Testing for Habit Formation, Autocorrelation and Theoretical Restrictions in Linear Expenditure Systems*

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I. Introduction

The purpose of this paper is to present full information maximum likelihood estimates of linear expenditure systems (LES) and to test for habit formation, autocorrelation and regularity conditions using the likelihood ratio procedure. Estimators for the model and associated tests are based on annual Canadian data for the years 1957–1972. The results obtained provide a basis for helping to clarify concerns with the appropriateness of the LES and an evaluation of alternative characterizations for persistence in consumption patterns.

The linear expenditure system has been used in many empirical demand studies. Stone [18], Parks [13], Yoshihara [22], Pollak and Wales [15], Phlips [14], Powell [16] and Deaton [6] and others have presented estimates of the static and simple habit formation variants of the linear expenditure system. Lluch and Williams [11], MacKinnon [12], and Green, Hassan, and Johnson [8] have provided estimates of the LES assuming autoregressive errors. More recently, Howe, Pollak, and Wales [10] estimated the LES considering simultaneously effects of autocorrelation and habits. In this paper we extend our earlier work on autocorrelation and habit formation by developing results which permit a broader basis for evaluating the implications of persistence in consumption behavior for the LES.

The plan of the study is as follows. Section II contains a brief review of the LES including the introduction of habit effects. Section III discusses the stochastic specification and estimation techniques employed. The data and the commodity classifications are described in

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Section IV. Comparisons of the results for various versions of the linear expenditure system are given in Section V together with an indication of how they relate with those from other studies. Concluding remarks and suggestions of avenues for future exploration are offered in Section VI.

II. Model

The linear expenditure system can be derived from the Stone-Geary utility function

$$U = \sum_{i=1}^{n} a_i \ln (q_i - b_i) \tag{1}$$

where $a_i \geq 0$, $\sum_{k=1}^{n} a_k = 1$, and $q_i - b_i \geq 0$.[1] Maximizing the utility function (1) subject to the budget constraint $\sum_{k=1}^{n} p_k b_k = \mu$ yields the system of demand functions

$$q_i = b_i + a_i/p_i[\mu - \textstyle\sum p_k b_k];\ i = 1, \cdots, n \tag{2}$$

where the q_i's are quantity flows, the p_i's are prices, the b_i's are interpreted as minimum required quantities, μ is income and the summation index is assumed understood. Multiplying equation (2) by p_i yields the expenditure relations

$$p_i q_i = p_i b_i + a_i[\mu - \textstyle\sum p_k b_k];\ i = 1, \cdots, n. \tag{3}$$

It can be shown [7] that the LES globally satisfies the additivity, homogeneity, and symmetry restrictions.[2] That is, for any set of values of the endogenous variables (commodity expenditures) and exogenous variables (prices and total expenditures) the LES satisfies the above properties. In addition, the substitution matrix is negative semidefinite if the a_i's are positive and income μ is greater than $\sum p_k b_k$. This condition means that compensated price effects must be negative for all pairs of goods and assures that the consumer described is maximizing rather than minimizing utility.

Stone [18] incorporated changes in tastes in the LES by allowing the a and b parameters to vary linearly with time and previous consumption levels, q_{t-1}. Other variants on this essentially ad hoc characterization for persistence in consumption behavior are of course possible. Habit formation schemes described by the following structure will be considered in the present context

$$b_{it} = b_i^* + \beta_i q_{it-1}. \tag{4}$$

Substituting expression (4) into equation (3) yields a generalized expenditure system (i = 1,

1. For a detailed discussion of the theoretical properties of Stone's LES, see, e.g., Goldberger [7], Brown and Deaton [4], or Powell [16].

