Input-Displacing Technologies in GreenREFORM

Theory and calibration

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1 Introduction

This note presents a modelling strategy for endogenizing input-displacing technologies in a constant elasticity of substitution (CES) production function framework. Input-displacing technologies allow firms to change their dependency on some inputs at the expense of others by making capital investments. The general CES-based theory presented in this paper can be employed using a technology catalog which describes the available technologies that firms can chose between.

The model endogenizes the choice of which technologies to employ, a choice which comes down to simply choosing the technologies which minimize the unit costs of the firm, taking input prices as given. The framework can be used both in partial equilibrium models as well as general equilibrium models. We developed it with the intention of incorporating it in the CGE model GreenREFORM. It allows the model to make a better and more precise prediction about the development of energy related emissions, such as CO_2 emissions. We use two different technology catalogs, both supplied by the Danish Energy Agency, see the companion note Beck and Ladefoged (Forthcoming) for details. All technologies in this catalog work in broadly the same way: They allow firms to reduce their usage of some energy input, say Liquefied Petroleum Gas (LPG), by investing in a particular production technology. Many technologies also involve an increase of a different type of energy input, usually electricity. A particular example could be the use of "catalytic infrared drying" in the paper industry. This allows firms to reduce their use of diesel fuel while increasing their use of electricity by purchasing capital, with the cost of this capital measured as an amount DKK per GJ saved from the installation of the technology.[1]

The note proceeds as follows: In section 2, we show how we modify the regular production structure to allow for input-displacing technologies. Section 3 takes a step back, and shows how one calibrates the technology parameters given the information provided in the technology catalog. Afterwards, section 4 presents the profit maximization problem of the representative firm and how its solution provides a straightforward way of endogenizing the installation of all the technologies in the catalog. Section 6 provides a few results from a partial equilibrium simulation of the model using counterfactual experiments and data. Section 5 presents a few modifications of the modelling setup which makes it appropriate for use in large-scale CGE models, such as GreenREFORM.

[1] Naturally, most technology catalogs are constructed at a different level of aggregation compared to the model that one wants to use it for. For the case of GreenREFORM, this aggregation procedure is described in a companion note (Beck and Ladefoged Forthcoming). The companion note also describes the procedures employed to convert the actual technology catalog into one directly compatible with the modelling framework presented in this note, see section 3.1.

2 The model

This section presents the modelling framework for incorporating input-displacing technologies and is divided into two parts. The first sets up a standard CES framework and introduces the notation employed in the rest of the paper, while the second presents how we extend the standard framework to be able to accommodate endogenous technology adoption. The actual endogenization is postponed to section 4.

2.1 A standard CES framework

Consider a standard constant elasticity of substitution (CES) production function of a representative firm as the one depicted above the dashed line in the top of figure 1. Every node is a CES aggregate from the inputs in the level below it, e.g. capital K and energy E combine into KE, the E_e combine into E and so forth. We interpret the different E_e as different energy purposes such as heavy process, EU ETS and heating and the B_{ei} as activities related to specific energy goods, for example oil or electricity. We will therefore refer to these as e.g. "oil-based energy input activities" or simply "oil-based activities". There are $\mathcal{E}$ energy purposes each indexed by e.[2] Each of these energy purposes, E_e, is the aggregation of N energy activites B_{ei} each indexed by i. This aggregation takes the standard CES form with some elasticity denoted σ_e and individual share parameters of μ_{ei}^B, with other nests following the same notation.[3] In a regular CGE model a specific B_{ei} would simply refer to a specific energy good. However, this is not the case in the present model as our introduction of input-displacing technologies involves adding an additional layer to the production structure, the subject of section 2.2. With the above notation, the CES demand for some energy purpose E_e is given by

$$E_e = \mu_e^E \left(\frac{P_e^E}{P^E}\right)^{-\sigma_E} E \tag{1}$$

and the demand for some energy input activity B_{ei} by

$$B_{ei} = \mu_{ei}^B \left(\frac{P_{ei}^B}{P_e^E}\right)^{-\sigma_e} E_e \tag{2}$$

where P^E, P_e^E and P_{ei}^B denote the price indexes/unit costs of E, E_e and B_{ei} respectively. Generally, we refer to "prices" as P when these are price indices of aggregates and p when these are actual prices (hence p will only be used for inputs at the very bottom of the production structure). The price index takes the standard CES form, e.g. for P_e^E:

$$P_e^E = \left(\sum_i^N \mu_{ei}^B \left(P_{ei}^B\right)^{1-\sigma_e}\right)^{\frac{1}{1-\sigma_e}}. \tag{3}$$

[2] We introduce notation gradually when relevant. For a quick reference on the full notation, see appendix A.1.

