a relatively high marginal welfare weight on household j, i.e. $\frac{\partial W}{\partial v^j}$ is relatively large. Households whose utility contributes more to social welfare thus have to bear less of the tax burden. Second, (13) is more likely to hold, the larger the households marginal utility of income λ^j is. Since marginal utility is decreasing, the government has a motive to put less tax burden on poorer households. Third, the government wants to shift energy consumption towards the household that generates most utility from a given quantity of energy, i.e. the household with the highest α_j. It holds that $\frac{\partial \lambda^j}{\partial \alpha_j} > 0$ and thus, the higher α_j, the more likely it is that (13) holds and household j is taxed below the Pigouvian level. Fourth, the lower the social marginal value of public funds γ, the more likely (13) will hold. If, in contrast, γ is very high and, hence, additional public funds would contribute relatively strongly to social welfare, the optimal tax is less likely to be below the Pigouvian level. □

Corollary 2. *When the government is constrained to use the uniform lump-sum transfer R instead of a household-specific transfer but can implement household-specific carbon taxes, it cannot achieve the first-best allocation.*

Proof. See appendix.

The type-specific carbon taxes derived in Proposition 3 require that governments can identify the type of the household, i.e. that the energy efficiency type is a public information. The optimal taxation literature, by focusing on individual labor productivity, has established for a long time tax designs which do not depend on this assumption. When productivity is a private information, some (notably more productive) households do not have an incentive to reveal their productivity. They rather pretend to be of low-productivity type to avoid higher tax payments. To address this problem, the government could consider so-called incentive compatibility constraints in their tax optimization problem (Mirrlees 1971, Aronsson and Sjögren 2018), which require that no household can gain from pretending to be another household type. A key insight from the optimal taxation literature is further that optimized non-linear tax schedules that depend only on observable economic decisions (like the amount of energy consumed or the income generated) are also incentive compatible (Hammond 1979, Bierbrauer 2016).

In our context, non-observability of energy efficiency types implies that the governments' optimization problem for type-specific carbon taxes should be complemented by incentive constraints. Clearly, such a setting can never create higher social welfare than the case in Proposition 3 (without the additional incentive constraint). Hence, we can understand type-specific carbon taxes to constitute an upper bound for welfare losses of optimal (incentive-compatible) carbon tax schedules. In particular, any non-linear carbon tax $t_E^j = t_E(\tilde{E}^j)$ that depends on households' (observable) energy consumption, $\tilde{E}^j$, can never outperform an optimized type-specific carbon tax.

A non-linear energy tax or subsidy requires substantial monitoring capacity by the state as the individual energy use needs to be assessed. While this might be easy for grid-based electricity and natural gas consumption, it requires a personalized recording of individual consumption for gasoline, diesel or heating oil. An optimized policy that consists of a uniform (per-capita) transfer together with a uniform carbon tax does even not require to monitor households individual energy consumption.

Corollary 3. *When energy efficiency is a private information and individual energy consumption cannot be observed, the government is constrained to use the uniform carbon taxes and uniform lump-sum transfer R. This can, in general, not achieve the first-best allocation.*

Proof. The result follows directly from Corollary 2.

The cascade of policy approaches emphasizes that lower informational requirements imply more restrictions to the set of available policies, which inevitably implies larger welfare losses compared to a first-best policy with full information. In the analytical part, we do not formally characterize all these variants of second-best policies but we will consider them in the subsequent numerical analysis.

3.5 Extensions

Two extensions to the basic model merit further attention. In the following, we therefore first discuss the possibility for households to increase their energy efficiency by investing part of their income in efficiency enhancing capital. It turns out that this extension does not yield any further qualitative insights. Second, we introduce a simple energy production sector which takes renewable and fossil resources as inputs (Section 3.5.2). We maintain the latter extension in our subsequent numerical analysis as it improves the calibration of the model to the data.

3.5.1 Energy efficiency enhancing investments by households

Assuming that households' energy efficiency α_j is fixed, limits the model's applicability to the short run. To relax this assumption, one could think of households as being able to invest part of their income to improve their energy efficiency α_j. Examples include investments into building insulation or moving house to locations closer to the work-place or with better access to public transport infrastructure. Final energy services consumed by household j would then be $E^j = \alpha_j \widetilde{E}^j = f(x_0^j + x^j)\widetilde{E}^j$, where $x_0^j, x^j > 0$ and $f' > 0 > f''$. In addition to the carbon tax and the lump-sum transfer, the government could implement a subsidy on efficiency-enhancing investments.

However, extending the model to allow for household investments in efficiency-enhancing capital yields very similar results to the ones obtained from the basic model. For example, Proposition 2 still remains valid, that is, the first-best can be implemented with a uniform carbon tax and household-specific transfers. If the government is restricted to a uniform carbon tax and a uniform transfer, it is welfare enhancing to allow for a subsidy on efficiency-enhancing investments. For details, see Appendix, Section B.

3.5.2 Decarbonization of final energy production

In the preceding analysis we have abstracted from the possibility to decarbonize energy production. The only mitigation option to achieve the environmental target E^* was to reduce energy consumption, for which the government used the energy tax t_E. The model's production side, however, can be extended to include a production function for energy that takes renewable X and non-renewable (fossil) resources Z as input factors where factor prices p_X and p_Z are fixed. Then, the environmental target consists of keeping the use of fossil resources below a certain threshold, $Z \leq Z^*$.

Now, the government can use a uniform carbon tax τ_Z on the use of Z in production and a uniform subsidy s_X for renewables to incentivize the decarbonization of energy production. Household-specific carbon taxes as considered in the simple model above cannot be modeled under this setup. Instead, we can consider household-specific subsidies s_E^i for energy expenditures that take over the role of energy taxes t_E^j in the preceding analysis.

The only additional equations that are added by this extension are the production function $E(X, Z)$, which we assume to satisfy the Inada conditions and to have constant returns to scale, and the competitive energy producer's first order conditions associated with its profit maximization:

$$p_E \frac{\partial \tilde{E}}{\partial X} = p_X - s_X \tag{14}$$

$$p_E \frac{\partial \tilde{E}}{\partial Z} = p_Z + \tau_Z \tag{15}$$

Using the analytical model, we can already obtain one result on the use of subsidies for renewable energy:

Proposition 4. *Consider the case where renewable energy can substitute fossil inputs in energy production and the government uses only linear and uniform tax policies. In this case, it is, in general, welfare-improving to add a renewable energy subsidy (or tax) to the carbon price and the uniform transfer.*

Proof. See Appendix 4. □

The key intuition of this proposition is that a government should also exploit the possibility to change relative prices between renewable and fossil energy, to influence overall energy prices. This, in turn, reduces the welfare losses of energy-intensive households which benefit from lower energy prices.

4 Numerical Model Set-up

To quantify the implications of our analysis for optimal climate policy, we develop a numerical model and calibrate it to German household data. In the following, we will describe the parametrization of the numerical model, the calibration procedure and our results. We abstract from including household investments in energy efficiency (as described in Section 3.5.1), because it is less relevant for the considered time period (5-10 years) and because household-specific data on energy efficiency capital and its energy demand effect is lacking. However, we do include the extension of the basic model to the energy production sector such that decarbonization of the latter is possible, as described in Section 3.5.2.

4.1 Functional Forms

4.1.1 Utility and Social Welfare

We assume Stone-Geary type utility functions for the households of the form

$$u^j(c^j, \tilde{E}^j) = \frac{\left(c^{j\beta}\left(\alpha_j \tilde{E}^j - \bar{E}\right)^{1-\beta}\right)^{1-\eta}}{1-\eta} \tag{16}$$

where $\bar{E}$ denotes a subsistence level of utility-relevant energy consumption and α_j the conversion efficiency for raw energy $\tilde{E}$ to utility-enhancing energy-intensive services such that $\alpha_j \tilde{E}^j = E^j$. The subsistence requirement enables us to model non-constant energy-expenditure shares over different income deciles; moreover, non-homethetic utility functions allow to consider horizontal inequality due to different energy efficiency technologies.