2. The symmetry condition is not apparent, however, it can be derived by recalling that the Slutsky compensated cross-price effects are given by

$$k_{ij} = \partial q_i/\partial p_j + q_j \partial q_i/\partial \mu.$$

Differentiating q_i in equation (2) by p_j and μ and substituting in the above expression yields $k_{ij} = a_i/p_i[q_j - b_j]$. Differentiation of q_j by p_i and μ similarly gives $k_{ji} = a_j/p_j[q_i - b_i]$. But $q_i - b_i = \mu - \Sigma p_k b_k$ in the LES, thus after making this substitution,

$$k_{ij} = a_i a_j/p_i p_j(\mu - \Sigma p_k b_k) = a_j a_i/p_j p_i(\mu - \Sigma p_k b_k) = k_{ji}.$$

$\cdots, n)$ of the form,

$$p_{it}q_{it} = [(1 - a_i)b_i^*]p_{it} - \sum_{j \neq i} (a_i b_j^*)p_{jt} + [(1 - a_i)\beta_i]p_{it}q_{it-1} - \sum_{j \neq i} (a_i\beta_j)p_{jt}q_{jt-1} + a_i\mu_t, \qquad (5)$$

where time subscripts have been added to distinguish observations for different periods.

III. Estimation Techniques

For specializing the static linear expenditure system to sample data an error term is added to equation (3), giving the tth observation

$$p_{it}q_{it} = p_{it}b_i + a_i[\mu_t - \sum p_{kt}b_k] + \epsilon_{it}. \qquad (6)$$

It is assumed, using vector notation, that

$$E(\epsilon_t) = 0 \qquad \text{and} \qquad (\epsilon_t\epsilon_{t'}') = \delta_{tt'}\Omega$$

where E is the expectation operator for $\delta_{tt'}$ the Kronecker product. That is, the error term has expectation zero, is temporally uncorrelated, and has a contemporaneous variance-covariance matrix Ω.

Since the sum of individual expenditures equals the total, it follows that the contemporaneous covariance matrix is singular. Barten [1] has shown that maximum likelihood estimates of the parameters can still be obtained by arbitrarily deleting an equation and that these estimates are invariant to the equation deleted. This result does not hold if autocorrelated disturbances are present [3]. If autocorrelation is present, i.e.,

$$\epsilon_t = R\epsilon_{t-1} + v_t \qquad (7)$$

for $t = 2, \cdots, T$ and where $v_2, \cdots, v_T$ are independently, identically distributed normal random vectors with mean vector zero and contemporaneous covariance matrix Σ, then other estimation methods must be used. Firstly, the variables in the system can be replaced by their first-order transforms, i.e., if Z_t is the original variable then it is replaced by $Z_t - \rho Z_{t-1}$ where ρ is the autocorrelation coefficient. Secondly, from the results of Berndt and Savin [3] the autocorrelation coefficient must be the same for each equation if it is assumed as for all the models considered in the present context, that there does not exist autocorrelation across equations, i.e., R is assumed to be diagonal.

With these stochastic assumptions, a program developed by Hall and Hall [9] and discussed in Berndt et al. [2] was used to obtain full information maximum likelihood estimators of the parameters of the linear expenditure system.[3]

IV. Data and Commodities

Annual time series data available from Statistics Canada [17] for the years 1947–1972 were used to estimate the models used for the various hypotheses tests.[4] Consumer expenditure

3. The gradient method of Hall and Hall's TSP program was used to obtain maximum likelihood estimates of the parameters of the various models. In a previous paper [8] we used a program developed by Wegge [20; 21]; however, for the most general linear habit formulation with autocorrelation we were not able to obtain convergence. Thus, we decided to use the TSP program with which convergence was attained.

4. For a more detailed discussion of the data, see Green et al. [8].

data for three aggregate groups were used: durables and semi-durables, nondurables, and services. The commodity classification was chosen due to the directly additive utility function associated with the LES and its restrictive conditions, e.g., of not allowing for complements. Furthermore, convergence problems encountered with using full information maximum likelihood methods prohibited a less aggregated commodity grouping.[5]

V. Results

The major emphasis of the analysis is the tests of restrictions afforded by the comparisons of models formulated under alternative habit formation and serial correlation hypotheses. As a preliminary to the presentation of results for these tests, the structural parameter estimates and implied elasticities are examined for six models. These models are then compared as a basis for generating the hypothesis tests concerning habit formation, serial correlation and the classical consumer demand formulation.

