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AGGREGATION, DISTRIBUTION AND DYNAMICS IN THE LINEAR AND QUADRATIC EXPENDITURE SYSTEMS

Adolf Buse*

Abstract—Using Canadian data (1965–86) we confirm and extend Stoker's (1986) results on the rule of distributional effects in demand systems. The confirmation consists of evidence from the LES model showing that distributional effects are statistically significant and can displace AR(1) dynamics in the disturbances. The extension is made to the QES model and an argument is advanced that standard habit formation dynamics may reflect omitted distributional effects. The evidence supports this conjecture. This suggests that we may have been drawing the wrong conclusions from expenditure studies. Rather than inferring dynamic behaviour we should have been concluding that these models are misspecified.

I. Introduction

IN his survey of demand analysis Deaton (1986, pp. 1819–1822) concludes the section on aggregation across consumers with the following observation:

> Much of the work reported in this section, by Muellbauer, Lau and Stoker, can be regarded as developing the appropriate techniques for the impacts of distribution on aggregate demand functions. That such effects could be potentially important has been known for a long time, see de Wolff (1941) for an early contribution. What seems to be lacking so far is empirical evidence that such effects are actually important.

The absence of significant distributional effects in aggregate equations can, perhaps, be taken as confirmation of a research strategy by those who construct macro models by appealing to the optimizing behaviour of representative agents. On the other hand, the literature on exact aggregation (see, for example, Heineke and Shefrin (1988)) makes it clear that, with the exception of the linear case, indices of the income distribution other than the mean must appear in properly aggregated equations. The specification errors that arise from the omission of these indices create aggregation biases that will distort inferences and policy analyses.[1] Deaton's comments suggest that the evidence of such aggregation biases has been less than persuasive and it is in this context that the recent paper by Stoker (1986) takes on particular significance. Stoker has devised tests for distributional effects in macro equations which are in essence tests for aggregation bias. Furthermore, he has shown that such distributional effects are statistically significant and that these effects can also account for model dynamics.

Stoker's results have two important implications. First, insofar as the representative agent paradigm assumes away aggregation biases the demonstration that those biases are present places that paradigm in question. At the very least the policy conclusions derived from such models should be advocated with considerable more circumspection. Second, in terms of modelling systems of demand equations, the finding that the estimated distributional effects can account for model dynamics (broadly interpreted to encompass both behavioural and disturbance components) suggests that the widely-held view, exemplified by the work of Anderson and Blundell (1983), that model failure is almost invariably due to inadequate dynamics is an unduly narrow diagnosis.

These two points, if generally true, thus undermine a good bit of conventional wisdom. It is

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[1] This literature also encompasses the case of heterogeneous households in which household attributes such as demographic characteristics are part of the aggregation problem. Neglect of such heterogeneity can also lead to aggregation biases but the present study focuses on distributional issues.

vital, therefore, to determine whether Stoker's results can be replicated using other data sets. With this in mind the present study attempts a replication using annual Canadian data covering the period 1965–1986. We do not, however, confine our tests to the linear expenditure system (LES) used by Stoker but extend the analysis to the quadratic expenditure system (QES) to determine whether his conclusions are robust to alternative demand systems. In conjunction with these models the question of dynamics is explored more fully by specifying a simple habit formation model and enquiring whether, as in Stoker's AR(1) specification, the distributional effects can account for this type of dynamic structure.

II. Modelling Distributional Effects

To provide a framework for the empirical analysis we need an outline of Stoker's analytical results and our extension of them to the quadratic case. The starting point for the aggregation analysis is a simple behavioural micro relation $y = f(x)$ where $f(x)$ is constant across agents and stable through time. For expository purposes we take y as household expenditure on a commodity and x as household income so that $f(x)$ is the cross-section Engel curve. The macro-economic counterparts of y and x are population averages defined by $E_t(y) = \int f(x)p_t(x)\,dx$ and $E_t(x) = \int xp_t(x)\,dx$, where $p_t(x)$ is the probability density function of income.

The key to Stoker's test for distributional effects in macro equations lies in the observation that if the Engel curve, $f(x)$, is nonlinear then macro equations cannot be formulated only in terms of the aggregates $E_t(y)$ and $E_t(x)$. This can most readily be seen if we decompose $y = f(x)$ into a linear component and a remainder. Thus, let

$$y = \alpha + \beta x + r(x) \tag{1}$$

where $r(x) \equiv f(x) - \alpha - \beta x$ so that

$$E_t(y) = \alpha + \beta E_t(x) + \int r(x)p_t(x)\,dx. \tag{2}$$

It is immediately clear that a macro equation that omits the expected value of the remainder term is mis-specified. This mis-specification is the aggregation bias and it depends on the departure of $f(x)$ from linearity and the distribution of income. If $f(x)$ is linear then $r(x) = 0$ and there is no aggregation bias; or equivalently, there are no distributional effects.

