CHAPTER 22

The New Keynesian Approach to Dynamic General Equilibrium Modeling: Models, Methods and Macroeconomic Policy Evaluation

Sebastian Schmidt, Volker Wieland
Goethe University of Frankfurt and Institute for Monetary and Financial Stability

Abstract

This chapter aims to provide a hands-on approach to New Keynesian models and their uses for macroeconomic policy analysis. It starts by reviewing the origins of the New Keynesian approach, the key model ingredients and representative models. Building blocks of current-generation dynamic stochastic general equilibrium models are discussed in detail. These models address the famous Lucas critique by deriving behavioral equations systematically from the optimizing and forward-looking decision making of households and firms subject to well-defined constraints. State-of-the-art methods for solving and estimating such models are reviewed and presented in examples. The chapter goes beyond the mere presentation of the most popular benchmark model by providing a framework for model comparison along with a database that includes a wide variety of macroeconomic models. Thus, it offers a convenient approach for comparing new models to available benchmarks and for investigating whether particular policy recommendations are robust to model uncertainty. Such robustness analysis is illustrated by evaluating the performance of simple monetary policy rules across a range of recently estimated models, including some with financial market imperfections, and by reviewing recent comparative findings regarding the magnitude of government spending multipliers. The chapter concludes with a discussion of important objectives for ongoing and future research using the New Keynesian framework.

Keywords

Monetary macroeconomics, Keynesian models, New Keynesian models, dynamic stochastic general equilibrium models, New Neoclassical synthesis, model comparison, rational expectations, policy evaluation, policy robustness, monetary and fiscal policy

JEL classification codes

C51, C52, C61, C68, E12, E17, E52, E63

22.1 INTRODUCTION

What is New Keynesian economics? In their 1991 introduction to a collection of seminal contributions Greg Mankiw and David Romer gave the following answer: (i) New Keynesian theories of business cycles posit that fluctuations in nominal variables like the

Handbook of CGE Modeling - Vol. 1 SET
ISSN 2211-6885, http://dx.doi.org/10.1016/B978-0-444-59568-3.00022-5

money supply influence fluctuations in real variables, like output and employment, and (ii) real market imperfections such as imperfect competition or imperfect information also have an important influence on economic fluctuations. At the time, they contrasted New Keynesian thought with real business cycle theory that emphasized technological disturbances and perfect markets (*cf*. Kydland and Prescott, 1982). Constraints on price or wage adjustment constituted a central element of New Keynesian models of the economy. A first wave of New Keynesian models following the 1970s rational expectations revolution, such as Fischer (1977), Phelps and Taylor (1977), and Taylor (1979a, 1979b), used long-term nominal contracts to explain how demand shifts cause real fluctuations even if expectations are rational and the shifts are anticipated.

The ensuing debate between real business cycle and New Keynesian theorists, and the successive extension and empirical application of both types of models, eventually triggered a second wave of New Keynesian models or monetary business cycle models that aimed to marry key ingredients of both approaches. The small-scale model of Goodfriend and King (1997) and Rotemberg and Woodford (1997) was quickly extended with additional decision aspects and constraints. These models, which are frequently referred to as New Keynesian dynamic stochastic general equilibrium (DSGE) models, are exemplified by the medium-scale model of the US economy of Christiano *et al*. (2005). Nowadays, medium- to large-scale DSGE models are routinely used by economists at central banks and international institutions to evaluate monetary and fiscal stabilization policies.

The objective of this chapter is to explain how to build current-generation New Keynesian DSGE models, how to estimate them and how to use them for policy design. Given their influence on current macroeconomic thinking and policy analysis such a hands-on introduction should be useful for any reader interested in macroeconomics. However, several of the topics addressed in this chapter should also be of interest to a wider readership that uses computable general equilibrium (CGE) modeling in many other areas of economic policy making. For example, the systematic handling of optimizing and forward-looking decision making by economic agents subject to a variety of constraints that is practiced in the macroeconomic DSGE literature may be usefully applied in other areas. Furthermore, the methods used for approximating the solutions of non-linear dynamic and stochastic models and for estimating them with economic data may easily be applied elsewhere. Finally, we review a new approach to model comparison that helps to identify robust policies under model uncertainty (see Wieland *et al*., 2012). Comparative robustness analyses appear particularly urgent to us, as commentators have criticized macroeconomists, in general, and DSGE modelers, in particular, for relying too much on a specific model and failing to foresee or warn of the risk of global financial crisis and recession. Additionally, such a comparative approach could benefit practical model-based policy making in other fields, including international trade, economic development and climate change.

The remainder of this chapter proceeds as follows. Section 22.2 offers a brief history of thought and additional references regarding the development of New Keynesian macroeconomic models. Section 22.3 begins with a detailed presentation of a small-scale New Keynesian model. Emphasis is laid on the microeconomic foundations of the model and the implied cross-equation restrictions on the reduced-form system. We then discuss various extensions that improve its empirical performance and are regularly included in medium- to large-scale DSGE models used for practical policy analysis. The section concludes with an illustration of the Lucas critique in Section 22.3.3. Section 22.4 discusses methods for solving dynamic general equilibrium models and provides an introduction to Bayesian methods for model estimation. An example is given by the estimation of the small-scale New Keynesian model on data of the US economy. In the last part of this section (Section 22.4.3) we address some remaining challenges for model estimation. Section 22.5 presents our approach to model comparison that allows for systematic and straightforward comparison and evaluation of macroeconomic models and alternative policies. Section 22.6 applies the comparative approach to evaluate the performance and robustness of monetary policy rules when the true model underlying the economy is unknown and the policy maker is instead confronted with a range of competing models. The second part of this section (Section 22.6.2) reviews recent comparative findings regarding the effectiveness of government spending stimulus programs such as the US American Recovery and Reinvestment Act (ARRA) of 2009. Section 22.7 concludes with an outlook on further research.

22.2 THE NEW KEYNESIAN APPROACH TO MONETARY ECONOMICS: A BRIEF HISTORY OF THOUGHT

The common characteristic of New Keynesian monetary models, compared to earlier models, is the combination of rational expectations, staggered price and wage setting, and policy rules. The term is also used to contrast such models with traditional Keynesian models that do not allow for rational expectations. New Keynesian models rather than the traditional Keynesian models are the ones commonly taught in graduate schools because they capture how people's expectations and microeconomic behavior change over time in response to policy interventions, and because they are empirically estimated and fit the data. They are therefore viewed as better for policy evaluation. In assessing the effect of government actions on the economy, for example, it is important to take into account how households and firms adjust their spending decisions as their expectations of future government policy changes.

In Section 22.1 we distinguished two waves of New Keynesian modeling in the last 35 years. Key driving factors of this scientific process included empirical failures of traditional approaches, intellectual challenges such as the Lucas critique, theoretical innovations such as the combination of nominal rigidities with forward-looking and optimizing behavior of economic agents, and the invention of new modeling and estimation techniques. The first wave of New Keynesian models took off in the late 1970s. The apparent failure of traditional Keynesian models to satisfactorily explain the 1970s stagflation raised many questions about the connection between inflation and economic activity, and the role of monetary policy in stabilizing the economy. The famous Lucas critique underscored the need to account for the forward-looking and optimizing behavior of households and firms in macroeconomic models intended to be used for policy evaluation. Traditional Keynesian models were typically lacking these elements. Expectations were modeled as backward-looking, i.e. fixed combinations of past values of the respective variables and the models' behaviorial equations were not being directly related to individual optimization.

Innovations in the late 1970s and 1980s led to the development of the first generation of New Keynesian models with rational expectations and nominal rigidities that allowed for interesting interactions between (systematic) monetary policy and real economic activity. These innovations included modeling of menu costs and overlapping wage and price contracts (Fischer, 1977; Taylor, 1979b; Calvo, 1983), new methods for solving linear and non-linear dynamic models with rational expectations as well as successful estimation of such models using maximum likelihood techniques (Hansen and Sargent, 1980; Fair and Taylor, 1983). First-generation New Keynesian models were extended, enlarged and eventually applied rather intensively in practical monetary policy analysis at central banks. We highlight the following three models from the 1990s that played an important role for US monetary policy: Taylor's (1993) model of the G-7 economies, Fuhrer and Moore's (1995) model with relative-real-wage staggered contracts that helped explain US inflation persistence, and the Federal Reserve's FRB-US model described, for example, in Reifschneider *et al.* (1999). All three models are available for comparison and policy evaluation exercises from the model archive that is discussed in more detail in Section 22.5.

Another challenge for Keynesian-style macroeconomic modeling arose from the real business cycle (RBC) approach to macroeconomic fluctuations propounded by Kydland and Prescott (1982). Their extension of the neoclassical growth model to study the real (rather than monetary) sources of business cycles delivered a modeling approach that stringently enforced all the restrictions following from the utility maximization of representative households and the profit maximization of representative firms on the dynamics of macroeconomic variables. At the same time the RBC approach put technological innovations forth as the main drivers of business cycles. Monetary policy has no real effects in the RBC world and, therefore, stabilization policy is of minor concern. Goodfriend and King (1997) and Rotemberg and Woodford (1997) presented a first monetary business cycle model using the approach to microeconomic foundation practiced in RBC research, but including also nominal rigidities and imperfect competition. In this manner, New Keynesian research aims to incorporate Keynesian ideas into the dynamic general equilibrium frameworks used in the RBC literature. For this reason, the above-mentioned monetary business cycle model is alternatively referred to as the New Neoclassical Synthesis model or the New Keynesian DSGE model. The inclusion of nominal rigidities and imperfect competition had also been motivated by the failure of RBC models – as seen by part of the New Keynesian literature – to account for certain empirical regularities (Rotemberg and Woodford, 1996; Galí, 1999).

Recent years have witnessed an explosion in New Keynesian modeling. Importantly, Christiano *et al.* (2005) developed and estimated a medium-sized DSGE model with capital accumulation, utilization and investment, monopoly power in goods and labor markets, price and wage rigidities, and a number of additional frictions, i.e. adjustment costs or constraints on household and firm decision making. While Christiano *et al.* (2005) used impulse response function matching techniques in order to choose values of the model parameters, Smets and Wouters (2003, 2007) showed how the parameters can be estimated more easily and effectively with Bayesian methods. This approach was quickly popularized and led to widespread New Keynesian model building at central banks around the world. Levin *et al.* (2003) and Taylor and Wieland (2012) provide systematic comparisons of these models with earlier New Keynesian models and assess their implications for monetary policy rules. New Keynesian models offer many uses for practical policy analysis. They can be utilized to evaluate the desirability of different policy strategies and of institutional developments such as the creation of a common currency area in Europe. Medium-scale models exhibiting a wide range of frictions have been deployed as tools for forecasting, for evaluating the effects of policy changes and for elucidating the sources of macroeconomic fluctuations by means of historical decompositions (e.g. Christiano *et al.*, 2005; Smets and Wouters, 2003, 2007; Adolfson *et al.*, 2007).

22.3 BUILDING NEW KEYNESIAN MODELS

New Keynesian business cycle models are characterized by a set of key assumptions and ingredients. Similar to real business cycle models, modern New Keynesian models are general equilibrium models. Equilibrium conditions are explicitly derived from the optimization problems of consumers and producers. A standard assumption is that agents have rational expectations, i.e. agents form model-consistent expectations conditional on the information available. Producers have market power over prices that facilitates the introduction of short-run nominal price rigidities. The presence of nominal rigidities is the key ingredient that distinguishes New Keynesian models from RBC models and that assigns an explicit stabilization role to monetary policy.

22.3.1 Simple model with microeconomic foundations

This section briefly reviews the small-scale stochastic New Keynesian model that has become a much-used workhorse model and is now widely taught in the first-year macro sequence in graduate school (see Galí, 2008; Galí and Gertler, 2007; Goodfriend and King, 1997; Walsh, 2010; Woodford, 2003). The model economy is inhabited by households, monopolistically competitive firms, the monetary authority and a government sector. Households decide how much to consume rather than save and how much labor to supply in order to maximize their lifetime utility. In turn, firms hire labor in order to produce differentiated goods. In contrast to the RBC literature, firms do not act under perfect competition, but under monopolistic competition, which converts them from price takers to price setters. This assumption is necessary to be able to introduce price stickiness. Specifically, firms can reset prices only once in a while. Due to these nominal rigidities the monetary authority can affect real activity in the short run because the real interest rate will no longer be insensitive to movements in the monetary policy instrument – the short-term nominal interest rate. The government collects lump-sum taxes and consumes part of the final good. Finally, the model is augmented with a set of stochastic shocks.

22.3.1.1 Households

The model economy contains a large number of identical households. The representative household is characterized by the following *preferences* regarding consumption, labor and real money balances:

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(C_t, M_t/P_t) - V(H_t)]. \tag{22.1}$$

Equation (22.1) represents households' expected discounted *lifetime utility*, where C_t denotes the household's consumption of a basket of differentiated goods, M_t measures the end-of-period money balances, P_t is the price of the consumption good basket in terms of money and H_t denotes the number of hours worked. The inclusion of real money balances in the utility function is a standard short-cut to capture their transaction services (see, e.g. Woodford, 2003).[1] The consumption goods basket C_t consists of a continuum of differentiated goods:

[1] Alternatively, one could model transactions frictions explicitly by introducing a cash-in-advance constraint on household consumption.

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{22.2}$$

where $\varepsilon > 1$ and $C_t(i)$ denotes consumption of good i. The price index P_t is then defined as the minimum expenditure at which the household can buy one unit of C_t:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} \mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}, \tag{22.3}$$

where $P_t(i)$ denotes the price of good i. One can show that $P_t C_t = \int_0^1 P_t(i) C_t(i)\mathrm{d}i$ and:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t. \tag{22.4}$$

Thus, household demand for good i depends on its relative price, $p_t(i) = \frac{P_t(i)}{P_t}$, with ε representing the elasticity of demand. A 1% increase in the relative price of good i leads to a reduction in the demand for this good of ε%.

The period utility function $U(C, M/P)$ is assumed to be strictly increasing and concave in each of its arguments, and $V(H)$ is assumed to be increasing and convex. Finally, $0 < \beta < 1$ denotes the subjective discount factor. Under rational expectations, the representative household maximizes (22.1) subject to a sequence of budget constraints:

$$P_t C_t + M_t + E_t Q_{t,t+1} B_t \leq W_t H_t + M_{t-1} + B_{t-1} + T_t + \Gamma_t, \tag{22.5}$$

for all t. B_t represents the quantity of a one-period, riskless, nominal government bond paying one unit of money per bond in period $t+1$. Its price is denoted by $E_t Q_{t,t+1}$. $E_t Q_{t,t+1}$ is equal to $1/R_t$, where R_t is the riskless one-period gross nominal interest rate. The nominal wage rate is denoted by W_t, T_t are (possibly negative) lump-sum transfers of the government and Γ_t denotes firms' profits distributed to the household sector. The optimality conditions of the households' expected utility maximization problem correspond to:

$$\frac{1}{R_t} = \beta E_t \frac{U_C(C_{t+1}, m_{t+1})/P_{t+1}}{U_C(C_t, m_t)/P_t} \tag{22.6}$$

$$\frac{V_H(H_t)}{U_C(C_t, m_t)} = w_t \tag{22.7}$$

$$\frac{U_m(C_t, m_t)}{U_C(C_t, m_t)} = \frac{R_t - 1}{R_t}, \tag{22.8}$$

where $w_t = W_t/P_t$ is the real wage. U_C and U_m with $m = M/P$ denote the marginal utility of consumption and real money balances, respectively, and V_H measures the marginal disutility of labor. We will interpret these optimality conditions when summarizing the complete set of model equations.

22.3.1.2 Firms

The economy is inhabited by a continuum of firms of measure one. Each firm i possesses a production technology:

$$Y_t(i) = A_t N_t(i), \tag{22.9}$$

where A_t denotes a common technology shock and $N_t(i)$ denotes labor demand by firm i. In this simple model, labor is the only production input. Demand for good i is given by:

$$Y_t(i) = C_t(i) + G_t(i), \tag{22.10}$$

where $G_t(i)$ denotes government purchases of good i, satisfying:

$$G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} G_t. \tag{22.11}$$

The public consumption good basket G_t is defined equivalently to the private consumption good basket (22.2).

Firms are price setters. However, following Calvo (1983), it is assumed that in a given period each firm can reset its price $P_t(i)$ only with probability $1 - \theta$. Therefore, each period a fraction $1 - \theta$ of firms reoptimizes its price while the remaining fraction θ of firms keep their price unchanged. Importantly, the probability of a change in the price of a firm i is independent of the time elapsed since its last price change. This price stickiness is an important feature of the model because it allows monetary policy to affect real variables in the short run.

To produce output $Y_t(i)$, firms have to hire labor. Minimizing production costs for a given demand level subject to the production technology leads to:

$$MC_t(i) = \frac{W_t}{A_t}, \tag{22.12}$$

where MC_t is the Lagrange multiplier representing marginal costs. In equilibrium, marginal costs of firm i equal the wage divided by the marginal product of labor. Note, that in our model marginal costs are identical across firms, $MC_t(i) = MC_t$. We can then formulate the optimization problem of firm i that resets its price in period t, taking into account that the price set today might be effective for some time and taking as given the demand for its good, as:

$$\max_{P_t(i)} \sum_{j=0}^{\infty} E_t Q_{t,t+j}\theta^j Y_{t+j}(i)\left[P_t(i) - MC_{t+j}\right], \tag{22.13}$$

subject to household and government demand functions, i.e. Equations (22.4) and (22.11), respectively. Note:

$$Q_{t,t+j} = \beta^j \frac{U_C(C_{t+j}, m_{t+j})/P_{t+j}}{U_C(C_t, m_t)/P_t}, \tag{22.14}$$

is the stochastic discount factor. The first-order condition then corresponds to:

$$\sum_{j=0}^{\infty} E_t Q_{t,t+j}\theta^j Y_{t+j} P_{t+j}^{\varepsilon}\left[P_t^*(i) - \frac{\varepsilon}{\varepsilon - 1} MC_{t+j}\right] = 0, \tag{22.15}$$

where $P_t^*(i)$ is the optimal price set by firm i in period t. Equation (22.15) reveals that all firms reoptimizing their price in a given period will set the same price, $P_t^*(i) = P_t^*$. In the case of flexible prices, Equation (22.15) reduces to $P_t^* = \frac{\varepsilon}{\varepsilon - 1} MC_t$. In this case, the optimal price is a constant markup over contemporaneous marginal costs. In the sticky-price model, the optimal price is instead a markup over a weighted sum of current and expected future marginal costs. From the definition of the price index in equation (22.3) it follows that the aggregate price level is given by:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{22.16}$$

22.3.1.3 Government

The government consumes part of the produced goods. Market clearing of all goods markets implies:

$$Y_t = C_t + G_t, \tag{22.17}$$

where $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right)^{\frac{\varepsilon}{\varepsilon-1}}$. When simulating the model, we will assume that deviations of government spending from its share in steady state output follow a simple AR(1) process. The government budget identity is given by:

$$P_t G_t + B_{t-1} = \frac{B_t}{R_t} - T_t + M_t - M_{t-1}. \tag{22.18}$$

Hence, government spending is financed by a combination of one-period nominal government bonds, lump-sum taxes (negative transfers) and seigniorage revenues. Note, since optimizing households base their consumption and savings decision only on the expected present value of lifetime income and taxes are raised in a lump-sum fashion, this model exhibits Ricardian equivalence. In other words, household decisions and government solvency only depend on the present discounted values of household income and government revenues, respectively, and not on the particular path of taxes and government debt. Thus, the modeler does not need to keep track of the timing of taxation and the path of government debt. Assuming that the government adjusts the present value of lump-sum tax revenue to ensure that its intertemporal budget constraint is satisfied taking as given government spending, nominal prices and seigniorage from money creation, it follows that the central bank is free to set money growth independently from fiscal policy considerations.

22.3.1.4 Monetary policy

Rather than assuming that the monetary authority controls money growth directly, it is more consistent with standard policy practice to model monetary policy with an operating target for the short-term nominal interest rate, R_t. The central bank then conducts open-market operations to achieve the operating target for the interest rate in the money market. Here, we define this interest rate target by means of a simple monetary policy rule that depends on inflation and the output gap:

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\tau_\pi} \left(\frac{Y_t^{\text{gap}}}{Y^{\text{gap}}}\right)^{\tau_Y} \upsilon_t, \tag{22.19}$$

where $\pi_t = P_t/P_{t-1}$ denotes the gross inflation rate between period $t-1$ and t, and Y_t^{gap} is the output gap, i.e. the deviation of actual output from some natural level, which will be defined explicitly further below. Variables without a time subscript denote steady-state values of the respective variable. Unsystematic components of interest rate policy are captured by the monetary policy shock υ_t. This interest rate rule is assumed to be known by all agents in the economy. As to the stock of money in the economy, the central bank supplies money to the extent demanded by households at the current levels of the nominal interest rate, income and prices.

22.3.1.5 Log-linearized system of equations

It remains to impose market clearing also on the labor market, and the markets for money balances and government bonds. Then, all model equations may be summarized to discuss the solution of the model, and its implications for aggregate fluctuations and macroeconomic policy design. For convenience, we proceed directly to a log-linear approximation of the model. Non-linear approximation methods will be discussed in Section 22.4.

