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Introduction

We begin this chapter with an extended example based on the linear expenditure system (LES). We use the LES to introduce the four aspects of demand system specification that we discuss in detail in the next four chapters: functional form specification (Chapter 2), the role of demographic variables (Chapter 3), dynamic structure (Chapter 4), and stochastic structure (Chapter 5). In Chapters 6 and 7 we present estimates based on these specifications using household budget data and per capita time series data.

1. FUNCTIONAL FORM: THE LES

In this section we introduce the LES and establish the terminology and notation we use throughout the book. Consider a direct utility function, U(X), of the form

$$(1) \quad U(X) = \sum_{k=1}^{n} a_k \log(x_k - b_k), \qquad a_i > 0, \quad (x_i - b_i) > 0, \qquad \sum_{k=1}^{n} a_k = 1,$$

where x_i denotes the quantity of good i and n the number of goods. It is useful to think of the goods as broad aggregates such as food and clothing, rather than as narrowly specified commodities such as bread, butter, and jam, although the theory does not require this or any other interpretation. We shall use the terms "goods" and "commodities" interchangeably.[1]

We write the budget constraint as

$$(2) \qquad \sum p_k x_k = \mu.$$

[1] Distinguishing between goods and commodities is important when discussing household production. Becker's [1965] household production model postulates that households combine "market goods" such as food or clothing (bread or running shoes) with time to produce "basic commodities" such as health and prestige; these commodities are the arguments of the household's utility function. Becker's terminology is now firmly established and, although ordinary English usage suggests reversing Becker's use of "goods" and "commodities," it is now too late. Michael and Becker [1973] provide a sympathetic restatement of the household production model; Pollak and Wachter [1975] emphasize its limitations.

When writing summation signs, we shall often omit the index and limits of summation, adopting the conventions that the omitted index is k and that the summation runs from 1 to n (i.e., over all goods).

Maximizing the utility function (1) subject to the budget constraint (2) yields the ordinary (i.e., Marshallian) demand functions, $h^i(P, \mu)$,

$$x_i = h^i(P, \mu) = b_i - \frac{a_i}{p_i} \sum p_k b_k + \frac{a_i}{p_i} \mu \tag{3}$$

where the p's denote prices and μ denotes total expenditure on the n goods, hereafter called "expenditure." At first glance the LES appears to have 2n parameters (n b's and n a's) but, because the a's must satisfy the normalization rule $\sum a_k = 1$, only $2n - 1$ of the LES parameters are independent. We shall often write demand functions in "expenditure form"

$$p_i x_i = p_i b_i + a_i\left(\mu - \sum p_k b_k\right). \tag{4}$$

Indeed, the LES owes its name to the fact that expenditure on each good is a linear function of all prices and expenditure.

As our use of the term "expenditure" in place of "income" suggests, we interpret demand theory as a model of expenditure allocation among an exhaustive set of consumption categories. Our terminology, although not yet standard, is convenient in empirical demand analysis because it enables us to distinguish between a household's receipts in a particular period (i.e., its "income," as reported in household budget data) and its total spending on the consumption categories included in the analysis (i.e., its "expenditure" in our terminology). Somewhat inconsistently, we defer to well-established usage and refer to "income–consumption" curves rather than "expenditure–consumption" curves.

A demand system consistent with utility maximization is said to be "theoretically plausible." The LES is the only theoretically plausible demand system for which expenditure on each good is a linear function of all prices and expenditure. This result was established by Klein and Rubin [1947–1948] in their paper introducing the LES. Samuelson [1947–1948] subsequently showed that the LES could be derived from the direct utility function (1). Because the type of argument used to establish the Klein–Rubin theorem can be applied to other demand system specifications, we prove this result in Appendix A.

The budget share devoted to good i, which we denote by w_i, is obtained by dividing the expenditure form of the demand equation by μ; we denote the budget share equations by $\omega^i(P, \mu)$. For the LES the budget share equations are given by

$$w_i = \omega^i(P, \mu) = \frac{p_i b_i}{\mu} + a_i\left[1 - \frac{\sum p_k b_k}{\mu}\right]. \tag{5}$$

The LES is transparent in the sense that its parameters have straightforward behavioral interpretations. A household whose demand system is an LES is often described as first purchasing "necessary," "subsistence," or "committed" quantities of each good ($b_1, \ldots, b_n$), and then dividing its remaining or "supernumerary" expenditure, $\mu - \sum p_k b_k$, among the goods in fixed proportions ($a_1, \ldots, a_n$). Thus, the quantities ($b_1, \ldots, b_n$) can be interpreted as a "necessary basket." In any demand system the "marginal budget shares" are defined as the fractions of an additional dollar of expenditure spent on each good:

$$\frac{\partial p_i h^i(P, \mu)}{\partial \mu}. \tag{6}$$

Marginal budget shares must sum to 1 and, for noninferior goods, are nonnegative. For the LES the marginal budget shares are constants, that is, they are independent of prices and expenditure, and they are equal to the a's. Goldberger [1969] has proposed a useful characterization of the LES in terms of marginal budget shares: the LES is the only demand system generated by an additive direct utility function that exhibits constant marginal budget shares.