To illustrate, suppose $f(x) = \alpha + \beta x + \gamma x^2$, a quadratic. In this case the correctly specified macro equation is given by

$$E_t(y) = \alpha + \beta E_t(x) + \gamma(E_t(x))^2 + \gamma V_t(x) \tag{3}$$

where V_t is the variance operator and we have substituted $E_t(x^2) = V_t(x) + (E_t(x))^2$. The typical macro equation motivated by the representative agent paradigm would estimate (3) in per capita terms and omit the variance. The usual specification error analysis would apply.

Thus behavioural nonlinearities generate distributional effects and these effects are captured by the expected value of departures from linearity, $E(r(x))$. Stoker has shown how these departures from linearity can be operationalized using cell proportion data. Let P_{jt} be the proportion of households in cell j at time t ($j = 1, \ldots, N$; $t = 1, \ldots, T$) after the distribution has been partitioned into N income groups. Then average expenditure and average income are defined by $E_t(y) = \sum_j E_t(y/j)P_{jt}$ and $E_t(x) = \sum_j E_t(x/j)P_{jt}$ where $E_t(y/j)$ and $E_t(x/j)$ are the within cell averages. If we assume that the within cell averages are constant through time[2] we can define the vectors $Y' = (Y_1, \ldots, Y_N)$ and $X' = (X_1, \ldots, X_N)$ where $E_t(y/j) = Y_j$ and $E_j(x/j) = X_j$. Correspondingly, set $P_t' = (P_{1t}, \ldots, P_{Nt})$. In vector notation we therefore have $E_t(y) = Y'P_t$ and $E_t(x) = X'P_t$.

To obtain a test for distributional effects we need to construct the discrete equivalent of (2); that is, we need to decompose the vector of micro expenditures into a linear part which depends on X and a remainder. If we take the orthogonal decomposition of Y as $Y = RY + (I - R)Y$, R being the projection matrix $R = X_c(X_c'X_c)^{-1}X_c'$ with $X_c = (\iota \;\; X)$ and ι a vector of ones, we can

[2] Since all households are identical, constancy of within cell means for income will guarantee constancy of within cell mean expenditure. Given a time varying income distribution the constancy of within cell mean income must be considered an approximating assumption unless income is uniformly distributed within cells.

write[3]

$$E_t(y) = Y'P_t = Y'RP_t + Y'(I - R)P_t$$
$$= Y'X_c(X_c'X_c)^{-1}X_c'P_t + \tilde{Y}'P_t \qquad (4)$$

where $\tilde{Y} = (I - R)Y$ is the vector of residuals in the cross-section regression of Y on X_c. It can readily be verified that $Y'X_c(X_c'X_c)^{-1}X_c' = \alpha + \beta E_t(x)$ and on appending the usual disturbance term (4) becomes

$$E_t(y) = \alpha + \beta E_t(x) + \tilde{Y}'P_t + v_t \qquad (5)$$

where v_t captures the aggregated micro disturbances.[4] Thus aggregate expenditure can be written as a linear function of aggregate income and the proportion of income recipients in each group. The coefficients of the proportions measure the deviations from linearity of the micro equation so that if $\tilde{Y} = 0$ exact linear aggregation is possible. We will refer to the vector $\tilde{Y}$ as distributional effects. The relevant F statistic for H_1: $\tilde{Y} = 0$ would have $N - 2$ numerator degrees of freedom as there are only $N - 2$ independent residuals in the cross-section regression (Stoker (1986, p. 778)).

The decomposition of Y into a linear component in X and a residual can also be effected by defining a linear equation which joins two (arbitrary) values of Y, say Y_1 and Y_2, then defining the residuals as deviations from this line. (See figure 3 in Stoker, p. 779.) Thus, if α_* and β_* are the coefficients of this linear equation, then we can write $E_t(y) = \alpha_* + \beta_* E_t(x) + \tilde{Y}_*' P_{*t} + v_t$ where $\tilde{Y}_*' = (\tilde{Y}_{*3}, \ldots, \tilde{Y}_{*N})$ and $P_{*t}' = (P_{3t}, \ldots, P_{Nt})$ are both $N - 2$ vectors because by construction $\tilde{Y}_{*1} = \tilde{Y}_{*2} = 0$. We will use this approach in testing for distributional effects.

Stoker's test is a test for aggregation bias relative to a particular functional form. Given the wealth of cross-section evidence on the nonlinearity of Engel curves the test is bound to reject in most cases. It would be of some interest to construct tests for other functional forms by treating deviations from specific functional forms as distributional effects. A step in this direction can be made by considering the case of a quadratic micro relation in which $f(x) = \alpha + \beta x + \gamma x^2$. If the vector Y is now orthogonally decomposed as $Y = RY + (I - R)Y$, with $R = X_c(X_c'X_c)^{-1}X_c'$ and $X_c = (\iota \;\, X \;\, X^2)$, where X^2 represents the elements of X squared, then by a simple generalization of (4)

$$E_t(y) = \alpha + \beta E_t(x) + \gamma\big(E_t(x)\big)^2$$
$$+ \gamma V_t(x) + \tilde{Y}'P_t. \qquad (6)$$

In this case $\tilde{Y}$ represents the distributional effects measured relative to a quadratic micro relation (see (3)).

