CHAPTER 17

An Econometric Approach to General Equilibrium Modeling

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Abstract

The first objective of this chapter is to present a new approach to econometric modeling of producer behavior. Our key contribution is to represent the rate and biases of technical change by unobservable or latent variables. We also divide the rate of technical change between components that are induced by changes in prices and those that are autonomous and not affected by prices. In our dataset, production is disaggregated into 35 separate commodities produced by one or more of the 35 industries making up the US economy. Our second objective is to present a new econometric model of aggregate consumer behavior. The model allocates full wealth among time periods for households distinguished by demographic characteristics, and determines the within-period demands for leisure, consumer goods and services. An important feature of our approach is the development of a closed-form representation of aggregate demand and labor supply that accounts for the heterogeneity in household behavior that is observed in micro-level data. Our model of producer behavior is the supply side of general equilibrium models of the US. The aggregate demand functions are important components of the demand side. These general equilibrium models are used to analyze the consequences of a broad spectrum of public policies. These applications are discussed in more detail in Chapter 8 of this Handbook. The third objective of the chapter is to demonstrate an important benefit of the econometric approach to parameterization. The parameter covariances obtained in the course of estimation can be used to construct confidence intervals for endogenous variables in general equilibrium models. Confidence intervals characterize the precision of modeling results more rigorously and systematically than traditional sensitivity analysis.

Keywords

Rate and bias of technical change, latent variables, Kalman filter, aggregate demand, labor supply, confidence intervals, outcome variables

JEL classification codes

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17.1 INTRODUCTION

The first objective of this chapter is to present a new approach to econometric modeling of producer behavior. The index number approach to productivity measurement has been the work horse of empirical research for half a century.[1] This salient concept has generated a vast literature on productivity measurement, recently surveyed by Jorgenson (2005). The key idea is to treat the level of technology as an unobservable or latent variable in a neoclassical production function. Under appropriate assumptions the rate of technical change is the residual between the growth rate of output and the growth rate of inputs. Using index numbers for these growth rates, the level of technology can be recovered without estimating the unknown parameters of the production function.

Recently, attention has shifted to the biases of technical change.[2] This shift is motivated by the fact that the rate of technical change accounts for only a modest proportion of economic growth.[3] In addition, biases of technical change have found a wide range of applications, such as changes in the distribution of income, emphasized in the survey by Acemoglu (2002b), and determinants of energy conservation, highlighted in the survey by Jaffe *et al.* (2003). However, biases of technical change are not directly observable. In this chapter we present a new econometric approach to measuring both the rate and the biases of technical change. Our key contribution is to represent the rate and biases by unobservable or latent variables.

The standard econometric approach to modeling the rate and biases of technical change was introduced by Binswanger (1974a,b), and described in the surveys by Binswanger and Ruttan (1978), Jorgenson (1986), and Ruttan (2001). Binswanger's approach is to represent price effects by the translog function of the input prices introduced by Christensen *et al.* (1973). He represents the rate and biases of technical change by constant time trends and fits the unknown parameters by econometric methods. This approach to modeling technical change is widely employed, for example, by Jorgenson and Fraumeni (1983), Jorgenson *et al.* (1987, Chapter 7), and, more recently, by Feng and Serletis (2008).

Binswanger's approach exploits the fact that price effects depend on observable variables, such as the prices of output and inputs and the shares of inputs in the value of output. The key to modeling these effects is to choose a flexible functional form that admits a variety of substitution patterns.[4] Our model of substitution, like Binswanger's, is

1 For further details, see Diewert and Morrison (1986).

2 Acemoglu (2002a) presents models of biased technical change, and reviews applications to macroeconomics, development economics, labor economics and international trade. Acemoglu (2007) surveys more recent developments in the literature, and presents detailed results on relative and absolute biases of technical change.

3 See Jorgenson (2009a).

4 Additional details are given by Jorgenson (2000b). Barnett and Serletis (2008) provide a detailed survey of flexible functional forms used in modeling consumer demand, including parametric, semiparametric and non-parametric approaches.

based on the translog price function, giving the price of output as a function of the prices of inputs. The measures of substitution are unknown parameters that can be estimated from observable data on prices and value shares.

Our novel contribution is to replace the constant time trends that describe the rate and biases of technical change in Binswanger's model by latent or unobservable variables. An important advantage of the translog price function in this setting is that the resulting model is linear in the latent variables. We recover these variables by applying the Kalman (1960, 1963) filter – a standard statistical technique in macroeconomics and finance, as well as many areas of engineering.

An important feature of the Kalman filter is that latent variables representing the rate and biases of technical change can be recovered for the sample period. A second and decisive advantage of the Kalman filter is that the latent variables can be projected into the future, so that the rate and biases of technical change can be incorporated into econometric projections.[5] The rate of technical change captures trends in productivity, while biases of technical change describe changes in the structure of production.

We implement our new approach for modeling substitution and technical change for the post-war US economy, 1960–2005. This period includes substantial changes in the prices of fossil fuels and the wage rate. Energy crisis periods with dramatic increases in energy prices alternating with periods of energy price collapse are particularly valuable for our purposes. By modeling substitution and technical change econometrically, we are able to decompose changes in the price of output and the input value shares between price effects and the effects of technical change. Empirically, these two sets of effects are comparable in magnitude.

We also decompose the rate of technical change between an autonomous part, unaffected by price changes, and an induced part, responsive to price changes. The rate of induced technical change links the rate and biases of technical change through the correlation between the input prices and the latent variables representing biases. Efforts to economize on an input that has become more expensive or to increase the utilization of an input that has become cheaper will affect the rate of technical change. Although modest in size, rates of induced technical change are generally opposite in sign to rates of autonomous technical change.

The second objective of this chapter is to present a new econometric model of aggregate consumer behavior for the US. The model allocates full wealth among time periods for households distinguished by demographic characteristics, and determines the within-period demands for leisure, consumer goods and services. An important feature

[5] An intertemporal general equilibrium model of the US, incorporating the rate and biases of technical change based on the Kalman filter, is presented by Jorgenson *et al.* (2012). This model also incorporates the dynamics of capital accumulation and asset pricing, so that we do not include these features in the specification of our models of producer and consumer behavior.

of our approach is the development of a closed-form representation of aggregate demand and labor supply that accounts for the heterogeneity in household behavior that is observed in micro-level data. Aggregate demand functions are important components of general equilibrium models that are used to analyze the macroeconomic consequences of a broad spectrum of public policies.

We combine expenditure data for over 150,000 households from the consumer expenditure surveys (CEX) with price information from the consumer price index (CPI) between 1980 and 2006. Following Slesnick (2002) and Kokoski *et al.* (1994), we exploit the fact that the prices faced by households vary across regions of the US as well as across time periods. We use the CEX to construct quality-adjusted wages for individuals with different characteristics that also vary across regions and over time. In order to measure the value of leisure for individuals who are not employed, we impute the opportunity wages they face using the wages earned by employees.

Cross-sectional variation of prices and wages is considerable and provides an important source of information about patterns of consumption and labor supply. The demographic characteristics of households are also significant determinants of consumer expenditures and the demand for leisure. The final determinant of consumer behavior is the value of the time endowment for households. Part of this endowment is allocated to labor market activities and reduces the amount available for consumption in the form of leisure.

We employ a generalization of the translog indirect utility function introduced by Jorgenson *et al.* (1997) in modeling household demands for goods and leisure. This indirect utility function generates demand functions with rank two in the sense of Gorman (1981). The rank-extended translog indirect utility function proposed by Lewbel (2001) has Gorman rank three. We present empirical results for the original translog demand system as well as the rank-extended translog system and conclude that the rank-three system more adequately represents consumer behavior although the differences are not large.

Our model of consumption and labor supply is based on two-stage budgeting, and is most similar to the framework described and implemented by Blundell *et al.* (1994). However, this is limited to consumption goods alone and does not include labor supply. The first stage allocates full wealth, including assets and the value of the time endowment, among time periods using the standard Euler equation approach introduced by Hall (1978). Since the CEX does not provide annual panel data at the household level, we employ synthetic cohorts, introduced by Browning *et al.* (1985), and utilized, for example, by Attanasio *et al.* (1999), Blundell *et al.* (1994) and many others.

The third objective of the chapter is to demonstrate an important benefit of the econometric approach to parameterization. The parameter covariances obtained in the course of estimation can be used to construct confidence intervals for endogenous variables in general equilibrium models. Confidence intervals characterize the precision of modeling results more rigorously and systematically than traditional sensitivity analysis.

Confidence intervals have three main strengths that are difficult to reproduce via sensitivity analysis: (i) They capture perturbations in all parameters, not just a subset selected by an analyst, (ii) they account for probability weightings of perturbations in the parameters and (iii) they do not treat perturbations in the parameters as independent; rather, they correctly capture the effects of covariances between the estimates. We present an approach for computing confidence intervals that is based on the delta method. It produces results comparable to Monte Carlo analysis, but is far faster and more scalable for large models. It also allows very fine-grained analysis of the sources of uncertainty in any given endogenous variable.

In Section 17.2 we present our econometric model of producer behavior. We augment the translog price function by introducing latent variables that represent the rate and biases of technical change. In Section 17.3 we apply an extension of the Kalman filter to estimate the unknown parameters of the model and generate the latent variables. In Section 17.4 we extend the standard framework for the Kalman filter to include endogenous prices by introducing instrumental variables. We propose a two-step procedure based on two-step maximum likelihood estimation and derive two diagnostic tests for the validity of the instruments.

In Section 17.5 we present our empirical results on producer behavior. We find that substitution and technical change are both important in representing changes in patterns of production. In particular, biases of technical change are quantitatively significant for all inputs. The rates of technical change decompose neatly between a negative rate of induced technical change and a positive rate of autonomous technical change, which generally predominates. This implies that biased technical change, a change in technology directed to a particular input, reduces the rate of technical change.

We introduce our model of consumer behavior in Section 17.6. We first consider the second stage of the model, which allocates full consumption among leisure, goods and services. We subsequently present the first stage of the consumer model that describes the allocation of full wealth across time periods. In Section 17.7 we discuss data issues including the measurement of price and wage levels that show substantial variation across regions and over time.

In Section 17.8 we present the estimation results for the rank-two and rank-three specifications of our second-stage model. We present estimates of price and income elasticities for goods and services, as well as leisure. We find that the wage elasticity of household labor supply is essentially zero, but the compensated elasticity is large and positive. Leisure and consumer services are income elastic, while capital services and non-durable goods are income inelastic. Perhaps most important, we find that the aggregate demands and labor supplies predicted by our model accurately replicate the patterns in the data despite the relatively straightforward representation of household labor supply.

In Section 17.9 we estimate a model of the intertemporal allocation of full consumption. We partition the sample of households into 17 cohorts based on the birth year of the head of the household. There are 27 time series observations from 1980 through 2006 for all but the oldest and youngest cohorts, and we use these data to estimate the remaining unknown parameters of the Euler equation using methods that exploit the longitudinal features of the data.

In Section 17.10 we present our method for computing confidence intervals. We then apply this to illustrative simulations using IGEM (Intertemporal General Equilibrium Model) model and show how it can be used to analyze the underlying sources of uncertainty in any variable of interest. Section 17.11 concludes the chapter.

17.2 ECONOMETRIC MODELING OF PRODUCER BEHAVIOR

In our dataset, production is disaggregated into 35 separate commodities produced by one or more of the 35 industries making up the US economy and listed in Table 17.1. The industries generally match two-digit sectors in the North American Industry Classification System (NAICS). Industries produce a primary product and may produce one or more secondary products. Each industry is modeled by a system of equations that represents possible substitutions among the inputs of capital, labor, energy and materials, and the rate and biases of technical change.

Our focus on the US economy is motivated by the availability of a new dataset constructed by Jorgenson *et al.* (2009). The EU released similar datasets for the 25 member states prior to the enlargement to include Bulgaria and Romania on 30 June 2008.[6] The Research Institute for Economy, Trade and Industry in Japan has developed datasets of this type for mainland China, Japan, Korea and Taiwan.[7] Our new methods for modeling substitution and technical change can be applied to these economies and others with similar datasets.

The production function expresses output as a function of capital, labor, m intermediate inputs, non-competing imports (X_N) and technology (t); for industry j:

$$Q_j = f\left(K_j, L_j, X_{1,j}, X_{2,j}, \ldots X_{m,j}, X_{Nj}, t\right),\ j = 1,\ 2,\ \ldots,\ 35. \tag{17.1}$$

At the first stage the value of each industry's output is allocated to four input groups – capital, labor, energy and non-energy materials:

$$Q_j = f\left(K_j, L_j, E_j, M_j, t\right)\ , \tag{17.2}$$

[6] See Timmer *et al.* (2010).
[7] See Jorgenson *et al.* (2007).

Table 17.1 List of sectors

Sector number	Sector name
1	Agriculture
2	Metal mining
3	Coal mining
4	Petroleum and gas
5	Nonmetallic mining
6	Construction
7	Food products
8	Tobacco products
9	Textile mill products
10	Apparel and textiles
11	Lumber and wood
12	Furniture and fixtures
13	Paper products
14	Printing and publishing
15	Chemical products
16	Petroleum refining
17	Rubber and plastic
18	Leather products
19	Stone, clay and glass
20	Primary metals
21	Fabricated metals
22	Industrial machinery and equipment
23	Electronic and electric equipment
24	Motor vehicles
25	Other transportation equipment
26	Instruments
27	Miscellaneous manufacturing
28	Transport and warehouse
29	Communications
30	Electric utilities
31	Gas utilities
32	Trade
33	Finance, insurance and real estate
34	Services
35	Government enterprises

The second stage allocates the energy and non-energy materials groups to the individual intermediate commodities. This stage is not discussed further in this chapter.[8]

[8] In the data set constructed by Jorgenson *et al.* (2009) the energy and non-energy aggregates in (17.2) are assumed to be homothetically separable within the production function Equation (17.1). More details are given by Jorgenson *et al.* (2005).

Assuming constant returns to scale and calculating the cost of capital as the residual that exhausts the value of output, the value of output is equal to the value of the four inputs:

$$P_{Q\,jt}Q_{jt} = P_{Kjt}K_{jt} + P_{Ljt}L_{jt} + P_{Ejt}E_{jt} + P_{Mjt}M_{jt}. \tag{17.3}$$

In representing substitution and technical change it is more convenient to work with the dual price function instead of the production function in Equation (17.2).[9] The price function expresses the unit output price as a function of all the input prices and technology, $P_{Qj} = p(P_{Kj}, P_{Lj}, P_{Ej}, P_{Mj}, t)$.

Dropping the industry subscript j for simplicity, we assume that the price function has the *translog* form:

$$\begin{aligned} \ln P_{Qt} &= \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln P_{it} + \frac{1}{2}\sum_{i,k} \beta_{ik} \ln P_{it} \ln P_{kt} + \sum_{i=1}^{n} \ln P_{it} f_{it} + f_{pt}, \\ i, k &= \{K, L, E, M\}. \end{aligned} \tag{17.4}$$

We refer to the translog price function Equation (17.4) as the *state-space model of producer behavior*. The parameters α_0,α_i and β_{ik} are estimated separately for each industry. The latent variables f_{it} and f_{pt} describe the *state* of the system and are also estimated separately for each industry, using the Kalman filter described in Section 17.3 below. Changes in the latent variables f_{it} represent biases of technical change and the latent variable f_{pt} represents the level of technology.

An important advantage of the translog price function in this application is that it generates input share equations that are linear in the latent variables representing the biases of technical change. Differentiation of the price function Equation (17.4) with respect to the log of input prices yields the input share equations. For example, the demand for capital is derived from the capital share equation:

$$v_{Kt} = \frac{P_K K}{P_Q Q} = \alpha_K + \sum_{k} \beta_{Kk} \ln P_{kt} + f_{Kt}. \tag{17.5}$$

The share of capital is a linear function of the logarithms of the input prices and a latent variable corresponding to the bias of technical change.

The biases of technical change are the changes in the shares of inputs, holding the input prices constant, for example:

$$\Delta v_{Kt} = f_{Kt} - f_{K,t-1}. \tag{17.6}$$

9 The dual price function is equivalent to the primal production function in that all the information expressed in one is recoverable from the other. Further details are given by Jorgenson (2000b).

The biases capture patterns of increasing or decreasing input use over time after accounting for price changes. If the latent variable f_{Kt} in Equation (17.5) is increasing with time, the bias of technical change is "capital-using." For a given set of input prices the share of capital is higher as a consequence of the change in technology. Alternatively, if f_{Kt} is decreasing, the bias of technical change is "capital-saving." It is important to emphasize that technical change may be capital-using at one point of time and capital-saving at another. This would be ruled out by the constant time trends used in Binswanger's approach. There is a separate bias for each of the productive inputs − capital, labor, energy and materials.

The rate of technical change between t and $t-1$ is the negative of the rate of change in the price of output, holding the input prices constant:

$$\Delta T_t = -\sum_{i=1}^{n} \ln P_{it}\left(f_{it} - f_{i,t-1}\right) - \left(f_{pt} - f_{p,t-1}\right). \tag{17.7}$$

As technology progresses for a given set of input prices, the price of output falls. The first term in the rate of technical change Equation (17.7) depends on the prices and the biases of technical change. We refer to this as the rate of *induced* technical change. If, for example, the price of capital input falls and the bias of technical change Equation (17.6), corresponding to a change in the latent variable f_{Kt}, is capital-using, the rate of technical change in Equation (17.7) will increase. However, if the bias of technical change is capital-saving, a decrease in the price for capital will retard the rate of productivity growth. The second term in Equation (17.7) depends only on changes in the level of technology f_{pt}, so that we refer to this as the rate of *autonomous* technical change.

The rate of technical change Equation (17.7) is the sum of induced and autonomous rates of technical change. Ordinarily, the autonomous rate of technical change would be positive, while the induced rate of technical change could be positive or negative. The rate of induced technical change is simply the negative of the covariance between the logarithms of the input prices and the biases of technical change. If lower input prices are correlated with higher biases of technical change, then the rate of induced technical change is positive.

The parameters β_{ik} capture the price responsiveness of demands for inputs for a given state of technology. These parameters are called *share elasticities* and represent the degree of substitutability among the inputs. For example, a lower price of capital leads to greater demand for capital input. This may lead to a higher or lower share of capital input, depending on the substitutability of other inputs for capital; this substitutability is captured by the share elasticity for capital input. Share elasticities may be positive or negative, so that the share of capital may increase or decrease with the price of capital input. When all share elasticities β_{ik} are zero, the cost function reduces to the Cobb−Douglas or linear logarithmic form and the shares are independent of input prices.

In estimating the unknown share elasticities, restrictions derived from production theory must be imposed on the translog price function Equation (17.4). In more compact vector notation the price function and input share equations can be written as:

$$\ln P_{Qt} = \alpha_0 + \boldsymbol{\alpha}'\ln \mathbf{p}_t + \frac{1}{2}\ln \mathbf{p}_t{}'\mathbf{B}\ln \mathbf{p}_t + \ln \mathbf{p}_t{}'\mathbf{f}_t + f_{pt} + \varepsilon_t^p \tag{17.4$'$}$$

$$\mathbf{v}_t = \boldsymbol{\alpha} + \mathbf{B}\ln \mathbf{p}_t + \mathbf{f}_t + \boldsymbol{\varepsilon}_t^v \tag{17.5$'$}$$

where $\mathbf{p} = (P_K, P_L, P_E, P_M)'$, $\mathbf{v} = (v_K, v_L, v_E, v_M)'$, $\mathbf{f}_t = (f_{Kt}, f_{Lt}, f_{Et}, f_{Mt})'$ and $\mathbf{B} = [\beta_{ik}]$. We have added disturbance terms ε_t^p and $\boldsymbol{\varepsilon}_t^v$, random variables with mean zero, to represent shocks to producer behavior for a given state of technology.

Homogeneity restrictions on the price function imply that doubling of all input prices doubles the output price, so that:

$$\alpha_K + \alpha_L + \alpha_E + \alpha_M = 1 \tag{17.8}$$

$$\sum_i \beta_{ik} = 0 \text{ for each } k.$$

In addition, the matrix of share elasticities must be symmetric, so that:

$$\beta_{ik} = \beta_{ki}. \tag{17.9}$$

Finally, the price function must be "locally concave" when evaluated at the prices observed in the sample period; note that this does not imply that the cost function is "globally concave" at all possible prices. The concavity condition implies that:

$$\mathbf{B} + \mathbf{v}_t\mathbf{v}_t{}' - \mathbf{V}_t, \tag{17.10}$$

must be non-positive definite at each t in the sample period,[10] where $\mathbf{B}$ is the matrix of parameters in Equation (17.4′) and $\mathbf{V}_t$ is a diagonal matrix with the shares along the diagonal. These restrictions on the parameter estimates are easily implemented by means of standard optimization code.[11]

Since the shares for all four inputs sum to unity, the latent variables representing biases of technical change f_{it} must sum to zero. Similarly, the shocks to producer behavior for a given state of technology $\boldsymbol{\varepsilon}_t^v$ sum to zero. We solve out these

10 More detail on the implications of imposing concavity at all data points in the sample is provided by Gallant and Golub (1984).

11 For additional details, see Gallant and Golub (1984).

constraints on the shocks, as well as the homogeneity constraints Equation (17.8), by expressing the model Equation (17.4′) and Equation (17.5′) in terms of relative prices and dropping one of the equations, Equation (17.5′) for the shares and one of the latent variables representing biases of technical change.

We assume that the latent variables corresponding to biases of technical change f_{it} are stationary, since the value shares $\mathbf{v}_t$ are non-negative and sum to unity. We assume, further, that the level of technology is non-stationary but the first difference, $\Delta f_{pt} = f_{pt} - f_{pt-1}$, is stationary, so that technology evolves in accord with a stochastic trend or unit root. To implement a model of production based on the price function Equation (17.4), we express the technology state variables as a vector autoregression (VAR).

Let $\mathscr{F}_t = (1, f_{kt}, f_{lt}, f_{et}, \Delta f_{pt})'$ denote the vector of stationary state variables. The *transition equation* is:

$$\mathscr{F}_t = \mathbf{\Phi} \mathscr{F}_t + u_t, \tag{17.11}$$

where u_t is a random vector with mean zero representing technology shocks and $\mathbf{\Phi}$ is a matrix of unknown parameters of a first-order VAR. The transition Equation (17.11) determines a vector of latent variables, including the biases as well as the determinants of the rate of technical change. This equation is employed in projecting the vector of latent technology variables, given the values of these variables during the sample period and estimates of the unknown parameters of the coefficient matrix $\mathbf{\Phi}$.

17.3 APPLICATION OF THE KALMAN FILTER

The econometric technique for identifying the rate and biases of technical change is a straightforward application of the Kalman filter, introduced by Kalman (1960, 1963), and presented in detail by Hamilton (1994, Chapter 13) and others. In the empirical research described in the following section, the Kalman filter is used to model production in each of the 35 sectors of our dataset. The latent variables in the state-space specification of the price function Equation (17.4) determine current and future patterns of production along with relative prices, which are the covariates of the Kalman filter.

The model underlying the Kalman filter is as follows:

$$\underset{(r\times 1)}{\xi_t} = \underset{(r\times r)}{F}\ \underset{(r\times 1)}{\xi_{t-1}} + \underset{(r\times 1)}{v_t} \tag{17.12}$$

$$\underset{(n\times 1)}{y_t} = \underset{(n\times k)}{A'}\ \underset{(k\times 1)}{x_t} + \underset{(n\times r)}{H'}\ \underset{(r\times 1)}{\xi_t} + \underset{(n\times 1)}{w_t}, \tag{17.13}$$

where ξ_t, $t = 0, 1, 2, \ldots, T$, is the vector of unobserved latent variables and y_t, $t = 1, 2, \ldots, T$ is the vector of observations on the dependent variables. The vector y_t is determined by ξ_t and x_t, the vector of observations on the explanatory variables. The subscript t denotes time and indexes the observations.

In the model underlying the Kalman filter the *state equation* is Equation (17.12) and the *observation Equation* is Equation (17.13), where x_t is exogenous, i.e. uncorrelated with the disturbance w_t. The shocks v_t and w_t are assumed uncorrelated at all lags, and:

$$E(v_t v_\tau{}') = \begin{cases} \underset{(r\times r)}{Q} & t = \tau \\ 0 & \text{otherwise} \end{cases}$$

$$E(w_t w_\tau{}') = \begin{cases} \underset{(n\times n)}{R} & t = \tau \\ 0 & \text{otherwise} \end{cases},$$

where Q and R are the covariance matrices for the disturbances. The matrices A, H, F, R and Q include unknown parameters, but some of their elements may be known. For simplicity, we denote the unknown components of these matrices by the parameter vector $\boldsymbol{\theta}$.

Computation of the standard Kalman filter involves two procedures, *filtering* and *smoothing*. In filtering we use the maximum likelihood estimator (MLE) to estimate the unknown parameter vector $\boldsymbol{\theta}$. The log-likelihood function, based on the normal distribution, is computed by the forward recursion described by Hamilton (1994):

$$\max_\theta l(\theta|Y_T) = \max_\theta \sum_{t=1}^{T} \log N\left(y_t \middle| \hat{y}_{t|t-1}, V_{t|t-1}\right),$$

where the matrix:

$$Y_t = (y_t{}', y_{t-1}{}', \ldots y_1{}', x_t{}', x_{t-1}{}', \ldots x_1{}')',$$

consists of the observations up to time t and the mean and variance are:

$$\hat{y}_{t|t-1} = E(y_t|Y_{t-1});\ V_{t|t-1} = E\left[\left(y_t - \hat{y}_{t|t-1}\right)\left(y_t - \hat{y}_{t|t-1}\right)'\right].$$

Both are functions of $\boldsymbol{\theta}$ and the data, calculated in the forward recursion. We use numerical methods to calculate the covariance matrix of the maximum likelihood estimator $\hat{\theta}$. In smoothing, we estimate the latent vector ξ_t, given the MLE, using the backward recursion described by Hamilton (1994).