Structural Estimates

The estimates of the structural parameters for the six different versions of the linear expenditure system are presented in Table I. The marginal budget shares, a_i, are all positive, less than one, and sum to one for each of the models. They are significantly different from zero in every case. The only borderline values are in the linear habit models with and without autocorrelation for the nondurables commodity. The calculated t values are slightly larger than two for the a_i even in these situations.

The coefficients interpreted as minimum required quantities, b_i, are all positive except for the durables and semi-durables and the services groups in the linear habit formation cases. They are not, however, generally significantly different from zero in the various models. This may be symptomatic of a multicollinearity problem arising due to misspecification of persistence relationships and/or insufficient variation in the sample data.[6] The presence of lagged consumption is apparently picking-up the minimum required quantities effect since in Cases V and VI for nondurables and services, the β_i's are significant.

For the proportional habit formation systems, Cases III and IV, the β_i's are highly significantly different from zero for nondurables and services. The exception occurs in Cases III and IV for the durables and semi-durables group. This suggests that persistence in consumption patterns is present in the Canadian data, a claim that will be subsequently more systematically investigated using a test for habit effects based on the likelihood ratio.

Comparisons across models provide several interesting observations. Firstly, consider Cases I and II, the static LES with and without an autocorrelated error scheme. The autocor-

5. Some of the results similar to those presented have been developed for a four commodity classification. The estimates for this less aggregated grouping can be obtained from the authors.

6. The maximum likelihood estimation method of Wegge [20; 21] gave somewhat more encouraging results in this regard for Cases I and II. Specifically, the estimated mean values for a_i and b_i^* were approximately the same as are reported in Table I but the estimated standard errors for the b_i^*'s were much smaller. In particular, for the durables and semi-durables, nondurables and services commodity groups the values of b_i^*, with standard errors in parentheses, were for Case I, 208.63 (95.26), 428.46 (82.54) and 359.91 (223.69) and for Case II 213.56 (103.88), 384.69 (88.59) and 289.04 (236.73), respectively. As previously indicated, the gradient method of Hall and Hall was employed for the full set of results since it would converge for the most general case (VI).

Table I. Maximum Likelihood Estimates of the Structural Parameters for the Linear Expenditure Systems, 1947–1972[a]

	CASE I		CASE II		CASE III		CASE IV		CASE V			CASE VI		
	No Autocorrelation ($\rho = 0$)		Autocorrelation ($\rho \neq 0$)		Proportional Habit Formation With No Aurocorrelation ($b_{it} = \beta_i q_{it-1}$) $\rho = 0$		Proportional Habit Formation With Autocorrelation ($b_{it} = \beta_i q_{it-1}$) $\rho \neq 0$		Linear Habit Formation With No Autocorrelation ($b_{it} = b_i^* + \beta_i q_{it-1}$) $\rho = 0$			Linear Habit Formation With Autocorrelation ($b_{it} = b_i^* + \beta_i q_{it-1}$) $\rho \neq 0$		
	a_i	b_i^*	a_i	b_i^*	a_i	β_i	a_i	β_i	a_i	b_i^*	β_i	a_i	b_i^*	β_i
Durables and Semi-Durables	0.297[b] (0.032)	208.627 (225.291)	0.292 (0.050)	163.278 (312.659)	0.651 (0.066)	0.332 (0.239)	0.647 (0.054)	0.049 (0.296)	0.574 (0.089)	-75.039 (811.534)	-0.023 (0.512)	0.550 (0.068)	-93.523 (817.720)	-0.190 (0.739)
Nondurables	0.253 (0.026)	428.458 (145.371)	0.251 (0.023)	335.126 (291.419)	0.207 (0.054)	0.841 (0.043)	0.201 (0.053)	0.773 (0.077)	0.207 (0.102)	21.987 (315.054)	0.638 (0.276)	0.218 (0.091)	31.617 (405.460)	0.523 (0.493)
Services	0.450 (0.041)	359.909 (387.033)	0.457 (0.070)	152.695 (769.586)	0.142 (0.045)	0.916 (0.043)	0.152 (0.053)	0.859 (0.087)	0.219 (0.060)	-148.563 (554.518)	0.869 (0.313)	0.232 (0.005)	-192.092 (616.925)	0.824 (0.393)
ρ	0		0.569 (0.266)		0		0.350 (0.314)		0			0.319 (0.304)		
$-2 \ln \lambda$	78.378		42.962		15.772		38.310		22.084			52.226		
$\chi^2_{.05}$, Restr.	7.815 (3 restr.)		23.685 (14 restr.)		7.815 (3 restr.)		23.685 (14 restr.)		12.592 (6 restr.)			35.173 (23 restr.)		