Following Kaplow (2010) we assume a Bergson-Samuelson social welfare function of the form

$$W(u) = \sum_j (1-\eta)^{-\gamma} \frac{{u^j}^{1-\gamma}}{1-\gamma}, \tag{17}$$

where $u = (u^1, ..., u^n)$. This function reduces to a simple utilitarian social welfare function $W(u) = \sum_j u^j$ for $\gamma = 0$. The parameter η determines the curvature of the individual household utility function u, i.e. the elasticity of the marginal utility of comprehensive household consumption. The parameter γ measures governmental inequality aversion as reflected by the curvature of the social welfare function $W(u)$. Kaplow (2010) showed that the combined concavity parameter is then given by $\epsilon = \eta + (1-\eta)\gamma$.[4] Details on how this formula affects ϵ for different values of η and γ can be found in Table C.1 in Appendix C.

We use equivalent variation (EV) to monetize households' policy costs. Given household j's exogenously given income y^j (excluding transfer payments) and energy price $q = p_E - s_E$, EV^j is obtained by the following indifference condition:

$$v^j(y^j + EV^j, p_E) = v^j(y^j, q). \tag{18}$$

The aggregate social welfare loss is then given by the sum of the individual household's EV^j weighted by the marginal utility of income $v^j_{b^j}$ and the household's welfare weight W_{v^j} (see for example Fankhauser et al. 1997). In order to calculate the monetized social welfare loss we then divide the aggregate social welfare loss by the product of the welfare weight and the marginal utility of income of the average household for the case without climate policy $W_{\bar{v}}\bar{v}_{\bar{b}}$.

[4]Note that every ϵ can also be obtained by varying only η while holding γ constant equal to zero. We follow this strategy in the numerical optimization to obtain the results in Section 5.

$$\frac{\sum_j W_{v^j} v_{b^j}^j EV^j}{W_{\bar{v}} \bar{v}_{\bar{b}}} \tag{19}$$

4.1.2 Energy Demand

Let $b^j = c^j + q^j \tilde{E}^j$ be total expenditures and $m_E^j = \frac{q_E^j \tilde{E}^j}{b^j}$ the energy expenditure share. With $\frac{\partial u^j}{\partial E^j}\alpha_j = q^j \frac{\partial u^j}{\partial c^j}$ from the first order conditions of the household optimization problem (equations (4) - (5)), we obtain for the energy expenditure share:

$$m_E^j = 1 - \beta + \frac{\beta q \bar{E}}{\alpha_j b^j} \tag{20}$$

Hence, for a homothetic utility function (with $\bar{E} = 0$), energy expenditure shares are constant and equal $1 - \beta$. Because of subsistence energy consumption $\bar{E}$, energy expenditure shares decrease with rising total expenditure. Further, if energy conversion efficiency α_j is high, energy expenditure share is lower. Importantly, there is no horizontal heterogeneity in energy expenditure shares if preferences are homothetic and $\bar{E} = 0$.

4.1.3 Energy Production

For the energy production sector, we consider a constant-elasticity-to-scale (CES) function with substitution elasticity $\sigma \in \mathbb{R}_0^+$, share parameter $a \in (0, 1)$ and scaling parameter A:

$$E(X, Z) = A\left(aX^\rho + (1-a)Z^\rho\right)^{\frac{1}{\rho}}, \text{ where } \rho = \frac{\sigma - 1}{\sigma}.$$

4.2 Calibration

We determine the structural parameters of the utility function based on official German household data that includes income and expenditure information (EVS 2018). We split households in 10 income deciles, based on their adult-equivalent household expenditures, as expenditures are a better proxy for permanent-income than annual income, which changes strongly over the life-cycle of an individual. We then calculate for each income decile ten energy expenditure deciles. This creates a grid of 10×10 household types that differ in the two dimensions: income and energy efficiency. To depend less on data outliers, we consider the median income and energy expenditure share within each decile as the value for the specific grid-cell household. The left panel in Fig. 1 shows this heterogeneity for German household data over different income deciles. Although the later analysis will only consider the different energy-efficiency deciles within the median income decile, the full grid is used to estimate the demand function.

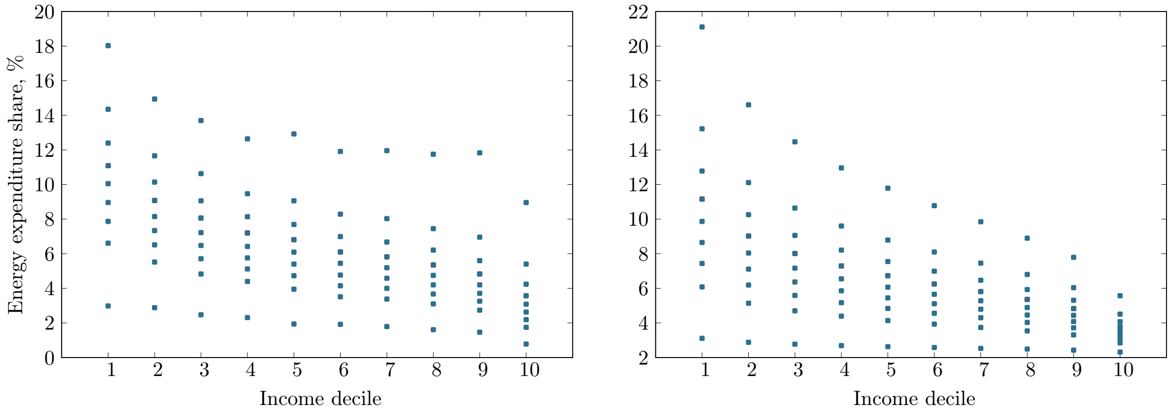

Figure 1: Hetereogeneity of households. Left: mean expenditure shares in 10x10 grid based on EVS 2018 data. Right: mean expenditure shares from fitted model.

For calibrating the model, we estimate the parameters α_j and β in Eq. (20) based on a linear regression with energy expenditure decile dummies that are interacted with household's expenditure b^j. Hence, we obtain for each energy-efficiency type the coefficient $\frac{\beta q \bar{E}}{\alpha_j}$. For calculating α_j, we further need to impose values on q and $\bar{E}$. Because the α_j scale with $\bar{E}$, there is an additional degree of freedom and we can set $\bar{E} = 1$ without loss of generality of the model.[5] From the environmental accounting data (Destatis 2020)

[5] To see this, define $\hat{\alpha}_j := \frac{\alpha_j}{\bar{E}}$. Considering the utility function (16), we get $u^j(c^j, E^j) = u^j(c^j, \alpha_j \tilde{E}^j) = \frac{\left(c^{j\,\beta}\left(\alpha_j \tilde{E}^j - \bar{E}\right)^{1-\beta}\right)^{1-\eta}}{1-\eta} = \frac{\left(c^{j\,\beta}\left(\hat{\alpha}_j \tilde{E}^j - 1\right)^{1-\beta}\right)^{1-\eta}}{1-\eta} \bar{E}^{(1-\beta)(1-\eta)} = u^j(c^j, \hat{\alpha}_j \tilde{E}^j) \bar{E}^{(1-\beta)(1-\eta)}$ Hence, changes in the value of $\bar{E}$ will only scale the vector $u \in \mathbb{R}^n$ of all households' utility levels by the factor $\bar{E}^{(1-\beta)(1-\eta)}$.

and energy price data, we calculated an average energy price (weighted by consumption shares) of $q = 462$ Euro$/tCO_2$ (see Tab. D.2). This allows us to calculate the different energy efficiency levels (see Tab. D.3). The common demand parameter is further estimated to be $\beta = 0.9786$. Note that an unconstrained OLS regression gives a negative estimate for α^1 (i.e. for the highest energy efficiency type) which would be inconsistent with our model assumptions. We therefore constrain that energy efficiency of the highest efficiency type such that it is twice the second-highest efficiency type, $\alpha^1 = 0.5\alpha^2$. The calibration of the demand function can explain very well the heterogeneity of energy consumption across income and energy efficiency types, as shown in the right panel in Fig. 1.

For calibrating the energy production sector, we set the elasticity of substitution between fossil and carbon-free energy to $\sigma = 4$.[6] In order to match the share of renewable energy production on total primary energy production of 21 percent in 2018 (Destatis 2020), the renewable energy price p_X needs to be set 56 percent higher than the price of fossil energy. To allow for a straight-forward model comparison between a model with and with-out the mitigation sector with equal final energy prices before carbon pricing, we set, without loss of generality, $p_Z = p_E = 462$, implying $A = 2.33$. The carbon price needed to reduce carbon emissions by 30 percent is in our model approx. 140 €/tCO_2, which is a plausible number lying well in the range of existing estimates for the EU climate policy.[7]

[6]Based on global input-output data, Papageorgiou et al. (2017) estimate an elasticity of substitution between clean and dirty inputs of approximately 2. As this estimate is based on past production data, it disregards the large role of sector-coupling between the electricity sector and the transport and heating sector, which will play an important role in the future. Moreover, from a physical point of view, energy production technologies are close to perfect substitutes.