The econometric model we have presented in Section 17.2 above can be expressed in the form required by the Kalman filter with the following definitions:

$$y_t = \begin{bmatrix} v_{Kt} \\ v_{Lt} \\ v_{Et} \\ \ln \dfrac{P_{Qt}}{P_{Mt}} \end{bmatrix}, \quad x_t = \begin{bmatrix} 1 \\ \ln \dfrac{P_{Kt}}{P_{Mt}} \\ \ln \dfrac{P_{Lt}}{P_{Mt}} \\ \ln \dfrac{P_{Et}}{P_{Mt}} \\ \frac{1}{2}\left(\ln \dfrac{P_{Kt}}{P_{Mt}}\right)^2 \\ \frac{1}{2}\left(\ln \dfrac{P_{Lt}}{P_{Mt}}\right)^2 \\ \frac{1}{2}\left(\ln \dfrac{P_{Et}}{P_{Mt}}\right)^2 \\ \ln \dfrac{P_{Kt}}{P_{Mt}} \ln \dfrac{P_{Lt}}{P_{Mt}} \\ \ln \dfrac{P_{Kt}}{P_{Mt}} \ln \dfrac{P_{Et}}{P_{Mt}} \\ \ln \dfrac{P_{Lt}}{P_{Mt}} \ln \dfrac{P_{Et}}{P_{Mt}} \end{bmatrix},$$

$$A' = \begin{bmatrix} \alpha_K & \beta_{KK} & \beta_{KL} & \beta_{KE} & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_L & \beta_{KL} & \beta_{LL} & \beta_{LE} & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_E & \beta_{KE} & \beta_{LE} & \beta_{EE} & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & \alpha_K & \alpha_L & \alpha_E & \beta_{KK} & \beta_{LL} & \beta_{EE} & \beta_{KL} & \beta_{KE} & \beta_{LE} \end{bmatrix}, \quad \xi_t = \begin{bmatrix} 1 \\ f_{Kt} \\ f_{Lt} \\ f_{Et} \\ f_{pt} \\ f_{pt-1} \end{bmatrix},$$

$$H' = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \ln\dfrac{P_{Kt}}{P_{Mt}} & \ln\dfrac{P_{Lt}}{P_{Mt}} & \ln\dfrac{P_{Et}}{P_{Mt}} & 1 & 0 \end{bmatrix}, \; w_t = \begin{bmatrix} \varepsilon_{Kt} \\ \varepsilon_{Lt} \\ \varepsilon_{Et} \\ \varepsilon_{pt} \end{bmatrix},$$

$$v_t = \begin{bmatrix} 0 \\ u_{Kt} \\ u_{Lt} \\ u_{Et} \\ u_{dpt} \\ 0 \end{bmatrix}, \; F' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \chi_K & \delta_{KK} & \delta_{KL} & \delta_{KE} & \delta_{Kp} & -\delta_{Kp} \\ \chi_L & \delta_{LK} & \delta_{LL} & \delta_{LE} & \delta_{Lp} & -\delta_{Lp} \\ \chi_E & \delta_{EK} & \delta_{EL} & \delta_{EE} & \delta_{Ep} & -\delta_{Ep} \\ \chi_p & \delta_{pK} & \delta_{pL} & \delta_{pE} & \delta_{pp}+1 & -\delta_{pp} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

17.4 INSTRUMENTAL VARIABLES AND SPECIFICATION TESTS

We require two modifications of the standard Kalman filter. First, we impose the concavity constraints Equation (17.10) at each data point in the sample period by simply adding these constraints to the computation of the MLE, converting this from an unconstrained to a constrained optimization. Second, the explanatory variables are prices determined by the balance of demand and supply, so that they may be endogenous. We introduce exogenous instrumental variables, say z_t, to deal with the potential endogeneity of the prices.[12] We assume that the vector z_t includes the observations on these variables at time t and satisfies the equation:

$$\underset{(k\times 1)}{x_t} = \underset{(k\times m)}{\Pi} \; \underset{(m\times 1)}{z_t} + \underset{(k\times 1)}{\eta_t}, \tag{17.14}$$

where z_t is uncorrelated with η_t and w_t, and η_t is correlated with w_t but uncorrelated with v_t.

[12] Input and output prices for each of the 35 sectors are determined within an intertemporal general equilibrium model like those presented by Jorgenson (1998) and employed by Jorgenson *et al.* (2012).

Combining Equation (17.14) with the observation equation and the state equation:

$$\underset{(n\times 1)}{y_t} = \underset{(n\times k)}{A'}\ \underset{(k\times 1)}{x_t} + \underset{(n\times r)}{H'}\ \underset{(r\times 1)}{\xi_t} + \underset{(n\times 1)}{w_t}$$

$$\underset{(r\times 1)}{\xi_t} = \underset{(r\times r)}{F}\ \underset{(r\times 1)}{\xi_{t-1}} + \underset{(r\times 1)}{v_t},$$

we can construct a new observation equation:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} A'\Pi \\ \Pi \end{bmatrix} z_t + \begin{bmatrix} H' \\ O \end{bmatrix} \xi_t + \begin{bmatrix} A'\eta_t + w_t \\ \eta_t \end{bmatrix},$$

or

$$\underset{[(n+k)\times 1]}{\tilde{y}_t} = \underset{[(n+k)\times m]}{\tilde{A}'}\ \underset{(m\times 1)}{\tilde{x}_t} + \underset{[(n+k)\times r]}{\tilde{H}'}\ \underset{(r\times 1)}{\xi_t} + \underset{[(n+k)\times 1]}{\tilde{w}_t},$$

leaving the state equation unchanged. The new model satisfies the exogeneity requirement of the Kalman filter. This would be a promising approach if the size of $\mathbf{\Pi}$ were small; however, in our application, this matrix involves 120 unknown parameters.

A more tractable approach is the two-step Kalman filter, obtained by a direct application of the two-step MLE (Wooldridge, 2010, Chapter 13). If the parameter $\mathbf{\Pi}$ were known, we could replace x_t with $\Pi z_t + \eta_t$ and formulate a new observation equation, $y_t = A'\Pi z_t + H'\xi_t + (A'\eta_t + w_t)$, where z_t is the exogenous explanatory variable. Motivated by this idea, we proceed in two steps:

Step 1 Estimate $\hat{\Pi} = XZ'(ZZ')^{-1}$ using OLS to obtain a consistent estimator of $\mathbf{\Pi}$, where X and Z represent the matrices of observations on x_t and z_t, $t = 1, 2, \ldots, T$.

Step 2 Replace X in the standard Kalman filter with $\hat{X} = \hat{\Pi}Z$, i.e. replace x_t with the fitted value $\hat{x}_t$ at time t, and use the standard filtering procedure to obtain the two-step MLE of the unknown parameters in the matrices A, H, F, R and Q.[13]

Wooldridge (2010, Chapter 13) shows that $\hat{\theta}$ is a consistent estimator of the parameter θ. In addition, it is asymptotically normal with:

[13] A similar approach for estimation of models with time-varying parameters has been introduced by Kim (2006) and Kim and Nelson (2006).

$$\begin{aligned}
\sqrt{N}\left(\hat{\theta}-\theta_0\right) &= \frac{A_0^{-1}}{\sqrt{N}}\sum\nolimits_{t=1}^{N}(-g_t(\theta_0;\Pi_0))+o_p(1)\\
&= -A_0^{-1}\left[\frac{1}{\sqrt{N}}\sum\nolimits_{t=1}^{N}\left(\frac{\partial \log f\left(y_t|x_t,y_{t-1};\theta_0,\Pi_0\right)}{\partial\theta}\right)\right.\\
&\qquad \left. +\sqrt{N}E\left(\frac{\partial^2 \log f\left(y_t|x_t,y_{t-1};\theta_0\right)}{\partial\theta\partial\Pi}\right)\times\left(\hat{\Pi}-\Pi_0\right)\right]+o_p(1)\\
&= -A_0^{-1}\left[\frac{1}{\sqrt{N}}\sum\nolimits_{t=1}^{N}S_t+\sqrt{N}F_0\left(\hat{\Pi}-\Pi_0\right)\right]+o_p(1)\\
&= -A_0^{-1}\left\{\frac{1}{\sqrt{N}}\sum\nolimits_{t=1}^{N}S_t+\sqrt{N}F_0\left[\left(\sum\nolimits_{t=1}^{N}Z_t'Z_t\right)^{-1}\left(\sum\nolimits_{t=1}^{N}Z_t'\eta_t\right)\right]\right\}+o_p(1)\\
&= -A_0^{-1}\left\{\frac{1}{\sqrt{N}}\sum\nolimits_{t=1}^{N}S_t+\frac{1}{\sqrt{N}}F_0\left[\left(N^{-1}\sum\nolimits_{t=1}^{N}Z_t'Z_t\right)^{-1}\left(\sum\nolimits_{t=1}^{N}Z_t'\eta_t\right)\right]\right\}+o_p(1)\\
&= -\frac{A_0^{-1}}{\sqrt{N}}\sum_{t=1}^{N}\left[s_t+F_0\left(N^{-1}\sum_{t=1}^{N}Z_t'Z_t\right)^{-1}Z_t'\eta_t\right]+o_p(1)\\
&= -\frac{A_0^{-1}}{\sqrt{N}}\sum_{t=1}^{N}[s_t+F_0r_t]+o_p(1),
\end{aligned}$$

where we employ the following definitions:

$$\begin{aligned}
A_0 &= E\left(\frac{\partial^2 \log f(y_t|x_t,y_{t-1};\theta_0)}{\partial\theta\partial\theta'}\right)\\
F_0 &= E\left(\frac{\partial^2 \log f(y_t|x_t,y_{t-1};\theta_0,\Pi_0)}{\partial\theta\partial\Pi}\right)\\
r_t &= \left(N^{-1}\sum_{t=1}^{N}Z_t'Z_t\right)^{-1}Z_t'\eta_t\\
s_t &= \left(\frac{\partial \log f(y_t|x_t,y_{t-1};\theta_0,\Pi_0)}{\partial\theta}\right).
\end{aligned}$$

So $\sqrt{N}(\hat{\theta}-\theta_0)$ can be expressed as:

$$\sqrt{N}(\hat{\theta}-\theta_0) = -\frac{A_0^{-1}}{\sqrt{N}}\sum_{t=1}^{N}[s_t+F_0r_t]+o_p(1).$$

The asymptotic variance of the unknown parameter $\hat{\theta}$ is:

$$AVar\sqrt{N}(\hat{\theta}-\theta_0) = \frac{1}{N}A_0^{-1}E\Big[(-g_t(\theta_0;\Pi_0))(-g_t(\theta_0;\Pi_0))'\Big]A_0^{-1}$$

$$= \frac{1}{N}A_0^{-1}D_0A_0^{-1},$$

given:

$$D_0 = E\Big[\Big(-g_t(\theta_0;\Pi_0)(-g_t(\theta_0;\Pi_0))'\Big].$$

Therefore, the variance of the unknown parameter θ should be:

$$Var(\hat{\theta}-\theta_0) = \frac{1}{N^2}A_0^{-1}D_0A_0^{-1}.$$

To estimate the asymptotic covariance matrix consistently, we apply the law of large numbers to the following sample statistics:

$$\hat{A}_0 = \frac{1}{N}\sum_{t=1}^{N}\frac{\partial \log f\Big(y_t\Big|x_t,y_{t-1};\hat{\theta},\hat{\Pi}\Big)}{\partial\theta\partial\theta'}$$

$$\hat{F}_0 = \frac{1}{N}\sum_{t=1}^{N}\frac{\partial^2 \log f\Big(y_t\Big|x_t,y_{t-1};\hat{\theta},\hat{\Pi}\Big)}{\partial\theta\partial\Pi}$$

$$\hat{r}_t = \left(N^{-1}\sum_{t=1t}^{N'}Z_t'Z_t\right)^{-1}Z_t'\eta_t$$

$$\hat{s}_t = \left(\frac{\partial \log f\Big(y_t|x_t,y_{t-1};\hat{\theta},\hat{\Pi}\Big)}{\partial\theta}\right)$$

$$\hat{D}_0 = \frac{1}{N}\sum_{t=1}^{N}\Big[(-\hat{g}_t(\theta_0;\Pi_0))(-\hat{g}_t(\theta_0;\Pi_0))'\Big]$$

$$= \frac{1}{N}\sum\nolimits_{t=1}^{N}\Big[\Big(\hat{\theta}+\hat{F}_0\hat{r}_t\Big)\Big(\hat{s}_t+\hat{F}_0\hat{r}_t\Big)'\Big],$$

so that a consistent estimator of the covariance matrix is:

$$\widehat{Var}(\hat{\theta}-\theta_0) = \frac{1}{N^2}\hat{A}_0^{-1}\hat{D}_0\hat{A}_0^{-1}.$$

Table 17.A1 of the Appendix provides a list of the instrumental variables and Figure 17.A1 displays the instruments graphically. These are treated as exogenous variables in the intertemporal general equilibrium model employed by Jorgenson

et al. (2012). We employ two tests to check the validity of our instrumental variables. Fortunately, we have more instrumental variables than endogenous explanatory variables; in fact, there are 11 non-constant instruments in z_t and nine endogenous explanatory variables in x_t. This enables us to conduct a test of over-identifying restrictions to confirm the exogeneity of the instruments.

We carry out the test of over-identifying restrictions as follows. (i) We select any two non-constant instrumental variables out of the eleven $z_t^{(m-k)}$, where $m-k=12-10=2$. (ii) In the second-stage Kalman filter, we include $z_t^{(m-k)}$ as an exogenous variable in the observation equation and keep the state equation the same as before:

$$\underset{(n\times 1)}{y_t} = \underset{(n\times k)}{A'}\underset{(k\times 1)}{\hat{x}_t} + \underset{[n\times (m-k)]}{C'}\underset{[(m-k)\times 1]}{z_t^{(m-k)}} + \underset{(n\times r)}{H'}\underset{(r\times 1)}{\xi_t} + \underset{(n\times 1)}{w_t^{(m-k)}}$$

$$\underset{(r\times 1)}{\xi_t} = \underset{(r\times r)}{F}\underset{(r\times 1)}{\xi_{t-1}} + \underset{(r\times 1)}{v_t}.$$

Note that $\hat{X}$, the observation matrix of $\hat{x}_t$, $t=1,2,\ldots,T$, satisfies $\hat{X} = \hat{\Pi}Z = XZ'(ZZ')^{-1}Z$ with rank $k = 10$; therefore, selection of any two non-constant instrumental variables yields the same test statistic. Moreover, if our null hypothesis that z_t is uncorrelated with w_t is true, the addition of $z_t^{(m-k)}$ to the observation equation will not affect the original Kalman filter. We perform a likelihood ratio test of the hypothesis that C is zero by comparing l and l_g, the log-likelihood values before and after the introduction of $z_t^{(m-k)}$. Under the null hypothesis of exogeneity the difference is asymptotically χ^2:

$$2\left(l_g - l\right) \overset{a}{\sim} \chi^2_{n\times(m-k)}.$$

The results presented in Table 17.A2 of the Appendix show that the instrumental variables are exogenous.

(ii) We apply a likelihood ratio test to the hypothesis of zero correlation between endogenous explanatory variables and instrumental variables. Let $\hat{\Sigma}$ represent the empirical covariance matrix of $\dot{x}_t$, the nine non-constant elements of x_t, and $\tilde{\Sigma}$ represent the corresponding empirical covariance matrix of $\dot{x}_t - \dot{\Pi}z_t$, the residuals from the fitted values of the $\dot{x}_t$'s in the linear regression, where $\dot{\Pi}$ represents the corresponding submatrix of Π. The log-likelihood for the later is:

$$\begin{aligned}\ln\tilde{L} &= -\frac{(k-1)T}{2}\ln|2\pi| - \frac{T}{2}\ln|\tilde{\Sigma}| - \frac{1}{2}\sum_{t=1}^{T}\left(\dot{x}_t - \dot{\Pi}z_t\right)'\tilde{\Sigma}^{-1}\left(\dot{x}_t - \dot{\Pi}z_t\right)\\ &= -\frac{(k-1)T}{2}\ln|2\pi| - \frac{T}{2}\ln|\tilde{\Sigma}| + \frac{(k-1)T}{2}.\end{aligned}$$

The quadratic term is replaced by a constant due to the maximum likelihood process of the linear regression. For $\hat{\Sigma}$, we can derive a similar log-likelihood:

$$\ln\hat{L} = -\frac{(k-1)T}{2}\ln|2\pi| - \frac{T}{2}\ln|\hat{\Sigma}| + \frac{(k-1)T}{2}.$$

This is a linear regression, where the parameters before the constant term in z_t are unconstrained and all other parameters in $\dot{\Pi}$ are fixed at zero.

The likelihood ratio test statistic is:

$$LR = -2\left(\ln\hat{L} - \ln\tilde{L}\right) = T\left(\ln|\tilde{\Sigma}| - \ln|\hat{\Sigma}|\right).$$

This statistic is asymptotically χ^2, where the number of degrees of freedom is equal to the number of parameters that are constrained, $(m-1)*(k-1) = 11*9 = 99$ in our model. The results presented in Table 17.A3 of the Appendix show that the instrumental variables are highly correlated with the endogenous variables. We conclude that both diagnostic tests confirm the validity of our instruments.

17.5 EMPIRICAL RESULTS ON PRODUCER BEHAVIOR

In this section we present the rate and biases of technical change for the state-space model of producer behavior Equation (17.4) for each of the 35 sectors of the US economy listed in Table 17.1. In Figures 17.1–17.4 we give the changes in input shares of the four inputs – capital, labor, energy and materials – over the period 1960–2005.[14] These are the dependent variables for the value share of capital input in Equation (17.5) and the remaining value shares. The industries are ordered by the magnitude of the changes. In general, the capital input shares have increased, some of them very substantially. With some notable exceptions the labor shares have decreased and the energy shares have increased slightly. The materials shares are almost evenly divided between increases and decreases.

We next allocate changes in the input shares between a price effect, corresponding to the second term in Equation (17.5), and the bias of technical change Equation (17.6), corresponding to the third term in Equation (17.5). The price effects presented in Figures 17.5–17.8 represent the responses of production patterns to price changes through substitution among inputs. These responses are substantial, but appear to be evenly balanced between negative and positive effects for capital and energy. The labor price effects are predominantly negative, reflecting increases in wages relative to prices, while the materials price effects are predominantly positive. These price effects rule out a Cobb–Douglas or linear logarithmic specification for the price function Equation (17.4).

[14] Formulas for the figures presented in this section are given in Table 17.A4 in the Appendix.

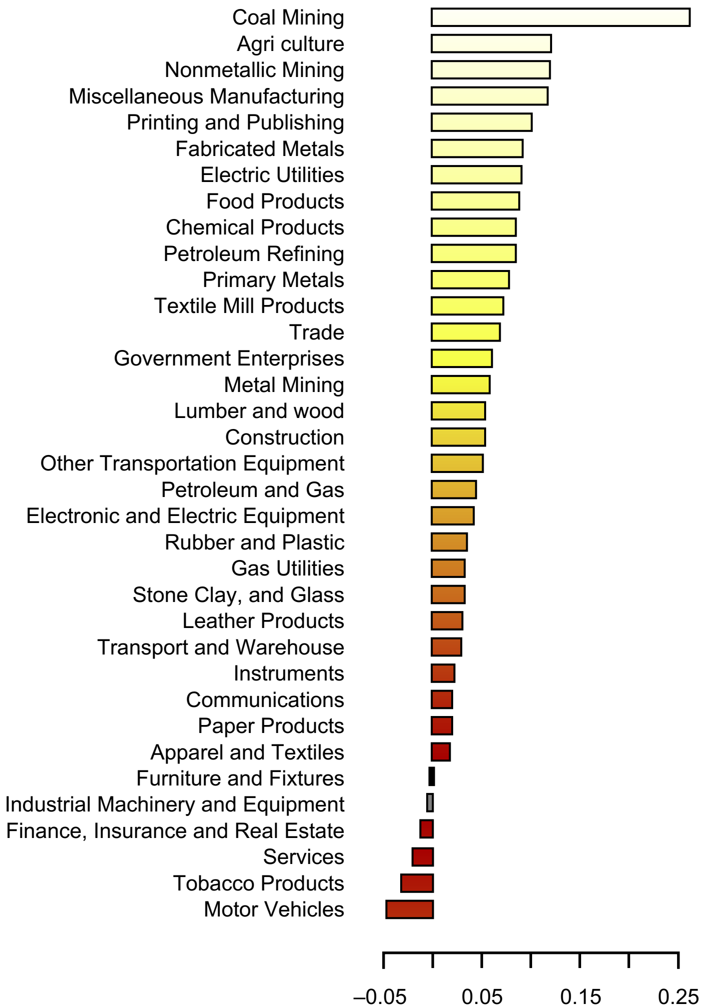

Figure 17.1 Change of capital input share: US 1960–2005.

We present the biases of technical change Equation (17.6) in Figures 17.9–17.12. The biases of technical change for capital input are predominantly capital-using and substantial in magnitude, especially for Coal Mining and Government Enterprises. The biases are capital-saving but relatively small for Paper Products and Services. The biases of technical change for labor input are divided between labor-saving technical change for industries like Leather Products and Coal Mining and labor-using change for industries such as Food Products and Textile Mill Products.

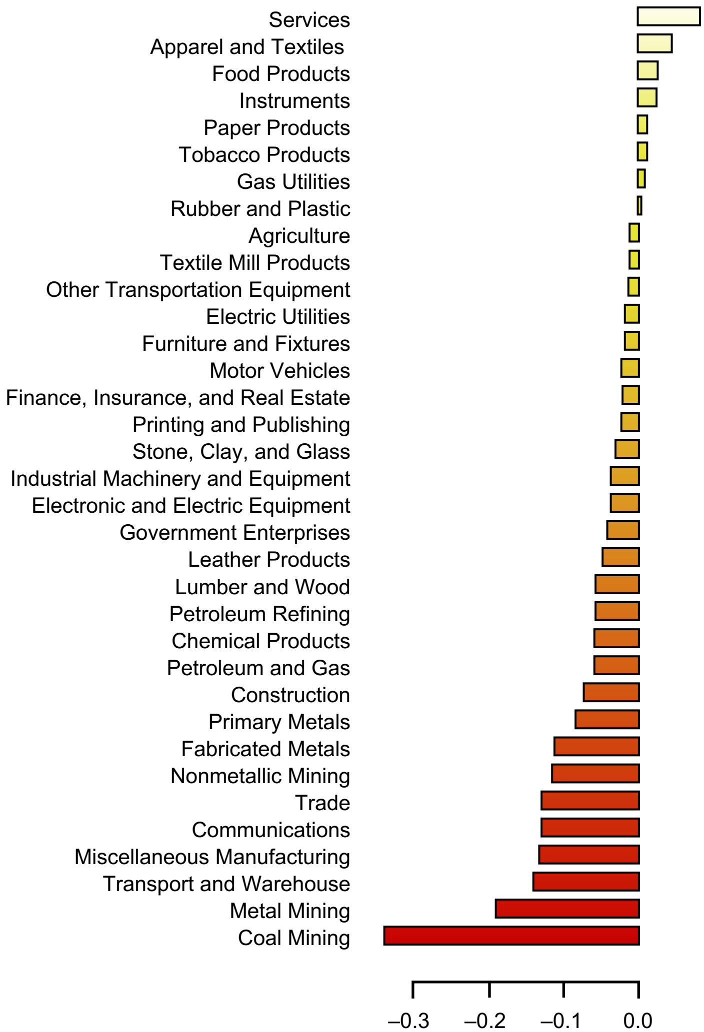

Figure 17.2 Change of labor input share: US 1960–2005.

The biases of technical change for energy are relatively small in magnitude, reflecting the small size of the energy shares for most industries. The bias for energy is energy-using for Transportation and Warehousing and energy-saving for Electric and Gas Utilities, Coal Mining, and Petroleum and Gas Mining. Finally, the biases of technical change for materials are predominantly materials-saving and substantial in size, especially for Government Enterprises and Food Products. However, the biases are materials-using for Petroleum and Gas Mining and Gas Utilities.

We conclude that the biases of technical change are comparable in magnitude to the price effects. Substitution among inputs and biased technical change are both important

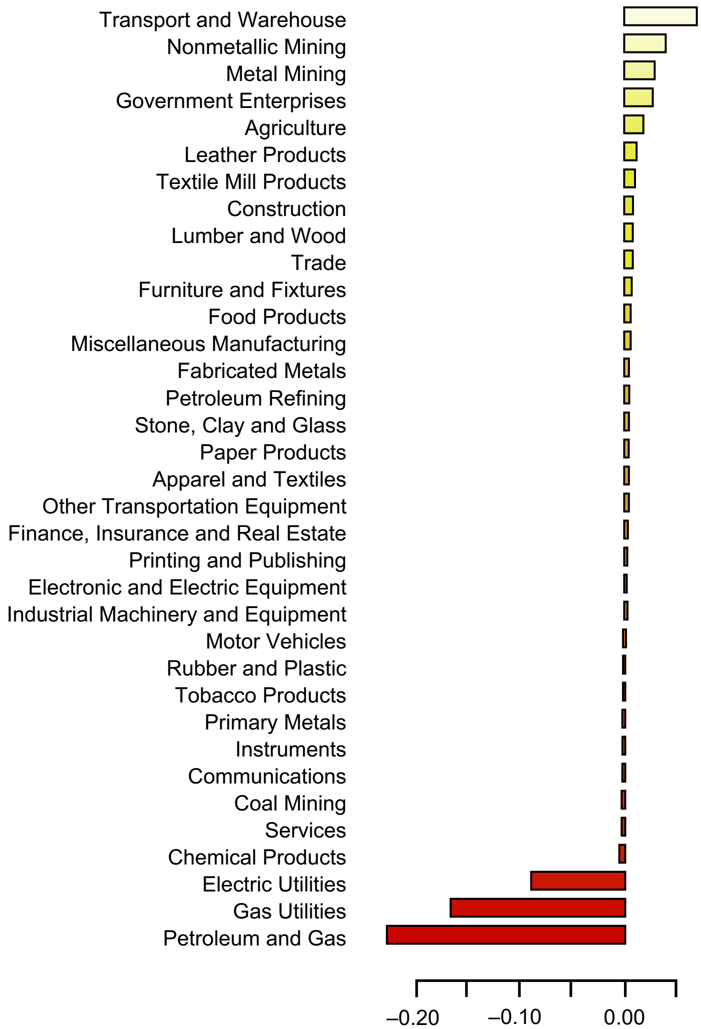

Figure 17.3 Change of energy input share: US 1960–2005.

determinants of changes in the input shares. However, the biases also play a significant role in our state-space model of producer behavior as determinants of the rate of induced technical change. We turn next to changes in the price of output and its decomposition into a price effect, corresponding to the second and third terms in Equation (17.4), and the rates of induced and autonomous technical change in Equation (17.6).