[a] See Green et al [8] for a partial comparison of these results using TSP with the full information maximum likelihood results using a package by Wegge [21].
[b] Estimated standard errors in parentheses.

relation coefficient is significantly different from zero in Case II. The value of ρ is 0.569 with an estimated asymptotic standard error of 0.266. Estimated marginal budget shares differ very little between Cases I and II but the subsistence quantities are appreciably smaller when the autocorrelation parameter is not constrained to zero.

The habit formation versions of the LES reveal that β_i's are higher for the proportional structures (Cases III and IV) than for the linear structures (Cases V and VI). When the autocorrelation parameter is not constrained to zero, it is estimated at 0.319 with a standard error of 0.304 for the linear habit model and 0.350 with a standard error of 0.314 with the proportional habit formation. In both instances the autocorrelation parameter is not significantly different from zero. Values of the parameters defining the marginal budget shares are similar for the two habit formation versions of the LES. The β_i's are similar for the linear habit models with and without an autocorrelation, Cases V and VI, and for the proportional habit models with and without autocorrelation, Cases III and IV.

With respect to the minimum subsistence quantities, the implied values are less than consumption quantities for every commodity and time period. For the static versions the minimum quantities are larger than consumption for a few commodities for the first several observations in the time series. Similar results in this regard were obtained by Pollak and Wales [15].

Elasticities

Price and income elasticities calculated using the structural parameters and the sample means are presented in Table II. Comparing the income elasticities for the six cases shows that for the nondurables group, quite similar results were obtained. For the durables and semi-durables group, the income elasticity nearly doubled between the static models (Cases I and II) and the models incorporating a structure for habitual behavior (Cases III–VI). The reverse was true for the services commodity group.

Except for the durables and semi-durables commodity group, the uncompensated direct price elasticities were generally larger for the static models than for the models with structures admitting persistence. The larger variations in the elasticities are for the services group. The lower price elasticities and lower income elasticities were observed for this group when the static structure was modified to allow for persistence in consumption behavior, better reflecting the secular growth in expenditures for this commodity group. Converse but less pronounced results were obtained from the durables and semi-durables group where inventories are of obvious importance in influencing expenditure decisions. The most general case (VI) showed for the durables and semi-durables commodity group a somewhat lower income elasticity and a higher price elasticity than for other models in which the persistence hypothesis was implemented either on the systematic or error structure of the model.

Tests of Restrictions

To test for habit effects, autocorrelation, and the validity of the restrictions implied by the models a likelihood ratio test was used where

$$\lambda = \max_{w} L / \max_{W} L. \tag{8}$$

That is, the statistic in (8) is the ratio of the maximum value of the likelihood function L subject to restrictions to the maximum value of the likelihood function without restrictions.