Own-price, cross-price, and expenditure elasticities of demand for the LES are easily calculated. Let $E^i_j(P, \mu)$ denote the elasticity of demand for good i with respect to p_j and $E^i_\mu(P, \mu)$ the expenditure elasticity:

$$E^i_i(P, \mu) = \frac{p_i b_i (1 - a_i)}{p_i b_i + a_i\left(\mu - \sum p_k b_k\right)} - 1 \tag{7}$$

$$E^i_j(P, \mu) = \frac{- a_i b_j p_j}{p_i b_i + a_i\left(\mu - \sum p_k b_k\right)} \tag{8}$$

$$E^i_\mu(P, \mu) = \frac{a_i \mu}{p_i b_i + a_i\left(\mu - \sum p_k b_k\right)}. \tag{9}$$

Because the LES price and expenditure elasticities are functions of all prices and expenditure rather than constants, these elasticities do not provide a transparent summary of the behavior implied by a particular set of LES parameters. Indeed, because the LES parameters have a straightforward behavioral interpretation, the parameter values themselves provide the most transparent summary statistics for the LES.

What does the LES indifference map look like? We begin with the Cobb–Douglas, which is a very simple special case of the LES. The Cobb–Douglas direct utility function can be written as

$$U(X) = \sum a_k \log x_k, \qquad a_i > 0, \qquad \sum a_k = 1, \tag{10a}$$

or, equivalently, in its more familiar "constant returns to scale" form

$$\text{(10b)} \qquad V(X) = \prod x_k^{a_k}, \qquad a_i > 0, \qquad \sum a_k = 1.$$

Figure 1 illustrates the indifference map corresponding to the Cobb–Douglas. The corresponding ordinary demand functions are given by

$$\text{(11)} \qquad x_i = h^i(P, \mu) = \frac{a_i}{p_i}\mu,$$

or, in expenditure form,

$$\text{(12)} \qquad p_i x_i = p_i h^i(P, \mu) = a_i \mu.$$

The Cobb–Douglas demand functions imply that consumption of each good is proportional to expenditure or, equivalently, that the income–consumption curves are rays from the origin. In the Cobb–Douglas case, the marginal budget shares are constants and are equal to the average budget shares.

The indifference map corresponding to the LES utility function, (1), is a Cobb–Douglas indifference map with the origin "translated" to the point $(b_1, \ldots, b_n)$. This indifference map is shown in Figure 2.

The indifference map in Figure 2 helps clarify two problems we have thus far ignored. The first, the "limited-domain problem," arises because the LES utility function is defined only in the region of the commodity space northeast of $(b_1, \ldots, b_n)$; the income–consumption curves are straight

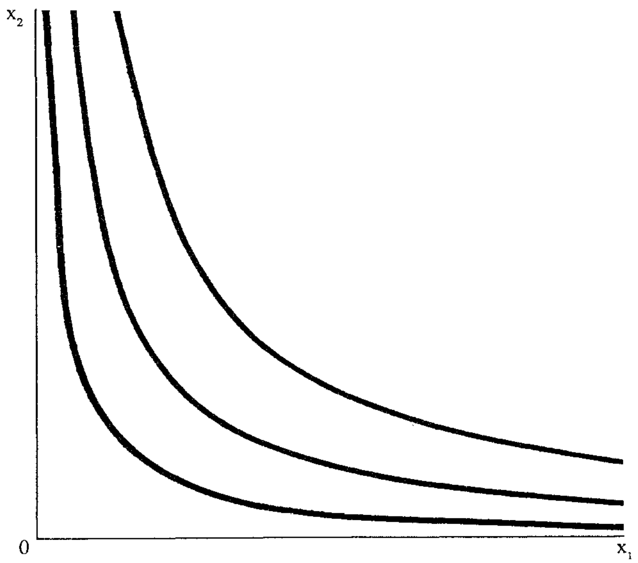

Figure 1 Cobb-Douglas indifference map

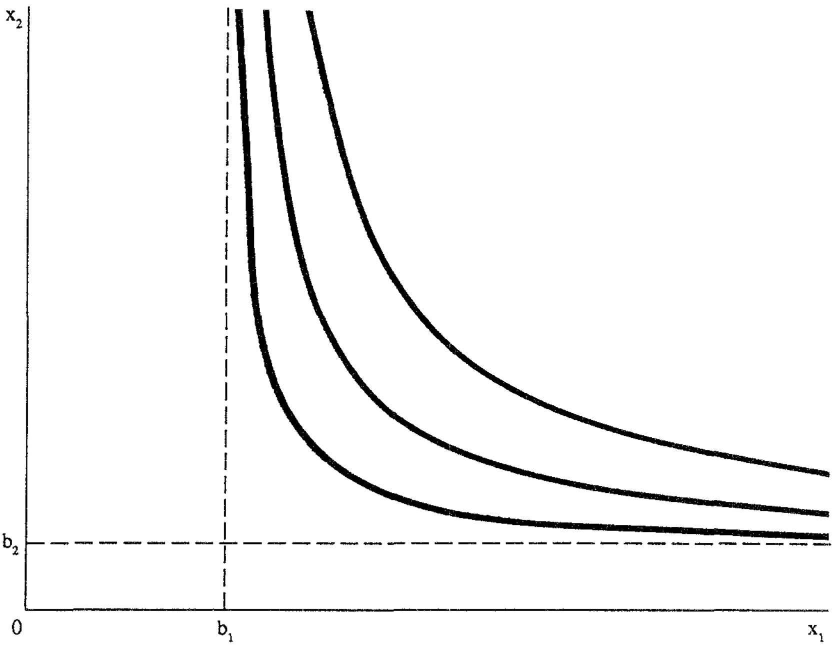

Figure 2 LES indifference map

lines radiating upward from this point. If the household can purchase a commodity bundle in this region, then it will do so; if it cannot, then, because the utility function is not defined if $x_i \leqslant b_i$ for any i, we can say nothing about the household's behavior.