Figure 17.13 presents reductions in the logarithms of prices of the outputs of the 35 industries, relative to the prices of materials inputs in each sector. Not surprisingly, these price changes are almost evenly divided between positive and negative values with the

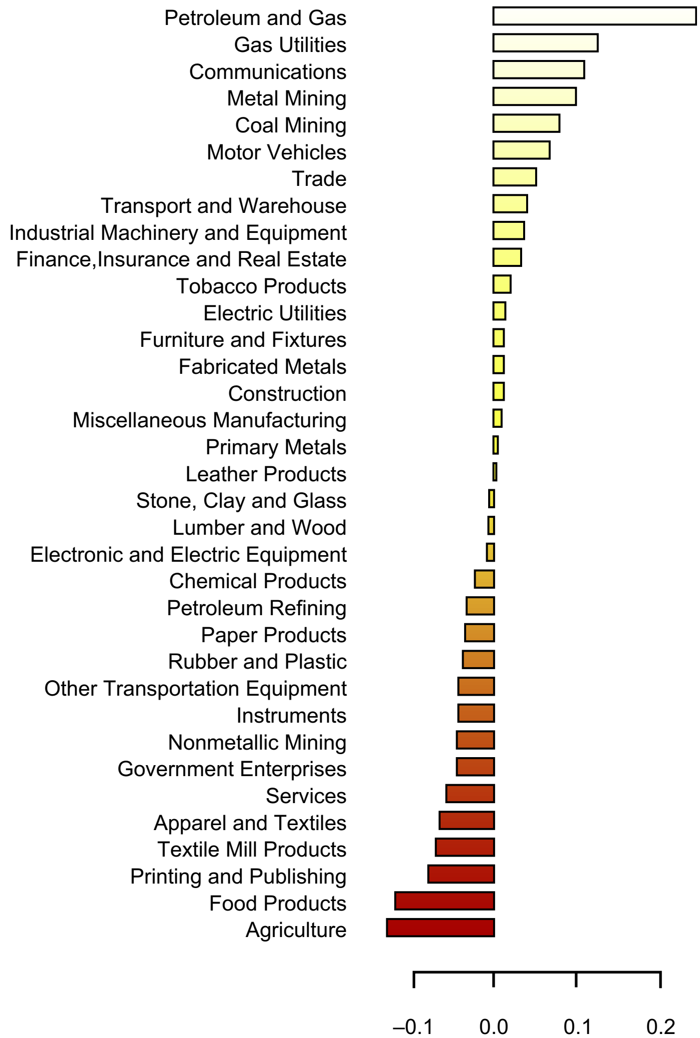

Figure 17.4 Change of material input share: US 1960–2005.

large reductions for Electronic and Electrical Equipment and Industrial Machinery and Equipment as the outstanding exceptions. The Electronic and Electrical Equipment industry produces semiconductor components for computers and other electronic equipment, while Industrial Machinery and Equipment includes computers. Technical change has resulted in a very dramatic fall in the prices of outputs for these industries, relative to the materials they consume.

The price effects presented in Figure 17.14 are differences between the price reductions in Figure 17.13 and the rates of technical change in Equation (17.7). These

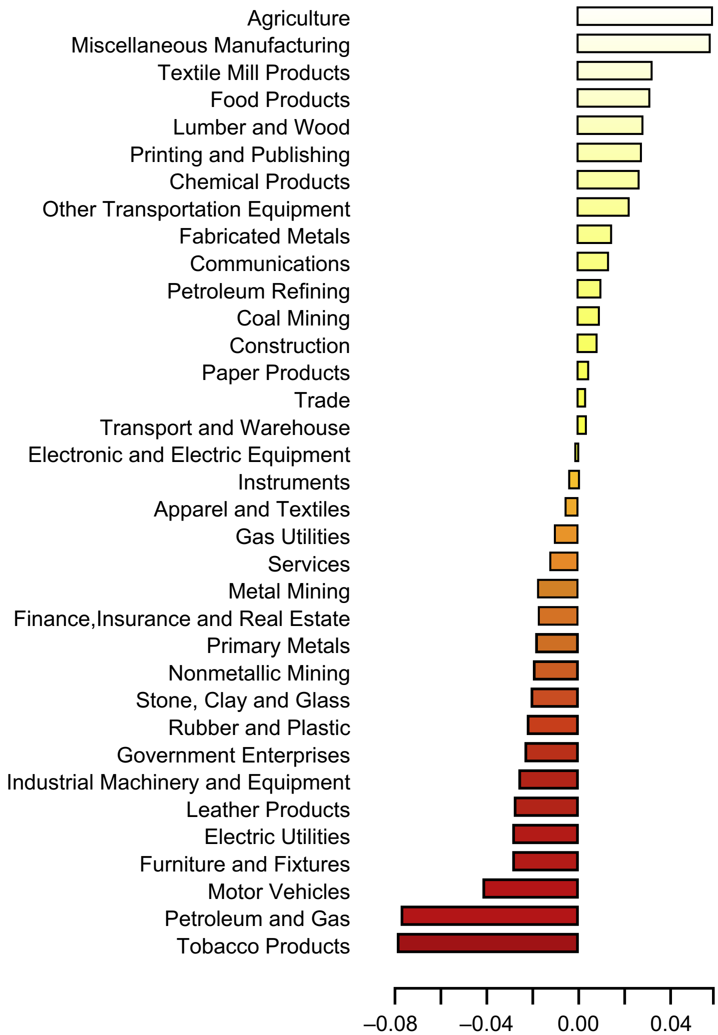

Figure 17.5 Price effect of capital input share change: US 1960–2005.

price effects result from substitution among inputs and are dominated by increases in wage rates, relative to prices of materials inputs. Using estimates of the biases of technical change, we can divide the rate of technical change between the rate of induced technical change, corresponding to the first term in Equation (17.7), and the rate of autonomous technical change, corresponding to the second term in this equation.

The rates of induced technical change in Figure 17.15, corresponding to the first term in Equation (17.7), depend on the correlations between prices of inputs and biases of technical change. If this correlation is negative, input-using technical change

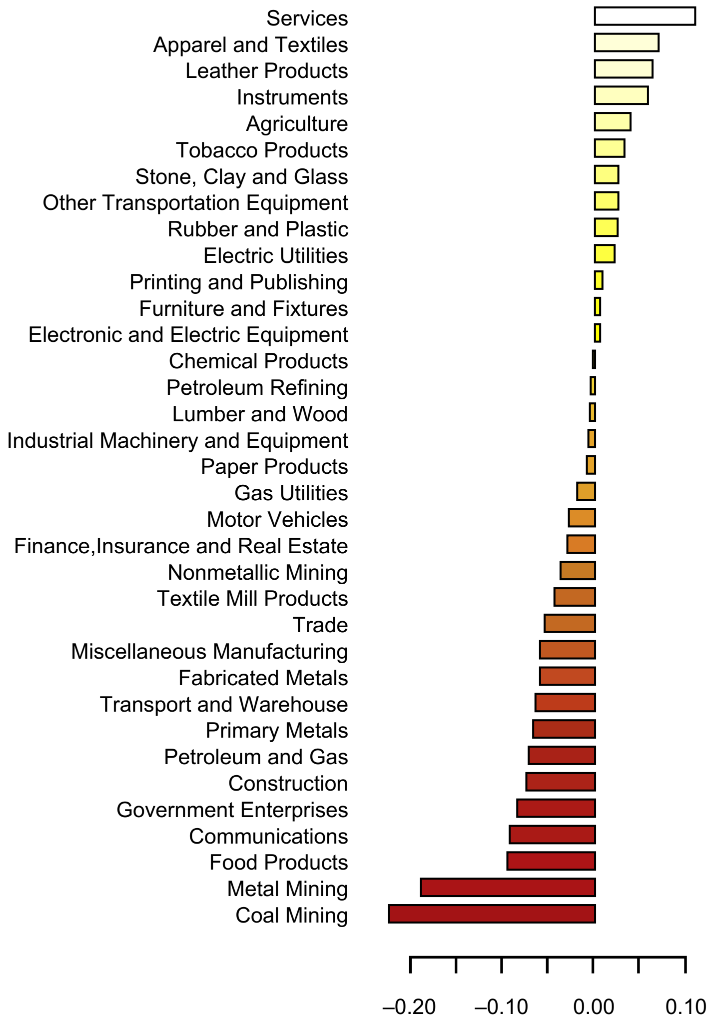

Figure 17.6 Price effect of labor input share change: US 1960−2005.

corresponds to low input prices and input-saving change to high input prices, so that the rate of induced technical change is positive. The rates of induced technical change presented in Figure 17.15 are predominantly negative, so that input-using technical change is correlated with high input prices and input-saving technical change with low input prices.

The rates of autonomous technical change given in Figure 17.16 are predominantly positive and substantial in magnitude. Electronic and Electric Equipment leads all other industries with a rate of autonomous technical change that exceeds even the very

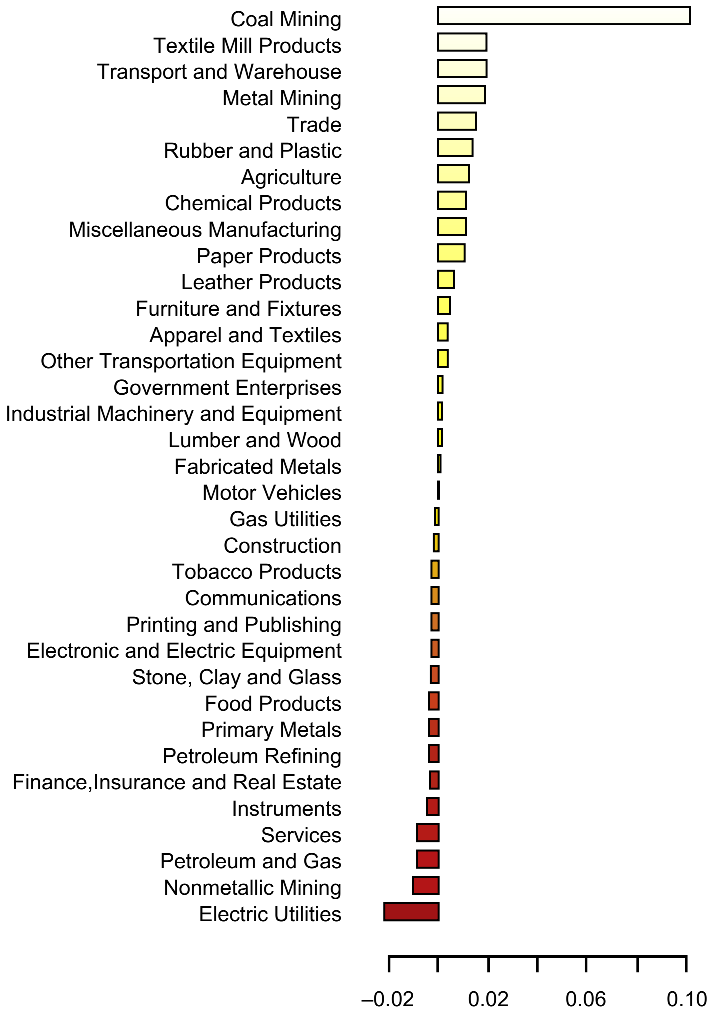

Figure 17.7 Price effect of energy input share change: US 1960–2005.

dramatic rate of decline of the relative price of the industry's output. Industrial Machinery and Equipment, the industry that includes computers, has the second largest rate of autonomous technical change. The Tobacco Products industry and Petroleum and Gas Mining have sizable negative rates of autonomous technical change.

Our overall conclusion from the empirical results presented in Figures 17.13–17.16 is that rates of autonomous and induced technical change are substantial in magnitude and opposite in sign. However, autonomous technical change predominate, so that rates of technical change are positive for most industries. Rates of technical change are

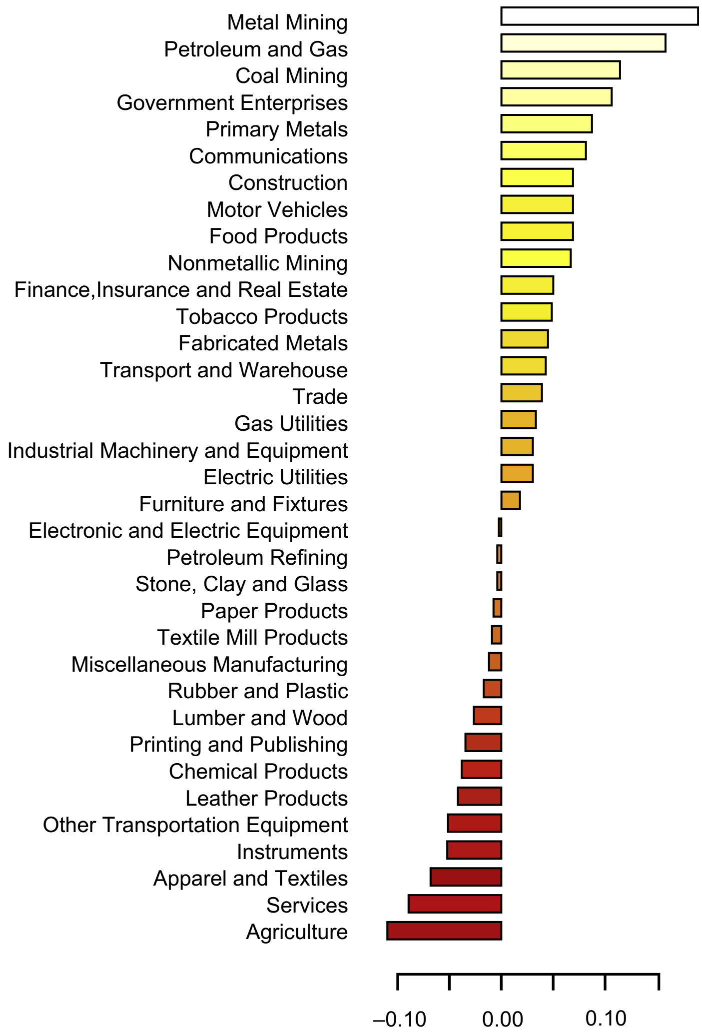

Figure 17.8 Price effect of energy input share change: US 1960—2005.

large relative to the price effects associated with substitution among inputs. The price effects exert upward pressure on output prices while induced technical change exerts pressure in the same direction, but both are offset by positive rates of autonomous technical change.

In order to explore changes in the direction and magnitude of biases in technical change in greater detail, we subdivide the biases for energy input into two subperiods, 1960—1980 and 1980—2005. Recall that the biases of technical change are first differences of the latent variables, as in Equation (17.6). In Figures 17.17 and 17.18 the biases are both energy-saving and energy-using for the seven most intensive energy-using

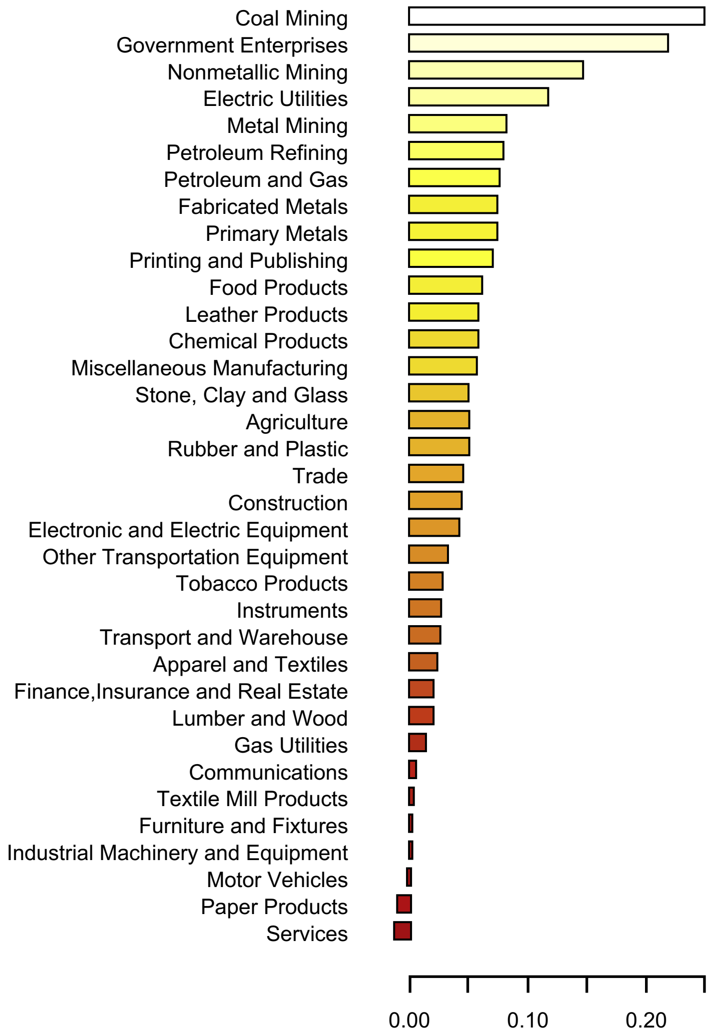

Figure 17.9 Bias of technical change for capital input: US 1960–2005.

sectors during the sample period — Petroleum Refining, Electric and Gas Utilities, Transportation and Warehousing, Coal Mining, Chemical Products, and Stone, Clay and Glass. This pattern would have been concealed by constant time trends.

There is a common pattern of energy-using change from 1960 to 1980 and energy-saving change from 1980 to 2005, except for Metal Mining. The turning point was the Second Oil Crisis, when energy prices reached their postwar peaks in real terms. We conclude that high energy prices after 1980 are correlated with energy-saving change, while low energy prices before 1980 are correlated with energy-using change. This pattern would also have been concealed by constant time trends.

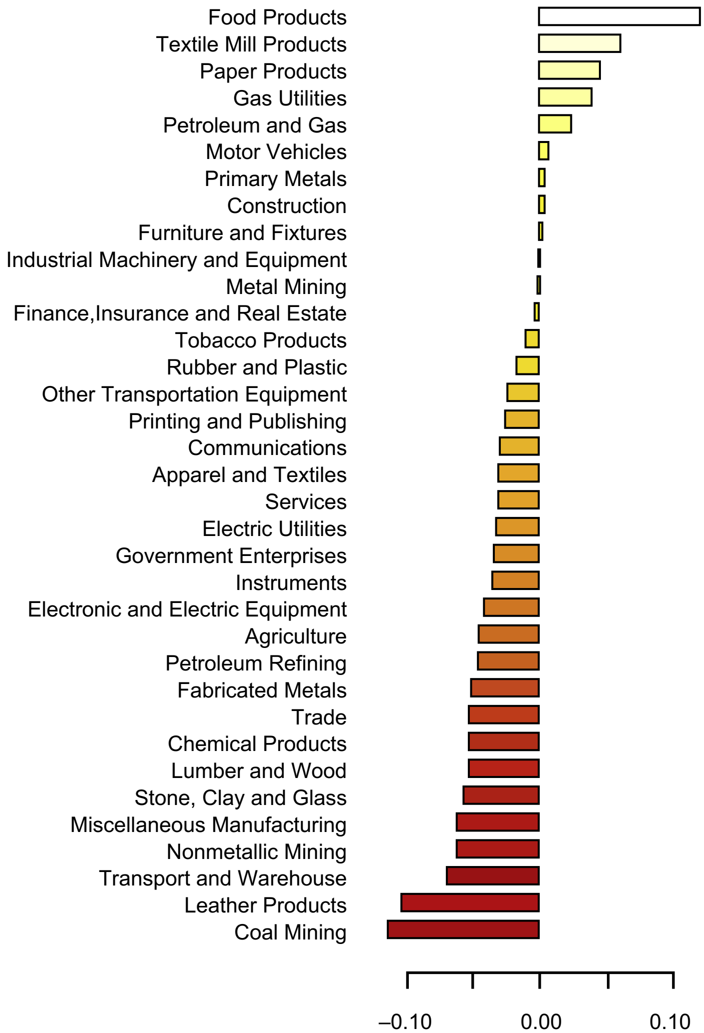

Figure 17.10 Bias of technical change for labor input: US 1960–2005.

The latent variables f_{it} converge to constants, so that biases of technical change corresponding to changes in these variables converge to zero. In Figure 17.19 we present projections of the biases of technical change for energy for the period 2006–2030. Note that the projected biases for the seven most energy-intensive industries are not simple extrapolations of the trends toward energy conservation we have identified after 1980. Projected biases are energy-saving for Gas Utilities, Transportation and Warehousing, and Chemical Products. However, projected biases are energy-using for Electric Utilities, Petroleum Refining, Stone, Clay and Glass, and Coal Mining. As before, these projections are inconsistent with the constant time trends in Binswanger's approach.

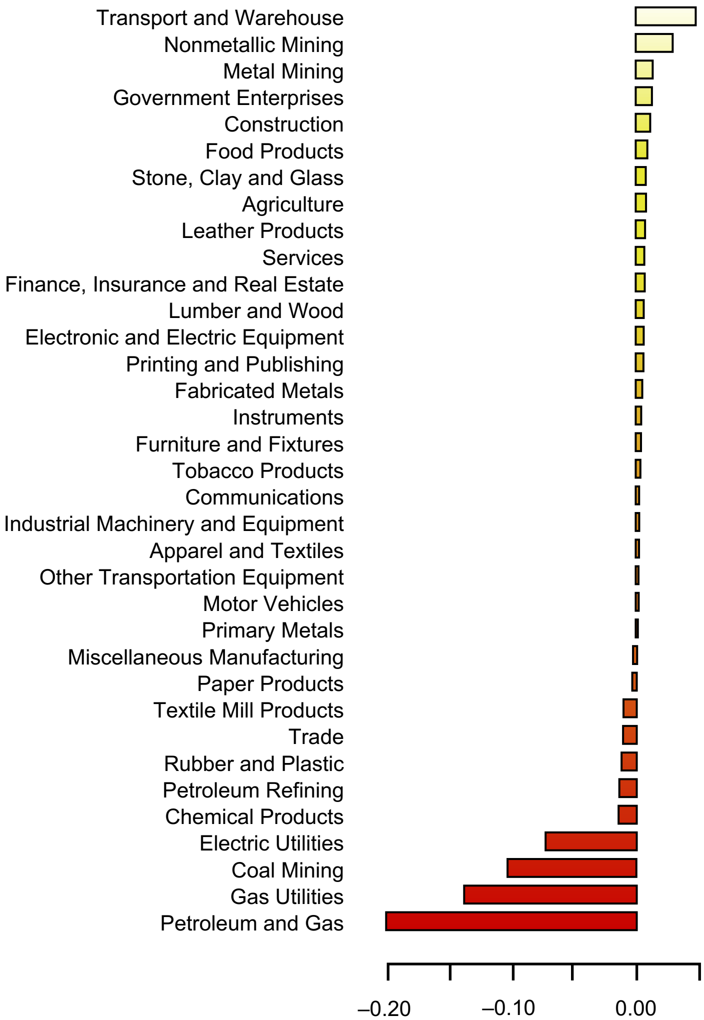

Figure 17.11 Bias of technical change for energy input: US 1960–2005.

In Figures 17.20–17.22 we give projections of biases of technical change for capital, labor, and materials for the period 2006–2030. These projections are not simple extrapolations of biases during the sample period and many alternate between input-using and input-saving bias. This variation is particularly pronounced in the case of energy input and characterizes biases of technical change during the sample period, 1960–2005, and the projection period, 2006–2030. We conclude that the latent variables representing biases of technical change must be sufficiently flexible to capture variations between input-using and input-saving technical change.

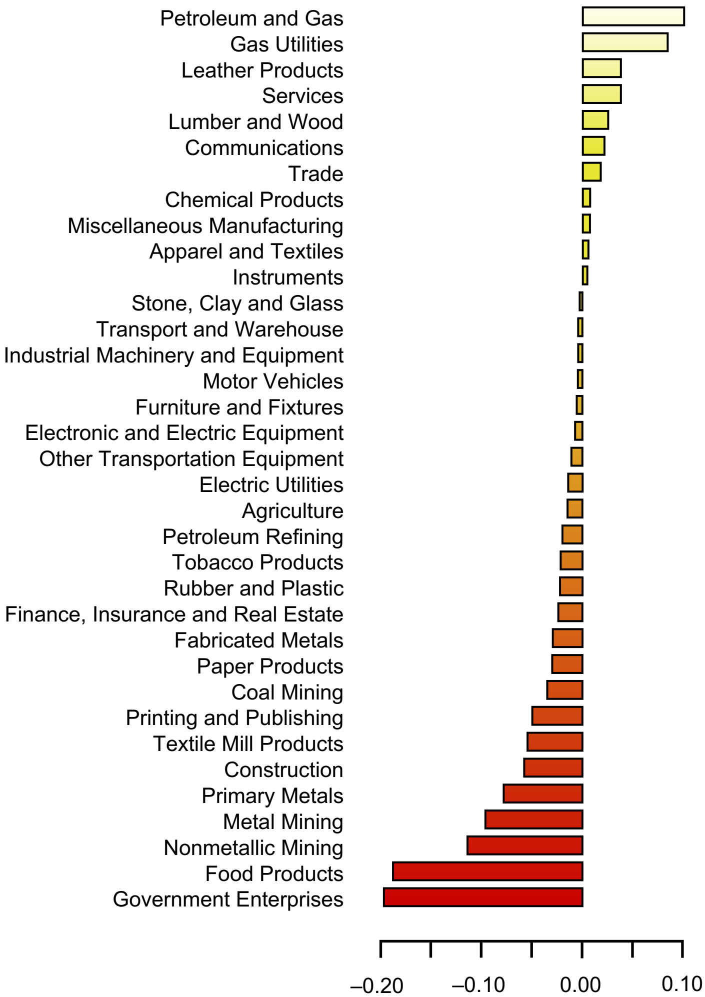

Figure 17.12 Bias of technical change for material input: US 1960–2005.

