

# The MONASH Style of Computable General Equilibrium Modeling: A Framework for Practical Policy Analysis

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## Abstract

MONASH models are descended from Johansen's 1960 model of Norway. The first MONASH model was ORANI, used in Australia's tariff debate of the 1970s. Johansen's influence combined with institutional arrangements in their development gave MONASH models distinctive characteristics, facilitating a broad range of policy-relevant applications. MONASH models currently operate in numerous countries to provide insights on a variety of questions including:

*the effects on:*

macro, industry, regional, labor market, distributional and environmental variables

*of changes in:*

taxes, public consumption, environmental policies, technologies, commodity prices, interest rates, wage-setting arrangements, infrastructure and major-project expenditures, and known levels and exploitability of mineral deposits (the Dutch disease).

MONASH models are also used for explaining periods of history, estimating changes in technologies and preferences and generating baseline forecasts. Creation of MONASH models involved a series of enhancements to Johansen's model, including: (i) a computational procedure that eliminated Johansen's linearization errors without sacrificing simplicity; (ii) endogenization of trade flows by introducing into computable general equilibrium (CGE) modeling imperfect substitution between imported and domestic varieties (the Armington assumption); (iii) increased dimensionality allowing for policy-relevant detail such as transport margins; (iv) flexible closures; and (v) complex functional forms to specify production technologies. As well as broad theoretical issues, this chapter covers data preparation and introduces the GEMPACK purpose-built CGE software. MONASH modelers have responded to client demands by developing four modes of analysis: historical, decomposition, forecast and policy. Historical simulations produce up-to-date data, and estimate trends in technologies, preferences and other naturally exogenous but unobservable variables. Decomposition simulations explain historical episodes and place policy effects in historical context. Forecast simulations provide baselines using extrapolated trends from historical simulations together with specialist forecasts. Policy simulations generate effects of policies as deviations from baselines. To emphasize the practical orientation of MONASH models, the chapter starts with a MONASH-style policy story.

## Keywords

MONASH computable general equilibrium models, flexible closures, computable general equilibrium forecasting, policy-oriented modeling, telling a computable general equilibrium story, Johansen's computable general equilibrium influence

## JEL classification codes

C68, C63, D58, F16, F14

## 2.1 INTRODUCTION

This chapter describes the MONASH style of CGE modeling, which started with the ORANI model of Australia (Dixon *et al.*, 1977, 1982). MONASH models are directly descended from the seminal work of Leif Johansen (1960). The influence of Johansen combined with the institutional arrangements under which MONASH models have been developed has given them distinctive technical characteristics, facilitating a broad range of policy-relevant and influential applications. MONASH models are now operated on behalf of governments and private sector organizations in numerous countries, including Australia, the US, China, Finland, Netherlands, Malaysia, Taiwan, Brazil, South Africa and Vietnam.<sup>1</sup> The MONASH approach underlies the worldwide Global Trade Analysis Project (GTAP) network (see Chapter 12 by Hertel in this Handbook).

The practical focus of MONASH models reflects their history. ORANI was created in the IMPACT Project — a research initiative of the Industries Assistance Commission (IAC). The IAC was the agency of the Australian government with responsibility for advising policy makers on the economic and social effects of tariffs, quotas and other protective devices against imports.

To understand why the IMPACT Project was established and why it produced a model such as ORANI, we need to go back to the federation of the Australian colonies. Before federation in 1901, the dominant British colonies on the Australian continent were New South Wales, which followed a free trade policy, and Victoria, which adopted high tariffs against manufactured imports. After a heated debate,<sup>2</sup> the federated nation adopted what was close to the Victorian policy, setting protection of manufacturing industries at an average rate of about 23%. Protection increased during the 1930s and continued to rise after World War II, reaching rates of more than 50% for some industries. Resentments about protection persisted and intensified as rates rose, especially in the export-oriented state of Western Australia. By the 1960s,

<sup>1</sup> A complete technical exposition of a modern MONASH model is Dixon and Rimmer (2002) together with supporting web material at <http://www.monash.edu.au/policy/monbook2.htm>. See also Honkatukia (2009).

<sup>2</sup> See Glezer (1982, chapter 1).

Australia's protectionist stance was being challenged analytically by leading economists such as Max Corden (e.g. Corden, 1958) and by the 1970s, there was demand by policy makers for a quantitative tool for analyzing protection. Policy makers wanted to know how people whose jobs would disappear with lower protection could be reabsorbed into employment. The IAC responded in 1975 by setting up the IMPACT Project with the task of building an economy-wide model that could be used to trace out the effects of changes in protection for one industry on employment prospects for other industries.

The arrangements for the IMPACT Project maximized the probability of a successful outcome. It had a sharp focus on a major policy problem (protection), thereby attracting policy-oriented, ambitious economists. It had an outstanding initial director, Alan A. Powell, who was a highly respected applied econometrician. Without blunting its practical orientation, Powell conducted the Project at arms length from the bureaucracy. Under his leadership, IMPACT was an open environment that allowed talented young economists to flourish. The outcome was the ORANI model, the first version of which was operational in 1977. By providing detailed quantification of the effects of cuts in protection on winners as well as losers, and by showing where jobs would be gained as well as lost, ORANI helped in the formation of an anti-protection movement that eventually prevailed and converted Australia from high protection in the mid-1970s to having almost free trade by the end of the century.

With changes in political circumstances, the IMPACT Project left the IAC in 1979. The IMPACT team was split between the University of Melbourne and La Trobe University. Nevertheless, it continued to work as a group and in 1984 was reunited at the University of Melbourne. Since 1991 the team has operated as the Centre of Policy Studies (CoPS) at Monash University. Throughout its 37-year history, CoPS/IMPACT has maintained an extraordinary level of staff cohesion. Three researchers have been with the Project continuously for 37 years, six others have devoted more than 20 years to the Project, while many others have served more than 10 years. Seven researchers have been promoted at the Project to the rank of Full Professor. What explains the Project's success and longevity?

One factor is that since its beginning, when Powell set the standards, the Project has been an enjoyable place to work with high levels of cooperation between researchers and generous treatment of colleagues. A second factor is that the Project has generated a continuous stream of challenging, satisfying inter-related activities. These include: data management and preparation; formulation of solution algorithms; development of software; translation of policy questions into forms amenable to modeling; creation of theoretical specifications to broaden the range of CGE applications and improve existing applications; checking of model solutions; interpretation of results and deducing their policy significance; and delivery of persuasive reports.

The third and perhaps the most important factor underlying the Project's success and longevity is the adaptability of MONASH models, which are its main product. After the initial work on protection, these models have provided insights on an enormous variety of questions including:

*the effects on:*

macro, industry, regional, labor market, distributional and environmental variables

*of changes in:*

taxes, public consumption, social-security payments, environmental policies, technologies, international commodity prices, interest rates, wage-setting arrangements and union behavior, infrastructure and major-project expenditures, and known levels and exploitability of mineral deposits (the Dutch disease).

In addition, MONASH models are used for: explaining periods of history; estimating changes in technologies, consumer preferences and other unobservable variables; and creating baseline forecasts. With this flexibility of its central product, the Project has maintained the long-term interest of its researchers. Model flexibility has also been critical to the financing of the Project. Starting in the 1980s, the Project has been increasingly reliant on the sale of contract and subscription services. In 2010, these services accounted for 80% of the budget of CoPS, which had a professional and support staff of 20. Without application flexibility, CoPS could not have remained viable as a predominantly commercial entity within a university.<sup>3</sup>

The rest of this chapter is organized as follows. In [Section 2.2](#) we tell a MONASH-style policy story. We start this way for four reasons.

- (i) We want to emphasize that the primary purpose of MONASH models is practical policy analysis.
- (ii) Before presenting technical details, we want to demonstrate the ability of MONASH models to generate policy-relevant results that can be communicated in a convincing way to people without CGE backgrounds.
- (iii) We want to illustrate the technique of explaining CGE results in a macro-to-micro manner that avoids circularity. This requires finding an 'exogenous' starting point.
- (iv) We want to motivate the study of CGE modeling by providing a thought-provoking analysis that illustrates its strengths.

[Section 2.3](#) starts by outlining Johansen's model. It then describes Johansen's legacy to MONASH modelers. This includes: the representation of models as rectangular systems of linear equations in changes and percentage changes of the variables; a transparent solution procedure that directly generates a solution matrix showing the elasticities of endogenous variables with respect to exogenous variables; a mode of analysis and result

<sup>3</sup> More detailed accounts of the history of the IMPACT Project and its reincarnation at CoPS can be found in [Powell and Snape \(1993\)](#) and [Dixon \(2008\)](#).

interpretation built around the solution matrix; and the use of back-of-the-envelope (BOTE) models to aid interpretation and management of the huge volume of results that flow from a full-scale CGE model. The final part of [Section 2.3](#) describes five innovations that were made in the journey from Johansen's model to ORANI: (i) implementation of a Johansen/Euler procedure that eliminates linearization errors without sacrifice of simplicity and interpretability; (ii) endogenization of trade flows by the introduction of imperfect substitution between imported and domestic varieties of the same commodity and downward-sloping foreign demand curves; (iii) vastly increased dimensionality that allows the incorporation of policy-relevant detail such as transport and trade margins; (iv) flexible closures; and (v) the use of complex functional forms such as CRESH to specify production technologies.

[Section 2.4](#) is the most technically demanding part of the chapter. [Sections 2.4.1 and 2.4.2](#) set out the mathematical structure of MONASH models. A supporting [Appendix](#) contains the mathematics that underlies the multistep Johansen/Euler solution procedure. [Section 2.4.3](#) demonstrates that we can always find an initial solution for a MONASH model mainly via the input-output database. There is no need to explicitly locate this solution, but the fact that it exists means that derivative methods, such as the Johansen/Euler procedure, can be used to compute required solutions (i.e. solutions with required values for the exogenous variables). [Section 2.4.4](#) shows how the percentage change equations that form Johansen's rectangular system are derived from levels equations. [Section 2.4.5](#) is an overview of GEMPACK.<sup>4</sup> This suite of programs solves MONASH models, and is used for interrogating data and results. [Section 2.4.6](#) discusses problems in transforming published input-output tables into databases for policy-relevant CGE models. We take as an example the transition from tables published by the Bureau of Economic Analysis (BEA) to a database for USAGE, a MONASH-style model of the US. Conventions and definitions adopted in published input-output tables vary from country to country. Consequently, the specifics of our experience are not immediately transferable outside the US. However, the general principle that CGE modelers need to work hard to understand input-output conventions is broadly applicable. Among other things, they need to figure out conventions adopted by their statistical agency concerning: valuations (basic prices versus producer prices versus purchasers prices); reconciliation with the national accounts; imports (direct or indirect allocation); investment (commodity versus industry); self-employment; and the treatment of imputed rents in housing.

[Section 2.5](#) describes how MONASH models have evolved in response to demands by consumers of CGE modeling services. These consumers are concerned primarily with current policy proposals. From modelers they want results showing policy effects on finely defined constituent groups, not just effects on macro variables and coarsely defined sectors. They want results from models that have up-to-date data, detailed disaggregation

<sup>4</sup> GEMPACK is described fully in Horridge *et al.* in Chapter 20 of this Handbook.

and accurate representation of relevant policy instruments. In trying to satisfy these demands, MONASH modelers have developed four modes of analysis: historical, decomposition, forecast and policy. Historical simulations are used to produce up-to-date data for MONASH models as well as estimates of trends in technologies, preferences and other naturally exogenous but unobservable variables. Decomposition simulations are used to explain periods of history and to place the effects of policy instruments in an historical context. Forecast simulations provide a baseline picture of likely future developments in the economy using extrapolated trends from historical simulations and forecasts from specialists on different parts of the economy. Policy simulations generate the effects of policies as deviations from baseline forecasts.

Section 2.6 summarizes the main ideas in the chapter.

To a large extent the sections are self-contained. Consequently, readers can choose their own path through the chapter. Some of the material in Section 2.4, particularly Section 2.4.6 on input-output accounting, would be difficult to read passively straight through. Input-output conventions are important, but tortuous and slippery. We hope that by scanning this subsection readers will get an idea of what is involved. They may then find it useful to return to the material if they are constructing or assessing a detailed policy-relevant model.

## 2.2 TELLING A CGE STORY

One of our graduate students recently asked us how to cope with skeptics: who will not believe anything from a model unless all the parameters are estimated by time-series econometrics; who harp on about the input-output data being outdated; who highlight what they see as the absurdity of competitive assumptions and constant returns to scale; who insist that general equilibrium means that all markets clear, thus ruling out real-world phenomena such as involuntary unemployment; and who claim that, like a chain, CGE models are only as strong as their weakest part.

Our advice is to get the results up front. Do not start by telling the audience about general features of the model. The idea is to tell a story that is so interesting and engaging that general-purpose gripes about CGE modeling are at least temporarily forgotten in favor of genuine enquiry about the application under discussion. The assumptions that really matter for the particular application can then be drawn out. The aim is to lead the audience to an understanding of what specific things they need to believe about behavior and data if they are to accept the results and policy conclusions being presented.

Here, we will try to follow our own advice. We will tell a CGE story without explicitly describing the model. We will use BOTE calculations to identify assumptions and data items that matter for the results. We will rely on explanatory devices such as demand and supply diagrams that are accessible to all economists, not just those with a CGE background. Only when we have given an illustration of what a MONASH-style

CGE application can deliver will we turn our attention in the rest of the chapter to the technicalities of MONASH modeling.

Our illustrative CGE story concerns the effects on the US economy of tighter border security to restrict unauthorized immigration. This is a good CGE topic for two reasons. First, it is a contentious policy issue with many people in the political debate demanding greater government efforts to improve border security and reduce unauthorized immigration. Popular opinion is that unauthorized immigrants do economic damage to legal residents of the US by generating a need for increased public expenditures and by taking low-skilled jobs. However, these opinions may not be the whole story. This brings us to the second reason that tighter border security is a good CGE topic. To get beyond popular opinions we need to look at interactions between different parts of the economy (i.e. we need to adopt a general equilibrium approach). We need to quantify the effects of varying the supply of unskilled foreign workers; on wage rates and employment opportunities of US workers in different occupations; on output, employment and international competitiveness in different industries; on public sector budgets; and on macroeconomic variables including the welfare of legal US residents.

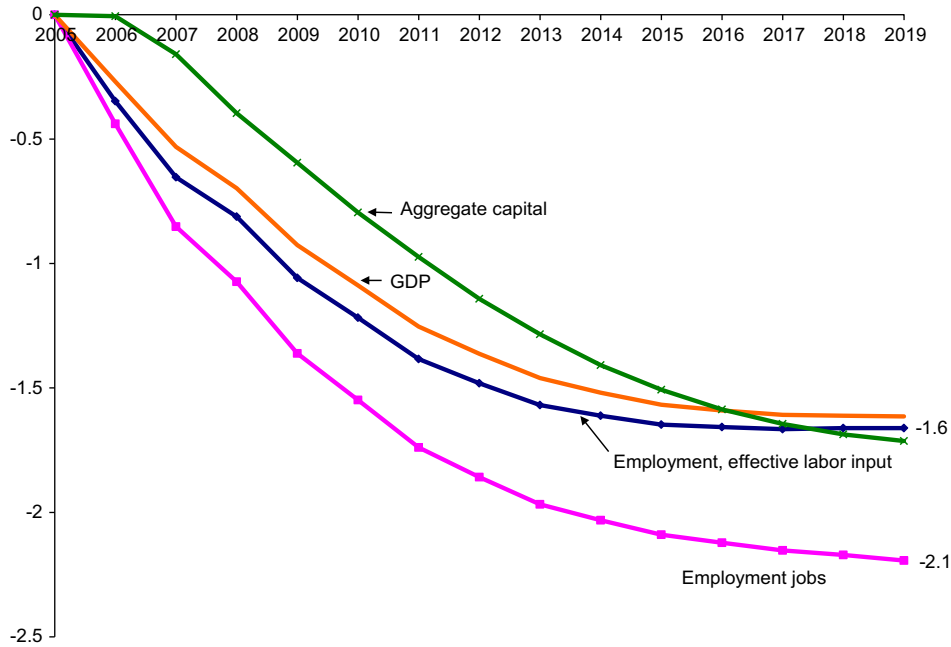
### **2.2.1 Tighter border security**

In 2005 there were about 7.3 million unauthorized foreign workers holding jobs in the US, about 5% out of total employment of 147 million. On business-as-usual assumptions unauthorized employment was expected to grow to about 12.4 million in 2019, about 7.2% out of total employment of 173 million. As unauthorized immigrants have low-paid jobs, their share in the total wage bill is less than their employment share. In the business-as-usual forecast, their wage bill share goes from 2.69% in 2005 to 3.64% in 2019.

In our CGE policy simulation we analyze the effects of a reduction in unauthorized employment caused by a restriction in supply. Specifically, we imagine that starting in 2006 the US implements a successful policy of tighter border security that has a long-run effect (2019) of reducing unauthorized employment by 28.6%: from 12.4 million in the baseline (business-as-usual situation) to 8.8 million in the policy situation. We have in mind policies that increase the costs and dangers of unauthorized entry to the US. These policies are represented in our model as a preference shift by foreign households against US employment. However, the exact nature and size of the policy is not important. Our focus is on the long-run effects of a substantial reduction in supply of unauthorized employment, however caused.

#### **2.2.1.1 Macroeconomic effects**

In the long run, we would not expect a policy implemented in 2006 to have a significant effect on the employment rate of legal workers. Thus, we would expect the policy to reduce total employment in 2019 by about 3.6 million ( $= 12.4 - 8.8$ ). That is, we would expect a reduction in total employment in the US of about 2.1% ( $= 100 * 3.6/173$ ).



**Figure 2.1** GDP, employment and capital with tighter border security (percentage deviations from baseline).

This is confirmed by the ‘Employment jobs’ line in Figure 2.1 that shows results from our CGE model for the effects on employment of the tighter border security policy as percentage deviations from the business-as-usual forecast.

Higher up the page in Figure 2.1 we can see the line showing deviations in ‘Employment, effective labor input’. In this measure, aggregate employment falls if the economy gains a job in a low-wage occupation but loses a job in a high-wage occupation. Under the assumption that wage rates reflect the marginal product of workers, deviations in effective labor input show the effects of a policy on the productive power of the labor input. With unauthorized immigrants concentrated mainly in low-wage occupations it is not surprising that Figure 2.1 shows smaller percentage reductions in effective labor input than in number of jobs. Whereas our tighter-border policy reduces jobs in the long run by 2.1%, it reduces effective labor input by only 1.6%.

In the long run, we would not expect a tighter border-security policy to have an identifiable effect on the US capital/labor ratio, that is the amount of buildings and machines used to support each unit of effective labor input. Underlying this expectation is the assumption that rental per unit of capital equals the value of the marginal product of capital:

$$\frac{Q}{P_g} = A * F_k \left( \frac{K}{L} \right), \quad (2.1)$$



where  $Q$  is the rental rate for a unit of capital,  $P_g$  is the price of a unit of output (the price deflator for GDP),  $A$  represents technology,  $K$  and  $L$  are aggregate inputs of capital and effective labor, and  $A * F_k$  is a monotonically decreasing function derived by differentiating an aggregate constant-returns-to-scale production function  $[A * F(K,L)]$  with respect to  $K$ .

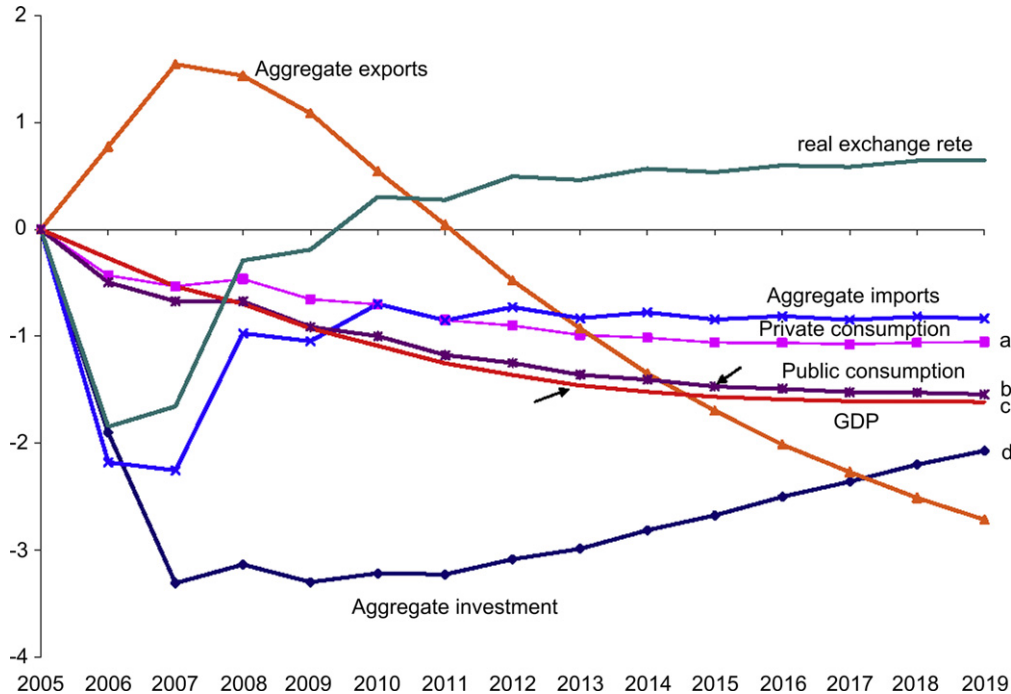
On the assumption that the cost of making a unit of capital (the asset price) moves in line with the price of a unit of output, the left-hand side of (2.1) is closely related to the rate of return on capital. In the long run we would not expect changes in border policy to affect rates of return.<sup>5</sup> These are determined by interest rates and perceptions of risk, neither of which is closely linked to border policy. Thus we would expect little long-run effect on the left-hand side of (2.1). On the right-hand side we would not expect any noticeable impact of border policy on US technology, represented by  $A$ . We can conclude that  $K/L$  will not be affected noticeably by changes in border policy. This is confirmed in Figure 2.1 where the long-run reduction of 1.6% in effective labor input is approximately matched by the long-run percentage reduction in capital. Figure 2.1 shows that the long-run deviation in GDP is also about  $-1.6\%$ . This is consistent with both  $K$  and  $L$  having long-run deviations of about  $-1.6\%$ , together with our assumption that border policy does not affect technology ( $A$ ).

Figure 2.2 shows the deviations in the expenditure components of GDP. In the long run these are all negative and range around that for GDP. The long-run deviations in private consumption, public consumption and imports are less negative than that for GDP while those for exports and investment are more negative than that for GDP.

We can understand these results as a sequence. The first element is that tighter border security improves the US terms of trade. This is a benefit from having a 1.6% smaller economy that demands less imports (thereby lowering their price) and supplies less exports (thereby raising their price). The second element is that terms-of-trade improvement allows private and public consumption to rise (as shown in Figure 2.2) relative to GDP. This is because an improvement in the terms of trade increases the prices of the goods and services produced by the US relative to prices of the goods and services consumed by the US, allowing the US to sustain a higher level of consumption for any given level of output (GDP).

The third element in understanding the long-run results in Figure 2.2 concerns investment. In the very long run, the change in immigration policy that we are considering will have little identifiable effect on the growth rate of labor input. Consequently it will have little effect on the growth rate of capital and therefore on the

<sup>5</sup> As will be explained shortly, there is a long-run increase in the terms of trade. Despite this, the cost of making a unit of capital (which includes import prices but not export prices) does not fall relative to  $P_g$  (which includes export prices but not import prices). This is mainly because in the long run the construction industry suffers an increase in its labor costs relative to other industries reflecting its intensive use of unauthorized labor.



**Figure 2.2** Expenditure aggregates with tighter border security (percentage deviations from baseline).

ratio of investment to GDP.<sup>6</sup> As can be seen in Figure 2.1, the deviation line for capital is still falling slightly in 2019 indicating that the capital stock has not fully adjusted to the 1.6% reduction in labor input.<sup>7</sup> With capital still adjusting downwards in 2019, the investment to GDP ratio is still below its eventual long-run level. However, in terms of contributions to GDP, the positive gaps in 2019 between the consumption and GDP deviations outweigh the negative gap between the investment and GDP deviation (the **ac** and **bc** gaps in Figure 2.2 weighted by private and public consumption easily outweigh the **dc** gap weighted by investment). This explains why the long-run deviation in imports in Figure 2.2 is less negative than that in exports.

The fourth element concerns the long-run relationships between the GDP deviation and the trade deviations. Although we have now understood why Figure 2.2 shows a long-run increase in imports relative to exports, we have not explained why these two

<sup>6</sup> The rate of growth of capital is given by  $k = (I/Y) * (Y/K) - \delta$ , where  $I$  and  $Y$  are investment and GDP, and  $\delta$  is the rate of depreciation (treated as a constant). Under our assumptions, the change in immigration policy does not affect  $k$  or  $Y/K$  in the long run. Therefore, it does not affect  $I/Y$ .