[7]The carbon price of 150 €/tCO_2 is higher than that in the EU impact assessment (60€) (European Commission 2020), but lower than carbon prices calculated by integrated assessment models like REMIND/LIMES (150-300€).

4.3 Policies

For the numerical analysis, we calculate different policy packages that differ in their welfare implications and their informational requirements.

Uniform carbon tax and household-specific transfers This is the first-best policy which sets the benchmark for an outcome that maximizes social welfare. It requires, however, that the government can fully observe the energy efficiency type of each household.

Uniform carbon tax, household-specific energy subsidies and uniform transfers As emphasized in Proposition 3, carbon taxes that are differentiated by household type, can be welfare-optimal if household-specific transfers are not possible. Because of the additional mitigation sector in the numerical application, we cannot differentiate carbon prices by households but rather implement household-specific energy subsidies (or taxes). Such a policy ensures that households face differentiated incentives to reduce energy demand (while the carbon tax works entirely in the energy production sector by substituting fossil with renewable energy).

The previous two approaches rely on perfect observability of energy efficiency types (or the absence of any self-selection constraints). Linear policies or taxes/subsidies, that are non-linear in the consumed demand, do not suffer from this problem. We therefore study an additional set of second-best policies:

Uniform carbon tax, non-linear energy subsidies and uniform transfers Here, energy subsidies (or taxes, in case of negative subsidies) take the functional form $\hat{s}_E^j = s_{E_0} + s_{E_1} \times \tilde{E}^j$, which approximates well the household-specific energy subsidies of the previous policy case.

Uniform carbon tax, uniform renewable energy subsidies and uniform transfers This policy mix relies only on linear taxes and has therefore low informational requirements for the government.

Uniform carbon tax and uniform transfers This is the simplest revenue-neutral policy in our setting that achieves a given carbon emission target.

4.4 Numerical Solution Technique

We numerically calculate the optimal tax and transfer policies that maximize social welfare, considering the first-order conditions determining energy demand and energy production. The model is written in the AMPL programming language and solved with the Knitro optimization solver (version 10.2).

5 Results

In the following, we present the results of the numerical optimization of climate policy instruments that achieve a 30% reduction in aggregate carbon emissions. We allow for substitution of fossil energy by renewable energy production. Otherwise, climate targets can only be achieved by energy demand reduction, implying significant welfare losses and requiring high carbon prices. In order to purely focus on horizontal inequality, we only analyze the variation of energy expenditures within the median income decile.[8] Socially optimal policy instruments are then calculated for ten different efficiency deciles α_j within that median income household group. Efficiency decile 1 includes the most energy efficient households ($\alpha^1 = 5.01$) whereas efficiency decile 10 captures the least energy efficient households ($\alpha^{10} = 0.26$). In addition we vary the households' elasticity of the marginal utility of comprehensive consumption η and thereby also the combined concavity parameter ϵ. It measures the combined social aversion to inequality, from 0.1 (low inequality aversion) to 2 (high inequality aversion), which allows us to capture different degrees of social preferences for horizontal equity. In the following we will first present results for the first-best optimum (uniform carbon taxation and household-specific transfers) and subsequently move to an analysis of second-best optima, while

[8] The household at the 5th income decile has a median income (adult-equivalent expenditure) of $\tilde{y} = 18318$ €/year.

the order of presentation reflects increasing mitigation welfare costs as compared to the first-best case.

5.1 First-best optimum

As a first step we consider the first-best optimum that can be achieved with a uniform carbon tax that must equal the social cost of energy consumption (Pigouvian component), while the resulting tax revenue should be redistributed household-specifically (see Proposition 2). For that case an optimal uniform carbon tax of $\tau_Z^* = 139$ €/tCO_2 achieves achieves the 30% reduction in aggregate household fossil energy consumption. Figure 2 shows how the resulting carbon tax revenue should be optimally redistributed to the ten household types in order to take horizontal inequality into account.

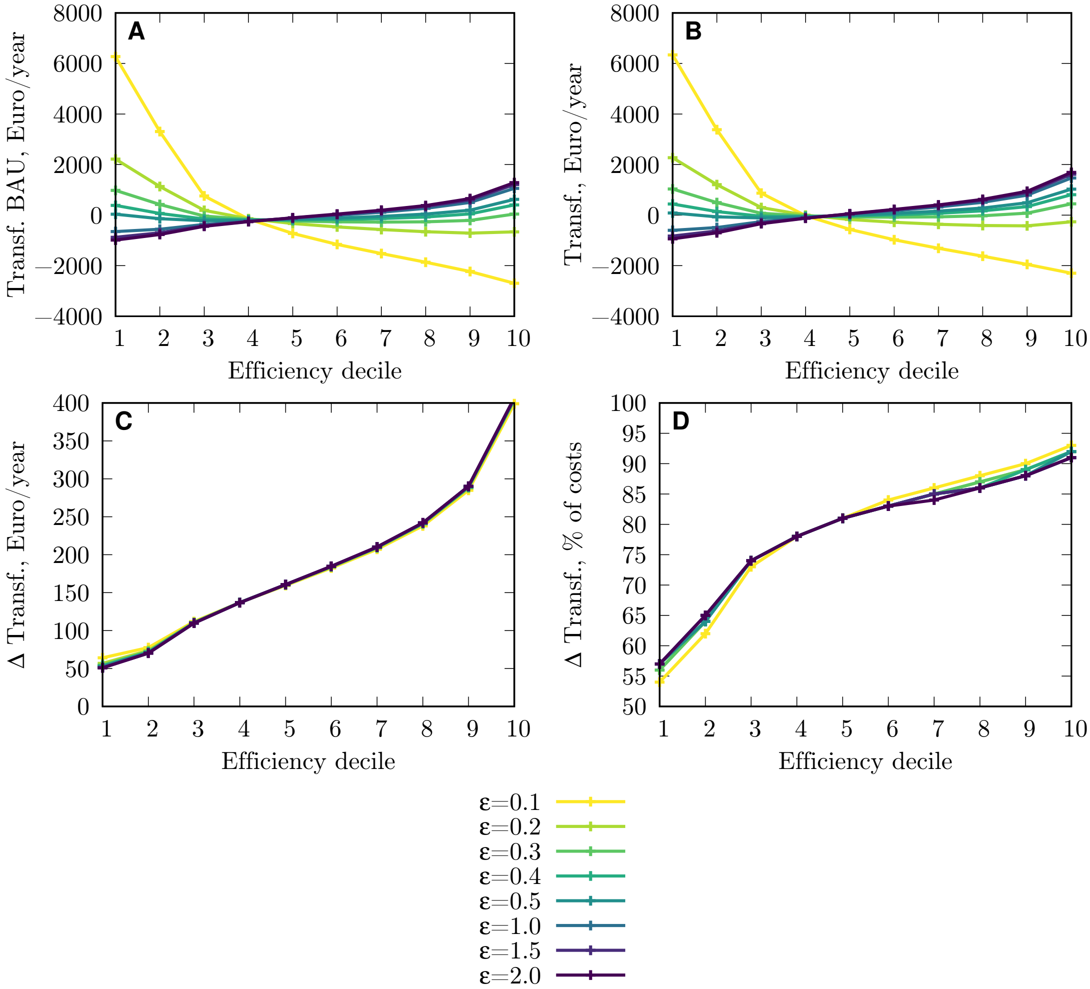

Figure 2: **Optimal household-specific transfers** for energy-efficiency deciles (1=high, 10=low) and inequality aversion $\epsilon = [0.1; 2]$. **A** shows transfers without climate target and policy (BAU - Business as Usual), **B** depicts transfers with an optimal uniform carbon tax $\tau_Z^* = 139$ €/tCO_2 set to achieve a 30% reduction in total carbon emissions, **C** shows the additional transfer due to the introduction of climate policy (as comapared to BAU) and **D** expresses this change as a percentage of households' policy costs.