The levels of technology f_{pt} converge to linear trends, corresponding to constant rates of autonomous technical change. Recalling that we employ the dual representation of technology Equation (17.4), falling trends correspond to positive rates of technical change, while rising trends represent negative rates. In Figures 17.23 and 17.24 we give projections of the rates of induced and autonomous technical change. Rates of induced technical change are relatively small in magnitude and are evenly divided between positive and negative values. Rates of autonomous technical change are predominantly positive in sign and substantial in magnitude. The projections for Electronic and Electric Equipment, including semiconductors, have very rapid rates of technical change. Projected rates of technical change for Industrial Machinery and Equipment, including

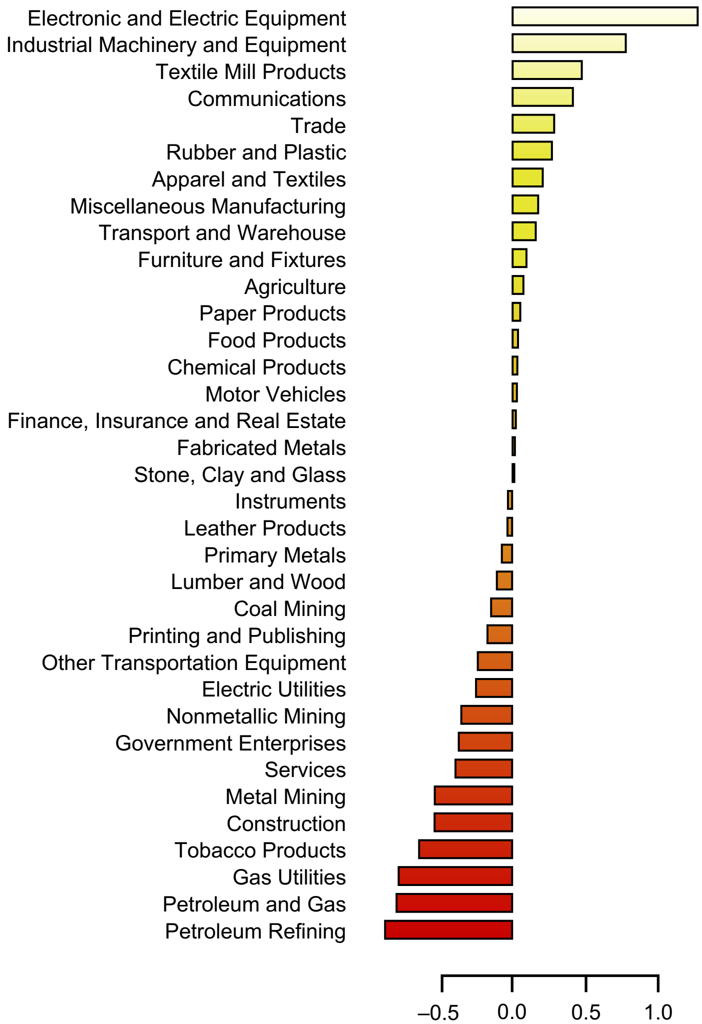

Figure 17.13 Reduction of log relative output price: US 1960–2005.

computers, are the next most rapid, also extrapolating trends during the sample period. Negative rates of autonomous technical change are substantial in magnitude for Coal Mining and Petroleum and Gas Mining.

17.6 ECONOMETRIC MODELING OF CONSUMER BEHAVIOR

We turn next to econometric modeling of consumer behavior. We assume that household consumption and labor supply are allocated in accord with two-stage budgeting. In the first stage, full expenditure is allocated over time so as to maximize a lifetime utility function subject to a full wealth constraint. Conditional on the chosen

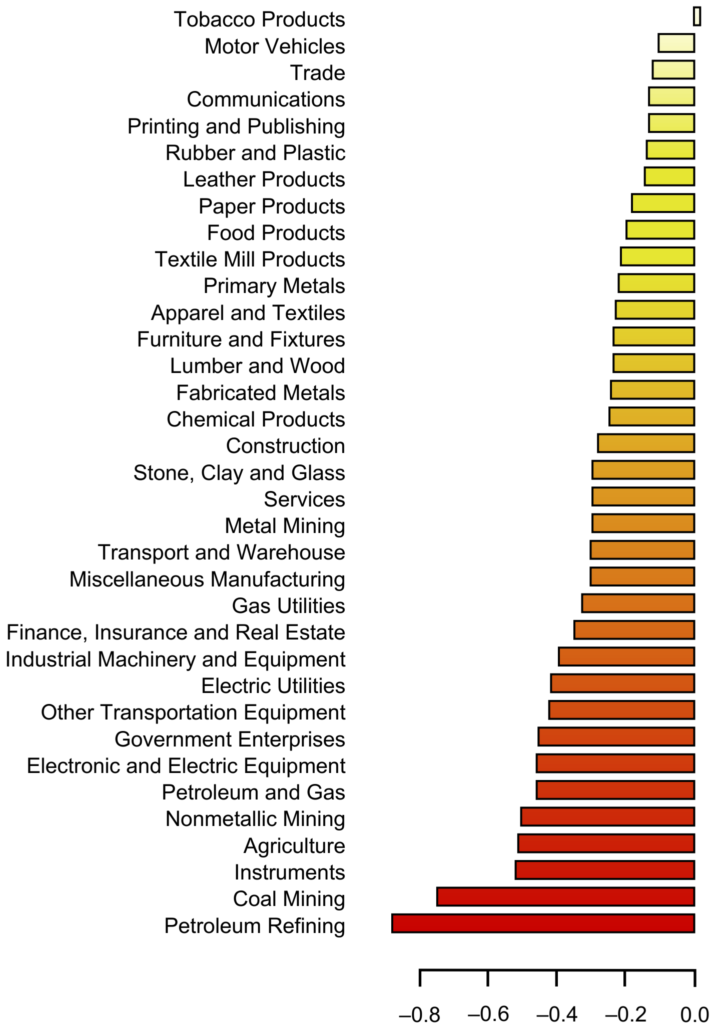

Figure 17.14 Price effect of log relative output price change: US 1960–2005.

level of full expenditure in each period, households allocate expenditures across consumption goods and leisure so as to maximize a within-period utility function.

To describe the second stage model in more detail, we assume that households consume n consumption goods in addition to leisure. The within-period demand model for household k can be described using the following notation:

$\mathbf{x}_k = (x_{1k}, x_{2k}, \ldots, x_{nk}, R_k)$	quantities of goods and leisure
$\boldsymbol{\rho}_k = (\mathbf{p}_k, p_{Lk})$	prices and wages faced by household k; these prices vary across geographic regions and over time

(*Continued*)

—cont'd

$w_{ik} = p_{ik}x_{ik/F_k}$	expenditure share of good i for household k
$\mathbf{w}_k = (w_{1k}, w_{2k}, \ldots, w_{nk}, w_{Rk})$	vector of expenditure shares for household k
$\mathbf{A}_k$	vector of demographic characteristics of household k
$F_k = \sum p_{ik}x_{ik} + p_{Lk}R_k$	full expenditure of household k, where p_{Lk} is the wage rate and R_k is the quantity of leisure consumed.

In order to obtain a closed-form representation of aggregate demand and labor supply, we use a model of demand that is consistent with exact aggregation as originally defined by Gorman (1981). Specifically, we focus on models for which the aggregate demands are the sums of micro-level demand functions rather than the typical

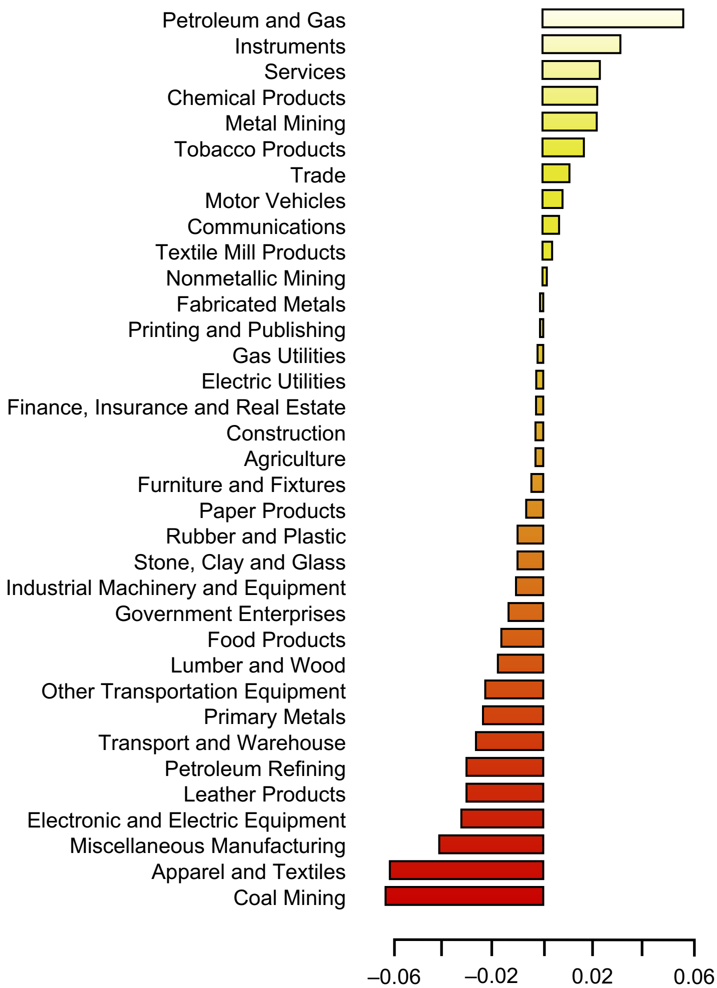

Figure 17.15 Rate of induced technical change: US 1960–2005.

assumption that they are generated by a representative consumer. Exact aggregation is possible if the demand function for good i by household k is of the form:

$$x_{ik} = \sum_{j=1}^{J} b_{ij}(\rho)\psi_j(F_k).$$

Gorman showed that if demands are consistent with consumer rationality, the matrix $\{b_{ij}(\rho)\}$ has rank that is no larger than three.[15]

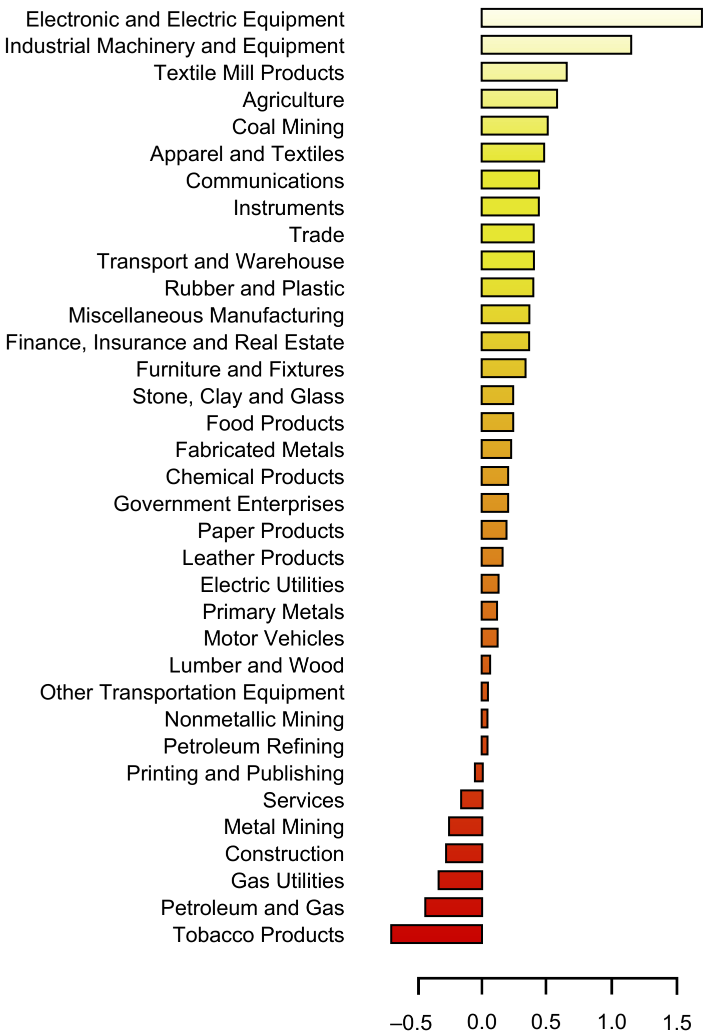

Figure 17.16 Rate of autonomous technical change: US 1960–2005.

15 See Blundell and Stoker (2005) for further discussion.

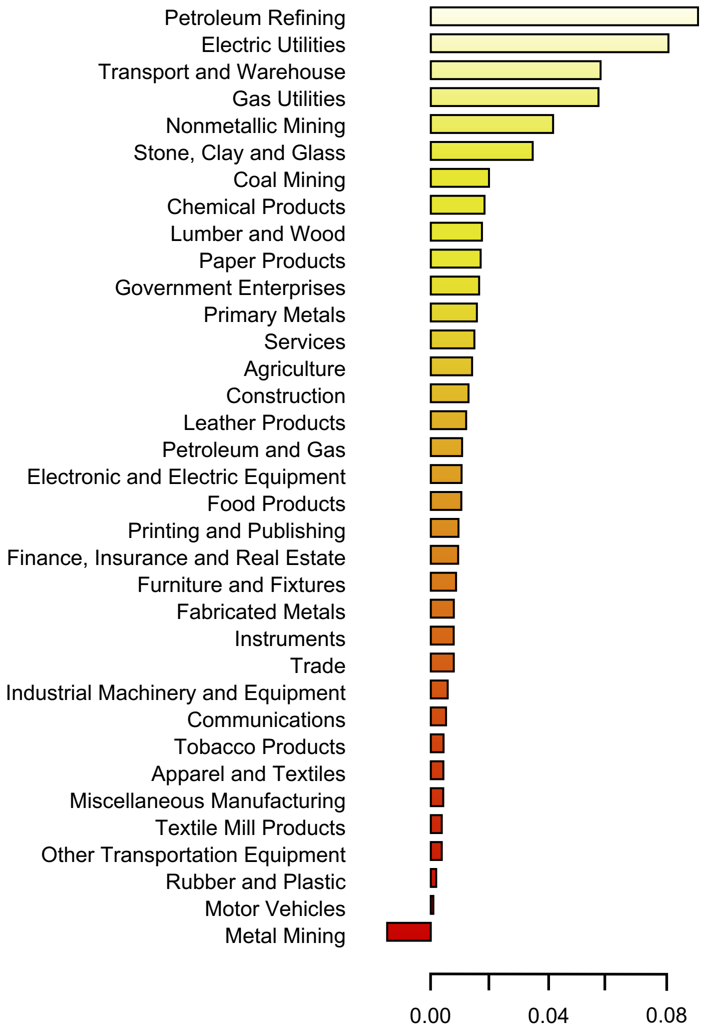

Figure 17.17 Bias of technical change for energy input: US 1960–1980.

We assume that household preferences can be represented by a translog indirect utility function that generates demand functions of rank three. Lewbel (2001) has proposed such an indirect utility function of the form:

$$(\ln V_k)^{-1} = \left[\alpha_0 + \ln\left(\frac{\boldsymbol{\rho_k}}{F_k}\right)'\boldsymbol{\alpha_P} + \frac{1}{2}\ln\left(\frac{\boldsymbol{\rho_k}}{F_k}\right)' B_{pp}\ln\left(\frac{\boldsymbol{\rho_k}}{F_k}\right) + \ln\left(\frac{\boldsymbol{\rho_k}}{F_k}\right)' B_{pA}\boldsymbol{A_k}\right]^{-1} - \ln\left(\frac{\boldsymbol{\rho_k}}{F_k}\right)'\boldsymbol{\gamma_p}, \tag{17.15}$$

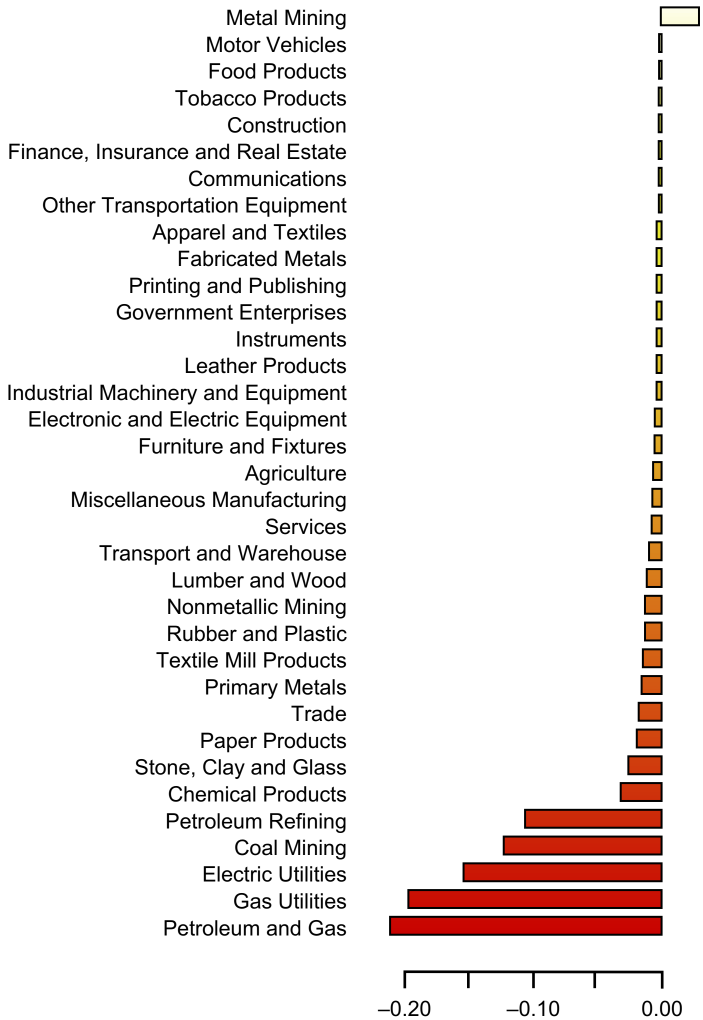

Figure 17.18 Bias of technical change for energy input: US 1980−2005.

where we assume $B_{pp} = B'_{pp}$, $\boldsymbol{i}'B_{pA} = 0$, $\boldsymbol{i}'B_{pp}\boldsymbol{i} = 0$, $\boldsymbol{i}'\boldsymbol{\alpha_p} = -1$ and $\boldsymbol{i}'\boldsymbol{\gamma_p} = 0$. To simplify the notation, we define $\ln G_k$ as:

$$\ln G_k = \alpha_0 + \ln\left(\frac{\boldsymbol{\rho_k}}{F_k}\right)'\boldsymbol{\alpha_P} + \frac{1}{2}\ln\left(\frac{\boldsymbol{\rho_k}}{F_k}\right)'B_{pp}\ln\left(\frac{\boldsymbol{\rho_k}}{F_k}\right) + \ln\left(\frac{\boldsymbol{\rho_k}}{F_k}\right)'B_{pA}\boldsymbol{A_k}. \tag{17.16}$$

Application of Roy's Identity to Equation (17.15) yields budget shares of the form:

$$\boldsymbol{w_k} = \frac{1}{D(\boldsymbol{\rho_k})}\left(\boldsymbol{\alpha_p} + B_{pp}\ln\frac{\boldsymbol{\rho_k}}{F_k} + B_{pA}\boldsymbol{A_k} + \boldsymbol{\gamma_p}[\ln G_k]^2\right), \tag{17.17}$$

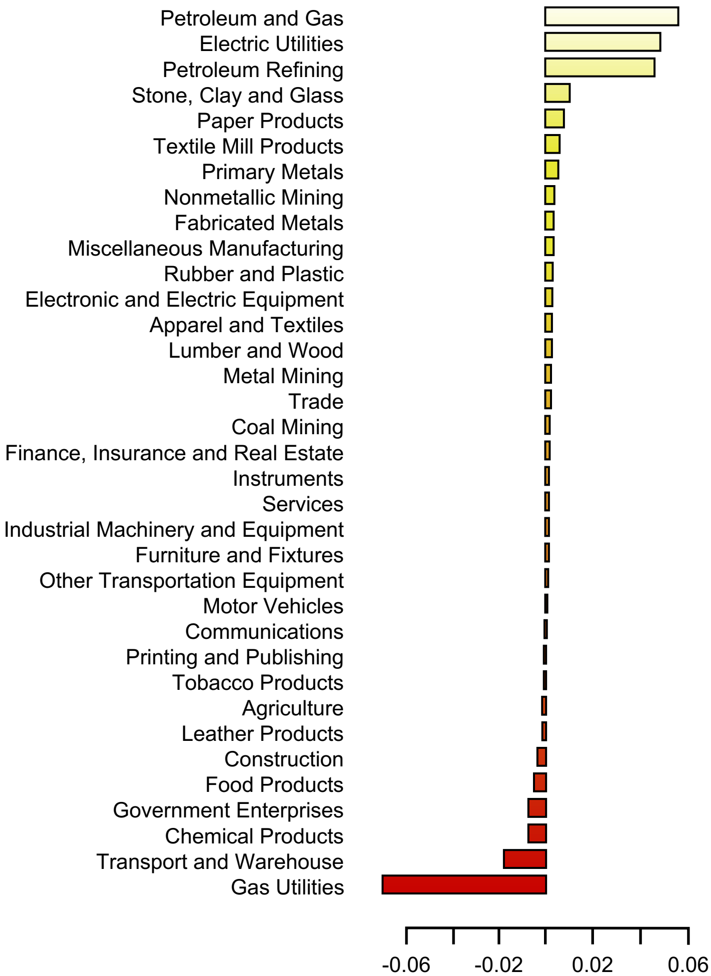

Figure 17.19 Projection of bias of technical change for energy input: US 2006–2030.

where:

$$D(\boldsymbol{\rho_k}) = -1 + \boldsymbol{i}' B_{pp} \ln \boldsymbol{\rho_k}.$$

With household demand functions of the form (17.17), the aggregate budget shares, denoted by the vector $\boldsymbol{w}$, can be represented explicitly as functions of prices and summary statistics of the joint distribution of full expenditure and household attributes:

$$\boldsymbol{w} = \frac{\sum_k F_k \boldsymbol{w_k}}{\sum_k F_k} = \frac{1}{D(\boldsymbol{\rho})}\left[\boldsymbol{\alpha_p} + B_{pp} \ln \boldsymbol{\rho} - \boldsymbol{i}' B_{pp} \frac{\sum F_k \ln F_k}{\sum F_k} + B_{pA} \frac{\sum F_k \boldsymbol{A_k}}{F_k} + \boldsymbol{\gamma_p} \frac{\sum F_k (\ln G_k)^2}{\sum F_k}\right].$$

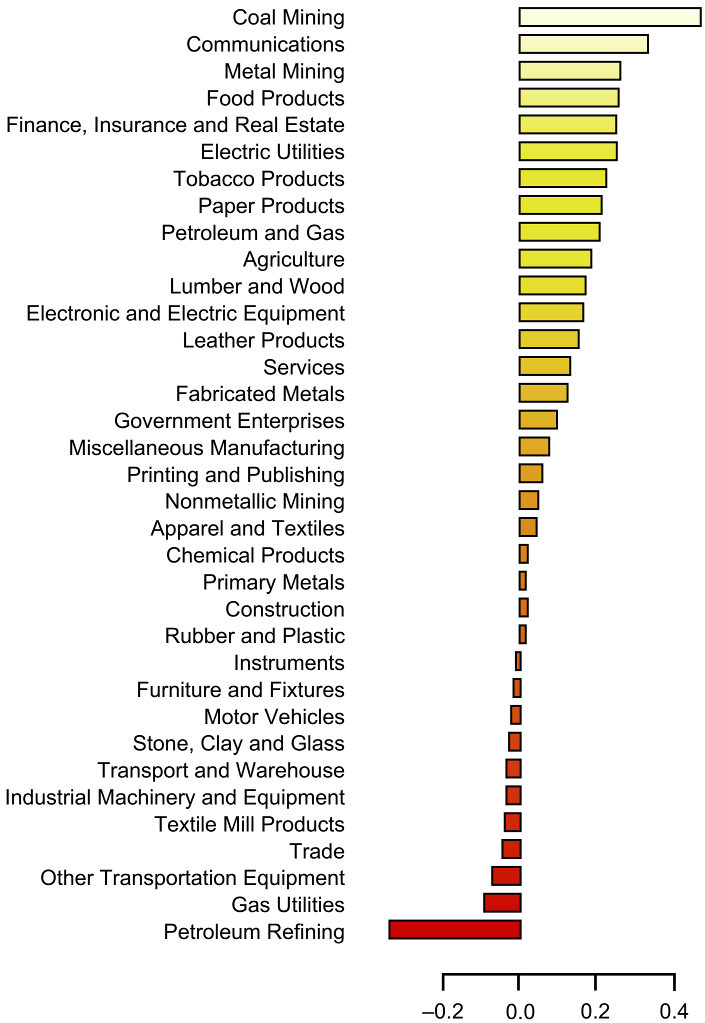

Figure 17.20 Projection of bias of technical change for capital input: US 2006–2030.