<sup>7</sup> It is apparent that the  $K/L$  ratio is heading to a slightly lower long-run value than in the baseline. This is because the cost of making a unit of capital increases slightly in the long run relative to the price deflator for GDP, reflecting the intensive use of unauthorized labor in the construction industry.

deviations straddle the GDP deviation. If a reduction in the supply of unauthorized immigrants were particularly harmful (cost increasing) to export-oriented industries then it would be possible for both the import and export deviation lines at 2019 to be below that of GDP. Similarly, if a reduction in the supply of unauthorized immigrants were particularly harmful to import-competing industries then it would be possible for both the import and export deviation lines at 2019 to be above that of GDP. As shown in [Dixon \*et al.\* \(2011\)](#), the long-run effects of reduced unauthorized immigration on the industrial composition of activity are quite small, with no pronounced bias against or in favor of either export-oriented or import-competing industries. The lack of bias is a consequence of unauthorized employment being spread over many industries and representing only a small share of costs in almost all industries. Thus, the required gap between imports and exports is achieved with the import deviation line above that of GDP in the long run and the export deviation line below that of GDP.

Two further points about the effects of reduced unauthorized immigration on trade are worth noting. (i) The increase in imports relative to exports is in quantity terms. With an improvement in the terms of trade there is no deterioration in the balance of trade. (ii) The increase in imports relative to exports is facilitated, as shown in [Figure 2.2](#), by a long-run increase in the real exchange rate (an increase in the nominal exchange rate relative to the foreign/US price ratio).

The final aspect of the long-run results in [Figure 2.2](#) that we will explain is the relative movements in public and private consumption. Public consumption falls relative to private consumption because consumption of public goods by unauthorized immigrants is high relative to their consumption of private goods. In the baseline forecast for 2019, unauthorized immigrants account for 3.7% of public consumption, but only 2.4% of private consumption.

The short-run results in [Figure 2.2](#) are dominated by the need for the economy to adjust to a lower capital stock than it had in the baseline forecast. In the short run, the policy causes a relatively sharp reduction in investment. With US investment at the margin being financed mainly by foreigners, a reduction in investment weakens demand for the US dollar. Consequently, a reduction in investment weakens the US exchange rate. This temporarily stimulates exports and inhibits imports. As the downward adjustment in the capital stock is completed, investment recovers, causing the real exchange rate to rise, exports to fall and imports to rise.

### **2.2.1.2 Effects on the occupational composition of legal employment**

The starting point for the explanation of the long-run macroeconomic results was the finding that a 28.7% cut in unauthorized employment reduces effective labor input by 1.6%. But why 1.6? Recall that in the baseline forecast the share of the US wage bill accounted for by unauthorized employment in 2019 is 3.64%. This suggests that a 28.7% cut in unauthorized employment should reduce effective labor input by only

1.0% ( $= 3.64 * 0.287$ ). The explanation of the discrepancy (1.0 versus 1.6) hinges on changes in the occupational mix of legal US employment.

Table 2.1 gives occupational data for 2005 and deviation results for 2019. Column (1) shows the share of unauthorized immigrants in the wage bill of each US occupation. The occupational classification was chosen to give maximum detail on employment of unauthorized immigrants, with about 90% of their employment being spread across the first 49 occupations. The last occupation, ‘Services, other’, accounts for about 60% of US employment, but only 10% of unauthorized employment. Columns (2) and (3) show the long-run effects of the supply-restriction policy on employment and real wage rates of legal US workers by occupation. In broad terms, the employment results in Table 2.1 show a long-run transfer of legal workers from ‘Services, other’, an amalgam of predominantly high-skilled, high-wage jobs, to the occupations that currently employ large numbers of unauthorized immigrants. The correlation coefficient between the deviations in jobs for legal workers (column 2) and unauthorized shares (column 1) is close to one. In occupations vacated by unauthorized immigrants, legal workers not only gain jobs, but also benefit from significant wage increases. The correlation coefficient between the employment and wage results in columns (2) and (3) is also close to one. The long-run change in occupational mix implied by column (2) does not mean that existing US workers change their occupations. For each occupation, restricting the supply of unauthorized workers presents legal workers with opportunities to replace unauthorized workers. On the other hand, the economy is smaller, generating a negative effect on employment opportunities for legal workers. The positive replacement effect dominates in the low-wage occupations that currently employ large numbers of unauthorized immigrants. The negative effect of a smaller economy dominates in high-wage occupations that currently employ few unauthorized immigrants. Thus, there is an increase in vacancies in low-wage occupations relative to high-wage occupations, allowing low-wage occupations to absorb an increased proportion of new entrants to the workforce and unemployed workers. Another way of understanding the change in the occupation mix of legal workers is to recognize that the labor market involves job shortages. At any time, not everyone looking for a job in a given occupation can find a job in that occupation. So people settle for second best. The college graduate who wants to be an economist settles for a job as an administrative officer; the high-school graduate who wants to be a police officer settles for a job in private security; the unemployed person who wants to be a chef settles for a job as a short-order cook; and so on. Through this shuffling process, a reduction in supply of unauthorized immigrants reduces the skill composition of employment of legal workers. It lowers the contribution of these workers to effective labor input, explaining the 1.0 versus the 1.6 discrepancy. We refer to this as a negative Occupation-mix effect. The idea of an Occupation-mix effect will be familiar to students of the history of US immigration. As described by Griswold (2002, p. 13), the inflow of low-skilled immigrants early in the twentieth

**Table 2.1** Occupational data for 2005 and deviation results for 2019

Occupation	Unauthorized immigrants: % of labor costs in 2005	% deviation in 2019	
		Legal jobs	Legal real wage
	(1)	(2)	(3)
1. Cooks	15.6	4.20	1.89
2. Grounds maintenance	24.8	7.45	3.19
3. House keeping and cleaning	22.0	6.56	2.82
4. Janitor and building cleaner	10.4	2.31	1.19
5. Miscellaneous agriculture worker	34.3	10.70	4.55
6. Construction laborer	23.9	7.10	3.16
7. Transport packer	24.6	7.37	3.19
8. Carpenter	15.1	3.90	1.92
9. Transport laborer	7.2	1.09	0.71
10. Cashier	4.7	0.31	0.43
11. Food serving	6.4	0.88	0.62
12. Transport driver	4.0	−0.09	0.25
13. Waiter	5.7	0.64	0.53
14. Production, miscellaneous assistant	8.3	1.07	0.72
15. Food preparation worker	13.3	3.42	1.61
16. Painter	24.9	7.46	3.31
17. Dishwasher	22.7	6.83	2.86
18. Construction, helper	24.8	7.42	3.30
19. Retail sales	2.4	−0.50	0.11
20. Production, helper	20.4	5.54	2.52
21. Packing machine operator	23.6	6.88	3.01
22. Butchers	21.0	6.20	2.74
23. Stock clerk	4.6	0.26	0.40
24. Child care	5.2	0.56	0.51
25. Miscellaneous food preparation	14.5	3.80	1.74
26. Dry wall installer	35.8	11.43	4.87
27. Nursing	2.8	−0.01	0.29
28. Industrial truck operator	8.5	1.47	0.87
29. Transport, cleaners	15.8	4.24	1.93
30. Automotive repairs	6.3	0.88	0.64
31. Sewing machine operator	18.8	4.95	2.39
32. Concrete mason	22.6	6.61	3.00
33. Roofers	28.2	8.64	3.78
34. Plumbers	7.1	1.07	0.80
35. Personal care	5.7	0.91	0.66
36. Shipping clerk	5.2	0.35	0.43
37. Brick mason	22.5	6.56	2.97
38. Carpet installer	21.4	6.21	2.82
39. Laundry	15.5	4.22	1.93

(Continued)

**Table 2.1** Occupational data for 2005 and deviation results for 2019—cont'd

Occupation	% deviation in 2019		
	Unauthorized immigrants:		
	% of labor costs in 2005	Legal jobs	Legal real wage
	(1)	(2)	(3)
40. Other production workers	9.1	1.57	0.91
41. Maintenance and repairs	2.2	−0.71	−0.01
42. Repair, helper	16.8	4.56	2.09
43. Welder	6.2	0.31	0.41
44. Supervisor, food preparation	3.4	−0.20	0.22
45. Construction supervisors	3.4	−0.27	0.27
46. Farm/food/clean, other	6.1	0.61	0.53
47. Construction, other	5.5	0.38	0.49
48. Production, other	4.6	−0.11	0.21
49. Transport, other	3.2	−0.40	0.13
50. Services, other	0.4	−1.27	−0.13
Total	2.6	−0.16	−0.46

century induced native-born US residents to complete their education and enhance their skills. In our terms, that was a positive Occupation-mix effect.

Before leaving Table 2.1, it is worth commenting on the deviations shown in the ‘Total’ row. The reduction (0.16%) in employment of legal workers is caused by the shift in the composition of their employment towards low-skilled occupations. These occupations have relatively high equilibrium rates of unemployment, which we have assumed are unaffected by immigration policy. It is sometimes asserted that cuts in employment of unauthorized immigrants would reduce unemployment rates of low-skilled legal workers. While our modeling suggests that there would be increases in the number of jobs for legal workers in low-skilled occupations, this does not mean that unemployment rates in these occupations would fall. With cuts in unauthorized immigration, low-skilled legal workers might find themselves under increased pressure from higher-skilled workers who can no longer find vacancies in higher-skilled occupations.

The overall reduction of 0.46% in the wage rate of legal workers seems surprising at first glance. Column (3) of Table 2.1 shows an increase or a negligible decrease in wage rates for legal workers in all occupations except ‘Services, other’ in which the wage rate is reduced by 0.13%. However, the average hourly wage rate of legal workers is reduced by the shift in the occupational composition of their employment to low-wage jobs.

### 2.2.1.3 Effects on the welfare of legal households

The headline number that policy makers are often looking for from a CGE study is the effect on aggregate welfare. In the present study, we take this as referring to long-run

**Table 2.2** Long-run (2019) percentage effects of tighter border security on consumption of legal residents

F1	Direct effect	−0.29
F2	Occupation-mix effect	−0.31
F3	Legal-employment effect	−0.11
F4	Capital effect	−0.24
F5	Public-expenditure effect	0.17
F6	Terms-of-trade effect	0.23
BOTE totals		−0.55
CGE result		−0.52

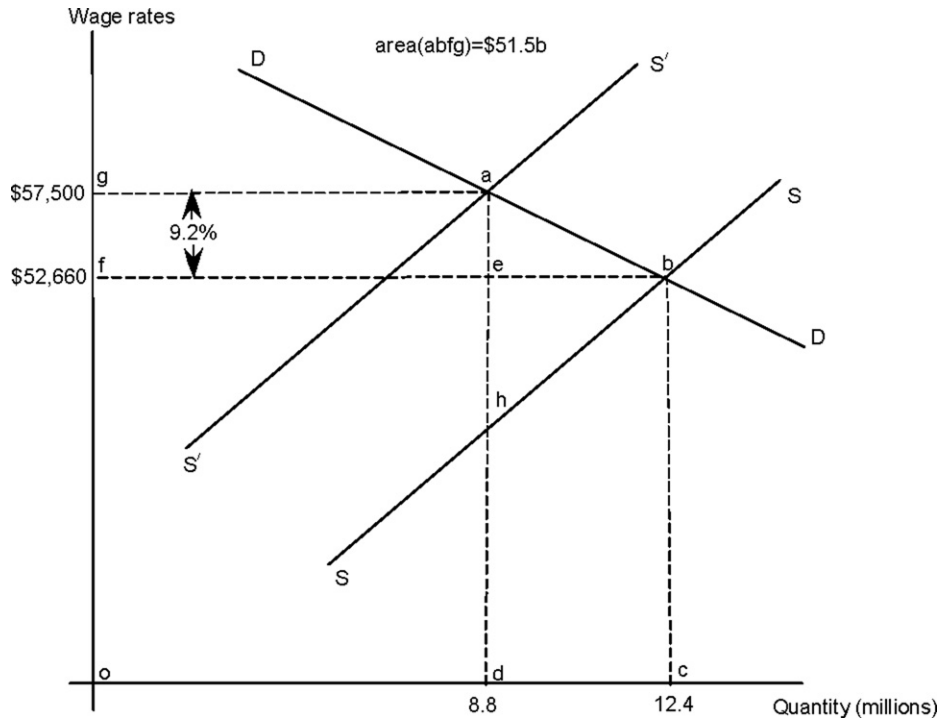
(2019) private and public consumption by legal US residents. We find that a reduction of 28.7% in unauthorized employment caused by tighter border security would generate a sustained annual welfare loss for legal residents of 0.52% (about \$80 billion in 2009 dollars).

This result can be explained in terms of the six factors indicated in Table 2.2 by F1–F6. As detailed in Dixon *et al.* (2011), each of these factors can be quantified by a BOTE calculation. The total of the BOTE calculations (−0.55) is an accurate estimate of the CGE result (−0.52). This gives us confidence that we have adequately identified the data and mechanisms in our model that explain the result. Here, we briefly describe the factors and their quantification.

#### F1: Direct effect

With a reduction in supply, the wage rate of unauthorized workers will rise. This is illustrated in Figure 2.3 in which DD is the demand curve for unauthorized labor in 2019, SS is the supply curve in the baseline forecast and S'S' is the supply curve with the tighter security policy in place. The numbers shown in the diagram are taken from our simulation: the policy reduces unauthorized employment in 2019 from 12.4 to 8.8 million and increases annual wage rates for unauthorized workers by 9.2%, from \$52,660 to \$57,500 (2019 dollars).

If workers are paid according to the value of their marginal product then the loss in output (represented by GDP) from reducing employment is the change in the area under the demand curve, area (abcd) in Figure 2.3. The change in the total cost to employers of unauthorized immigrants is area (gaef), the increase in costs associated with the increase in the unauthorized immigrant wage rate, minus area (ebcd), the reduction in costs associated with employment of less unauthorized immigrants. Ignoring taxes, the analysis so far suggests that the Direct effect of cutting illegal employment (the change in GDP less the change in the costs of employing illegal workers) is a loss represented by area (abfg). As indicated in Figure 2.3, this area is worth \$51.5 billion. Taxes complicate the situation in two ways. (i) The change in the area under the demand curve is an



**Figure 2.3** Demand for and supply of illegal immigrants in 2019.

underestimate of the loss in GDP because indirect taxes mean that wage rates are less than the value of the marginal product of workers. (ii) Unauthorized immigrants pay income taxes. Consequently, area **(ebcd)** overstates the saving to the US economy associated with paying wages to 28.6% fewer unauthorized immigrants and area **(gaef)** overstates the cost to the US economy of paying higher wage rates to unauthorized immigrants who remain in employment. After adjusting for taxes, the final estimate that [Dixon \*et al.\* \(2011\)](#) obtained for the Direct effect was a loss of \$77.3 billion. This causes a 0.29% reduction in consumption by legal households (row 1, [Table 2.2](#)).

## F2: Occupation-mix effect

Restricting the supply of unauthorized immigrants changes the occupational mix of employment of legal workers, reducing their average hourly wage rate by 0.46% ([Table 2.1](#)). In the baseline forecast for 2019, wages are 66% of the total income of legal residents.<sup>8</sup> A 0.46% reduction in average wage rates translates into a 0.31% ( $= 0.46 \times 0.66$ ) reduction in the ability of legal residents to consume private and public goods.

<sup>8</sup> This is GNP (i.e. GDP less net income flowing to foreign investors) minus post-tax income accruing to unauthorized immigrant.



**F3: Legal-employment effect**

As explained in [Section 2.2.1.2](#), we assume that equilibrium rates of unemployment are higher for low-skilled occupations than for high-skilled occupations, leading in our simulation to a reduction in legal employment of 0.16% ([Table 2.1](#)). With wages being 66% of the total income of legal residents, this reduces their income and consumption by 0.11%.

**F4: Capital effect**

If a change in immigration policy had no effect on savings by legal residents (including the government) *up to* 2019, then it would have no effect on US ownership of capital *in* 2019. In this case, if a change in immigration policy led to a reduction in the stock of capital in the US, then it would lead to a corresponding reduction in the stock of foreign-owned capital, with little effect on capital income accruing to legal households. Nevertheless, they would suffer a welfare loss because the US treasury would lose taxes paid by foreign owners of US capital. Via the Direct effect and other negative effects in [Table 2.1](#), the tighter border-security policy reduces saving by legal households throughout the simulation period. Thus, in the policy run, legal households own less US capital in 2019 than they had in the baseline forecast and lose the full-income of this lost capital. As explained in [Section 2.2.1.1](#), the policy causes a long-run reduction in US capital stock of about 1.6%. This is split approximately evenly between reductions in foreign-owned and US-owned capital. Taking account of the tax effects of the loss of foreign-owned capital and the full-income effects of the loss of US-owned capital, we find that the 1.6% reduction in capital contributes  $-0.24\%$  to sustained long-run welfare of legal households.

**F5: Public-expenditure effect**

With a reduction in the number of unauthorized immigrants working in the US, the public sector would cut its expenditures, particularly on elementary education, emergency healthcare and correctional services. This would allow either cuts in taxes or increased provision of public services to legal households. This effect contributes 0.17% to sustained long-run welfare of legal households.<sup>9</sup> It should be noted that F5 encompasses only public sector expenditures and does not take account of taxes paid by illegal immigrants. These taxes are accounted for in F1 where we compute the Direct contribution of illegal immigrants to GDP net of their *post-tax* wages.

**F6: Terms-of-trade effect**

In our simulation, the cut in unauthorized immigration reduces the prices of the goods and services that are consumed in the US relative to the prices of goods and services that

<sup>9</sup> The underlying data on public expenditures on unauthorized immigrants were taken from [Rector and Kim \(2007\)](#) and [Strayhorn \(2006\)](#).

are produced in the US. In 2019, the policy-induced reduction in the price index for private and public consumption relative to that for GDP is 0.23%. This increases the consuming power of legal households by 0.23%. The main reason for the relative decline in the price of consumption is the improvement in the terms of trade, discussed in [Section 2.2.1.1](#). A terms-of-trade improvement generally reduces the price index for consumption (which includes imports, but not exports) relative to that for GDP (which includes exports, but not imports).

## 2.2.2 Engaging the audience: hoped-for reactions

When we present a CGE story to an audience we are hoping for certain reactions. We want the audience to engage on the topic, not on prejudices and general views about CGE modeling. Whether we get the desired reaction depends on how well we have told the story in terms of mechanisms that are accessible to people without a CGE background.

In the case of our tighter border-security story, we hope that the audience is enthusiastic enough to want to know about extensions. For example, what would happen if the reduction in unauthorized employment were achieved by restricting demand through more rigorous prosecution of employers rather than restricting supply? More radically, what would happen if we replaced unauthorized immigrants by low-skilled guest workers? If we have told our story sufficiently well then audiences or readers of our papers can go a long way towards answering these questions without relying on our model.

The effects of demand restriction can be visualized in terms of [Figure 2.3](#) as an inward movement in **DD** rather than **SS**. If the demand policy were scaled to achieve the same reduction in unauthorized employment as in the supply policy, then we would expect similar results for F2–F6, which depend primarily on the reduction in the number of unauthorized workers. At first glance we might expect the Direct effect (F1) for demand-side restriction to be more favorable than that for supply-side restriction: with demand-side restriction, wage rates for unauthorized workers fall rather than rise. However, when we think of the gap between the supply-restricted wage (**da** in [Figure 2.3](#)) and the demand-restricted wage (**dh**) as being absorbed by prosecution-avoiding activities, then we can conclude that even the Direct effect will be similar under the demand- and supply-side policies. Thus, on the basis of F1–F6 we would expect little difference in the effects of equally scaled demand- and supply-side policies. This is confirmed in [Dixon \*et al.\* \(2011\)](#).

The guest-worker question arose from comments by Dan Griswold of the Cato Institute (a free-trade think tank in Washington, DC). After seeing a presentation on the negative welfare result in [Table 2.2](#), he asked whether welfare would increase if there were more low-skilled immigrants employed in the US rather than less. This led to a consideration of a program under which US businesses could obtain permits to legally

employ low-skilled immigrants. In terms of [Figure 2.3](#), we can envisage such a program as shifting the supply curve outward — at any given wage, more low-skilled immigrants would be willing to enter the US under a guest-worker program than under the present situation in which they incur considerable costs from illegal entry. The outward shift in the supply curve would reverse the signs of the six effects identified in [Table 2.2](#). As shown in [Dixon and Rimmer \(2009\)](#), a permit charge paid by employers could be used to control the number of low-skilled immigrants. It would also be a useful source of revenue, effectively transferring to the US treasury what are currently the costs to immigrants of illegal entry.

A second hoped-for reaction from audiences is well-directed questions about robustness and sensitivity. By this we mean questions about data items and parameter values that can be identified from our story as being important for our results. In presentations of our work on unauthorized immigration, we welcome questions concerning: our baseline forecasts for unauthorized employment (7.3 million in 2005 growing to 12.4 million in 2019); our data on the occupational and industrial composition of unauthorized and legal employment; our assumptions about the level of public expenditures associated with unauthorized employment; our choice of values for the elasticities of demand and supply for unauthorized workers; our adoption of a one-country framework that ignores effects outside the US; and other key ingredients of our story. When questions are asked about the existence, uniqueness and stability of competitive equilibria, then we suspect that our presentation has not effectively led the audience to understand how they should assess what we are saying.

A third satisfying reaction is curiosity about results for other dimensions. In our story here we have concentrated on macro and occupational results but for an interested audience we could also report industry results: our simulations were conducted at a 38-industry level. Greater industrial disaggregation can be introduced for organizations with a particular industry focus. For example, in a current study on unauthorized workers in agriculture, the US Department of Agriculture has expanded the industrial dimension to 70, emphasizing agricultural activities.

For readers of this chapter, we hope that our story has done two things. (i) We hope that it has demonstrated how CGE results can be explained in non-circular, macro-to-micro fashion. As in this story, we have found that in explaining most CGE results the best starting point is the inputs to the aggregate production function. In this case we started with what a cut in unauthorized immigration would do to aggregate employment. We then moved to the effect on aggregate capital. From there we went to the expenditure side of GDP. Eventually we worked down to employment by occupation. (ii) We hope that our story has aroused curiosity about some methodological issues. We have mentioned the business-as-usual forecast or baseline. How is this created in a MONASH model? We have reported policy-induced deviations. How are policy simulations conducted and what is their relationship to baseline simulations? We have

worked with considerable labor market disaggregation: 50 occupations by two birth-places by two legal statuses by 38 industries. How do we cope with large dimensions? We have shown year-by-year results. How do we handle dynamic mechanism such as capital accumulation that provide connections between years?<sup>10</sup> These are among the issues discussed in the rest of this chapter.

## 2.3 FROM JOHANSEN TO ORANI

Modern CGE modeling has not evolved from a single starting point and there are still quite distinct schools of CGE modeling. While [Johansen \(1960\)](#) made the first CGE model, there were several other largely independent starting points including the contributions of [Scarf \(1967, 1973\)](#), Jorgenson and associates (e.g. [Hudson and Jorgenson, 1974](#)) and the World Bank group (e.g. [Adelman and Robinson, 1978](#); [Taylor \*et al.\*, 1980](#)). Each of these later contributors adopted a style quite distinct from that of Johansen: different computational techniques, estimation methods, approaches to result analysis and issue focuses. In the case of the MONASH models, the ancestor is Johansen. His style was simple, effective and adaptable. It facilitated the inclusion of policy-relevant detail in CGE models and opened a path to result interpretation via BOTE explanations. In this section we describe Johansen's model and the extensions that were made in creating the ORANI model.

### 2.3.1 Johansen model

Johansen presented his 22-commodity/20-industry model of Norway as a system of 86 linear equations connecting 86 endogenous and 46 exogenous variables:

$$A_X * x + A_Y * y = 0, \quad (2.2)$$

where  $x$  and  $y$  are  $46 \times 1$  and  $86 \times 1$  vectors of exogenous and endogenous variables, and  $A_X$  and  $A_Y$  are matrices of coefficients of dimensions  $86 \times 46$  and  $86 \times 86$  built mainly from Norwegian input-output data for 1950 supplemented by estimates of income elasticities for consumer demand.

The 46 exogenous variables are: aggregate employment (1); aggregate capital (1); population (1); Hicks-neutral primary factor technical change in each industry (20); exogenous demand for each commodity (22); and the price of non-competing imports (1).<sup>11</sup> The 86 endogenous variables are: labor input and capital input by industry ( $2 \times 20$ ); output and prices by commodity ( $2 \times 22$ ); the average rate of return on capital

<sup>10</sup> For dynamic mechanisms that are specific to our immigration work, such as vacancy-induced occupational shuffling, we refer readers to [Dixon and Rimmer \(2010a\)](#).

<sup>11</sup> We are sometimes asked about the numéraire in Johansen's model. It is the nominal wage rate which is exogenously fixed on zero growth and then omitted from the model.