III. Testing for Distributional Effects Under Alternative Dynamic Structures

As noted in the introduction, our objective, in the first instance, is the replication of Stoker's tests to determine whether his conclusions have a wider validity. To this end we assembled an appropriate data set for Canada covering the period 1965–86. While the expenditure categories (Food, Clothing, Shelter and Other) correspond quite closely to those used by Stoker our choice of time period was severely constrained by the lack of a continuous series on the distribution of income prior to 1971. We were able to extend the income data back to 1965 only by interpolating for the years 1966, 1968 and 1970.[5] The estimated series on the variance of the income distribution was also derived from these data. It must also be noted that the income distribution is a proxy variable for the distribution of total expenditures because the budget constraint for the models analyzed here is defined in terms of total expenditure on the commodities included in the analysis. In the absence of data on the distribution of total expenditure we have followed Stoker (and others; see for example, Berndt, Darrough and Diewert (1977), Lewbel (1988), Van Daal and Merkies (1984)) in assuming (implicitly) that the expenditure distribution is proportional to the

[3] Apparently (4) differs from Stoker's (p. 776) equation (17) since he has $\tilde{Y}'\tilde{P}_t$ where we have $\tilde{Y}'P_t$ as the last expression on the right hand side. Note, however, that $\tilde{Y}'\tilde{P}_t = Y'(I - R)P_t = \tilde{Y}'P_t$ as $(I - R)$ is idempotent.

[4] At this point our notation deviates from Stoker who writes $\bar{Y}_t = E_t(y) + v_t$. We prefer to treat both $E_t(y)$ and $E_t(x)$ as observables. If we can write $E_t(y) = (Y + \epsilon)'P_t = Y'P_t + \epsilon'P_t$, then v_t is the weighted sum of the micro disturbances. This implies, however, that the disturbances are heteroskedastic. Estimation of our models with the appropriate heteroskedastic adjustment did not affect any of our conclusions.

[5] Details of the data sources are available on request. Income distribution interpolations are not without precedent; see, for example, Berndt, Darrough and Diewert (1977).

income distribution. The decision-making unit is taken to be the household, the total number of households being the sum of the number of families and unattached individuals.

Distributional tests were carried out on the LES and QES models which form a nested structure. The particular QES specification that we chose to investigate is given by (Howe, Pollak and Wales (1979), p. 1238)

$$y_{it} = \gamma_i p_{it} + \beta_i\left(x_t - \sum_j^4 \gamma_j p_{jt}\right) + \left(\alpha_i p_{it} - \beta_i \sum_j^4 \alpha_j p_{jt}\right) \times \prod_j^4 p_{jt}^{-2\beta_j}\left(x_t - \sum_j^4 \gamma_j p_{jt}\right)^2,$$
$$\sum \beta_j = 1, \tag{7}$$

where y_{it} is expenditure on the i^{th} commodity group, p_{it} is a price index of the group and x_t is total expenditure. The LES model is nested within the QES by imposition of the restrictions $\alpha_i = 0$ $(i = 1, \ldots, 4)$.

Using (6) the corresponding aggregate estimating equation for (7), with distributional effects included, is obtained when y_{it} and x_t are replaced by average per household values, $E_t(y_i)$ and $E_t(x)$, respectively, so that

$$E_t(y_i) = g_i p_{it} + b_i\left(E_t(x) - \sum_j^4 g_j p_{jt}\right) + \left(a_i p_{it} - b_i \sum_j^4 a_j p_{jt}\right) \times \prod_j^4 p_{jt}^{-2b_j}\left(E_t(x) - \sum_j^4 g_j p_{jt}\right)^2 + \sum_k^{N-2} d_{ik} P_{kt} + d_{io} + e_i V_t(x) + v_{it},$$
$$\sum_j^4 b_j = 1. \tag{8}$$

Setting $a_i = e_i = 0$ in (8) gives us the macro LES specification.