In the first stage of the household model, full expenditure F_{kt} is allocated across time periods so as to maximize lifetime utility U_k for household k:

$$\max_{F_{kt}} U_k = E_t\left\{\sum_{t=1}^{T}(1+\delta)^{-(t-1)}\left[\frac{V_{kt}^{(1-\sigma)}}{(1-\sigma)}\right]\right\}, \qquad (17.18)$$

subject to:

$$\sum_{t=1}^{T}(1+r_t)^{-(t-1)}F_{kt} \le W_k,$$

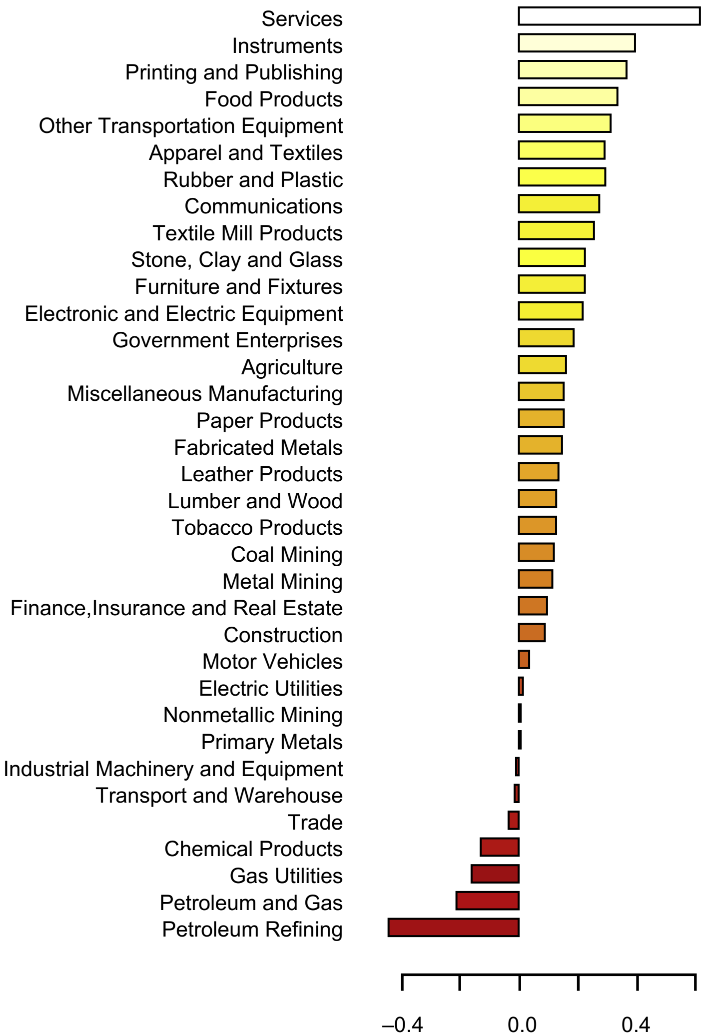

Figure 17.21 Projection of bias of technical change for labor input: US 2006–2030.

where r_t is the nominal interest rate, σ is an intertemporal curvature parameter, and δ is the subjective rate of time preference. We expect δ to be between zero and one and the within-period utility function is logarithmic if σ is equal to one.

The first-order conditions for this optimization yield Euler equations of the form:

$$(V_{kt})^{-\sigma}\left[\frac{\partial V_{kt}}{\partial F_{kt}}\right] = E_t\left[\left(V_{k,t+1}\right)^{-\sigma}\left[\frac{\partial V_{k,t+1}}{\partial F_{k,t+1}}\right]\frac{(1+r_{t+1})}{(1+\delta)}\right]. \tag{17.19}$$

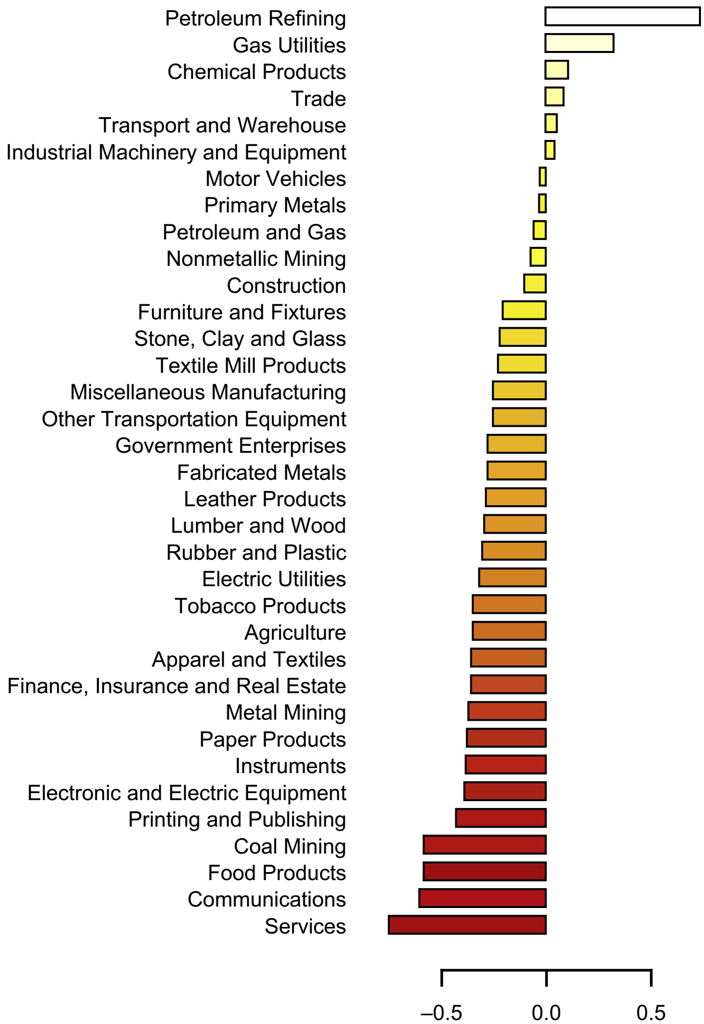

Figure 17.22 Projection of bias of technical change for material input: US 2006–2030.

If the random variable η_{kt} embodies expectational errors for household k at time t, Equation (17.19) becomes:

$$(V_{kt})^{-\sigma}\left[\frac{\partial V_{kt}}{\partial F_{kt}}\right] = \left[\left(V_{k,t+1}\right)^{-\sigma}\left[\frac{\partial V_{k,t+1}}{\partial F_{k,t+1}}\right]\frac{(1+r_{t+1})}{(1+\delta)}\right]\eta_{k,t+1}. \tag{17.20}$$

We can simplify this equation by noting that, for the rank-three specification of the indirect utility function given in Equation (17.15), we obtain:

$$\frac{\partial V_{kt}}{\partial F_{kt}} = \frac{V_{kt}}{F_{kt}}(-D(\boldsymbol{\rho_{kt}}))\left[1-\left(\boldsymbol{\gamma}'_{\boldsymbol{p}}\ln\boldsymbol{\rho_{kt}}\right)*G_{kt}\right]^{-2}.$$

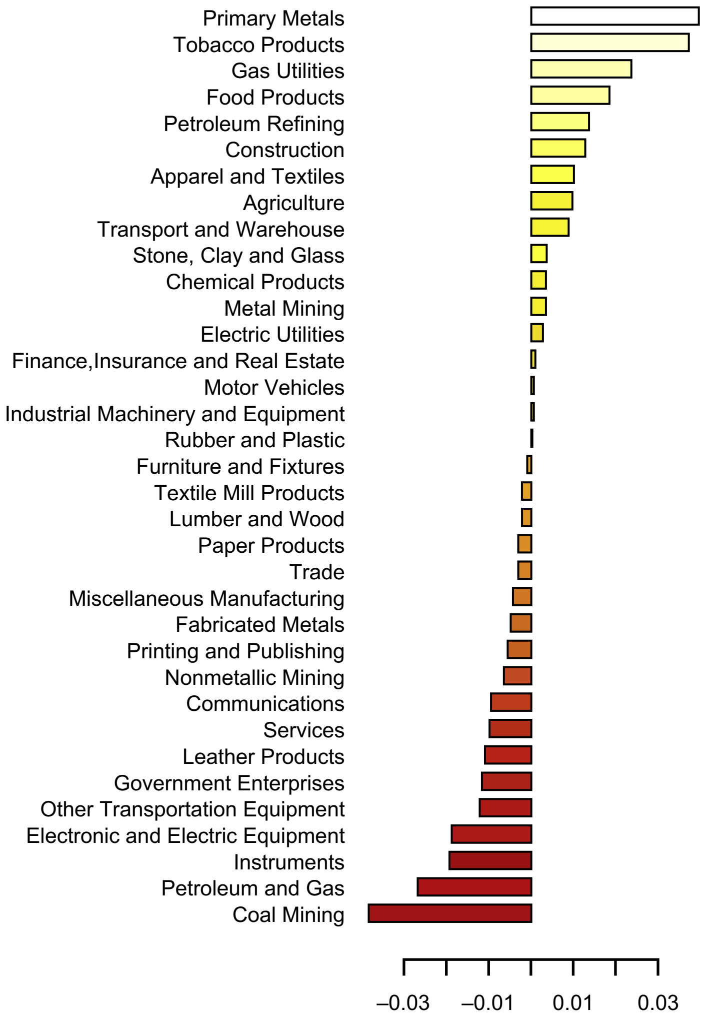

Figure 17.23 Projection rate of induced technical change: US 2006–2030.

The last term in the square bracket is approximately equal to one in the data, so that taking logs of both sides of Equation (17.20) yields:

$$\begin{aligned}\Delta \ln F_{k,t+1} &= (1-\sigma)\Delta \ln V_{k,t+1} + \Delta \ln\left(-D\left(\boldsymbol{\rho}_{\boldsymbol{k},\boldsymbol{t}+\mathbf{1}}\right)\right) \\ &+ \ln(1+r_{t+1}) - \ln(1+\delta) + \ln \eta_{kt},\end{aligned} \tag{17.21}$$

Equation (17.21) serves as the estimating equation for σ and the subjective rate of time preference δ.

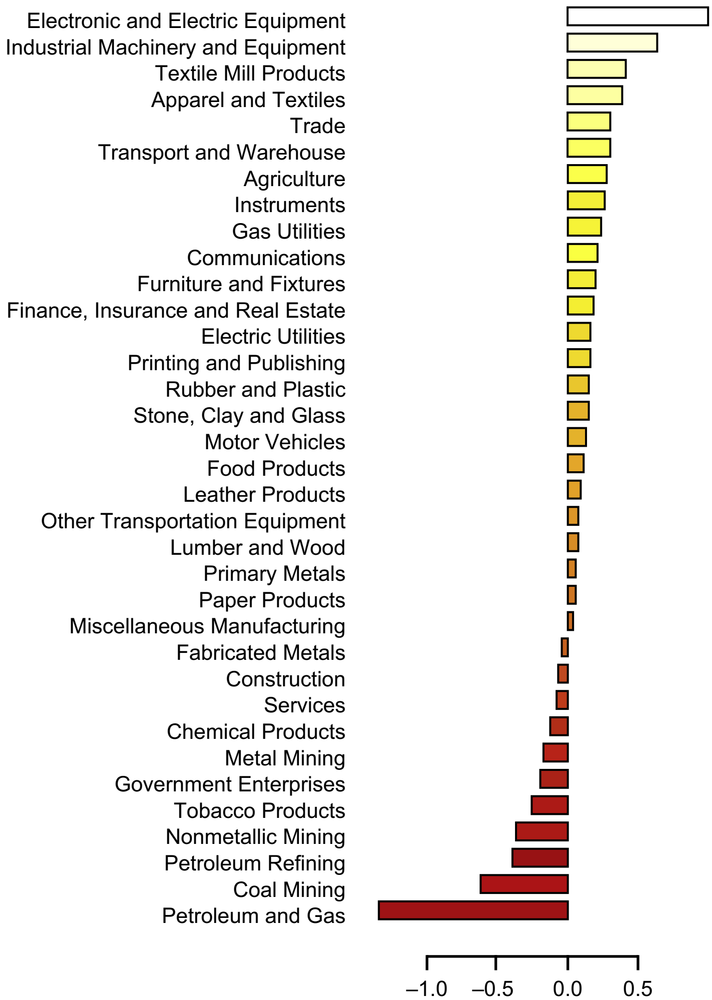

Figure 17.24 Projection rate of autonomous technical change: US 2006–2030.

17.7 DATA ISSUES IN MODELING CONSUMER BEHAVIOR

17.7.1 CEX

In the US, the only comprehensive sources of information on expenditure and labor supply are the CEX data published by the Bureau of Labor Statistics (BLS). These surveys are representative national samples that are conducted for the purpose of computing the weights in the CPI. The surveys were administered approximately every 10 years until 1980 when they were given every year. Detailed information on labor supply is provided only after 1980 and, as a result, we use the sample that covers the period from 1980 through 2006. Expenditures are recorded on a quarterly basis and our sample sizes range from between 4000 and 8000 households per quarter. To avoid issues related to the seasonality of expenditures, we use only the set of households that were interviewed in the second quarter of each year.[16]

In order to obtain a comprehensive measure of consumption, we modify the total expenditure variable reported in the surveys. We delete gifts and cash contributions as well as pensions, retirement contributions and Social Security payments. Outlays on owner occupied housing such as mortgage interest payments, insurance and the like are replaced with households' estimates of the rental equivalents of their homes. Durable purchases are replaced with estimates of the services received from the stocks of goods held by households.[17] After these adjustments, our estimate of total expenditure is the sum of spending on non-durables and services (a frequently used measure of consumption) plus the service flows from consumer durables and owner-occupied housing.

17.7.2 Measuring price levels

The CEX records the expenditures on hundreds of items, but provides no information on the prices paid, which makes it necessary to link the surveys with price data from alternative sources. While the BLS provides time series of price indexes from the CPI for different cities and regions, they do not publish information on price levels. Kokoski *et al.* (1994) use the 1988 and 1989 CPI database to estimate the prices of a variety of goods and services in 44 urban areas. We use their estimates of prices for rental housing, owner occupied housing, food at home, food away from home, alcohol and tobacco, household fuels (electricity and piped natural gas), gasoline and motor oil, household furnishings, apparel, new vehicles, professional medical services and entertainment.[18] Given price levels for 1988 and 1989, prices both before and after this period are extrapolated using price indexes published by the BLS. Most of these indexes cover the period from December 1977 to the present at either monthly or bimonthly frequencies depending on the year and the commodity group.[19]

The prices from the CPI are linked to the expenditure data in the CEX. Although Kokoski *et al.* (1994) provide price estimates for 44 urban areas across the US, the publicly available CEX data do not report households' cities of residence in an effort to preserve the confidentiality of survey participants. This necessitates

16 Surveys are designed to be representative only at a quarterly frequency. We use the second quarter to avoid seasonality of spending associated with the summer months and holiday spending at the end of the calendar year.

17 The methods used to compute the rental equivalent of owner-occupied housing and the service flows from consumer durables are described in Slesnick (2001).

18 In 1988 and 1989 these items constituted approximately 75% of all expenditures.

19 A detailed description of this procedure can be found in Slesnick (2002).

aggregation across urban areas to obtain prices for the four major Census regions: the Northeast, Midwest, South and West. As the BLS does not collect non-urban price information, rural households are assumed to face the prices of Class D-sized urban areas.[20]

17.7.3 Measuring wages in efficiency units

The primitive observation unit in the CEX is a "consumer unit" and expenditures are aggregated over all members. We model labor supply at the same level of aggregation by assuming that male and female leisure are perfect substitutes when measured in quality-adjusted units. The price of leisure (per efficiency unit) is estimated using a wage equation defined over "full-time" workers, i.e. those who work more than 40 weeks per year and at least 30 hours per week. The wage equation for worker i is given by:

$$\ln P_{Li} = \sum_j \beta_j^z z_{ji} + \sum_j \beta_j^s \left(S_i^* z_{ji}\right) + \sum_j \beta_j^{nw} \left(NW_i^* z_{ji}\right) + \sum_l \beta_l^g g_{li} + \varepsilon_{it}, \quad (17.22)$$

where P_{Li} is the wage of worker i, $\mathbf{z}_i$ is a vector of demographic characteristics that includes the age, age squared, years of education and years of education squared of worker i, S_i is a dummy variable indicating whether the worker is female, NW_i is a dummy variable indicating whether the worker is nonwhite, and $\mathbf{g}_i$ is a vector of region-year dummy variables.

The wage equation is estimated using the CEX from 1980 through 2006 using the usual sample selection correction, and the quality-adjusted wage for a worker in region-year s is given by $p_L^s = \exp(\hat{\beta}_s^g)$. The parameter estimates (excluding the region-year effects) are presented in Table A5 in the Appendix.

In Figure 17.25(a) we present our estimates of quality-adjusted hourly wages in the urban Northeast, Midwest, South and West as well as rural areas from 1980 through 2006. The reference worker, whose quality is normalized to one, is a white male, age 40, with 13 years of education. The levels and trends of the wages generally accord with expectations; the highest wages are in the Northeast and the West and the lowest are in rural areas. Perhaps most surprising is the finding that real wages, shown in Figure 17.25(b), have decreased over the sample period and exhibit substantially less variation across regions. This suggests that more accurate adjustments for differences in the cost of living across geographical regions reduce the between-region wage dispersion to a large degree.

[20] These areas correspond to nonmetropolitan urban areas and are cities with less than 50 000 persons. Examples of cities of this size include Yuma, Arizona in the West, Fort Dodge, Iowa in the Midwest, Augusta, Maine in the Northeast and Cleveland, Tennessee in the South.

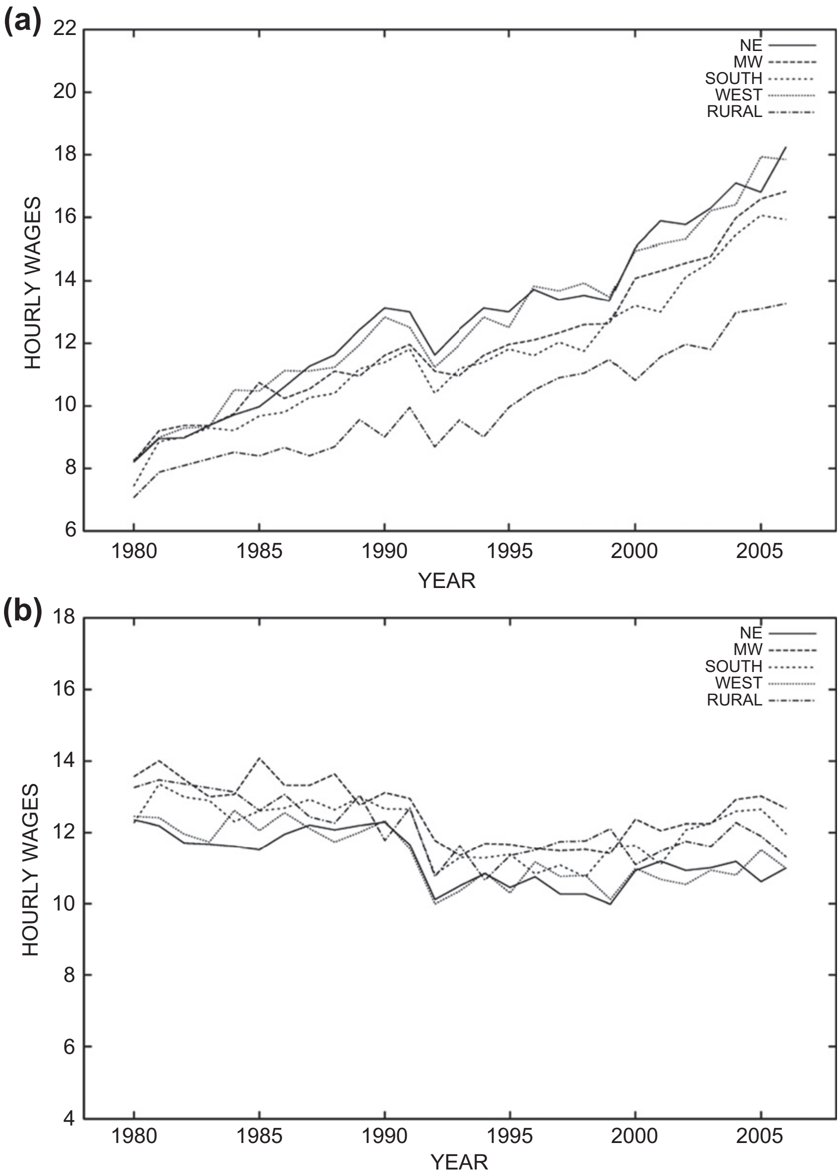

Figure 17.25 (a) Regional wages (current dollars). (b) Regional real wages (NE 1989 dollars).

17.7.4 Measuring quality-adjusted household leisure

For workers, estimates of the quantity of leisure consumed are easily obtained. The earnings of individual m in household k at time t are:

$$E_{kt}^{m} = p_{Lt} q_{kt}^{m} H_{kt}^{m},$$

where p_{Lt} is the wage at time t per efficiency unit, q_{kt}^m is the quality index of the worker and H_{kt}^m is the observed hours of work. With observations on wages and the hours of work, the quality index for worker m is:

$$q_{kt}^m = \frac{E_{kt}^m}{p_{Lt} H_{kt}^m}.$$

If the daily time endowment is 14 hours, the household's time endowment measured in efficiency units is $T_{kt}^m = q_{kt}^m * (17.14)$ and leisure consumption is $R_{kt}^m = q_{kt}^m (14 - H_{kt}^m)$.

For nonworkers, we impute a nominal wage for individual m in household$k$, $\hat{p}_{Lkt}^m$, using the fitted values of a wage equation similar to Equation (17.22). The quality adjustment for non-workers is:

$$\hat{q}_{kt}^m = \frac{\hat{p}_{Lkt}^m}{p_{Lt}},$$

and the individual's leisure consumption is calculated as $R_{kt}^m = \hat{q}_{kt}^m * (17.14)$. Given estimates of leisure for each adult in the household, full expenditure for household k is computed as:

$$F_{kt} = p_{Lt} R_{kt} + \sum_i p_{ik} x_{ik},$$

where:

$$R_{kt} = \sum_m R_{kt}^m,$$

is total household leisure computed as the sum over all adult members.

In Figure 17.26(a) we present tabulations of *per capita* full consumption (goods and household leisure) as well as *per capita* consumption (goods only). For both series, expenditures are deflated by price and wage indexes that vary overtime and across regions. Over the period from 1980 through 2006, *per capita* consumption grew at an average annual rate of 1.1% per year compared to 1.0% per year for *per capita* full consumption. Figure 17.26(b) shows the average level of quality-adjusted leisure consumed per adult. The average annual hours increased by approximately 18% over the 26 years from 2656 in 1980 to 3177 in 2006. Figure 17.26(c) shows that the inclusion of household leisure has the effect of lowering the dispersion in consumption in each year. The variance of log *per capita* full consumption is approximately 25% lower than the variance of log *per capita* consumption. The trends of the two series, however, are similar.

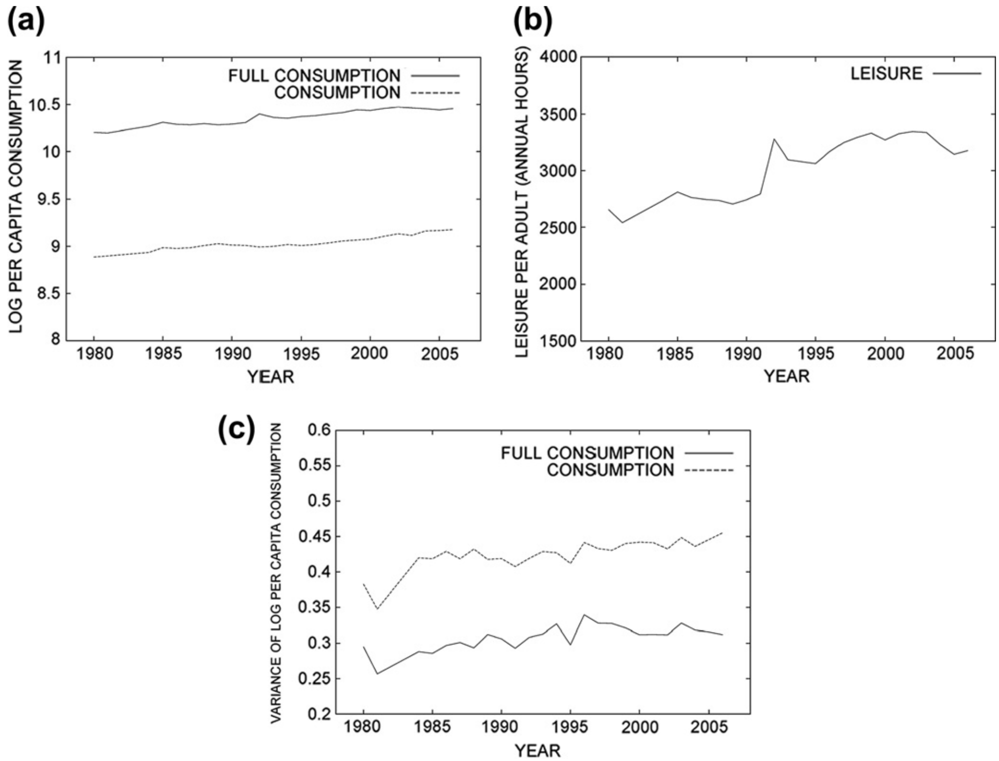

Figure 17.26 (a) Log consumption *per capita* (constant dollars). (b) Quality-adjusted leisure for adults. (c) Variance of log *per capita* consumption.

17.8 AGGREGATE DEMANDS FOR GOODS AND LEISURE

We estimate the parameters of the second stage model using a demand system defined over four commodity groups:

- *Non-durables*: Energy, food, clothing and other consumer goods.
- *Consumer Services:* Medical care, transportation, entertainment, and so on.
- *Capital Services:* Services from rental housing, owner-occupied housing and consumer durables.
- *Household Leisure:* Sum of quality-adjusted leisure over all of the adult members of the household.

The demographic characteristics that are used to control for heterogeneity in household behavior include:

- *Number of adults:* A quadratic in the number of individuals in the household who are age 18 or older.
- *Number of children:* A quadratic in the number of individuals in the household who are under the age of 18.

- *Gender of the household head:* Male, female.
- *Race of the household head:* White, nonwhite.
- *Region of residence:* Northeast, Midwest, South and West.
- *Type of residence:* Urban, rural.

In Table 17.2 we present summary statistics of the variables used in the estimation of the demand system. On average, household leisure comprises almost 70% of full expenditure although the dispersion is greater than for the other commodity groups. As expected, the price of capital (which includes housing) shows substantial variation in the sample as does the price of consumer services. The average number of adults is 1.9 and the average number of children is 0.7. Female headed households account for over 28% of the sample and almost 16% of all households have nonwhite heads.