(1); and aggregate private consumption (1). All of the variables in Johansen's system are growth rates or percentage growth rates. Johansen derived the equations in (2.2) from underlying levels forms. For example, in (2.2) he represented the Cobb–Douglas relationship:

$$Z_j = N_j^{\gamma_j} * K_j^{\beta_j} * e^{\epsilon_j * t}, \quad (2.3)$$

between the output in industry  $j$  ( $Z_j$ ) and labor and capital inputs ( $N_j$  and  $K_j$ ) as:

$$z_j - \gamma_j * n_j - \beta_j * k_j - \epsilon_j = 0, \quad (2.4)$$

where  $z_j$ ,  $n_j$  and  $k_j$  are percentage growth rates in  $Z_j$ ,  $N_j$  and  $K_j$ , and  $\epsilon_j$  is the rate of technical progress.

From (2.2) Johansen solved his model, i.e. expressed growth rates in endogenous variables in terms of exogenous variables, as:

$$y = b * x, \quad (2.5)$$

where  $b$  is the  $86 \times 46$  matrix given by:

$$b = -A_Y^{-1} * A_X. \quad (2.6)$$

Johansen was fascinated by the  $b$  matrix in (2.6) and devoted much of his book to discussing it. The  $b$  matrix shows the sensitivity (usually an elasticity) of every endogenous variable with respect to every exogenous variable. Johansen regarded the 3956 entries in the  $b$  matrix as his basic set of results and he looked at every one of them. His management strategy for coping with 3956 results was to use a simple one-sector BOTE model as a guide. The BOTE model told him what to look for and what to expect in his full-scale model.

For example, the BOTE model suggested that the entries in the  $b$  matrix referring to the elasticities of industry outputs with respect to movements in aggregate capital and labor should lie in the (0,1) interval. This follows from a macro version of equation (2.4).<sup>12</sup> With one exception, this expectation was fulfilled: the  $b$  matrix shows a negative entry for the elasticity of equipment output with respect to an increase in aggregate employment. Following up and explaining exceptions is an important part of the BOTE methodology. In this way we can locate result-explaining mechanisms in the full model that are not present in the BOTE model. In other words, we can figure out what the full model knows that the BOTE model does not know. In the case of the equipment-output/employment elasticity, the explanation of the negative result is that an increase in aggregate employment changes the composition of the economy's capital stock in favor of structures and against equipment. This morphing of the capital stock reduces

<sup>12</sup> That is  $z = \gamma * n + \beta * k + \epsilon$ , where  $\gamma$  and  $\beta$  are parameters with values between 0 and 1.

maintenance demand for equipment, thereby reducing the output of the equipment industry.<sup>13</sup>

One of the most interesting parts of  $b$  is the submatrix relating movements in industry outputs to movements in exogenous demands. At the time when Johansen was writing, Leontief's input-output model, with its emphasis on input-output multipliers, was the dominant tool for quantitative multisectoral analysis. In Leontief's model, if an extra unit of output from industry  $j$  is required by final users, then production in  $j$  must increase by at least one unit and production in other industries will increase to provide intermediate inputs to production in  $j$ . Further rounds of this process can be visualized with suppliers to  $j$  requiring extra intermediate inputs. Thus, in Leontief's picture of the economy, developed in the depressed 1930s,<sup>14</sup> industries are in a complementary relationship, with good news for any one industry spilling over to every other industry. Johansen, working in the booming 1950s challenged this orthodoxy. His industry-output/exogenous-demand submatrix implies diagonal effects that in most cases are less than one and off-diagonal effects and are predominantly negative. Rather than emphasizing complementary relationships between industries, Johansen emphasized competitive relationships. In Johansen's model, expansion of output in one industry drags primary factors away from other industries. Only where there are particularly strong input-output links did Johansen find that stimulation of one industry (e.g. food) benefits another industry (e.g. agriculture).

Having examined the  $b$  matrix, Johansen used it to decompose movements in industry outputs, prices and primary factor inputs into parts attributable to observed changes in exogenous variables. In making his calculations he shocked all 46 exogenous variables with movements representing average annual growth rates for the period around 1950. In discussing the results of his decomposition exercise, Johansen paid particular attention to agricultural employment. This was a contentious issue among economists in 1960. On the one hand, diminishing returns to scale suggested that relative agricultural employment would grow with population and perhaps even with income despite low expenditure elasticities for agricultural products. On the other hand, agriculture was experiencing rapid technical progress, suggesting that employment in agriculture might not only fall as a share of total employment but might even fall in absolute terms. Johansen was able to separate and quantify these conflicting forces. He found that growth in capital, employment and population around 1950 caused relatively strong increases in agricultural employment, consistent with diminishing returns to scale interacting with increased consumption of food. However, the dominant effect on agricultural employment was technical change. This was strongly negative, leaving agriculture with net declining employment.

<sup>13</sup> For a fuller explanation of Johansen's negative equipment-output/employment elasticity, see Dixon and Rimmer (2010c, p. 7).

<sup>14</sup> Leontief (1936).

In another exercise, Johansen performed a validation test. He compared observed average annual growth rates around 1950 in endogenous variables such as industry outputs, employment and capital inputs with the total effects calculated in his decomposition exercise. He used this comparison to pinpoint weaknesses in his model and to organize a discussion of real-world developments. For agriculture, he found that the computed growth rate in output closely matched reality, but that the computed growth rate in employment was too high while that in capital was too low. This led to a discussion of reasons, not accounted for in the model, for exodus of rural workers to the cities. For forestry, the computed growth rates for output and primary factor inputs were too high. He thought that the income elasticity of demand for forestry products may have been set too high and also that there may have been a taste change, not included in his model, against the use of forestry products as fuel. By going through his results in this way, Johansen developed an agenda for model improvement.

### 2.3.2 Building on the Johansen legacy: creating the ORANI model

MONASH modelers owe an enormous intellectual debt to Johansen.<sup>15</sup> (i) They presented their models as linear systems in changes and percentage changes, Johansen's equation (2.2). This simplified the interpretation of their models and facilitated teaching. (ii) They adopted Johansen's solution equations (2.5) and (2.6). This enabled them to solve models in the 1970s and 1980s with much larger dimensions than was possible with other styles of CGE modeling. Johansen's use of BOTE models was also taken up and extended in the MONASH paradigm. (iii) MONASH modelers followed Johansen in using the  $b$  matrix to understand properties of their models, to explain periods of history via decomposition analysis (see Section 2.5) and to perform validation exercises (see Dixon and Rimmer in Chapter 19 of this Handbook).

While Johansen's techniques were simple and effective, adopting them came at a cost. His solution equations (2.5) and (2.6) give only an approximation to effects implied by the underlying non-linear model: (2.5) and (2.6) produce solutions that are subject to linearization errors.<sup>16</sup> At the time that Johansen was developing his model, incurring this cost was a computational necessity. Later CGE pioneers were keen to avoid linearization errors and perhaps this caused them to overlook the strengths of Johansen's approach to CGE modeling. In any case, sustained development of Johansen's style of CGE modeling was not undertaken until work commenced in Australia on the ORANI model, a decade

<sup>15</sup> In their overview of the IMPACT Project's first 10 years of operation, Powell and Lawson (1990, pp. 265–266) identify the decision to use Johansen strategies as a key ingredient in the Project's success.

<sup>16</sup> Johansen recognized this problem and reported (Johansen, 1974) experience with a method implemented in the late 1960s by Spurkland (1970) for calculating accurate solutions. Spurkland's method used (2.5) and (2.6) to obtain an approximate solution, and then moved to an accurate solution via a general non-linear equation method such as Newton's algorithm. Spurkland's method sacrificed Johansen's simplicity and was rather awkward to implement. It was not widely adopted.

and a half after the publication of Johansen's book.<sup>17</sup> As described in [Section 2.4.1](#), linearization errors were avoided in ORANI and later MONASH models without sacrificing Johansen's simplicity and interpretability. This was done by adopting a multistep extension of Johansen's solution method.

The multistep solution method was not the only innovation in the creation of ORANI. As outlined in [Sections 2.3.2.1–2.3.2.4](#), other innovations were: the treatment of imports and competing domestic products as imperfect substitutes; the incorporation of policy-relevant detail requiring large dimensionality; allowance for closure flexibility; and inclusion of complex functional forms.

### **2.3.2.1 Imperfect substitution between imports and domestic products: the Armington specification**

Johansen paid little attention to trade, simply setting net exports exogenously for all commodities except non-competing imports, which were handled as Leontief inputs to production.<sup>18</sup> For a trade-focused model, a more elaborate approach is required. The builders of ORANI turned to [Armington \(1969, 1970\)](#) who had built a 15-country trade model in which each country produced just one good, but consumed all 15 goods, treating the goods from different countries as imperfect substitutes. In ORANI, imports were disaggregated by commodity rather than country of origin. For each using agent (industries, capital creators, households and government), imports of a commodity were specified as constant elasticity of substitution (CES) substitutes for the corresponding domestic commodity.

The import/domestic substitution elasticities (named Armington elasticities in [Dixon \*et al.\* 1982](#)) were econometrically estimated for about 50 commodities by [Alaouze \*et al.\* \(1977\)](#) and [Alaouze \(1977\)](#) using a quarterly database assembled for this purpose on import and domestic prices and quantities for the period 1968(2) to 1975(2). This work is summarized in [Dixon \*et al.\* \(1982, pp. 181–189\)](#).<sup>19</sup> With its Armington specification, ORANI produced results in which imports responded in a realistic manner to changes in the relative prices of imported and domestic goods, avoiding import-domestic 'flip-flop'. This refers to extreme and unrealistic movements in the share of a country's demands for a commodity that are satisfied by imports. It occurs in long-run simulations with models in which import/domestic price ratios are allowed to play a role in import/domestic

<sup>17</sup> There were some important one-off flurries using Johansen's techniques in the mid-1970s (see, e.g. [Taylor and Black, 1974](#); [Staelin, 1976](#); [Bergman, 1978](#); and [Keller, 1980](#)).

<sup>18</sup> This exogenous treatment of trade was also the approach of [Hudson and Jorgenson \(1974\)](#) for the US. [Adelman and Robinson \(1978\)](#) in their study of Korea set exports of some commodities exogenously and fixed the share of exports in domestic output for other commodities. For most imports, Adelman and Robinson fixed the import share in domestic demand. [Taylor \*et al.\* \(1980, chapter 7\)](#) in their study of Brazil exogenized exports and related imports to industry outputs and final demands via exogenous coefficients.

<sup>19</sup> For an overview on recent work on Armington elasticities and other elasticities used in modeling international trade, see [Hillberry and Hummels](#) in Chapter 18 of this Handbook.



choice and imported and domestically produced units of a commodity are treated as perfect substitutes. Flip-flop can also be a problem with exports. When export prices are taken as given and long-run supply curves are flat, there is a tendency for models to show extreme and unrealistic specialization in the commodity composition of exports. The ORANI modelers avoided export flip-flop by the introduction of downward-sloping export demand curves (Dixon *et al.* 1982; pp. 195–196; Dixon and Rimmer, 2002; pp. 222–225).<sup>20</sup> Even for a small country, downward-sloping export demand curves can be justified by attributing Armington behavior to foreigners (i.e. by assuming that they treat imports of any given commodity from different countries as imperfect substitutes).

Following ORANI, the Armington specification has been adopted almost universally in CGE models, although there is some dissatisfaction with this approach. The Armington specification with elasticity values in the empirically relevant range leads to negative terms-of-trade effects that outweigh efficiency gains for countries undertaking unilateral tariff cuts even from quite high levels (e.g. 30%) (Brown, 1987). This is worrying to people who believe that low tariffs are always better than high tariffs. For a discussion of the relevant issues, see Dixon and Rimmer (2010b). While no alternative to Armington for practical CGE modeling has emerged, incorporation of ideas from Melitz (2003) seems promising, see Fan (2008) and Balistreri and Rutherford in Chapter 24 in this Handbook. The Melitz specification introduces productivity differences between firms within industries. Efficiency effects of tariff cuts are increased by allowing for elimination of low-productivity firms. However, potentially large terms-of-trade effects remain.

### **2.3.2.2 Incorporation of policy-relevant detail requiring large dimensionality**

Policy makers want detail. They want results for identifiable industries (e.g. motor vehicle parts), not vague aggregates (e.g. manufacturing). They want results for regions, not just the nation. Consequently, ORANI was designed from its outset to encompass considerable detail. The first version had 113 industries (Dixon *et al.*, 1977) and was quickly endowed with a facility for generating results for Australia's eight states/territories (Dixon *et al.*, 1978).<sup>21</sup> Later, this facility was extended to 56 substate regions, Fallon (1982). The imperative of providing results that were persuasive in policy circles meant that ORANI was equipped not only with industry and regional detail, but also with detail in other areas that were normally ignored by academics. For example, from its outset ORANI had a detailed specification of margin costs (road transport, rail transport, air transport, water transport, wholesale trade and retail trade). Recognition of margin

<sup>20</sup> As recognized by Johansen (1974), Taylor and Black (1974) avoided extreme flip-flop in their trade-oriented model by adopting a short-run closure (fixed capital in each industry), thereby giving supply curves a positive slope. With his focus on long-run tendencies, Johansen could not adopt the Taylor and Black approach. Instead, he continued to treat exports and competitive imports exogenously.

<sup>21</sup> Also see Giesecke and Madden in Chapter 7 of this Handbook.

costs is important in translating the effects of tariff changes (that impact basic prices) into implications for purchasers prices (that influence demand responses). Attention to such details was important in providing results that could be believed by policy makers.

Detail expands dimensionality. In ORANI, the dimensions of the  $A_Y$  matrix in (2.2) were far too large to allow direct solution via (2.6). This dimensionality problem was overcome by a process of condensation in which high-dimension variables were substituted out of the computational form of the model. For example, consider the variable  $x(i, s, j, k, r)$ , which represents the percentage change in the use of margin commodity  $r$  (e.g. road transport) to facilitate the flow of commodity  $i$  from source  $s$  (domestic or imported) to industry  $j$  for purpose  $k$  (current production or capital creation). In a model with 100 commodities/industries and 10 margin commodities this variable has 400,000 components. These were explained in ORANI by 400,000 Johansen-style linear percentage change equations:

$$x(i, s, j, k, r) = x(i, s, j, k) + a(i, s, j, k, r), \quad (2.7)$$

where  $x(i, s, j, k)$  is the percentage change in flow  $(i, s, j, k)$  and  $a(i, s, j, k, r)$  is the percentage change in the use of margin  $r$  per unit of flow  $(i, s, j, k)$ . In many ORANI simulations variables such as  $a(i, s, j, k, r)$  were interpreted as changes in technology.

To reduce the computational dimensions of ORANI, (2.7) was used to substitute out  $x(i, s, j, k, r)$ , i.e. (2.7) was deleted and  $x(i, s, j, k, r)$  was replaced by the right-hand side of (2.7) wherever it appeared in the rest of the model. By this process, the dimensions of the matrix to be inverted in (2.6) were reduced to a manageable size: about  $200 \times 200$  in the 1977 version of ORANI and about  $400 \times 400$  in the 1982 version.<sup>22</sup>

While variables and equations disappear from a model during condensation, no information is lost. Results for eliminated variables can be recovered by backsolving using the eliminated equations. One implication of this is that eliminated variables are necessarily endogenous.

Through condensation in a Johansen linear framework, problems of dimensionality were largely removed. This gave two advantages: (i) the full dimensionality of available input-output tables could be used and (ii) computational—theoretical compromises were reduced. For example, in ORANI there was no inhibition on computational grounds about including a high-dimension equation such as (2.7) if this was considered the theoretically appropriate specification.

### 2.3.2.3 Closure flexibility in the Johansen framework

Johansen used just one closure. However, his framework was readily extended to encompass closure flexibility. This was done in ORANI by leaving the allocation of

<sup>22</sup> The condensations of the two versions are described in Sutton (1976, 1977) and Dixon *et al.* (1982, pp. 207–229).

variables between  $\gamma$  (endogenous) and  $x$  (exogenous) in Johansen's equation (2.2) as a user choice. This imparted an important degree of flexibility.

If, for example, the focus was on the short-run effects of a policy, then capital in each industry was treated exogenously, unaffected in the short-run. At the same time, rates of return were treated endogenously. Simulations conducted under this closure were thought to reveal effects that would emerge after about 2 years.<sup>23</sup> If a long-run focus were required, then the closure was reversed. It was assumed that deviations in rates of return would be temporary. Thus, in long-run simulations, rates of return were exogenous while capital stocks adjusted endogenously to allow rates of return to be maintained at their initial levels.

An early ORANI study that took advantage of closure flexibility was that by [Dixon et al. \(1979\)](#). This was commissioned by the Crawford Group, set up by the Australian Government in 1977 to report on macro and industry policies to achieve a broad-based industry and regional recovery from what was then a deeply recessed situation. To widen the appeal of the ORANI results and defuse criticism, simulations were conducted under two closures. In both closures real wages were treated exogenously, reflecting their determination in what was at the time a legalistic, centralized system that could produce outcomes with little resemblance to those that would be expected from market forces. The closures differed with respect to capital utilization and exports.

In what was referred to as a neoclassical closure, capital used in each industry was set exogenously to fully employ the capital available to the industry. Rental rates adjusted endogenously to ensure compatibility between demand for capital and the exogenously given levels of capital usage. Exports in the neoclassical closure were determined by the interaction of production costs in Australia and price-elastic foreign demands.

In what was referred to as a neo-Keynesian closure,<sup>24</sup> the rental rate on capital was treated as a profit mark-up and linked exogenously in each industry to variable costs per unit of production.<sup>25</sup> Capital in use was treated endogenously. Exports were assumed rigid and to make this computationally possible, a phantom export subsidy was endogenized for each commodity.

Despite these seemingly radical differences in closure, the policy implications of the two sets of simulations were the same: a combination of reduction in the real costs of employing labor and an expansion in demand offered the best prospect for a broad-based recovery. Real cost reduction would stimulate trade-exposed industries and regions while demand expansion would stimulate the rest of the economy. Naturally, the

<sup>23</sup> This was worked out by [Cooper and McLaren \(1983\)](#) who compared ORANI comparative static short-run results with those produced by a continuous-time macro model. See also [Breece et al. \(1994\)](#) and [Dixon \(1987\)](#).

<sup>24</sup> The terms neoclassical and neo-Keynesian have been used by a number of authors, but never in quite the same way. For a discussion of closure possibilities in early CGE models with associated nomenclature, see [Rattso \(1982\)](#) and [Robinson \(2006\)](#).

<sup>25</sup> For a more recent application of this idea in a dynamic setting, see [Dixon and Rimmer \(2011\)](#).

question arose as to what policies could reduce labor costs in an acceptable way while expanding demand. One answer, illustrated by ORANI simulations in [Corden and Dixon \(1980\)](#), was a wage-tax bargain under which workers forego wage increases in return for tax cuts or improvements in social capital. Such bargains were an important part of Australian economic policy in the 1980s.

In the 1990s, the idea of flexible closures was extended in dynamic MONASH models to allow for four modes of analysis: historical, decomposition, forecast and policy. These are described in [Section 2.5](#).

#### 2.3.2.4 Complex functional forms in the Johansen framework

Early CGE modelers outside the Johansen school worried that the use of the Johansen linear percentage-change format was limiting with respect to model specification. For example, [Dervis \*et al.\* \(1982, p. 137\)](#) comment that:

*Johansen linearized the general equilibrium model (in logarithms) and so was able to solve it by simple matrix inversion. ... Since then there have been advances in solution methods that permit CGE models to be solved directly for the levels of all endogenous variables and so permit model specifications that cannot easily be put into log-linear forms.*

Far from being limiting, the Johansen framework simplified the introduction into CGE modeling of the advanced functional forms that were being developed in this period. For example, consider the CRESH<sup>26</sup> cost-minimizing problem:

choose  $X_i, i = 1, \dots, n$

to minimize :

$$\sum_{i=1}^n P_i * X_i \quad (2.8)$$

subject to:

$$\sum_{i=1}^n \left( \frac{X_i}{Z} \right)^{h_i} * \frac{Q_i}{h_i} = \alpha, \quad (2.9)$$

where  $Z$  is output,  $P_i$  and  $X_i$  are input prices and quantities, and the  $Q_i$ ,  $h_i$  and  $\alpha$  are parameters, with the  $Q_i$  values being positive and summing to one and the  $h_i$  values being less than one but not precisely zero.

On the basis of problem (2.8)–(2.9) it is difficult to obtain an intuitive understanding of the input-demand functions: they have no closed form levels representation. Given values for  $h_i$ , values for  $Q_i$  and  $\alpha$  can be determined on the basis of input-output data, but

<sup>26</sup> CRESH was introduced as a generalization of CES by [Hanoch \(1971\)](#).

this is technically awkward. By contrast, the Johansen-style percentage change representation of the input-demand functions is readily interpretable and easily calibrated. As shown in Section 2.4.4, (2.8)–(2.9) leads to:

$$x_i = z - \sigma_i * (p_i - p), \quad i = 1, \dots, n, \quad (2.10)$$

where  $x_i$ ,  $z$  and  $p_i$  are percentage changes in the variables represented by the corresponding uppercase symbols,  $\sigma_i$  is a positive substitution parameter defined by  $\sigma_i = 1/(1 - h_i)$ , and  $p$  is a weighted average of the percentage changes in all input prices defined by:

$$p = \sum_{k=1}^n S_k^{\#} * p_k. \quad (2.11)$$

The weights  $S_k^{\#}$  are modified cost shares of the form:

$$S_k^{\#} = \frac{S_k * \sigma_k}{\sum_{i=1}^n S_i * \sigma_i}. \quad (2.12)$$

where  $S_k$  is the share of  $k$  in costs.

Reflecting constant returns to scale, (2.10) implies that a 1% increase in output, holding input prices constant, causes a 1% increase in demand for all inputs. An increase in the price of  $i$  relative to the average price of all inputs causes substitution away from  $i$  and towards other inputs. The sensitivity of demand for  $i$  with respect to its relative price is controlled by the parameter  $\sigma_i$ . If this parameter has the same value for all  $i$ , then (2.10) takes the familiar CES form. However, if we wish to introduce differences between inputs in their price sensitivity then this can be done by adopting different values for  $\sigma_i$ . Once values have been assigned for  $\sigma_i$ , calibration can be completed on the basis of cost shares ( $S_k$ ) from input-output data.

Dixon *et al.* (1992, pp. 124–148) give derivations of Johansen-style demand and supply equations for a variety of optimizing problems based on CES, CET, Translog, CRESH and CRETH functions. In all these cases, the Johansen-style input-demand functions or output-supply functions look like (2.10): the percentage change in the particular input (output) equals the percentage change in the relevant activity variable minus (plus) a substitution (transformation) term that compares the percentage change in the particular price with a share-weighted average over the percentage changes in the prices of all the substitutes (transformates).

More generally, all differentiable input demand functions and output supply functions can be represented in a Johansen format. Usually the Johansen representation is more transparent than the levels representation. Perhaps reflecting this, rapid progress was made in the adoption of sophisticated functional forms in the ORANI model.

## 2.4 EXTENDING JOHANSEN'S COMPUTATIONAL FRAMEWORK: THE MATHEMATICAL STRUCTURE OF A MONASH MODEL

This section is a broad technical overview of MONASH modeling. We start in [Section 2.4.1](#) by describing a MONASH model as a system of  $m$  equations in  $n$  variables. We emphasize two points: (i) the variables and equations are concerned with a single period, usually thought of as year  $t$ , and (ii) we always have an initial solution, i.e. a set of values for the  $n$  variables that satisfy the  $m$  equations. Starting from the initial solution, other solutions can be obtained by derivative methods. We describe the Johansen/Euler method used for MONASH models. [Section 2.4.2](#) shows how periods are linked in MONASH models to make them dynamic. [Section 2.4.3](#) establishes the existence of the initial solution for each year  $t$ . MONASH models are written largely as equations in which the variables are percentage changes in prices and quantities in year  $t$  away from their initial solution. The derivation of percentage change equations from underlying levels equations is discussed in [Section 2.4.4](#). An overview of the GEMPACK software used in building, solving and analyzing MONASH models is presented in [Section 2.4.5](#). [Section 2.4.6](#) provides some notes on the creation of a database for a MONASH-style CGE model.

### 2.4.1 Theory of the Johansen/Euler solution method

A MONASH model can be represented as a system of  $m$  equations in  $n$  variables:

$$F(X, Y) = 0, \quad (2.13)$$

where  $F$  is a vector of  $m$  functions,  $X$  is the vector of  $n - m$  variables chosen to be exogenous and  $Y$  is the vector of  $m$  variables chosen to be endogenous.

In discussing (2.13) it is convenient to assume that we are dealing with a national model with annual periodicity.<sup>27</sup> In such a model the vector  $(X, Y)$  includes flow variables for year  $t$  at the national level representing quantities and values of demands and supplies. Other variables in  $(X, Y)$  refer to stocks or levels at an instant of time, examples being capital stocks at the start of year  $t$  and at the end of year  $t$ , and the level of the exchange rate at the start and end of year  $t$ .  $(X, Y)$  also includes lagged variables, e.g. the lagged consumer price index for year  $t$ , which is the consumer price index for year  $t - 1$ .