[3] Generally, subscripts reflect where in the nest the variable belongs, while the superscript reflects the type of the variable. Later, there will be examples where only the combination of both the subscript and the superscript will uniquely identify the underlying variable which motivates our need for heavy notation.

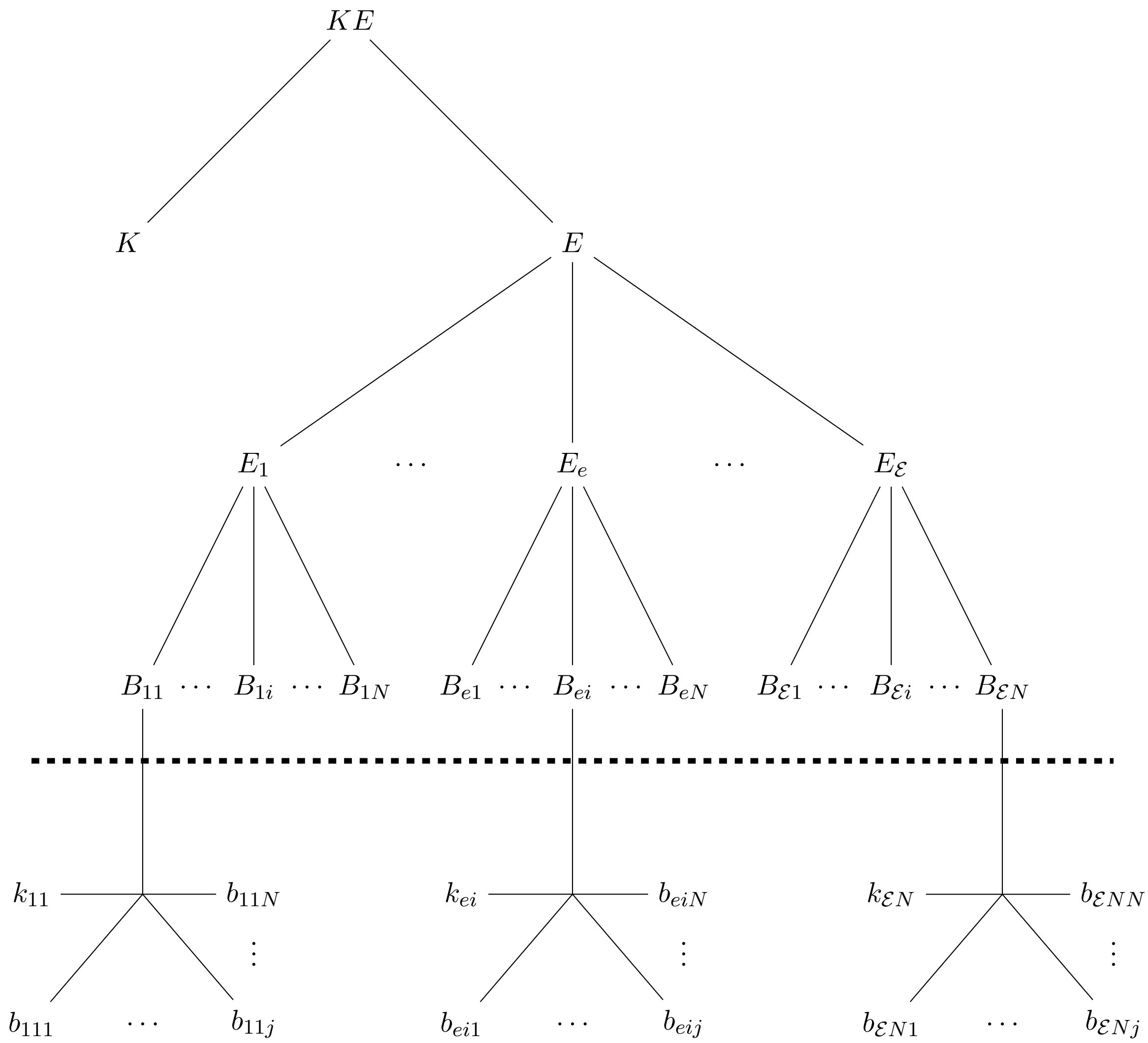

Figure 1: The extended CES production function

Note: This figure presents the structure of the CES production function that we use to incorporate input-displacing technologies. Our extension of the model is implemented in the section below the dashed line, and this is where all technologies take their effect. All nests below the dashed line are Leontief. Technology adoption is endogenous and affects share parameters in the bottom nests.