We model the within-period allocation of expenditures across the four commodity groups using the rank-extended translog model defined in Equation (17.17). We assume that the disturbances of the demand equations are additive so that the system of estimating equations is:

$$\mathbf{w}_k = \frac{1}{D(\boldsymbol{\rho_k})}\left(\boldsymbol{\alpha_p} + B_{pp}\ln\frac{\boldsymbol{\rho_k}}{F_k} + B_{pA}\mathbf{A}_k + \boldsymbol{\gamma_p}[\ln G_k]^2\right) + \boldsymbol{\varepsilon_k},$$

Table 17.2 Summary statistics (sample size = 154 180)

Variable	Mean	Standard error	Minimum	Maximum
Share NON	0.101	0.052	0.0009	0.695
Share CAP	0.133	0.076	0.0001	0.895
Share CS	0.072	0.054	0.00004	0.787
Share LELS	0.694	0.123	0.0001	0.991
Log PNON	0.116	0.212	−0.510	0.877
Log PCAP	−0.090	0.280	−1.101	0.526
Log PCS	0.144	0.333	−0.828	0.702
Log wage	−0.304	0.234	−0.933	0.137
Log full expenditure	11.547	0.605	8.241	15.281
No. children	0.717	1.121	0.000	12.000
No. adults	1.887	0.841	1.000	13.000
White dummy	0.844	0.363	0.000	1.000
Nonwhite dummy	0.156	0.363	0.000	1.000
Male dummy	0.715	0.451	0.000	1.000
Female dummy	0.285	0.451	0.000	1.000
Urban dummy	0.905	0.293	0.000	1.000
Rural dummy	0.095	0.293	0.000	1.000
Northeast dummy	0.198	0.398	0.000	1.000
Midwest dummy	0.249	0.433	0.000	1.000
South dummy	0.311	0.463	0.000	1.000
West dummy	0.242	0.428	0.000	1.000

where the vector ε_k is assumed to be mean zero with variance-covariance matrix $\boldsymbol{\Sigma}$. We compare these results to those obtained using the rank-two translog demand system originally developed by Jorgenson *et al.* (1997):

$$\mathbf{w}_k = \frac{1}{D(\boldsymbol{\rho_k})}\left(\boldsymbol{\alpha_p} + B_{pp}\ln\frac{\boldsymbol{\rho_{kk}}}{F_k} + B_{pA}\mathbf{A}_k\right) + \boldsymbol{\mu_k}.$$

Note that the two specifications coincide if the elements of the vector $\boldsymbol{\gamma_p}$ are equal to zero.

Both the rank-two and rank-three demand systems are estimated using nonlinear full information maximum likelihood with leisure as the omitted equation of the singular system. The parameter estimates of both models are presented in Table 17.A5 and 17.A6 in the Appendix. The level of precision of the two sets of estimates is high as would be expected given the large number of observations. Less expected is the fact that the rank-two and rank-three estimates are similar for all variables other than full expenditure. Note, however, that the parameters $\boldsymbol{\gamma_p}$ are statistically significant and that any formal test would strongly reject the rank-two model in favor of the rank-three specification (i.e. the likelihood ratio test statistic is over 978).

In Table 17.3 we compute price and income elasticities for the three consumption goods and leisure. In all cases the elasticities are calculated for a particular type of household: two adults and two children, living in the urban Northeast, with a male, white head of the household with \$100,000 of full expenditure in 1989. Both non-durables and consumer services are price inelastic, while capital services have elasticities exceeding unity. The own compensated price elasticities are negative for all goods and the differences between the rank-two and rank-three models are small. The uncompensated wage elasticity of household labor supply is negative but close to zero while the expenditure elasticity is quite high. The compensated wage elasticity is around 0.70 and, as with the consumption goods, the differences between the two types of demand systems are small.[21]

If the rank-two and rank-three models are to differ, they most likely differ in terms of their predicted effects of full expenditure on demand patterns. To assess this possibility, we present the fitted shares from both systems at different levels of full expenditure for the reference household in Table 17.4. The predicted shares from the two models are similar for levels of full expenditure in the range between \$25,000 and \$150,000. They diverge quite sharply, however, in both the upper and lower tails of the expenditure distribution. For example, when full expenditure is \$7500, the share of non-durables in the rank-two model is 0.227 compared with 0.268 for the rank-three model. At high

21 In the calculation of the wage elasticities, unearned income is assumed to be zero the value of the time endowment is equal to full expenditure.

Table 17.3 Price and income elasticities

Good	Uncompensated price elasticity		Compensated price elasticity		Full expenditure elasticity	
	Rank 2	Rank 3	Rank 2	Rank 3	Rank 2	Rank 3
Non-durables	−0.918	−0.903	−0.822	−0.809	0.722	0.724
Capital services	−1.428	−1.432	−1.314	−1.319	0.926	0.930
Consumer services	−0.613	−0.614	−0.548	−0.548	1.088	1.096
Leisure	0.012	0.014	−0.323	−0.314	1.059	1.056
Labor supply	−0.026	−0.030	0.698	0.698	−2.289	−2.342

Table 17.4 Fitted budget shares

Expenditure level ($)	Non-durables share		Capital services share		Consumer services share		Leisure share	
	Rank 2	Rank 3	Rank 2	Rank 3	Rank 2	Rank 3	Rank 2	Rank 3
7500	0.227	0.268	0.147	0.183	0.047	0.073	0.579	0.476
25000	0.183	0.192	0.136	0.145	0.053	0.060	0.627	0.603
75000	0.143	0.140	0.126	0.125	0.059	0.058	0.672	0.677
150000	0.117	0.116	0.120	0.119	0.063	0.062	0.700	0.702
275000	0.095	0.100	0.114	0.119	0.066	0.070	0.725	0.711
350000	0.086	0.095	0.112	0.120	0.067	0.073	0.734	0.711

levels of full expenditure ($350,000) the fitted share of household leisure is 0.734 in the rank-two model and 0.711 in the rank-three model.

Both the rank-two and rank-three demand systems are consistent with exact aggregation and can provide closed-form representations of aggregate demands for the four goods:

$$\boldsymbol{w} = \frac{\sum_k F_k \boldsymbol{w_k}}{\sum_k F_k}$$

$$= P_t + Y_t + D_t,$$

where P_t, Y_t and D_t are summary statistics similar to the aggregation factors described by Jorgenson *et al.* (1997). Specifically, the price factor is the full expenditure weighted average of the price terms in the share equations in each time period:

$$P_t = \frac{\sum_k F_{kt} D(\boldsymbol{\rho_{kt}})^{-1}\left(\boldsymbol{\alpha_p} + B_{pp}\ln \boldsymbol{\rho_{kt}}\right)}{\sum_k F_{kt}},$$

and Y_t and D_t are defined similarly for the full expenditure and demographic components of the aggregate demand system:

$$Y_t = \frac{\sum_k F_{kt} D(\boldsymbol{\rho_{kt}})^{-1}\left(\boldsymbol{\gamma_p}(\ln G_{kt})^2 - \boldsymbol{i}' B_{pp} \ln F_{kt}\right)}{\sum_k F_{kt}}$$

$$D_t = \frac{\sum_k F_{kt} D(\boldsymbol{\rho_{kt}})^{-1}\left(B_{pA}\boldsymbol{A_{kt}}\right)}{\sum_k F_{kt}}.$$

How well do the fitted demands reflect aggregate expenditure patterns and their movements over time? In Table 17.5 we compare the fitted aggregate shares for the rank-three system with sample averages tabulated for each of the four commodity groups. The rank-three demand system provides an accurate representation of both the levels and movements of the aggregate budget shares overtime. With few exceptions, the fitted shares track the sample averages closely in terms of both the absolute and relative differences. Table 17.5 also reports the R^2 statistic to assess the normalized within-sample performance of the predicted household-level budget shares. At this level of disaggregation, the non-durables and leisure demand equations fit better than the other two commodity groups in most years.

The aggregation factors show that essentially all of the movement in the aggregate shares was the result of changes in prices and full expenditure; the demographic factors showed very little movement over time for any of the four commodity groups. This is especially true of leisure where the effects of prices and full expenditure on the aggregate shares changed significantly (in opposite directions), while the influence of demographic variables showed little temporal variation.

As a final assessment of our within-period demand model, we examine the statistical fit of the leisure demand equations for subgroups of the population for whom our model might perform poorly. Recall that in order to develop a model of aggregate labor supply, we have made the simplifying assumption that quality-adjusted male and female leisure are perfect substitutes within the household. If this turns out to be overly strong, we might expect the demand system to predict less well for groups for which this assumption is likely to be counterfactual.

In Table 17.6 we compare the aggregate leisure demands of households with at least two adults. It seems reasonable to expect that the presence of children almost certainly complicates the labor supply decisions of adults and, given that we do not explicitly model this interaction, our model might not fit the data as well for this subgroup as for others. Instead, we find that for both types of households, the fitted aggregate demands

Table 17.5 Aggregate budget shares

Year	Sample shares	Fitted shares	R^2	Aggregation factors: Price	Expenditure	Demographics
Non-durables						
1980−1981	0.1145	0.1074	0.1273	0.3985	−0.3009	0.0098
1985−1986	0.0993	0.1003	0.1609	0.4000	−0.3090	0.0084
1990−1991	0.0967	0.0990	0.1793	0.4051	−0.3141	0.0080
1995−1996	0.0898	0.0892	0.2198	0.3996	−0.3181	0.0077
2000−2001	0.0846	0.0852	0.1910	0.4011	−0.3235	0.0076
2005−2006	0.0864	0.0845	0.1806	0.4055	−0.3279	0.0068
Capital services						
1980−1981	0.0956	0.1162	0.0296	0.2100	−0.0141	−0.0797
1985−1986	0.1134	0.1178	0.1003	0.2103	−0.0143	−0.0782
1990−1991	0.1186	0.1213	0.1292	0.2132	−0.0145	−0.0774
1995−1996	0.1222	0.1240	0.1161	0.2161	−0.0150	−0.0771
2000−2001	0.1306	0.1272	0.1226	0.2193	−0.0154	−0.0766
2005−2006	0.1403	0.1344	0.1134	0.2255	−0.0155	−0.0756
Customer services						
1980−1981	0.0566	0.0561	0.0018	−0.0439	0.1202	−0.0202
1985−1986	0.0626	0.0668	0.0111	−0.0370	0.1236	−0.0199
1990−1991	0.0706	0.0678	0.0317	−0.0379	0.1258	−0.0201
1995−1996	0.0734	0.0750	0.0318	−0.0326	0.1270	−0.0193
2000−2001	0.0724	0.0747	0.0420	−0.0350	0.1289	−0.0192
2005−2006	0.0748	0.0678	0.0245	−0.0434	0.1308	−0.0195
Leisure						
1980−1981	0.7333	0.7203	0.1506	0.4354	0.1948	0.0902
1985−1986	0.7247	0.7151	0.1532	0.4257	0.1997	0.0897
1990−1991	0.7141	0.7119	0.1804	0.4197	0.2028	0.0895
1995−1996	0.7146	0.7117	0.1791	0.4170	0.2060	0.0887
2000−2001	0.7124	0.7129	0.1758	0.4147	0.2100	0.0882
2005−2006	0.6985	0.7133	0.1458	0.4124	0.2126	0.0883

Table 17.6 Group budget shares for leisure

	At least one child			No children		
Year	Sample share	Fitted share	R^2	Sample share	Fitted share	R^2
1980−1981	0.7347	0.7236	0.1144	0.7522	0.7412	0.0277
1985−1986	0.7221	0.7167	0.0774	0.7467	0.7391	0.0171
1990−1991	0.7171	0.7136	0.0730	0.7342	0.7366	0.0811
1995−1996	0.7161	0.7138	0.0799	0.7364	0.7343	0.0612
2000−2001	0.7115	0.7158	0.0633	0.7405	0.7358	0.0528
2005−2006	0.7029	0.7161	0.0421	0.7200	0.7360	0.0349

for leisure are quite close to the sample averages for the subgroups. Moreover, the R^2 computed for households with children is actually higher than that computed for those without.

17.9 INTERTEMPORAL ALLOCATION OF FULL CONSUMPTION

In this section we describe the intertemporal allocation of full consumption. Equation (17.21) serves as the basis for the estimation of the curvature parameter σ and the subjective rate of time preference δ. However, because we do not have longitudinal data on full consumption, we create synthetic panels from the CEX as described by Blundell *et al.* (1994) and Attanasio and Weber (1995). The estimating equation for this stage of the consumer model is:

$$\begin{aligned}\Delta \ln F_{c,t+1} &= (1-\sigma)\Delta \ln V_{c,t+1} + \Delta \ln\left(-D\left(\boldsymbol{\rho}_{\boldsymbol{c},\boldsymbol{t}+\mathbf{1}}\right)\right) \\ &\quad + \ln(1+r_{t+1}) - \ln(1+\delta) + v_{ct},\end{aligned} \tag{17.23}$$

where:

$$\Delta \ln F_{c,t+1} = \frac{\sum_{k\epsilon c} \ln F_{k,t+1}}{n_{c,t+1}} - \frac{\sum_{k\epsilon c} \ln F_{k,t}}{n_{c,t}}$$

$$\Delta \ln V_{c,t+1} = \frac{\sum_{k\epsilon c} \ln V_{k,t+1}}{n_{c,t+1}} - \frac{\sum_{k\epsilon c} \ln V_{k,t}}{n_{c,t}}$$

$$\Delta \ln\left(-D\left(\boldsymbol{\rho}_{c,t+1}\right)\right) = \frac{\sum_{k\epsilon c}\left(-D(\boldsymbol{\rho}_{k,t+1})\right)}{n_{c,t+1}} - \frac{\sum_{k\epsilon c}\left(-D(\boldsymbol{\rho}_{k,t})\right)}{n_{c,t}}$$

where the summations are over all households in cohort c at time t.

To create the cohorts, we partition the sample of households in the CEX into birth cohorts defined over five year age bands on the basis of the age of the head of the household. In 1982 and 1983 the BLS did not include rural households in the survey and, to maintain continuity in our sample, we use data from 1984 through 2006. The characteristics of the resulting panel are described in Table 17.7. The oldest cohort was born between 1900 and 1904 and the youngest cohort was born between 1980 and 1984. The cell sizes for most of the cohorts were typically several hundred households, although the range is substantial.

The age profiles of full consumption *per capita*, consumption *per capita*, and household leisure *per capita* are presented in Figure 17.27(a–c) for the cohorts in the sample. Not surprisingly, the profile of *per capita* full consumption is largely determined by the age

Table 17.7 Characteristics of cohorts

Cohort	Cohort birth year	Average no. of observations	Range of no. of observations	Years covered
1	1900–1904	108	52–169	1980–1989
2	1905–1909	158	78–229	1980–1994
3	1910–1914	195	92–305	1980–2000
4	1915–1919	261	176–347	1980–2000
5	1920–1924	284	53–415	1980–2005
6	1925–1929	337	234–417	1980–2006
7	1930–1934	337	272–469	1980–2006
8	1935–1939	354	289–446	1980–2006
9	1940–1944	437	341–554	1980–2006
10	1945–1949	546	432–705	1980–2006
11	1950–1954	622	457–817	1980–2006
12	1955–1959	650	382–910	1980–2006
13	1960–1964	580	120–870	1980–2006
14	1965–1969	484	103–768	1985–2006
15	1970–1974	464	83–742	1990–2006
16	1975–1979	397	71–594	1995–2006
17	1980–1984	331	45–473	2000–2006

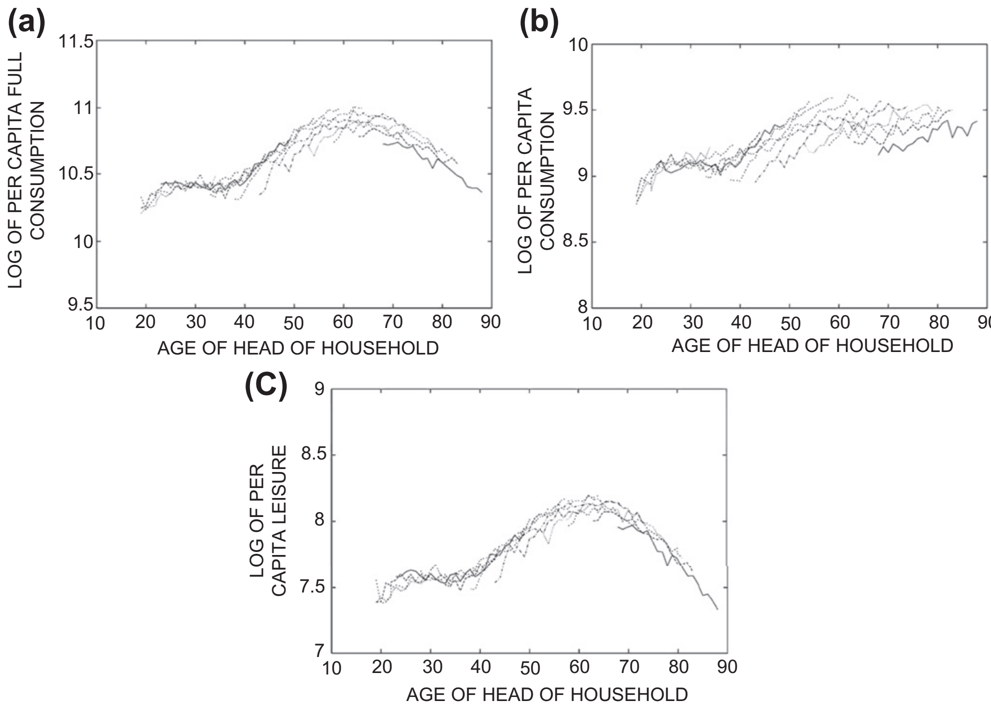

Figure 17.27 Age profile of (a) *per capita* full consumption, (b) *per capita* consumption and (c) *per capita* leisure.

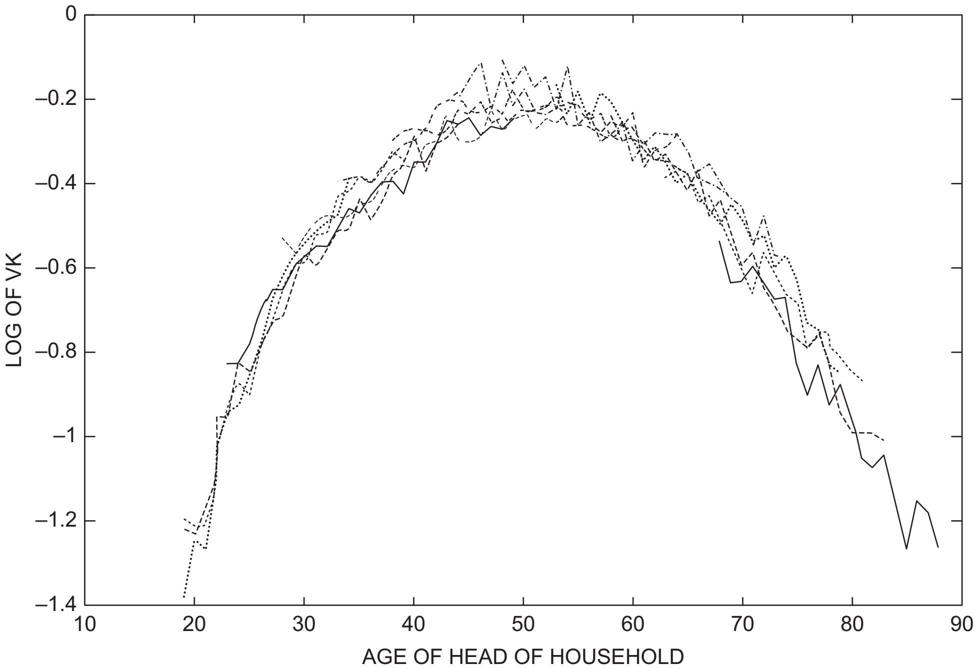

Figure 17.28 Age profile of the average within period utility levels ($\ln V_k$).

profile of household leisure. *Per capita* full expenditure remains relatively constant until age 35, increases until age 60 and then decreases. Figure 17.28 shows the age profile of the average within period utility levels ($\ln V_k$), which plays a critical role in the estimation of Equation (17.23).

The statistical properties of the disturbances v_{ct} in Equation (17.23) that are used with synthetic panels are described in detail by Attanasio and Weber (1995). They note that the error term is the sum of expectational error as well as measurement error associated with the use of averages tabulated for each cohort. We present estimates of δ and σ using ordinary least squares, least squares weighted by the cell sizes of each cohort in each year, and a random effects estimator that exploits the panel features of our synthetic cohort data. The first panel in Table 17.8 shows that estimates of δ are consistently around 0.015 while the estimates of σ are approximately 0.1.

We re-estimate Equation (17.23) using a variety of instruments to account for expectational and measurement error associated with synthetic cohorts. The results shown in the second panel of Table 17.8 are based on different sets of instruments. The first estimator (IV1) uses a constant, the average age of the cohort, a time trend, and the two period lagged average marginal tax rate on earnings as instruments. The second estimator (IV2) uses, in addition, the two period lags of wages, interest rates, and prices of capital services and consumer services. The third estimator (IV3) also includes the third period lags. Regardless

Table 17.8 Parameter estimates

Least squares estimates

	OLS		**Weighted OLS**		**Random effects**	
Variable	**Estimate**	**SE**	**Estimate**	**SE**	**Estimate**	**SE**
δ	0.01471	0.0011	0.01185	0.0011	0.01460	0.0016
σ	0.08226	0.0194	0.11280	0.0218	0.10183	0.0202

Instrumental variables estimators

	IV1		**IV2**		**IV3**	
Variable	**Estimate**	**SE**	**Estimate**	**SE**	**Estimate**	**SE**
δ	0.01253	0.0012	0.01251	0.0012	0.01249	0.0011
σ	0.03414	0.0357	0.05521	0.0350	0.08150	0.0337

SE = Standard Error

of the instrument set, the point estimates of the subjective rate of time preference remains around 0.0125 while the estimates of σ are in the range between 0.0341 and 0.0815.

17.10 COMPUTING CONFIDENCE INTERVALS

In this section we present a fast, highly-scalable method for computing confidence intervals for results from general equilibrium models. We follow the delta-method-based approach used by Tuladhar and Wilcoxen (1999) and Tuladhar (2003) but apply it to IGEM, a much larger model with far more parameters and equations. We compute confidence intervals for model results, compare them to the results from a Monte Carlo simulation, and then show how the method can be used to carry out very fine-grained analysis of the underlying sources of uncertainty in any given result.

17.10.1 Methodology

Let Y be a vector of n_Y endogenous variables that depends on a vector of n_X exogenous variables X and a vector of n_β parameters β via vector function F:

$$Y = F(X, \beta).$$

Suppose β has been estimated by adding a vector of n_ε stochastic disturbances ε to the system of equations and running a regression. Let the vector of estimated parameters be $\hat{\beta}$. The vector of prediction errors e will be given by the following expression:

$$e = Y - \hat{Y} = F(X, \beta, \varepsilon) - F(X, \hat{\beta}, 0).$$

In general, the function F will be nonlinear. However, it can be approximated using a first-order Taylor Series expansion. Expanding F, where J_β and J_ε denote the Jacobian matrices of partial derivatives of F with respect to the elements of β and ε:

$$F(X, \hat{\beta}, 0) \approx F(X, \beta, \varepsilon) + J_\beta(\hat{\beta} - \beta) - J_\varepsilon \varepsilon.$$

Matrices J_β and J_ε will be $n_Y \times n_\beta$ and $n_Y \times n_\epsilon$, respectively. The vector of prediction errors can thus be approximated as follows:

$$e \approx -J_\beta(\hat{\beta} - \beta) + J_\varepsilon \varepsilon.$$

Taking the expectation of e:

$$E(e) \approx -J_\beta E(\beta - \hat{\beta}) + J_\varepsilon E(\varepsilon) = 0.$$

If the $n_\beta \times n_\beta$ covariance matrix for $\hat{\beta}$ is Σ_β and the $n_\varepsilon \times n_\varepsilon$ covariance matrix of ε is Σ_ϵ, the $n_Y \times n_Y$ covariance of e, Σ_Y, can be written:

$$\Sigma_Y \approx J_\beta \Sigma_\beta J_\beta{}' + J_\varepsilon \Sigma_\varepsilon J_\varepsilon{}'.$$

This approach to approximating a variance-covariance matrix is known as the Delta Method.[22]

The first term on the right-hand side is the covariance matrix for the vector of predicted means of the endogenous variables, which can be denoted by $\Sigma_{\bar{Y}}$:

$$\Sigma_{\bar{Y}} = J_\beta \Sigma_\beta J_\beta{}'.$$

The diagonal elements of $\Sigma_{\bar{Y}}$ will be the variances of the mean values of the model's endogenous variables and can be used to compute confidence intervals. Computing $\Sigma_{\bar{Y}}$ itself requires two matrices: Jacobian matrix J_β and parameter covariance matrix Σ_β. Obtaining the parameter covariance matrix is usually straightforward: it is generally computed in the course of estimating the parameters. The Jacobian matrix, however, must be computed by symbolic or numeric differentiation of the model.

17.10.2 Application to IGEM

In this section we illustrate how the method can be used by evaluating the sensitivity of IGEM's steady state to uncertainty in its household demand parameters.[23] The top tier of the household model has 48 independent parameters. A small subset is show in Table 17.9. The corresponding portion of the parameter covariance matrix Σ_β is shown in Table 17.10.

Numerical differentiation of IGEM with respect to the parameters produces the Jacobian matrix J_β. Table 17.11 shows selected columns of the Jacobian for a range of endogenous variables: consumption expenditure, gross domestic product, the capital stock, consumption of leisure, carbon emissions, and the values of output and prices for three industries: petroleum refining, electric utilities and services. For reference, the base case value of each variable is shown as well.