The  $m$  equations include links between flow variables in year  $t$  provided by market-clearing conditions, zero-pure-profit conditions and demand and supply equations derived from optimizing problems. The equations also impose links between stock and

<sup>27</sup> MONASH-style regional and multicountry models are discussed by Giesecke and Madden in Chapter 7 and Hertel in Chapter 12 of this Handbook. MONASH models with quarterly periodicity can be found in [Adams \*et al.\* \(2001\)](#) and [Dixon \*et al.\* \(2010\)](#).

flow variables. For example, end-of-year capital stocks are linked to start-of-year capital stocks via investment and depreciation during the year. Lagged adjustment processes may be included among the equations. For example, wage rates in year  $t$  might be related to lagged consumer prices in year  $t$ .

This brings us to the first critical point in understanding the MONASH paradigm. A MONASH model is a system of equations connecting variables for year  $t$ . These can be current variables, lagged variables, stock variables or flow variables, but they are all variables for year  $t$ .

To solve the model for year  $t$  we need a method for computing the value for the  $Y$  vector in (2.13) corresponding to the year  $t$  value for the  $X$  vector. If (2.13) were small and sufficiently simple we might contemplate solving it explicitly to obtain the relationship:

$$Y = G(X), \quad (2.14)$$

However, in realistic practical CGE models (2.13) consists of many thousands of variables connected by non-linear relationships. In these circumstances, solution via discovery of an explicit form for  $G$  is out of the question.

This brings us to the second critical point in understanding the MONASH paradigm. While we can rarely have an explicit form for the solution function  $G$ , we can always have an initial solution, i.e. a vector  $(\bar{X}(t), \bar{Y}(t))$  that satisfies:

$$F(\bar{X}(t), \bar{Y}(t)) = 0 \text{ or equivalently } \bar{Y}(t) = G(\bar{X}(t)). \quad (2.15)$$

As will be discussed in Section 2.4.3,  $(\bar{X}(t), \bar{Y}(t))$  usually represents the situation in a particular year, often year  $t - 1$ . With an initial solution in place, and assuming that  $F$  is differentiable,<sup>28</sup> further solutions can be computed by derivative methods. These involve estimation of the partial derivatives,  $G_X$ , of the  $G$  function but not explicit representation of  $G$  itself. With  $G_X$  we can estimate the effects on  $Y$  of moving  $X$  from its initial value,  $\bar{X}(t)$ , to its required value for year  $t$ ,  $X(t)$ .

The derivative method used in MONASH models to move from the initial solution for year  $t$  to the final solution is the Johansen/Euler method.<sup>29</sup> We named this method in recognition of the contributions of Johansen (1960) who, as described in Section 2.3, applied a one-step version of it to solve his CGE model of Norway, and Euler, the eighteenth century mathematician who set out the theory of the method as an approach to numerical integration.<sup>30</sup>

<sup>28</sup> Non-differentiabilities associated with complementarity conditions are discussed in Horridge *et al.* in Chapter 20 of this Handbook.

<sup>29</sup> Other derivative methods are described in Dervis *et al.* (1982, pp. 491–496).

<sup>30</sup> Early followers of Johansen's one-step method include Taylor and Black (1974), Staelin (1976), Dixon *et al.* (1977), and Keller (1980). The multistep version or the Johansen/Euler method was developed by Dixon *et al.* (1982). Another early application of the multistep approach is Bovenberg and Keller (1984).

In Johansen/Euler computations we start by replacing the original system of non-linear equations in  $X$  and  $Y$  with a linear system in which the variables are changes in  $X$  and  $Y$ :

$$F_X(\bar{X}(t), \bar{Y}(t)) * \Delta X + F_Y(\bar{X}(t), \bar{Y}(t)) * \Delta Y = 0, \quad (2.16)$$

where  $F_X(\bar{X}(t), \bar{Y}(t))$  and  $F_Y(\bar{X}(t), \bar{Y}(t))$  are the  $m \times (n - m)$  and  $m \times m$  matrices of first-order partial derivatives of  $F$  with respect to  $X$  and  $Y$  evaluated at  $(\bar{X}(t), \bar{Y}(t))$  and  $\Delta X$  and  $\Delta Y$  are  $(n - m) \times 1$  and  $m \times 1$  vectors of deviations in the values of the variables away from  $(\bar{X}(t), \bar{Y}(t))$ . The left-hand side of (2.16) is an approximation to the vector of changes in the  $F$  functions caused by changing the variable values from  $(\bar{X}(t), \bar{Y}(t))$  to  $(\bar{X}(t) + \Delta X, \bar{Y}(t) + \Delta Y)$ . As we are looking for a new solution to (2.13), we put the vector of approximate changes in  $F$  equal to zero. We recognize that in going from the initial solution for year  $t$  to the new solution, we must leave the values of the  $F$  functions unchanged from zero.

From (2.16), we obtain:

$$\Delta Y = B(\bar{X}(t), \bar{Y}(t)) * \Delta X, \quad (2.17)$$

where  $B(\bar{X}(t), \bar{Y}(t))$  is the  $G_X$  matrix evaluated at  $(\bar{X}(t), \bar{Y}(t))$  and computed according to:<sup>31</sup>

$$B(\bar{X}(t), \bar{Y}(t)) = -F_Y(\bar{X}(t), \bar{Y}(t))^{-1} * F_X(\bar{X}(t), \bar{Y}(t)). \quad (2.18)$$

Equation (2.17) is a version of Johansen's linear approximation of the true relationship between changes in  $X$  and changes  $Y$ . By setting  $\Delta X$  at  $X(t) - \bar{X}(t)$  we can estimate the required value for  $Y$  in year  $t$  as:

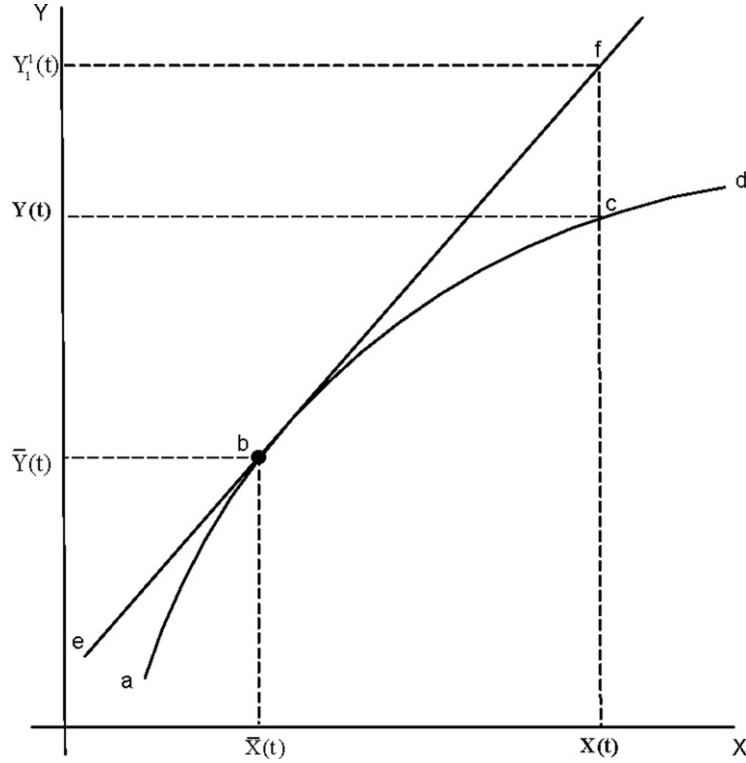
$$Y_1^1(t) = \bar{Y}(t) + B(\bar{X}(t), \bar{Y}(t)) * \Delta X, \quad (2.19)$$

where  $Y_1^1(t)$  is the estimate obtained in the first step (superscript) of a one-step (subscript) procedure.

The linearization errors generated in the application of (2.19) are illustrated in Figure 2.4 for a two-variable, one-equation model in which **abcd** is the true relationship between  $X$  and  $Y$  and **ebf** is Johansen's linear approximation — a straight line tangent to the true relationship at the initial solution. In using (2.19) to compute the effect of moving  $X$  from its initial value  $\bar{X}(t)$  to its final value  $X(t)$ , the linearization error is **fc**, i.e. the gap between the Johansen solution,  $Y_1^1(t)$ , and the true solution,  $Y(t)$ .

<sup>31</sup> We assume that  $F_Y(\bar{X}(t), \bar{Y}(t))$  is non-singular. Via the implicit functions theorem, this is equivalent to assuming the existence of a unique  $G$  function such that  $F(X(t), G(X(t))) = 0$  for all  $X$  in a neighborhood of  $\bar{X}(t)$ . If  $F_Y(\bar{X}(t), \bar{Y}(t))$  is singular, then the Johansen method will fail. However, this is not a computational problem. Any method should fail because the model does not imply that movements in  $Y$  are uniquely determined by movements in  $X$  in the neighborhood of  $\bar{X}(t)$ .





**Figure 2.4** Johansen solution.

The reason for linearization errors in the use of (2.19) to solve (2.13) can be found in (2.16), which led to (2.19). The left-hand side of (2.16) only approximates the vector of changes in the  $F$  functions caused by changes in  $X$  and  $Y$ . As we move away from  $(\bar{X}(t), \bar{Y}(t))$ , the partial derivatives of the  $F$  functions are also moving. On the left-hand side of (2.16), we evaluate the effects on the  $F$  functions of movements in the variables with the partial derivatives fixed at their initial values. More accurate solutions of (2.13) can be achieved by multistep Johansen/Euler calculations where we allow for changes in the partial derivatives of  $F$ .

In a two-step Johansen/Euler computation, we impose the change,  $\Delta X$ , in the exogenous variables in two equal steps. In the first step we use (2.19) to compute:

$$Y_2^1(t) = \bar{Y}(t) + B(\bar{X}(t), \bar{Y}(t)) * \frac{\Delta X}{2}. \quad (2.20)$$

$Y_2^1(t)$  is an estimate of the solution of (2.13) at  $X = X_2^1(t)$ , where:

$$X_2^1(t) = \bar{X}(t) + \Delta X/2, \quad (2.21)$$

and the superscript 1 and the subscript 2 in (2.20) and (2.21) denote values reached at the end of the first step in a two-step procedure. In the second step we re-evaluate the partial derivatives of  $F$  at  $(X_2^1(t), Y_2^1(t))$ , recompute the  $B$  matrix according to:

$$B(X_2^1(t), Y_2^1(t)) = -F_Y(X_2^1(t), Y_2^1(t))^{-1} * F_X(X_2^1(t), Y_2^1(t)), \quad (2.22)$$

and obtain our two-step estimate  $[Y_2^2(t)]$  of the required year  $t$  value of  $Y$  as:

$$Y_2^2(t) = Y_2^1(t) + B(X_2^1(t), Y_2^1(t)) * \frac{\Delta X}{2}. \quad (2.23)$$

We can expect the two-step answer,  $Y_2^2(t)$ , to be considerably closer to one-step than the one-step answer,  $Y_1^1(t)$ . This is illustrated in Figure 2.5 in which the linearization error for the two-step computation, **sc**, is much smaller than that for the one-step computation, **fc**. In drawing Figure 2.5 we have assumed that  $B(X_2^1(t), Y_2^1(t))$  is a good approximation to  $B(X_2^1(t), G(X_2^1(t)))$ . In other words, we have assumed that moving off the solution line **abwcd** in the first step does not invalidate formula (2.22) as an

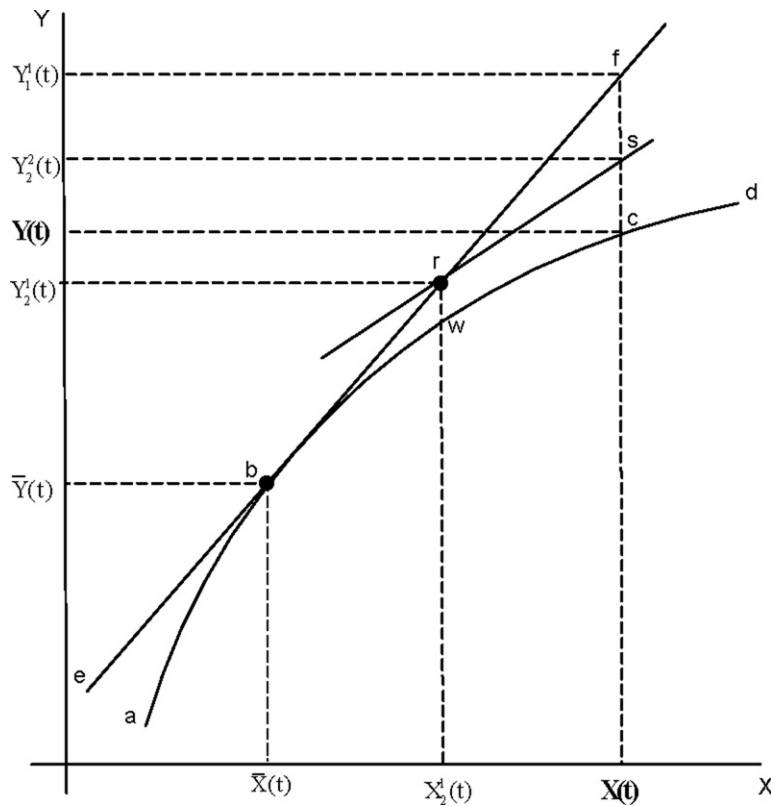


Figure 2.5 Two-step Johansen/Euler solution.

approximation to the slope of the solution line at  $X_2^1(t)$ . The formal justification of this assumption is given in the [Appendix](#): it depends on the derivatives of  $B$  with respect  $Y$  being bounded so that  $B$  does not move too far away from the slope on the solution line as  $Y$  moves away from the solution line.

Another feature of [Figure 2.5](#) worthy of comment is that **sc** is about half of **fc**, i.e. as we double the number of steps (from 1 to 2) the linearization error is halved. This is not just an artifact of the particular illustration in [Figure 2.5](#). As discussed in the [Appendix](#), it is a quite general phenomenon. It can be exploited via Richardson's extrapolation to obtain highly accurate solutions from a small number of low-step computations. For example, with:

$$Y_2^2(t) - Y(t) \approx 0.5(Y_1^1(t) - Y(t)), \quad (2.24)$$

we can often generate a highly accurate extrapolated estimate of  $Y(t)$  on the basis of one- and two-step solutions as:

$$Y_{\text{extrap}}^{1,2}(t) = 2 * Y_2^2(t) - Y_1^1(t). \quad (2.25)$$

Even more accurate solutions can be obtained via higher-step computations and associated extrapolations.<sup>32</sup>

As an alternative to working with a system of equations such as (2.16) connecting *changes* in variables, it is usually more convenient to work with a system in which most of the variables are *percentage changes*. Johansen's model was mainly in percentage changes as are MONASH models. The advantage of percentage changes is that they are immediately interpretable without worrying about units. However, for some variables, those that may pass through zero in a simulation, percentage changes are not an option.

Starting from (2.16) we can produce a mixed system in which some variables are changes and some are percentage changes by replacing relevant components,  $\Delta X_i$  and  $\Delta Y_j$ , of  $\Delta X$  and  $\Delta Y$  with percentage change variables:

$$x_i = 100 * \frac{\Delta X_i}{\bar{X}_i(t)} \quad \text{and} \quad y_j = 100 * \frac{\Delta Y_j}{\bar{Y}_j(t)}, \quad (2.26)$$

and relevant columns of  $F_X$  and  $F_Y$  by:

$$[0.01 * \bar{X}_i(t) * F_{X,i}(\bar{X}(t), \bar{Y}(t))] \quad \text{and} \quad [0.01 * \bar{Y}_j(t) * F_{Y,j}(\bar{X}(t), \bar{Y}(t))], \quad (2.27)$$

where  $F_{X,i}(\bar{X}(t), \bar{Y}(t))$  and  $F_{Y,j}(\bar{X}(t), \bar{Y}(t))$  are the  $m \times 1$  vectors of derivatives of  $F$  with respect to the  $i$ th component of  $X$  and the  $j$ th component of  $Y$ .

<sup>32</sup> Equation (2.25) is the simplest version of Richardson's extrapolation. For other versions, see [Dahlquist et al. \(1974, p. 269\)](#).

Using the mixed system we can proceed to a linearized form of our model (corresponding to equation 4.5) that can be written as:

$$y = b(\bar{X}(t), \bar{Y}(t)) * x, \quad (2.28)$$

where  $b(\bar{X}(t), \bar{Y}(t))$  is derived using matrices incorporating (2.27), and  $y$  and  $x$  are deviation vectors with percentage changes for most variables but changes for some variables such as the balance of trade for which zero is a realistic value.

With minor changes in the calculation of  $(X, Y)$  at the end of each step, the mixed linearized system can be used in a multistep Johansen/Euler computation in the same way as the pure change system.

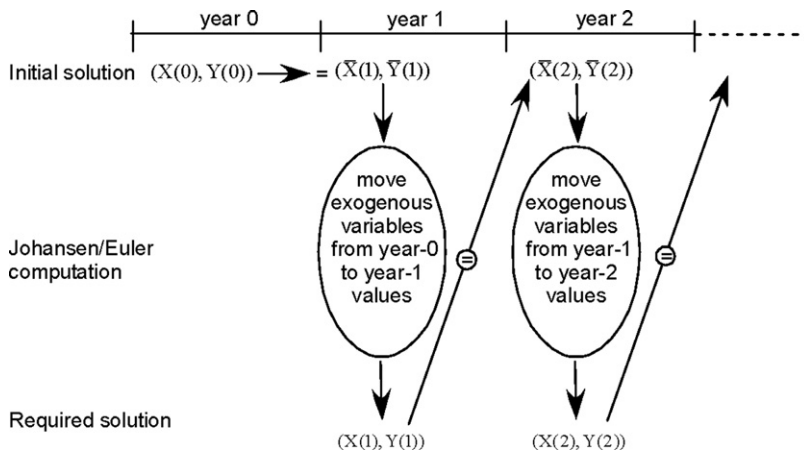
## 2.4.2 Linking the periods: dynamics

Assume that we have a solution,  $(X(0), Y(0))$ , for our model depicting the situation in year 0. Then we can use this as an initial solution for year 1:

$$(\bar{X}(1), \bar{Y}(1)) = (X(0), Y(0)). \quad (2.29)$$

From here we can use the Johansen/Euler technique to generate the required solution for year 1 by applying shocks reflecting the difference between  $X(0)$  and  $X(1)$ . The changes  $dY$  in the endogenous variables generated in this process can be interpreted as growth between year 0 and year 1. As shown in Figure 2.6, we can create a sequence of solutions showing year-on-year growth through any desired simulation period.

In a year-on-year sequence of solutions, start-of-year stock variables in the required solution for year  $t$  adopt the values of end-of-year stock variables in the required solution



**Figure 2.6** Sequence of solutions using the required solution for year  $t - 1$  as the initial solution for year  $t$ .

for year  $t - 1$ . Consider, for example, a situation in which the start-of-year and end-of-year quantities of capital in industry  $j$  in year  $t - 1$  are given by:

$$K_j^{\text{start}}(t - 1) = 10 \quad \text{and} \quad K_j^{\text{end}}(t - 1) = 12. \quad (2.30)$$

In the initial solution for year  $t$ , we have:

$$\bar{K}_j^{\text{start}}(t) = 10 \quad \text{and} \quad \bar{K}_j^{\text{end}}(t) = 12. \quad (2.31)$$

In using the Johansen/Euler method to generate the required solution for year  $t$ , we must make sure that the start-of-year capital stock for industry  $j$  moves up by 20%, from its initial value of 10 to its required value of 12. If we include start-of-year stock variables among the components of  $X$ , then the required year-to-year changes can be imposed exogenously via shocks. More convenient methods are available via the use of homotopy equations. For example, we can include in (2.13) equations of the form:

$$K_j^{\text{start}}(t) = \bar{K}_j^{\text{start}}(t) + (\bar{K}_j^{\text{end}}(t) - \bar{K}_j^{\text{start}}(t)) * U. \quad (2.32)$$

where the barred coefficients referring to the initial solution are treated as parameters, and  $U$  is a variable (known as a homotopy variable) whose initial value is zero and final value is one.

With  $U$  on zero, (2.32) is satisfied by the initial solution (i.e.  $K_j^{\text{start}}(t) = \bar{K}_j^{\text{start}}(t)$ ).

When  $U$  moves to one,  $K_j^{\text{start}}(t)$  moves to its required value,  $K_j^{\text{start}}(t) = \bar{K}_j^{\text{end}}(t)$ .

The sequence of annual solutions depicted in Figure 2.6 is recursive (i.e. the solution for year 1 uses year 0 as a starting point, the solution for year 2 uses year 1 as a starting point, etc.) In models with forward-looking expectations, a simple recursive approach will not work: in computing the solution for year 1 we need information on year 2. Nearly all MONASH calculations have been conducted with static or adaptive expectations so that the recursive approach is adequate. However, as described in Dixon *et al.* (2005), it is possible to handle forward-looking expectations by an iterative method while retaining an essentially recursive approach. First, we set the model up with static expectations and solve it recursively for years 1, 2, ...,  $T$ . This gives us the basis for guessing values for variables in years  $t + 1$  and beyond when we are computing the solution for year  $t$ . With these guesses in place, we repeat the recursive sequence of solutions. The guesses for forward-looking variables are refined from sequence to sequence.<sup>33</sup>

<sup>33</sup> Another method of solving models with forward-looking variables is to compute all years simultaneously. This method was developed by Wilcoxon (1985, 1987) and Bovenberg (1985). See also Malakellis (1998, 2000). A disadvantage of simultaneous-solution methods is that they are feasible only if the underlying model is small.

Many MONASH computations are not concerned with the year-on-year evolution of the economy. For example, in a decomposition analysis we may wish to use a MONASH simulation to explain economic developments across a period of several years, say 1992–1998. In this case, the initial solution for 1998 is the situation in 1992, i.e.:

$$(\bar{X}(1998), \bar{Y}(1998)) = (X(1992), Y(1992)), \quad (2.33)$$

and the simulation consists of looking at the effects on the endogenous variables of moving the exogenous variables from their 1992 values to their 1998 values. In such a simulation, it is no longer appropriate to assume that start-of-year stock values in the required solution equal end-of-year stock values in the initial solution. In our example, this would entail the unwarranted assumption that stock values at the start of 1998 were the same as stock values at the end of 1992.

## 2.4.3 Developing a solution for year 0 from the input-output data

### 2.4.3.1 Solution for year 0: overview

To implement the Johansen/Euler method (or any other derivative method) we need a starting point,  $(X(0), Y(0))$ , which is a solution for year 0. As explained earlier, once we have a starting solution we can generate other solutions. However, how do we get a starting solution?

Most of the components in  $(X(0), Y(0))$  can be derived from input-output or social accounting data for year 0. We start by explaining this in general terms. Then we will look more specifically at the input-output database for a typical MONASH model.

Input-output data are normally given as values. To separate out prices and quantities we can adopt quantity units that are compatible with all prices in year 0 being one. For example, if the price of a bushel of wheat is \$4, then we adopt the quarter bushel as the quantity unit for wheat. If the input-output data shows a flow of \$1 billion of wheat from farmers to bakers, then we say that 1 billion units (quarter bushels) of wheat are sold to bakers.

Given the balance conditions in input-output data, we can be sure that the quantities and prices derived in this way are compatible with demand/supply equality and zero pure profits. What about equations derived from utility maximization and cost minimization problems? These are satisfied with prices on one and the resulting quantities implied by input-output data via calibration of the parameters or the introduction of shift variables. For example, if households maximize a Cobb–Douglas utility function so that demand for commodity  $i$  ( $C_i$ ) is related to the price of commodity  $i$  ( $P_i$ ) and to total consumption ( $CTOT$ ) by:

$$C_i = \alpha_i * \frac{CTOT}{P_i} \text{ for all commodities } i, \quad (2.34)$$

then the parameter  $\alpha_i$  is calibrated or estimated as:

$$\alpha_i = \frac{C_i(0) * P_i(0)}{CTOT(0)} \text{ for all commodities } i, \quad (2.35)$$

where  $P_i(0)$  is set at one, and  $C_i(0)$  and  $CTOT(0)$  are obtained from the household column of the input-output data. With  $\alpha_i$  set via (2.35), it is clear that the input-output values for  $C_i$ ,  $P_i$  and  $CTOT$  satisfy (2.34). More generally, all of the demand and supply equations in MONASH models (and models built in other input-output/social-accounting-matrix traditions) contain sufficient free parameters and shift variables so that they can be satisfied by the initial input-output data.

Input-output tables may not cover all of the flow variables in a model. For example, MONASH models include variables making up the balance of payments and the public sector budget. Additional data tables are necessary to provide an initial solution for these variables (see Dixon and Rimmer, 2002; pp. 212–219). As well as flow variables, MONASH models contain stock variables. Year 0 data are required for variables such as start-of-year capital stocks by industry, start-of-year foreign debts and assets and start-of-year public sector liabilities. Values for end-of-year stock variables in year 0 can be derived from start-of-year values and relevant year 0 flow variables.