How this 'should' be done depends on the overall social aversion to horizontal inequality reflected by ϵ that is influenced by both private inequality aversion η and governmental inequality aversion γ. Panel 2A isolates the transfer payments without carbon tax and without climate target (BAU = business as usual), as also in this case transfers are used as an optimal instrument to target horizontal inequality. Panel 2B depicts the transfer payments in the climate policy case, where the additional effect on horizontal inequality through the introduction of carbon taxation is considered. Panel 2C shows the difference between Panel B and A, hence the additional (Δ) transfer targeting the pure effect of carbon taxation on horizontal inequality, while in panel 2D these additional transfers are expressed as a percentage of household's climate policy costs as measured by the equivalent variation (EV).

Comparing panels 2A and 2B with panels 2C and 2D reveals that while transfers that target horizontal equity in the absence of climate policy vary with inequality aversion ϵ, the additional transfers due to the introduction of climate policy are almost independent of ϵ. In the first case (panels 2A and 2B) the equity-efficiency trade-off in allocating transfers to households becomes clearly visible: When ϵ is rather low, higher transfers are given to the most energy-efficient households as they can best convert the additional income into well-being. The higher ϵ, i.e. the higher the social preference to care about (horizontal) equity for a given level of private household inequality aversion, the more it is socially optimal to allocate higher transfers to less energy-efficient households. In the second case (panels C and D) the additional transfers increase in the energy-efficiency decile almost irrespective of social inequality aversion. When climate policy is introduced, it is thus always socially optimal to take horizontal inequality into consideration by redistributing a higher fraction of the carbon tax revenue to less energy-efficient households. For the most energy efficient households the additional transfer payment amounts to 64 (51) €/year for $\epsilon = 0.1$ ($\epsilon = 2$) and covers between 54% and 57% of households' policy welfare costs (measured in EV) due to introducing carbon pricing. The least energy efficient household should receive an additional transfer of 399 (406) €/year for $\epsilon = 0.1$ ($\epsilon = 2$), which covers 91–93% of policy costs.

5.2 Second-best optima

5.2.1 Household-specific energy consumption subsidies

We now turn to the results for household-specific energy consumption subsidies to target horizontal inequality summarized in figure 3. Compared to the first-best optimum mitigation welfare costs (i.e. the welfare costs of achieving the aggregate emission target) increase by 2% (10%) for $\epsilon = 0.1$ ($\epsilon = 2$). The uniform carbon tax amounts to 137 (145) €/tCO_2 for $\epsilon = 0.1$ ($\epsilon = 2$) to achieve the climate target. The carbon tax revenue is redistributed to households through (i) uniform transfers of 196 (156) €/year for $\epsilon = 0.1$ ($\epsilon = 2$), and (ii) household-specific energy subsidies. Figure 3A depicts the optimal energy subsidy (+) or tax (-), measured in percent of the energy price p_E, for each efficiency decile and different degrees of social inequality aversion. Similarly to the case of household-specific transfers, for low values of ϵ, efficiency considerations dominate the socially optimal amount of the subsidy, i.e. households that are very energy efficient receive a preferential treatment by the government in form of a higher subsidy. Contrary, equity considerations dominate for higher ϵ implying higher subsidies for less energy-efficient households.

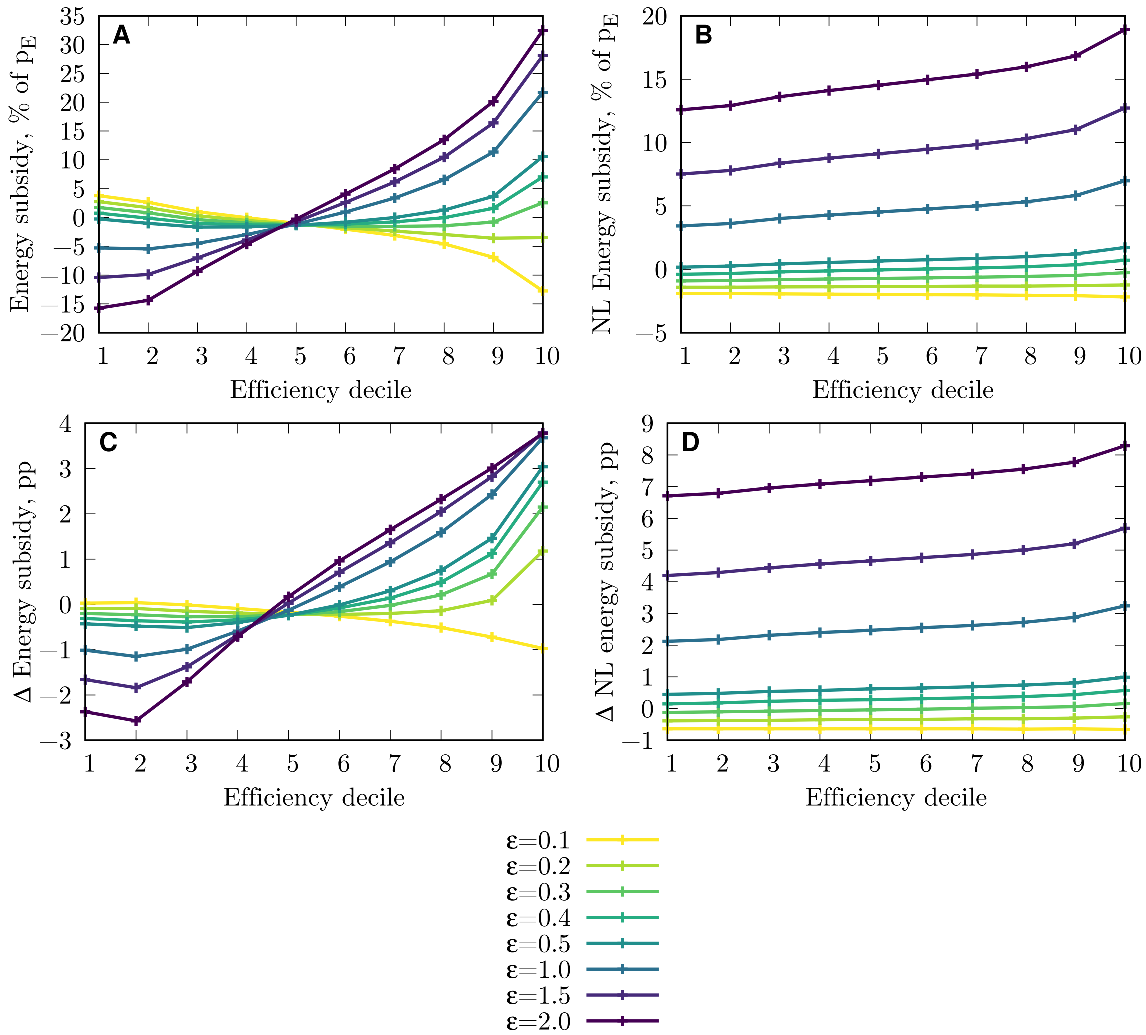

Figure 3: **Optimal household-specific energy consumption subsidies(+)/taxes(-)** for energy-efficiency deciles (1=high, 10=low) and inequality aversion $\epsilon = [0.1; 2]$. Panel **A** shows the optimal energy subsidy (+) or tax (-) with an optimal carbon tax set to achieve a 30% reduction in total carbon emissions, **B** depicts non-linear (NL) subsidies/taxes when the energy efficiency levels cannot be observed by the government, **C** shows the additional subsidy/tax due to the introduction of climate policy and **D** the additional non-linear subsidy/tax for that case. Note the different y-ranges when comparing panel A and B (full subsidies) to panel C and D (Δ subsidies).

In panel 3B we additionally calculate non-linear energy subsidies of the form $\hat{s}_E^j = s_{E_0} + s_{E_1} \times \tilde{E}^j$ for the case when the government cannot directly observe the households' heterogeneous energy-efficiency levels α_j. This is of particular relevance since the energy-efficiency of households will only partly be observable by the government and observation might be subject to measurement error, transaction costs or self-selection problems. Thus, the suggested simple rule for differentiated energy subsidies would have the merit of being able to target horizontal inequality – and thus likely increase the political feasibility of carbon taxation – without knowing the determinants of horizontal inequality in the background. Compared to panel 3A, panel 3B shows that that the simple rule for the carbon tax captures the trade-off between efficiency and horizontal equity in a similar but less pronounced way. Contrary to the household-specific energy subsidy, households facing a non-linear energy subsidy anticipate that their marginal subsidy rate changes with the amount of energy consumed. As marginal energy subsidies increase in energy consumption, the non-linear policy induces excessive energy consumption, implying further dead-weight losses to society. This explains while the marginal subsidy increases in panel 3B are less pronounced than in panel 3A.