Let $\hat{x}_t = \log(x_t) - \log(x)$, for some variable x_t, where x denotes the corresponding steady-state level. Thus, the variable is expressed in terms of percentage deviations from its steady state. Section 22.4.1.1 provides an introduction to the method of (log)-linearization. We log-linearize Equations (22.6), (22.7), (22.8), (22.9), (22.15), (22.16), (22.17) and (22.19) around the non-stochastic steady state with zero inflation, i.e. a gross steady-state inflation rate of $\pi = 1$. In this manner we obtain the following set of linear equations that define a local approximation of the complete model near its steady state:

$$\hat{C}_t = E_t\hat{C}_{t+1} - \frac{1}{\sigma}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) \tag{22.20}$$

$$\hat{H}_t = \frac{1}{\eta}\hat{w}_t - \frac{\sigma}{\eta}\hat{C}_t \tag{22.21}$$

$$\hat{m}_t = \frac{\sigma}{\sigma_m}\hat{C}_t - \frac{1}{\left(\beta^{-1}-1\right)\sigma_m}\hat{R}_t \tag{22.22}$$

$$\hat{Y}_t = \hat{A}_t + \hat{H}_t \tag{22.23}$$

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \hat{g}_t \tag{22.24}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{Y}_t^{\text{gap}} \tag{22.25}$$

$$\hat{R}_t = \tau_\pi\hat{\pi}_t + \tau_Y\hat{Y}_t^{\text{gap}} + \hat{v}_t, \tag{22.26}$$

The parameters σ, η,σ_m and κ are defined as follows: $\sigma \equiv -\frac{U_{C,C}(C,m)}{U_C(C,m)}C$, $\eta \equiv \frac{V_{H,H}(H)}{V_H(H)}H$, $\sigma_m \equiv -\frac{U_{m,m}(C,m)}{U_m(C,m)}m$ and $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}\left(\frac{\sigma}{C/Y}+\eta\right)$. We assumed that $U_{C,M}(C,m) = U_{M,C}(C,m) = 0$, meaning that utility is separable between consumption and real money balances.

Equation (22.20) is the log-linearized consumption Euler Equation (22.6). It states that consumption increases when expected future consumption increases or the *ex ante* real interest rate, $\hat{R}_t - E_t\hat{\pi}_{t+1}$, decreases. It is often referred to as the New Keynesian IS curve once consumption is substituted with aggregate demand. Equation (22.21) can be interpreted as a labor supply equation. It indicates that the number of hours worked depends positively on the equilibrium real wage and negatively on the level of consumption. Equation (22.22) defines the demand for real money balances. In this model, the money demand function only serves the purpose to determine the amount of money that the central bank has to supply at the nominal interest rate implied by the monetary policy rule. Equation (22.23) represents the production technology aggregated over all firms. Equation (22.24) is the resource constraint, where $\hat{g}_t \equiv \frac{G_t - G}{Y}$. Equation

(22.25) results from combining the optimal price set by adjusting firms, i.e. Equation (22.15), with the aggregate price defined by Equation (22.16). This is the so-called New Keynesian Phillips curve. It indicates that current inflation depends on expected future inflation and the contemporaneous output gap. The gap is defined as $\hat{Y}_t^{\text{gap}} = \hat{Y}_t - \hat{Y}_t^{\text{nat}}$, i.e. the percentage deviation of output from its natural level, which would be obtained under price flexibility in the absence of the Calvo-constraint on price adjustment. This natural output level is given by:

$$\hat{Y}_t^{\text{nat}} = \frac{1}{\sigma/(C/Y) + \eta}\left[(1+\eta)\hat{A}_t + \frac{\sigma}{C/Y}\hat{g}_t\right]. \tag{22.27}$$

Finally, Equation (22.26) is the log-linearized monetary policy rule. The percentage deviation of government spending from its steady-state level (as a share of total output), aggregate technology and the monetary policy shock are assumed to follow AR(1) processes:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g \tag{22.28}$$

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_t^A \tag{22.29}$$

$$\hat{v}_t = \rho_v \hat{v}_{t-1} + \varepsilon_t^R, \tag{22.30}$$

where $\varepsilon_t^j, j \in \{g, A, R\}$, are zero mean, constant variance *iid* innovations. The above set of linear equations can then easily be solved to obtain the solution functions describing the equilibrium dynamics of the endogenous variables. Importantly, unlike the structural equations these reduced-form solution functions are not independent of monetary policy. Methods for model solution are discussed in Section 22.4.1. Here, we proceed instead with an analysis of the model dynamics.

22.3.1.6 Model dynamics

The mechanisms by which random innovations propagate into persistent fluctuations in endogenous model variables may be illustrated by impulse−response functions. They isolate the impact of a particular shock throughout the economy. In the following we present impulse responses for a specific parameterization of the linear approximation of the small-scale New Keynesian model presented in Sections 22.3.1.1−22.3.1.5. The values of the model parameters are chosen as follows. Assuming that the period length is one quarter, the subjective discount factor is set to $\beta = 0.99$, which implies a steady-state annualized interest rate of around 4%. The parameters in the consumption demand and labor supply equations are set to $\sigma = 1.5$ and $\eta = 1$ as in Ravenna and Walsh (2006). The preference parameter regarding real money balances is set to a rather high value of $\sigma_m = 110$, consistent with the empirical evidence presented in Andres *et al.* (2006). The

Calvo parameter is fixed at $\theta = 0.75$, implying that prices are reset on average every four quarters. Steady-state government spending as a share of GDP is set to 0.2. Finally, the response parameters in the monetary policy function, $\tau_\pi = 1.5$ and $\tau_Y = 0.5/4$, are chosen in accordance with Taylor's rule (see Taylor, 1993b). The ARcoefficients of the three structural shocks in our model are fixed at $\rho_g = 0.85$, $\rho_A = 0.9$ and $\rho_v = 0.5$.

Figure 22.1 displays the dynamic responses of output, inflation, hours, real money balances, nominal and real interest rates to a monetary policy shock. Since prices are sticky, the increase in the nominal interest rate fosters an increase in the real rate as well. Higher real interest rates induce households to reduce current consumption. Faced with reduced household demand, firms in turn require less labor. As a result, equilibrium marginal costs decline and create downward pressure on inflation. Note that monetary policy only affects the deviation of economic activity from its natural level. Thus, the responses of the output gap and output (not shown) are identical. Finally, the rise in the nominal interest rate and the reduction in output both serve to decrease households' demand for real money balances.

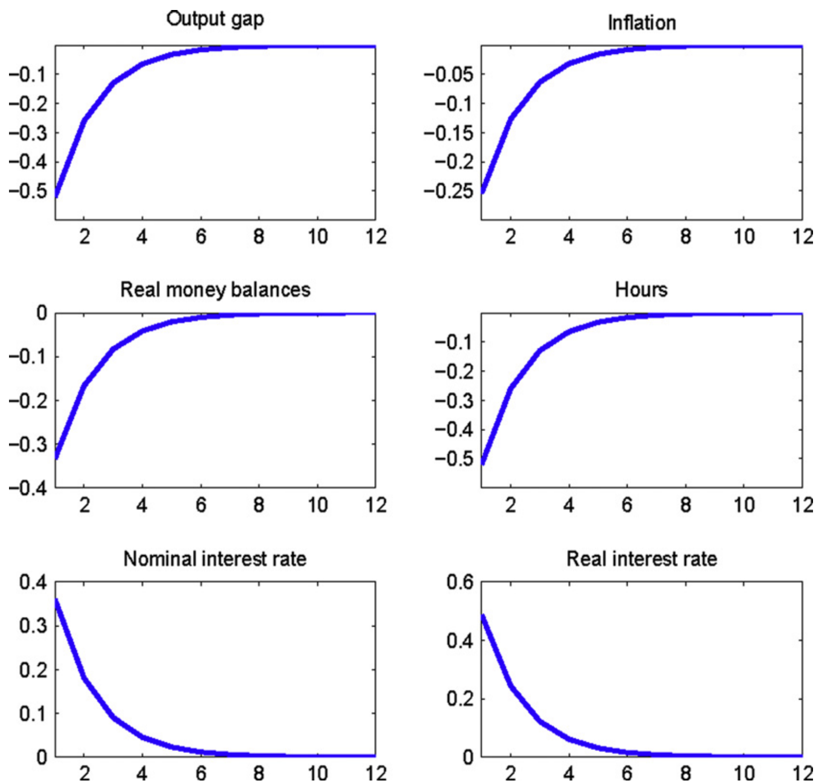

Figure 22.1 Monetary policy shock.

Figure 22.2 reports the dynamic responses to a technology shock. In this case, both the deviation of output from steady state and the output gap, i.e. the deviation of output from the flexible-price level, are shown. The improvement in firms' production technology reduces production costs. However, due to price stickiness, only a share of the firms can lower their prices immediately. Hence, the increase in aggregate demand and output is less than proportional to the improvement in technology. Therefore, equilibrium hours of work decline temporarily and the output gap turns negative. The reductions in inflation and the output gap induce the monetary authority to lower interest rates. However, interest rates do not fall by as much as would be needed in order to completely offset the decline in the two target variables. In the empirical literature, the directions of the effects of exogenous changes in technology on various macroeconomic variables are controversial (see, e.g. the literature overview by Galí and Rabanal, 2004).

Figure 22.3 displays dynamic responses to a government spending shock. The increase in government demand for the composite consumption good stimulates aggregate demand. However, we observe that private consumption is partially crowded out and hence total output rises by less than government spending. The fall in private

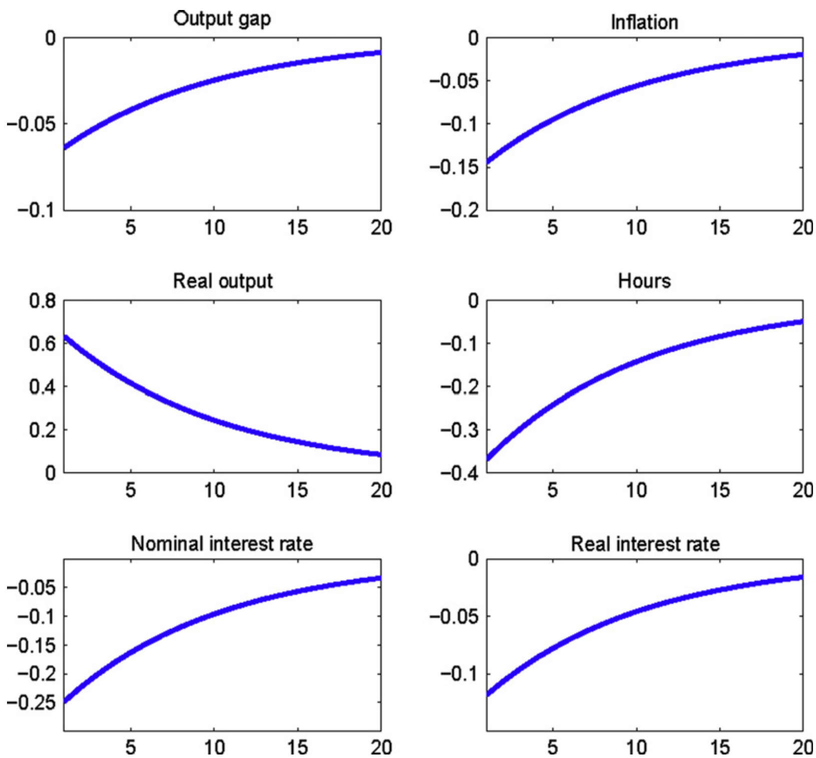

Figure 22.2 Technology shock.

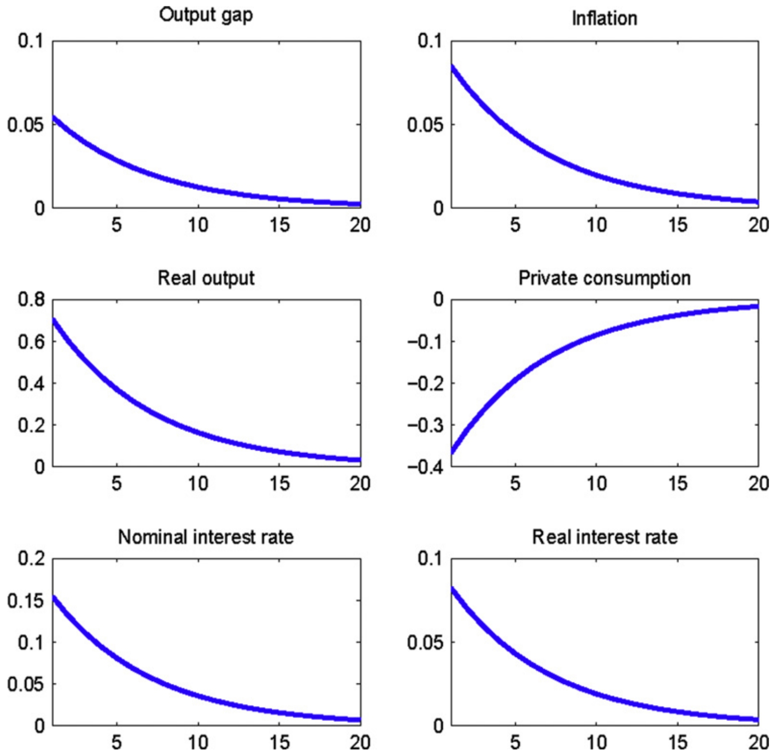

Figure 22.3 Government spending shock.

consumption results from anticipated higher taxes that reduce lifetime household income and from an intertemporal substitution effect due to the increase in interest rates following the greater need for financing of government spending. The interest rate response, however, is not aggressive enough to prevent small increases in the output gap and inflation. The empirical literature is characterized by an active debate on the size of government spending multipliers. Galí *et al.* (2007) have shown that an extension of the small-scale New Keynesian model of this section with rule-of-thumb consumers can induce a temporary increase in household consumption following a government spending shock. The consumption of the rule-of-thumb consumers is simply determined by current income. We will return to the question of the magnitude of government spending multipliers in Section 22.6, where we will review recent findings with empirically estimated DSGE models regarding the impact of the 2009 US fiscal stimulus program.

22.3.2 Medium-scale models for policy analysis

While the small-scale model described in Section 22.3.1 forms a useful starting point for understanding some of the principal ingredients of New Keynesian modeling, more elaborate models are needed to perform forecast exercises and policy simulations. In particular, the small baseline model does not capture the high degree of persistence observed for many macroeconomic variables. It has long been noted that many macroeconomic variables appear to exhibit hump-shaped responses to shocks. For example, the well-known graduate macroeconomics textbook of Blanchard and Fischer (1989), which was widely used in the 1990s, motivated theoretical modeling of business cycles by pointing out that US GNP deviations from trend are well described by an ARMA(2,2) process that implies hump-shaped output fluctuations following random shocks. Such hump-shaped dynamics also arose naturally in traditional Keynesian-style empirical macroeconomic models that included multiple lags of endogenous variables in estimated behavioral equations.

More recently, structural vector autoregression (SVAR) models have been employed to identify specific economic shocks on the basis of minimal structural assumptions. Again, such empirical methods revealed that many macroeconomic variables exhibit hump-shaped responses to demand-side shocks such as monetary policy innovations (see Christiano *et al.* 1999). In fact, the New Keynesian DSGE model of Christiano *et al.* (2005), which was the first medium-scale model to fully incorporate recent advances in terms of microeconomic foundations, was estimated by minimum distance methods using an empirical SVAR model as benchmark. Specifically, they minimized the distance between the model and the empirical impulse response to a monetary policy shock. Thus, their model's impulse response function to a monetary policy shock exemplifies the empirical SVAR evidence on hump-shaped dynamics for the US economy (shown in Section 22.3.2.6). Altig *et al.* (2005) extend the impulse response function matching approach to include general technology and investment-specific shocks along with the monetary policy shock. Again, most empirical dynamic responses of US macroeconomic variables follow a hump-shaped pattern. For recent evidence on the dynamic persistence in euro area macroeconomic aggregates, see Smets and Wouters (2003) and Coenen and Wieland (2005). Smets and Wouters (2003, 2007) employed Bayesian estimation techniques to estimate medium-scale models similar to the Christiano *et al.* (2005) model for the euro area and the US, respectively. This Bayesian likelihood approach rather than the impulse response function matching method has become the estimation tool of choice among academics and central bank economists working with DSGE models.

Recently, Taylor and Wieland (2012) have compared impulse responses of the current generation New Keynesian models of Altig *et al.* (2005) and Smets and Wouters (2007) with those of an influential first-generation New Keynesian model by Taylor (1993a). While both types of models deliver hump-shaped dynamics of macroeconomic aggregates after a variety of structural shocks, the most striking finding is that the US output response to a monetary policy shock is almost identical across the three models. Apparently, the impact of an unexpected change in the US federal funds rate on GDP is the same in spite of 15 years of additional data, new estimation methods and structural assumptions. The model archive and software presented in Section 22.5 allows readers to explore and compare impulse responses and serial correlations in all of the above-mentioned models and many more.

In the remainder of this section, we present the additional frictions that are typically introduced in medium-scale New Keynesian DSGE models to account for the empirical dynamics of key macroeconomic aggregates. For example, the model of Christiano *et al.* (2005) features additional nominal rigidities not present in the small baseline model such as staggered wage contracts, and price and wage indexation. Indexation implies that firms that cannot reoptimize their prices in a given period instead let their prices evolve with a prespecified aggregate index such as the preceding period's rate of inflation. Additionally, Christiano *et al.* (2005) employ real frictions. They introduce capital accumulation along with investment adjustment costs and variable capital utilization. Furthermore, household preferences are modeled with habit formation in consumption. These frictions and others are described in the following.

22.3.2.1 Capital and investment

In medium-scale models, production of the consumption good typically uses not only labor, but also capital services as inputs. Here, we follow the assumption of Christiano *et al.* (2005) that households own the economy's capital stock and rent capital services to firms in an economy-wide rental market for capital. Thus, the simple production function of the small-scale model, Equation (22.9), is replaced with:

$$Y_t(i) = A_t F\left(N_t(i), K_t^s(i)\right), \tag{22.31}$$

Here, K_t^s denotes capital services rented from households and F represents a Cobb–Douglas production function, $F(N_t(i), K_t^s(i)) = N_t(i)^{1-\alpha}(K_t^s(i))^{\alpha}$, where $\alpha \in (0,1)$ denotes the capital services share in production. The stock of physical capital, K_t, follows:

$$K_t = (1-\delta)K_{t-1} + S(i_t, i_{t-1}), \tag{22.32}$$

where the parameter δ denotes the capital depreciation rate, i_t refers to purchases of the investment good, and the function S represents the technology for the production of new capital goods as a function of current and past investment. The latter function is meant to capture investment adjustment costs. Christiano *et al.* (2005) assume $S(i_t, i_{t-1}) = \left(1 - \tilde{S}\left(\frac{i_t}{i_{t-1}}\right)\right) i_t$, where $\tilde{S}(1) = \tilde{S}'(1) = 0$ and $\tilde{S}''(1) > 0$. New capital becomes productive with a lag of one period. Thus, the amount of capital services in the current period can only be varied by changing the utilization rate of capital, u_t, which is set by the representative household:

$$K_t^s = u_t K_{t-1}. \tag{22.33}$$

Variation in the utilization rate is subject to a cost, $a(u_t)K_{t-1}$. a is an increasing, convex function with $a(1) = 0$. The steady-state utilization rate is given by $u = 1$. R_t^{k} denotes the rental rate of capital. Utilization costs and household's earnings from renting capital services to the firms, $R_t^{\mathrm{k}} u_t K_{t-1}$, both, enter the budget constraint. Cost minimization by the household then requires that the marginal benefit of raising the utilization rate equals marginal costs, i.e. $R_t^{\mathrm{k}} = a'(u_t)$. Log-linearization then implies $\frac{1}{\sigma_a} R_t^{\mathrm{k}} = \hat{u}_t$, where $\sigma_a = \frac{a''(1)}{a'(1)} > 0$. Empirical estimates of σ_a tend to be fairly small (see Christiano *et al.*, 2005). In turn, the elasticity of the capital utilization rate with respect to the rental rate of capital tends to be large. Christiano *et al.* (2005) find that variable capital utilization is crucial to allow their model to generate the desired inertia in the inflation response to a monetary policy shock together with a persistent output response. Without variable capital utilization, firms' cost of capital would be more sensitive to an expansionary monetary policy shock, resulting in stronger inflationary pressures and weaker effects on real output. In the literature, modeling assumptions such as a variable utilization rate are referred to as real rigidities.

Some studies replace the assumption of competitive markets for production inputs, with the assumption that these inputs are firm-specific. For instance, if capital is firm-specific, then each individual firm accumulates capital only for its own use. This specificity represents another real rigidity. It is used, for example, in Sveen and Weinke (2005) and Woodford (2005). With an economy-wide market for capital, an increase in demand in a part of the firm sector will increase the rental price for capital for all firms. By contrast, with firm-specific capital, the individual firm's variable production costs are less affected by an increase in demand for some other firms' products. Importantly, firm-specific production inputs help dampen the effect of an expansionary shock on inflation as shown by Eichenbaum and Fisher (2007).

Turning to the evolution of investment, the introduction of investment adjustment costs implies that the household's first order condition with respect to investment involves lagged as well as expected future investment:

$$\hat{i}_t = \frac{1}{1+\beta}\hat{i}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{i}_{t+1} + \frac{1}{(1+\beta)\tilde{S}''(1)}\hat{P}_i^k, \tag{22.34}$$

in addition to the real value of the existing capital stock, which is denoted by $\hat{P}_t^{\mathrm{k}}$. The lag of investment helps to generate endogenous persistence that is not present in the purely forward-looking baseline New Keynesian model presented earlier. Specifically, investment and therefore output, exhibit hump-shaped responses to an expansionary monetary policy shock.