#### **2.4.3.2 Solution for year 0 and the input-output database for a MONASH model**

The input-output database for a typical MONASH model is illustrated in Figure 2.7. These data not only provide the bulk of the year 0 solution, but they also give an immediate impression of the model's properties. By looking at the input-output data we can see the levels of commodity, industry and occupational disaggregation. We can also see: whether imported and domestic good  $i$  are treated as distinct varieties; whether margins and indirect taxes are taken seriously and a distinction is made between purchasers' and producer prices; and whether there are industries that produce more than one commodity (multiproduct industries) and commodities that are produced by more than one industry (multi-industry products).

The data in Figure 2.7 has three parts: an absorption matrix; a joint-production matrix; and a vector of import duties. The first row of matrices in the absorption matrix, BAS1, ..., BAS6, shows flows in year 0 of commodities to producers, investors, households, exports, public consumption and inventory accumulation. Each of these matrices has  $C \times S$  rows, one for each of  $C$  commodities from  $S$  sources.  $C$  can be large. For example in USAGE, a MONASH-style model of the US, there are over 500 commodities.<sup>34</sup>  $S$  is usually 2: domestic and imported. However, it can be larger to facilitate analyses in which it is important to identify imports from different countries. For example, the US International Trade Commission (2007, 2009) uses a version of

<sup>34</sup> See US International Trade Commission (2004) and Dixon and Rimmer (2004).

		Absorption Matrix					
		1	2	3	4	5	6
		Prod- ucers	Invest- ors	House- holds	Exports	Govern- ment	Invent- ories
Size		$\leftarrow I \rightarrow$	$\leftarrow I \rightarrow$	$\leftarrow 1 \rightarrow$	$\leftarrow 1 \rightarrow$	$\leftarrow 1 \rightarrow$	$\leftarrow 1 \rightarrow$
Basic Flows	$\begin{matrix} \uparrow \\ C \times S \\ \downarrow \end{matrix}$	BAS1	BAS2	BAS3	BAS4	BAS5	BAS6
Margins	$\begin{matrix} \uparrow \\ C \times S \times N \\ \downarrow \end{matrix}$	MAR1	MAR2	MAR3	MAR4	MAR5	MAR6
Sales Taxes	$\begin{matrix} \uparrow \\ C \times S \\ \downarrow \end{matrix}$	TAX1	TAX2	TAX3	TAX4	TAX5	TAX6
Labor	$\begin{matrix} \uparrow \\ M \\ \downarrow \end{matrix}$	LABOR	C = Number of commodities I = Number of industries S = Number of sources, usually 2 (dom & imp) M = Number of occupations N = Number of commodities used as margins				
Capital	$\begin{matrix} \uparrow \\ 1 \\ \downarrow \end{matrix}$	CAPITAL					
Land	$\begin{matrix} \uparrow \\ 1 \\ \downarrow \end{matrix}$	LAND					
Production Taxes	$\begin{matrix} \uparrow \\ 1 \\ \downarrow \end{matrix}$	TAX0					

Joint Production Matrix	
Size	$\leftarrow I \rightarrow$
$\begin{matrix} \uparrow \\ C \\ \downarrow \end{matrix}$	MAKE

Import Duty	
Size	$\leftarrow 1 \rightarrow$
$\begin{matrix} \uparrow \\ C \\ \downarrow \end{matrix}$	TARIFF

Figure 2.7 Input-output database for a typical MONASH model.

USAGE with 23 import sources ( $S=24$ ) to capture the effects of country-specific import quotas.

BAS1 and BAS2 each have  $I$  columns, where  $I$  is the number of industries (usually approximately the same as the number of commodities). The typical component of BAS1 is the value of good  $i$  from source  $s$  [good  $(i,s)$ ] used by industry  $j$  as an input to current production, and the typical component of BAS2 is the value of  $(i,s)$  used to create



capital for industry  $j$ . As shown in Figure 2.7, BAS3, ..., BAS6 each have one column. Most MONASH-style models recognize one household, one foreign buyer, one category of public demand and one category of inventory demand. These dimensions can be extended in work concerned with income distribution, free trade agreements and multiple levels of government.

All of the flows in BAS1, ..., BAS6 are valued at basic prices. The basic price of a domestically produced good is the price received by the producer (that is the price paid by users excluding sales taxes, transport costs and other margin costs). The basic price of an imported good is the landed-duty-paid price, i.e. the price at the port of entry just after the commodity has cleared customs.

Costs separating producers or ports of entry from users appear in the input-output data in the margin matrices and in the row of sales-tax matrices. The margin matrices, MAR1, ..., MAR6, show the values of  $N$  margin commodities used in facilitating the flows identified in BAS1, ..., BAS6. Typical margin commodities are the domestic varieties of wholesale trade, retail trade, road transport, rail transport, water transport, air transport, natural gas and other pipelines. Each of the matrices MAR1, ..., MAR6 has  $C \times S \times N$  rows corresponding to the use of  $N$  margin commodities in facilitating flows of  $C$  commodities from  $S$  sources. The sales tax matrices TAX1, ..., TAX6 show collections of sales taxes (positive) or payments of subsidies (negative) associated with each of the flows in the BAS matrices.

Payments by industries for  $M$  occupational groups are recorded in Figure 2.7 in the matrix LABOR. In models and applications focusing on labor market issues, such as training needs and immigration,  $M$  can be large. For example, some versions of the USAGE model distinguish 750 occupations.

In most MONASH models, payments by industries for the use of capital and land are recorded in the input-output data as vectors: CAPITAL and LAND in Figure 2.7. However in studies concerned with food security and biofuels, the land dimension has been disaggregated (see, e.g. Winston, 2009). The vector TAX0 shows collections of taxes net of subsidies on production.

The final two data items in Figure 2.7 are TARIFF and MAKE. TARIFF is a  $C \times 1$  vector showing tariff revenue by imported commodity. The joint-product matrix, MAKE, has dimensions  $C \times I$ . Its typical component is the output (valued in basic prices) of commodity  $c$  by industry  $i$ .

Together, the absorption and joint-production matrices satisfy two balance conditions. (i) The column sums of MAKE, which are values of industry outputs, are identical to the values of industry inputs. Hence, the  $j$ th column sum of MAKE equals the  $j$ th column sum of BAS1, MAR1, TAX1, LABOR, CAPITAL, LAND and TAX0. (ii) The row sums of MAKE, which are basic values of outputs of domestic commodities, are identical to basic values of demands for domestic commodities. If  $i$  is a non-margin commodity, then the  $i$ th row sum of MAKE is equal to the sum across the  $(i, \text{'dom'})$ -rows

of BAS1 to BAS6. If  $i$  is a margin commodity, then the  $i$ th row sum of MAKE is equal to the direct uses of domestic commodity  $i$ , i.e. the sum across the  $(i, \text{'dom'})$ -rows of BAS1 to BAS6, plus the margins use of commodity  $i$ . The margins use of  $i$  is the sum of the components in the  $(c, s, i)$ -rows of MAR1 to MAR6 for all commodities  $c$  and sources  $s$ .

To obtain a year 0 solution for MONASH flow variables from a database such as that in Figure 2.7, we start by defining quantity units for commodities as the amounts that had a *basic* price of one. Now we can read from BAS1, ..., BAS6 and MAR1, ..., MAR6 both values and quantities of commodity demands. Similarly, we can read from MAKE both values and quantities of commodity supplies. With basic prices of commodities assigned the value one, the input-output data quickly reveals purchasers prices for year 0. For example, the year 0 purchasers price for good  $i$  from source  $s$  bought by industry  $j$  for use in current production is

$$P1(i, s, j) = [\text{BAS1}(i, s, j) + \sum_{n=1}^N \text{MAR1}(i, s, j, n) + \text{TAX1}(i, s, j)] / \text{BAS1}(i, s, j). \quad (2.36)$$

Do year 0 prices and quantities defined in this way satisfy MONASH equations specifying that:

$$\text{Quantity demanded of domestic product } i = \text{quantity supplied} \quad (2.37)$$

$$\text{Value of output from industry } j = \text{value of } j\text{'s inputs plus production taxes} \quad (2.38)$$

$$\text{Purchasers values} = \text{basic values plus margins and sales taxes?} \quad (2.39)$$

The balancing properties of the input-output data ensure that the values we have assigned to year 0 prices and quantities satisfy (2.37) and (2.38). Equation (2.39) is satisfied via definitions of year 0 purchasers prices such as (2.36).

MONASH models contain many more equations connecting input-output variables than those indicated by (2.37)–(2.39). All of these additional equations contain either free parameters and/or free variables. That is, they contain parameters or variables for which we are free to assign values that allow the equations to be satisfied by our year 0 values for prices and quantities. For example, consider the equation:

$$X1MARG(i, s, j, r) = X1(i, s, j) * A1MARG(i, s, j, r), \quad (2.40)$$

where  $X1MARG(i, s, j, r)$  is the use of margin-commodity  $r$  (e.g., road transport) to facilitate the flow of intermediate input  $i$  from source  $s$  (domestic or imported) to industry  $j$ ,  $X1(i, s, j)$  is the use of good  $i$  from source  $s$  by industry  $j$  as an intermediate input and  $A1MARG(i, s, j, r)$  is the use of margin-commodity  $r$  per unit of flow of intermediate input  $(i, s)$  to industry  $j$ .

From our input-output data, we have already assigned year 0 values to  $X1MARG(i, s, j, r)$  and  $X1(i, s, j)$ . However,  $A1MARG(i, s, j, r)$  is free. If  $X1(i, s, j)$  is non-zero, then we ensure that (2.40) is satisfied by the year 0 quantities read from our input-output data by choosing the year 0 value for  $A1MARG(i, s, j, r)$  to be the ratio of the year 0 values of  $X1MARG(i, s, j, r)$  and  $X1(i, s, j)$ . If  $X1(i, s, j)$  is zero, then  $A1MARG(i, s, j, r)$  can be assigned any value provided  $X1MARG(i, s, j, r)$  is also zero. If  $X1(i, s, j)$  is zero but  $X1MARG(i, s, j, r)$  is not zero, then we have a data error requiring correction.

Now consider a less trivial example. Part of the theory of MONASH models is that industry  $j$  chooses its current inputs of domestic and imported good  $i$  to minimize costs subject to a CES constraint in which the industry's requirements for good  $i$  are proportional to its activity level,  $Z(j)$ , i.e. industry  $j$  chooses:

$$X1(i, s, j), s = \{\text{dom}, \text{imp}\},$$

to minimize:

$$\sum_s P1(i, s, j) X1(i, s, j), \quad (2.41)$$

subject to:

$$Z(j) = \left[ \sum_s X1(i, s, j)^{-\rho(i, j)} \delta 1(i, s, j) \right]^{-1/\rho(i, j)}, \quad (2.42)$$

where the  $\delta 1(i, s, j)$  are non-negative parameters summing to one over  $s$  and  $\rho(i, j)$  is a substitution parameter assigned a value greater than  $-1$  (but not precisely zero) reflecting econometric estimates or views about import/domestic substitution.

Problem (2.41)–(2.42) leads to equations for the ratio of domestic to imported inputs of the form:

$$\frac{X1(i, \text{dom}, j)}{X1(i, \text{imp}, j)} = \left[ \frac{\delta 1(i, \text{dom}, j)}{\delta 1(i, \text{imp}, j)} * \frac{P1(i, \text{imp}, j)}{P1(i, \text{dom}, j)} \right]^{1/(1+\rho(i, j))}. \quad (2.43)$$

Values can be assigned to the parameters  $\delta 1(i, s, j)$ ,  $s = \text{dom}$  and  $\text{imp}$ , to ensure that (2.43) is satisfied by the year 0 values for  $X1(i, s, j)$  and  $P1(i, s, j)$ , together with the value for the substitution parameter  $\rho(i, j)$ .

A few examples is not a proof of the existence of a year 0 solution,  $(X(0), Y(0))$ , to (2.13). A complete proof for any model involves working through every equation, identifying free parameters or variables. This is not difficult, but it is tedious.

#### 2.4.4 Deriving change and percentage change equations

MONASH models are represented as linear systems of the form:

$$A(V) * v = 0, \quad (2.44)$$

where  $V$  is an  $n \times 1$  vector of initial values or values generated during a multistep process for the variables (denoted as  $(X, Y)$  in the previous subsection),  $A$  is an  $m \times n$  matrix of coefficients each of which is a function of  $V$  and  $v$  is a vector of changes and percentage changes in the variables away from their values in  $V$ .

In this subsection we describe how equations that make up the change/percentage change system (2.44) can be derived from the levels system (2.13).

Most equations in (2.44) can be derived from the corresponding equation in (2.13) by the application of the three rules in Table 2.3. For example, the multiplication and power rules applied to (2.43) give the percentage change equation:

$$x1(i, \text{dom}, j) - x1(i, \text{imp}, j) = \sigma(i, j) * (p1(i, \text{imp}, j) - p1(i, \text{dom}, j)), \quad (2.45)$$

where the lowercase  $x$  and  $p$  are percentage changes in the variables represented by the corresponding uppercase symbols and  $\sigma(i, j)$ , which equals  $1/(1 + \rho(i, j))$ , is the elasticity of substitution in industry  $j$  between domestic and imported units of commodity  $i$ .

In representing optimization problems in system (2.44), it is often convenient to use percentage change versions of the first-order conditions. For example, in the optimization problem (2.8)–(2.9), the first-order conditions are:

$$P_i = \Lambda * \frac{X_i^{h_i-1}}{Z^{h_i}} * Q_i, \quad i = 1, \dots, n, \quad (2.46)$$

where  $\Lambda$  is the Lagrangian multiplier, together with the constraint (2.9). These conditions can be represented in (2.44) as:

$$p_i = \lambda + (h_i - 1) * x_i - h_i * z, \quad i = 1, \dots, n, \quad (2.47)$$

$$\sum_j (x_j - z) * S_j = 0, \quad (2.48)$$

**Table 2.3** Rules for deriving percentage-change equations

	Representation in		
	Levels		Percentage changes
Multiplication rule	$U = RW$	$\Rightarrow$	$u = r + w$
Power rule	$U = R^\alpha$	$\Rightarrow$	$u = \alpha r$
Addition rules	$U = R + W$	$\Rightarrow$	$Uu = Rr + Ww$ or $u = S_r r + S_w w$

$U$ ,  $R$  and  $W$  are levels of variables,  $u$ ,  $r$  and  $w$  are percentage changes,  $\alpha$  is a parameter and  $S_r$  and  $S_w$  are shares evaluated at the current solution. In the first step of a Johansen/Euler computation, the current solution is the initial solution. Hence,  $S_r = R(0)/U(0)$  and  $S_w = W(0)/U(0)$ . In subsequent steps,  $S_r$  and  $S_w$  are recomputed as  $U$ ,  $R$  and  $W$  move away from their initial values.

where again we use lowercase symbols to represent percentage changes in the variables denoted by the corresponding uppercase symbols and:

$$S_j = \left(\frac{X_j}{Z}\right)^{h_j} * Q_j / \left[ \sum_k \left(\frac{X_k}{Z}\right)^{h_k} * Q_k \right], \quad j = 1, \dots, n. \quad (2.49)$$

It is apparent from (2.46) that  $S_j$  is the share of total costs accounted for by input  $j$ .

While (2.47) and (2.48) can be used in (2.44) to represent optimization problem (2.8)–(2.9), we may prefer to eliminate the percentage change in the Lagrangian multiplier,  $\lambda$ . After a little algebra we can obtain (2.9) and (2.11).<sup>35</sup>

Not all of the equations in (2.44) can be derived by the simple rules in Table 2.3. Occasionally more complicated differentiations are required. For example, consider the levels equation

$$R = R_{\text{norm}} + C * \ln \left\{ \frac{G - G_{\min}}{G_{\max} - G} * \frac{G_{\max} - T}{T - G_{\min}} \right\}. \quad (2.50)$$

Variants of this equation are used in MONASH models to relate an industry's capital growth ( $G$ ) through year  $t$ <sup>36</sup> to the industry's expected rate of return ( $R$ ) and its normal rate of return ( $R_{\text{norm}}$ ).<sup>37</sup> In (2.50),  $G_{\min}$ ,  $G_{\max}$ ,  $T$  and  $C$  are parameters with  $C$  positive and  $G_{\min} < T < G_{\max}$ .  $T$  represents a trend rate of growth of capital. If  $R$  equals  $R_{\text{norm}}$  then via (2.50) the growth in capital through the year is at its trend value ( $G = T$ ). Capital growth will exceed trend ( $G > T$ ) if the expected rate of return is greater than the normal rate of return ( $R > R_{\text{norm}}$ ). Capital growth will never move above  $G_{\max}$  (as  $R \rightarrow \infty$ ,  $G \rightarrow G_{\max}$ ). Similarly capital growth will never move below  $G_{\min}$  (as  $R \rightarrow -\infty$ ,  $G \rightarrow G_{\min}$ ). By choosing suitable values for  $G_{\min}$  and  $G_{\max}$ , we can ensure that our model always implies growth rates for capital in a realistic range. To obtain a form of (2.50) suitable for inclusion in (2.44) we can totally differentiate both sides. This gives

$$\text{del}_R - \text{del}_R - \text{del}_R - C * \left\{ \frac{1}{G - G_{\min}} + \frac{1}{G_{\max} - G} \right\} * \text{del}_G = 0. \quad (2.51)$$

In (2.51),  $\text{del}_R$ ,  $\text{del}_R$  and  $\text{del}_G$  are *change* variables and form part of the  $\nu$  vector in (2.44). We use change variables because  $R$ ,  $R_{\text{norm}}$  and  $G$  are variables for which zero is a sensible value. The coefficients on  $\text{del}_R$ ,  $\text{del}_R$  and  $\text{del}_G$  in the relevant row of the  $A$  matrix are 1,  $-1$  and  $-C * \{1/(G - G_{\min}) + 1/(G_{\max} - G)\}$ .

<sup>35</sup> The first move in deriving (2.10) and (2.11) from (2.47) and (2.48) is to multiply (2.47) by  $S_i/(h_i - 1)$  and then sum over  $i$ .

<sup>36</sup>  $G$  is  $J - 1$ , where  $J$  the ratio of capital at the end of year  $t$  to capital at the start of year  $t$ .

<sup>37</sup> Details on the MONASH treatment of capital growth are in Dixon and Rimmer (2002, pp. 189–198).

### 2.4.5 Introduction to the GEMPACK programs for solving and analyzing MONASH models

MONASH models are built, solved and analyzed using GEMPACK. The existence of this software explains much of the popularity of MONASH-style models throughout the world. GEMPACK is described in detail by Horridge *et al.* in Chapter 20 of this Handbook. In this subsection we start with some brief comments on GEMPACK's history. Then we look at the structure of how model solutions are computed via GEMPACK. This will be helpful in summarizing the technical material that has been covered so far in this section.

The computer code for the first MONASH model (the ORANI model) was developed by Sutton (1977). This code was effective and handled what was for the time a very large CGE model. However, Sutton's programs were model specific: they solved the ORANI model. In 1980 Ken Pearson started work on GEMPACK. His objective was to create a suite of programs that could be used to solve any CGE model in the Johansen/MONASH tradition. The first version of GEMPACK was used for teaching in 1984 and shortly after that was adopted by Australian CGE modelers. The first GEMPACK manuals were published in 1986 (see Codsì and Pearson, 1986). Early journal descriptions of GEMPACK are Pearson (1988) and Codsì and Pearson (1988). Evidence of the success of GEMPACK is its adoption over the last 25 years by thousands of CGE modelers.

The structure of a GEMPACK solution of a MONASH model is illustrated in Figure 2.8. GEMPACK users start by presenting their model in TABLO code. This is a language close to ordinary algebra. For example, assume that model (2.13) consists of three equations and nine variables:

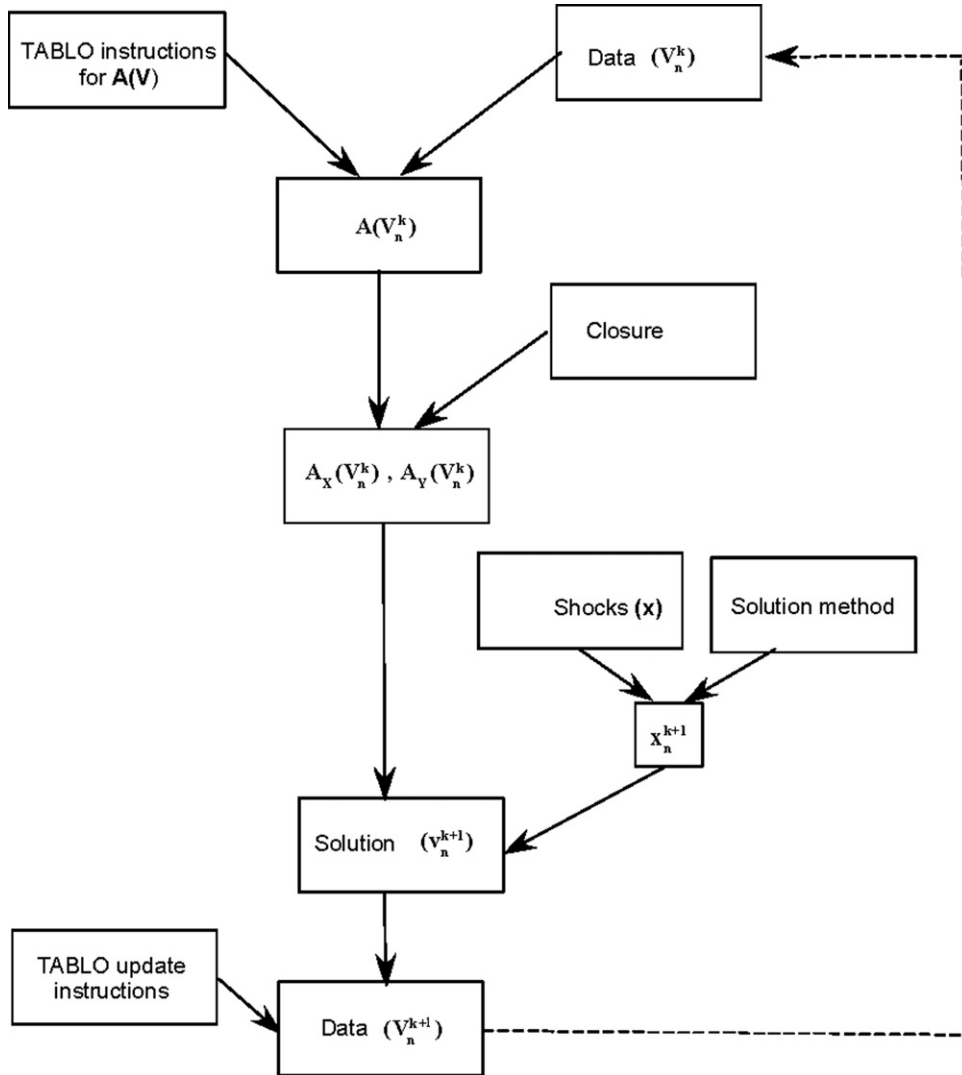
$$DTOT(i) = \sum_{j=1}^2 D(i,j), \quad i = 1, 2, 3, \quad (2.52)$$

where  $DTOT(i)$  is total demand for good  $i$  and  $D(i,j)$  is demand for  $i$  by user  $j$ . In percentage-change form the model is:

$$dtot(i) = \sum_{j=1}^2 S(i,j) * d(i,j), \quad i = 1, 2, 3, \quad (2.53)$$

where lowercase symbols are percentage changes in variables represented by the corresponding uppercase symbols and  $S(i,j)$  is the share of user  $j$  in the total demand for good  $i$ . Largely self-explanatory TABLO code for the model is shown in Table 2.4.

TABLO code has two main roles in GEMPACK. The first is to give a set of instructions for reading a database, which we can think of as revealing a value for  $V_i$  and using it to evaluate  $A(V)$  in (2.44). In the example in Table 2.4 the declaration of



**Figure 2.8** GEMPACK solution of a MONASH model.

variables is an instruction to create an  $A$  matrix with nine columns, called  $d(C1,U1)$ , ...,  $d(C3,U2)$ ,  $dtot(C1)$ , ...,  $dtot(C3)$ . The command starting with 'Equation' is an instruction that the  $A$  matrix should have three rows called  $E\_dtot(C1)$ , ...,  $E\_dtot(C3)$ . The equation itself is an instruction that the row  $E\_dtot(Ci)$  should contain 1 in the  $dtot(i)$  column and  $-S(i,j)$  in the  $d(i,j)$  column. The *Read* and *Formula* commands contain instruction on how to evaluate  $S(i,j)$  from a dataset (e.g. input-output data of the form shown in Figure 2.7). In Figure 2.8 we show the data to be used in the  $k + 1$ th step in an  $n$ -step sequence as  $Data(V_n^k)$ ,  $k = 0, \dots, n - 1$ .