The expenditure variables which appear in (8) are measured in nominal dollars, and deviations from an underlying linear or quadratic Engel curve as measured by distributional effects will also be in nominal terms. The time series of the proportions of households in each income class should therefore be taken in nominal dollars.[6] From the data on the nominal distribution of income in Canada it was possible to construct a time series of cell proportions for only five cells; namely, $0-5$, $5-10$, $10-15$, $15-25$ and $25+$ (thousands of dollars). Thus $N-2=3$ in the results reported below with the second $(5-10)$ and third cell $(10-15)$ omitted.[7]

In his investigation of the LES model Stoker found that distributional effects are statistically significant and, intriguingly, that these distributional effects could account for AR(1) dynamics in the disturbances. But dynamics can be introduced into both the LES and QES models in other, possibly economically more meaningful, ways. The habit formation hypothesis is a leading example. In light of Stoker's results it is natural to speculate whether the evidence that has been adduced for habit formation may in fact be a proxy for distributional effects which, if included in the equations in the first instance, would make the habit formation variable redundant. In our empirical investigation we have, therefore, introduced dynamics using in turn the AR(1) and habit formation specifications and this is the order in which the results are presented.[8]

(i) Distribution and AR(1) Dynamics

The most unrestricted version of the model is obtained when the disturbances in (8) follow the AR(1) process $v_{it} = rv_{i,t-1} + w_{it}$. This unrestricted model will be denoted by QES(r, d) where d is the vector of distributional effects. Restricted versions are obtained by setting the appropriate parameters to zero; thus QES$(0, 0)$ will be the basic quadratic expenditure model without autoregression or distributional effects.

[6] The results using deflated variables are available upon request.

[7] The observant reader will note that only two restrictions $(k = 1, \ldots, N-2)$ have been imposed on the vector d in (8) whereas the analytical arguments made earlier call for three when fitting the quadratic. This specification has been chosen to preserve the nesting of the LES within the QES model.

[8] Given our comparatively small sample size we have not attempted to estimate AR(1) and habit formation jointly in one specification but treated them as alternative dynamic structures. Howe, Pollak and Wales (1979, pp. 1241–1243) estimate AR(1) and habit formation jointly but their preferred models typically include only one of the dynamic structures.

TABLE 1.—LES MODEL

	LES(0, 0)	LES(r, 0)	LES(0, d)	LES(r, d)	LES(h, 0)	LES(h, d)
b_1[a]	−0.3323	0.1886	0.1056	0.1196	0.2277	0.0988
	(0.0968)	(0.0374)	(0.0284)	(0.0297)	(0.0357)	(0.0296)
b_2	0.2143	0.2191	0.2173	0.2174	0.1985	0.2434
	(0.0117)	(0.0151)	(0.0146)	(0.0138)	(0.0175)	(0.0204)
b_3	0.6975	0.3350	0.4415	0.4304	0.2606	0.3996
	(0.0899)	(0.0449)	(0.0187)	(0.0214)	(0.0305)	(0.0313)
b_4[b]	0.4205	0.2573	0.2357	0.2326	0.3132	0.2582
	(0.0270)	(0.0252)	(0.0151)	(0.0155)	(0.0304)	(0.0223)
g_1^*[c]	4.1115	2.5970	2.8790	2.7277	−0.0216	3.3698
	(0.0978)	(0.2705)	(0.1936)	(0.2206)	(0.1867)	(0.5089)
g_2^*	1.4317	0.3236	0.6241	0.4281	0.0675	0.9382
	(0.0516)	(0.2647)	(0.3272)	(0.3604)	(0.194)	(0.3510)
g_3^*	5.5117	4.9439	6.0098	5.8827	−0.5355	5.2113
	(0.1807)	(0.4022)	(0.3223)	(0.3516)	(0.2320)	(0.9132)
g_4^*	3.0927	2.2684	2.1144	2.0371	−0.1639	2.2530
	(0.1066)	(0.1929)	(0.1330)	(0.1481)	(0.2581)	(0.3017)
h_1	—	—	—	—	0.9330	−0.1141
					(0.0471)	(0.1455)
h_2	—	—	—	—	0.8242	−0.0934
					(0.0799)	(0.1453)
h_3	—	—	—	—	1.0632	0.1883
					(0.0358)	(0.1477)
h_4	—	—	—	—	0.9568	−0.0073
					(0.0659)	(0.1157)
r	—	0.9922	—	0.2216	—	—
	—	(0.0156)	—	(0.1323)	—	—
log L	102.967	157.033	190.253	190.814	163.510	192.561
Reg[d]	0	15	1	2	21	0
log $L(T)$[e]	111.681	158.715	196.996	197.073	164.309	206.123

[a] Asymptotic standard errors in parentheses.
[b] Calculated from other estimates.
[c] When $h_i = 0$, $g_i^* \equiv g_i$, $i = 1, \ldots, 4$.
[d] Number of sample points (out of 21) at which regularity is satisfied.
[e] Log-likelihood with time trend in model.

Although the LES is nested within the QES it will be notationally more convenient to adopt the equivalent but separate notation LES(r, d).