Table II. Income and Price Elasticity Estimates, Cases I–VI, for the Linear Expenditure Models[a]

Commodity Group	Average Expenditure Proportions	Income Elasticity						Direct Price Elasticity[b]					
		I	II	III	IV	V	VI	I	II	III	IV	V	VI
Durables and Semi-Durables	0.2810	1.06	1.04	2.32	2.30	2.04	1.96	-0.71	-0.77	-0.82	-0.92	-1.01	-1.37
Nondurables	0.3446	0.73	0.73	0.60	0.58	0.60	0.63	-0.49	-0.60	-0.30	-0.35	-0.44	-0.53
Services	0.3744	1.20	1.22	0.38	0.41	0.58	0.62	-0.79	-0.91	-0.21	-0.27	-0.44	-0.53

[a] For the habit models, the short run elasticities are reported.
[b] Uncompensated.

It can be shown that

$$-2 \ln \lambda = T(\ln |\hat{\Sigma}_w| - \ln |\hat{\Sigma}_W|) \tag{9}$$

under the null hypothesis is distributed asymptotically, as chi-square with the number of degrees of freedom equal to the number of restrictions to be tested where $\hat{\Sigma}_w$ is the restricted estimator of the covariance matrix and $\hat{\Sigma}_W$ is the unrestricted estimator [19, 396–97].

The approach employed to conduct the tests was to use the general form of the expenditure function (equation 5) plus autocorrelation (equation 7) or equivalently Case VI defined in Table I as the general hypothesis. Then the following null hypotheses were tested:

H_1: β_i's = 0 (no habit persistence)

H_2: $\rho = 0$ (no autocorrelation)

H_3: β_i's = 0, $\rho = 0$ (no habit persistence and autocorrelation).

If H_1 were rejected then there would be empirical support for habit persistence. The next step would consist of testing the restrictions from standard consumption theory. Assuming H_1, H_2, and H_3 were all rejected, the appropriate maintained hypothesis for testing the theoretical restrictions would again be as specified in equations (5) and (7).

To test for habit persistence, the likelihood ratio test was applied to models as indicated above. Results of these tests are presented in Table III. The static models are restricted forms, $\beta_i = 0$, of the linear habit persistence model specified in equations (5) and (7). The computed value of $-2 \ln \lambda$ was 25.03 for Case VI versus Case II, which is larger than the chi-square value, $\chi^2_{.05,3} = 7.81$. (Note that there are three restrictions and thus three degrees of freedom.) Similar results, though not reported since the restriction $\rho = 0$ was rejected, were obtained for Case V versus Case I with $-2 \ln \lambda = 98.01$. Thus, the null hypothesis was rejected and habit persistence was assumed present.

To test which structure for habit formation better fitted the data, the linear habit model or the proportional habit case, likelihood ratio tests were again performed. Cases III and IV

Table III. Computed Values of $-2 \ln \lambda$ Used in Hypothesis Tests Across the Six LES Models

	Hypothesis Tests				
Test Statistics	Case VI vs. Case I	Case VI vs. Case II	Case VI vs. Case III	Case VI vs. Case IV	Case VI vs. Case V
$-2 \ln \lambda$	100.44	25.03	16.89	14.16	2.44
$\chi^2_{.05}$, Restr.	9.49 (4 restr.)	7.81 (3 restr.)	9.49 (4 restr.)	7.81 (3 restr.)	3.84 (1 restr.)
$\ln L$[a]	-199.96[b] (Case I)	-162.25 (Case II)	-158.19 (Case III)	-156.82 (Case IV)	-150.96 (Case V)

[a] Represents the value, excluding irrelevant constants, of the log likelihood functions for the six demand systems. The value of ln L for Case VI is 149.741.
[b] In order to obtain $-2 \ln \lambda$ for each combination of the various hypothesis tests, just take the difference of the ln L's and multiply by two. Also note that, e.g., Case II is not a "nested" version of Case IV, nor is Case I a "nested" version of Case III. Hence, the likelihood ratio procedure cannot be used to test these cases.

are restricted versions of Cases V and VI, respectively where $b_i^* = 0$. The value of $-2 \ln \lambda$ of 14.16 for Case VI versus Case IV is greater than $\chi^2_{.05,3} = 7.81$, thus indicating a rejection of the null hypothesis, i.e., the linear habit formation model appeared to describe the data better than the proportional version. Also, although not reported in Table III, a value of $-2 \ln \lambda$ of 14.45 for Case V versus Case III was greater than $\chi^2_{.05,3} = 7.81$, again indicating the same result but with ρ set to zero, i.e., the linear habit model is preferred over the proportional formulation.