**Table 2.4** Sample of TABLO code

---

```

File DATA # Data file #;
Set COM # Commodities # (C1–C3);
Set USER # User # (U1–U2);

Coefficient ! Declaration of coefficients !
  (A11,i,COM)(A11,j,USER) BAS(i,j) # Data for demand for i by j #;
  (A11,i,COM) TBAS(i) # Total demand for i #;
  (A11,i,COM)(A11,j,USER) S(i,j) # Share of j in demand for i #;

Read BAS from file DATA Header“BAS”;

Formula
  (A11,i,COM) TBAS(i) = Sum(j,USER, BAS(i,j) );
  (A11,i,COM)(A11,j,USER) S(i,j) = BAS(i,j)/TBAS(i);

Variable ! Declaration of variables !
  (A11,i,COM)(A11,j,USER) d(i,j) # Demand for i by j #;
  (A11,i,COM) dtot(i) # Total demand for i #;

Equation E_dtot
  (A11,i,COM) dtot(i) = Sum(j,USER, S(i,j)*d(i,j));

Update
  (A11,i,COM)(A11,j,USER) BAS(i,j) = d(i,j);

```

---

Having evaluated  $A(V)$  with the initial database to obtain  $A(V_n^0)$ , we introduce the closure. This can be done via a subprogram that lists the exogenous variables. GEMPACK can now split the  $A$  matrix into  $A_X(V_n^0)$  and  $A_Y(V_n^0)$ .

The next two subprograms introduce the shocks (movements in exogenous variables) and specify the solution method (e.g. Johansen/Euler with  $n$  steps). From these subprograms, GEMPACK can compute the shocks to be applied in the first step of the  $n$ -step sequence. All the information has now been assembled to enable GEMPACK to compute the movements in the endogenous variables in the first step of the  $n$ -step sequence.

This brings us to the second main role of TABLO. It provides instructions for updating to a new database, Data  $(V_n^1)$ , that incorporates the movements in the variables imposed and generated in the first step. In Table 2.4, the update instruction says that the data item  $BAS(i,j)$  should be increased by  $d(i,j)\%$ . Once Data  $(V_n^1)$  has been created, GEMPACK is ready to undertake the second step of the  $n$ -step sequence, and so on through the  $n$  steps.<sup>38</sup>

The core set of programs outlined in Figure 2.8 has been supplemented since the mid-1980s by an ever-expanding set of wonderfully useful GEMPACK features contributed by Ken Pearson and his colleagues including George Codsì, Mark

<sup>38</sup> Apart from the two roles described here, TABLO also provides instructions for condensation, see Section 2.3.2.2.



Horridge, Jill Harrison and Michael Jerie. For example, AnalyseGE allows GEMPACK users to see the value of any coefficient (i.e. any function of database items) or any variable in a particular solution via point-and-click applied to the TABLO representation of the model. ViewSOL allows GEMPACK users to see a series of simulation results in a variety of styles (e.g. year-on-year growth, cumulative growth from an initial year or cumulative difference between two series of results) and a variety of formats (graphs or numbers). These aids greatly enhance the user's ability to undertake MONASH-style modeling.

## 2.4.6 Creating a database for a MONASH model

One of the most difficult and least teachable CGE skills is the compilation of a database for year 0. For MONASH models, the central components are a set of input-output accounts as illustrated in [Figure 2.7](#) and estimates of capital stock by industry.<sup>39</sup> What makes compilation of these data so hard is that, for the most part, they must be gleaned from bulletins prepared by statistical agencies for purposes far removed from CGE modeling. The accounting conventions adopted by the agencies are often opaquely documented and tortuous to follow.

Here, we flag some of the difficulties, drawing on our experience in preparing a year 0 database for the USAGE model of the US. However, experience for the US may not be directly relevant for other countries. In creating a practical, policy-relevant CGE model for any country, there is no substitute for the time-consuming work of looking at the data presented by the statistical agencies, thinking about what it means, working out the underlying accounting conventions, and contacting the agencies and asking questions.<sup>40</sup> The practice adopted by some CGE modelers of delegating the data preparation task to research assistants is inappropriate for models intended for serious policy analysis.

### 2.4.6.1 Input-output data published by the BEA

The starting point for the USAGE database was the  $498 \times 498$  input-output data for 1992 published by the BEA ([Bureau of Economic Analysis, 1998](#)). The first challenge in using these data was to sort out the meaning of mysterious rows and columns designed by the BEA to give desired row and column sums. For example, the BEA wanted the household consumption and export columns to add to total household consumption and total exports as shown in the National Income and Product Accounts (NIPA) for 1992.

<sup>39</sup> Other data items required for year 0 include the balance of payments and the public sector budget. These are not discussed here but are described in [Dixon and Rimmer \(2002, pp. 212–219\)](#).

<sup>40</sup> For the US, the main statistical agency supplying data relevant for CGE modeling is the BEA. Their officers, particularly Karen Horowitz, were extremely helpful in answering the numerous questions that arose as we prepared the database for the USAGE model.

In making estimates for their input-output tables of consumption expenditures disaggregated by commodity, the BEA felt unable to distinguish between expenditures by residents and expenditures by visitors. They recorded all consumption expenditures on each commodity in the household consumption column and did not include expenditures by visitors in the export column. On the other hand, NIPA data excludes total visitor expenditure from the estimate of total household consumption and includes it in total exports. To achieve their objective of NIPA compatibility, the BEA included in their input-output tables a row and corresponding user column labeled 'Rest-of-world adjustment to final use'. The row contains two non-zero entries: a negative entry in the household column representing expenditures by visitors and a positive entry in the export column representing the same thing. The column consists entirely of zeros. Initially in using the BEA data we deleted both the column and row for 'Rest-of-world adjustment to final use'. Eventually we dealt with the issue satisfactorily by using data from the BEA's Tourism Satellite Accounts (Okubo and Planting, 1998) to itemize visitor expenditures which we reallocated from the household consumption column to a new industry, Export tourism. We modeled the output of this industry as being entirely exported.

After further adjustments we were able to present the BEA input-output data in the form shown in Figure 2.9 where: PV1, ..., PV6 represent direct uses of commodities (not identified by import/domestic source) valued in producer prices; MAR1, ..., MAR6 represent margins on the flows in PV1, ..., PV6; PVM represents imports; LAB, TAX0 and OVA are a breakdown of value added into compensation of employees, indirect taxes and other value added. MAKE represents commodity outputs by industries. The sum down a column of [PV1, MAR1, LAB, TAX0, OVA] matches the corresponding column sum of MAKE. For non-margin commodities the row sums of [PV1, PV2, ..., PV6, -PVM] match the corresponding row sums of MAKE. Finally, for any commodity  $n$  which is a margin, the sum across the  $n$ -row of PV1, ..., -PVM plus all the  $n$ -entries in MAR1, ..., MAR6 matches the commodity  $n$ -row sum in MAKE.

To move from Figure 2.9 to a MONASH-style input-output database of the form shown in Figure 2.7 it was necessary to consider conventions in the BEA data concerning: valuation of flows and the recording of indirect taxes; imports; public sector demands, particularly the use of negative entries; investment; and value added. The following subsections describe some of these conventions and our efforts to cope with them. However, in the space available we cannot be comprehensive. Many important details must be omitted concerning, for example, the BEA treatment of: real estate agents and home ownership; royalties; scrap and used and second hand goods; auto rental; secondary production; capital stocks in public sector enterprises; and foreign ownership of US capital.<sup>41</sup>

<sup>41</sup> Documentation on these issues is available from the authors.

		Absorption Matrix						
		Prod-ucers	Invest-ors	House-holds	Exports	Govern-ment	Invent-ories	-Imports
Size		$\leftarrow I \rightarrow$	$\leftarrow 1 \rightarrow$	$\leftarrow 1 \rightarrow$	$\leftarrow 1 \rightarrow$	$\leftarrow 35 \rightarrow$	$\leftarrow 1 \rightarrow$	$\leftarrow 1 \rightarrow$
Commod-ity flows	$\uparrow$ $C$ $\downarrow$	PV1	PV2	PV3	PV4	PV5	PV6	-PVM
Margins	$\uparrow$ $C \times N$ $\downarrow$	MAR1	MAR2	MAR3	MAR4	MAR5	MAR6	0
Labor	$\uparrow$ 1 $\downarrow$	LAB	<p> <math>C</math> = Number of commodities (= 483)  <math>I</math> = Number of industries (= 493)  <math>N</math> = Number of commodities used as margins (= 8) </p>					
Taxes	$\uparrow$ 1 $\downarrow$	TAX0						
Other value added	$\uparrow$ 1 $\downarrow$	OVA						

Joint Production Matrix	
Size	$\leftarrow I \rightarrow$
$\uparrow$ $C$ $\downarrow$	MAKE

**Figure 2.9** Schematic representation of BEA benchmark input-output data for 1992.

#### 2.4.6.2 Valuation and treatment of indirect taxes

All commodity flows in the BEA data, and therefore in Figure 2.9, are valued at producer prices, i.e. basic values (prices accruing to producers) plus sales and excise taxes. In tables at producer prices, the indirect tax row (TAX0) normally represents taxes paid on the sales of the industry's products together with production taxes and taxes on the use of primary factors.<sup>42</sup> Hence, we expected to find large entries in the Tobacco and Petrol columns of TAX0. However, these entries were only moderate, whereas the entries in the Wholesale and Retail columns were surprisingly large. We found that in the BEA tables, taxes are recorded in the column of the industry that collects the taxes. Apparently, tobacco and petrol taxes are collected largely by wholesalers and retailers.

In these circumstances, we expected large amounts of wholesale and retail margins to be associated with sales of tobacco and petrol, reflecting large taxes associated with the wholesaling and retailing of these products. For sales to consumers we did, in fact, find large wholesale and retail margins in MAR3 associated with sales of tobacco and

<sup>42</sup> For a description of the standard types of input-output tables (basic values, producer values and purchasers values with either direct or indirect allocation of imports), see Dixon *et al.* (1992, chapter 2).

petrol. For example, wholesale and retail margins on petrol sales to households were \$58.7 billion on a producer value of only \$51.4 billion. We suspected that most of the \$58.7 billion was tax paid by wholesalers and retailers on their sales of petrol to households. On petrol sales to industries the ratio of wholesale and retail margins to producer value was only about 28%, i.e. about a quarter of the value of the ratio applying to petrol sales to consumers. We suspected that this was the result of two factors: (i) lower taxes on industry use of petrol than on household use, and (ii) lower payments by industry than by households to wholesalers and retailers per gallon of petrol, i.e. genuinely lower margins.

More generally, the practice of allocating taxes to the collecting industry is unsatisfactory for CGE purposes. For example, without knowing the tax content of retail and wholesale margins associated with consumer purchases of tobacco and petrol, we cannot project effects on tax collections and retail and wholesale activity of changes in consumer demands for these products. For USAGE we needed to reclassify indirect taxes so that they were excluded from wholesale and retail margins and so that they were associated (as in [Figure 2.7](#)) with the purchases which give rise to them. In doing this we were assisted by the BEA who gave us about 10 000 items of unpublished data showing indirect taxes by commodity and user. In most cases the BEA indicated where the item was placed in their published producer value input-output tables. This enabled us to work out, for example, how much of PV3(Cigarettes), MAR3(Cigarettes, Wholesale) and MAR3(Cigarettes, Retail) in [Figure 2.9](#) were in fact sales taxes. With this information, we reduced these three flows to basic values and made corresponding adjustments to TAX3(Cigarettes) and to the values of Cigarettes, Wholesale and Retail outputs in the MAKE matrix.

### 2.4.6.3 Imports

The BEA input-output tables adopt indirect allocation of imports. Consequently in PV1, ..., PV6 in [Figure 2.9](#), competing imports are aggregated with output from domestic producers. Imports valued at producer prices (which for the BEA tables are landed-duty-paid prices) are shown as negative entries in a single import column (–PVM). The first problem we noticed with the BEA's treatment of imports is that three of the entries in the import column were positive, seemingly implying negative levels of imports for Wholesale trade, Water transport and Non-ferrous metal ores.

After inquiries with the BEA we found that import duties were recorded as if they were negative imports of Wholesale trade. This treatment has a column and row logic, but it is opaque for CGE modeling. The column logic is that the BEA wanted the total of the import column to reflect the cost to the US of imports (payment to foreigners). As part of achieving this they needed to deduct duties from the total of the landed-duty-paid values recorded in the import column. The row logic is that the BEA

recorded duties as a tax (part of TAX0 in Figure 2.9) on the sales of the Wholesale industry, the industry they deemed to have collected the import duties. With the producer value of the output of the Wholesale industry inflated in this way, the BEA needed to inflate the value of sales of the Wholesale industry. Negative imports of wholesale services achieved this purpose. The BEA supplied us with an unpublished disaggregation of import duties by commodity, enabling us to form the Tariff vector in Figure 2.7, and to undo the BEA's treatment of import duties by zeroing out the Wholesale entry in the import column and making corresponding deductions from TAX0 in the Wholesale column and from the (Wholesale, Wholesale) entry in the MAKE matrix.

For Water transport, the seemingly negative value of imports arose from the BEA's treatment of water transport services provided by US shipping in delivering imports to US ports. The cost of these services is embedded in the landed-duty-paid value of imports recorded in the import column. Treating US-provided water transport services on imports as negative imports rather than as exports was motivated by the objective of ensuring that the total for the import column reflected the cost to the US of imports. Using unpublished data provided by the BEA we were able to reclassify negative imports of Water transport as positive exports, leaving only genuine imports of Water transport (e.g. cruises by US residents on foreign ships) in the import column. Similar adjustments were necessary for Air transport, although the problem was less obvious in the BEA data because the Air transport entry in the import column had the expected sign, negative: genuine imports of Air services outweighed US-provided air services embedded in US imports.

In the case of Non-ferrous metal ores the negative value for imports was due to the BEA's treatment of gold. For this item, the BEA is willing to estimate net imports, but not imports and exports separately. For 1992 the BEA estimated that net imports of gold were negative. We reclassified these negative imports as exports.

The second problem with the BEA's treatment of imports is that it provides no disaggregation of imports by using industry or final demander. Such a disaggregation is required for a MONASH-style database, see Figure 2.7. Again the BEA came to the rescue with unpublished data enabling us to turn the import column into an import matrix.

#### **2.4.6.4 Public sector demands**

The BEA tables give 35 columns of government expenditures: there are 35 columns in PV5 and MAR5 in Figure 2.9. Fourteen of these columns refer to government consumption activities and have labels such as *Federal Government consumption expenditures, national defense*. The remaining 21 columns refer to government investment activities and have labels such as *Federal Government gross investment, national defense*. Of the 21 investment activities, 14 are investment counterparts of the 14 consumption activities.

For each of the 35 activities the corresponding column in Figure 2.9 shows the commodity composition of public expenditure. For example, the column for *State and local government consumption expenditures, elementary and secondary public school systems* (column 9800C1) shows expenditures totaling \$224.107 billion accounted for mainly by expenditure of \$186.326 billion on *General government* (commodity 820000). While most of the expenditures in PV5 are positive, some are negative. In column 9800C1 of PV5, for example, there are expenditures of:  $-\$2.680$  billion on *Eating and drinking places* (commodity 740000),  $-\$3.078$  billion on *Elementary and secondary schools* (commodity 770401), and  $-\$0.002$  billion on *Pens, etc.* (commodity 640501). In response to our queries the BEA explained that negative entries in the government vectors are government sales, e.g. sales of Eating and drinking (lunch program) by State and local government schools. In their input-output tables, the BEA follows the convention of making the row sum for a commodity across PV1, ..., PV6,  $-PVM$ ,<sup>43</sup> equal to the value of domestic *non-government* production.<sup>44</sup> The restriction to non-government production is achieved by the negative entries in PV5.

For CGE modeling it is inappropriate to treat only non-government enterprises as producers and it is counter-intuitive to treat government outputs as negative demands. Consequently we dropped the BEA convention. We converted the 14 columns for government consumption activities and the seven investment columns with no consumption counterpart into industries. (The treatment of the remaining 14 government investment activities is described in Section 2.4.6.5.) In creating these 21 new government industries we regarded the positive entries from the original BEA columns as inputs and the negative entries as outputs. Thus, we interpreted the data in column 9800C1 of PV5 and MAR5 as showing that government industry 9800C1 produced output of \$229.867 billion. This was the sum of the positive entries in column 9800C1 of PV5 and MAR5. The negative entries were entered as positives in the MAKE matrix and interpreted as showing that industry 9800C1 produced \$2.680 billion of *Eating and drinking places* (commodity 740000), \$3.078 billion of *Elementary and secondary schools* (commodity 770401) and \$0.002 billion of *Pens, etc.* (commodity 640501). In addition, industry 9800C1 produced \$224.107 billion of its principal product ( $= 229.867 - 2.680 - 3.078 - 0.002$ ), which was designated as commodity 9800C1. We assumed that the sales of industry 9800C1's principal product were entirely to a single category of government final demand. Sales of industry 9800C1's other outputs were already accounted for in Figure 2.9 in purchases by households and other demanders of commodities 740000, 770401 and 640501.

<sup>43</sup> Here we consider only non-margin commodities.

<sup>44</sup> An exception is the output of *General government* where the commodity row sum is entirely government production.

The BEA's government columns (PV5, MAR5) did not include any purchases of labor or other primary factor inputs. These inputs were accounted for via purchases of *General government* (commodity 820000), which is produced by industry 820000 entirely from labor and other value added. This left us unable to distinguish in our modeling between the composition of primary factor inputs in different government activities.

#### **2.4.6.5 Investment by investing industry**

The investment column in Figure 2.9 shows gross private fixed investment by commodity. For a MONASH-style model we need to give investment an industry dimension, see Figure 2.7. Our main data source for doing this in the case of the USAGE model was a 163 commodity by 64 industry matrix of private investment expenditures published by the BEA (BEA product NDN-0224).<sup>45</sup> The commodities in this matrix mapped easily to the input-output commodities for which the BEA's input-output tables showed non-zero investment. Thus, the investment matrix provided an adequate basis for giving the input-output investment column a 64-industry dimension. Within the 64 industries we allocated investment expenditures on each commodity to component industries at the detailed USAGE level (approximately 500 industries) using indicators such as other value added and employment. Thus, we assumed that all component industries within a 64-order industry had the same commodity composition of investment expenditures.

This procedure covered only private sector industries, not our 21 government industries. The BEA government investment expenditure vectors became the investment vectors for 14 of the government industries. We left the other seven government industries with no investment (zero entries in their columns of BAS2, MAR2 and TAX2 in Figure 2.7). With investment in 14 government industries, we recognized that these industries must have capital stocks with corresponding rentals. In handling this we replaced expenditures on *General government* by these 14 industries with entries in the LABOR and CAPITAL vectors in Figure 2.7. These entries represented the primary factor constituents of *General government*<sup>46</sup> expenditures by the 14 industries.

#### **2.4.6.6 Value added, self-employment and capital stocks**

The value-added section of input-output table provides the main data for CGE models on resource constraints. Perhaps reflecting the interests and times of Wassily Leontief, the originator of input-output economics, published input-output tables often lack adequate detail on value added for CGE modeling. Writing in the 1930s,<sup>47</sup> Leontief saw his input-

<sup>45</sup> See Bonds and Aylor (1998).

<sup>46</sup> Recall that the *General government* industry has only primary factor inputs.

<sup>47</sup> See, e.g. Leontief (1936).

output system as a means of estimating the effects on employment by industry of demand stimulation policies in an environment of high unemployment and excess capacity, a situation in which resource constraints are unimportant. Consequently, relative to the demand side of his model, Leontief gave little emphasis to value added. This bias in the presentation of input-output tables has continued even in countries in which full employment and inflationary conditions were present for much of the second half of the twentieth century making resource constraints of paramount interest. Apart from taxes which we have already discussed, the BEA input-output tables divide value added for each industry in the US into only two categories: *Compensation of employees* and *Other value added* (LAB and OVA in Figure 2.9).

For a CGE model we require the measure of labor input in each industry to be compensation of employees *plus* the value of non-payroll labor (the self employed and family helpers). Data from the Bureau of Labor Statistics (BLS) indicates that about 10% of all jobs are held by non-payroll workers and that for some industries this percentage is much higher. For example, non-payroll workers hold about half the jobs in agriculture. In developing the USAGE database, we imputed a wage (discussed below) to non-payroll workers in each industry. We then adjusted the BEA value-added data by removing the estimated values of non-payroll labor from the OVA row and adding them to the LAB row. BLS data also allowed us to disaggregate the adjusted LAB row into a large number of occupations. This has been important in studies such as that described in Section 2.2 concerned with immigration and other labor market issues.

We interpreted the entries in the adjusted OVA vector as rental on capital.<sup>48</sup> However, in many cases further adjustments to OVA were necessary so that our database implied reasonable values for rates of return on capital. In working out the implied rate of return for industry  $j$ , we divided  $OVA_j$  by the value of capital stock ( $K_j$ ) and deducted the rate of depreciation ( $D_j$ ).

Estimates of capital stocks and depreciation rates are important in dynamic CGE models but unfortunately relevant data are scarce. For the USAGE model the main data source was the BEA's dataset NDN-0216 (see Bureau of Economic Analysis, 1999) which gives usable data for capital stocks, investment and depreciation rates for the economy divided into 55 sectors.

An unattractive feature of these data is that they are classified to sectors on a company and ownership basis. For example, the capital stock for the construction sector in NDN-0216 refers to fixed capital owned by companies whose principal activity is construction. For modeling purposes we want to know how much capital is used in construction

<sup>48</sup> In the initial version of USAGE, rental on land was not modeled. Agricultural land was included in later versions concerned with biofuels; see, e.g. Winston (2009) and Gehlhar *et al.* (2010).



activities. Capital used in construction activities can differ sharply from the NDN-0216 concept for several reasons:

- Non-construction companies may undertake construction (e.g. mining companies may drill new wells, a construction activity) and therefore own capital that is used for construction activities.
- Construction companies may operate across several non-construction activities and therefore own capital that is used for non-construction activities.
- Construction companies may hire capital from financial institutions and therefore use capital in construction activities that is not owned by construction companies.

While the capital and investment data in NDN-0216 are on a company and ownership basis, the investment data in NDN-0224 (used in our estimation of investment by industry, see [Section 2.4.6.5](#)) are on an activity basis.<sup>49</sup> By comparing sectoral investment from NDN-0216 with a 55 sector aggregation of our input-output investment estimates, we made an assessment of the extent to which the company/ownership capital data in NDN-0216 was likely to be a satisfactory basis for estimating capital stocks by industry defined on an activity basis. For most sectors investment on the two bases was reasonably compatible. However, for some sectors the differences were dramatic. For example, NDN-0224 showed investment in the construction sector of \$32 billion, whereas NDN-0216 showed \$6 billion. It appears that construction in the US is carried out to a large extent by companies that do not specialize in construction or by construction companies using rented capital.

Despite the NDN-0216 and NDN-0224 incompatibilities, we had no choice but to use NDN-0216 as the basis for the USAGE capital stock estimates. To estimate capital stocks on an activity basis at the 55 sector level, we assumed that depreciation rates ( $D_j$ ) and capital growth ( $I_j/K_j - D_j$ ) for sector  $j$  calculated on an ownership basis from NDN-0216 also applied on an activity basis.

With sectoral capital stocks on an activity basis estimated in this way, we were able to calculate implied sectoral rates of return. This calculation gave rates of return for 26 of the 55 sectors outside the range 0–20. We considered estimates outside this range to be unrealistic and likely to cause difficulties in simulations.

Our first step in dealing with this problem was to revisit the issue of self-employment. For our initial estimates of imputed wages of self-employed workers we used the average wage rate of employees. Now for each of the 55 sectors we looked at the effects on OVA and implied rates of return of varying the self-employed/employee wage ratio between 0.5 and 4. In most sectors self-employment is

<sup>49</sup> NDN-0224 is compiled using input-output conventions. Under input-output conventions, capital is assigned to the industrial activity for which it is used. NDN-0216 is compiled using NIPA conventions. Under these conventions, capital is assigned to owning industries regardless of how it is used.

unimportant and variations in the wage ratio have little effect on estimated rates of return. However, for some sectors, we were able to make a plausible change in the wage ratio and at the same time produce a more realistic rate-of-return estimate. For health services, we raised the wage ratio to 2, thereby recognizing that self-employed health professionals are likely to be paid considerably more than health employees. This reduced the estimated rate of return in health services from 21% to a more reasonable 14%. For construction, on the other hand, we lowered the wage ratio, to 0.5. This seems reasonable because self-employed construction contractors (which include handymen) are likely to be paid considerably less than construction employees of major firms. The adjustment in the construction wage ratio increased the estimated rate of return in for the sector from an unlikely  $-14.3\%$  to a less unreasonable  $-3.5\%$ .

Having done as much as we could with the wage ratio, we were still had 18 sectors with estimated rates of return outside the range 0–20. For each of these 18 sectors we reset the value of capital stock. For sectors having initial estimated rates of return of over 20%, we raised our estimate of their capital stocks so that their rates of return fell to 20%. For sectors having initial estimated rates of return below zero, we lowered our estimate of their capital stocks so that their rates of return rose to zero. We then spread these final estimates of activity-based sectoral capital stocks to constituent USAGE industries mainly according to our estimates of other value added.