2.2 Extending the standard framework

2.2.1 An additional Leontief nest

To prepare for accommodating input-displacing technologies, we add an additional layer to the production structure at the very bottom (below the dashed line in figure 1), and assume that the elasticity of substitution is zero between inputs in this nest, i.e. the nests are Leontief. The inputs in this bottom nest are the firm's actual usage of energy goods such as oil and electricity, plus an additional input which we refer to as "technology capital" and denote k_{ei}. In other words, energy input activity B_{ei} is now a Leontief aggregator of $N+1$ inputs; the N energy goods denoted b_{eij} indexed by j and the technology capital k_{ei}. This does not alter the form of the demand for B_{ei} (it is still given by equation (2)), but explains why its price is denoted P^B_{ei} rather than p^B_{ei}. The Leontief form in the lowest nests implies that the demand for some k_{ei} and some b_{eij} are given by, respectively:

$$k_{ei} = \mu^k_{ei} B_{ei} \tag{4}$$

and

$$b_{eij} = \mu^b_{eij} B_{ei}. \tag{5}$$

Before introducing technologies formally, let's consider how it makes sense for oil, biogas, electricity etc. to combine into "oil-based activities". Think of the B_{ei} which refers to oil, i.e. $B_{e,\text{oil}}$, as reflecting the firm's need for different industrial processes which require oil; that might be drying, melting, boiling etc. The fact that the firm uses oil in the first place reflects that it needs to perform these activities in order to produce, and the data dictates how much oil is needed in a baseline period. Now, in a situation with no technologies installed, the entirety of oil-based activities are made up of oil, essentially making the lower Leontief nests redundant. This corresponds to a case where the specific share parameter (μ^b_{eij}) is one, and thus $b_{eij} = B_{ei}$. However, as the firm installs a technology which substitutes e.g. oil for electricity, this changes. Suppose the firm installs a boiler which runs on electricity rather than oil. This shifts the energy goods needed to produce what are still the oil-based activities, towards electricity and away from oil itself. Usually, this shift will be small relative to total initial consumption, so that only a few percent of initial oil consumption is replaced by electricity. Mathematically, the share-parameters for energy goods other than oil change, in this example it increases for electricity, while it decreases for oil. A technology catalog such as the one we use from the Danish Energy Agency delivers a list of e.g. boilers running on different energy goods and with different energy savings potentials. It is this information which we use to calibrate parameters related to a specific technology, but the low-level assumption is that the underlying energy activities that the firm wishes to perform are the same. Therefore we interpret technologies as investments which allow the firm to use, say, 0.9 GJ of oil, 0.05 GJ of electricity and some units of capital for each 1 GJ of oil it used before. Hence, the initial oil consumption is gradually changed into consumption of oil, electricity and other energy goods, as the relevant technologies are installed, such that oil-based activities are performed not only from the consumption of oil, but also from the consumption of other energy goods, provided that technologies that substitute towards these are installed. To summarize, we have energy purposes, E_e, energy input activities, B_{ei}, which reflect underlying industrial processes although these are not explicitly defined in the model, and energy goods, b_{eij}, which measure actual use of the N energy goods indexed by j here. The following section formalizes the ideas presented above into the framework.

2.2.2 Technologies

Suppose that the firm has U technologies available indexed by u, with an indicator function f^u reflecting whether a particular technology is installed. Endogenizing technology adoption in this model comes down to endogenizing these f^u functions, something we postpone to section 4. Each technology has a necessary capital investment cost c^u measured in DKK per GJ saved. As motivated above, a technology alters the energy input mix below the different B_{ei}, which we model as linear shifts in the respective μ parameters seen in equations (4) and (5):

$$\begin{aligned}\mu^b_{eij} &= \mathbb{1}\{i=j\} + \theta^1_{eij} f^1 + \ldots + \theta^u_{eij} f^u + \ldots + \theta^U_{eij} f^U \\ &= \mathbb{1}\{i=j\} + \sum_{u=1}^{U} \theta^u_{eij} f^u \quad \forall j = 1, \ldots, N\end{aligned} \tag{6}$$