If we try to finesse the limited-domain problem by beginning with the demand functions rather than with the utility function, then the limited-domain problem manifests itself in a different form—as a violation of "regularity conditions." The Slutsky matrix is defined as the $n \times n$ matrix of substitution terms $S(P, \mu) = [s^{ij}(P, \mu)]$, where

$$(13) \qquad s^{ij}(P, \mu) = \frac{\partial h^i(P, \mu)}{\partial p_j} + h^j(P, \mu)\frac{\partial h^i(P, \mu)}{\partial \mu}.$$

A preference ordering is said to be "well-behaved" if it is representable by a direct utility function that is monotonically increasing in all its arguments, has continuous second partial derivatives, and is strictly quasi-concave.[2] A demand system derivable from a well-behaved preference ordering satisfies three regularity conditions. First, the budget

[2]The requirement that the utility function be strictly quasi-concave, $U(X^a) \geqq U(X^b)$, $X^a \neq X^b$, implies $U[\lambda X^a + (1-\lambda)X^b] > U(X^b)$ for all λ, $0 < \lambda < 1$, corresponds to the requirement that the indifference curves have the "right" curvature and no linear segments.

constraint holds as an identity

$$(14) \qquad \sum p_k h^k(P, \mu) \equiv \mu.$$

Second, the demand system is homogeneous of degree 0 in prices and expenditure

$$(15) \qquad h^i(\lambda P, \lambda\mu) = h^i(P, \mu), \qquad \text{for all } \lambda > 0.$$

Third, the Slutsky matrix, $S(P, \mu)$, is symmetric and negative semidefinite. Furthermore, any demand system satisfying these regularity conditions is theoretically plausible, that is, it can be rationalized by a well-behaved preference ordering. For the LES the substitution terms are given by

$$(16) \qquad s^{ii}(P, \mu) = -\frac{a_i}{p_i^2}(1 - a_i)\left(\mu - \sum p_k b_k\right)$$

$$(17) \qquad s^{ij}(P, \mu) = \frac{a_i\, a_j}{p_i\, p_j}\left(\mu - \sum p_k b_k\right), \qquad i \neq j.$$

At price–expenditure situations for which μ is less than $\sum p_k b_k$, the LES own-substitution terms are positive, a violation of regularity conditions.

The second problem that the indifference curve diagram helps clarify, the "nonnegativity problem," arises because the conventional LES demand functions, (3), were derived without regard for nonnegativity constraints on consumption; hence, in some price–expenditure situations, the conventional LES demand functions predict negative consumption of some goods. Of course, this is nonsense: the conventional demand functions are correct only when they coincide with the true LES demand functions (i.e., those derived taking proper account of the nonnegativity conditions) and this occurs at and only at those price–expenditure situations for which the conventional demand functions predict non-negative consumption of all goods. The case in which all of the b's are negative provides the clearest example; with two goods, the point (b_1, b_2) lies in the third quadrant. The income–consumption curves in the first quadrant are linear, and their linear extensions pass through the point (b_1, b_2); thus, even when the b's are negative we can describe the income–consumption curves as radiating upward from the point $(b_1, \ldots, b_n)$. When the b's are negative, however, we must abandon our heuristic interpretation of these parameters as necessary, subsistence, or committed quantities.

To find the true demand functions corresponding to price–expenditure situations for which the conventional LES demand functions predict negative consumption of some goods, we must determine which goods will not be consumed, drop them from the analysis, and use the suitably reduced LES to calculate the demand for the remaining goods. Both theoretical and empirical analysis are simplified if we ignore the nonnegativity problem and restrict our attention to a region of the price–expenditure space in which only the goods in some prespecified subset are consumed in strictly positive quantities. To achieve this simplification, we appeal to the friendly fairy who helps economists: she tells us at the outset which goods to drop from the analysis. In empirical work nonnegativity constraints can be ignored when using data for broad commodity groups that have been aggregated over households. At the other extreme, we must confront the nonnegativity problem when using household level data for narrowly defined goods.

When all the b's are positive, the conventional LES demand functions predict positive consumption at price–expenditure situations for which the regularity conditions hold. Hence, when all the b's are positive, the limited-domain problem takes precedence over the nonnegativity problem. When all the b's are negative, the LES utility function defines a well-behaved preference ordering over the entire commodity space, so the limited-domain problem does not arise. In this case, however, there must exist a region of the price–expenditure space in which the nonnegativity problem arises.

All the b's need not be of the same sign. Examination of the elasticity formulas (7), (8), and (9), confirms that demand for the ith good is inelastic if b_i is positive and elastic if b_i is negative. When the x's are broad commodity groups, we would expect inelastic demand and, hence, positive b's. The signs of the b's are an empirical question, however, and cannot be determined on the basis of a priori speculation.