22 See Oehlert (1992) for more details.
23 A detailed discussion of IGEM is presented in Jorgenson, *et al.* (2012).

Table 17.9 Selected household parameters

Parameter	Value	Standard error
α_1^p	−0.5299	0.0028
α_2^p	−0.2559	0.0043
…	…	…
β_{12}^{pp}	−0.0237	0.0013
…	…	…
β_3^m	0.0051	0.0003

Table 17.10 Covariance matrix for selected household parameters

	α_1^p	α_2^p	…	β_{12}^{pp}	…	β_3^m
α_1^p	7.92E-06	1.30E-06	…	−3.47E-07	…	−1.54E-07
α_2^p	1.30E-06	1.87E-05	…	−4.38E-07	…	−6.03E-08
…	…	…	…	…	…	…
β_{12}^{pp}	−3.47E-07	−4.38E-07	…	1.58E-06		−3.78E-09
…	…	…	…	…	…	…
β_3^m	−1.54E-07	−6.03E-08	…	−3.78E-09	…	8.20E-08

Table 17.11 Jacobian matrix for selected variables and parameters

		Parameter name and value					
Variable	**Base case**	α_1^h	α_2^h	…	β_{12}^{pp}	…	β_3^m
Consumption	13,492	−42,338	−45,283	…	122,802	…	481,184
GDP	21,517	−76,228	−89,655	…	231,205	…	787,157
Capital	281,488	−692,303	−1,518,460	…	2,985,210	…	7,879,670
Leisure	30,088	40,930	33,094	…	−105,463	…	−489,414
Carbon	42,157	−282,492	−87,024	…	552,590	…	1,315,570
Value, refined	896	−5839	−1945	…	11,516	…	29,893
Value, elect	666	−4357	−1336	…	8526	…	17,967
Value, services	7875	−11,501	−13,331	…	14,231	…	530,444
Price, refined	0.701	−1.078	−0.458	…	2.356	…	1.686
Price, elect	0.226	−0.036	−0.021	…	0.088	…	−0.381
Price, services	0.415	−0.028	−0.006	…	0.049	…	−0.563

Matrices Σ_β and J_β allow the covariance matrix for the model as a whole, $\Sigma_{\overline{Y}}$, to be computed. A portion of $\Sigma_{\overline{Y}}$ is shown in Table 17.12.

The standard error of each variable is the square root of the corresponding diagonal element of the matrix in Table 17.12. These are shown in Table 17.13, along with each variable's base case. Also shown is the standard error expressed as a percentage of the base-case value.

Table 17.12 Selected elements from the endogenous variable covariance matrix

	Consumption	GDP	Capital	Leisure	Carbon	Value, refined	Value, elect	Value, services	Price, refined	Price, elect	Price, services
Consumption	519,630	958,068	12,761,500	−450,858	1,777,510	38,105	26,753	256,268	6.777	0.134	−0.044
GDP	958,068	1,774,430	23,954,700	−824,322	3,243,470	69,524	48,907	455,230	12.593	0.275	−0.052
Capital	12,761,500	23,954,700	339,708,000	−10,664,500	39,954,800	859,790	604,255	5,569,680	162.361	4.271	−0.132
Leisure	−450,858	−824,322	−10,664,500	397,622	−1,591,570	−34,086	−23,896	−235,300	−5.893	−0.098	0.057
Carbon	1,777,510	3,243,470	39,954,800	−1,591,570	7,214,770	152,809	109,210	847,685	27.597	0.604	0.030
Value, refined	38,105	69,524	859,790	−34,086	1,52,809	3240	2311	18,378	0.582	0.012	0.000
Value, elect	26,753	48,907	604,255	−23,896	1,09,210	2311	1655	12,459	0.421	0.010	0.001
Value, services	256,268	455,230	5,569,680	−235,300	847,685	18,378	12,459	174,150	2.637	−0.020	−0.115
Price, refined	6.777	12.593	162.361	−5.893	27.597	0.582	0.421	2.637	<0.001	<0.001	<0.001
Price, elect	0.134	0.275	4.271	−0.098	0.604	0.012	0.010	−0.020	<0.001	<0.001	<0.001
Price, services	−0.044	−0.052	−0.132	0.057	0.030	0.000	0.001	−0.115	<0.001	<0.001	<0.001

Table 17.13 Base case values and standard errors of selected variables

	Base case	Standard error	Percent of base
Consumption	13,492	721	5.3
GDP	21,517	1332	6.2
Capital	281,488	18,431	6.5
Leisure	30,088	631	2.1
Carbon	42,157	2686	6.4
Value, refined	896	57	6.3
Value, elect	666	41	6.1
Value, services	7875	417	5.3
Price, refined	0.7014	0.0106	1.5
Price, elect	0.2264	0.0004	0.2
Price, services	0.4149	0.0004	0.1

Standard errors as percentages are also shown in Figure 17.29. Uncertainties in the model's household parameters imply standard errors of 5—6% in many of the model's endogenous variables. However, some variables are considerably less sensitive. The standard error for leisure is about 2% and the errors for most prices are even smaller: about 1.5% for the price of refined petroleum products and only 0.1% for the price of services.

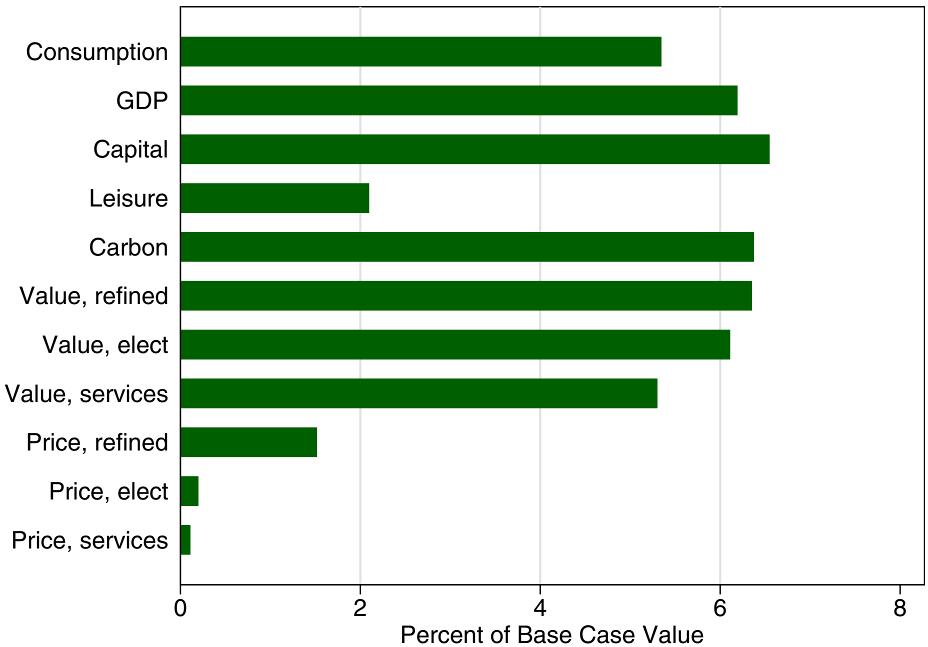

Figure 17.29 Standard error as a percent of base-case value.

To confirm that the method produces results that closely approximate the true nonlinear standard errors of the variables, we carried out a Monte Carlo analysis using 10,000 draws from the joint distribution of the household parameters. IGEM was solved for each draw and the values of key results were retained in a database. The variance of each variable was then computed. The Monte Carlo results and the delta method results are shown in Figure 17.30. Overall, the two methods are very similar. The Monte Carlo analysis produces slightly smaller standard errors because second order effects not captured by the delta method tend to tighten the model's confidence intervals. The delta method results can thus be regarded as conservative: they tend to understate the model's accuracy. Moreover, The delta method is far faster: the number of simulations needed is equal to the number of parameters; the number of runs required for the Monte Carlo analysis is several orders of magnitude higher.

17.10.3 Determining sources of uncertainty

An added benefit of this methodology is that the resulting confidence intervals can be decomposed to determine the key sources of uncertainty in the endogenous variables. In scalar notation, the variance of variable i is given by:

$$\sigma_i^2 = \sum_{j=1}^{n_\beta} \sum_{k=1}^{n_\beta} J_{ik} \Sigma_{kj} J_{ij}.$$

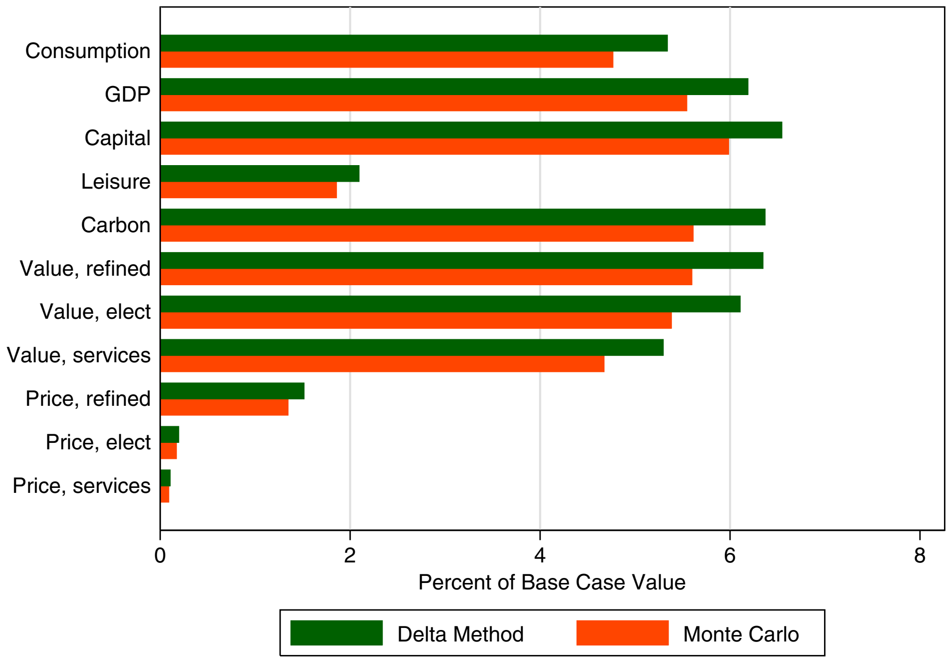

Figure 17.30 Comparison of Delta method and Monte Carlo results.

The relative contribution of term kj (the covariance between parameters k and j) to σ_i^2 can be summarized by computing its share, ω_{ijk}, in the overall variance.

Figure 17.31 shows the 10 largest ω_{ijk} terms, in order of importance, for the variance of consumer expenditure in IGEM. Each is expressed as a percentage of σ_i^2 and is weighted appropriately for the number of times the underlying covariance term occurs in the sum above (once for diagonal elements of $\Sigma_{\overline{Y}}$, twice for off-diagonal terms other than between elements of the β^{pp} matrix, and four times for covariances between off-diagonal elements of β^{pp} due to symmetry restrictions). Each bar is labeled with the names of the parameters corresponding to row k and column j of the parameter covariance matrix; for variances, the two labels will be identical.

The most important term is the variance of β_{12}, which accounts for more than 18% of the variance in consumption. Next most important is the covariance between α_2 and β_2^m, which accounts for slightly more than 14% of the variance in consumption. This term illustrates an important advantage of this approach over sensitivity analysis, which ordinarily involves one-by-one perturbations of parameters and hence cannot capture effects associated with covariances. In fact, almost half of the top 10 components in Figure 17.31 stem from covariances. Third in importance is the variance of β_{23}^{pp} at 12%. Together, the first three terms account for almost half of the variance.

The individual terms can be further decomposed to determine whether the underlying source of uncertainty is imprecision in the estimates (that is, a large value of

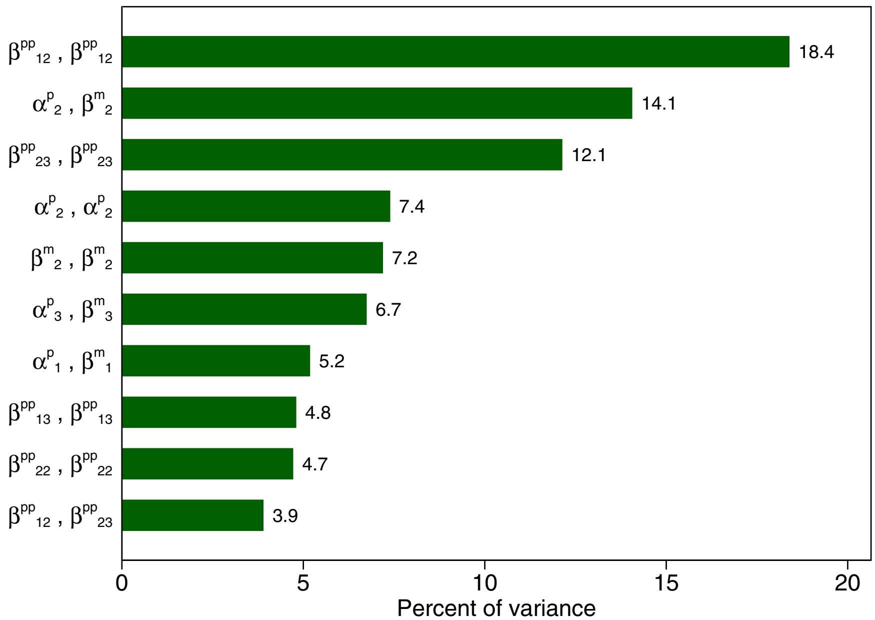

Figure 17.31 Contributions to the variance of consumption expenditure.

Σ_{kj}) or unusually high sensitivity of the model to those terms (large values of J_{ik} or J_{ij}). For example, Figure 17.32 shows the Jacobian terms and covariances underlying Figure 17.31. The Jacobian terms appear in Figure 17.32(A) as a pair of partial elasticities: the upper bar in each pair shows the partial elasticity of consumption with respect to the first parameter in the covariance and the lower bar shows the partial elasticity with respect to the second parameter. The covariances themselves appear in Figure 17.32(B).

For the first term, which is associated with the variance in β_{12}, Figure 17.32(A) shows that the partial elasticity is about 9, i.e. a unit change in β_{12} would change consumption expenditure by a factor of nine. Referring to the standard errors in Table 17.9, β_{12}^{pp} has a standard error of 0.0013. Thus, the partial elasticity of 9 implies that increasing β_{12} by one standard error would increase consumption by about 0.0013×9 or about 1%.

As shown by the second pair of partial elasticities in Figure 17.32(A), the effects of the two parameters may differ in both sign and magnitude. In this case, a unit increase in parameter α_2^p would *decrease* consumption by a factor of 3.4. Referring to Table 17.9, an increase in α_2^p of one standard error would reduce consumption by about 1.5%. By

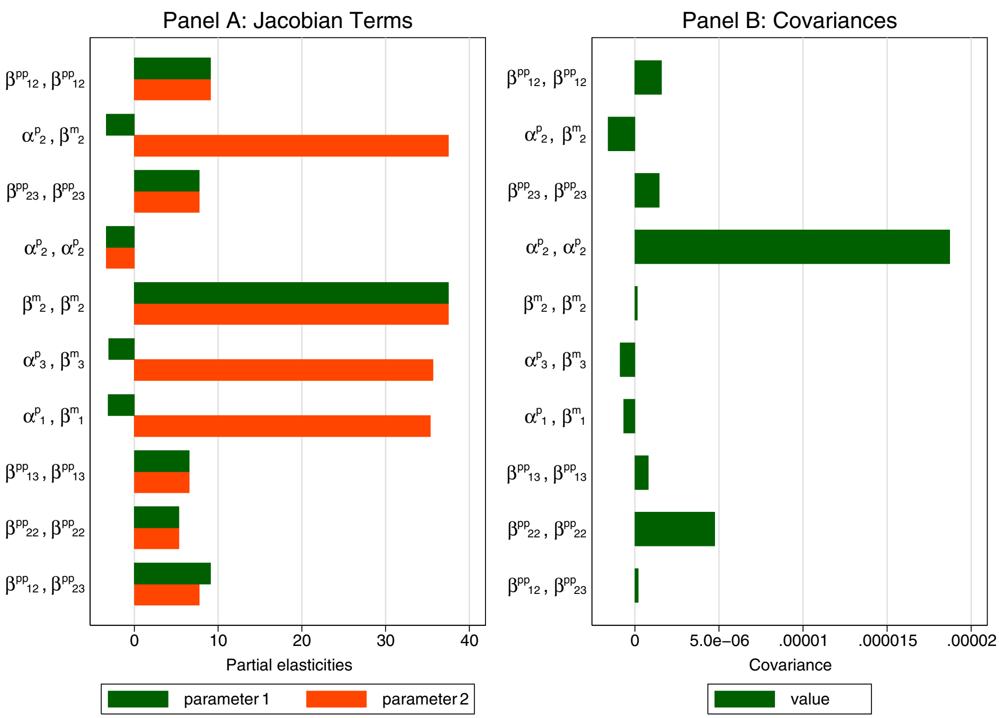

Figure 17.32 Variance components for consumption expenditure.

contrast a unit increase in β_2^m would *raise* consumption by a factor of 37.5. However, because the standard error of β_2^m is very small, a one standard error increase in β_2^m would increase consumption by about 1.5%.

The covariances Σ_{kj} associated with terms in Figure 17.31 are shown in Figure 17.32(B), and differ substantially from one another in both magnitude and sign. As expected, all variances are positive and range in magnitude from that associated with α_2 at the high end to that associated with β_2^m at the low end. In contrast, the covariances between each of the α terms and the corresponding elements of β^m are all negative.

It is interesting to note that all of the terms in Figure 17.31 are positive even though some of the covariances are negative. The reason is the alternating signs of the corresponding partial elasticities shown in Figure 17.32(A). For example, the covariance between α_2^p and β_2^m is negative but the partial elasticities of consumption with respect to the two parameters differ in sign. As a result, the product is positive and the term contributes positively to the overall variance in consumption.

A second important observation is that the decomposition shows that the uncertainties could be classified into two rough categories: (i) Those stemming from terms like β_{12}^{pp}, which have relatively low variance but have large partial elasticities (i.e. parameters that relatively certain but are particularly important to the model) and (ii) those stemming from terms like α_2^p that are determined less precisely, but also have less overall impact on the model.

For comparison, a decomposition of the variance in the capital stock into its 10 largest components is shown in Figure 17.33. Many of the key covariance terms are the same as those for consumption, although their order and magnitude differ. For the capital stock, the most important term is the covariance linking α_2^p and β_2^m, which is responsible for more than 24% of the variance in the stock. One notable difference between Figure 17.32 and 17.33 is the presence of two negative terms. For both terms, the estimated covariances between the parameters are negative and the partial elasticities associated with all three parameters (β_{23}^{pp}, β_2^m and β_{22}^{pp}) are positive. Hence, the two covariances tend to offset some of the others and decrease the overall variance in the capital stock.

A decomposition of the variance in leisure — which is unusually small as a percentage of the base case value — is shown in Figures 17.34 and 17.35. Many of the 10 most important terms are the same as those for consumption and the capital stock. However, as shown by Figure 17.35(A), the partial elasticities of leisure are generally much smaller. Hence the corresponding parameter covariances make smaller contributions to the overall variance of leisure.

Finally, the sources of variance in the model's estimate of carbon emissions are shown in Figure 17.36. Among all household parameters, the term associated with the variance in β_{12} is most important and accounts for about 27% of the variance. Next most important, at almost 17%, is the covariance between α_1^p and β_1^m. The variances of

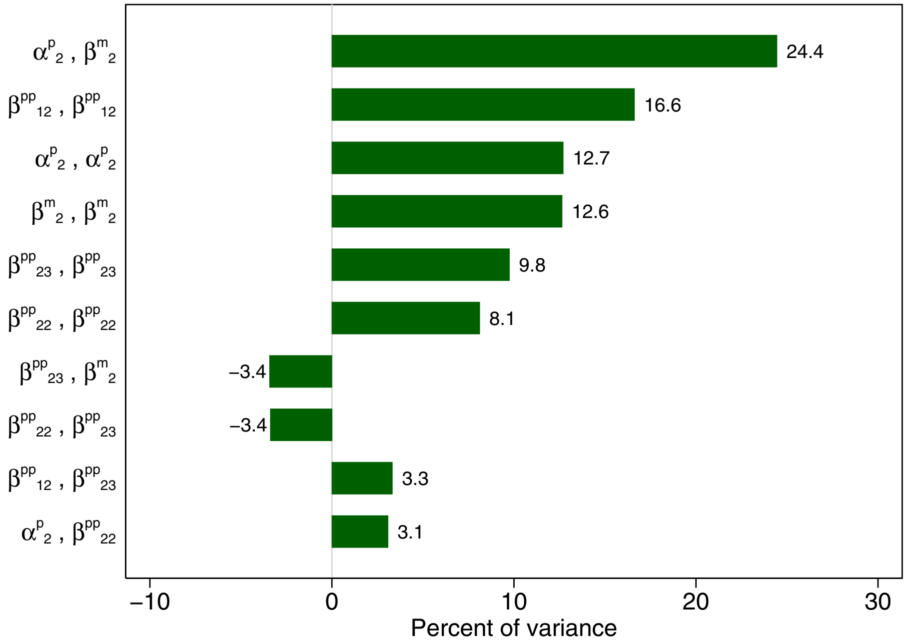

Figure 17.33 Contributions to the variance in capital.

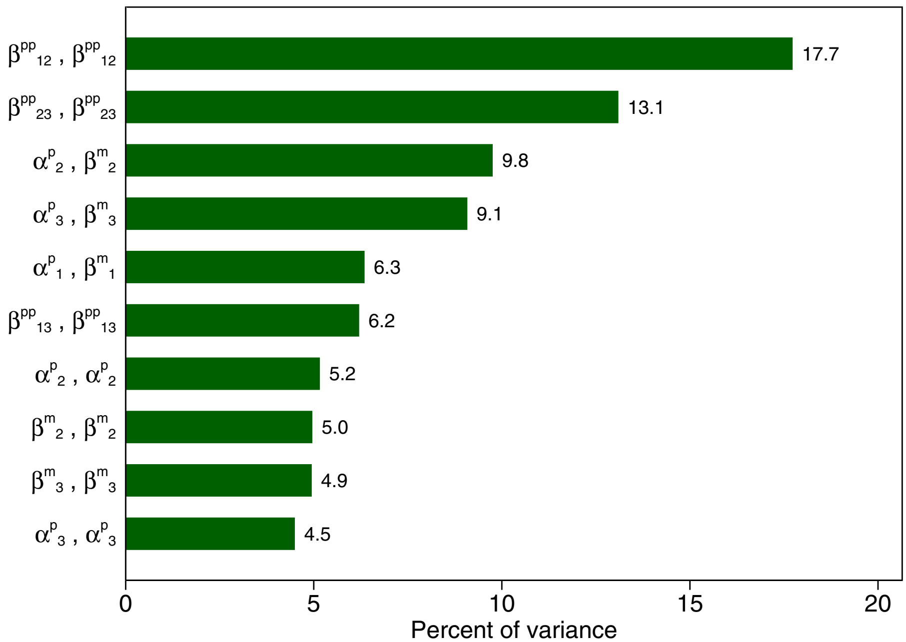

Figure 17.34 Contributions to the variance in leisure.

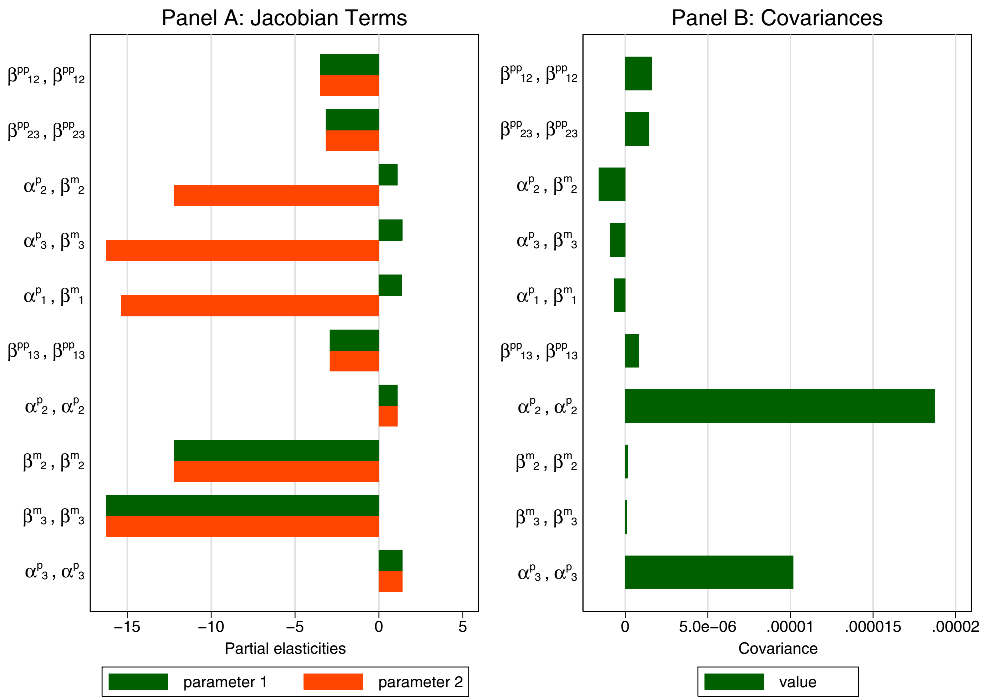

Figure 17.35 Variance components for leisure.