While the energy subsidy is almost uniform across household types for low inequality aversion, it favors less-energy efficient households through higher subsides for higher inequality aversion. Panels 3C and 3D present the additional household-specific (3C) and non-linear energy subsidy (3D) solely taking into account the additional effect due to the emission target and measured in terms pf percentage points (pp) increase or decrease as compared to the case without climate policy. The allocation dynamics of subsidies between deciles follows a similar pattern as compared to the case with full subsidies. The optimal range of subsidies between efficiency deciles is reduced by 78% (72%) for $\epsilon = 0.1$ ($\epsilon = 2$) in panel 3C, while it is reduced by 56% (44%) for $\epsilon = 0.1$ ($\epsilon = 2$) in panel 3D.

5.2.2 Subsidy on renewable energy production

The previous policies either relied on perfect observability of households' energy efficiency types or on personalized energy consumption (in case of non-linear energy taxes). If the government can only implement linear (uniform) policies, it can still subsidize the production of renewable energy and thereby alleviate the carbon tax burden for energy-intensive households. This second-best policy results in an increase of mitigation welfare costs by 1% (22%) for $\epsilon = 0.1$ ($\epsilon = 2$). For different levels of inequality aversion, figure

4 shows how the revenue from uniform carbon taxation should be optimally allocated between uniform transfers and subsidies on renewable energy production.

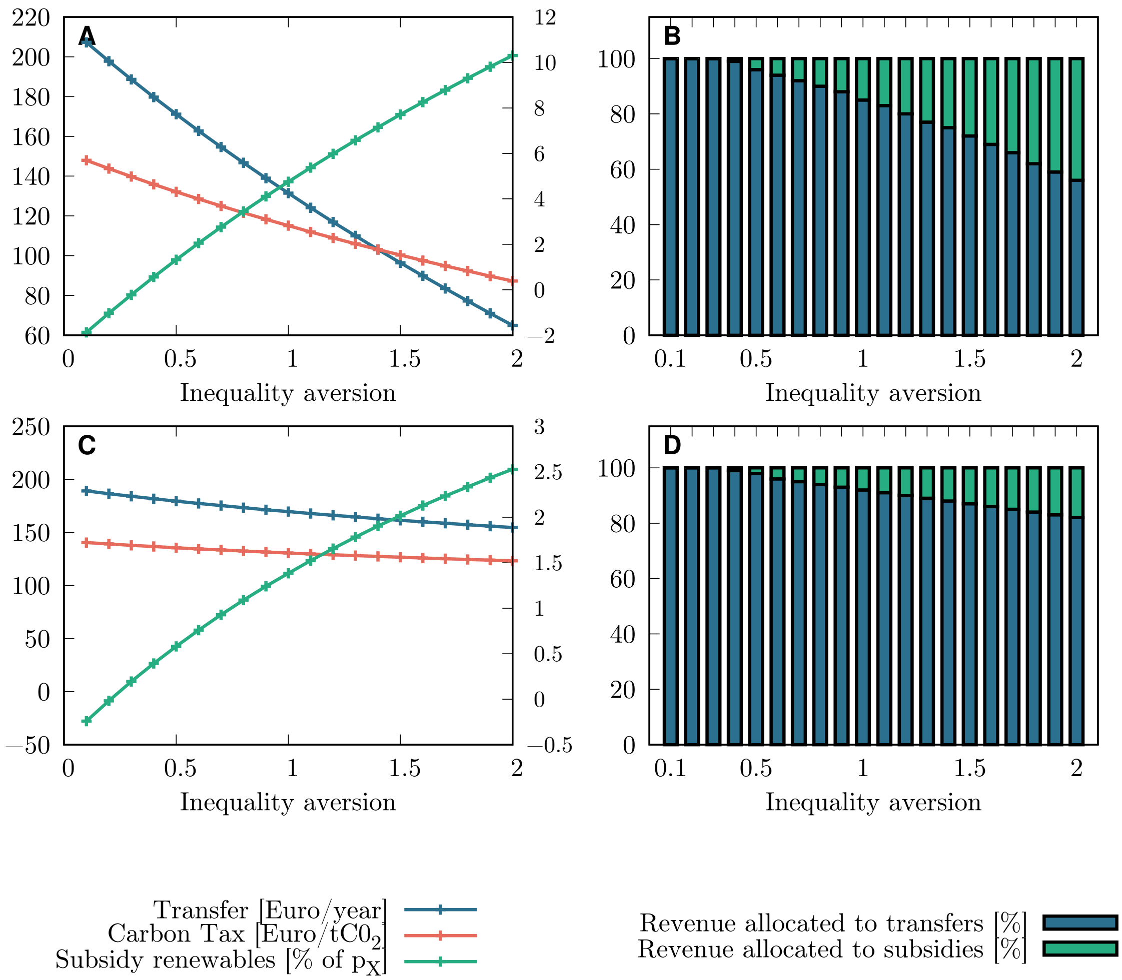

Figure 4: **Optimal uniform carbon tax, uniform transfer and uniform renewable energy subsidies** for different levels of inequality aversion $\epsilon = [0.1; 2]$. Panel **A** shows the optimal uniform carbon taxes, subsidies on renewable energy production and transfers that achieve a 30% reduction in total carbon emissions while **B** depicts the resulting optimal allocation of the government's carbon tax revenue. Panels **C** and **D** present the same information for the additional tax/subsidy/transfer due to the introduction of climate policy.

While panel 4A and 4B depict the full size of policy instruments, panels 4C and 4D isolate the additional effect due to the introduction of climate policy. With increasing social inequality aversion the uniform transfer decreases from 207 to 65 €/year while the uniform carbon tax decreases from 148 to 87 €/tCO_2. The uniform subsidy on renewable energy production, however, increases from -1.9% (i.e. a tax) of the renewable energy price p_X for $\epsilon = 0.1$ to 10.3% for $\epsilon = 2$ (4A). This translates into an optimal allocation of the government's carbon tax revenue between uniform transfers and subsidies (4B). The higher the social inequality aversion, the higher the share of tax revenue allocated to subsidies on renewable energy production. While for low inequality aversion ($\epsilon = 0.1$) 100% of tax revenue is redistributed via equal-per-household transfer payments, 44% of the revenue should be used for renewable energy subsidies for high inequality aversion ($\epsilon = 2$). Panel 4C and 4D present the same results but, similar to the previous analyses, focus on the additional effect due to the introduction of climate policy.[9] The dynamics follow a similar although less pronounced pattern such that for $\epsilon = 2$, 18% of carbon tax revenue should be used for renewable energy subsidies. If climate policy is introduced and the government puts a relatively high value on social equity ($\epsilon = 2$), it is thus optimal to increase the share of governmental revenue spent on subsidies for renewable energy by almost 70% as compared to the case without climate policy.

5.2.3 Comparison of policy instruments to target horizontal inequality

Table 1 provides an overview of the numerical results for the four policy instrument packages to target horizontal equity analysed in this paper. In a last step we now compare the suggested policy instruments in terms of their implied impact on monetized aggregate social welfare by calculating the increase in mitigation welfare costs relative to the first-best optimum. Figure 4 illustrates the results of Propositions 2, 3 and 4 as well as Corollary 2.

The first best solution to tackle horizontal inequality can be achieved by household-specific redistribution of the revenue from uniform carbon taxation. In this case the social welfare impact due to the introduction of climate policy can be minimized at around -0.21% of total household income before redistribution. Without any redistri-

[9]Note that in this case we only eliminate the climate target to calculate the Δ.