22.3.2.2 Habit formation in consumption

When consumers form habits, their preferences depend not only on current, but also on past levels of consumption. Specifically, we assume that household period utility from consumption depends on the difference between the level of current consumption and previous period's consumption.[2] Modelers distinguish between external and internal habits. External habits relate the current level of consumption to aggregate past consumption, internal habits refer to individual past consumption. Assuming internal habits, the representative household's objective function (1) is replaced with:

$$E_0 \sum_{t=0}^{\infty} \beta^t [U(C_t - bC_{t-1}, M_t/P_t) - V(H_t)], \tag{22.35}$$

where $b \in [0, 1]$ is referred to as the habit parameter. In the case of $b = 0$, (22.35) reduces to the standard utility function without habits. With $b > 0$, aggregate current consumption no longer depends solely on expected future consumption, but also on past consumption. Consequently, the consumption Euler Equation of the baseline model (22.20) has to be modified to include past consumption. Christiano *et al.* (2005) report a point estimate of $b = 0.65$. Similar to investment adjustment costs, consumption habits help to increase the degree of endogenous model persistence. Specifically, the response of consumption to exogenous shocks becomes more inertial and exhibits a hump-shaped pattern. Without habit formation, impulse responses of consumption to a monetary policy shock peak in the initial period and then decline monotonically towards the steady state as in the small-scale model.[3] To gain intuition, reconsider the Euler equation from the small baseline model, equation (22.20), with expected future consumption brought on the left-hand side:

$$E_t \Delta \hat{C}_{t+1} = \frac{1}{\sigma}\left(\hat{R}_t - E_t \hat{\pi}_{t+1}\right). \tag{22.36}$$

Here, Δ denotes the first difference operator. Equation (22.36) indicates that expected consumption growth in the small baseline model is low, whenever interest rates are low (e.g. due to an expansionary shock). This relationship requires that the response to an expansionary shock starts from a high consumption level and then decreases monotonically in subsequent periods. Instead, with habit formation, it is the change in the growth rate of consumption that is related to the interest rate. In this case, low interest rates are consistent with an expectation of a decline in an initially positive growth rate of

2 Alternatively, some models assume that the ratio between current consumption and previous period's consumption enters the utility function. See Schmitt-Grohe and Uribe (2005) for an overview of modeling approaches to habit formation.

3 The response of consumption to a monetary policy shock in the small-scale model is qualitatively equivalent to the response of the output gap shown in Figure 22.1.

consumption. This expectation translates into a hump-shaped pattern of the impulse response of consumption.

22.3.2.3 Price indexation

In the small-scale model, the rate of inflation in the New Keynesian Phillips curve (22.25) is a purely forward-looking variable. It depends on the current output gap and expected future inflation. Solving forward, it is easy to show that current inflation is a function of current and expected future output gaps. Absent substantial inertia in the output gap or *ad hoc* shocks in the baseline model cannot replicate the empirical degree of inflation persistence. Empirical estimates of Philips curves support the inclusion of a lagged inflation term on the right-hand side of (22.25) as shown, for example, by Galí and Gertler (1999). However, such hybrid backward- and forward-looking Phillips curves have been suggested at least since the early 1990s. As it became apparent that the staggered contracts suggested in the first wave of New Keynesian modeling by Taylor (1980) and Calvo (1983) did not match the inflation persistence in the data, researchers such as Fuhrer and Moore (1995) proposed structural interpretations of staggered relative price contracts that introduced additional lags of the price level in standard contracting specifications.[4]

A rationale for including lagged inflation in the small-scale New Keynesian model of the preceding section is obtained by introducing price indexation as in Christiano *et al.* (2005). Under (partial) price indexation, firms that do not receive a Calvo signal to reoptimize their price in a given period instead increase previous period's price mechanically by an amount proportional to past inflation. Thus, indexation gives rise to the following New Keynsian Phillips curve:

$$\hat{\pi}_t = \frac{\gamma}{1+\beta\gamma}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\gamma}E_t\hat{\pi}_{t+1} + \frac{\kappa}{1+\beta\gamma}\hat{Y}_t^{\text{gap}}, \tag{22.37}$$

where the parameter γ represents the degree of price indexation to past inflation. A similar expression can be derived under the assumption that a share of the firms follow simple rules of thumb when setting their price as in Galí and Gertler (1999). While price indexation helps to produce additional inertia in the rate of inflation, there is little microeconomic evidence that firms change prices continuously as pointed out by Klenow and Malin (2010) and the references therein.

22.3.2.4 Sticky wages

In the baseline model presented before, households and firms interact in a perfectly competitive labor market. This assumption has rather unrealistic implications for wage dynamics. In fact, the earliest New Keynesian contributions cited in Section 22.1 tended

[4] Coenen and Wieland (2002) and Coenen and Wieland (2005) contrast estimates of these different specifications with US, euro area and Japanese data.

to focus on staggered wage rather than price contracts motivated by widespread use of nominal contracts in labor markets. Medium-scale DSGE models therefore generally feature some form of staggered wage setting. Erceg *et al.* (2000) introduced sticky wages into the small-scale model of the preceding section. They assumed that households supply differentiated labor services over which they exhibit some monopolistic power. While each household supplies one type of labor service, firms employ all types of labor to produce consumption goods. As in Calvo (1983), each period a randomly drawn fraction of households is allowed to reset their nominal wage while the remaining fraction demands the same wage as in the previous period. The optimization problem of a household that is allowed to reset its nominal wage in the current period then consists of choosing the nominal wage that maximizes its expected discounted lifetime utility, taking into account that it might not be able to reoptimize its nominal wage for some time in the future.[5] The optimality condition for the wage setting decision results in a Phillips curve for wage inflation:

$$\hat{\pi}_t^{\mathrm{W}} = \beta E_t \hat{\pi}_{t+1}^{\mathrm{W}} + \kappa_{\mathrm{W}} \hat{Y}_t^{\mathrm{gap}} - \xi_W \hat{w}_t^{\mathrm{gap}}, \tag{22.38}$$

where $\hat{\pi}_t^{\mathrm{W}} = \hat{W}_t - \hat{W}_{t-1}$ is the wage inflation rate, and $\hat{w}_t^{\mathrm{gap}}$ denotes the real wage gap, i.e. the deviation of the actual real wage from its natural level that would be obtained in the absence of price and wage rigidities. The wage inflation equation replaces the labor supply Equation (22.21) in the baseline model. The presence of sticky wages induces a more muted response of real wages to monetary policy shocks.

22.3.2.5 Financial market frictions

The recent global financial crisis has drawn attention to the need for an explicit modeling of financial market imperfections in New Keynesian DSGE models. Fortunately, this research need not start from ground zero. A prominent starting point for integrating financial frictions into micro-founded models of the macroeconomy is the so-called financial accelerator model of Bernanke *et al.* (1999). We first present its main features and then discuss some more recent extensions.

Bernanke *et al.* (1999) introduce credit market imperfections into an otherwise standard New Keynesian model with variable capital, and demonstrate that these frictions contribute to propagating and amplifying the response of key macroeconomic variables to nominal and real shocks. Specifically, they consider an agency problem that is due to information asymmetries in borrower–lender relationships. Their model of the economy is inhabited by three types of agents – risk-averse households, risk-neutral entrepreneurs and retailers. Entrepreneurs use capital and labor to produce wholesale

[5] Since labor income differs across households, individual private consumption need not be the same across all households. A short-cut for avoiding this source of heterogeneity that is often used in the literature is to assume that households have access to complete asset markets that allow for full consumption risk sharing.

goods that are sold to the retail sector. The retail market is characterized by monopolistic competition. Each period, entrepreneurs accumulate capital that becomes productive one period later. Bernanke *et al.* (1999) assume that entrepreneurs have finite horizons, thereby precluding the possibility that aggregate entrepreneur wealth increases without bounds. Entrepreneurs have to borrow from households via a financial intermediary to finance part of the new capital. The agency problem arises because the return to capital is prone to idiosyncratic risk and can only be observed by the financial intermediary if it pays an auditing cost.[6] Therefore, the entrepreneurs' net worth becomes a crucial determinant of their borrowing costs. If net worth is high, less of the capital acquisition has to be financed via external borrowing, thereby reducing the severity of the agency problem. The optimal contract in this environment turns out to be similar to a standard debt contract. The contract is characterized by a non-default loan rate, Z_t^j, and a threshold value for the idiosyncratic shock, ω^j, denoted by $\bar{\omega}^j$. This threshold is defined as the minimum realization of the idiosyncratic shock required in order for the entrepreneur to be able to repay the loan:

$$\bar{\omega}^j R_{t+1}^{\mathrm{k}} Q_t K_t^j = Z_t^j B_t^j, \tag{22.39}$$

where R_t^{k} is the gross return to capital averaged across firms, Q_t is the price per unit of capital and K_t^j denotes the amount of capital acquired by entrepreneur j in period t for production in period $t+1$. The funds that have to be borrowed equal $B_t^j = Q_t K_t^j - N_t^j$, where N_t^j is net worth of entrepreneur j at the end of period t. Entrepreneurs accumulate net worth primarily from profits from capital investment and to a minor extent also from the supply of labor. If the realization of the idiosyncratic shock lies below the contractual threshold level, the entrepreneur defaults, the financial intermediary pays the auditing cost and takes over the entrepreneur's remaining wealth. Since the idiosyncratic loan risk can be diversified perfectly, the opportunity cost of the financial intermediary equals the risk-free nominal interest rate. Any aggregate risk is absorbed by the risk-neutral entrepreneurs as specified in the contract. Each entrepreneur then has to choose the amount of capital to buy. The optimality condition relates the ratio of external finance costs to the riskless rate and the ratio of capital expenditures to net worth. The aggregated log-linearized condition corresponds to:

$$E_t\left(\hat{R}_{t+1}^{\mathrm{k}}\right) - \hat{R}_t = \chi\left(\hat{Q}_t + \hat{K}_t - \hat{N}_t\right), \tag{22.40}$$

where the parameter $\chi > 0$ is a function of the structural model parameters.[7] Equation (22.40) indicates that the so-called external finance premium, i.e. the difference between the cost of external funding and the opportunity cost of internal funds, rises with the

6 This framework refers to the so-called costly state verification problem of Townsend (1979).

7 See Bernanke *et al.* (1999) for the details of the derivation and conditions that permit aggregation.

amount of external borrowing. As Bernanke *et al.* (1999) have shown, unexpected movements in the price of capital can have considerable effects on entrepreneurs' financial conditions. The entrepreneurs' net worth affects borrowing conditions, which in turn influence investment decisions. For instance, an unexpected drop in the return to capital reduces net worth of a leveraged entrepreneur by more than one-for-one. The external finance premium rises, demand for capital decreases, investment decreases and the price of capital falls, which reduces entrepreneurial net worth even further. Importantly, the credit market feeds back into the real economy. The countercyclical movement in the external finance premium serves to amplify the response of macroeconomic aggregates such as output and investment to economic shocks.

In recent years, a number of extensions of the basic financial accelerator model have been developed in response to the observation of the global financial crisis. These extensions include, for example, the consideration of nominal instead of real financial contracts (e.g. Christensen and Dib, 2008), the incorporation of the financial accelerator in a small open economy model, (e.g. Gertler *et al.* 2007) and in a medium-scale New Keynesian DSGE model (e.g. De Graeve, 2008). Meh and Moran (2010) consider the role of financial frictions in a DSGE model that introduces an agency problem between banks and entrepreneurs as in Bernanke *et al.* (1999), together with an agency problem between banks and their creditors, i.e. households. In this two-sided agency problem, not only entrepreneurs' wealth influences business cycle movements, but also the capital position of banks. Furthermore, Iacoviello (2005) has developed a financial accelerator model with financing constraints at the household level in the form of collateral constraints tied to housing values (see also Kiyotaki and Moore, 1997). Allowing for nominal debt contracts, he shows that the feedback of the financial market friction on the economy depends on the type of shock. Responses of output and consumer price inflation to a demand shock get amplified and propagated, whereas the output response to supply shocks is mitigated. In the case of an unexpected increase in aggregate demand, goods prices and housing prices rise, which increases borrowers collateral value and reduces the real value of their debt. Since the borrowers in the model have a higher propensity to consume than the lenders, the net effect of this resource transfer from creditors to debtors is positive and serves to amplify the output response. By contrast, a negative supply shock decreases inflation and therefore raises borrowers' real value of debt, leading to a mitigated output response.

22.3.2.6 Model dynamics

How are model dynamics affected by these additional frictions? In order to answer this question we compare impulse responses in a prototypical medium-scale model estimated by Smets and Wouters (2007) with Bayesian methods on US data to the impulse responses in the small-scale model of Section 22.3.1 displayed in Figure 22.1. The Smets-Wouters model of the US economy incorporates capital accumulation with investment

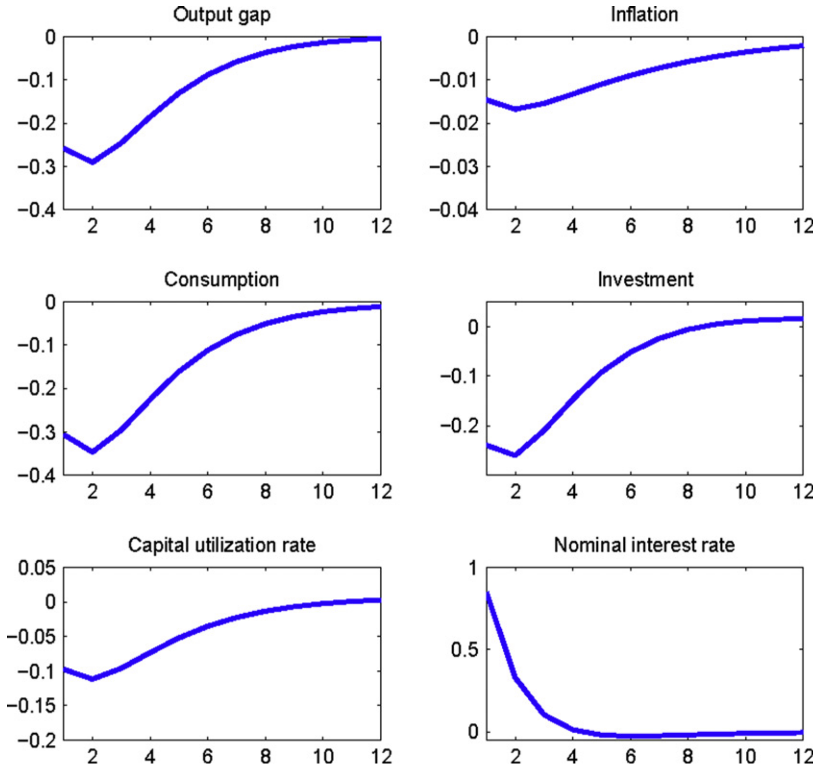

Figure 22.4 Monetary policy shock in the Smets–Wouters (2007) model.

adjustment costs and variable capital utilization, habit formation in consumption, partial price indexation, and sticky wages. Figure 22.4 displays the dynamic responses of several variables to a monetary policy shock in the Smets–Wouters model. To facilitate the comparison with the baseline model, monetary policy is assumed to follow the policy rule defined by equation (22.26) in both models. In other words, we have replaced the estimated rule from the Smets–Wouters model with Equation (22.26):[8] Indeed, we observe hump-shaped impulse responses of consumption and investment. The responses of the output gap and inflation are more persistent than in the small-scale model, respectively. In fact, the effect of the policy shock on the considered variables persists beyond the effect on the nominal interest rate. Also, the inflation response is particularly subdued, being much smaller in magnitude than in the small-scale model.

Figure 22.5 compares the impulse responses of output and inflation in the Smets–Wouters model to those from another medium-scale DSGE model estimated by

[8] The model archive and software presented in Section 22.5 allows readers to conduct such comparisons rather easily with a range of different policy rules and many more macroeconomic models.

Altig *et al.* (2005). This comparison is particularly interesting because Altig *et al.* (2005) selected the values of the model parameters in order to minimize the differences between the model's impulse responses and corresponding impulse responses from a structural vector autoregression on US data. Thus, the impulse responses of the Altig *et al.* (2005) model are close to the empirical responses to such shocks when they are identified with minimal structural assumptions. However, it is important to note that in order to obtain impulse responses to a particular shock, say a monetary policy shock, from a vector autoregression one has to make identifying assumptions. Thus, also a vector autoregression is not free of structural assumptions when it comes to shock identification. Altig *et al.* (2005) choose identification assumptions for the SVAR model that are consistent with the structure of their DSGE model. Finally, note that we have used the monetary policy rule estimated by Christiano *et al.* (2005) in the Smets–Wouters model and the Altig *et al.* (2005) model. Any differences in the impulse response to a monetary policy shock between the two models must be due to the other structural equations and parameters. Figure 22.5 shows that except for the response in the first period, the impulse response of output in the Smets–Wouters model is very close to the response in the Altig *et al.* (2005) model. The impact on inflation is somewhat different though small in magnitude in both models.

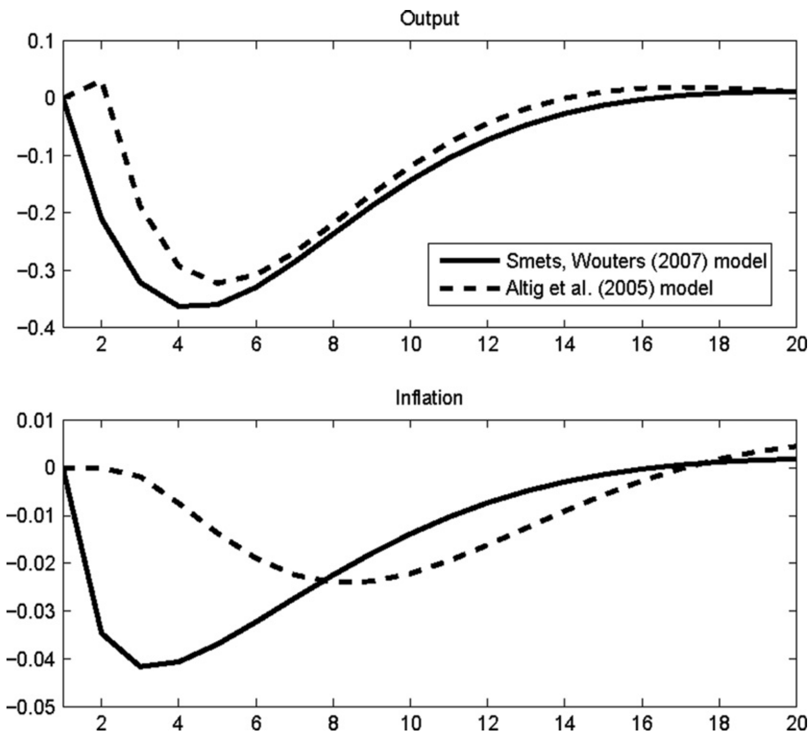

Figure 22.5 Monetary policy shock in two medium-scale DSGE models.

While the government sector is fairly rudimentary in the models considered so far, it is worth noting that a number of DSGE models have been built with much more detailed characterizations of the government sector in order to allow more extensive analysis of fiscal shocks and fiscal rules. Examples are the QUEST III model of the European Commission described in Ratto *et al.* (2009) or the European Central Bank's New Area Wide Model described in Coenen *et al.* (2008). Such models include not only lump-sum taxation, but also distortionary labor income, capital income and value-added taxation. Furthermore, they differentiate between government consumption and government investment, and include reaction functions for tax and debt dynamics. We will return to a more detailed discussion of fiscal shocks and policy issues in Section 22.6.

22.3.3 Using structural models for policy analysis: the Lucas critique

Two key ingredients of New Keynesian modeling that distinguish it from the traditional Keynesian paradigm are that: (i) The decision rules of economic agents are based on optimization subject to constraints and that (ii) the agents' view of the future behavior of variables is formed under rational expectations. Importantly then, agents' decision rules inevitably vary with changes in policy. This dependence becomes explicit in the system of reduced-form equations. The Lucas critique, named after economist Robert E. Lucas and formulated in Lucas (1976), questions the validity of policy evaluation exercises based on estimated reduced-form relationships that – while potentially successful in short-term forecasting – fail to recognize this dependence. Lucas argues that such econometric models are not suitable for policy analysis because the estimated parameters are not policy-invariant.

To give an example, let us reconsider the hybrid New Keynesian Phillips curve introduced in Section 22.3.2.3:

$$\hat{\pi}_t = \frac{\gamma}{1+\beta\gamma}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\gamma}E_t\hat{\pi}_{t+1} + \frac{\kappa}{1+\beta\kappa}\hat{Y}_t^{\mathrm{gap}} + \frac{1}{1+\beta\kappa}\varepsilon_t^{\pi}, \tag{22.41}$$

where we have added a zero mean, constant variance *iid* shock ε_t^{π}. For simplicity, let us assume that the policy maker can directly control the output gap $\hat{Y}_t^{\mathrm{gap}}$ and monetary policy is described by the rule:

$$\hat{Y}_t^{\mathrm{gap}} = \tau\hat{\pi}_t, \tag{22.42}$$

where $\tau < 0$. Substituting the policy rule into (22.41), we obtain:

$$\hat{\pi}_t = \frac{\gamma}{1+\beta\gamma}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\gamma}E_t\hat{\pi}_{t+1} + \frac{\kappa\tau}{1+\beta\kappa}\hat{\pi}_t + \frac{1}{1+\beta\kappa}\varepsilon_t^{\pi}. \tag{22.43}$$

We guess the following solution for inflation:

$$\hat{\pi}_t = a\hat{\pi}_{t-1} + b\varepsilon_t^{\pi}, \tag{22.44}$$

where the solution function parameters a and b remain to be determined. This guess implies $E_t\hat{\pi}_{t+1} = a\hat{\pi}_t$. Substituting this expression for inflation expectations into Equation (22.43) and collecting terms, one can show that $a(\tau)$ is defined by the stable solution of the polynomial $a^2 - \frac{1}{\beta}(1+\beta\gamma-\kappa\tau)\ a + \frac{\gamma}{\beta} = 0$ and the second parameter is defined by $b(\tau) = \frac{1}{1+\beta\gamma-\kappa\tau-\beta a(\tau)}$. As the notation emphasizes, the coefficients $a(\tau)$ and $b(\tau)$ depend on the policy rule parameter τ.