The indirect utility function provides a representation of preferences that permits a straightforward derivation of the ordinary demand functions using Roy's identity. An indirect utility function represents a preference ordering over the price–expenditure space in the same way that a direct utility function represents a preference ordering over the commodity space. The indirect utility function, $\psi(P, \mu)$, can be obtained by substituting the ordinary demand functions into the direct utility function:

$$(18a) \qquad \psi(P, \mu) = \max_{\sum p_k x_k \leqslant \mu} U(X) = U[h(P, \mu)].$$

It is often convenient to define "normalized" prices, $y_i = p_i/\mu$, and to write the indirect utility function in normalized price form as

$$(18b) \qquad \phi(Y) = U[h(Y, 1)].$$

An indirect utility function (written in terms of normalized prices) corresponds to a well-behaved preference ordering if it is monotonically decreasing in all its arguments, has continuous second partials, and is strictly quasi-convex.[3] A major advantage of the indirect utility function

[3] A function $\phi(Z)$ is said to be strictly quasi-convex if $-\phi(Z)$ is strictly quasi-concave. Thus, if the function $\phi(Y)$ is a well-behaved indirect utility function, then the function $-\phi(X)$ is a well-behaved direct utility function. Except in very special cases, these two utility functions correspond to different preference orderings; we return to this issue in Chapter 2.

is that closed-form expressions for the ordinary demand functions can be obtained from it using Roy's identity:

$$\text{(19a)} \qquad h^i(P, \mu) = -\frac{\partial\psi(P, \mu)/\partial p_i}{\partial\psi(P, \mu)/\partial\mu}.$$

In terms of normalized prices, Roy's identity becomes

$$\text{(19b)} \qquad h^i(Y, 1) = \frac{\partial\phi(Y)/\partial y_i}{\sum y_k[\partial\phi(Y)/\partial y_k]}.$$

The indirect utility function corresponding to the LES is a good illustration. It is easily verified that applying Roy's identity to the indirect utility function

$$\text{(20)} \qquad \psi(P, \mu) = \frac{\mu - \sum p_k b_k}{\prod p_k^{a_k}}$$

yields the LES ordinary demand functions (3). Hence, (20) is the indirect utility function corresponding to the LES. To obtain this indirect utility function by substituting the ordinary demand functions into the direct utility function, it is convenient to begin by subjecting the LES direct utility function (1) to the increasing transformation $V(X) = T[U(X)]$, where $T(z) = e^z$. The original utility function $U(X)$ and the transformed utility function

$$\text{(21)} \qquad V(X) = \prod(x_k - b_k)^{a_k}, \qquad a_i > 0, \qquad (x_i - b_i) > 0, \qquad \sum a_k = 1$$

are equally valid representations of the same underlying preference ordering. Substituting the ordinary demand functions (3) into (21) yields

$$\text{(22)} \qquad V[h(P, \mu)] = \frac{\prod a_k^{a_k}}{\prod p_k^{a_k}}\left(\mu - \sum p_k b_k\right).$$

Dropping the irrelevant constant factor, we obtain the LES indirect utility function (20). It is unnecessary to transform the direct utility function in this or any other way in order to obtain the indirect utility function, but doing so yields a form of the indirect utility function compatible with forms that appear in Chapter 2.

As we shall see repeatedly in Chapter 2, functional form specification for a demand system usually begins with an assumption about the form of the indirect utility function and uses Roy's identity to derive the ordinary demand functions. Similarly, theorems that characterize the class of preferences compatible with some class of ordinary demand functions (e.g., demand functions linear in expenditure) usually characterize preferences in terms of the indirect utility function.

We consider now very briefly some results obtained from estimating the LES using U.K. budget study data. The sources and definitions of the variables are discussed in detail in Chapter 6. Briefly we have three broad

Table 1 Own-Price Elasticities: Static Model without Demographic Effects

S. per week	Food	Clothing	Miscellaneous
200	−.66	−1.41	−1.53
300	−.73	−1.23	−1.26
400	−.78	−1.15	−1.17

Notes:
1. These elasticities are calculated on the basis of LES parameters estimated from U.K. budget study data (81 observations).
2. Own-price elasticities are evaluated at 1970 prices.
3. Mean expenditure in 1970 is 315 S. per week.

consumption categories—food, clothing, and miscellaneous—for the years 1968–1972. These data are cross-classified by income level and family size, yielding a total of 81 observations. For the moment we ignore the information about family size and attempt to explain the observed differences in consumption patterns using only prices and total expenditure as explanatory variables. We obtain a stochastic form for the LES by adding a disturbance term to each of the share equations given in (5). We denote the 3×1 vector of disturbances corresponding to the ith cell by $u_i = (u_{i1}, u_{i2}, u_{i3})'$ and assume that $E(u_i) = 0$, that $E(u_i u_j') = \Omega$ for all i and j, and that the u_i are independently normally distributed.

In Table 1 we present the own-price elasticities obtained from estimating the LES with the specification just described using the maximum likelihood procedure. The demand for food is inelastic at all three expenditure levels, while demand for the other two goods is elastic. Further, the demand for food becomes less inelastic as expenditure rises, while the demands for clothing and for miscellaneous become less elastic. Although not shown in Table 1, the (constant) marginal budget shares for these three goods are .33, .22, and .45, respectively, while the values of b are 66.4, − 14.4, and 41.4 shillings (S.) per week.