PI PACKAGE ⇒	**ΔT, €/year** (% cost cover)	**ΔESUB, pp**	**ΔNL-ESUB, pp**	**Δ RESUB, pp**
Low aversion to horizontal inequality $\epsilon = 0.1$				
H-type 1	64 (54%)	0.03	-0.64	-0.24
H-type 5	159 (81%)	-0.17	-0.64	-0.24
H-type 10	399 (93%)	-0.97	-0.66	-0.24
Δ Transfer €/year	Specific	Uni, 196	Uni, 200	Uni, 189
Δ Carbon tax, €/tCO_2	Uni, 139	Uni, 137	Uni, 141	Uni, 141
Medium aversion to horizontal inequality $\epsilon = 1$				
H-type 1	51 (57%)	-1.01	2.12	1.38
H-type 5	161 (81%)	-0.13	2.47	1.38
H-type 10	406 (91%)	3.68	3.24	1.38
Δ Transfer, €/year	Specific	Uni, 164	Uni, 140	Uni, 170
Δ Carbon tax, €/tCO_2	Uni, 139	Uni, 142	Uni, 132	Uni, 131
High aversion to horizontal inequality $\epsilon = 2$				
H-type 1	70 (57%)	-2.37	6.71	2.53
H-type 5	161 (81%)	0.17	7.19	2.53
H-type 10	406 (91%)	3.78	8.29	2.53
Δ Transfer, €/year	Specific	Uni, 156	Uni, 62	Uni, 155
Δ Carbon tax, €/tCO_2	Uni, 139	Uni, 145	Uni, 121	Uni, 123

Table 1: **Policy instrument (PI) packages to target horizontal equity**. Household-types: 1= Most efficient; 5 = Medium efficient; 10 = Least efficient. Policy instrument packages: ΔT: Transfer payments (€/year); ΔESUB: Energy consumption subsidies; ΔNL-ESUB: Non-linear energy consumption subsidies; Δ RESUB: Subsidies for renewable energy consumption.'pp' refers to percentage points increase as compared to the case without climate policy.

bution of carbon tax revenue the increase in mitigation welfare costs is around 500% higher as compared to the first-best optimum. Between these two extreme options the government has a number of policy instrument packages at its disposal that each include redistribution via equal-per-household transfers resulting in increases in mitigation welfare costs ranging between 2% and 29% relative to first-best optimum depending on the policy instrument package and the level of social inequality aversion. The policy that performs best among the analysed second-best options is to allocate part of the carbon tax revenue to household-specific subsidies on energy consumption, which results in increasing mitigation welfare costs of 2% (10%) for $\epsilon = 0.1$ ($\epsilon = 2$). Non-linear subsidies on energy consumption, that the government could implement without perfect knowledge about the households' energy efficiency decile, perform only slightly worse

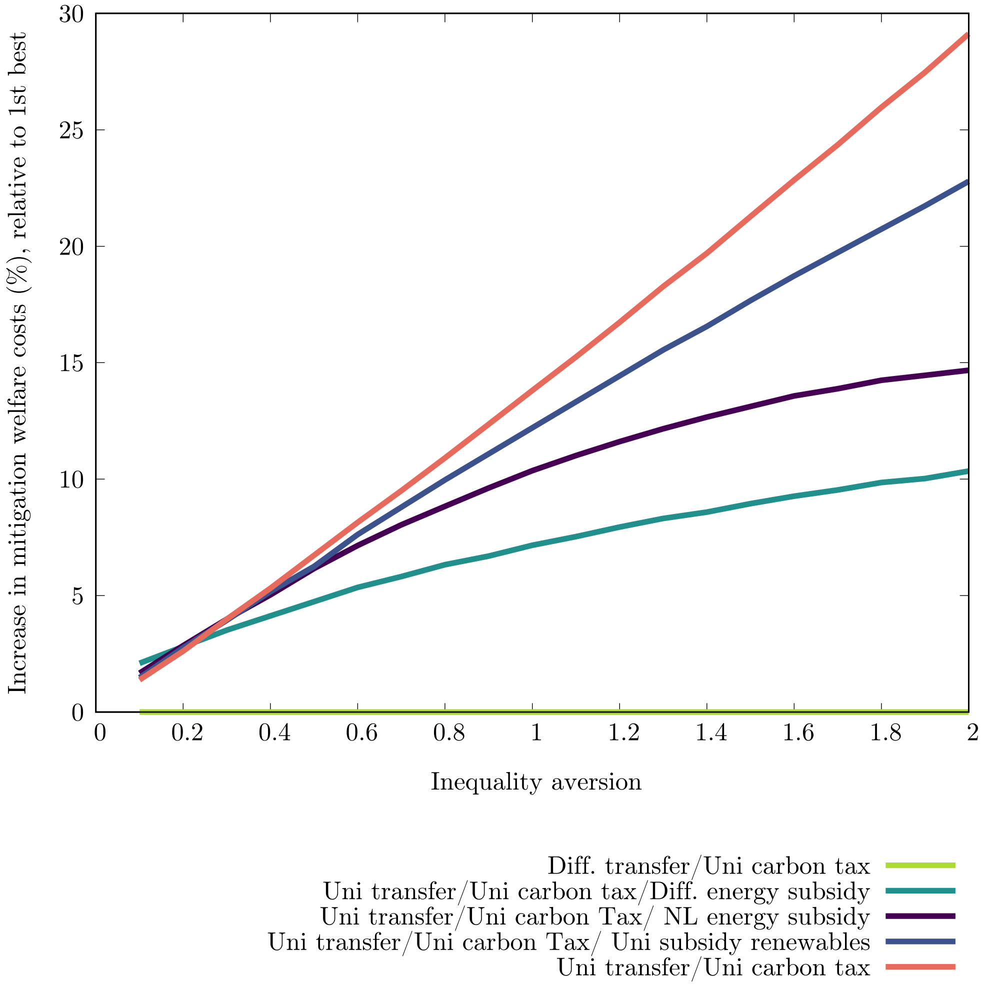

Figure 5: **Increase in mitigation welfare costs.** Percentage change relative to the first-best case (household-specific transfers and uniform carbon tax). Without redistribution of carbon tax revenue the increase in welfare costs amounts to between 481% ($\epsilon = 0.1$) and 523% ($\epsilon = 2$).

and imply rising welfare cost of 2% (15%) for $\epsilon = 0.1$ ($\epsilon = 0.1$). While it might be possible to find other non-linear energy subsidy rules that perform better in terms of welfare than our simple rule, non-linear policies can never outperform the differentiated energy subsidy policy. Hence, welfare improvements due to other functional forms are quantitatively very moderate.

In case the government cannot implement any of the preceding household-specific policy instrument packages, it can still dampen the adverse effects of carbon taxation on hardship households by implementing subsidies on renewable energy production. This will help to reduce the carbon-intensive lifestyle of households by accelerating the transformation towards a carbon-free energy system. In this case the increase in mitigation welfare costs amounts to 1% (22%) for $\epsilon = 0.1$ ($\epsilon = 2$).

6 Conclusions

In this paper we have developed a welfare-theoretic model of optimal carbon taxation and redistribution when households differ in their ability to convert energy to well-being. This is, for example, the case for commuting households or households living in badly insulated homes or households living in areas with poor access to public transportation infrastructure (like rural areas). While these conditions can change in the very long-run due to investments, they are rather inflexible in the short to medium-run. Our approach allows to derive optimal climate policy in a first-bet setting (with perfect information by governments) and various second-best settings (with imperfect information by governments) that take into account this heterogeneity – and the associated horizontal distributional effects of climate policy.

The key findings can be summarized as follows. Within a standard social welfare framework, horizontal inequality effects are already accounted for. Nevertheless, it is not always socially optimal to reduce horizontal inequality due to equity-efficiency trade off. If and only if the social inequality aversion is sufficiently large, the equity motive dominates and households with low ability to convert energy into well-being receive larger redistributive resources like transfers and subsidies. Otherwise, energy-efficient households receive larger transfers as they are better capable of converting scarce resources into well-being.

When the government can observe the energy efficiency type, the first-best policy that maximizes social welfare and optimally addresses the (horizontal) equity–efficiency trade-off is a uniform carbon tax which is combined with household-specific transfer payments. When governments cannot observe the household type, a number of second-best approaches remain: Uniform carbon taxes and transfers can be combined with non-linear energy subsidies, which change with households' energy consumption. Such a policy would also be compatible with potential mimicking behavior of households (i.e. it satisfies the self-selection constraints as households do not have incentives to pretend to be another household type). Non-linear energy subsidies require, however, a fraud-proof monitoring of personalized energy use. This may be easy for grid-based energy consumption (electricity and natural gas) but might also involve substantial transaction costs for monitoring gasoline, fuel or heating oil consumption.