If we use the solution function to substitute out the expected inflation term in (22.41) and collect terms, we arrive at:

$$\hat{\pi}_t = d_1\hat{\pi}_{t-1} + d_2\hat{Y}_t^{\text{gap}} + d_3\varepsilon_t^{\pi}, \tag{22.45}$$

where:

$$d_1 = \frac{\gamma}{1+\beta[\gamma - a(\tau)]} \tag{22.46}$$

$$d_2 = \frac{\kappa}{1+\beta[\gamma - a(\tau)]} \tag{22.47}$$

$$d_3 = \frac{1}{1+\beta[\gamma - a(\tau)]}. \tag{22.48}$$

Equation (22.45) is reminiscent of a traditional Phillips curve. However, Equation (22.45) is a reduced-form relationship, not a structural equation. Policy analysis based on empirical estimates of $d_j, j = 1, 2, 3$, that fails to recognize the parameter restrictions imposed by (22.46)–(22.48) is misleading, because the parameters d_j, $j = 1, 2, 3$, are not invariant, but will change in response to changes in the policy rule parameter τ. By contrast, when using structural models to analyze changes in systematic monetary policy, one incorporates the parameter restrictions on the optimal decision rules of economic agents and therefore automatically takes into account changes in behavioral reduced-form parameters resulting from changes in policy making.

22.4 METHODS FOR MODEL SOLUTION AND ESTIMATION

22.4.1 Solving New Keynesian models

Here, we aim to inform the reader of some commonly used approaches for obtaining approximate solutions of New Keynesian DSGE models such as log-linearization, first- and second-order numerical approximation, and certain non-linear methods. References to available software are also provided.

Consider a particular model m defined by the following system of non-linear difference equations:

$$E_t\left[\psi_m\left(x_{t+1}^m, x_t^m, \nu_t^m, \mu^m\right)\right] = 0, \tag{22.49}$$

where the x^ms are $n \times 1$ vectors of endogenous model variables. The model may include current values, lags and leads of endogenous variables. Such higher-order systems can be written as a first-order system by augmenting the x^m vectors accordingly. The model variables are functions of each other, of structural shocks, ν_t^m, and of model parameters μ^m. A variety of approaches exists to approximate and solve the model in (22.49). We present three different solution procedures. The first procedure consists of two steps, i.e. constructing a linear approximation of the system of non-linear equations and then obtaining the exact solution of the linear system. The second procedure presented is the extended path (EP) solution method that does not require prior linearization of the non-linear system. Finally, the value function iteration procedure is presented in the context of a linear quadratic dynamic programming problem.

22.4.1.1 Linear approximation

In presenting the derivation of the linear approximation of system (22.49) we abstract from the stochastic model components:

$$\psi_m\left(x_{t+1}^m, x_t^m, \mu^m\right) = 0. \tag{22.50}$$

A first-order Taylor series approximation around the non-stochastic steady state yields:

$$0 \approx \psi(\bar{x}) + \frac{\partial \psi}{\partial x_t}(\bar{x}) \times (x_t - \bar{x}) + \frac{\partial \psi}{\partial x_{t+1}}(\bar{x}) \times (x_{t+1} - \bar{x}), \tag{22.51}$$

where we have simplified notation in that the model index m and the dependence of $\psi(x_{t+1}, x_t)$ on the model parameters, μ, are suppressed for the moment. The $n \times n$ matrix $\frac{\partial \psi}{\partial x_t}(\bar{x})$ constitutes the Jacobian matrix of $\psi(x_{t+1}, x_t)$ with respect to x_t evaluated at the steady state $\bar{x}$. In order to derive such a log-linear approximation for the case of the small-scale New Keynesian model in Section 22.3.2, we define:

$$A \equiv \frac{\partial \psi}{\partial x_{t+1}}(\bar{x}) \times \mathrm{diag}(\bar{x}), \quad B \equiv -\frac{\partial \psi}{\partial x_t}(\bar{x}) \times \mathrm{diag}(\bar{x}),$$

where $\mathrm{diag}(\bar{x})$ is an $n \times n$ matrix with the elements of $\bar{x}$ on the main diagonal. The log-linear approximation of the non-linear system then corresponds to:

$$A\hat{x}_{t+1} = B\hat{x}_t. \tag{22.52}$$

Following the notation in Section 22.3.2, $\hat{x}_t$ denotes the percentage deviation of variable x_t from its steady state.

22.4.1.2 Solving a system of linear difference equations

Various methods are available for solving a system of linear (stochastic) difference equations such as (22.52) exactly. References include Blanchard and Kahn (1980), Uhlig (1999), Klein (2000), Sims (2002), and King and Watson (1998, 2002). In the following we use the well-known method of Blanchard and Kahn (1980). Their method requires that the system of linear difference equations may be reformulated as:

$$\begin{pmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{pmatrix} = \Omega \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} + \Gamma f_t, \tag{22.53}$$

where the vector of possibly (log-)linearized endogenous variables x_t has been divided into an $n_1 \times 1$ vector of predetermined endogenous variables, $x_{1,t}$, and an $n_2 \times 1$ vector of free endogenous variables, $x_{2,t}$, with $n = n_1 + n_2$. The vector f_t contains exogenous forcing variables such as the exogenous technology variable in the small-scale model presented in Section 22.3.2. The idea is to transform the variables in a way that facilitates the solution of the system. First, one applies a Jordan decomposition to the matrix Ω:

$$\Omega = \Lambda^{-1} J \Lambda. \tag{22.54}$$

Matrix J is called the Jordan canonical form of Ω and contains the eigenvalues of Ω on its diagonal. Then matrix J is partitioned such that J_1 contains the eigenvalues that lie inside or on the unit circle and J_2 contains the eigenvalues that lie outside the unit circle:

$$J = \begin{pmatrix} J_1 & 0 \\ 0 & J_2 \end{pmatrix}. \tag{22.55}$$

The matrices Λ and Γ are then partitioned conformably:

$$\Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix}. \tag{22.56}$$

A unique solution of the model exists if the number of eigenvalues outside the unit circle equals the number of free endogenous variables, n_2.[9] If this condition is satisfied, the vector $X_t = [x'_{1,t}\, x'_{2,t}]'$ may be transformed according to:

$$z_t \equiv \Lambda x_t. \tag{22.57}$$

Then, the linear system (22.53) can be rewritten as:

$$\begin{pmatrix} z_{1,t+1} \\ E_t z_{2,t+1} \end{pmatrix} = \begin{pmatrix} J_1 & 0 \\ 0 & J_2 \end{pmatrix} \begin{pmatrix} z_{1,t} \\ z_{2,t} \end{pmatrix} + \begin{pmatrix} \tilde{\Gamma}_1 \\ \tilde{\Gamma}_2 \end{pmatrix} f_t, \tag{22.58}$$

[9] See Blanchard and Kahn (1980) for further details.

where $\tilde{\Gamma} \equiv \Lambda\Gamma$. The system of the transformed variables is diagonal.[10] Therefore, one can consider the subsystem related to the vector $z_{2,t}$ separately from the rest of the system. Solving for $z_{2,t}$, one obtains:

$$z_{2,t} = J_2^{-1} E_t z_{2,t+1} - J_2^{-1} \tilde{\Gamma}_2 f_t. \tag{22.59}$$

Iterating forward on this equation, one arrives at:

$$z_{2,t} = -J_2^{-1} \sum_{j=0}^{\infty} \left(J_2^{-1}\right)^j \tilde{\Gamma}_2 E_t f_{t+j}, \tag{22.60}$$

by application of the Law of Iterated Expectations. If one transforms again the endogenous variables in (22.60), one obtains the solution functions for the free endogenous variables:

$$x_{2,t} = -\Lambda_{22}^{-1} \Lambda_{21} x_{1,t} - \Lambda_{22}^{-1} J_2^{-1} \sum_{j=0}^{\infty} \left(J_2^{-1}\right)^j \tilde{\Gamma}_2 E_t f_{t+j}. \tag{22.61}$$

Finally, the solution functions for the predetermined endogenous variables can be obtained by substituting (22.61) into (22.53):

$$\begin{aligned} x_{1,t+1} = {} & (\Omega_{11} - \Omega_{12}\Lambda_{22}^{-1}\Lambda_{21}) x_{1,t} + \left(\Gamma_1 - \Omega_{12}\Lambda_{22}^{-1} J_2^{-1} \tilde{\Gamma}_2\right) f_t - \Omega_{12}\Lambda_{22}^{-1} J_2^{-1} \\ & \times \sum_{j=1}^{\infty} \left(J_2^{-1}\right)^j \tilde{\Gamma}_2 E_t f_{t+j}. \end{aligned} \tag{22.62}$$

Here, matrix Ω has been partitioned conformably:

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}. \tag{22.63}$$

22.4.1.3 EP solution method for non-linear models

In certain cases, linear approximations are not even locally accurate. An example would be the zero floor on nominal interest rates that arises from the availability of cash as a zero-interest-paying asset to savers and limits the central bank's ability to cut interest rates in deep recessions. In this case, it is essential to use a method that respects the non-linearity of interest rates. An example of such a method is the EP method of Fair and Taylor (1983). It can be used to obtain a numerical solution without prior linearization of the model equations and allows for the correct consideration of non-linear equations

[10] If the number of distinct real eigenvalues is smaller than n so that Ω is not diagonalizable, then the superdiagonal elements corresponding to repeated eigenvalues are ones rather than zeros.

such as the above-mentioned non-negativity constraint on nominal interest rates.[11] Thus, it avoids the approximation error that would arise in the context of the log-linearization method that allows for negative nominal interest rates. The basic idea of the approach is to solve (22.49) iteratively for $t = 1, \ldots, T$, each time setting future innovations to their expected value of zero.

The EP solution method proceeds according to the following five steps:

(i) Begin by choosing the initial length of a forecast horizon n and the initial conditions for the state variables. Then, set the innovations $\nu_{t+i} = 0$, for $i = 1, \ldots, n$.

(ii) Next, guess values $\tilde{x}^j_{t+i}$ for x_{t+i}, $i = 0, \ldots, n$.

(iii) Solve the resulting nonlinear equation system:

$$\psi\left(\tilde{x}^j_{s+1}, x_s, \nu_s, \mu\right) = 0, \tag{22.64}$$

for x_s, $s = t, \ldots, t + n - 1$, e.g. by iterative methods. Standard methods for nonlinear equation solution that can be used are described in Judd (1998) and Heer and Maussner (2005).[12]

(iv) Check whether the obtained values for x_{t+i} are within a selected tolerance criterion of the guesses $\tilde{x}^j_{t+i}$, for $i = 0, \ldots, n - 1$. If not, return to Step (ii) and update your guesses $\tilde{x}^{j+1}_{t+i} = x_{t+i}$. Otherwise, continue with Step (v).

(v) Denote the values obtained for x_t by x^k_t, where k counts how many times Step (v) has been reached. If $k = 1$, increase the forecast horizon n by one period and return to Step (ii). Otherwise, check whether the values x^k_t are within a selected tolerance criterion of the values x^{k-1}_t. If so, x^k_t is the numerical approximate solution for x_t. If not, increase the forecast horizon n by one period and return to Step (ii).

Given a sequence of innovations $\{\nu_t\}^T_{t=1}$, a time series $\{x_t\}^T_{t=1}$ can then be generated using the described algorithm for $t = 1, 2, \ldots, T$.

Approximation error arises due to the assumption of setting future shocks equal to zero. This assumption is also made in the context of the log-linearization method. However, in that case it does not add further measurement error because the resulting linear approximate solution exhibits certainty equivalence. In the case of the EP method, which respects the non-linearity of the model equations, setting future zero-mean shocks equal to zero introduces an approximation error because it neglects Jensen's inequality. For example, in the presence of the zero bound on nominal interest rates, the expected mean interest and inflation rates will exhibit a positive bias that increases with the

11 We will discuss an application of this method in an analysis of implications of the zero bound on nominal interest rates for the effectiveness of fiscal stimulus in Section 22.6.2. For an early application regarding the effectiveness of monetary policy in deep recessions, the reader is referred to Orphanides and Wieland (1998).

12 Juillard (1996) describes the version of the method that is implemented in DYNARE, and may be used together with the model archive and software presented in Section 22.5.

variance of future shocks.[13] The magnitude of this error depends on the degree of non-linearity in the model. Examinations of the accuracy of numerical solutions obtained with the EP algorithm are documented in Gagnon (1990) and Taylor and Uhlig (1990) for the case of the stochastic neoclassical growth model. For a numerical approximation method that accounts for Jensen's inequality in non-linear models, see Section 22.4.1.5.

22.4.1.4 Linear quadratic dynamic programming: Value function iteration

Linear quadratic dynamic programming procedures are useful for solving problems that can be recast as an optimization problem with quadratic objective function and linear constraints. For example, consider the following optimization problem:

$$\max_{\{u_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t x_t' Q x_t \tag{22.65}$$

$$\text{s.t.} \quad s_{t+1} = As_t + Bu_t, \tag{22.66}$$

where (22.65) represents a quadratic objective function or a quadratic approximation to the non-linear non-quadratic objective function. The $(n-m)\times 1$ vector u_t contains the control variables and the $m \times 1$ vector s_t the state variables. Let $x_t = [s_t' \quad u_t']'$ and Q a conformable $n \times n$ matrix. Maximization takes place subject to the linear constraints in (22.66). An appropriate guess regarding the functional form of the value function is that it is quadratic. The Bellman equation for the linear quadratic optimization problem is:

$$s^{\mathrm{T}} Vs = \max_u \left\{ x^{\mathrm{T}} Qx + \beta(s')^{\mathrm{T}} Vs' \right\}, \tag{22.67}$$

subject to (22.66), where we have changed notation in line with the literature on dynamic programming. Notably, we ignore the time subscript. Leads are marked by a prime and a transpose is denoted by a T superscript. Matrix V of value function $s^{\mathrm{T}} Vs$ is a negative semidefinite. Let us rewrite the Bellman equation as:

$$\max_u \left\{ \begin{pmatrix} x \\ s' \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} Q & 0_{n\times m} \\ 0_{m\times n} & \beta V \end{pmatrix} \begin{pmatrix} x \\ s' \end{pmatrix} \right\}, \tag{22.68}$$

subject to (22.66). Define:

$$C = \begin{pmatrix} A & B \end{pmatrix}, \quad R = \begin{pmatrix} I_{n\times n} \\ C \end{pmatrix}. \tag{22.69}$$

13 See Orphanides and Wieland (1998) for this example and for a discussion how to reduce this type of approximation error by means of repeated stochastic simulation.

Matrix $I_{n\times n}$ denotes the identity matrix of dimension n. We can then incorporate the linear constraints (22.66) as:

$$\max_{u}\left\{x^{\mathrm{T}}R^{\mathrm{T}}\begin{pmatrix} Q & 0_{n\times m} \\ 0_{m\times n} & \beta V \end{pmatrix}Rx\right\}. \tag{22.70}$$

Define:

$$W = R^{\mathrm{T}}\begin{pmatrix} Q & 0_{n\times m} \\ 0_{m\times n} & \beta V \end{pmatrix}R, \tag{22.71}$$

and differentiate (22.70) with respect to the control vector u to get the first-order condition for the maximization problem. Solving the resulting condition for u, we obtain:

$$u = -W^{-1}_{[m+1:n]\times[m+1:n]}W_{[m+1:n]\times[1:m]}s. \tag{22.72}$$

Let:

$$F = -W^{-1}_{[m+1:n]\times[m+1:n]}W_{[m+1:n]\times[1:m]}. \tag{22.73}$$

Substituting the feedback rule (22.72) into (22.70), we obtain the following algebraic matrix Riccati equation:

$$V = \begin{pmatrix} I_{m\times m} \\ F \end{pmatrix}^{\mathrm{T}} R^{\mathrm{T}}\begin{pmatrix} Q & 0_{n\times m} \\ 0_{m\times n} & \beta V \end{pmatrix}R\begin{pmatrix} I_{m\times m} \\ F \end{pmatrix}. \tag{22.74}$$

Value-function iteration involves iterations on:

$$V^{(j+1)} = \begin{pmatrix} I_{m\times m} \\ F^{(j)} \end{pmatrix}^{\mathrm{T}} R^{\mathrm{T}}\begin{pmatrix} Q & 0_{n\times m} \\ 0_{m\times n} & \beta V^{(j)} \end{pmatrix}R\begin{pmatrix} I_{m\times m} \\ F^{(j)} \end{pmatrix}. \tag{22.75}$$

Under particular conditions, (22.75) converges to a unique solution as $j\rightarrow\infty$ (see Ljungqvist and Sargent (2004) and references therein). In practice, one iterates until $\left|V^{(j+1)} - V^{(j)}\right| < \varepsilon$, for some small $\varepsilon > 0$. This procedure also provies an approximation of the solution for the policy function F.

22.4.1.5 Perturbation methods for higher-order approximations

Perturbation methods constitute a generalized approach to obtain linear or higher-order local approximations of the true model solution. As in the case of the linear approximation method presented above, the true solution is approximated in the neighborhood of a particular point. The basic idea consists of finding a special case of the general problem for which the exact solution is known. The special case and its known solution is then used to compute approximate solutions of the general problem for points in the neighborhood of the special case with known solution.

The following illustration of the approach is based on Judd (1998, Chapter 13). Consider the univariate problem:

$$f(x,\sigma)=0, \tag{22.76}$$

where σ denotes some known parameter. It is assumed that for each value of σ, there exists a solution of (22.76) for x. Hence, (22.76) describes a system of equations in x, for which the solution is unknown. Suppose, however, that this equation can be solved for a specific value of σ, say $\sigma=0$. Let $x(\sigma)$ be an unknown function that satisfies $f(x(\sigma),\sigma)=0$. If f is differentiable, implicit differentiation of (22.76) leads to:

$$f_x(x(\sigma),\sigma)x'(\sigma)+f_\sigma(x(\sigma),\sigma) \;=\; 0. \tag{22.77}$$

For $\sigma=0$, we can find:

$$x'(0)=-\frac{f_\sigma(x(0),0)}{f_x(x(0),0)}. \tag{22.78}$$

Furthermore, differentiating (77) leads to:

$$\begin{aligned}&f_{xx}(x(\sigma),\sigma)(x'(\sigma))^2+f_x(x(\sigma),\sigma)x''(\sigma)+2f_{x\sigma}(x(\sigma),\sigma)x'(\sigma)\\&+f_{\sigma\sigma}(x(\sigma),\sigma) \;=\; 0,\end{aligned} \tag{22.79}$$

from which we obtain:

$$x''(0)=-\frac{f_{xx}(x(0),0)(x'(0))^2+2f_{x,\sigma}(x(0),0)x'(0)+f_{\sigma\sigma}(x(0),0)}{f_x(x(0),0)}. \tag{22.80}$$

This allows us to compute a quadratic approximation to the solution using a second-order Taylor series expansion around $\sigma=0$:

$$\hat{x}(\sigma)=x(0)+x'(0)\sigma+\frac{1}{2}x''(0)\sigma^2, \tag{22.81}$$

with $x'(0)$ and $x''(0)$ given by (22.78) and (22.80), respectively. Similarly, one can find higher-order approximations of $x(\sigma)$ using higher-order derivatives of $x(\sigma)$ obtained from further differentiation.

Judd (1998) contains a detailed treatment of perturbation methods. Algorithms for quadratic approximations and corresponding applications can be found in Collard and Juillard (2001), Kim *et al.* (2005), and Schmitt-Grohe and Uribe (2004). A ready-to-use computer implementation for MATLAB programmed by Michel Juillard and his collaborators is available within DYNARE, and may be downloaded from www.dynare.org.

22.4.2 Estimating New Keynesian models

Medium-size New Keynesian DSGE models are typically developed with the objective of taking them to the data. Nowadays, the most prominent method for estimating DSGE models is the Bayesian approach. A convenient software implementation is also available within DYNARE. This section aims to provide a short introduction to Bayesian estimation and an illustration with respect to the small-scale New Keynesian model of Section 22.3.2. Examples for the estimation of small-scale New Keynesian models by means of impulse response function matching or maximum likelihood methods are available from Rotemberg and Woodford (1997) and Ireland (2004b), respectively. The following Section 22.4.3 then concludes with an overview of prevailing challenges for model estimation.