To test whether or not autocorrelation was present, the likelihood ratio was again used but with one degree of freedom it is equivalent to an asymptotic t test. For the static model (Case II) the autocorrelation parameter is significantly different from zero. When autocorrelation is not constrained to zero in the linear habit model (Case VI) and the proportional habit model (Case IV) it is not significantly different from zero. As added evidence, the more general comparison, Case VI versus Case V, is reported in Table III with $-2 \ln \lambda = 2.44$. Thus, the habit formation structures are apparently accounting for autocorrelation observed with the static LES within the systematic components of the models.

Finally, two more general tests of the composite habit formation and autocorrelation formulations are presented for reference in Table III. The comparison of the static model with no autocorrelation (Case I) with the most general formulation (Case VI), gives a very large value for the test statistic as would be expected given the foregoing results. A less pronounced difference, although still statistically significant, is obtained by comparing Cases VI and III.

To test for the full set of restrictions implied by each version of the LES, the models were compared with the unrestricted reduced forms. Degrees of freedom are calculated as the difference between unrestricted reduced form parameters and those required to specify the various versions of the LES model. Note that the "restrictions implied by the theory" are assumed by the imposition of the LES parameter restrictions. In each case, as indicated by the values of the likelihood ratio statistics at the bottom of Table I, the restrictions implied by the theory were not appropriate. These conclusions are not surprising. Many others have obtained similar results. An implication, however, is that standard applied theory of consumption may be lacking as a guide to the specification of aggregate expenditure models even when corrected for irregularities in the error term and incorporating simple structures for reflecting persistence in consumption behavior.

Comparison of Results With Other Studies

These results can be compared with those of several recent studies, MacKinnon [12], Lluch and Williams [11], Green et al [8] and Howell, Pollak and Wales [10]. In most cases the autocorrelation coefficient as estimated in these studies had a rather high value. The values were −0.30 [12], 0.65 [11], 0.85 [8] and 0.99 [10]. Thus, for the static LES, autocorrelation appears to be present. When habits are introduced into the model structure, the autocorrelation becomes insignificant in the present analysis (see Cases IV and VI, Table I). For the Howe et al [10] model, it was negative and significantly different from zero. Clearly, a more thorough treatment of habit effects and autocorrelation is needed. Sometimes, for example, the inclusion of a time trend works about as well as lagged quantities in describing changes in tastes [15].

Finally, the income and price elasticities we obtained appear to be consistent with those from other studies. For example, for durables and semi-durables, our estimate of the own uncompensated price elasticity was −0.71 compared to a value of −0.825 found by Lluch and Williams [11] for durables. Thus, the major source of variation among the studies appears to be with the characterization of persistence in consumption behavior.

VI. Conclusions

This paper presented some empirical results for several habit variants of the LES. Three general conclusions are suggested by this and related work. Firstly, habit persistence appears to be important for the various versions of the LES based on different data sets, time periods, and countries. Secondly, the autocorrelation which is present in most of the models likely reflects misspecification problems and/or persistence in consumption patterns not adequately described by simple habit formation structures. In the present analysis these conclusions are based on the correctly specified autoregressive scheme [3]. Thirdly, theoretical restrictions consistently appear to be violated. These recurring results may be due to small sample properties, but a more significant implication is that the direct additivity assumption may be too restrictive. More flexible demand forms [5] may be required to properly model the aggregate annual data.

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