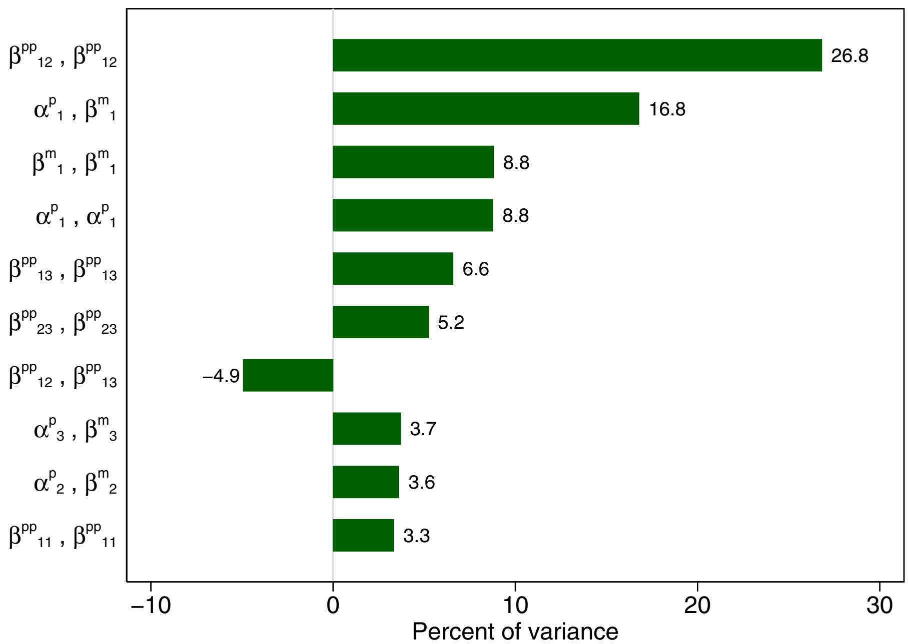

Figure 17.36 Contributions to the variance in carbon emissions.

β_1^m and α_1^p are each responsible for nearly 9% of the variance and after that the terms drop off substantially.

17.10.4 Relationship to sensitivity analysis

Confidence intervals are superior to traditional sensitivity analysis in several respects.

(i) They systematically capture uncertainties in large subsets (or all) of a model's parameters. By contrast sensitivity analysis is usually done for only a handful of parameters selected by the analyst. Sensitivity analysis thus runs the risk of missing key parameters that were not expected to be important by the analyst.

(ii) Confidence intervals correctly capture the likelihood of deviations in the model's results. They provide a rigorously determined range of variation with an explicit probability that the range will contain the true result. In contrast, sensitivity analysis examines the effect of relatively arbitrary perturbations in particular parameters and does not provide systematic probabilities.

(iii) Confidence intervals correctly account for covariances between parameters. This feature is particularly important because parameter estimates within a subsection of a model — such as the representation of producer behavior in a given sector — are usually correlated. Under sensitivity analysis, however, the reported perturbations of parameters generally do not take such correlations into account. Indeed, because parameter covariances are not usually reported in the literature used for calibration-based modeling, analysts carrying out sensitivity analysis may not even be aware of them.

Failing to account for covariances can cause results from sensitivity analysis to be misleading because some reported combinations of perturbations may be highly unlikely. For example, an analysis might report the effects of increasing two parameters simultaneously. If the covariance between the two is negative, a simultaneous increase in both would be highly unlikely. However, a user of the analysis would have no way of knowing that the results for that case have very low probability. The confidence intervals, in contrast, fully account for such covariances.

Finally, although confidence intervals are superior to sensitivity analysis for characterizing the precision of modeling results, they also allow detailed analysis of the sources of uncertainty underlying any given result. Thus, they can be used by model builders for the kinds of diagnostic assessments now carried out by sensitivity analysis.

17.10.5 Summary

A key advantage of the econometric approach to parameterization is that it allows construction of standard errors and confidence intervals for modeling results.

Confidence intervals are valuable to users of models because they provide concise and transparent summaries of the effects of uncertainties in the model's parameters. Moreover, they characterize the precision of modeling results more rigorously and systematically than traditional sensitivity analysis: (i) They capture perturbations in all parameters, (ii) they account for probability weightings of perturbations and (iii) they correctly capture the effects of covariances between the estimates.

In addition, computing standard errors via the methodology presented above provides additional benefits to model builders: by decomposing standard errors into key components, it is straightforward to see which parameters should be highest priority for improved estimation. Finally, our approach to computing confidence intervals achieves results comparable to full Monte Carlo simulation but with a small fraction of that method's computational resources. It is highly scalable and suitable for very large models.

17.11 CONCLUSIONS

Our principal innovation in modeling producer behavior is to represent the rate and biases of technical change as latent or unobservable variables, while retaining flexibility in modeling substitution in response to changes in prices. We find that biases of technical change are substantial in magnitude, comparable to responses to price changes. These biases are generally capital-using and materials-saving; biases for energy alternate between energy-using before 1980 and energy-using afterward. Projections of the biases of technical change are inconsistent with the constant time trends employed in Binswanger's approach.

The rate of induced technical change captures the correlation between the biases of technical change and the prices of inputs. Perhaps surprisingly, this correlation is positive, so that rates of induced technical change are predominantly negative. Technical change directed toward increasing or decreasing the utilization of a particular input generally reduces the rate of technical change. However, rates of autonomous technical change are predominantly positive and much greater in magnitude. Projections of rates of technical change are positive and substantial, suggesting a relatively optimistic outlook for future US economic growth.

Our approach to econometric modeling of consumer behavior integrates 27 years of repeated cross sections with information on the levels of prices and wages that vary across regions and over time. The resulting dataset is comprised of over 150,000 households and allows us to model the joint determination of the allocation of full expenditure across goods and leisure. The large sample sizes and lengthy time series enable us to create synthetic cohorts that facilitate the estimation of the allocation of full wealth, including the assets and time endowment of each household, over time.

We find that the cross-sectional and intertemporal variation of prices and wages is substantial which allows us to estimate the price and wage elasticities very precisely. We find that wage elasticities of labor supply are negative but close to zero while the price elasticities of demand for non-durables and consumer services are price inelastic. Household heterogeneity is important in explaining consumption patterns and these effects would be missed with the use of micro-level data.

For example, we find that the numbers of adults and children in each household have an important impact on the allocation of full consumption between leisure and goods.

We extend the within-period model of consumer behavior by utilizing a less restrictive approach for representing income effects. We estimate a translog demand system of Gorman rank three and compare it with a demand system of rank two. We find that the average income and price elasticities of goods and services, as well as leisure, are very similar. However, over the entire range of full consumption, the new rank-three translog demand system better describes the income effects than the earlier rank-two system implemented by Jorgenson *et al.* (1997) and Jorgenson and Slesnick (1997).

The novel feature of our model of the joint determination of leisure and goods demand is consistency with exact aggregation. As illustrated by Jorgenson and Wilcoxen (1998), aggregate commodity demands and labor supply play crucial roles in general equilibrium models used to evaluate the macroeconomic consequences of energy and environmental policies. As noted by Browning *et al.* (1999), the challenge is to capture the heterogeneity of household behavior in a tractable way.

The exact aggregation framework presented in Section 17.8 incorporates household heterogeneity, as well as the price and income variation included in models based on the highly oversimplified theory of a representative consumer. Aggregate demand functions can be represented in closed form and are obtained by simply summing over all of the households in the sample. We find that the aggregate expenditure patterns and leisure demands are largely determined by movements in prices and incomes. While demographic characteristics have significant cross-sectional impacts, they show little movement over time and have a small impact on variations in aggregate demand.

A final important aspect of the econometric approach is that it allows for fast, scalable and systematic computation of confidence intervals for general equilibrium results. These confidence intervals are critical to model users because they concisely summarize the precision of any given result. At the same time, they are a very valuable tool for model builders because they facilitate fine-grained analysis of the sources of uncertainty in a model.

APPENDIX

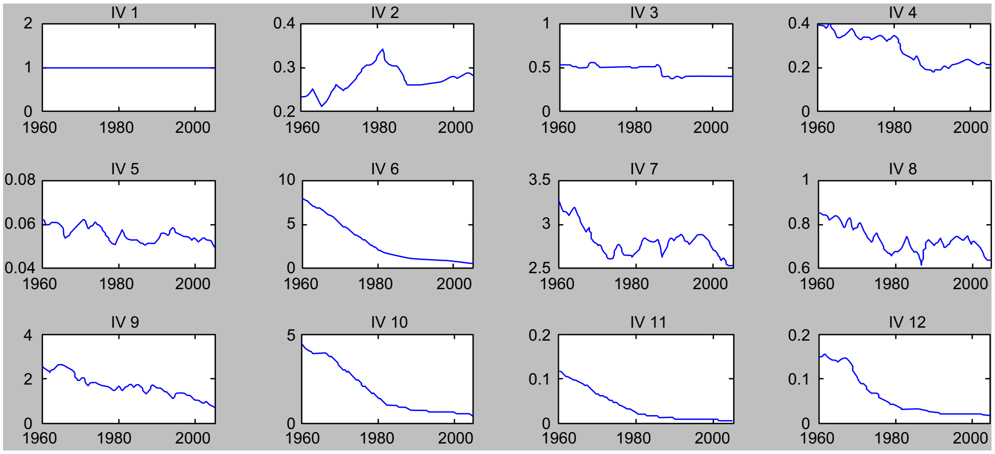

Figure 17.A1 Instrumental variables.

Table 17.A1 Instrumental variables

1	Constant
2	Average marginal tax rate on personal labor income
3	Effective corporate income tax rate
4	Average marginal tax rate on dividends
5	Rate of taxation on consumption goods
6	Time endowment in 2000 dollars/lagged private wealth including claims on government and the rest of the world
7	Lagged price of personal consumptions expenditure/lagged price index of private domestic labor input
8	Lagged price of leisure and unemployment/lagged price index of private domestic labor input
9	Lagged price of capital services for household/ lagged price index of private domestic labor input
10	Lagged real full consumption/lagged private wealth including claims on government and the rest of the world
11	US population/lagged private wealth including claims on government and the rest of the world
12	Government demand/lagged private wealth including claims on government and the rest of the world

Table 17.A2 Tests for over-identification

Sector	l_g	l	$2(l_g - l)$	*p*-value	*p*-value * 35
1	563.48	562.65	1.67	0.990	34.63
2	454.21	447.07	14.28	0.075	2.62
3	481.81	478.00	7.61	0.473	16.55
4	509.39	508.64	1.50	0.993	34.74
5	550.44	547.61	5.66	0.685	23.97
6	714.71	712.61	4.19	0.839	29.38
7	730.60	727.74	5.72	0.679	23.75
8	604.98	604.94	0.07	1.000	35.00
9	704.31	702.58	3.47	0.902	31.56
10	709.93	708.88	2.11	0.978	34.21
11	638.50	637.66	1.68	0.989	34.62
12	691.26	688.36	5.82	0.668	23.37
13	630.64	627.61	6.07	0.640	22.40
14	722.78	719.49	6.59	0.581	20.33
15	620.06	618.78	2.55	0.959	33.57
16	531.17	526.66	9.01	0.342	11.96
17	702.20	699.77	4.86	0.772	27.03
18	597.88	596.47	2.82	0.945	33.08
19	660.17	658.94	2.47	0.963	33.71
20	647.79	641.26	13.07	0.109	3.83
21	702.88	700.86	4.03	0.854	29.90
22	701.61	697.96	7.30	0.505	17.67
23	648.23	648.00	0.47	1.000	35.00
24	674.59	670.90	7.38	0.496	17.36
25	648.00	642.70	10.61	0.225	7.87
26	700.77	695.14	11.26	0.187	6.56
27	673.38	669.86	7.06	0.530	18.56
28	607.55	602.54	10.01	0.264	9.26
29	781.95	776.59	10.72	0.218	7.64
30	595.61	594.48	2.25	0.972	34.03
31	560.82	552.14	17.35	0.027	0.93
32	703.84	699.63	8.43	0.392	13.73
33	765.86	760.74	10.24	0.248	8.69
34	726.58	721.86	9.43	0.307	10.76
35	572.33	564.66	15.34	0.053	1.85

The number of degrees of freedom for the likelihood ratio test for each sector is 8. The null hypothesis is that the instrumental variables are exogenous. High *p*-values indicate that we cannot reject the null hypothesis of exogeneity. The last column presents *p*-values adjusted for simultaneous inference.

Table 17.A3 Tests of validity of the instrumental variables

Sector	Likelihood ratio	p-value
1	677.89	<0.001
2	580.45	<0.001
3	679.11	<0.001
4	762.29	<0.001
5	717.90	<0.001
6	646.73	<0.001
7	672.32	<0.001
8	646.00	<0.001
9	782.78	<0.001
10	643.17	<0.001
11	541.68	<0.001
12	600.82	<0.001
13	668.19	<0.001
14	743.66	<0.001
15	732.65	<0.001
16	692.95	<0.001
17	734.26	<0.001
18	625.93	<0.001
19	829.69	<0.001
20	626.03	<0.001
21	726.69	<0.001
22	696.75	<0.001
23	791.19	<0.001
24	601.69	<0.001
25	596.51	<0.001
26	777.20	<0.001
27	588.84	<0.001
28	568.53	<0.001
29	762.82	<0.001
30	657.10	<0.001
31	748.91	<0.001
32	856.78	<0.001
33	755.03	<0.001
34	715.89	<0.001
35	764.75	<0.001

Number of degrees of freedom for the likelihood ratio test for each sector is 99. The null hypothesis is that instrumental variables are uncorrelated with the endogenous independent variables. Low p-values indicate that we can reject the null hypothesis of no correlation.

Table 17.A4 Formulas for the figures

Figure 17.1	$v_{KT} - v_{K1}$
Figure 17.2	$v_{LT} - v_{L1}$
Figure 17.3	$v_{ET} - v_{E1}$
Figure 17.4	$v_{MT} - v_{M1}$
Figure 17.5	$\begin{aligned}&\left(\beta_{KK}\ln\frac{P_{KT}}{P_{MT}} + \beta_{KL}\ln\frac{P_{LT}}{P_{MT}} + \beta_{KE}\ln\frac{P_{ET}}{P_{MT}}\right) - \left(\beta_{KK}\ln\frac{P_{K1}}{P_{M1}} + \beta_{KL}\ln\frac{P_{L1}}{P_{M1}} + \beta_{KE}\ln\frac{P_{E1}}{P_{M1}}\right)\\ &= (\beta_{KK}\ln P_{KT} + \beta_{KL}\ln P_{LT} + \beta_{KE}\ln P_{ET} + \beta_{KM}\ln P_{MT})\\ &\quad -(\beta_{KK}\ln P_{K1} + \beta_{KL}\ln P_{L1} + \beta_{KE}\ln P_{E1} + \beta_{KM}\ln P_{M1})\end{aligned}$
Figure 17.6	$\begin{aligned}&\left(\beta_{KL}\ln\frac{P_{KT}}{P_{MT}} + \beta_{LL}\ln\frac{P_{LT}}{P_{MT}} + \beta_{LE}\ln\frac{P_{ET}}{P_{MT}}\right) - \left(\beta_{KL}\ln\frac{P_{K1}}{P_{M1}} + \beta_{LL}\ln\frac{P_{L1}}{P_{M1}} + \beta_{LE}\ln\frac{P_{E1}}{P_{M1}}\right)\\ &= (\beta_{KL}\ln P_{KT} + \beta_{LL}\ln P_{LT} + \beta_{LE}\ln P_{ET} + \beta_{LM}\ln P_{MT})\\ &\quad -(\beta_{KL}\ln P_{K1} + \beta_{LL}\ln P_{L1} + \beta_{LE}\ln P_{E1} + \beta_{LM}\ln P_{M1})\end{aligned}$
Figure 17.7	$\begin{aligned}&\left(\beta_{KE}\ln\frac{P_{KT}}{P_{MT}} + \beta_{LE}\ln\frac{P_{LT}}{P_{MT}} + \beta_{EE}\ln\frac{P_{ET}}{P_{MT}}\right) - \left(\beta_{KE}\ln\frac{P_{K1}}{P_{M1}} + \beta_{LE}\ln\frac{P_{L1}}{P_{M1}} + \beta_{EE}\ln\frac{P_{E1}}{P_{M1}}\right)\\ &= (\beta_{KE}\ln P_{KT} + \beta_{LE}\ln P_{LT} + \beta_{EE}\ln P_{ET} + \beta_{EM}\ln P_{MT})\\ &\quad -(\beta_{KE}\ln P_{K1} + \beta_{LE}\ln P_{L1} + \beta_{EE}\ln P_{E1} + \beta_{EM}\ln P_{M1})\end{aligned}$
Figure 17.8	$(\beta_{KM}\ln P_{KT} + \beta_{LM}\ln P_{LT} + \beta_{EM}\ln P_{ET} + \beta_{MM}\ln P_{MT}) - (\beta_{KM}\ln P_{K1} + \beta_{LM}\ln P_{L1} + \beta_{EM}\ln P_{E1} + \beta_{MM}\ln P_{M1})$
Figure 17.9	$f_{KT} - f_{K1}$
Figure 17.10	$f_{LT} - f_{L1}$
Figure 17.11	$f_{ET} - f_{E1}$

Figure 17.12 $f_{MT} - f_{M1}$

Figure 17.13 $-\left(\ln\dfrac{P_{QT}}{P_{MT}} - \ln\dfrac{P_{Q1}}{P_{M1}}\right)$

Figure 17.14

$$-\left\{\begin{array}{l} \left[\alpha_K \quad \alpha_L \quad \alpha_E \quad \beta_{KK} \quad \beta_{LL} \quad \beta_{EE} \quad \beta_{KL} \quad \beta_{KE} \quad \beta_{LE}\right]\left(\begin{bmatrix} \ln\dfrac{P_{KT}}{P_{MT}} \\ \ln\dfrac{P_{LT}}{P_{MT}} \\ \ln\dfrac{P_{ET}}{P_{MT}} \\ \frac{1}{2}\left(\ln\dfrac{P_{KT}}{P_{MT}}\right)^2 \\ \frac{1}{2}\left(\ln\dfrac{P_{LT}}{P_{MT}}\right)^2 \\ \frac{1}{2}\left(\ln\dfrac{P_{ET}}{P_{MT}}\right)^2 \\ \ln\dfrac{P_{KT}}{P_{MT}}\ln\dfrac{P_{LT}}{P_{MT}} \\ \ln\dfrac{P_{KT}}{P_{MT}}\ln\dfrac{P_{ET}}{P_{MT}} \\ \ln\dfrac{P_{LT}}{P_{MT}}\ln\dfrac{P_{ET}}{P_{MT}} \end{bmatrix} - \begin{bmatrix} \ln\dfrac{P_{K1}}{P_{M1}} \\ \ln\dfrac{P_{L1}}{P_{M1}} \\ \ln\dfrac{P_{E1}}{P_{M1}} \\ \frac{1}{2}\left(\ln\dfrac{P_{K1}}{P_{M1}}\right)^2 \\ \frac{1}{2}\left(\ln\dfrac{P_{L1}}{P_{M1}}\right)^2 \\ \frac{1}{2}\left(\ln\dfrac{P_{E1}}{P_{M1}}\right)^2 \\ \ln\dfrac{P_{K1}}{P_{M1}}\ln\dfrac{P_{L1}}{P_{M1}} \\ \ln\dfrac{P_{K1}}{P_{M1}}\ln\dfrac{P_{E1}}{P_{M1}} \\ \ln\dfrac{P_{L1}}{P_{M1}}\ln\dfrac{P_{E1}}{P_{M1}} \end{bmatrix}\right) + \\ \displaystyle\sum_{t=2}^{T} f_{Kt}\left(\ln\frac{P_{Kt}}{P_{Mt}} - \ln\frac{P_{Kt-1}}{P_{Mt-1}}\right) + f_{Lt}\left(\ln\frac{P_{Lt}}{P_{Mt}} - \ln\frac{P_{Lt-1}}{P_{Mt-1}}\right) + f_{Kt}\left(\ln\frac{P_{Et}}{P_{Mt}} - \ln\frac{P_{Et-1}}{P_{Mt-1}}\right) \end{array}\right\}$$

(*Continued*)

Table 17.A4 Formulas for the figures—cont'd

Figure 17.15	$-\left[\sum_{t=2}^{T}\ln\frac{P_{Kt}}{P_{Mt}}(f_{Kt}-f_{Kt-1})+\ln\frac{P_{Lt}}{P_{Mt}}(f_{Lt}-f_{Lt-1})+\ln\frac{P_{Et}}{P_{Mt}}(f_{Et}-f_{Et-1})\right]$ $=-\left[\sum_{t=2}^{T}\ln P_{Kt}(f_{Kt}-f_{Kt-1})+\ln P_{Lt}(f_{Lt}-f_{Lt-1})+\ln P_{Et}(f_{Et}-f_{Et-1})+\ln P_{Mt}(f_{Mt}-f_{Mt-1})\right]$
Figure 17.16	$-(f_{pT}-f_{p1})$
Figure 17.17	$f_{E1980}-f_{E1}$
Figure 17.18	$f_{ET}-f_{E1980}$
Figure 17.19	$f_{E2030}-f_{ET}$
Figure 17.20	$f_{K2030}-f_{KT}$
Figure 17.21	$f_{L2030}-f_{LT}$
Figure 17.22	$f_{M2030}-f_{MT}$
Figure 17.23	$-\left[\ln\frac{P_{KT}}{P_{MT}}(f_{K2030}-f_{KT})+\ln\frac{P_{LT}}{P_{MT}}(f_{L2030}-f_{LT})+\ln\frac{P_{ET}}{P_{MT}}(f_{E2030}-f_{ET})\right]$
Figure 17.24	$-(f_{p2030}-f_{pT})$

Note: Year $1=1960$, Year $T=2005$.

Table 17.A5 Parameter estimates of wage equation

Variable	Estimate	Standard Error
CONST	0.04507	0.0783
AGE	0.06014	0.0024
AGESQ	−0.00056	0.00003
EDUC	0.03609	0.0058
EDUCSQ	0.00118	0.00022
FEM * AGE	−0.02322	0.0028
FEM * AGESQ	0.00022	0.00003
FEM * EDUC	−0.00808	0.0082
FEM * EDUCSQ	0.00075	0.0003
NW * AGE	−0.01340	0.0035
NW * AGESQ	0.00014	0.00004
NW * EDUC	−0.02971	0.0088
NW * EDUCSQ	0.00127	0.0003
MAR	0.09257	0.0044
NW	0.38577	0.0882
FFM	0.30830	0.0772
INVMILLS	−0.21600	0.0204

Table 17.A6 Parameter estimates—rank-two model

	Nondurables		Capital services		Consumer services		Leisure	
Variable	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
CONST	−0.53660	0.0027	−0.29696	0.0041	−0.03032	0.0030	−0.13615	0.0063
PNON	−0.01576	0.0014	−0.02430	0.0013	0.05024	0.0009	−0.04689	0.0019
PCAP	−0.02430	0.0013	0.05172	0.0022	−0.04291	0.0012	0.00632	0.0026
PSERV	0.05024	0.0009	−0.04291	0.0012	−0.02309	0.0013	0.02110	0.0018
WAGE	−0.04689	0.0019	0.00632	0.0026	0.02110	0.0018	0.05999	0.0045
FULLC	0.03670	0.0002	0.00916	0.0004	−0.00535	0.0003	−0.04052	0.0006
CHILD	−0.01482	0.0002	−0.00256	0.0004	−0.00077	0.0003	0.01815	0.0006
CHILDSQ	0.00152	0.0001	0.00066	0.0001	0.00053	0.0001	−0.00271	0.0002
ADULT	0.00360	0.0006	0.04338	0.0007	0.01814	0.0007	−0.06511	0.0011
ADULTSQ	−0.00048	0.0001	−0.00407	0.0001	−0.00139	0.0001	0.00594	0.0002
REGMW	−0.00390	0.0004	0.00865	0.0007	−0.01280	0.0005	0.00805	0.0010
REGS	−0.00824	0.0004	0.01163	0.0007	−0.01857	0.0005	0.01519	0.0010
REGW	0.00057	0.0004	−0.00753	0.0005	−0.00216	0.0004	0.00913	0.0009
NONWHITE	0.01115	0.0003	0.01341	0.0005	0.01139	0.0004	−0.03594	0.0008
FEMALE	0.00823	0.0003	0.00285	0.0004	−0.00776	0.0003	−0.00333	0.0007
RURAL	−0.01264	0.0004	0.03158	0.0008	−0.01141	0.0005	−0.00752	0.0011

SE = standard error.

Table 17.A7 Parameter estimates—rank-three model (A0 = −12.1089; SE = 1.2657)

	Nondurable		Capital services		Consumer services		Leisure	
Variable	**Estimate**	**SE**	**Estimate**	**SE**	**Estimate**	**SE**	**Estimate**	**SE**
CONST	−0.42468	0.2143	−0.20158	0.1835	0.03746	0.1305	0.41120	0.5281
PNON	−0.01508	0.0039	−0.02232	0.0043	0.05108	0.0018	−0.04136	0.0086
PCAP	−0.02232	0.0043	0.05330	0.0039	−0.04132	0.0034	0.00886	0.0053
PSERV	0.05108	0.0018	−0.04132	0.0034	−0.02203	0.0022	0.02310	0.0046
WAGE	−0.04136	0.0086	0.00886	0.0053	0.02310	0.0046	0.02773	0.0608
FULLC	0.02768	0.0177	0.00148	0.0151	−0.01082	0.0107	−0.01833	0.0435
FULLCSQ	−0.00678	0.0002	−0.00590	0.0003	−0.00428	0.0002	0.01696	0.0005
CHILD	−0.01474	0.0004	−0.00257	0.0004	−0.00080	0.0003	0.01811	0.0008
CHILDSQ	0.00150	0.0001	0.00065	0.0001	0.00052	0.0001	−0.00267	0.0002
ADULT	0.00112	0.0007	0.04090	0.0011	0.01649	0.0008	−0.05851	0.0020
ADULTSQ	0.00003	0.0001	−0.00360	0.0001	−0.00106	0.0001	0.00462	0.0002
REGMW	−0.00400	0.0004	0.00851	0.0007	−0.01275	0.0005	0.00824	0.0011
REGS	−0.00816	0.0005	0.01151	0.0007	−0.01847	0.0006	0.01512	0.0012
REGW	0.00054	0.0004	−0.00747	0.0006	−0.00215	0.0004	0.00907	0.0009
NONWHITE	0.01079	0.0005	0.01306	0.0006	0.01113	0.0005	−0.03497	0.0013
FEMALE	0.00848	0.0003	0.00312	0.0004	−0.00748	0.0003	−0.00412	0.0007
RURAL	−0.01274	0.0005	0.03112	0.0008	−0.01150	0.0005	−0.00687	0.0011

SE = standard error.

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