22.4.2.1 Bayesian methods

Bayesian methods allow us to estimate model parameters, to construct model forecasts and to conduct model comparisons. Here, we focus on model estimation. Typically, Bayesian estimation is implemented as a full information approach, i.e. the econometrician's inference is based on the full range of empirical implications of the structural model that is to be estimated. In the Bayesian context, a model is defined by a likelihood function and a prior. The likelihood function represents the data-generating process; more specifically, it is the density of the data conditional on the structure of the model and conditional on the model parameters. Under the maximum likelihood approach, the model parameters are interpreted as fixed and the observed data represents a particular draw from the likelihood function. Parameter estimation then requires the maximization of the likelihood function. By contrast, the Bayesian approach interprets the parameters as random variables. Let μ represent model parameters and let y be a sample of data observations to be explained by a model M. Employing the rules of probability, the joint probability of (y, μ) conditional on model M is given by:

$$p(y, \mu|M) = L(y|\mu, M)p(\mu|M), \tag{22.82}$$

or, alternatively by:

$$p(y, \mu|M) = p(\mu|y, M)p(y|M). \tag{22.83}$$

Here, $L(y|\mu, M)$ denotes the likelihood function. Combining both equations in order to eliminate the joint probability terms results in Bayes' rule:

$$p(\mu|y) = \frac{L(y|\mu)p(\mu)}{p(y)}, \tag{22.84}$$

keeping in mind that here this expression refers to a particular model M. The term $p(\mu|y)$ denotes the posterior distribution and $p(\mu)$ is the prior distribution. The posterior distribution may be used to make probabilistic statements with respect to the model parameters conditional on the model, the data and the prior. The posterior kernel is given by:

$$p(\mu|y) \propto L(y|\mu)p(\mu), \tag{22.85}$$

where the prior $p(\mu)$ contains any information about the parameters μ available to the econometrician that is not based on the sample of data observations. Thus, Equation (22.85) may be interpreted as an updating rule that uses data observations to update the econometrician's prior belief regarding the model parameters.

In practice, the posterior distribution does not have a simple known form for most applications of interest. Suppose we are interested in a point estimate of the model parameters μ. One candidate would be the mean of the posterior distribution:

$$E(\mu) = \int \mu p(\mu|y)\mathrm{d}\mu. \tag{22.86}$$

In most cases, it is not possible to derive an analytical expression for this integral. Instead, one has to rely on computational methods. Markov chain Monte Carlo (MCMC) methods are particularly popular in the context of the estimation of DSGE models. The goal is to generate a Markov chain $\{\mu_j\}$ that has the ergodic distribution $p(\mu|y)$, i.e. the posterior. Various algorithms exist to generate $\{\mu_j\}$. Here, we present the Metropolis–Hastings (MH) algorithm. A more detailed description can be found in Chib and Greenberg (1995). As it is not possible to draw directly from the posterior distribution, instead a stand-in density, $q(\mu|\mu_{j-1})$, needs to be used. Let the candidate draw from this the stand-in density be denoted by μ^*. The candidate's draw is accepted to be the next drawing μ_j with probability:

$$\alpha\left(\mu^*\middle|\mu_{j-1}\right) = \min\left\{1, \frac{p(\mu^*|y)q\left(\mu_{j-1}\middle|\mu^*\right)}{p\left(\mu_{j-1}\middle|y\right)q\left(\mu^*\middle|\mu_{j-1}\right)}\right\}. \tag{22.87}$$

Importantly, it is sufficient to employ the posterior kernel (22.85). Let τ denote the draw from a uniform distribution over the interval $[0, 1]$. The candidate draw μ^* is then accepted if $\alpha(\mu^*|\mu_{j-1}) > \tau$, otherwise we set $\mu_j = \mu_{j-1}$. This procedure is repeated J times. Note that the acceptance probability will be relatively low if $q(\mu^*|\mu_{j-1})$ is rather high and *vice versa*, and it will be relatively high if $p(\mu^*|y)$ is rather high. The acceptance probability therefore adjusts for the fact that the stand-in density is different from the posterior density. If the stand-in density equals the posterior density, the acceptance probability will be unity. To initialize the algorithm, one needs to specify a starting value μ_0. Typically, numerical optimization is used to determine the maximizer of the (log-)posterior kernel, which is then used as a starting value.

Having obtained a sequence of accepted draws $\{\mu_j\}$, one can approximate the mean of the posterior distribution by:

$$\bar{\mu}_J = \frac{1}{J}\sum_{j=1}^{J}\mu_j. \tag{22.88}$$

More generally, let $f(\mu)$ be a function of the model parameters. The conditional expected value of this function can then be approximated by:

$$\bar{f}_J = \frac{1}{J}\sum_{j=1}^{J}f(\mu_j). \tag{22.89}$$

Various instruments exist to assess the convergence of $\bar{f}_J$. For a comparison of different convergence diagnostics the reader is referred to Cowles and Carlin (1996). A remaining question is how to choose the stand-in density $q(\mu|\mu_{j-1})$. A widely used variant of the algorithm is the Random Walk Chain MH algorithm. The idea is to explore the neighborhood of an accepted draw. In this case, the candidate draw is generated from:

$$\mu^* = \mu_{j-1} + \varepsilon, \tag{22.90}$$

where $\varepsilon \sim iid(0, \Sigma)$. This implies $q(\mu^*|\mu_{j-1}) = q(\mu_{j-1}|\mu^*)$, hence (22.87) simplifies to:

$$\alpha\left(\mu^*\middle|\mu_{j-1}\right) = \min\left\{1, \frac{p(\mu^*|y)}{p\left(\mu_{j-1}\middle|y\right)}\right\}. \tag{22.91}$$

The choice of Σ is crucial for the efficiency of the sampler. A common approach is to use an estimate of the posterior covariance matrix scaled by some constant.

Another variant of the MH algorithm is called the Independence Chain MH algorithm. In this case, the stand-in density has the property $q(\mu|\mu_{j-1}) = q(\mu)$. In practice, it is important to select a stand-in density that has fatter tails than the posterior. For more detailed expositions of the Bayesian approach in the context of DSGE models, see An and Schorfheide (2007) and Del Negro and Schorfheide (2010).

22.4.2.2 Estimating a small New Keynesian model

To demonstrate the Bayesian approach for estimating DSGE models, we estimate the small New Keynesian model of Section 22.3.2. We start by consolidating the log-linearized model equations as follows. First, we combine the dynamic IS Equation (22.20) with the aggregate resource constraint (22.24):

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - E_t\Delta\hat{g}_{t+1} - \frac{1}{\tilde{\sigma}}\left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right), \tag{22.92}$$

where $\frac{1}{\tilde{\sigma}} = \frac{1}{\sigma} C/Y$. This IS relation may also be expressed in terms of the output gap:

$$\hat{Y}_t^{\mathrm{gap}} = E_t \hat{Y}_{t+1}^{\mathrm{gap}} - \frac{1}{\tilde{\sigma}} \left(\hat{R}_t - E_t \hat{\pi}_{t+1} - \hat{R}_t^{\mathrm{nat}} \right). \tag{22.93}$$

Equation (22.93) contains a composite shock term:

$$\hat{R}_t^{\mathrm{nat}} = \tilde{\sigma} \left[E_t \left(\hat{Y}_{t+1}^{\mathrm{nat}} - \hat{g}_{t+1} \right) - \left(\hat{Y}_t^{\mathrm{nat}} - \hat{g}_t \right) \right], \tag{22.94}$$

which represents the natural rate of interest.[14] Next, we consider the New Keynesian Phillips curve

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \tilde{\kappa} (\tilde{\sigma} + \eta) \hat{Y}_t^{\mathrm{gap}}, \tag{22.95}$$

where $\tilde{\kappa} = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. Finally, we use a slightly modified version of the interest rate rule (22.26)

$$\hat{R}_t = \tau_R \hat{R}_{t-1} + (1-\tau_R) \left(\tau_\pi \hat{\pi}_t + \tau_Y \hat{Y}_t^{\mathrm{gap}} \right) + \varepsilon_t^{\mathrm{R}}, \tag{22.96}$$

which includes an interest rate smoothing term. The vector of endogenous model variables then consists of the inflation rate, the nominal interest rate, output and the output gap. The exogenous variables $\hat{g}_t$ and $\hat{A}_t$ are again specified as AR(1) processes as in Equations (22.28) and (22.29). The natural level of output and the natural rate of interest are defined by (22.27) and (22.94), respectively.

We estimate this version of the model using quarterly data on the US economy for 1966:1 to 2007:2. The three data series employed comprise real *per capita* quarter-to-quarter GDP growth (in percent), the average effective federal funds rate (in percent) and the quarter-to-quarter inflation rate based on the GDP Implicit Price Deflator (in percent). Real GDP *per capita* is constructed by dividing real GDP by the level of the civilian non-institutional population over 16 from the FRED database of the St Louis Fed. Prior to estimation the mean has been removed from all three data series. A detailed description of the construction of the data series is provided in the Appendix.

As discussed before, the Bayesian approach allows specifying particular prior distributions. To the extent the priors provide information, they add curvature to the likelihood function. In principle, economic theory is a valuable source of priors. Thus, an important advantage of structural models relative to reduced-form specifications is that *a priori* information regarding the structural model's parameterization is more readily available. Additionally, prior distributions may also be based on pre-sample data or on microeconomic data.

[14] Equation (22.93) reveals that if the policy maker keeps the real interest rate equal to the natural rate of interest, real output, Y_t, will always equal the natural level of output, Y_t^{nat}. Given the New Keynesian Phillips curve, Equation (22.25), inflation will also be fully stabilized in this case. A tradeoff between output and inflation arises in the presence of cost-push shocks or when wages are also sticky.

Table 22.1 Prior distribution

Parameter	Density	Mean	Standard deviation
$\tilde{\kappa}$	Gamma	0.08	0.1
$\tilde{\sigma}$	Gamma	1	0.5
τ_π	Gamma	1.5	0.25
τ_Y	Gamma	0.5	0.25
τ_R	Beta	0.5	0.2
ρ_g	Beta	0.8	0.2
ρ_A	Beta	0.8	0.2
σ_R	InvGamma	1	4
σ_g	InvGamma	1.5	4
σ_A	InvGamma	1.5	4

Turning to the application, we start by imposing fixed values for some of the parameters *ex ante*. Specifically, the subjective discount value is pegged at $\beta = 0.99$, which is consistent with a steady-state real interest rate of 4% in annualized terms. Furthermore, the data employed for estimation is unlikely to contain much information about the inverse of the elasticity of labor supply, η. We impose $\eta = 1$, which is a value often used in model calibrations.

With regard to other parameters we pick particular prior distributions. A complete summary of the chosen priors is given by Table 22.1. We assume $\tilde{\kappa} \sim Gamma(0.08, 0.1)$ centered around a value in line with a Calvo parameter of $\theta = 0.75$ as in our baseline calibration. For the inverse of the inter temporal elasticity of substitution, we assume $\tilde{\sigma} \sim Gamma(1, 0.5)$, where the prior mean is in line with a log-utility specification for consumption and zero steady-state government spending. The priors for the policy rule parameters are loosely centered around values often used in the literature. A normal distribution is used for the response coefficients to inflation and the output gap and a Beta distribution for the response coefficient to the lagged interest rate. Relatively uninformative priors are used for the standard errors of the three exogenous innovations, each being described by an Inverse Gamma distribution.

The estimation is conducted using the DYNARE software package.[15] The MH algorithm is used to generate 250,000 draws of which 33% are discarded as burn-in replications. We select the step size of the algorithm in line with an average acceptance ratio of around 35%. Table 22.2 shows the resulting mean, and the 5th and 95th percentiles of the posterior distribution of the model parameters.

The mean of the posterior estimates of $\tilde{\kappa}$ and $\tilde{\sigma}$ deviates quite substantially from the mean of the prior, which lies outside of the reported confidence interval in both cases. The means of the estimated monetary policy rule parameters reflect a high degree of interest rate smoothing, a more than one-for-one long-run response to inflation and

[15] We employ DYNARE version 4.1.3, see Juillard (1996, 2001) for a general description of the software package.

Table 22.2 Posterior distribution

Parameter	Mean	5th percentile	95th percentile
$\tilde{\kappa}$	0.0375	0.0133	0.0615
$\tilde{\sigma}$	5.4574	3.9660	6.9018
τ_π	1.2607	1.1248	1.3881
τ_Y	0.3117	0.0666	0.5412
τ_R	0.7730	0.7311	0.8161
ρ_g	0.9492	0.9145	0.9861
ρ_A	0.9308	0.9003	0.9616
σ_R	0.2851	0.2541	0.3144
σ_g	0.9760	0.8749	1.0798
σ_A	1.6186	1.1403	2.0933

a positive response coefficient on the output gap. However, the latter is estimated less precisely than most of the other parameters. The two exogenous processes for the government spending shock and the technology shock are estimated to be very persistent with AR(1) coefficients of 0.95 and 0.93, respectively. Finally, the estimated standard errors of the government spending shock and the technology shock are much larger than the standard error of the monetary policy shock. Overall, the data appears to be quite informative regarding the model parameters not fixed prior to the estimation procedure. However, much of the persistence in the data is attributed to the serial correlation in exogenous shock processes that was introduced into the model in an *ad hoc* manner, rather than to the behaviorial dynamics of endogenous variables. The standard deviation of the posterior distribution is lower than the one of the prior distribution for all estimated parameters except of the inverse of the intertemporal elasticity of substitution, $\tilde{\sigma}$.

Additional estimation output from DYNARE is shown in Figure 22.6. It shows the prior (solid gray line) and posterior (solid black line) distributions together with the posterior mode (dashed grey line) for all parameters. In each case, the posterior distribution has a single peak which is always close to the posterior mode.

22.4.3 Challenges for model estimation

Despite the enormous progress in the development of computational methods that allow for increasing complexity of macroeconomic models, several important issues remain to be resolved. First, it is important to recognize that any DSGE model represents at best an approximation of the law of motion of the economy. An important source of misspecification is the cross-equation restrictions imposed by the structural assumptions of DSGE models. Consider, for instance, the point estimate of the Phillips curve parameter $\tilde{\kappa}$ in our small-scale model. From $\tilde{\kappa} = \dfrac{(1-\beta\theta)(1-\theta)}{\theta}$ it follows that a point estimate of $\tilde{\kappa} = 0.0375$ is consistent with $\beta = 0.99$ and $\theta = 0.8279$. This value of the Calvo parameter θ implies that firms reoptimize their prices on average about every six

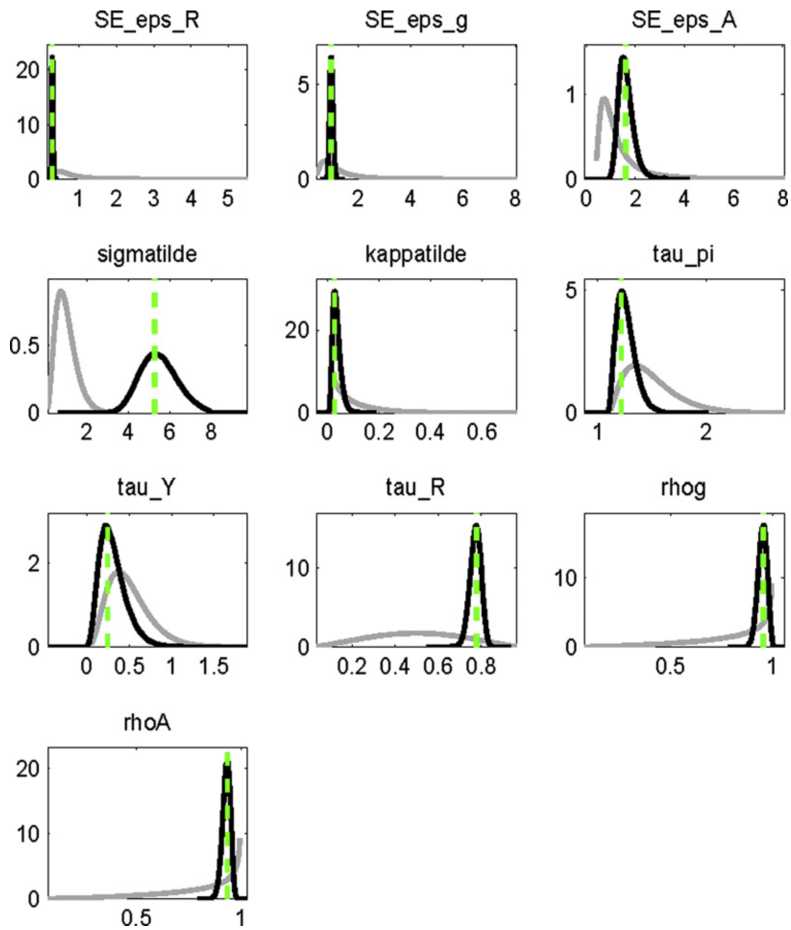

Figure 22.6 Prior and posterior distribution.

quarters. Unfortunately, microeconomic evidence on price setting points towards much more frequent average changes in prices as shown by the survey of Klenow and Malin (2010). Many of the frictions in medium-scale New Keynesian models have been introduced to improve model fit. In this manner, the incompatibility of our point estimate for $\tilde{\kappa}$ with micro evidence on price changes can be partly overcome by the introduction of firm-specific production inputs as in Eichenbaum and Fisher (2007). However, one should not conclude that less abstraction and adding more detail should in general be preferred. As Kydland and Prescott (1996) put it: "To criticize or reject a model because it is an abstraction is foolish: all models are necessarily abstractions. A model environment must be selected based on the question being addressed." Even so, it is important to keep in mind, when engaging in policy experiments, that structural parameter estimates obtained via Bayesian or maximum likelihood estimation will only be invariant to changes in the policy regime if the model is not misspecified. From a practical perspective, the parameter estimates of models used for policy analysis should be approximately invariant to shifts in policy parameters as computational power is in any case limited. An example is provided by Cogley and Yagihashi (2010). They consider two New Keynesian models: (i) The true data-generating model and (ii) an approximating model, and show that policy analysis based on the approximating model can still provide sensible recommendations for monetary policy.

A second problem that typically arises when estimating DSGE models is the lack of identification of some of the structural parameters. Identification problems arise if different parameterizations of a model generate the same probability distribution. Since the elements of the coefficient matrices of the model solution are usually highly nonlinear functions of the structural parameters, identification problems are of practical relevance. Canova and Sala (2009) categorize these problems as follows. (i) Observational equivalence occurs if the objective function, e.g. the likelihood function, does not have a unique maximum given the mapping of structural parameters. (ii) Under-identification arises when structural parameters do not appear in the solution such that the objective function is independent of these parameters. (iii) Structural parameters may enter the objective function only proportionally, thereby rendering them individually unidentifiable. This case is labeled partial identification by Canova and Sala (2009). (iv) Weak identification occurs when the objective function exhibits a unique maximum, but is rather flat in some regions of the parameter space. In this case it is rather difficult to identify the values of these parameters.

A comparison of the prior and posterior distributions of model parameters can provide some first insights regarding identification problems. However, even if the posterior is shifted away from the prior, identification problems cannot be definitely ruled out as the shift might be due to some stability constraints. Canova and Sala (2009) recommend considering a sequence of prior distributions with increasing variances to detect evidence of identification problems. In the literature, it is common practice to fix a subset of parameters in the estimation step as we did with the stochastic discount factor and the inverse of the elasticity of labor supply when estimating the small New Keynesian model. However, in the case of partial identification of this parameter together with another one, estimates of the latter parameter will depend on the calibration of the former. For a recent overview of techniques to determine conditions for identifiability, see Schorfheide (2011).

22.5 A NEW APPROACH TO MODEL COMPARISON AND POLICY EVALUATION

The two waves of New Keynesian modeling have generated a plethora of models, and more recently efforts to better understand the causes of the global financial crisis and recession have induced a further surge of macroeconomic model building. Model builders include not only academics, but also researchers at many central banks, treasuries and international organizations. Not surprisingly, the available models differ in terms of economic structure, estimation methodology and parameter estimates. Yet, systematic comparisons of the empirical implications of a large variety of available models are rare. One reason for the small number of significant model comparison projects surely has been that they required the input of many teams of researchers and multiple meetings to obtain a limited set of comparative findings. Examples include Bryant, Hooper and Mann (1993), Taylor (1999), and Hughes-Hallett and Wallis (2004). Here, we present a new systematic approach to macroeconomic model comparison developed by Wieland *et al*. (2012). This approach is based on a common computational platform that includes many well-known empirically estimated models, and enables individual researchers to conduct model comparisons easily and on a large scale.

The financial crisis and subsequent world recession has triggered much criticism of modern macroeconomics, including the New Keynesian approach and the stringent microeconomic foundations with representative agents and homogeneous expectations that are embodied in modern New Keynesian DSGE models. In this situation, taking a comparative approach is particulary useful as an avenue for setting different models against each other and for checking whether new modeling approaches perform equally well or better than existing approaches in fitting empirical benchmarks.

In Section 22.4, we have defined a model m by a system of nonlinear difference equations:

$$E_t\left[\psi_m\left(x_{t+1}^m, x_t^m, \nu_t^m, \mu^m\right)\right] = 0. \tag{22.97}$$

The letter m is used to refer to a specific model that we would like to compare to other models. The endogenous model variables are denoted by x_t^m, the structural shocks by ν_t^m and the model parameters by μ^m. For any model m, we distinguish between two types of equations. Policy rules are denoted by g_m, and the other model equations and identities by f_m. Similarly, we distinguish between policy shocks, η_t^m, and other shocks, ε_t^m, with $\nu_t^m = (\eta_t^m, \varepsilon_t^m)$, and between policy-rule parameters, γ^m, and the rest of the model parameters, β^m, with $\mu^m = (\gamma^m, \beta^m)$. Thus, (22.97) may be rewritten as:

$$E_t\left[g_m\left(x_{t+1}^m, x_t^m, \eta_t^m, \gamma^m\right)\right] = 0, \tag{22.98}$$

$$E_t\left[f_m\left(x_{t+1}^m, x_t^m, \varepsilon_t^m, \beta^m\right)\right] = 0. \tag{22.99}$$

Of course, a particular model may include lags and further leads of endogenous variables. In this case, x_t^m has to be augmented accordingly. The innovations ν_t^m have a zero mean and a constant covariance matrix Σ^m, which can be partitioned as:

$$\Sigma^m = \begin{pmatrix} \Sigma_\eta^m & \Sigma_{\eta,\varepsilon}^m \\ \Sigma_{\eta,\varepsilon}^m & \Sigma_\varepsilon^m \end{pmatrix}, \tag{22.100}$$

Table 22.3 Common comparable variables, shocks, equations and parameters

Notation	Description
z_t	Common variables in all models
η_t	Common policy shocks in all models
$g(.)$	Common policy rules
γ	Common policy rule parameters

where we distinguish the covariance matrices of policy shocks, Σ_η^m, and other economic shocks, Σ_ε^m. Unless policy shocks are correlated with the other shocks, we set $\Sigma_{\eta,\varepsilon}^m = 0$.