2. DEMOGRAPHIC SPECIFICATION

Demographic variables such as family size and age composition have traditionally played a major role in the analysis of household budget data. Family size and composition, race, religion, age, and education have all been used as demographic variables in demand studies, although only recently in the context of complete demand systems.

There are two ways to allow for demographic variables. First, given enough data, it is always possible to estimate separately the demand systems for subsamples of households with identical demographic profiles. For example, we might specify that each household's demand equations are given by an LES and estimate the $2n - 1$ independent parameters of that system separately for each household type. This approach allows all of the parameters of the demand system to depend on the demographic profile and does not require us to specify the form of the relationship between the parameters and the demographic variables. Under this approach the only data relevant to the analysis of households with a particular demographic profile are observations on households with that profile.

The second approach, which we discuss in Chapter 3, introduces specifications that relate the behavior of households with different demographic profiles. Here we describe only one such specification, "demographic translating," which we introduce in the context of a single demographic variable—for definiteness, household size, which we denote by η. We assume that each household's demand equations are given by an LES, that the b's depend linearly on the number of persons in the household,

$$b_i = b_i^* + \beta_i \eta, \tag{23}$$

and that the a's are independent of the demographic variables. The implied demand system (in expenditure form) is given by

$$x_i = h^i(P, \mu, \eta) = b_i^* - \frac{a_i}{p_i} \sum p_k b_k^* + \frac{a_i}{p_i} \mu + \beta_i \eta - \frac{a_i}{p_i} \sum p_k \beta_k \eta. \tag{24}$$

To examine the effect of household size on the consumption of good i we differentiate (24) with respect to η, obtaining

$$\frac{\partial h^i(P, \mu, \eta)}{\partial \eta} = \beta_i - \frac{a_i}{p_i} \sum p_k \beta_k. \tag{25}$$

The budget constraint implies that a change in household size that causes an increase in the consumption of one good must also cause offsetting decreases in the consumption of other goods. A major advantage of working with complete demand systems rather than analyzing the demand for each good separately is that the complete system approach restricts our attention to reallocations of expenditure that satisfy the budget constraint. The effect of an increase in family size on the consumption of a particular good must thus be viewed in the context of allocating expenditure among all goods. In terms of our LES example, even the direction of the effect of an increase in household size on consumption of the ith good cannot be inferred from the direction of the effect of such a change on the demand system parameter b_i. Indeed, there is no presumption that an increase in household size will increase rather than decrease b_i, since changes in the b's, regardless of their direction, imply a reallocation of expenditure among the goods but leave total expenditure unchanged.

To illustrate the translating procedure, we used the data set consisting

Table 2 Own-Price Elasticities: Static Model with Demographic Effects

Family size/ S. per week	Food	Clothing	Miscellaneous
One child			
200	−.73	−1.86	−1.83
300	−.78	−1.48	−1.42
400	−.81	−1.33	−1.28
Two children			
200	−.66	−1.83	−1.79
300	−.72	−1.45	−1.41
400	−.76	−1.31	−1.28
Three children			
200	−.60	−1.79	−2.23
300	−.67	−1.41	−1.49
400	−.71	−1.28	−1.30

Notes:
1. See Notes to Table 1.

of 81 observations introduced in Section 1 to estimate Eq. (24) in share form. We now use the cross-classification by family size as well as by income. In Table 2 we present the estimated own-price elasticities for different expenditure levels and family sizes. The pattern of results, for a given family size, is the same as that obtained without using family size as an explanatory variable: the demand for food is inelastic and becomes less inelastic as expenditure rises, while the demand for clothing and for miscellaneous become less elastic. A comparison of elasticities across family sizes indicates that the number of children has very little effect, although in all but two cases demand becomes slightly less elastic as family size increases, for given expenditure levels. Although not reported in the table, the marginal budget shares for the three goods are .29, .23, and .48, respectively. These are very close to the values obtained earlier with no demographics included, namely .33, .22, and .45.

The advantage of an approach that relates the behavior of households with different demographic profiles is that it enables us to draw inferences about the behavior of households with one demographic profile from observations on the behavior of households with different profiles. Inferences of this type are not possible if we estimate separately the demand systems for households with each type of demographic profile. This advantage is especially important when degrees of freedom are limited, either because the number of observations is small or because the number of demographic variables is large. In Chapter 3 we examine demographic translating in more detail and show that it is a "general procedure" in the sense that it can be used in conjunction with any complete demand system. We also examine other general procedures for incorporating demographic variables into complete demand systems and discuss the problem of comparing the welfare of households with different demographic profiles.

3. DYNAMIC SPECIFICATION

Empirical demand analysis must either assume that demand system parameters remain constant over time or specify how they change. If one takes the necessary basket interpretation seriously, then it seems plausible that the b's might vary over time. From a technical standpoint, it is relatively simple to incorporate changing b's into the LES because they enter the demand functions linearly.