The estimated results (ML, TSP 4.1) of the AR(1) specification can be found under the appropriate column headings in tables 1 and 2. We consider the LES results first in order to make direct comparisons with Stoker. In the present context detailed discussion of the estimated parameters would not serve the primary objective of testing for distributional effects. Nonetheless, we should note that in table 1 the most restricted specification, LES(0, 0), produces an unacceptable negative marginal share for food which can be taken as evidence of mis-specification. When autoregressive and/or distributional effects are added this anomaly disappears and all estimated parameters are economically plausible.

The LR tests associated with the LES(r, d) model can be found in table 3. Using a 1% level of significance LES(0, 0) is decisively rejected against both LES(r, 0) and LES(0, d). Furthermore, we cannot reject the null that $r = 0$ once the distributional effects are included. Thus not only are there departures from linearity in the Engel curve, the inclusion of these nonlinear components accounts for model dynamics up to the level of an AR(1) process. These are precisely the results obtained by Stoker. Our results differ only in that, unlike Stoker, we reject LES(r, 0) in favor of LES(r, d) and we cannot conclude that the AR(1) process and distributional effects are alternative ways of accounting for the data. What is clear, however, is that distributional effects are statistically important so that models estimated without these effects are mis-specified.

The estimated QES models can be found in table 2 and the relevant LR statistics are given in table 3. Note that each of the four LES models is rejected when tested against the corresponding QES model although the nesting parameters (a_i and e_i) are not particularly well determined. The

TABLE 2.—QES MODEL

	QES(0, 0)	QES(r, 0)	QES(0, d)	QES(r, d)	QES(h, d)	QES(h, 0)
a_1[a]	0.5360	−0.0097	−0.1171	−0.0229	−0.1637	0.0894
	(0.6353)	(0.0039)	(0.0737)	(0.0185)	(0.0819)	(0.1477)
a_2	−0.0935	−0.0038	−0.1094	−0.0307	−0.1609	−0.0476
	(0.0777)	(0.0017)	(0.0654)	(0.0153)	(0.0650)	(0.0543)
a_3	−1.3383	−0.0010	−0.1371	0.0724	−0.0497	−0.0526
	(1.1234)	(0.0021)	(0.0980)	(0.0255)	(0.0751)	(0.0570)
a_4	−0.6470	−0.0028	−0.0640	−1.6884	−0.2010	0.2127
	(0.3852)	(0.0017)	(0.0746)	(2.6220)	(0.1402)	(0.2063)
b_1	−0.3486	0.3501	0.2352	0.0248	0.2557	0.3195
	(0.1883)	(0.2154)	(0.0509)	(0.0435)	(0.0681)	(0.0763)
b_2	0.0903	0.2030	0.2317	0.0336	0.2575	0.0561
	(0.0171)	(0.0213)	(0.0343)	(0.0575)	(0.0421)	(0.0435)
b_3	0.8736	0.2411	0.3593	−0.0088	0.1823	0.0728
	(0.1853)	(0.0462)	(0.0528)	(0.0143)	(0.0793)	(0.0784)
b_4[b]	0.3846	0.2058	0.1738	0.9503	0.3044	0.5517
	(0.0414)	(0.0368)	(0.0847)	(0.0900)	(0.1145)	(0.0697)
g_1^{*}[c]	4.2767	3.3830	2.6526	2.7495	1.9319	−0.3282
	(0.2924)	(0.3242)	(0.1744)	(0.1550)	(0.6340)	(0.2839)
g_2^*	1.2378	0.4370	0.5230	0.5133	0.2384	−0.0855
	(0.0245)	(0.2154)	(0.2094)	(0.1269)	(0.2676)	(0.1678)
g_3^*	5.2340	4.0215	6.5871	7.2022	4.1446	−0.1430
	(0.4077)	(0.3476)	(0.2108)	(0.2036)	(0.5871)	(0.1604)
g_4^*	2.5684	2.0352	2.4769	0.1703	1.8037	0.2458
	(0.1538)	(0.1742)	(0.1801)	(0.6070)	(0.3589)	(0.2893)
h_1	—	—	—	—	0.2076	1.0157
					(0.1619)	(0.0471)
h_2	—	—	—	—	0.2538	1.0892
					(0.1320)	(0.1259)
h_3	—	—	—	—	0.4013	1.0374
					(0.0942)	(0.0362)
h_4	—	—	—	—	0.3765	1.3798
					(0.1531)	(0.2082)
e_1	0.0696	−0.0824	−0.0926	−0.0950	−0.1017	−0.0928
	(0.1764)	(0.0246)	(0.0392)	(0.0423)	(0.0379)	(0.0288)
e_2	0.0392	0.0707	0.0754	0.0785	0.0531	0.0006
	(0.0201)	(0.0196)	(0.0182)	(0.0138)	(0.0145)	(0.0202)
e_3	−0.2432	−0.0669	−0.0164	0.0592	0.0076	0.0215
	(0.1592)	(0.0414)	(0.0310)	(0.0285)	(0.0320)	(0.0310)
e_4[b]	0.1343	0.0786	0.0336	0.0758	0.0410	0.0708
	(0.0384)	(0.0317)	(0.0334)	(0.0367)	(0.0320)	(0.0313)
r	—	1.0299	—	−0.0124	—	—
		(0.0148)		(0.0162)		
Log L	134.364	177.729	203.355	208.246	209.290	171.852
Reg[d]	0	21	21	0	21	0
Log $L(S)$[f]	96.925	125.931	160.746	163.023	167.830	123.126
Log $L(T)$[e]	142.378	179.484	214.771	215.674	224.969	173.860