In general, the endogenous model variables, x_t^m, the structural shocks, ν_t^m, and the model parameters, μ^m, are not defined in a comparable manner across models. Thus, a comparison of the empirical implications of two different models, say $m \in \{1, 2\}$, cannot be based directly on either (22.97) or (22.98) and (22.99). First, it is necessary to augment all models with a set of common, comparable variables, parameters, shocks and equations. Table 22.3 summarizes our notation when referring to such common comparable elements. These common objects are not indexed by m, because they are defined coherently across all models included in a comparison exercise. The common policy rules, $g(.)$, replace the model-specific policy rules $g_m(.)$ so that model implications can be compared conditional on a particular common policy specification.

The model augmentation step also involves the definition of a set of additional model-specific equations. These equations define the common variables, z_t, in terms of the model-specific variables, x_t^m, and are denoted by $h_m(.)$. Importantly, the notation and definitions for all the other equations, variables, parameters and shocks is preserved. Consequently, an augmented model m consists of three components: (i) the original set of model equations, $f_m(.)$, determining endogenous variables, excluding the model-specific policy rules, $g_m(.)$, (ii) a set of new model-specific equations, $h_m(.)$, that define the common variables in terms of original model-specific endogenous variables with parameters θ^m, and (iii) the common policy rules $g(.)$ expressed in terms of common variables z_t, common policy shocks, η_t, and common policy rule parameters, γ. These three components comprise the system of difference equations defining the augmented model *m:*

$$E_t\left[f_m\left(x_{t+1}^m, x_t^m, \varepsilon_t^m, \beta^m\right)\right] = 0 \tag{22.101}$$

$$E_t\left[h_m\left(z_t, x_{t+1}^m, x_t^m, \theta^m\right)\right] = 0 \tag{22.102}$$

$$E_t[g(z_{t+1}, z_t, \eta_t, \gamma)] = 0. \tag{22.103}$$

Models augmented in this manner can be used for comparison exercises. For instance, one can compare the implications of a policy rule across models by constructing certain

Table 22.4 Comparable common variables

Notation	Description
i_t^z	Annualized quarterly money market rate
g_t^z	Discretionary government spending (share in GDP)
π_t^z	Year-on-year rate of inflation
p_t^z	Annualized quarter-to-quarter rate of inflation
y_t^z	Quarterly real GDP
q_t^z	Quarterly output gap (developed from flex-price level)

metrics based on the dynamics of the common endogenous variables in the different models. Before we consider such objects for comparison, we illustrate the model augmentation step with an example.

Let us suppose the vector of common comparable variables, z_t, consists of six variables, the annualized quarterly money market rate, i_t^z, discretionary government spending expressed as a share in GDP, g_t^z, the year-on-year rate of inflation, π_t^z, the annualized quarter-to-quarter rate of inflation, p_t^z, quarterly real GDP, y_t^z, and the output gap, q_t^z. The notation and the definitions for the common variables are also summarized in Table 22.4.

The common monetary and fiscal policy rules are assumed to be subject to random innovations $\eta_t = [\,\eta_t^i \quad \eta_t^g\,]'$. We are now ready to introduce the small-scale New Keynesian model that we have estimated in the previous section into our comparison framework. Let us denote this model by $m = 1$. The original model and the augmented model are presented in Table A1 in the Appendix. In the augmented version of the model the original equations, $f_1(.)$, are unchanged except for the original policy rule which is replaced by a common rule $g(.)$. The additional model-specific equations $h_1(.,\theta^1)$ define the common comparable variables in terms of the model-specific variables. In the case of this small-scale model, this augmentation step may seem rather trivial, but it is nevertheless necessary to avoid comparing apples and oranges.

In order to illustrate how to conduct a model comparison, we need at least one more model. Here, we take the business cycle model from Ireland (2004a) presented in Table A2 in the Appendix and abbreviated henceforth by $m = 2$. It represents a stylized New Keynesian model with real money balance effects and quadratic adjustment costs in price setting. The model is estimated by maximum likelihood methods using quarterly US data from 1980:1 to 2001:3. It consists of a dynamic IS equation, a New Keynesian Phillips curve, a demand equation for real money balances, $\hat{m}_t$, an interest-rate rule and AR(1) specifications for three non-policy shocks. All variables are log-linearized around the non-stochastic steady state.The version of the model we consider here is the one with household utility being non-separable in consumption and money balances.[16] The augmented model is shown in the lower part of Table A2. The original Ireland

16 It is the case of the constrained estimate in Ireland (2004a) with the parameter ω_2 fixed to a value of 0.25.

model does not feature an output gap and a government spending shock. The natural level of output and therefore the output gap can be derived in the common variables block based on the microeconomic foundations of the model. However, the model remains silent with regard to the common variable g_t^z.

The two augmented models can then be solved conditional on a range of common policy rules using the methods outlined in Section 22.4. The solution function for an augmented model m can be written as:

$$\begin{pmatrix} z_t \\ x_t^m \end{pmatrix} = K_m(\gamma) \begin{pmatrix} z_{t-1} \\ x_{t-1}^m \end{pmatrix} + D_m(\gamma) \begin{pmatrix} \eta_t \\ \varepsilon_t^m \end{pmatrix}. \tag{22.104}$$

The reduced-form matrices $K_m(\gamma)$ and $D_m(\gamma)$ are functions of the common policy parameters, γ, and the model-specific non-policy parameters, β^m. Having obtained the solution functions for models $m \in \{1, 2\}$, one can construct objects for comparison based on the common comparable variables z_t. For example, we could compare dynamic responses of the common variables to a common policy shock across models. The impulse response functions of an augmented model m to a common monetary policy shock, η_t^i, in period $t+j$, $j \geq 0$, are defined as:

$$IR_{t+j}^m\left(\gamma, \eta_t^i\right) = \begin{pmatrix} E_t\left(z_{t+j} \middle| z_{t-1}, x_{t-1}^m, \eta_t^i\right) - E_t\left(z_{t+j} \middle| z_{t-1}, x_{t-1}^m\right) \\ E_t\left(x_{t+j}^m \middle| z_{t-1}, x_{t-1}^m, \eta_t^i\right) - E_t\left(x_{t+j}^m \middle| z_{t-1}, x_{t-1}^m\right) \end{pmatrix}. \tag{22.105}$$

For some $\eta_t^i > 0$, $IR_{t+j}^1(\gamma, \eta_t^i)$ represents the impulse responses of the estimated small-scale New Keynesian model to a positive monetary policy shock and similarly $IR_{t+j}^2(\gamma, \eta_t^i)$ for the Ireland (2004a) model. Comparisons of impulse responses from different augmented models should be limited to common variables and common shocks. They could be based on common or on different policy rules. Such comparisons can provide interesting insights into the monetary policy transmission channels of the included models. One may evaluate the distance between several models for a given characteristic of the model dynamics by defining some metric s. For instance, one might consider the difference in the cumulative sum of the response of some common variable to a monetary policy shock:

$$s(\gamma, z) = \sum_{j=0}^{\infty} \left[IR_{t+j}^1\left(\gamma, \eta_t^i, z\right) - IR_{t+j}^2\left(\gamma, \eta_t^i, z\right) \right], \tag{22.106}$$

where the index z serves as a caveat that we can compare only the impulse responses of the common variables. To give an example, we impose the following common monetary policy rule from Smets and Wouters (2007) written in terms of common variables:

$$i_t^z = 0.81 i_{t-1}^z + 0.39 p_t^z + 0.97 q_t^z - 0.90 q_{t-1}^z + \eta_t^i, \tag{22.107}$$

and compare impulse responses of the output gap, q^z, to a unitary monetary policy shock. For the two models $m = 1, 2$ we obtain a cumulative difference in the impact on the output gap of $s(\gamma, q^z) = 1.14$. Further examples and a more detailed presentation of the formal approach to model comparison are provided in Wieland *et al.* (2012). We have also built a computational platform together with a model archive that includes many well-known empirically estimated models, and allows individual researchers to conduct model comparisons and quantitative analysis of stabilization policies easily and on a large scale. The Macroeconomic Model Data Base software and model archive can be downloaded from http://www.macromodelbase.com. The Appendix to this chapter contains a complete list of the 50 models available in version 1.2 of the model archive from October 2011. In addition, users may download information and software regarding the replication of published findings of the original model authors.

22.6 POLICY EVALUATION AND ROBUSTNESS UNDER MODEL UNCERTAINTY

In a situation where no model's structure is considered completely satisfactory from a theoretical perspective and many competing models fit the historical data of key aggregates reasonably well, it is not advisable to base real-world policy recommendations on a single preferred model. Instead, researchers should help policy makers to develop robust policies. This strategy for policy advice is well expressed by McCallum (1999), who proposes "to search for a policy rule that possesses robustness in the sense of yielding reasonably desirable outcomes in policy simulation experiments in a wide variety of models." In this vein, we will demonstrate how to use the above-mentioned model comparison platform for investigating the robustness of policies under model uncertainty.

22.6.1 Simple and robust monetary policy rules

We start by applying McCallum's advice to the design of simple rules for monetary policy. Specifically, we focus on rules for setting central banks' preferred policy instrument, i.e. the short-term nominal interest rate.[17] Such rules prescribe that the nominal interest rate responds systematically to a small number of variables. They are often referred to as Taylor-style rules, citing the influential contribution of Taylor (1993b), who proposed a simple rule for the US federal funds rate with only two variables, inflation and the output gap, and response coefficients of 1.5 and 0.5, respectively. Interestingly, Taylor (1993b) credits the model comparison project summarized in Bryant *et al.* (1993) as the crucial testing ground for this rule. The interest rate rules

17 See Wieland (2009a) for a recent discussion of the implications of the New Keynesian approach for the science and practice of monetary policy.

specified earlier in this chapter by Equations (22.26) and (22.96) also belong to the class of simple monetary policy rules.

In principle, economic models can also be used to evaluate much more complex rules that respond to a large number of state variables, employ different instruments and may also take non-linear functional forms. They could even be used to derive fully optimal but model-specific policies by means of optimal control methods. However, there are several reasons to focus on simple interest rate rules. (i) There is a broad consensus in the literature that an interest rate instrument is superior to a money supply instrument, at least in normal periods when the central bank has a non-negative operating target for the interest rate (see, e.g. the survey of Taylor and Williams, 2010). (ii) Earlier comparative research such as Levin *et al.* (1999, 2003) suggests that the gains from increasing either the number of leads and lags or the set of variables to which the instrument responds are rather small. (iii) This work also suggests that simple monetary policy rules are more robust to model uncertainty than more complicated model-specific rules.

22.6.1.1 Interest rate rules and central bank objectives

We begin with the consideration of rules that specify the interest rate as a linear function of two variables — the year-on-year inflation rate and the output gap. The above-mentioned research on simple rules suggests that responding to a smoothed inflation rate like the the year-on-year rate is more desirable in terms of stabilization performance than the one-period inflation rate even if the latter is the one that enters the central bank's policy objective. Using the definition of common variables introduced in the preceding section, this rule corresponds to:

$$i_t^z = \tau_\pi \pi_t^z + \tau_q q_t^z, \tag{22.108}$$

where i_t^z is the annualized quarterly nominal interest rate, π_t^z denotes the annual inflation rate and q_t^z is the output gap. Empirical estimates of policy rules typically indicate a substantial degree of interest rate smoothing of monetary policy in practice. Including the lagged interest rate as in:

$$i_t^z = \tau_i i_{t-1}^z + \tau_\pi \pi_t^z + \tau_q q_t^z, \tag{22.109}$$

was also found to improve stabilization performance in several of the models studied in Taylor (1999).

We assume that the central bank's objective is represented by the following quadratic loss function:

$$L = \mathrm{Var}(\pi^z) + \lambda_q \mathrm{Var}(q^z) + \lambda_{\Delta i} \mathrm{Var}(\Delta i^z), \tag{22.110}$$

where $\mathrm{Var}(.)$ denotes the unconditional variance operator. The parameters $\lambda_q \geq 0$ and $\lambda_{\Delta i} \geq 0$ represent the central bank's preferences for reducing the variability of the output gap and of changes in the nominal interest rate relative to inflation variability. There are a number of arguments for choosing such an objective function for the central bank. (i) The form as well as the targets entering the loss function have been widely used in previous model-based analyses of monetary policy rules, especially in the context of policy experiments based on competing models.[18] (ii) Stabilizing the rate of inflation and, in the short-run, also reducing output volatility tend to be at the forefront of central banks' concerns in actual policy practice and feature prominently as objectives in central bank laws and strategies, and central banks have a well-documented tendency to smooth interest rates. (iii) In the particular case of the small-scale model of Rotemberg and Woodford (1997), Equation (22.110) corresponds to a second-order approximation of household utility (for $\lambda_{\Delta i} = 0$ and the limiting case with the discount factor approaching unity, see Woodford, 2003).[19] With real money balances entering the utility function, also the level of the nominal interest rate appears in such a linear-quadratic approximation. To be sure, in more elaborated medium-size models, additional variables will enter a welfare-based loss function. Thus, an alternative approach would be to evaluate policy performance against the particular model-specific approximations of household utility. However, such an approach would restrict the permissible set of models to those that contain a well-defined measure of (representative) household welfare.

22.6.1.2 Range of estimated models of the US economy

In the following, we build on a recent comparison exercise by Taylor and Wieland (2012) that evaluated the performance of simple policy rules in three well-known models that are also available from the Macroeconomic Model Data Base. The first model, which is a multi country model of the G-7 economies built more than 15 years ago, has been used extensively in the earlier model comparison projects. It is described in detail in Taylor (1993a) and is labeled the TAY model in the following. The other two models are the best-known representatives of the most recent generation of empirically estimated New Keynesian models, the Christiano *et al.* (2005) (CEE/ACEL) model[20] and the Smets and Wouters (2007) (SW) model. We extend this analysis by including three recently built medium-scale New Keynesian DSGE models with financial frictions i.e. the models of De Graeve (2008), Iacoviello (2005) and Rabanal (2007).

18 See, e.g. Taylor (1999), Levin *et al.* (1999, 2003), Levin and Williams (2003), Taylor and Wieland (2012), and Taylor and Williams (2010).

19 The magnitude of the implied value of λ_q is very sensitive to the particular specification of staggered nominal contracts: random-duration Calvo-style contracts imply a very low value not far from zero, whereas fixed-duration Taylor-style contracts imply a value near unity (see Erceg and Levin, 2006).

20 Specifically, they use the version of this model estimated by Altig *et al.* (2005). This version, which was also compared to the Smets–Wouters model in Section 22.3.2.6, incorporates additional economic shocks other than the monetary policy shock.

(i) De Graeve (2008). This model incorporates the financial accelerator of Bernanke *et al*. (1999) as discussed in Section 22.3.2.4 into a New Keynesian model with nominal price and wage frictions, habit formation in consumption, price and wage indexation, and variable capital and investment adjustment costs. The model, henceforth referred to as the DG model, has been estimated on quarterly US data from 1954:1 to 2004:4 using Bayesian techniques. We also consider a second variant of the model, labeled DGnoff, in which we shut down the financial accelerator mechanism.

(ii) Iacoviello (2005). This model, referred to as IAC in the following, incorporates housing into a New Keynesian framework. A financial accelerator arises in the IAC model due to the presence of borrowing constraints. The value of housing serves as collateral for firms and for part of the households. Unlike in Bernanke *et al*. (1999), debt contracts are denominated in nominal terms. The model also features variable capital and adjustment costs for housing and for capital. Model estimation has been conducted using calibration and impulse response function matching based on quarterly US data from 1974:1 to 2003:2.

(iii) Rabanal (2007). This model, termed RB, is similar to the model of De Graeve (2008) in that it exhibits nominal rigidities in price and wage setting, price and wage indexation, variable capital utilization, investment adjustment costs, and habit formation in consumption. Unlike the DG model, there is no financial accelerator present in the RB model. However, part of the firms have to pay their wage bill prior to their sales receipts, which forces them to borrow from a financial intermediary. A cost channel of monetary policy transmission arises where changes in the nominal interest rate have a direct effect on firms' marginal costs. The model has been estimated on quarterly US data from 1959:1 to 2004:4 using Bayesian techniques.

22.6.1.3 Model-specific rules

Table 22.5 reports the model-specific optimized response coefficients of the two policy rules (22.108) and (22.109) for the three models considered in Taylor and Wieland (2012) and the four additional models listed above, i.e. the TAY, CEE/ACEL, SW, DG, DGnoff, IAC and RB models.

The left panel reports results for the case of equal weights on the variance of inflation and the change in the nominal interest rate and no weight on the output gap variance in the central bank loss function (22.110), whereas the right panel shows results for the case of equal weights on the variances of all three variables. All two-parameter rules satisfy the so-called Taylor principle, which postulates that the nominal interest rate should respond to more than one-for-one to changes in inflation, $\tau_\pi > 1$. In many New Keynesian models the Taylor principle is a necessary and often also sufficient condition for the existence of a unique rational expectations equilibrium. A second characteristic common to all optimized two-parameter rules with one exception is a strictly positive

Table 22.5 Characteristics of simple rules optimized in different models

	Rule					
	$\lambda_q = 0,\ \lambda_{\Delta i} = 1$			$\lambda_q = 1,\ \lambda_{\Delta i} = 1$		
Model	τ_i	τ_π	τ_q	τ_i	τ_π	τ_q
Two parameters						
TAY		2.54	0.19		3.00	0.52
SW		2.33	−0.10		2.04	0.26
CEE/ACEL		4.45	0.28		2.57	0.45
DG		1.45	0.70		1.46	1.60
DGnoff		1.82	0.47		1.39	1.99
IAC		2.12	0.07		1.31	0.49
RB		2.43	0.27		2.44	1.20
Three parameters						
TAY	0.98	0.37	0.09	0.98	0.21	0.53
SW	1.06	0.49	0.01	1.13	0.012	0.015
CEE/ACEL	0.97	0.99	0.02	2.84	7.85	−2.12
DG	1.00	0.28	0.01	0.90	0.46	0.68
DGnoff	1.01	0.22	0.01	0.98	0.16	0.87
IAC	1.14	0.75	−0.01	1.49	0.52	0.59
RB	1.05	0.66	0.12	1.07	0.54	0.56

Optimized response coefficients for the two-parameter rule $i_t^z = \tau_\pi \pi_t^z + \tau_q q_t^z$ and the three-parameter rule $i_t^z = \tau_i i_{t-1}^z + \tau_\pi \pi_t^z + \tau_q q_t^z$ are reported. The parameters λ_q and $\lambda_{\Delta i}$ denote the weight on the variance of the output gap and on the variance of the change in the nominal interest rate in the central bank's loss function, respectively.

response coefficient to the output gap. The one exception is the SW model with a negative but near-zero coefficient when the central bank loss function assigns no weight to output stabilization. In all other cases, the interest rate is increased in order to dampen aggregate demand whenever it exceeds the natural level of output and *vice versa*. In all cases, the coefficient on the output gap increases with a positive weight on output in the loss function. Despite these similarities of optimized simple rules across models, the response coefficients can differ quite substantially in terms of magnitude. The results for the three-parameter rules reveal that, in most of the cases, a response coefficient to the lagged interest rate near unity is desirable. Rules with $\tau_i > 1$ are often referred to as super-inertial rules in the literature. Rules that respond to the lagged interest rate introduce historical dependence because future policy actions will depend in part on current economic conditions. It should be noted, however, that super-inertial rules can lead to instability in models with primarily backward-looking dynamics.

In order to compare the stabilization performance of the two-parameter and three-parameter rules, Table 22.6 reports the increase in absolute loss when moving from the three-parameter to the two-parameter rule. Here we restrict attention to the DG, DGnoff, IAC and RB models and refer the reader to Taylor and Wieland (2012) for the TAY, CEE/ACEL and SW model results. The increase in absolute loss when moving

Table 22.6 Loss increase when reducing the number of parameters in the rule: IIP

Model	$\lambda_q = 0,\ \lambda_{\Delta i} = 1$	$\lambda_q = 1,\ \lambda_{\Delta i} = 1$
DG	0.41	0.49
DGnoff	0.51	0.75
IAC	0.58	1.43
RB	0.96	0.76

The increase in absolute loss when monetary policy follows the optimized two-parameter rule instead of the optimized three-parameter rule is reported in terms of the IIP. The IIP corresponds to the required increase in the standard deviation of the annual inflation rate that would imply an equivalent increase in absolute loss.

from three-to two-parameter rules is measured in terms of the implied inflation variability premium (IIP). The IIP, proposed by Kuester and Wieland (2010), translates the increase in the absolute loss into an equivalent increase in the standard deviation of inflation.[21] For instance, the entry in the third row and second column of Table 22.6 indicates that if the central bank objective considers inflation and output gap volatility equally, and the IAC model represents the economy, then employing the optimized two-parameter rule instead of the three-parameter rule will result in an absolute increase in the central bank's loss that is equivalent to an increase in the standard deviation of inflation of 1.43 percent. An increase in the standard deviation of inflation of this magnitude is of economic relevance. While the numbers are somewhat smaller for the other models, the results confirm the earlier finding that including the lagged interest rate in the policy rule leads to non-negligible improvements in the central bank's stabilization performance.