Although the b's in the LES can sometimes be interpreted as necessary quantities, there is no presumption that they are physiologically rather than psychologically necessary. Indeed, it seems plausible that the "necessary" quantity of a good should depend, at least in part, on past consumption of that good. The simplest assumption is that the necessary quantity of each good is proportional to consumption of that good in the previous period, that is,

$$(26) \qquad b_{it} = \beta_i x_{it-1}$$

where b_{it} is the value of b_i in period t and β_i a "habit formation coefficient." A more general assumption is that the necessary quantity of each good is a linear function of consumption of that good in the previous period, that is,

$$(27) \qquad b_{it} = b_i^* + \beta_i x_{it-1}.$$

Here b_i^* can be interpreted as the "physiologically necessary" component of b_{it} and $\beta_i x_{it-1}$ as the "psychologically necessary" component. This specification provides a simple example of a "habit formation" model.

If all goods are subject to habit formation of the type described by (27), then the utility function becomes

$$(28) \qquad U(X_t; X_{t-1}) = \sum a_k \log(x_{kt} - b_k^* - \beta_k x_{kt-1}).$$

The semicolon separating X_t and X_{t-1} indicates that the preference ordering over current consumption (X_t) is "conditional" on past consumption (X_{t-1}); we discuss the interpretation of this habit formation specification in Chapter 4. In period t the individual is supposed to choose $(x_{1t}, \ldots, x_{nt})$ to maximize (28) subject to the budget constraint

$$(29) \qquad \sum p_{kt} x_{kt} = \mu_t.$$

The resulting demand system (suppressing the time subscripts on the p's and μ) is of the form

$$(30) \qquad h^i(P, \mu; X_{t-1}) = b_i^* - \frac{a_i}{p_i} \sum p_k b_k^* + \frac{a_i}{p_i}\mu + \beta_i x_{it-1} - \frac{a_i}{p_i} \sum p_k \beta_k x_{kt-1}.$$

These short-run demand functions, like their static counterparts (3), are locally linear in expenditure. Because the b's are linear in past consumption and because current consumption depends linearly on the b's, present consumption of each good is a linear function of past consumption of all goods. Furthermore, provided the β's are positive, an increase in past consumption of a good implies an increase in present consumption of that good,

$$\frac{\partial h^{it}(P, \mu; X_{t-1})}{\partial x_{it-1}} = \beta_i - a_i\beta_i > 0, \tag{31}$$

and a decrease in present consumption of every other good,

$$\frac{\partial h^{jt}(P, \mu; X_{t-1})}{\partial x_{it-1}} = -a_j p_i \beta_i / p_j < 0. \tag{32}$$

We define "long-run" or "steady-state" demand functions corresponding to these short-run demand functions. For every price–expenditure situation, the short-run demand functions define a mapping of the consumption vector in period t into the consumption vector in period t + 1. A long-run equilibrium or steady-state consumption vector corresponding to a particular price–expenditure situation is a consumption vector that, if it prevailed in period t, will prevail in period t + 1. In Chapter 4 we show that if the short-run demand system is an LES, then the long-run demand system is also an LES, although the parameters of the long-run LES depend on the habit coefficients as well as on the other parameters of the short-run LES. We also show that these dynamic demand functions are locally stable provided the β's are all less than 1. In addition, Chapter 4 discusses alternative specifications of habit formation for the LES and for general demand systems.

An alternative dynamic specification, interdependent preferences, postulates that a household's preferences depend on the consumption of some (and perhaps all) other households. Again the LES provides a simple example. Specifications in which the demand system parameters depend on the lagged rather than the current consumption of others are more tractable and seem to capture the essential features of interdependent preferences; hence, in our discussion of interdependent preferences, we emphasize lagged interdependence. In Chapter 4 we discuss models of interdependent preferences, examining both their short-run and long-run implications for the LES and for more general demand systems.

To illustrate the dependence of the b's on some measure of past consumption, we use the U.K. data employed earlier and estimate equation (30) in share form. For the lagged quantities in dynamic demand equation (30) we use a weighted average over all observations in the sample in the previous year, because those data do not report average lagged consumption for each observation.[4] In Table 3 we present estimated own-price

[4] Because the observations are cell means, we weight each observation by cell size.

Table 3 Own-Price Elasticities: Dynamic Model

S. per week 2-child family	Food	Clothing	Miscellaneous
231	−1.13	−2.66	−2.66
259	−1.12	−2.41	−2.35
280	−1.11	−2.26	−2.19
320	−1.10	−2.06	−1.96
360	−1.09	−1.92	−1.82
476	−1.08	−1.65	−1.56

Notes
1. See Notes to Table 1.

elasticities for 1970 for families with two children for expenditure levels prevailing in that year. The elasticities are all higher in absolute value than those given in Tables 1 and 2—indeed the demand for food is now slightly elastic. The marginal budget shares, however, are identical to those in Table 1, namely .33, .22, and .45 for the three goods. The values of β, although not reported in Table 3, are .44, − .43, and .25, respectively. Thus the values for food and miscellaneous are consistent with habit formation, while the value for clothing is not.

We conclude Chapter 4 by arguing that, although endogenous preferences are tractable in both theoretical and empirical demand analysis, they cause serious difficulties for welfare analysis.

4. STOCHASTIC SPECIFICATION

Estimating a demand system requires assumptions about stochastic structure. The most common stochastic specification begins with the share form of the demand system and adds a disturbance term to each share equation. In the case of the LES this implies

$$(33) \qquad w_{it} = \frac{p_{it}b_i}{\mu_t} + a_i\left[1 - \sum \frac{p_{kt}b_k}{\mu_t}\right] + u_{it}.$$

Instead of adding disturbance terms to the share equations, one could add them to the expenditure equations. We prefer the specification based on the share form because we think it is likely to involve less heteroskedasticity.