[a–e] See table 1.
[f] Log-likelihood for subsample 1972–86.

pattern of QES LR statistics is similar to that observed for the LES model. QES(0, 0) is rejected against both QES(r, 0) and QES(0, d) but unlike the LES model QES(0, d) is rejected against QES(r, d), indicating that there are significant dynamic effects not accounted for by the distributional variables. Closer scrutiny of the QES(r, d) estimates suggests that this conclusion may be rather questionable. First, adding the autoregressive structure has a serious degrading effect on the first three b_i coefficients. Not only are they all insignificant, the third one is negative as well so that the derived value for the omitted "Other" commodity group is an implausible 0.9477. More importantly, the estimated autoregressive parameter is -0.0124 and under the null $r = 0$ the Wald χ^2 statistic is 0.584 whereas LR $= 9.782$. In the face of such conflicting evidence

TABLE 3.—LIKELIHOOD RATIO TESTS

H_0/H_1	DF/CV[a]	LR[b]
(a) AR(1) Dynamics		
LES(0, 0)/LES(r, 0)	1/6.635	108.132[c]
LES(0, 0)/LES(0, d)	12/26.217	174.572[c]
LES(r, 0)/LES(r, d)	12/26.217	67.562[c]
LES(0, d)/LES(r, d)	1/6.635	1.122
QES(0, 0)/QES(r, 0)	1/6.635	86.730[c]
QES(0, 0)/QES(0, d)	12/26.217	137.982[c]
QES(r, 0)/QES(r, d)	12/26.217	60.716[c]
QES(0, d)/QES(r, d)	1/6.635	9.469[c]
LES(0, 0)/QES(0, 0)	7/18.475	62.794[c]
LES(r, 0)/QES(r, 0)	7/18.475	41.392[c]
LES(0, d)/QES(0, d)	7/18.475	26.204[c]
LES(r, d)/QES(r, d)	7/18.475	34.546[c]
(b) Habit Formation		
LES(0, 0)/LES(h, 0)	4/13.277	121.086[c]
LES(h, 0)/LES(h, d)	12/26.217	58.102[c]
LES(0, d)/LES(h, d)	4/13.277	4.616
QES(0, 0)/QES(h, 0)	4/13.277	74.976[c]
QES(h, 0)/QES(h, d)	12/26.217	75.890[c]
QES(0, d)/QES(h, d)	4/13.277	12.884
LES(h, 0)/QES(h, 0)	7/18.475	16.684
LES(h, d)/QES(h, d)	7/18.475	34.472[c]

[a] Degrees of Freedom/Critical Value of χ^2 at 1%.
[b] Value of LR Statistic.
[c] Significant at 1% level.

it would seem best to withhold judgment about whether dynamics have a role in the model once the distributional effects are accounted for.[9]

In principle, it is possible to infer the shape of the underlying Engel curve by examining the pattern of the distribution parameters. Unfortunately, with our data we have only three points to infer the shape of the departure from the linear or quadratic function and this is insufficient to draw unambiguous conclusions. Accordingly, the question is not pursued nor are the estimated parameters reported here.[10]

(ii) Distribution and Habit Formation

Habit formation models have a long history in applied consumption analysis. Here we follow Pollak and Wales (1969) and specify

$$g_{it} = g_i^* + h_i q_{i,t-1}, \qquad 0 \le h_i < 1 \tag{9}$$

where $q_{i,t-1}$ is per household consumption of the last period.[11] The conjecture to be examined is that, as in the AR(1) set-up, the introduction of distributional effects will displace the habit formation effects. The notation for identifying models has been modified by replacing the autoregressive parameter with the habit formation vector, h; thus QES(h, d) is the quadratic model with both habit formation and distributional effects.

The last two columns of table 1 and 2 give the estimated coefficients of LES(h, 0) and LES(h, d), and QES(h, 0) and QES(h, d), respectively. Inspection of the coefficients for LES(h, 0) reveals two rather unsatisfactory features. First, there is some evidence of potential instability in the model as the habit formation parameter h_3 is significantly greater than one at the 5% level (but not at the 1% level). Secondly, although the habit formation coefficients are individually and collectively significant their introduction has caused three of the subsistence parameters to take on unexpected negative signs and one of these (g_3^*) is significant at the 1% level. In the LES(0, 0) estimation all the subsistence parameters were significant and of the right order of magnitude.[12] Thus the introduction of the lagged consumption variable appears to capture the effects previously attributed to the subsistence parameters. Similar effects can be observed in the results of Howe, Pollak and Wales (1979), p. 1242) and Green, Hassan and Johnson (1980, p. 436), the latter using annual Canadian data.