A natural question then is whether one should raise the number of parameters in the policy rule further. Taylor and Wieland (2012) consider four-parameter rules that include the lagged output gap in addition to the three variables already included in our three-parameter rules and find small gains in stabilization performance. Also, one might ask whether policy rules should respond to expectations of future inflation and the output gap instead of contemporaneous realizations. Levin *et al.* (2003) show that the benefits of such policy rules are in general limited. Furthermore, if the forecast horizon is too long, the models become highly susceptible to equilibrium indeterminacy under this class of rules. Instead of exploring these questions further with the additional DSGE models with financial frictions, we proceed to investigate the robustness of model-specific rules.

22.6.1.4 Robustness of model-specific rules

So far, we have implicitly assumed that the central bank knows the true model of the economy with certainty. What if the reference model of the central bank used for policy

[21] In the literature, the analysis of policy rules is often based on the percentage increase in the central bank's loss instead of the absolute increase. This measure of relative policy performance can lead to misleading signals, as demonstrated by Kuester and Wieland (2010) and Taylor and Wieland (2012).

analysis is not a good representation of the economy and one of the other models constitutes a more valid representation? To address this question we evaluate each rule optimized for a particular model in the competing models. We start by considering the performance of the two- and three-parameter rules optimized in the TAY, CEE/ACEL and SW models, in the new models with financial frictions. Table 22.7 reports the loss increase in terms of IIPs when a rule optimized for model Y is evaluated in the distinct model X relative to the performance of the model-consistent optimal rule in X of the same class.

The IIPs document that rules optimized for one model may exhibit poor performance in other models. Optimized model-specific rules may even lead to disastrous outcomes when the true model turns out to be different from the reference model. In the case of the two-parameter rules the SW rule leads to equilibrium indeterminacy in the DGnoff model allowing for a multiplicity of possible equilibria. In the case of the three-parameter rules and equal weights on all three target variables in the loss function, the rule optimized for the CEE model induces IIPs of tremendous size (between 12 and 18 percentage points) in the four models.

Table 22.8 reports the performance of optimized model-specific rules from the DG, DGnoff, IAC and RB models. The evaluation is restricted to the performance within this set of four models leaving out the TAY, SW and CEE/ACEL models. In this case, the IIPs tend to be somewhat smaller due to the greater similarity of these four models. Even

Table 22.7 Robustness of model-specific rules: IIP

Model	Two-parameter rules		Three-parameter rules	
	$\lambda_q = 0,\ \lambda_{\Delta i} = 1$	$\lambda_q = 1,\ \lambda_{\Delta i} = 1$	$\lambda_q = 0,\ \lambda_{\Delta i} = 1$	$\lambda_q = 1,\ \lambda_{\Delta i} = 1$
TAY rule				
DG	0.26	0.90	0.07	0.06
DGnoff	0.12	0.95	0.07	0.06
IAC	0.42	0.36	0.26	0.18
RB	0.01	0.31	0.11	0.55
CEE rule				
DG	0.79	0.92	0.16	14.19
DGnoff	0.71	1.01	0.17	18.18
IAC	0.87	0.24	0.05	12.00
RB	0.60	0.20	0.17	18.45
SW rule				
DG	0.53	1.29	0.04	1.12
DGnoff	IND	1.45	0.04	1.65
IAC	0.01	0.26	0.02	0.40
RB	0.07	0.24	0.12	0.77

The increase in absolute loss under optimized rules from Taylor and Wieland (2012) relative to the model-specific optimized rule is reported in terms of the IIP. The IIP corresponds to the required increase in the standard deviation of the annual inflation rate that would imply an equivalent increase in absolute loss. IND refers to the case where a rule induces equilibrium indeterminacy in a particular model.

Table 22.8 Robustness of model-specific rules from the DG, DGnoff, AC and RB models

	Two-parameter rules		Three-parameter rules	
Model	$\lambda_q = 0,\ \lambda_{\Delta i} = 1$	$\lambda_q = 1,\ \lambda_{\Delta i} = 1$	$\lambda_q = 0,\ \lambda_{\Delta i} = 1$	$\lambda_q = 1,\ \lambda_{\Delta i} = 1$
DG rule				
DGnoff	0.04	0.05	0.00	0.09
IAC	1.37	1.36	0.08	0.27
RB	0.82	0.85	0.19	0.23
DGnoff rule				
DG	0.06	0.06	0.00	0.18
IAC	0.93	1.80	0.10	0.34
RB	0.24	1.20	0.28	1.38
IAC rule				
DG	0.52	1.04	0.19	0.71
DGnoff	0.14	1.19	0.19	0.88
RB	0.05	0.92	0.31	0.24
RB rule				
DG	0.21	0.28	0.11	0.12
DGnoff	0.09	0.31	0.10	0.21
IAC	0.56	0.93	0.22	0.13

The increase in absolute loss when the policy rule optimized for one model is evaluated in the other models is reported in terms of the IIP. The IIP corresponds to the required increase in the standard deviation of the annual inflation rate that would imply an equivalent increase in absolute loss.

so, among the two-parameter rules with equal weights on inflation and output gap variation, five of the 12 IIPs exceed one percent.

Turning to the three-parameter rules, the corresponding IIPs turn out to be somewhat lower than under the two-parameter rules for the majority of experiments, although there are exceptions. Two three-parameter rules — the DG rule and the RB rule — turn out to be relatively robust to model uncertainty, leading only to small increases in the absolute loss when implemented in competing models.

For both types of rules and both parameterizations of the loss function we find that the rule optimized for the DG model with the financial friction shut down leads to almost no performance loss in the full DG model relative to the rule optimized in that model. The same result holds true when evaluating the rule optimized for the full DG model in the model variant without financial frictions. The IIP never exceeds 0.18%. Interestingly though, the rules optimized for the DG model with and without financial frictions do not perform equally well in other models. For instance, the DGnoff rule performs much worse in the RB model than the DG rule, when the central bank cares about output gap stabilization.

22.6.1.5 Robustness and model averaging

The lack of robustness of model-specific rules suggests that one should take model uncertainty explicitly into account when designing a simple rule for monetary policy.

This may be achieved by adopting a Bayesian perspective on the design of robust rules, following the approach proposed by Levin *et al.* (1999, 2003) and Brock *et al.* (2003). Under this approach the policy rule parameters are optimized by minimizing a weighted average of losses across models.

$$\sum_{m\in M}\omega_m L_m = \sum_{m\in M}\omega_m\left[\mathrm{Var}\left(\pi_m^z\right)+\lambda_q\mathrm{Var}\left(q_m^z\right)+\lambda_{\Delta i}\mathrm{Var}\left(\Delta i_m^z\right)\right], \tag{22.111}$$

where M refers to the set of models considered by the policy maker and the parameters ω_m denote the weights on the models. Under a Bayesian perspective these weights would correspond to the central bank's priors on model probabilities.

Taylor and Wieland (2012) computed model-averaging rules using the TAY, CEE/ACEL and SW models. Here, we replicate this exercise with the other four models. Thus, in our exercise the set of models in Equation (22.111) is $M = \{\mathrm{DG}, \mathrm{DGnoff}, \mathrm{IAC}, \mathrm{RB}\}$. We consider flat priors, $\omega_m = 1/4$ for all m, so that (22.111) effectively implies simple model averaging. Table 22.9 reports the optimized response coefficients of the two- and three-parameter model-averaging rules for the case when output, inflation and interest volatility receive the same weight in the loss function. For the two-parameter rule, the optimized response coefficient to inflation lies close to two, being somewhat smaller than the optimal response coefficient in the RB model, but larger than the optimal parameter value in the remaining three models. The optimized three-parameter rule has a response coefficient to the lagged interest rate near unity, and (short-run) response coefficients to inflation and the output gap that are smaller than under the two-parameter rule and near to 0.5. Comparing this model-averaging rule with the corresponding model-specific optimal rules in Table 22.5 it turns out that the parameter values of the model-averaging rule are relatively close to the rule that is optimal in the RB model. This is not too surprising given that our earlier experiments showed that the RB rule performs quite well in the DG, DGnoff and IAC models.

What about the TAY, CEE/ACEL and SW models considered by Taylor and Wieland (2012)? Interestingly, the three-parameter model averaging rule reported in that study (shown in the bottom row of Table 22.9) is almost identical to the model averaging

Table 22.9 Characteristics of model-averaging rules ($\lambda_q = 1,\ \lambda_{\Delta i} = 1$)

Rule	τ_i	$\tau_{\hat{\pi}}$	τ_q
$M = \{\mathrm{DG}, \mathrm{DGnoff}, \mathrm{IAC}, \mathrm{RB}\}$			
Two parameters		2.06	0.91
Three parameters	1.05	0.49	0.60
$M = \{\mathrm{TAY}, \mathrm{CEE/ACEL}, \mathrm{SW}\}$			
Three parameters	1.05	0.41	0.23

Optimized response coefficients for the two-parameter rule $i_t^z = \tau_\pi \pi_t^z + \tau_q q_t^z$ and the three-parameter rule $i_t^z = \tau_i i_{t-1}^z + \tau_\pi \pi_t^z + \tau_q q_t^z$ in case of equal weights on the variance of the output gap and the variance on the change in the nominal interest rate, $\lambda_q = 1, \lambda_{\Delta i} = 1$, are reported.

Table 22.10 Optimized model-averaging rules: IIP

Model	Two-parameter rule	Three-parameter rule
DG	0.35	0.07
DGnoff	0.43	0.15
IAC	0.55	0.14
RB	0.05	0.02

The increase in absolute loss in each model under a rule optimized by averaging over all models relative to the model-specific optimized rule of the same class is reported in terms of the IIP. The IIP corresponds to the required increase in the standard deviation of the annual inflation rate that would imply an equivalent increase in absolute loss.

rule obtained here. Only the coefficient on the output gap is a bit smaller. Thus, we refrain from extending the model-averaging exercise to the full set of seven models.

In a final exercise, we check the differences between the model-averaging rule and the model-specific rules in the DG, DGnoff, RB and IAC models. Table 22.10 reports the corresponding IIPs for the two classes of rules. Not surprisingly, the IIPs are strictly positive, i.e. the model-averaging rule always performs worse than the model-specific rules when those are applied in the correct model. However, the magnitude of this difference is very small. Even more so, in the case of the more effective three-parameter rules. In this case, the maximum IIP amounts to 0.15%.

In sum, our findings emphasize that rules fine-tuned to a particular model may lead to poor or even disastrous outcomes in other models. Policy recommendations should therefore be based on a broad range of alternative models. Here, we have focused on models with financial frictions, but in general it is desirable to consider substantial diversity in terms of modeling approaches to ensure that policy recommendations are robust. Our conclusion should not be understood as a proposal to maximize the number of potential models considered. Rather, we have focused on models that pass stringent tests in terms of empirical fit and economic theory.

22.6.2 Robustness of impact estimates of discretionary fiscal stimulus

New Keynesian DSGE models are not only useful for monetary policy analysis, but can also be employed for evaluating fiscal policy. In fact, many medium-and large-scale models used at policy institutions now contain a rather detailed treatment of the fiscal sector with various types of distortionary taxes and explicit modeling of the different components of government spending and transfers. These models can be used to evaluate discretionary as well as rule-based fiscal policy initiatives. They are best used to investigate questions concerning business cycle stabilization and the interaction of monetary and fiscal policy measures in the short to medium run. Of course, there are many fiscal policy questions that focus on distributional issues and longer-term impacts. Other computable general equilibrium models that are more appropriate for such questions are described in other chapters of this Handbook.

Here, we focus on a well-known Keynesian idea with great relevance for short-run fiscal policy, i.e. the Keynesian multiplier effect. This effect has been used as a justification for initiating major discretionary stimulus packages in the aftermath of the 2008−2009 recession. The Keynesian multiplier implies that an increase in government spending leads to a greater than one-for-one increase in overall GDP. It arises in the text book IS-LM model because the Keynesian consumption function implies a fixed relationship between consumption and current household income. Because additional government spending boosts income it also induces additional private consumption, and thus an effect on overall GDP that is greater than unity.

22.6.2.1 Debate on the GDP impact of the 2009 ARRA stimulus package

Specifically, we review evidence from New Keynesian models regarding the GDP impact of the ARRA legislated in February 2009 in the US. In a prominent paper from January 2009, Christina Romer, then-Chair of the President's Council of Economic Advisers, and Jared Bernstein, Chief Economist of the Office of the Vice-President, presented model-based evidence that a lasting increase in government purchases of 1% of GDP would lead to a rapid rise in real GDP of 1.6 % persisting for at least five years. On this basis, they estimated that the full stimulus package proposed would induce an increase in US GDP of 3.6% by the end of 2010 over a baseline without stimulus. However, Cogan *et al.* (2010) (first public working paper version issued on 2 March 2009) showed that the estimates of Romer and Bernstein (2009) were not robust. They estimated only one-sixth of the GDP impact of the stimulus package expected by Romer and Bernstein. While Romer and Bernstein based their analysis on results from Keynesian-style models used by private-sector forecasters and the Federal Reserve, Cogan *et al.* (2010) considered state-of-the-art medium-scale DSGE models estimated on US data.

Cogan *et al.* (2010) first evaluated the impact of the additional government spending announced with the ARRA legislation in the Smets−Wouters model of the US economy. This model, which was also part of the monetary policy exercise of the preceding section, features forward-looking, optimizing households as in the small-scale model presented in Section 22.3.1. As discussed in that section and confirmed by Figure 22.3 an increase in government spending leads to a crowding out of private consumption spending in that model. Higher than expected interest rates and the anticipation of a greater future tax burden induce forward-looking, rational households to reduce their expenditures. Cogan *et al.* (2010) showed that the announced ARRA spending would reduce private consumption and investment in the Smets−Wouters model even if one takes into account that interest rates might remain constant for up to two years. The assumption of constant interest rates was meant to capture the anticipation that the Federal Reserve's notional operating target for the federal funds rate would remain negative for some time during the recession and thereby suppress the upward pressure on interest rates due to increased government debt.[22]

In a further step, Cogan *et al.* (2010) extended the Smets–Wouters model to allow for the presence of rule-of-thumb households. Such households consume their current income as prescribed by the Keynesian consumption function. As a consequence, Ricardian equivalence fails, and it is necessary to account explicitly for tax and debt dynamics. They re-estimate the complete model together with a reaction function of taxes in response to government spending and debt with Bayesian methods. Their estimate of the share of rule-of-thumb households is 26.5%. Even so, the estimate of the government spending multiplier and the GDP impact of announced ARRA spending remain far below the estimates used by Romer and Bernstein (2009).

22.6.2.2 Recent findings from a major model comparison exercise

Rather than conducting additional simulations in other models, we review instead findings from a major comparison exercise sponsored by the International Monetary Fund (IMF) and reported in Coenen *et al.* (2012). Interestingly, the published version of this study by 17 authors also includes a robustness analysis of the Cogan *et al.* estimates of the GDP impact of the announced ARRA spending, which was not part of the initial IMF exercise. They simulate the time profile of ARRA spending documented by Cogan *et al.* in seven macroeconomic models that are currently used at the IMF, the Federal Reserve, the European Central Bank (ECB), the Bank of Canada, the Organization for Economic Cooperation and Development (OECD) and the European Commission, respectively. Then, they compare their results with those obtained in the models estimated by Cogan *et al.* (2010) and Christiano *et al.* (2005).

Figure 22.7 displays their findings. It reproduces Figure 7 from Coenen *et al.* (2012).[23] The bars shown in each panel are identical and indicate the time profile of ARRA government spending. The simulations are carried out under the assumption that market participants anticipate the execution of the announced purchases over the coming years. Other measures included in the ARRA such as tax rebates and certain transfers are not included. The different lines displayed in the panels indicate the estimated GDP impact of the additional government spending in different macroeconomic models. As some of the models are estimated with euro area data, there are two columns of panels. The left column reports the GDP impact in models of the US economy, while the right column labeled "Europe" refers to euro area models.[24]

22 Technically, they solve the model with the EP method reviewed in Section 22.4.1.3 so as to respect the non–linearity resulting from the zero-lower-bound on nominal interest rates.

23 We are grateful to the authors for supplying the simulation data for replicating this figure from their paper.

24 For an evaluation of the euro area stimulus packages in a range of models see Cwik and Wieland (2011).

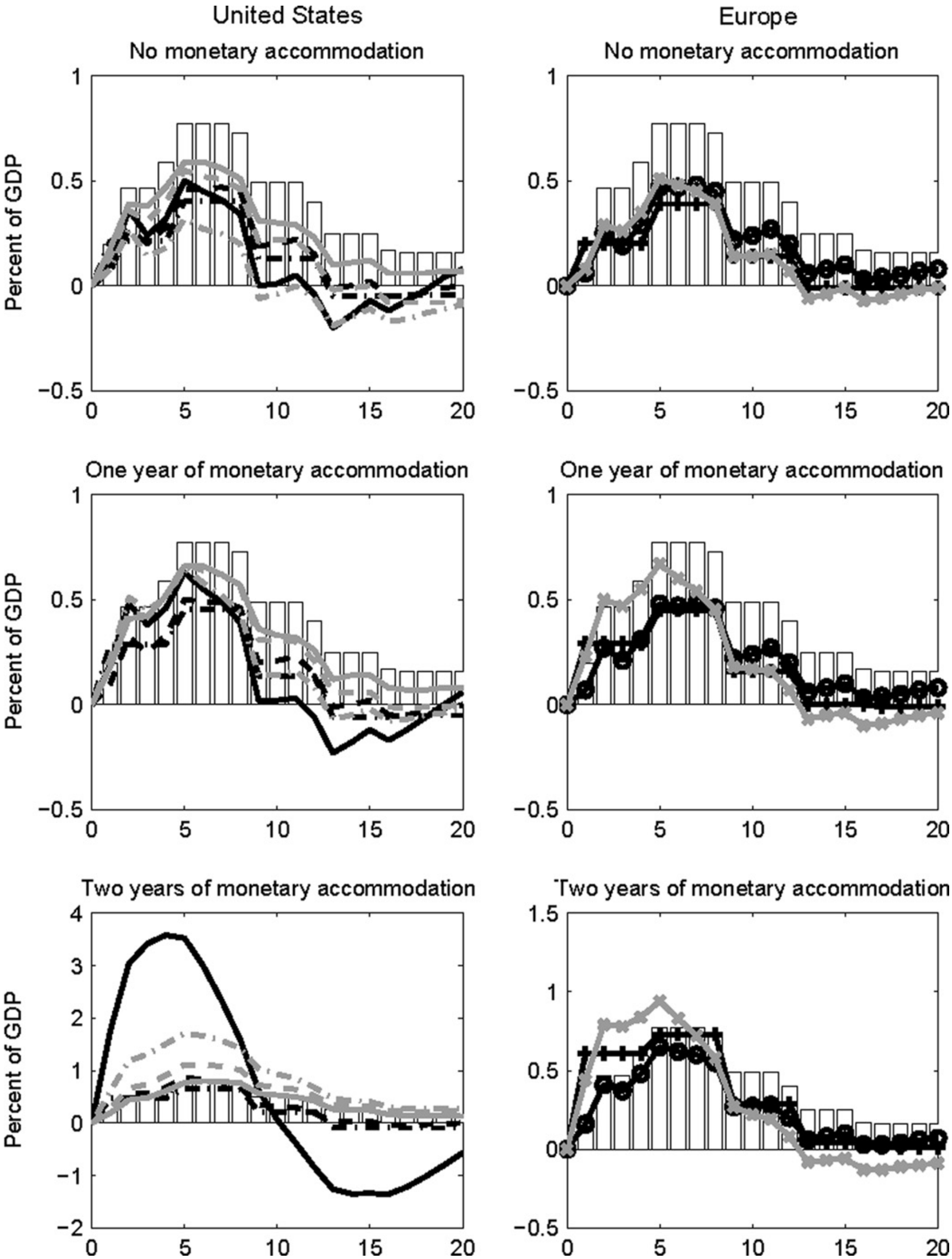

Figure 22.7 Estimated GDP effects of announced ARRA spending. Shown are estimated output effects of government purchases in the February 2009 US stimulus legislation for nine macroeconomic models. See text for details.

The models shown in the left column are the following: (1) Christiano *et al.* (CEE), solid black line, (2) Cogan *et al.* (CCTW), dashed black line, (3) the IMF's Global Integrated Monetary and Fiscal Policy model (GIMF), dashed-dotted black line, (4) the Federal Reserve's US model (FRB-US), solid grey line, (5) the Federal Reserve's international model (SIGMA), dashed grey line, (6) the Bank of Canada's Global Economic Model (GEM), dashed-dotted grey line. The models shown in the right column are: (7) the European Commissions QUEST model (EC-QUEST), solid black line with plus signs, (8) the European Central Bank's New Area-Wide Model (ECB-NAWM), solid black line with circles, and (9) the OECD's macroeconomic model (OECD), solid grey line with crosses.