## **2.5 RESPONDING TO THE NEEDS OF CGE CONSUMERS: THE FOUR CLOSURE APPROACH**

From their beginnings in the 1970s, MONASH models have been produced to satisfy the needs of consumers of CGE services in the public and private sectors. These are real needs expressed via willingness to pay from limited budgets. This means that the evolution of MONASH models has been largely demand driven. [Section 2.5.1](#) describes what it is that consumers of CGE services demand. [Section 2.5.2](#) then describes how, with MONASH models, we have tried to satisfy these demands via simulations conducted under four closures: historical, decomposition, forecast and policy.

### **2.5.1 What consumers of CGE services want**

Both public and private sector consumers of services based on MONASH models are mainly concerned with current policy proposals. In assessing the quality of modeling services they assign heavy weight to: up-to-date data; detailed disaggregation in the focus area and accurate representation of relevant policy instruments; and disaggregated results. While not directly demanded by consumers, we have found that servicing their needs is

made easier if we can produce forecasts showing likely developments in the economy with and without the policy under consideration and decomposition analyses quantifying the role of similar policy changes in the past.

#### ***2.5.1.1 Up-to-date data***

Consumers are often well-informed about the latest statistics for their particular industries of interest. If they see conflict between what they know and data in a model, they lose confidence in all aspects of the model and its results. We suffered an example of this in 2006 when we were working on the US International Trade Commission's flagship publication concerned with the economy-wide effects of removing all major import restraints (US International Trade Commission, 2007). At a late stage in this work a US International Trade Commission Commissioner, who was knowledgeable about the Textile and clothing sector, drew our attention to data showing that US Apparel output in 2004 was \$34 billion, yet our model's database was showing \$61 billion. Our \$61 billion was an estimate and overlooked data in the latest Annual Survey of Manufactures. In the context of the overall project, the problem seemed relatively minor. Nevertheless, although rectifying it involved considerable delays, this could not be avoided. It was essential for the credibility of the entire project. While satisfying consumer demands for accurate up-to-date data is a chore for CGE modelers, there is often a genuine payoff in terms of improved real-world relevance. In our example, failure to recognize that \$34 billion was the right number would have led to an overstatement of the welfare gain from removing import restraints on apparel.

#### ***2.5.1.2 Detailed disaggregation in the focus area and accurate representation of relevant policy instruments***

Policy proposals often call for the application of complicated instruments at a fine level of industry/commodity disaggregation. This sometimes causes consumers of modeling services to demand model features that stretch producers of these services to their limits or beyond. In the US, for example, our colleague Ashley Winston (Winston, 2009) has responded to demands by consumers interested in biofuel policy by extending a 500 commodity model to include as separate commodities: corn; switch grass; crop residue; cellulosic materials; organic byproducts; corn ethanol; dried distillers grains with solubles; cellulosic ethanol; advanced ethanol; gasoline; diesel; and Other fuels. Reflecting consumer demands, Winston also incorporated explicit complementarity conditions specifying the operation of tariff rate quotas on imports of Sugar and other agricultural products together with 72 types of agricultural land. To achieve all this required highly skilled theoretical, computing and data work over a long period of time. Being stretched to meet consumer demands can lead to productive and creative outcomes, as in Winston's case. However, being stretched can

cause difficulties for CGE modelers in terms of budgets, time constraints and research priorities. CGE modelers must sometimes be firm in asking their customers to set the problem (e.g. work out the effects of replacing  $x\%$  of imported oil with domestically produced biofuels) but not to dictate the way in which the modeler should tackle the problem. While a natural inclination of consumers is to think that highly elaborate modeling is called for, producers can often find shortcuts. [See, e.g. [Dixon \*et al.\* \(2007\)](#) in which the biofuel issue was tackled as a technological change in the production of motor fuels.] In these cases, the consumer is usually convinced of the adequacy of the short cut when defensible results are produced on time and within budget.

### 2.5.1.3 Disaggregated results

Consumers of modeling services want more than bottom-line aggregate welfare and GDP effects. The real policy debate is often about reallocating large revenues across industries and factors: there can be big winners and losers even when the bottom line is small. [Table 2.5](#) is an example of the kind of information that consumers find useful. It shows results from a US International Trade Commission study on the effects of imposing a Steel Safeguard tariff. The US International Trade Commission estimated that the imposition of the tariff would result in a net loss in GDP of \$30.4 million. This tiny net effect reflects large gains for the government in tariff revenue and for the Iron and steel industry in capital income, offset by losses in labor income and capital income in other industries, particularly those that use iron and steel inputs. Ability to provide disaggregated information is a CGE strength and demands by consumers for this information justifies the retention in CGE models of considerable detail.

### 2.5.1.4 Baseline forecasts

Many consumers of CGE analyses have little background in economics. It does not come naturally to them to think in terms of the effect on variable  $i$  of changes in policy  $j$

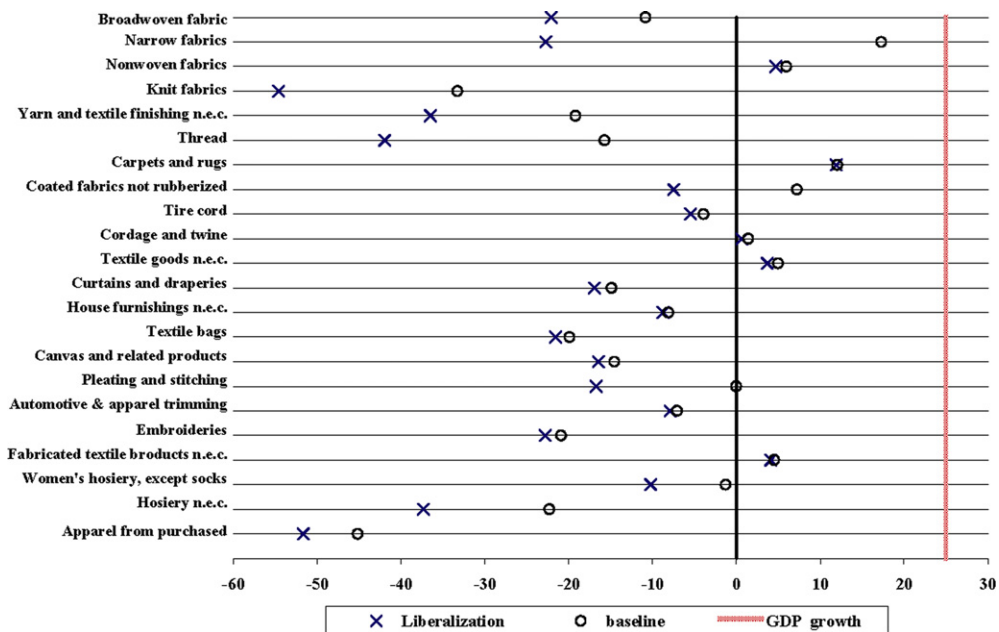
**Table 2.5** CGE results: income changes from an iron and steel safeguard tariff

		Income changes (\$ million)
Tariff revenue		649.9
Labor income		−386.0
Capital income		−294.3
Iron and steel industry	239.5	
Input suppliers to iron and steel	67.4	
Other industries (including steel users)	−601.2	
GDP		−30.4

Source: [US International Trade Commission \(2003, pp. 4–5\)](#).

holding all other exogenous variables constant. A current example in Australia is the heated debate concerning the government's buyback of water rights from farmers along the Murray and Darling rivers. The aim is to increase water flow in the rivers, thereby improving the downstream environment. Implementation of the policy coincided with a severe drought and depressed conditions in rural communities. Economists' estimates showing that the effects of buyback on rural economic activity are negligible are not accepted because community leaders and their constituents are not separating the effects of buyback from the effects of drought.

We have found that the presentation of a baseline without the policy change together with a projection with the policy change helps consumers to separate out the effects of the policy change from the effects of other factors. Figure 2.10 is a diagrammatic presentation that has been used to advantage by the US International Trade Commission to explain the effects of unilateral trade liberalization by the US involving the dismantling of all major import restraints. The figure shows USAGE results for percentage changes between 2005 and 2013 in outputs of textiles and apparel in a baseline without liberalization (circles) and an alternative projection that includes liberalization (crosses). The circles immediately tell consumers that most of the textile/apparel sector is in decline and that none of it is likely to achieve growth



**Figure 2.10** Percentage changes in outputs of textiles and apparel, baseline projection and liberalization, 2005–2013. Source: *USAGE results presented by the US International Trade Commission (2009, p. 49).*

to match that of GDP. For about half the industries in the sector, the crosses and circles are close together indicating that liberalization would have only a minor effect on their prospects. For five industries (Narrow fabrics, Thread, Knit fabrics, Yarn and textile finishing n.e.c., and Pleating and stitching), liberalization is projected to have a severely negative effect on output growth: the gap between the crosses and circles is more than 15 percentage points. In the case of Narrow fabrics, liberalization converts relatively strong growth into contraction. For the other four industries, liberalization converts poor prospects into substantially worse prospects. As explained by the [US International Trade Commission \(2009\)](#) and more fully in [Fox \*et al.\* \(2008\)](#), the five textile/apparel industries worst affected by liberalization would all suffer from loss of export markets. These markets depend on rules of origin which give some countries an incentive to import textile inputs from the US. With sufficient US content in their textile/apparel exports, these countries gain access to the US market at zero tariff. With liberalization, which reduces the tariff to zero on US imports from all countries, the incentive to source intermediate inputs from the US disappears.

While a baseline is valuable from a presentational point of view, its role goes deeper than that. As discussed in [Section 2.5.2.3](#), answers to policy questions can be improved by generating them as deviations around a realistic baseline forecast. There are at least three other reasons (discussed later in this Handbook<sup>50</sup>) for baseline forecasting.

- (i) Consumers are interested in the baseline: they want to know where we think the economy is going, not just how the economy will be affected by a particular policy change or other shock to the economy.
- (ii) A forecast is necessary in calculating adjustment costs associated with a policy change.
- (iii) Forecasting opens up a possibility for model validation and model improvement.

### **2.5.1.5 Historical decomposition analyses**

Another useful device for helping consumers to separate out the effects of policy changes from the effects of other factors is an historical decomposition. [Table 2.6](#) is an example. It shows results from a 1987–1994 decomposition simulation with Australia's MONASH model undertaken to support a report by the [Industry Commission \(1997\)](#). The Commission was investigating the effects of reductions, proposed for 2001, in the tariff applying to imports of Motor vehicles and parts (MVP).<sup>51</sup> The technique of historical decomposition is described in [Section 2.5.2.2](#). Here we will simply explain the results.

<sup>50</sup> See Section 19.6 of Dixon and Rimmer in Chapter 19 of this Handbook.

<sup>51</sup> [Dixon \*et al.\* \(1997\)](#) gives the details of the motor vehicle decomposition study. Another decomposition study, focused on the determinants of growth in Australia's international trade, is described in [Dixon \*et al.\* \(2000\)](#).

**Table 2.6** Output of the Australian MVP industry, 1987–1994

Driving factor	Percentage effect
1. Shifts in foreign demands and import supply curves	−4.8
2. Changes in protection	−5.6
3. Technical change	24.4
4. Growth in aggregate employment	16.7
5. Changes in import/domestic preferences	−4.0
6. Changes in required rates of return	−7.0
7. Other factors	−5.2
Total	14.5

*Source:* Extracted from results reported in table 5.5 of Dixon and Rimmer (2002).

As shown in the last row of Table 2.6, the output of Australia's MVP industry grew between 1987 and 1994 by 14.5%. Our historical decomposition simulation attributes this growth to seven factors.

The first is shifts in the positions of foreign demand curves for Australian exports and foreign supply curves for Australian imports. Between 1987 and 1994 these shifts were generally favorable. Holding constant all other exogenous variables (protection, technology, etc.) MONASH showed that the shifts in these curves gave Australia an improvement in its terms of trade of nearly 20%. However, this was bad for the MVP industry, reducing its output by 4.8%. The industry was damaged by good news for the rest of the economy via exchange rate effects. Improvement in the terms of trade strengthens Australia's real exchange rate. The MVP industry faces considerable competition from imports and real appreciation associated with terms-of-trade improvement weakened its competitive position.

The second factor is changes in protection. Between 1987 and 1994, tariff were reduced on almost all imports. The MVP tariff cut reduced the landed-duty-paid price of MVP imports by 6.5%. Although the import/domestic substitution elasticity for MVP products is high (2.55), the damage to MVP output was limited to 5.6% (row 2). The MVP industry benefited from cuts in tariffs on its inputs and from real exchange rate devaluation associated with tariff reductions more generally.

The third factor is technical change throughout the economy.<sup>52</sup> In the MVP industry, technical change favored intermediate inputs and capital relative to labor but there was almost no net improvement in total factor productivity. The large (24.4%) contribution to growth in MVP output attributed to technical change in Table 2.6 arises from two indirect sources. (i) Technical change is a major driver of GDP growth

<sup>52</sup> This is an amalgam of the effects of many types of technical change: input-saving for each intermediate and primary factor flow to each industry, margin-saving, and input-saving in the creation of units of capital.

which in turn contributes to growth in demand for MVP products. (ii) Between 1987 and 1994 there was a large increase throughout Australian industries in the use of MVP products per unit of output. In our calculations this was treated as an MVP-using technical change. Rather than being strictly technological, much of the increase in MVP inputs reflected the exploitation of a loophole in Australia's tax laws in this period which allowed employers to give workers tax-free use of company cars for private purposes in lieu of taxable income.

The fourth factor is growth in aggregate employment. Together with changes in technology, employment growth was responsible for most of the growth between 1987 and 1994 in GDP.<sup>53</sup> Thus, like row 3, row 4 of Table 2.6 shows a major contribution (16.7%) to the output of the MVP industry.

The fifth factor refers to changes in import/domestic preferences reflected by changes in import/domestic quantity ratios beyond those that can be explained by changes in import/domestic price ratios. Between 1987 and 1994, rationalization of the Australian MVP industry reduced the variety of Australian-produced cars. Simultaneously, there was an increase in the variety of imported cars available to Australian consumers.<sup>54</sup> This generated a strong twist in preferences in favor of imported cars. However the damage to MVP output shown in row 5 of Table 2.6 is only 4.0%. This entry includes not only the effects of the MVP twist, but the effects of all other import/domestic twists. These were generally in favor of imports. With a twist-induced increase in aggregate imports there was an associated real devaluation. This gave the MVP industry some relief against import competition.

The sixth factor is changes in required rates of return which increased between 1987 and 1994. This means that to support any given level of capital growth, investors needed to anticipate a higher rate of return in 1994 than they did in 1987. An increase in required rates of return can be caused by increases in interest rates and by reduced investor confidence. The increase in required rates of return between 1987 and 1994 reduced MVP output by 7%, mainly by reducing aggregate investment which is an MVP-intensive component of aggregate expenditure.

Decomposition simulations allow us to look at the effects of an overwhelming number of exogenous variables. Inevitably we must terminate the process by having an 'Other factors' row. In Table 2.6 this is row 7 which covers a variety of factors each having a minor effect on MVP output between 1987 and 1994. These factors include

<sup>53</sup> We can think of GDP as being specified by an aggregate production function:  $Y = A * F(K, L)$ . In the decomposition simulation technology ( $A$ , considered in row 3) and employment ( $L$ , considered in row 4) are exogenous.  $K$  is endogenous, determined by exogenously given rates of return that are held constant in all rows except 6.

<sup>54</sup> Up to the mid-1980s the MVP industry was protected by quantity quotas on imports which favored the import of a narrow range of large expensive cars. After 1985, quotas were replaced by tariffs, leading to a dramatic increase in the variety of imported cars.



changes in: the average propensity to consume out of household disposable income; the ratio of public to private consumption expenditure; the commodity composition of public consumption; and shifts in export supply curves associated with increased awareness by Australian businesses of export opportunities.

In the Australian policy debate, protection is often portrayed as *the* critical variable determining the welfare of the MVP industry. Each time there is a proposal for reduced protection, supporters of the industry argue that the industry's survival would be threatened. Results such as those in Table 2.6 help to diffuse such arguments by putting the effects of past cuts in tariffs into historical context. As shown in Table 2.6, the health of the industry depends on many factors apart from protection. These include international trading conditions, technology, economy-wide employment growth, import/domestic preferences and required rates of return on capital.

### 2.5.2 Four-closure approach

Satisfying consumer demands for a high level of industry disaggregation and for presentation of results in many dimensions is facilitated in MONASH-style modeling by the adoption of the Johansen/Euler computational procedure implemented in GEMPACK software. For many years this has allowed us to handle large models without serious computing limitations. High dimensionality is also useful in satisfying consumer demands for accurate representation of policy instruments. MONASH models are equipped with tax and technology variables associated with every commodity and factor flow. Availability in MONASH models of detailed technology variables has been particularly useful in policy work concerned with microeconomic reforms, trade promotion, the environment, greenhouse gases and other energy issues. Policies in these areas can often be represented as technological changes that alter the mix of inputs per unit of output in narrowly defined parts of the economy.

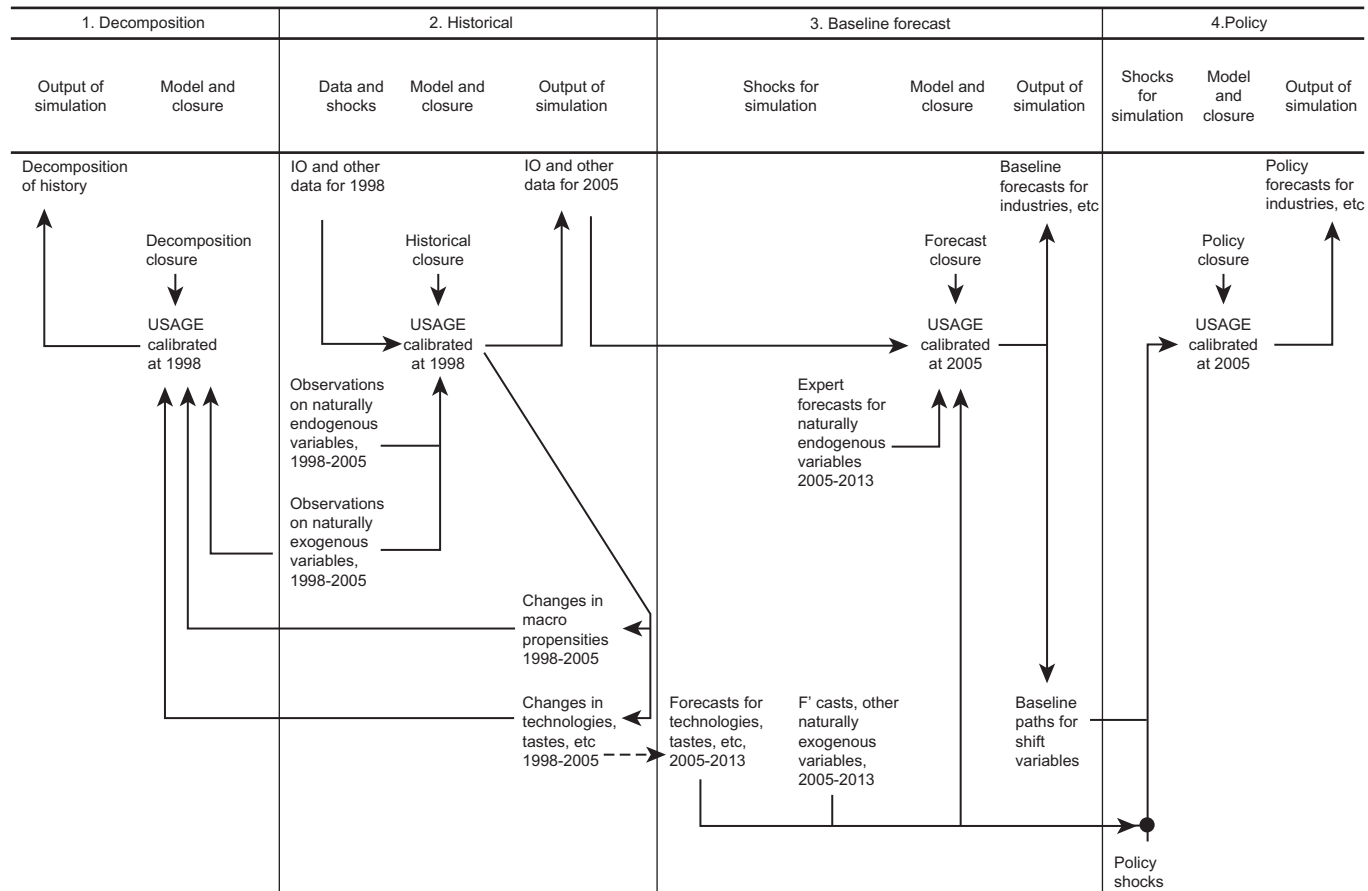
To satisfy needs for up-to-date data, baseline forecasts and historical disaggregation analysis, we have developed four broad modes of analysis with MONASH models. These are conducted with simulations under four different closures:

- The *historical* closure in which the exogenous variables are chosen so that observations at a detailed commodity/industry level on movements in consumption, investment, government spending, exports, imports, employment, capital stocks and many other variables can be introduced to the model as shocks. Computations with this closure can be used to generate up-to-date input-output tables that incorporate available statistics for years since the last published input-output table. Historical simulations also produce disaggregated estimates of movements in many naturally exogenous variables representing: industry technologies; household preferences; required rates of return on capital; and positions of export demand curves and import supply curves.

By naturally exogenous variables we mean those that are not normally explained in the type of CGE model being applied.

- The *decomposition* closure in which technology, preference and other naturally exogenous variables are exogenous so that they can be shocked with the movements estimated for them in an historical simulation. As we saw in Table 2.6, computations with this closure can be used to identify the roles in the growth of industry outputs and other naturally endogenous variables of changes in technology, changes in preferences, and changes in other naturally exogenous variables.
- The *forecast* closure which is used in simulations designed to produce a baseline picture (e.g. the circles in Figure 2.10) of the future evolution of the economy. The underlying philosophy of this closure is similar to that of the historical closure. In both closures, we exogenize variables for which we have information, with no regard to causation. Rather than exogenizing variables for which we have historical observations, in the forecast closure we exogenize variables for which we have forecasts. This might include macro variables, exports by commodity and demographic variables for which forecasts are provided by official organizations. Naturally exogenous technology, preference and trade variables in forecast simulations are largely exogenous and are given shocks that are informed by trends derived from historical simulations.
- The *policy* closure which is used in simulations designed to quantify the effects of changes in policies or other exogenous shocks to the economy. The underlying philosophy of this closure is similar to that of the decomposition closure. In both policy and decomposition closures, we are concerned with causation, with how tariff changes, for example, cause changes in employment. Thus, in policy closures, as in decomposition closures, naturally exogenous variables are exogenous and naturally endogenous variables are endogenous. In policy simulations, nearly all of the exogenous variables adopt the values that they had, either endogenously or exogenously, in the forecast simulation. The only exceptions are the policy variables of focus. For example, if we are interested in the effects of a tariff change, then the relevant tariff variable is moved away from its baseline forecast path. The effects of the tariff change on macro variables, exports by commodity and other endogenous variables are calculated by comparing their paths in the policy simulation with their paths in the baseline forecast simulation (e.g. the crosses in Figure 2.10 compared with the circles).

Connections between the four modes of analysis are illustrated in Figure 2.11. For concreteness we have drawn Figure 2.11 with reference to the US International Trade Commission study, conducted with the USAGE model, that produced the baseline forecast and policy results given in Figure 2.10.



**Figure 2.11** Connections between four modes of analysis with MONASH-style models.

### 2.5.2.1 MONASH-style historical simulations

When the US International Trade Commission study was undertaken, the latest USAGE database was for 1998 and there were no published input-output data for a year beyond that date. The US International Trade Commission required a baseline and policy simulation for 2005–2013. Thus, the first job was to move the USAGE database forward to 2005. To do this, we performed an historical simulation. As shown in panel 2 of Figure 2.11, we started with USAGE calibrated with input-output and other data for 1998, and shocked it with observed movements between 1998 and 2005 in both naturally exogenous and naturally endogenous variables.

As is typical in historical simulations, the shocked naturally exogenous variables included tax rates, tariff rates, public expenditure and population. The shocked naturally endogenous variables included standard macro variables and a large number of industry and commodity variables. Absorbing macro variables requires endogenization of naturally exogenous propensities. For example, to allow growth in household consumption to be set exogenously at its observed value requires endogenization of the average propensity to consume. Absorbing micro observations requires endogenization of corresponding naturally exogenous taste, technology and trade variables. For example, data on growth in consumption of tobacco products (a naturally endogenous variable) is absorbed by allowing the model to tell us endogenously that there was a change in consumer preferences (a naturally exogenous variable) against this product. As indicated in the output column of panel 2 in Figure 2.11, the historical simulation undertaken for the US International Trade Commission produced the required up-to-date data for 2005 (including an input-output table) which, in principal, incorporated all statistical information that was available in 2005. It also produced estimates of changes between 1998 and 2005 in tastes, technologies, required rates of return and positions of export demand and import supply curves.