The "standard" stochastic specification assumes that disturbances are independent across observations. In the case of time series data this implies independence over time periods; in the case of household budget data it implies independence over households.

In Chapter 5 we discuss in detail the standard specification and consider several alternative stochastic structures, including a serial correlation model, an error components model, and a random coefficients model. In a random coefficients model all observations correspond to the same parametric demand system, but some of the underlying demand system parameters are random variables. In the LES, for example, we could assume that the b's contain a random term

$$b_{it} = b_i^* + v_{it} \tag{34}$$

so that the demand system is given by

$$w_{it} = \frac{p_{it}b_i^*}{\mu_t} + a_i\left[1 - \frac{\sum p_{kt}b_k^*}{\mu_t}\right] + \frac{p_{it}v_{it}}{\mu_t} - a_i\frac{\sum p_{kt}v_{kt}}{\mu_t}. \tag{35}$$

We could obtain this form directly from (33) by specifying that the u's are given by

$$w_{it} = \frac{p_{it}v_{it}}{\mu_t} - a_i\frac{\sum p_{kt}v_{kt}}{\mu_t}. \tag{36}$$

Different assumptions about the distribution of the v's will, of course, have different implications for the covariance matrix of the u's. In Chapter 5 we discuss a number of such assumptions and their implications. In Chapter 6 we use the U.K. data to estimate this random coefficients specification under various assumptions about the distribution of the v's. We do not present the results here because, under what we think are the most plausible assumptions about the distribution of the v's, the estimated own-price elasticities and marginal budget shares are virtually identical to those reported in Table 1.

5. ESTIMATION

Data never speak for themselves. A unifying theme of this book is that empirical analysis depends on both the data and the specification of a theoretical model. In empirical demand analysis the selection of a theoretical model entails choosing a functional form, a demographic specification, a dynamic specification, and a stochastic specification. The necessity of choosing a functional form is inescapable, although proponents of flexible functional forms have sometimes suggested that, by choosing such a form, one can avoid imposing a priori restrictions on admissible demand behavior. Choosing a demographic specification can be avoided only by estimating separate demand systems for households with distinct demographic profiles. Choosing a dynamic specification cannot be avoided: the assumption that demand system parameters remain constant over time, which is the only alternative to modeling explicitly how they change, is itself a dynamic specification. The necessity of choosing a stochastic specification has been recognized only recently; mindlessly adding a disturbance term to a nonstochastic model is no longer professionally acceptable. In summary: theory must play a crucial role in structuring empirical research and we think that this role should be explicit rather than implicit.

In Chapters 6 and 7 we turn to estimation, in Chapter 6 using household budget data and in Chapter 7 using per capita time series data. Although household budget data have long played an important role in empirical demand analysis, the analysis of such data has traditionally focused on demographic effects and expenditure or income effects, and has ignored price effects. In Pollak and Wales [1978] we demonstrated that the LES and other interesting complete demand systems can be estimated using household budget data from as few as two periods, despite the limited price variability present in such data.

Because we expect household budget data will play an increasing role in empirical demand analysis, we have emphasized two aspects of demand system specification that are especially important in dealing with such data. In Chapter 2 we emphasize functional form specifications that yield Engel curves suitable for analyzing data in which expenditure variation is substantial relative to price variation, as is typically the case in household budget data. In Chapter 3 we discuss the treatment of demographic variables, an issue that has traditionally played a central role in the analysis of household budget data.

Five conclusions emerge from our analysis of household budget data in Chapter 6.

- Although the LES is an attractive functional form because of its theoretical and empirical tractability, it is not appropriate for the analysis of household budget data. Likelihood ratio tests reject the LES restrictions in all samples against a wide range of alternative specifications.
- Functional forms with three-parameter Engel curves are significantly superior to forms with two-parameter Engel curves. The likelihood ratio test not only rejects the LES against forms allowing quadratic Engel curves, but also rejects the translog, a form involving a two-parameter Engel curve, against a three-parameter generalization.
- Demographic variables matter. We reject the pooled specification, which assumes demographic variables have no effect, against each of five procedures we describe for incorporating demographic variables into complete demand systems.
- Dynamic specifications are superior to static ones. Although this conclusion need not be interpreted as evidence of taste change, it does indicate that the static specification is flawed.
- Stochastic and dynamic specifications often act as substitutes; because they are difficult to disentangle, they should be evaluated together.

In Chapter 7 we use per capita time series data from national product accounts to estimate complete demand systems. We begin by investigating alternative specifications of functional forms, dynamic structures, and stochastic structures. We cannot easily summarize our results here because we have not yet introduced the specifications being compared. We can say, however, that there is strong evidence that dynamic factors play an important role. As with household budget data, dynamic and stochastic specifications tend to act as substitutes and, hence, it is difficult to determine the extent to which dynamic factors operate through the nonstochastic rather than the stochastic portion of the model. On balance, our results confirm the importance of dynamic factors, lending support to the emphasis we have placed on habit formation and other dynamic specifications.