Coenen *et al.* (2012) consider three alternative assumptions regarding monetary policy. The first row displays results for the case of no monetary accommodation, i.e. monetary policy in each model is set according to an interest rate rule. Thus, interest rates will rise along with the increase in GDP and dampen the stimulative impact of the additional government spending. In this scenario, all the models considered deliver an increase in GDP over the first 2.5 years of the stimulus. However, the increase in GDP remains well behind the associated increase in government spending. Thus, the Keynesian multiplier effect is not working. Instead, private demand is crowded out and declines all throughout the period of government stimulus. Some of the models even predict an overall negative effect on GDP in the fourth year of the stimulus. The extended Smets−Wouters model estimated by Cogan *et al.* (dashed black line) lies well within the range of other model outcomes (left panel). This finding is particularly interesting because these other models are used at major policy institutions and contain much more thoroughly detailed fiscal sectors than the models considered in Cogan *et al.* The IMF's GIMF model is especially noteworthy as it contains overlapping generation households with finite planning horizons. This level of heterogeneity is rare in New Keynesian DSGE models, but relevant for many fiscal policy considerations (see Freedman *et al.* 2010). As to the stimulative effect of planned ARRA spending, however, the model's predictions remain close to the pessimistic assessment of Cogan *et al.*

For the simulations shown in the second row of Figure 22.7, nominal interest rates are held constant for one year and follow a nominal interest rate rule thereafter, and for the results shown in the third row, nominal interest rates are held constant for two years. These simulations illustrate the role of the monetary policy stance for the effectiveness of fiscal stimulus. If nominal interest rates are initially held constant, fiscal multipliers increase. With one year of anticipated monetary accommodation, multipliers remain below one in all of the models. Within two years, GDP exceeds government spending a little bit in some of the models, due to crowding-in of private consumption. There is one outlier. The CEE model exhibits a massive boost for two years, followed by a recession. As suggested in Christiano *et al.* (2011) with this model government spending multipliers may be large. However, in the case of the ARRA stimulus, this finding is not robust to model uncertainty. All other models considered by Coenen *et al.* (2012) induce much smaller GDP effects. Thus, their comparative findings support the skepticism regarding Romer and Bernstein's recommendation of an extensive sustained fiscal stimulus program expressed by Cogan *et al.* and others in 2009.

22.7 OPEN QUESTIONS AND FUTURE RESEARCH

In conclusion of this chapter, we point out some research questions that need to be addressed in New Keynesian modeling and list selected recent articles that suggest new avenues for extending the methodology. The New Keynesian approach to business cycle analysis initially benefitted tremendously from the concise modeling of imperfect competition, nominal rigidity and monetary policy in a context with optimizing households and firms. The clarity that rendered the type of small-scale model we presented in Section 22.3.1 so popular was made possible by some drastic shortcuts. Subsequent medium-size DSGE models built to explain the persistence in major macroeconomic aggregates mostly continued to employ these same simplifications. They ignored questions regarding the endogeneity of technology and long-run growth that had featured so prominently in the new growth theory and growth empirics of the 1980s and 1990s. They suppressed consumer and producer heterogeneity that is treated as a central issue in most of the other chapters in this Handbook. Imperfections in expectations formation were typically given short shrift. Perhaps most importantly, even DSGE models built at central banks prior to the global financial crisis did not contain a detailed formal treatment of the sources of disruption in financial markets and financial intermediation that eventually became widely apparent in the course of the financial crisis. Going forward, these deficiencies of current-generation New Keynesian DSGE models offer tremendous opportunity for productive innovation.

22.7.1 Long-run growth

Typically, business cycle models are either written without reference to the growth factors and then estimated on detrended data or they are stationary with respect to a balanced growth path and estimated as linear approximations around the deterministic steady state. More sophisticated versions take into account that the presence of shocks in non-linear models induces a difference between the deterministic and the stochastic steady state. For example, they employ perturbation methods for second-order approximation of the stochastic steady state. More generally, market participants' expectations of long-run growth have important implications for their current choices. Similarly, policy makers' decisions are based on deviations from perceived long-run equilibrium values. Thus, perceptions regarding long-run growth will influence short-run dynamics and policy responses. To give a recent example, changing estimates of US productivity growth in the context of advances in information technology as documented in Jorgenson *et al.* (2008) led to shifts in the perceived neutral setting of monetary policy. Further investigation of the interaction between perceived growth trends and the formulation of monetary and also fiscal policy is an important area of application for DSGE modeling. Central bank misperceptions regarding potential output and growth may induce trends in inflation (see Beck and Wieland, 2008). Similarly, changes in distortionary labor or capital income tax rates will influence long-run growth opportunities. Finally, incorporating advances from endogenous growth theory in DSGE models will help shed new light on the interaction of short-run fluctuations, investment and technological change.

22.7.2 Consumer and producer heterogeneity

While the baseline New Keynesian model assumes a single representative agent, more elaborate models allow for different types of economic agents. As noted in Section 22.6.2 on fiscal stimulus, many medium-size DSGE models additionally include consumers that spend according to a rule-of-thumb rather than solving an intertemporal optimization problem. The IMF's GIMF model even incorporates overlapping generations of households with finite planning horizons. Furthermore, certain models with financial frictions feature households with different degrees of impatience that separate them into borrowers and lenders. For instance, Iacoviello (2005) develops a model where lenders are borrowing-constrained and their collateral is tied to the value of housing. These frictions amplify the consequences of certain macroeconomic shocks while attenuating others. Other studies do away with the assumption of perfect risk sharing between different households. Consider, for example, a New Keynesian model with a staggered wage setting. Labor income then differs among households. The standard approach is to assume that households engage in complete risk sharing across households to simplify aggregation. By contrast, a recent paper by Lee (2012) develops a simple New Keynesian model in which households cannot perfectly insure against idiosyncratic labor income risk because of costs of moving resources between households. He finds that this real rigidity improves the model's ability to reconcile macroeconomic estimates of the slope of the New Keynesian Phillips curve with microeconomic data on the frequency of price changes.

22.7.3 Financial intermediation and regulatory policy

DSGE models used prior to the financial crisis typically did not include a realistic treatment of the banking sector and the involved macroeconomic risks. The crisis and ensuing criticism of macroeconomic modeling, however, has produced a burst of creative research from economists interested in the impact of a breakdown in financial intermediation and the integration of traditional monetary policy with financial regulation. Some of these advances have already been incorporated in DSGE models.

Here are some examples. Meh and Moran (2010) consider the role of financial frictions in a DSGE model that introduces an agency problem between banks and entrepreneurs as in Bernanke *et al.* (1999), together with an agency problem between banks and their creditors, i.e. households. In this two-sided agency problem, not only entrepreneurs' wealth influences business cycle movements but also the capital position of banks. Gerali *et al.* (2010) introduce an imperfectly competitive banking sector in a DSGE model with financial frictions and study the role of credit supply factors in business cycle fluctuations. They estimate this model with euro area data. Gertler and Kiyotaki (2010) also present a framework for studying credit market frictions in DSGE models, and use it to analyze the impact of the type of credit market interventions by the central bank and the Treasury seen in the recent crisis. Lambertini *et al.* (2011) study the performance of monetary and macro-prudential policies that lean against news-driven boom—bust cycles in housing prices and credit. Further extensions of DSGE models that incorporate recent advances in partial equilibrium modeling on the interaction of default in the credit sector and market liquidity will be of great interest.

22.7.4 Rational expectations versus learning

New Keynesian DSGE models typically impose rational expectations, as we do throughout this chapter. Agents are treated as if they are able to calculate expectations under complete knowledge about the economic structure. Critics correctly point out that market participants do not have access to such information and may never form expectations that achieve this level of rationality. A range of different approaches for modeling less-than-fully rational expectations have been proposed in the literature. A well-known case is adaptive learning. Under adaptive learning agents in the model economy estimate simple reduced-form specifications of model variables to form expectations of future variables. The parameter estimates of these reduced-form specifications are then updated as new data becomes available. In this sense the economic agent acts like an econometrician. Examples of recent examinations of the implications of adaptive learning for policy performance and business cycle dynamics in New Keynesian models include Orphanides and Williams (2006), Slobodyan and Wouters (2008), and Wieland (2009).

22.7.5 Heterogenous expectations and endogenous uncertainty

While adaptive learning relaxes the assumption of rational expectations, it maintains homogeneity across agents. By contrast, survey data from professional forecasters exhibits substantial diversity. Theoretical studies show that such heterogeneity of expectations can amplify economic fluctuations and may therefore have important implications for policy design. Recent contributions include Branch and McGough (2011), Branch and Evans (2011), De Grauwe (2011), Kurz *et al.* (2005), and Kurz (2009). An explicit treatment of belief diversity makes it possible to decompose the sources of economic volatility into exogenous uncertainty due to shocks and endogenous uncertainty due to disagreement in forecasts. Kurz (2011) constructs a New Keynesian model with diverse but rational beliefs and analyzes the implications of belief heterogeneity for monetary policy.

In sum, New Keynesian modeling is a thriving field of research. The global financial crisis has raised many questions that generate demand for improvement and modification with potentially important lessons for the design of monetary, fiscal, macro-prudential and regulatory policies.

APPENDIX

A.1 Data sources and treatment

The data series used for estimation of the small New Keynesian model in Section 22.4.2 are defined as follows:

$$YGR = (1-L)\ln(GDPC1/CNP16OV) * 100$$

$$INFL = GDPDEF$$

$$INT = FEDFUNDS/4,$$

where L denotes the lag-operator. The original data sources are:

- GDPC1: Real GDP (billions of chained 2005 dollars, seasonally adjusted annual rate). *Source*: US Department of Commerce, Bureau of Economic Analysis (via St Louis Fed FRED database).
- CNP16OV: Civilian non-institutional population (Thousands, not seasonally adjusted, average of monthly data). *Source*: US Department of Labor, Bureau of Labor Statistics (via St Louis Fed FRED database).
- GDPDEF: GDP implicit price deflator (percent change, seasonally adjusted). *Source*: US Department of Commerce, Bureau of Economic Analysis (via St Louis Fed FRED database).
- FEDFUNDS: Effective federal funds rate (percent, averages of daily figures). *Source*: Board of Governors of the Federal Reserve System (via St Louis Fed FRED database)

The data variables are related to the model variables via the following measurement equations:

$$YGR_t = \hat{Y}_t - \hat{Y}_{t-1} + \text{mean}(YGR_t)$$

$$INFL_t = \hat{\pi}_t + \text{mean}(INFL_t)$$

$$INT_t = \hat{R}_t + \text{mean}(INT_t).$$

A.2 Augmented models

Table 22.A1 The small-scale New Keynesian model

Description	Equations and definitions
Original model	
Variables	$x_t^1 = [\hat{R}_t \quad \hat{\pi}_t \quad \hat{Y}_t \quad \hat{Y}_t^{\text{gap}} \quad \hat{Y}_t^{\text{nat}} \quad \hat{R}_t^{\text{nat}} \quad \hat{g}_t \quad \hat{A}_t]'$
Shocks	$\varepsilon_t^1 = \varepsilon_t^A, \quad \eta_t^1 = [\varepsilon_t^R \quad \varepsilon_t^g]'$
Parameters	$\beta_1 = [\beta \quad \kappa \quad \tilde{\sigma} \quad \eta \quad \rho_g \quad \rho_A]', \quad \gamma_1 = [\tau_R \quad \tau_\pi \quad \tau_Y]'$
Model equations	
$g_1(.)$	$\hat{R}_t = \tau_R \hat{R}_{t-1} + (1-\tau_R)(\tau_\pi \hat{\pi}_t + \tau_Y \hat{Y}_t^{\text{gap}}) + \varepsilon_t^R$
$f_1(.)$	$\hat{Y}_t^{\text{gap}} = E_t \hat{Y}_{t+1}^{\text{gap}} - \frac{1}{\tilde{\sigma}}(\hat{R}_t - E_t \hat{\pi}_{t+1} - \hat{R}_t^{\text{nat}})$
	$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{Y}_t^{\text{gap}}$
	$\hat{Y}_t = \hat{Y}_t^{\text{gap}} + \hat{Y}_t^{\text{nat}}$
	$\hat{Y}_t^{\text{nat}} = \frac{1}{\tilde{\sigma}+\eta}[(1+\eta)\hat{A}_t + \tilde{\sigma}\hat{g}_t]$
	$\hat{R}_t^{\text{nat}} = \tilde{\sigma}[E_t(\hat{Y}_{t+1}^{\text{nat}} - \hat{g}_{t+1}) - (\hat{Y}_t^{\text{nat}} - \hat{g}_t)]$
	$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g$
	$\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_t^A$
Augmented model	
z_t, η_t	$z_t = [i_t^z \quad g_t^z \quad \pi_t^z \quad p_t^z \quad y_t^z \quad q_t^z]', \quad \eta_t = [\eta_t^i \quad \eta_t^g]'$
$\gamma, g(.)$	$i_t^z = 0.81 i_{t-1}^z + 0.39 p_t^z + 0.97 q_t^z - 0.90 q_{t-1}^z + \eta_t^i$
$f_1(.)$	As defined above in original model
$h_1(z_t, E_t x_{t+1}^1, x_t^1, \theta^1)$	$i_t^z = 4\hat{R}_t$
	$g_t^z = \hat{g}_t$
	$\pi_t^z = \hat{\pi}_t + \hat{\pi}_{t-1} + \hat{\pi}_{t-2} + \hat{\pi}_{t-3}$
	$p_t^z = 4\hat{\pi}_t$
	$y_t^z = \hat{Y}_t$
	$q_t^z = \hat{Y}_t^{\text{gap}}$

Table 22.A2 New Keynesian model of Ireland (2004a)

Description	Equations and definitions
Original model	
Variables	$x_t^2 = [\hat{r}_t \quad \hat{\pi}_t \quad \hat{y}_t \quad \hat{m}_t \quad \hat{a}_t \quad \hat{e}_t \quad \hat{z}_t]'$
Shocks	$\epsilon_t^2 = [\varepsilon_{at} \quad \varepsilon_{et} \quad \varepsilon_{zt}]', \quad \eta_t^2 = \varepsilon_{rt}$
Parameters	$\beta_2 = [\omega_1 \quad \omega_2 \quad \gamma_1 \quad \gamma_2 \quad \gamma_3 \quad \psi \quad \pi \quad r \quad \rho_a \quad \rho_e \quad \rho_z]',$
	$\gamma_2 = [\rho_r \quad \rho_\pi \quad \rho_y]'$
Model equations	
$g_2(.)$	$\hat{r}_t = \rho_r \hat{r}_{t-1} + \rho_y \hat{y}_{t-1} + \rho_\pi \hat{\pi}_{t-1} + \varepsilon_{rt}$

(*Continued*)

Table 22.A2 New Keynesian model of Ireland (2004a)—cont'd

Description	Equations and definitions
$f_2(.)$	$\hat{y}_t = E_t\hat{y}_{t+1} - \omega_1(\hat{r}_t - E_t\hat{\pi}_{t+1})$ $+\omega_2[(\hat{m}_t - \hat{e}_t) - (E_t\hat{m}_{t+1} - E_t\hat{e}_{t+1})]$ $+\omega_1(\hat{a}_t - E_t\hat{a}_{t+1})$
	$\hat{\pi}_t = \frac{\pi}{r}E_t\hat{\pi}_{t+1} + \psi\left[\frac{1}{\omega_1}\hat{y}_t - \frac{\omega_2}{\omega_1}(\hat{m}_t - \hat{e}_t) - \hat{z}_t\right]$
	$\hat{m}_t = \gamma_1\hat{y}_t - \gamma_2\hat{r}_t + \gamma_3\hat{e}_t$
	$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{at}$
	$\hat{e}_t = \rho_e\hat{e}_{t-1} + \varepsilon_{et}$
	$\hat{z}_t = \rho_z\hat{z}_{t-1} + \varepsilon_{zt}$
Augmented model	
z_t, η_t	$z_t = [i_t^z \quad \pi_t^z \quad p_t^z \quad y_t^z \quad q_t^z]', \quad \eta_t = \eta_t^i$
$\gamma, g(.)$	$i_t^z = 0.81 i_{t-1}^z + 0.39 p_t^z + 0.97 q_t^z - 0.90 q_{t-1}^z + \eta_t^i$
$f_2(.)$	As defined above in original model
$h_2(z_t, E_t x_{t+1}^2, x_t^2, \theta^2)$	$i_t^z = 4\hat{r}_t$
	$\pi_t^z = \hat{\pi}_t + \hat{\pi}_{t-1} + \hat{\pi}_{t-2} + \hat{\pi}_{t-3}$
	$p_t^z = 4\hat{\pi}_t$
	$y_t^z = \hat{y}_t$
	$q_t^z = \hat{y}_t - \frac{1}{1-\omega_2\gamma_1}[\omega_1\hat{z}_t + \omega_2(\gamma_3 - 1)\hat{e}_t]$

A.3 Database of macroeconomic models

Tables 22.A3 summarizes the models currently available in the database.

Table 22.A3 Models available in the Macroeconomic Model Database (version 1.2)

1. *Small calibrated models*		
1.1	NK_RW97	Rotemberg and Woodford (1997)
1.2	NK_LWW03	Levin *et al.* (2003)
1.3	NK_CGG99	Clarida *et al.* (1999)
1.4	NK_CGG02	Clarida *et al.* (2002)
1.5	NK_MCN99cr	McCallum and Nelson (1999) Calvo–Rotemberg model
1.6	NK_IR04	Ireland (2004a)
1.7	NK_BGG99	Bernanke *et al.* (1999)
1.8	NK_GM05	Galí and Monacelli (2005)
1.9	NK_GK09	Gertler and Karadi (2011)
1.10	NK_CK08	Christoffel and Kuester (2008)
1.11	NK_CKL09	Christoffel *et al.* (2009)
1.12	NK_RW06	Ravenna and Walsh (2006)

(*Continued*)

Table 22.A3 Models available in the Macroeconomic Model Database (version 1.2)—cont'd

2. *Estimated US models*		
2.1	US_FM95	Fuhrer and Moore (1995)
2.2	US_OW98	Orphanides and Wieland (1998) equivalent to MSR model in Levin *et al.* (2003)
2.3	US_FRB03	Federal Reserve Board model linearized as in Levin *et al.* (2003)
2.4	US_FRB08	Linearized by Brayton and Laubach (2008)
2.5	US_FRB08mx	Linearized by Brayton and Laubach (2008) mixed expectations
2.6	US_SW07	Smets and Wouters (2007)
2.7	US_ACELm	Altig *et al.* (2005) (monetary policy shock)
	US_ACELt	Altig *et al.* (2005) (technology shocks)
	US_ACELswm	No cost channel as in Taylor and Wieland (2012) (monetary policy shock)
	US_ACELswt	No cost channel as in Taylor and Wieland (2012) (technology shocks)
2.8	US_NFED08	based on Edge *et al.* (2008), version used for estimation in Wieland and Wolters (2011)
2.9	US_RS99	Rudebusch and Svensson (1999)
2.10	US_OR03	Orphanides (2003)
2.11	US_PM08	IMF projection model US, Carabenciov *et al.* (2008)
2.12	US_PM08fl	IMF projection model US (financial linkages), Carabenciov *et al.* (2008)
2.13	US_DG08	De Graeve (2008)
2.14	US_CD08	Christensen and Dib (2008)
2.15	US_IAC05	Iacoviello (2005)
2.16	US_MR07	Mankiw and Reis (2007)
2.17	US_RA07	Rabanal (2007)
2.18	US_CCTW10	Smets and Wouters (2007) model with rule-of-thumb consumers, estimated by Cogan *et al.* (2010)
2.19	US_IR11	Ireland (2011)
3. *Estimated euro area models*		
3.1	EA_CW05ta	Coenen and Wieland (2005) (Taylor-staggered contracts)
3.2	EA_CW05fm	Coenen and Wieland (2005) (Fuhrer–Moore-staggered contracts)
3.3	EA_AWM05	ECB's area-wide model linearized as in Dieppe *et al.* (2005)
3.4	EA_SW03	Smets and Wouters (2003)
3.5	EA_SR07	Sveriges Riksbank euro area model of Adolfson *et al.* (2007)

(*Continued*)

Table 22.A3 Models available in the Macroeconomic Model Database (version 1.2)—cont'd

3.6	EA_QUEST3	QUEST III Euro Area Model of the DG-ECFIN EU, Ratto *et al.* (2009)
3.7	EA_CKL09	Christoffel *et al.* (2009)
3.8	EA_GE10	Gelain (2010)
4. *Estimated/calibrated multicountry models*		
4.1	G7_TAY93	Taylor (1993a) model of G-7 economies
4.2	G3_CW03	Coenen and Wieland (2002) model of US, euro area and Japan
4.3	EACZ_GEM03	Laxton and Pesenti (2003) model calibrated to euro area and Czech republic
4.4	G2_SIGMA08	The Federal Reserve's SIGMA model from Erceg *et al.* (2008) calibrated to the US economy and a symmetric twin
4.5	EAUS_NAWM08	Coenen *et al.* (2008), New Area Wide model of euro area and US
4.6	EAES_RA09	Rabanal (2009)
5. *Estimated models of other countries*		
5.1	CL_MS07	Medina and Soto (2007) model of the Chilean economy
5.2	CA_ToTEM10	ToTEM model of Canada, based on Murchison and Rennison (2006), 2010 vintage
5.3	BRA_SAMBA08	Gouvea *et al.* (2008) model of the Brazilian economy
5.4	CA_LS07	Lubik and Schorfheide (2007) small-scale open economy model of the Canadian economy
5.5	HK_FPP11	Funke *et al.* (2011), open-economy model of the Hong Kong economy

ACKNOWLEDGMENTS

V.W. acknowledges funding support from European Community grant MONFISPOL under grant agreement SSH-CT-2009-225149. We are grateful for very helpful comments by Dale Jorgenson, Peter Dixon, Hans Fehr and conference participants at the Handbook of Computable General Equilibrium Modeling Conference at the World Bank, Washington, DC, 1—3 June 2011. All remaining errors are the authors' responsibility.

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