While the broad ideas underlying an historical simulation are straightforward, coping with the details of the data make the process time-consuming and difficult. For example, the USAGE model in our 1998–2005 historical simulation had 500 industries/commodities but data availability made it necessary to introduce micro shocks at a variety of different levels of disaggregation: 397-order export and import values from the US International Trade Commission; 160-order import prices from the BLS; 100-order export prices from the BLS; 56-order private consumption quantities and prices from the BEA; 20-order public consumption quantities and prices from the BEA; 68-order industry outputs and value-added prices from the BEA; 60-order occupational wage rates from the BEA; and 338-order industry employment from the BLS. Each of these micro data concepts were defined and absorbed in the USAGE historical simulation via special purpose equations. For example, to allow us to use BLS data on employment by 338 industries, we included in USAGE equations of the form:

$$l_{\text{BLS}}(q) = \sum_{i=1}^{500} S(i, q) * l_{\text{USAGE}}(i), \quad q = 1, 2, \dots, 338 \quad (2.54)$$

$$alab_{\text{USAGE}}(i) = \sum_{q=1}^{338} M(i, q) * f_{\text{BLS}}(q), \quad i = 1, 2, \dots, 500, \quad (2.55)$$

where  $l_{\text{BLS}}(q)$  is growth in employment in BLS sector  $q$ ,  $l_{\text{USAGE}}(i)$  is growth in employment in USAGE industry  $i$ ,  $S(i, q)$  is the share of BLS sector  $q$ 's employment accounted for by USAGE industry  $i$ ,  $alab_{\text{USAGE}}(i)$  is labor-saving technical change in USAGE industry  $i$ ,  $f_{\text{BLS}}(q)$  is a shift variable for BLS sector  $q$ , and  $M(i, q)$  is a coefficient that has value 1 if USAGE industry  $i$  is part of BLS sector  $q$  and zero otherwise. For simplicity we assume in this example that each USAGE industry is contained in just one BLS sector.

Equation (2.54) defines growth in employment by BLS sector  $q$  in terms of growth in employment in component USAGE industries. In the historical simulation the naturally endogenous variable  $l_{\text{BLS}}(q)$  was exogenized and shocked with the value implied by the BLS data on employment. Correspondingly,  $f_{\text{BLS}}(q)$  was endogenized. Via (2.55), we imposed the assumption that labor-saving technical change was the same in each USAGE industry  $i$  contained in BLS sector  $q$ .

The obvious alternative to within-model determination of USAGE industry employment growth via equations such as (2.54) and (2.55) is to assume, outside the model, that BLS sector  $q$ 's employment growth applied to each USAGE industry in the sector. However, we prefer the within-model approach because it allows the allocation of employment growth in sector  $q$  to component USAGE industries to be informed by other information used in the historical simulation. For example, if USAGE industries 1 and 2 are both in BLS sector  $q$  and other information in the historical simulation indicates that output in industry 1 grew rapidly relative that in industry 2, then it is reasonable to suppose that employment in industry 1 grew rapidly relative to employment in industry 2. This will be the result in an historical simulation under the uniform-within-sector technology assumption implemented in the model via (2.54) and (2.55) but not under the uniform-within-sector employment assumption implemented outside the model. For a more general discussion of the advantages of within-model allocation procedures, see Dixon and Rimmer (2002, pp. 200–201).

### 2.5.2.2 MONASH-style decomposition simulations

Once an historical simulation is completed, then we can perform a decomposition simulation. The decomposition simulation uses the same model and data as the historical simulation, but a different closure. All of the exogenous variables in the decomposition closure are *naturally* exogenous. As indicated by the arrows from panel 2 to panel 1 in Figure 2.11, these naturally exogenous variables were shocked in the decomposition simulation with the same values they had (either exogenously or endogenously) in the historical simulation. Consequently, the decomposition simulation produces the same results as the historical simulation.

The reason for performing decomposition simulations is that they allow us to decompose movements in macro and industry variables into parts attributable to different driving forces. This is done by partitioning the exogenous variables and separately computing the effects of the shocks for the each subset.<sup>55</sup> The results obtained in this way are a legitimate decomposition to the extent that the exogenous variables in the decomposition simulation can be thought of as varying independently of each other. In setting up the decomposition closure, the exogenous variables are chosen with exactly this property in mind. Thus, in the decomposition closure we find on the exogenous list variables representing policy instruments, technologies, tastes, required rates of return and positions of export demand and import supply curves. All of these can be considered as independently determined and all can be thought of as making their own contributions to movements in endogenous variables such as incomes, consumption, exports, imports, outputs, employment and investment.

### 2.5.2.3 *MONASH-style forecast simulations*

MONASH-style forecast simulations are conducted with models calibrated to data for a recent year. These data are often generated by an historical simulation. As indicated by the solid arrow from panel 2 to panel 3 in Figure 2.11, the 2005–2013 baseline forecast for the US International Trade Commission project was created by USAGE calibrated with data for 2005 created by our historical simulation for 1998–2005.

In creating shocks to generate a baseline forecast, we draw as much as possible on the work of specialist forecasting organizations. In many countries, well-informed forecasts are available from organizations covering different aspects of the economy. In Australia, for example, macro forecast are provided by Access Economics and the Australian Treasury; forecasts for volumes and prices of agricultural and mineral exports are provided by the Australian Bureau of Agricultural and Resource Economics; and forecasts for tourist numbers are provided by the Bureau of Tourism Research. In the US, macro forecasts are provided by the Congressional Budget Office, the US Department of Agriculture and the BLS; and forecasts for an array of energy variables are provided by the Energy Information Administration. All these forecasts are prepared by large teams of economists with considerable expertise. In forecast simulations with MONASH-style models we try to take advantage of their knowledge.

We do this by exogenizing variables for which there are reputable expert forecasts and using these forecasts as shocks. To accommodate macro forecasts we endogenize macro

<sup>55</sup> Because USAGE is a non-linear system, the effect on endogenous variable  $i$  of movements in exogenous variable  $j$  cannot be computed unambiguously: the effects of movements in any exogenous variable depend on the values adopted for other exogenous variables. To resolve this problem we, in effect, carry out decomposition simulations in a linear system in which derivatives of endogenous variables with respect to exogenous variables are evaluated at a half-way point between the initial and final values of the exogenous variables. The computations can be done conveniently in GEMPACK, see Harrison *et al.* (2000).

propensities. To accommodate micro forecasts we endogenize corresponding micro shift variables. For example, if forecasts are available from the Energy Information Administration on the sale of electricity to US industries and households, then we endogenize electricity-using technical change in US industries and an electricity preference variable for US households. In [Figure 2.11](#), the input of expert forecasts occurs mainly in the area in panel 3 marked ‘Expert forecasts for naturally endogenous variables, 2005–2013’. We may also have expert input in the area marked ‘Forecasts, other naturally exogenous variables, 2005–2013’ for tariff rates and other naturally exogenous variables.

Because we know less about the future than the past, MONASH-style forecast closures are more conventional than historical closures. In forecast closures most disaggregated technology and preference variables are exogenous. In setting their forecast values, we rely heavily on extrapolations from historical simulations. This is indicated by the dotted arrow connecting panel 2 with panel 3 in [Figure 2.11](#).

There are two outputs from a forecast simulation. The first is a baseline forecast for a potentially huge set of disaggregated variables. The forecasts start from an up-to-date database and incorporate technology, preference and trade trends derived from recent history together with expert forecasts for macro variables and for whatever micro variables are covered by specialist forecasting organizations. The second output is forecasts for movements in naturally exogenous shift variables, such as the average propensity to consume and electricity-saving technical change, that were endogenized to absorb expert forecasts.

#### **2.5.2.4 MONASH-style policy simulations**

Policy closures are similar to decomposition closures. In policy closures naturally endogenous variables (such as macro variables and sales of electricity) are endogenous. They must be allowed to respond to the policy change under consideration. Correspondingly, in policy closures naturally exogenous variables (such as the average propensity to consume and electricity-saving technical change) are exogenous. If there are no policy shocks, a policy simulation generates the same solution as the baseline forecast. With no policy shocks all of the exogenous variables would have the same values as in the baseline: this is indicated by the arrows from panel 3 to panel 4 in [Figure 2.11](#). Thus the differences between results in a policy simulation and the baseline forecast are entirely due to policy shocks. Under the assumption that the non-policy exogenous variables are genuinely independent of the policy, these differences can be interpreted as the effects of the policy.

The effects of any given policy depend on the structure of the economy. For example, the removal in the US of tariffs on imports of Textiles and apparel will have a different effect on the economy if the domestic sector accounts for 0.7% of aggregate employment (as it did in 1998) than if it accounts for only 0.2% of aggregate employment (as is likely in 2015). In considering policy proposals we want to know the likely effects in the future.

These depend on the structure of the economy in the future. Thus, for policy analysis, it is a major advantage to be able to calculate policy effects in the MONASH style as deviations from a baseline that gives a plausible picture of the future structure of the economy.

## 2.6 CONCLUDING REMARKS

Here are the ideas that we hope readers will take from this chapter.

First, results from detailed CGE modeling can be explained in a convincing manner to people without CGE backgrounds. We illustrated this in [Section 2.2](#) by explaining USAGE results for the effects of restricting the supply of unauthorized immigrants to the US workforce. Our explanation relied on elementary microeconomics (e.g. demand and supply curves) and on identifying key data items (e.g. numbers of unauthorized workers in different occupations). Explaining results in a way that is accessible to people with backgrounds in economics, but not CGE modeling, is necessary for CGE modeling to be influential in policy circles. Policy advisors cannot effectively carry our results forward unless they have confidence in them. They can only have sufficient confidence to defend our results if they understand them.

A second idea from [Section 2.2](#) is that CGE results are often best explained in a macro-to-micro, non-circular sequence. For example, our explanation of the USAGE results for the effects of restricting the supply of unauthorized immigrants started with aggregate employment and aggregate capital. Then we moved to the expenditure side of the national accounts and eventually to occupations.

A third idea illustrated in [Section 2.2](#) is that disaggregated CGE modeling can produce results that are credible, new, policy-relevant and not available from aggregated models. An example in [Section 2.2](#) is the Occupation-mix effect. Identifying this effect depended on having a model with considerable labor market disaggregation. Critics, with the benefit of our explanations, sometimes suggest that our results are obvious and did not require the application of a large-scale model. Our response is that it was the model that alerted us, and we suspect them, to the result. We would not have thought of the Occupation-mix effect and numerous other subtle results, let alone quantified them, without a detailed MONASH-style model.

The main idea in [Section 2.3](#) is that Johansen is still worth reading. His 1960 book sets out a simple effective computing technique based on a representation of a model as a rectangular system of linear equations in changes and percentage changes of the variables. He then introduces a BOTE method for interpreting results and applies it in an analysis of the matrix showing the elasticities of endogenous variables with respect to exogenous variables. He uses this matrix in several applications including a decomposition of history and a validation check of his model's forecasting performance. The addendum in his 1974 book shows Johansen's enthusiasm for having his model used, developed and scrutinized



in policy departments of the Norwegian government. By starting with Johansen's simple linear framework, MONASH modelers were able to make rapid progress in the 1970s with innovations in the specification of international trade, dimensionality, closure flexibility and the use of complex functional forms. They also eliminated Johansen's linearization errors. This was done without sacrificing simplicity and transparency by introducing the Johansen/Euler multistep method.

The first key idea in [Section 2.4](#) is that the initial solution is important. It can be derived mainly from the input-output database. Then derivative methods can be used to compute other solutions either for the same year (comparative statics) or for a linked sequence of years (dynamics). The derivative method used by MONASH models is Johansen/Euler. This can be applied routinely even for very large models using GEMPACK software.

Another idea in [Section 2.4](#) is that creation of a database from available statistics for a detailed policy-relevant CGE model is a major task requiring skill and perseverance. It is certainly too hard for a lightly supervised research assistant.

The central idea in [Section 2.5](#) is that the primary purpose of CGE modeling is to assist in policy formation. Policy advisors on trade, microeconomic reform, the environment, labor markets, natural resources and taxation want models with high levels of disaggregation, up-to-date data and accurate representations of policy instruments. These wishes should be respected by producers of CGE services. In trying to satisfy consumer demands, MONASH modelers have devised the four-closure approach: historical; decomposition; forecast; and policy.

The final idea is that research in CGE modeling benefits from a team environment. The enduring team at CoPS/IMPACT has facilitated the creation and application of MONASH models in several ways. First, it has allowed members of the team to adopt a degree of specialization in theory, data, computing and application. The most obvious payoff from specialization has been the development of the GEMPACK software alongside the models. The GEMPACK group, headed within CoPS/IMPACT by Ken Pearson and Mark Horridge, understands and responds to modeling needs as they emerge and anticipates future needs. However, this is not the only payoff from specialization. Team members specialize on particular countries (e.g. Australia, US and China) and particular issues (e.g. labor markets, energy and environment). While building their own specialist knowledge, they absorb general techniques (e.g. the four closure approach) from other members of the team. Transfer of knowledge within the team is of particular advantage to new members who start with fully functioning models and draw on many years of accumulated experience from people who know how to adapt models for particular applications. A second benefit of an enduring team has been the accumulation of modeling improvements. With a long collective memory, CoPS/IMPACT is able to maintain ambitious, large-scale, continuously improving models that frequently generate insights that are not available from small single-purpose models.

## APPENDIX: THEORETICAL JUSTIFICATION FOR THE JOHANSEN/EULER SOLUTION METHOD

For many people, the least convincing aspect of Figure 2.5 as an explanation of the Johansen/Euler method is the assumption that the slope of **rs** (an ‘off-solution’ slope) is a good approximation to the derivative of  $Y$  with respect to  $X$  on the solution line at **w**. Once they have doubts about that assumption, then their confidence in the theoretical underpinnings of the method is seriously eroded.

In this Appendix we provide some reassurance by proving a proposition concerning the convergence of Johansen/Euler solutions as the number of steps approaches infinity. This proposition was first proved in the context of an  $n$ -variable/ $m$ -equation CGE model by Dixon *et al.* (1982, section 35). Here we set out the proof for a two-variable/one-equation model. Nothing essential is lost from the mathematical argument by cutting down the dimensions. Being able to treat  $X$  and  $Y$  as scalars eliminates the need for some rather clumsy matrix notation. We also provide an explanation of the idea mentioned in Section 2.4.1 underlying Richardson’s extrapolation: doubling the number of steps in a Johansen/Euler computation tends to halve the linearization error.

### A.1 Convergence proposition for the Johansen/Euler method

**Proposition.** Assume that we are dealing with a two-variable/one-equation model in which the endogenous variable,  $Y$ , is a differentiable function of the exogenous variable,  $X$ :

$$Y = G(X). \quad (\text{A.1})$$

While we do not know the form of  $G$ , assume that we do know how to evaluate a function  $B(X, Y)$  which has the property that:

$$B(X, Y) = G_X(X) \quad \text{if } Y = G(X), \quad (\text{A.2})$$

where  $G_X(X)$  is the Jacobian matrix of  $G$  evaluated at  $X$ . In the scalar case we are considering,  $G_X(X)$  is simply  $\partial Y / \partial X$  where  $Y$  is given by (A.1).

Assuming that the derivative of  $G_X$  with respect to  $X$  and the derivative of  $B(X, Y)$  with respect to  $Y$  are bounded over the relevant domain of  $(X, Y)$ , then the Johansen/Euler method will converge, i.e. given a starting point  $(\bar{X}, \bar{Y})$  satisfying:

$$\bar{Y} = G(\bar{X}), \quad (\text{A.3})$$

then:

$$\lim_{n \rightarrow \infty} Y_n^n = G(\bar{X} + \Delta X), \quad (\text{A.4})$$

where  $\Delta X$  is any given change in  $X$  and  $Y_n^n$  is the  $n$ -step estimate of  $G(\bar{X} + \Delta X)$ .

**Proof.** We denote the values of  $X$  and  $Y$  reached in the  $r$ th step of an  $n$ -step computation by  $X_n^r$  and  $Y_n^r$ . Then:

$$X_n^r = X_n^0 + \left(\frac{r}{n}\right) * \Delta X, \quad r = 1, \dots, n, \quad (\text{A.5})$$

and:

$$Y_n^r = Y_n^{r-1} + \left(\frac{1}{n}\right) * B(X_n^{r-1}, Y_n^{r-1}) * \Delta X, \quad r = 1, \dots, n, \quad (\text{A.6})$$

where:

$$X_n^0 = \bar{X} \text{ and } Y_n^0 = \bar{Y}. \quad (\text{A.7})$$

We denote the true value of  $Y$  corresponding to  $X_n^r$  as  $\bar{Y}_n^r$ , i.e.:

$$\bar{Y}_n^r = G(X_n^r). \quad (\text{A.8})$$

Note that (A.8), (A.7) and (A.3) imply that:

$$\bar{Y}_n^0 = Y_n^0 = \bar{Y}. \quad (\text{A.9})$$

By applying Taylor's theorem we can relate the true value for  $Y$  in the first step of an  $n$ -step procedure to the starting value of  $Y$  by:

$$\bar{Y}_n^1 = \bar{Y}_n^0 + \left(\frac{1}{n}\right) * B(X_n^0, \bar{Y}_n^0) * \Delta X + \left(\frac{1}{2 * n^2}\right) * G_{XX}^{0,n}. \quad (\text{A.10})$$

In (A.10),  $G_{XX}^{0,n}$  is  $(\Delta X)^2$  multiplied by the derivative of  $G_X$  evaluated between  $X_n^0$  and  $X_n^1$ . More generally, we use the notation:

$$G_{XX}^{r,n} = G_{XX}(X_{r,n}) * (\Delta X)^2, \quad r = 0, 1, \dots, n-1, \quad (\text{A.11})$$

where  $G_{XX}$  is the derivative of  $G_X$ , that is the second derivative of  $G$ ; and  $X_{r,n}$  is a particular point between  $X_n^r$  and  $X_n^{r+1}$ . Combining (A.6) and (A.10), and using (A.7) allows us to relate the true value of  $Y$  in the first step of the  $n$ -step procedure to the estimated value by:

$$\bar{Y}_n^1 = Y_n^1 + \left(\frac{1}{2 * n^2}\right) * G_{XX}^{0,n}. \quad (\text{A.12})$$

Again applying Taylor's theorem, we relate the true value of  $Y$  in the second step in an  $n$ -step procedure to the true value in the first step by:

$$\bar{Y}_n^2 = \bar{Y}_n^1 + \left(\frac{1}{n}\right) * B(X_n^1, \bar{Y}_n^1) * \Delta X + \left(\frac{1}{2 * n^2}\right) * G_{XX}^{1,n}. \quad (\text{A.13})$$

Replacing  $\bar{Y}_n^1$  by the right-hand side of (A.12) and adding and subtracting  $(1/n) * B(X_n^1, Y_n^1) * \Delta X$  gives:

$$\begin{aligned} \bar{Y}_n^2 = & Y_n^1 + \left( \frac{1}{2 * n^2} \right) * G_{XX}^{0,n} + \left( \frac{1}{n} \right) * B(X_n^1, Y_n^1) * \Delta X + \left( \frac{1}{2 * n^2} \right) * G_{XX}^{1,n} \\ & + \left( \frac{1}{n} \right) * \left[ B(X_n^1, \bar{Y}_n^1) - B(X_n^1, Y_n^1) \right] * \Delta X. \end{aligned} \quad (\text{A.14})$$

Using (A.6) to replace the first and third terms on the right-hand side of (A.14) by  $Y_n^2$  and applying the mean value theorem to the last term, we obtain:

$$\bar{Y}_n^2 = Y_n^2 + \left( \frac{1}{2 * n^2} \right) * G_{XX}^{0,n} + \left( \frac{1}{2 * n^2} \right) * G_{XX}^{1,n} + \left( \frac{1}{n} \right) * B_Y^{1,n} * (\bar{Y}_n^1 - Y_n^1). \quad (\text{A.15})$$

In (A.15),  $B_Y^{1,n}$  is the derivative of  $B(X_n^1, Y)$  with respect to  $Y$  evaluated at a particular point between  $\bar{Y}_n^1$  and  $Y_n^1$ . More generally, we use the notation  $B_Y^{r,n}$  for  $r = 1, \dots, n-1$  to denote the derivative of  $B(X_n^r, Y)$  with respect to  $Y$  evaluated at a particular point between  $\bar{Y}_n^r$  and  $Y_n^r$ . Substituting from (A.12), we eliminate  $\bar{Y}_n^1 - Y_n^1$  from (A.15):

$$\bar{Y}_n^2 = Y_n^2 + \left( \frac{1}{2 * n^2} \right) * \left[ \left( 1 + \frac{1}{n} * B_Y^{1,n} \right) * G_{XX}^{0,n} + G_{XX}^{1,n} \right]. \quad (\text{A.16})$$

Continuing in this way, we can show that:

$$\begin{aligned} \bar{Y}_n^3 = & Y_n^3 + \left( \frac{1}{2 * n^2} \right) \\ & * \left[ \left( 1 + \frac{1}{n} * B_Y^{2,n} \right) * \left( 1 + \frac{1}{n} * B_Y^{1,n} \right) * G_{XX}^{0,n} + \left( 1 + \frac{1}{n} * B_Y^{2,n} \right) * G_{XX}^{1,n} + G_{XX}^{2,n} \right], \end{aligned} \quad (\text{A.17})$$

and ultimately that:

$$\bar{Y}_n^r = Y_n^r + \left( \frac{1}{2 * n^2} \right) * \sum_{k=0}^{r-1} \left[ \prod_{s=1}^{r-1-k} \left( 1 + \left( \frac{1}{n} \right) * B_Y^{r-s,n} \right) \right] * G_{XX}^{k,n}, \quad (\text{A.18})$$

where it is understood that  $\prod_{s=1}^0(\dots) = 1$ .

By setting  $r = n$ , we can use (A.18) to obtain an expression for the gap between the  $n$ -step estimate,  $Y_n^n$ , of  $G(\bar{X} + \Delta X)$  and the true value,  $\bar{Y}_n^n$ . To show that the limit of this gap as  $n$  approaches infinity is zero, we invoke our boundedness assumptions, i.e. there exists  $v$  and  $u$  sufficiently large that:

$$|G_{XX}| \leq u \quad \text{and} \quad |B_Y| \leq v, \quad (\text{A.19})$$

where  $G_{XX}$  and  $B_Y$  are evaluated at any point in the relevant domain of  $(X, Y)$ . We can think of this domain as covering economically meaningful prices and quantities. By this we mean prices and quantities that are greater than or equal to an arbitrarily small positive number or less than and equal to an arbitrarily large number.

By substituting from (A.19) into (A.18) we find that:

$$\begin{aligned} |\bar{Y}_n^n - Y_n^n| &\leq \left(\frac{1}{2 * n^2}\right) * \sum_{k=0}^{n-1} u * \prod_{s=1}^{n-1-k} \left(1 + \frac{v}{n}\right) \leq \left(\frac{u}{2 * n^2}\right) * \sum_{k=0}^{n-1} \left(1 + \frac{v}{n}\right)^{n-1-k} \\ &\leq \frac{u}{2 * n * v} * \left\{ \left(1 + \frac{v}{n}\right)^n - 1 \right\}. \end{aligned} \quad (\text{A.20})$$

Recalling that:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{v}{n}\right)^n = e^v \text{ and } \lim_{n \rightarrow \infty} [f(n) * g(n)] = \lim_{n \rightarrow \infty} [f(n)] * \lim_{n \rightarrow \infty} [g(n)], \quad (\text{A.21})$$

we can conclude that:

$$\lim_{n \rightarrow \infty} |\bar{Y}_n^n - Y_n^n| = 0. \quad (\text{A.22})$$

This is sufficient to complete our proof.

## A.2 Richardson's extrapolation

Why does doubling the number of steps tend to halve the linearization error, thus often allowing Richardson's extrapolation applied with low-step solutions to give highly accurate results? Assume that  $B_Y(X, Y)$  is zero. Under this assumption, (A.18) reduces for  $r = n$  to:

$$\bar{Y}_n^n = Y_n^n + \left(\frac{1}{2 * n^2}\right) * \sum_{k=0}^{n-1} G_{XX}^{k,n}, \quad (\text{A.23})$$

or:

$$\bar{Y}_n^n = Y_n^n + \left(\frac{1}{2 * n}\right) * G_{XX}^{\text{ave},n}, \quad (\text{A.24})$$

where  $G_{XX}^{\text{ave},n}$  is the average value over  $k$  of the  $G_{XX}^{k,n}$ s. We would not expect  $G_{XX}^{\text{ave},n}$  to be particularly sensitive to  $n$ . (In the case in which  $G$  is quadratic  $G_{XX}^{\text{ave},n}$  is completely insensitive to variations in  $n$ .) Thus, (A.24) suggests that the gap between the  $n$ -step estimate,  $Y_n^n$ , of  $G(\bar{X} + \Delta X)$  and the true value,  $\bar{Y}_n^n$  is proportional to  $1/n$ , implying that if we double the number of steps we will halve the error.

In CGE modeling we cannot expect  $B_Y(X, Y)$  to be zero and  $G_{XX}^{ave, n}$  will not be completely insensitive to  $n$ . However, experience over many years with Johansen/Euler computations confirms that the doubling/halving rule remains a useful approximation.

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