Even though the LR test decisively rejects LES(0, 0) against LES(h, 0), the discussion above casts doubt on the usefulness of the LES(h, 0) specification. These difficulties do not arise in the LES(h, d) specification. Although some of the habit formation parameters are negative none of them are significant and the LR test shows that we cannot reject LES(0, d) against LES(h, d). Thus we find that if distributional effects are

[9] The careful reader will note that in QES(r, 0) the autoregressive parameter is greater than one. A line search on r has shown that this is indeed the global maximum of the likelihood function with no local maxima in the (0, 1) interval. This problem disappears when the distribution variables are added.

[10] The detailed results are available from the author.

[11] The properties of these dynamic demand equations are analyzed in Pollak (1970).

[12] The subsistence interpretation holds only when $\gamma_i > 0$ and $x_t - \Sigma_j \gamma_j p_{jt} > 0$ whereas regularity requires that $q_{it} - \gamma_i > 0$. Given the broad level of commodity aggregation used in this study we would expect positive γ_i values.

included in the system the habit formation dynamics became statistically redundant.

The inferences using the QES model are almost identical to those for the LES model. In QES$(h, 0)$ all of the point estimates of the habit formation vector are greater than one but none are significantly so (1% level). There is the same degrading effect on the subsistence parameters when habit formation is introduced and this is again corrected when the distributional effects are included. The only difference is that in the QES(h, d) model one of the habit parameters is significant (1% level). However, we cannot reject QES$(0, d)$ against QES(h, d). The evidence appears to confirm our conjecture that traditional habit formation dynamics in aggregate equations may have appeared to be important because distributional variables have not been included in the model.

IV. Evaluations and Conclusions

The opening quote from Deaton suggested that the empirical evidence on distributional effects in aggregate equations was very weak. Insofar as our results clearly demonstrate the statistical significance of the distributional variables in all our specifications Deaton's assessment is open to question. Statistical significance does not, however, entail quantitative importance. One potentially useful way of assessing importance is to consider the impact of distributional variables on elasticities. In 1986 the uncompensated own price elasticities for clothing were -1.09, -0.82 and -0.20 for QES$(0, d)$, QES$(r, 0)$ and QES$(h, 0)$, respectively. Someone using the QES$(h, 0)$ parameters for tax policy would be very surprised in their revenue predictions if QES$(0, d)$ is the correct model.[13] Differences of the same order of magnitude exist for other commodity groups as well as for the income elasticities.

We have checked whether our principal results are robust to plausible alternative specifications for the degree of commodity aggregation and the choice of omitted distributional proportions. None of these alternative specifications provided any decisive evidence that the important rôle we have established for the distributional variables is dependent on the particular choices that we have made about commodity aggregation and omitted proportions. Furthermore, to allay any concern about the possible effects of our data interpolations we have also estimated the QES model for the period 1972–86. The penultimate row of table 2 gives the log-likelihood values for these subsample estimations. The test statistics derived from these likelihood values differ from the full sample only in that QES$(0, d)$ is now rejected against QES(h, d) but not against QES(r, d) whereas in the full sample the converse held.

It can also be argued that the distribution variables are simply picking up simple time trends due to the trending character of the distribution proportions. However, the argument can be placed on its head and one can argue with equal justification that the time trends, if used in lieu of the distribution variables, would pick up the effect of the relevant and omitted distribution effects. We have estimated all our models with a linear trend in addition to the distribution variables, and the values of the log-likelihoods are given in the last row of tables 1 and 2. In most cases the trend specification is significant and uniformly so whenever the distribution variables are included in the specification. Thus, the trend terms make an independent contribution over and above that captured by the distributional variables. If, following Blundell (1988, p. 28) we are willing to conjecture that household attributes such as demographic characteristics evolve slowly, our trend specification will capture these omitted variables and help to reduce a potential source of specification bias.