We conclude Chapter 7 by exploring the possibility of pooling data from different countries for demand system estimation. The obvious objection to pooling consumption data is that we would expect two countries facing the same prices and having the same level of expenditure to have systematic differences in their consumption patterns. We allow for these systematic differences in two ways. First, we use dynamic specifications that permit "transitory" or short-run differences across countries reflecting differences in past consumption patterns. Under the transitory difference specification, if two countries were to face the same price–expenditure situation for a number of successive periods, then their consumption patterns would converge to a common value. Pooling, however, does not require that all countries have identical consumption patterns, even in the long run. Our second specification, the "permanent difference" specification, allows a subset of the demand system parameters to differ across countries and is thus consistent with underlying demographic, climatic, or taste differences. We also estimate mixed specifications that permit both transitory and permanent differences among countries.

A second difficulty in pooling data from different countries is related to their use of different national currencies. We show that when quantities are measured in different value units (e.g., U.S. food in 1970 U.S. dollars; Belgian food in 1970 B. francs) the problem is essentially the same as when goods are measured in different physical units. Pooling requires transforming the data into common units, and commodity-specific purchasing power parities are the conversion factors required to do this. Despite the attractiveness of pooling data from different countries and our choice of three countries—Belgium, the U.K., and the U.S.—for which pooling seems plausible, our empirical results do not support pooling.

6. CONCLUSION

In this chapter we have used the LES to establish the terminology and notation that appears throughout the book. In addition, the LES illustrated four basic aspects of demand specification that provide context and motivation for the extensive discussions of functional form specification, the role of demographic variables, and basic elements of dynamic and stochastic structure in Chapters 2 through 5. Finally, we used empirical estimates of the LES based on household budget data and per capita time series data to foreshadow our concern with estimation in Chapters 6 and 7 and to illustrate a major theme of this book—the crucial role of economic theory as a foundation for empirical analysis.

APPENDIX A: THE KLEIN–RUBIN THEOREM

Theorem: Let $h(P, \mu)$ be a theoretically plausible demand system in which expenditure on each good is a linear function of prices and expenditure

$$\text{(A1a)} \qquad h^i(P, \mu) = \frac{1}{p_i} \sum p_k \hat{\alpha}^{ik} + \frac{\alpha^{i0}}{p_i} \mu$$

and for which the marginal budget shares are not equal to 0 or 1 for any good. Any such demand system can be written in the LES form

$$\text{(A2)} \qquad h^i(P, \mu) = b_i - \frac{a_i}{p_i} \sum p_k b_k + \frac{a_i}{p_i} \mu, \qquad \sum a_k = 1.$$

Proof: It is convenient to rewrite the demand system (A1a) as

$$\text{(A1b)} \qquad h^i(P, \mu) = \alpha^{ii} - \frac{a_i}{p_i} \sum \alpha^{ik} p_k + a_i \frac{\mu}{p_i},$$

where the new variables are defined by

$$a_i = \alpha^{i0}$$

$$\alpha^{ij} = -\frac{\hat{\alpha}^{ij}}{\alpha^{i0}} = -\frac{\hat{\alpha}^{ij}}{a_i}$$

$$\alpha^{ii} = \frac{\hat{\alpha}^{ii}}{1 - \alpha^{i0}} = \frac{\hat{\alpha}^{ii}}{1 - a_i}.$$

Since a theoretically plausible demand system must satisfy the budget constraint, we must have $\sum a_k = 1$, as the theorem requires.

Although rewriting (A1a) and (A1b) is not crucial to proving the theorem, it does simplify the analysis. As is so often the case, it is easier to prove a characterization result when you know where you are going. Thus, we define b_t by

$$b_t = \alpha^{1t}.$$

To prove the theorem, we must show that

(A3) $$\alpha^{jt} = \alpha^{it} = b_t$$

for all j and t.

A theoretically plausible demand system must satisfy the Slutsky symmetry conditions

$$\frac{\partial h^i}{\partial p_j} + h^j \frac{\partial h^i}{\partial \mu} = \frac{\partial h^j}{\partial p_i} + h^i \frac{\partial h^j}{\partial \mu}.$$

Letting $i = 1$, for the demand system (A1b) the Slutsky symmetry conditions become

(A4) $$-\frac{a_1}{p_1}\alpha^{1j} + h^j \frac{a_1}{p_1} = -\frac{a_j}{p_j}\alpha^{j1} + h^1 \frac{a_j}{p_j}.$$

Multiplying both sides by $p_1 p_j$ yields

(A5) $$-p_j a_1 \alpha^{1j} + p_j h^j a_1 = -p_1 a_j \alpha^{j1} + p_1 h^1 a_j$$

for all j. Differentiating (A5) with respect to p_t, $t \neq 1, j$, yields

$$-a_1 a_j \alpha^{jt} = -a_j a_1 \alpha^{1t}$$

so

$$\alpha^{jt} = \alpha^{1t}$$

for all $j \neq t$. Differentiating (A5) with respect to p_j yields

$$-a_1 a_j \alpha^{j1} = -a_j \alpha^{j1} + a_j \alpha^{11} - a_j a_1 \alpha^{11},$$

or, equivalently,

$$(a_j - a_1 a_j)\alpha^{j1} = (a_j - a_j a_1)\alpha^{11}$$

so

$$\alpha^{j1} = \alpha^{11}.$$

This establishes (A3). QED