Compared to these approaches, linear policies – i.e. uniform taxes or subsidies on factors – have the lowest informational and administrative challenges. As they are not well targeted to address household heterogeneity, they also perform worse in terms of social welfare. Recycling all revenues from carbon pricing back to households on an equal-per capita base would be the most straight-forward approach in this setting. Using some revenues from carbon pricing to subsidize renewable energy, however, increases welfare further because energy-intensive households benefit from cheaper (and cleaner) energy. Horizontal equity concerns may therefore constitute a new second-best rationale for renewable energy policies, besides technological innovation issues (Kalkuhl et al. 2012). In our calibrated numerical model, we show that the majority of carbon pricing revenues should still be transferred back to households on an equal per capita base.

With our numerical analysis, we can quantify the trade-offs between informationally demanding policies vs. simpler linear policy mixes. When social inequality aversion is small, the social-welfare adjusted mitigation costs of achieving an emission target increase only marginally, by less than 5 percent. When inequality aversion is large, linear policies increase the costs of reducing emissions by more than 25 percent. It can therefore be valuable to identify targeted transfers wherever this is possible at low administrative and incentive costs. In our numerical analysis, optimal targeted transfers from introducing a carbon price of 139 EUR/tCO_2 are six to eight times higher (399-406 €/year) for energy-intensive households compared to energy-efficient households (51–64 €/year). Examples for observable energy efficiency types are energy certificates of buildings or commuting distances – but targeted transfers should be designed to avoid perverse incentives that prevent the adoption of better technologies. This could be done by linking transfers to conditions at a specific closing date or by phasing out transfers over time.

These insights put existing climate policy packages also into a new light. The introduction of a national carbon price in Germany in 2021 was, for example, combined with reductions in energy prices, large subsidies for carbon-saving and low-carbon technologies (like electric vehicles, heat pumps, building insulation) and the introduction of a temporary long-distance commuting allowance which can be deduced from the income tax (Edenhofer et al. 2020). From a horizontal equity point of view, subsidy programs – consuming three quarters of the carbon price revenues – seem to be exaggerated. Other programs like the temporary long-distance commuting allowance, in turn, could conceptually be understood as a targeted transfer to particularly energy-intensive households.

For the sake of clarity, we focused in the numerical analysis on the horizontal dimension only. Our unique contributions were to show how horizontal equity can be integrated into a welfare-theoretic optimal taxation model and which implications follow from this for the design of optimal climate policy. Future research could extend our framework by adding vertical distributional effects for designing optimal tax reforms. In such a setting, carbon pricing will then not only impact the distribution of costs within income groups but also – through non-linear Engel curves – across income groups. Adding the vertical dimension requires to introduce a labor-leisure trade-off which will further generate heterogeneous effects on labor supply. A model in this direction could show very rich dynamics due to various mechanisms and channels. Ultimately, a proper modeling of vertical *and* horizontal equity effects can lay the foundation for a consistent and rational debate about fair climate policy and the just transition.

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Appendix A Proofs for Section 3

A.1 Proof of Proposition 2

Proof. The Lagrangian of the government's optimization problem is

$$L^G = W(v^1, ..., v^n) + \mu(E^* - \sum_j \widetilde{E}^j) + \gamma \sum_j \left(t_E \widetilde{E}^j - R^j\right) \tag{21}$$

The first order conditions for the optimal individual carbon taxes and lump-sum transfers read

$$\frac{\partial L^G}{\partial t_E} = \sum_j \frac{\partial W}{\partial v^j}\frac{\partial v^j}{\partial q} - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial q} + \gamma \left(t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial q} + \sum_j \widetilde{E}^j\right) = 0 \tag{22}$$

$$\frac{\partial L^G}{\partial R^j} = \frac{\partial W}{\partial v^j}\frac{\partial v^j}{\partial I^j} - \mu \frac{\partial \widetilde{E}^j}{\partial I^j} + \gamma \left(t_E \frac{\partial \widetilde{E}^j}{\partial I^j} - 1\right) = 0 \tag{23}$$

If the government sets $t_E = \frac{\mu}{\gamma}$, then by using (4) equation (23) can be shown to be identical to equation (8). By using (5), equation (23) can be shown to be identical to equation (7).

With $t_E = \frac{\mu}{\gamma}$, Roy's identity $\frac{\partial v^j}{\partial q} = -\lambda^j \widetilde{E}^j$, equation (5) and using (7), equation (22) can be shown to hold.

Using (7) and (8), we can derive two different expressions for W_u^j. Eliminating the latter yields (10). □

A.2 Proof of Corollary 2

Proof. The conditions for the first best to hold are (7) and (8). The latter can be shown to hold by reformulating the government's FOCs (11) and (12). The former, however, is violated as soon as $\Phi_j \neq 0$. To see this, recall (5), which can be plugged into (8) to yield

$$W_u^j u_E^j \alpha_j = \gamma p + \gamma t_E^j$$

If $\Phi_j = 0$, then $t_E^j = \frac{\mu}{\gamma}$ and (7) holds. Otherwise, this condition for the first best allocation does not hold. □

A.3 Proof of Proposition 4

Proof. By eliminating p_E from the energy producer's first-order conditions (14) and (15), we can obtain a relation between tax and subsidy.

$$\tau_Z = \frac{\tilde{E}_Z}{\tilde{E}_X}(p_X - s_X) - p_Z \tag{24}$$

The governments Lagrangian is

$$L^{gov} = W(v^1, ..., v^n) + \mu(Z^* - Z) + \gamma\left(\tau_Z Z - nR - s_X X\right)$$

and the first-order conditions are

$$0 = L^{gov}_{\tau_Z} = \sum_i W_{v^i} v^i_q q^i_{\tau_Z} - \mu Z_{\tau_Z} + \gamma(\tau_Z Z_{\tau_Z} + Z - s_X X_{\tau_Z})$$
$$0 = L^{gov}_{R} = \sum_i W_{v^i} v^i_b - \mu Z_R + \gamma(\tau_Z Z_b - n - s_X X_R)$$
$$0 = L^{gov}_{s_X} = \sum_i W_{v^i} v^i_q q^i_{s_X} - \mu Z_R + \gamma(\tau_Z Z_{s_X} - s_X X_{s_X} - X)$$

From this and equation (24) we get

$$s_X = \frac{\frac{\sum_i W_v^i \lambda^i \tilde{E}^i}{\tilde{E}_Z}\left(1 - (p_E - p_Z + p_X \frac{\tilde{E}_Z}{\tilde{E}_X})\frac{\tilde{E}_{XZ} X_{\tau_Z} + \tilde{E}_{ZZ} Z_{\tau_Z}}{\tilde{E}_Z}\right) + \mu Z_{\tau_Z}(1 + p_Z - p_X \frac{\tilde{E}_Z}{\tilde{E}_X})}{-\frac{\sum_i W_v^i \lambda^i \tilde{E}^i (E_{XZ} X_{\tau_Z} + \tilde{E}_{ZZ} Z_{\tau_Z})}{\tilde{E}_X \tilde{E}_Z} - \gamma\left(Z_{\tau_Z}\frac{\tilde{E}_Z}{\tilde{E}_X} + X_{\tau_Z}\right)}$$

which is non-zero in general. □

Appendix B Efficiency enhancing investments by households

In the long run, households can make investments in efficiency-enhancing capital, i. e. $x^j \in \mathbb{R}$.