To summarize, insofar as our results on the LES model with AR(1) errors and distributional effects are almost identical to Stoker's we can be satisfied that Stoker's results are neither country- nor period-specific. By extending the Stoker procedures to the QES model we have shown that distributional effects can be used to determine not just departures from linearity but from the quadratic as well. Not surprisingly, since linearity is rejected in all the tests using the LES specification that specification is (with one exception, LES$(h, 0)$) rejected when tested against the nesting QES alternative. The QES model does not, however, seem to capture the curvature of the underlying micro equations as there are significant distributional effects in all of the QES speci-

[13] The clothing elasticity is particularly topical for Canada because on January 1, 1991 the federal tax on clothing went from zero to 7%.

fications. As noted by a referee, semi-parametric procedures which are not tied to specific functional forms could be potentially useful in resolving the question of curvature. In this context the regularity conditions (see tables 1 and 2) are of some interest. In the LES model the "d" specifications fare badly whereas conventional dynamics do well. Under QES, on the other hand, only three specifications satisfy regularity everywhere and two of these are "d" specifications. It should be noted, however, that significant distributional effects indicate functional mis-specification so that failure of the regularity conditions does not necessarily constitute model failure.

There is an extensive literature documenting dynamic effects in consumer demand systems; consider, for example, Chambers' (1990) recent study in which an LES model with habit formation is found to have superior forecasting performance compared to five alternative specifications. In the light of our investigation we think a more skeptical view of this literature is in order. There are two interpretations of our results which can lead to this conclusion. The first suggests that conventional dynamics and distributional effects are simply alternative models equally capable of explaining the data.[14] This was Stoker's conclusion on observing that the log-likelihoods for LES$(r, 0)$ and LES$(0, d)$ were approximately equal. Our results appear to be somewhat stronger inasmuch as the distribution models always have substantially larger log-likelihoods than the dynamic models. Furthermore, the Akaike information criterion always picks LES$(0, d)$ and QES$(0, d)$ over their alternative AR(1) and habit formation counterparts.

The second interpretation, which we are inclined to favour, holds that the dynamic effects which have been observed in the literature have been picking up the effects of omitted distributional effects. In three of the four cases we have examined, the dynamics are not statistically significant once the distribution effects are in the model. In the fourth case (QES$(0, d)$ against QES(r, d)) the evidence is inconclusive. This suggests to us that we may have drawn the wrong conclusions from the literature. Rather than inferring the presence of dynamic behaviour we should perhaps have been concluding that the underlying behavioural equations are of a different functional form.

[14] A referee has argued that there may be a fundamental identification problem in distinguishing between habit formation and distributional effects that can only be resolved by panel data. This is a plausible conjecture and we would be happy to accept the verdict of such data when it becomes available. In the meantime we prefer to maintain a skeptical position, not least because the habit formation model appears to be applied indiscriminately. The case for habit formation in, say, detergent or floor coverings, has never seemed entirely persuasive to us.

REFERENCES

Anderson, Gordon, and Richard Blundell, "Testing Restrictions in a Flexible Dynamic Demand System: An Application to Consumer's Expenditure in Canada," *Review of Economic Studies* 50 (July 1983), 397–410.

Berndt, Ernst R., Masako N. Darrough, and W. Erwin Diewert, "Flexible Functional Forms and Expenditure Distributions: An Application to Canadian Consumer Demand Functions," *International Economic Review* 18 (Oct. 1977), 651–675.

Blundell, Richard, "Consumer Behaviour: Theory and Empirical Evidence—A Survey," *Economic Journal* 98 (Mar. 1988), 16–66.

Chambers, Marcus J., "Forecasting with Demand Systems: A Comparative Study, *Journal of Econometrics* 44 (June 1990), 363–376.

Deaton, Angus, "Demand Analysis," in Z. Giliches and M. D. Intriligator (eds.), *Handbook of Econometrics* vol. 3 (Amsterdam: North-Holland, 1986).

Green, Richard D., Zuhair A. Hassan, and Stanley R. Johnson, "Testing for Habit Formation Autocorrelation and Theoretical Restrictions in Linear Expenditure Systems," *Southern Economic Journal* 47 (Oct. 1980), 433–443.

Heineke, J. M., and H. M. Shefrin, "Exact Aggregation and the Finite Basis Property," *International Economic Review* 29 (Aug. 1988), 525–538.

Howe, Howard, Richard A. Pollak, and Terence J. Wales, "Theory and Time Series Estimation of the Quadratic Expenditure System," *Econometrica* 47 (Sept. 1979), 1231–1248.

Lewbel, Arthur, "Exact Aggregation, Distribution Parametrization, and a Nonlinear Representative Consumer," in *Advances in Econometrics*, vol. 7 (Greenwich, Conn: JAI Press, 1988).

Pollak, Robert A., "Habit Formation and Dynamic Demand Functions," *Journal of Political Economy* 78 (July/Aug. 1970), 745–763.

Pollak, Robert A., and Terence J. Wales, "Estimation of the Linear Expenditure System," *Econometrica* 37 (Oct. 1969), 611–628.

Stoker, Thomas M., "Simple Tests of Distributional Effects on Macro-Economic Equations," *Journal of Political Economy* 94 (Aug. 1986), 763–795.

VanDaal, J., and A. H. Q. M. Merkies, *Aggregation in Economic Research* (Dordrecht: D. Reidel, 1984).