B.0.1 Social planner economy

Analogously to the short-run described above, the social planner now chooses an allocation of numeraire consumption, energy services consumption and investments in energy efficiency to maximize welfare. The Lagrangian, hence, is

$$L^{SP} = W(u^1, ..., u^n) + \mu(E^* - \sum_j \widetilde{E}^j) + \gamma(\sum_j y^j - c^j - p\widetilde{E}^j - x^j), \tag{25}$$

Proposition 1. *In the long-run, the social optimum in our model can be achieved under the condition that*

$$0 = W_u^j u_E^j \alpha_j - \mu - \gamma p \quad \forall j \tag{26}$$

$$0 = W_u^j u_c^j - \gamma \quad \forall j \tag{27}$$

$$0 = W_u^j u_E^j \widetilde{E}^j f'(x_0^j + x^j) - \gamma \quad \forall j \tag{28}$$

B.0.2 Decentralized market economy

Household's optimization then yields the Lagrangian

$$L^H = u(c^j, E^j) + \lambda^j(y - c^j - \frac{q^j E^j}{\alpha_j} - (1 - s^j)x^j), \tag{29}$$

We assume that the government implements a combination of the following household-specific or uniform instruments: energy taxes t_E^j, transfers R^j and subsidies s^j on efficiency-enhancing investments. Household's optimization then yields FOCs

$$u_c^j = \lambda^j \tag{30}$$

$$u_E^j = \frac{\lambda^j q^j}{f(x_0^j + x^j)} \tag{31}$$

$$u_E^j \widetilde{E}^j f' = \lambda^j (1 - s^j) \tag{32}$$

Uniform tax, individual transfers, no subsidy. The government maximizes social welfare $W(u^1, ..., u^n)$, subject to the environmental target $\bar{E} = \sum_j \widetilde{E}^j$ and its budget constraint $\sum_j (t_E \widetilde{E}^j - R^j) = 0$. The government's Lagrangian is now

$$L^G = W(v^1, ..., v^n) + \mu(E^* - \sum_j \widetilde{E}^j) + \gamma \sum_j \left(t_E \widetilde{E}^j - R^j \right) \tag{33}$$

and the FOCs

$$L_{R^j}^G = W_{v^j} v_b^j - \mu \frac{\partial \widetilde{E}^j}{\partial b^j} - \gamma + \gamma t_E \frac{\partial \widetilde{E}^j}{\partial b^j} = 0 \quad \forall j \tag{34}$$

$$L_{t_E}^G = \sum_j W_{v^j} v_q^j - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial q} + \gamma \sum_j \widetilde{E}^j + \gamma t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial q} = 0 \tag{35}$$

Proposition 2. *In the long-run, the government can achieve the 1st-best optimum by using individual transfers and a uniform tax on energy use. The latter is determined by*

$$t_E^* = \frac{\mu}{\gamma} \tag{36}$$

The optimal transfers are determined indirectly by

$$\frac{\mu}{\gamma} + p = \alpha_j \frac{{u_E}^*}{{u_c}^*} = \alpha_j MRS^* \;\forall j \tag{37}$$

and the governments budget

$$t_E^* E^* = \sum_j R^{j*}. \tag{38}$$

Proof. The proof is analogous to Proposition 2 □

Uniform tax, uniform transfers, uniform subsidy. The government's Lagrangian is now

$$L^G = W(v^1, ..., v^n) + \mu(E^* - \sum_j \widetilde{E}^j) + \gamma \sum_j \left(t_E \widetilde{E}^j - R - s x^j \right) \tag{39}$$

and the FOCs

$$L_R^G = \sum_j W_{v^j} v_b^j - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial b^j} - n\gamma + \gamma t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial b^j} - \gamma s \sum_j \frac{\partial x^j}{\partial b^j} = 0 \tag{40}$$

$$L_{t_E}^G = \sum_j W_{v^j} v_q^j - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial q} + \gamma \sum_j \widetilde{E}^j + \gamma t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial q} - \gamma s \sum_j \frac{\partial x^j}{\partial q} = 0 \tag{41}$$

$$L_s^G = \sum_j W_{v^j} v_s^j - \mu \sum_j \frac{\partial \widetilde{E}^j}{\partial s} + \gamma t_E \sum_j \frac{\partial \widetilde{E}^j}{\partial s} - \gamma \sum_j x^j - \gamma s \sum_j \frac{\partial x^j}{\partial s} = 0 \tag{42}$$

Proposition 3. *With linear tax and transfer, additionally using a subsidy on x^j can be welfare enhancing.*

Proof. The FOCs can be reformulated

$$t_E = \frac{\mu}{\gamma} - \frac{\sum_j W_v^j \frac{\lambda^j}{\gamma}}{\sum_j \widetilde{E}_b^j} + \frac{n}{\sum_j \widetilde{E}_b^j} + s \frac{\sum_j x_b^j}{\sum_j \widetilde{E}_b^j}$$

$$t_E = \frac{\mu}{\gamma} + \frac{\sum_j W_v^j \frac{\lambda^j}{\gamma} \widetilde{E}^j}{\sum_j \widetilde{E}_q^j} - \frac{\sum_j \widetilde{E}^j}{\sum_j \widetilde{E}_q^j} + s \frac{\sum_j x_q^j}{\sum_j \widetilde{E}_q^j}$$

$$t_E = \frac{\mu}{\gamma} - \frac{\sum_j W_v^j \frac{\lambda^j}{\gamma} x^j}{\sum_j \widetilde{E}_s^j} + \frac{\sum_j x^j}{\sum_j \widetilde{E}_s^j} + s \frac{\sum_j x_s^j}{\sum_j \widetilde{E}_s^j}$$

Eliminating t_E by equalizing the first two expressions yields

$$s = \frac{\sum_j \widetilde{E}_q^j \left(\sum_j W_v^j \frac{\lambda^j}{\gamma} - n \right) - \sum_j \widetilde{E}_b^j \left(\sum_j W_v^j \frac{\lambda^j}{\gamma} \widetilde{E}^j + \sum_j \widetilde{E}^j \right)}{\sum_j x^j \sum_j \widetilde{E}_q^j - \sum_j x_q^j \sum_j \widetilde{E}_b^j}$$

In general, the expression for s is non-zero.

$\square$

Appendix C Curvature of the social welfare function

Households' utility and social welfare are given by

$$u^j(c^j, \tilde{E}^j) = \frac{\left(c^{j\beta}\left(\alpha_j \tilde{E}^j - \bar{E}\right)^{1-\beta}\right)^{1-\eta}}{1-\eta},$$
$$W(u) = \sum_j (1-\eta)^{-\gamma} \frac{u^{j\,1-\gamma}}{1-\gamma}$$

η	ϵ for $\gamma = 0$	ϵ for $\gamma = -1$	ϵ for $\gamma = -2$
0.1	0.1	-	-
0.2	0.2	-	-
0.3	0.3	-	-
0.4	0.4	-	-
0.5	0.5	-	-
0.6	0.6	0.2	-
0.7	0.7	0.4	0.1
0.8	0.8	0.6	0.4
0.9	0.9	0.8	0.7
1	1	1	1
1.1	1.1	1.2	1.3
1.2	1.2	1.4	1.6
1.3	1.3	1.6	1.9
1.4	1.4	1.8	2.2
1.5	1.5	2	2.5
1.6	1.6	2.2	2.8
1.7	1.7	2.4	3.1
1.8	1.8	2.6	3.4
1.9	1.9	2.8	3.7
2	2	3	4

Table C.1: Relationship between the curvature of the individual household utility funcion η, governmental inequality aversion γ and the combined concavity of the social welfare function ϵ determining social inequality aversion. Note that γ needs to be negative to ensure positive values for ϵ (Kaplow 2010). W.l.o.g. we use ϵ for $\gamma = 0$ for the numerical model in this paper.

Appendix D Additional data for calibration

	Price	Unit	Price (ct/kwh)	Emission factor (tCO2/TJ)	Price (€/tCO2)	Consumption share
Electricity	30.19	ct/kwh	30.19	130.00	645.11	0.22
Natural gas	6.53	ct/kwh	6.53	55.90	324.55	0.18
Coal	4.57	ct/kwh	4.57	97.75	129.86	0.08
Heating oil	69.40	€/100l	7.73	77.65	276.52	0.12
Gasoline	1.46	€/l	16.31	73.10	619.60	0.20
Diesel	1.32	€/l	13.23	74.00	496.57	0.16
Heat	23.28	€/GJ	8.38	63.89	364.32	0.04

Table D.2: Energy prices and consumption shares. Price data is taken from BMWI (Entwicklung von Energiepreisen und Preisindizes, https://www.bmwi.de/Redaktion/DE/Artikel/Energie/energiedaten-gesamtausgabe.html). Emission-weighted consumption shares for households are derived from the environmental national accounts (Destatis 2020).

j	α_j
1	5.007
2	2.504
3	1.233
4	0.915
5	0.745
6	0.628
7	0.538
8	0.456
9	0.371
10	0.256

Table D.3: Calibrated energy efficiency levels α_j.