where $\mathbb{1}(\cdot)$ denotes the indicator function. Technologies are therefore characterized by a set of θ-coefficients, reflecting the potential of each technology. Suppose as an example that some $\theta^u_{eij} = -0.1$ with j =oil. That implies that if this technology is fully installed ($f^u = 1$), the firm can produce the same as before using only 90 per cent of the initial oil use. Hence, the θ-parameters reflect the firm's options for reducing its dependence on specific fuels, usually at the expense of increased expenditure on electricity and investments in capital. Using the technology catalog to be described in section 3.1 we model the θ-coefficients such that any technology which reduces some energy input j does so in the nest of B_{eij} where $i = j$. Hence, $\theta^u_{eii} \leq 0$.[4] This means for example that all oil-reducing technologies work in the nest of oil-based activities (of which there are $\mathcal{E}$, the number of energy purposes). To understand equation (6), consider first the initial situation where no technologies are installed, $f^u = 0 \; \forall \; u$. In this case, the share parameter is simply equal to one or zero, depending on whether i equals j or not. Hence, initially, oil will have share 1 in the oil-based activities, just as described in the previous section. As new technologies are installed, the share parameters are scaled: Oil-displacing technologies reduce the share-parameter of oil in its own nest, while increasing the share-parameters of both k_{ei} and b_{eij} for $j \neq i =$ oil, with quantities depending on the particular technologies installed. The technologies stated in the catalog are described only in terms of their technical features and the catalog does not take firms' economic responses into account. In the model, we wish to include this economic response, so that e.g. firms reoptimize their input demands when technologies are installed. Our model does reflect this type of optimization, see e.g. sections 4 and 6, but we abstract from it when calibrating technologies, in line with the technical nature of the catalog. Furthermore, to ensure that the technologies described in this paper are consistent with the

[4]A more subtle but rather innocuous additional restriction is that no share-parameter can be negative or above 1.

notion of being "input-displacing", we wish to make sure that the firm's demand for any B_{ei} is not directly affected by adoption of a technology. With directly we mean non-price-related. In other words: Technologies will not affect the firm's demand for B_{ei} (e.g. oil-based activities) directly, but will indirectly through its price P^B_{ei}. As can be seen from equation (2), this will be fulfilled as long as μ^B_{ei} stays fixed, which it does by construction. The technologies only change the share-parameters which combine through Leontief production into B_{ei}. As mentioned earlier, installing a technology requires (technological) capital investments. We model these also through linear shifts in the share-parameters of k_{ei}, i.e.

$$\mu^k_{ei} = \theta^1_{eik} f^1 + \ldots + \theta^u_{eik} f^u + \ldots + \theta^U_{eik} f^U = \sum_u^U \theta^u_{eik} f^u. \tag{7}$$

Since all technologies require at least some capital investment, we have $\theta^u_{eik} > 0$ for all relevant triplets (e, i, u). The size of these θ-parameters is determined by calibration using the technology catalog, see section 3. The price of technology capital is the same as the price of "regular" capital i.e. the price of K in figure 1. Hence, we do not consider capital investments made for input-displacing technologies as different from regular capital investments.
We end this section with a definition which summarizes our notion of a technology:

Definition 1. A technology u consists of:

- A set of coefficients θ^u_{eij} across j and θ^u_{eik}. The former govern the changes in energy inputs, while the latter reflects the investments in capital that must be made in order to acquire the technology.
- A cost, c^u measured in DKK per GJ saved in total by adopting the technology.
- An indicator function, $f^u \in \{0, 1\}$, reflecting whether that particular technology is installed.

3 Calibration

To employ our model of endogenous technology adoption, one needs data on particular technologies, often collected in so-called technology catalogs.[5] Section 3.1 describes the data that our current catalog includes while section 3.2 explains the actual calibration of parameters from this data.

3.1 The Technology catalog

The technology catalog includes a list of industry-specific technologies. For each of these technologies u, the following variables are available:

- The industry it affects, e.g. "agriculture".
- What energy category it affects, e.g. "heavy process" corresponding to our index e.
- The energy input type it decreases, corresponding to an index combination ii.[6]
- The energy savings potential[7] (as a percentage of baseline energy usage, usually in some base year). We denote this reduction as $\%\Delta b^u_{eii} < 0$.[8]
- The energy input type it increases, and by how much, measured as a percentage of the initial energy use of the energy input it decreases (not relevant for all technologies).[9] We denote this $\%\Delta b^u_{eij} \geq 0$ where $j \neq i$.
- The net investment cost measured in "DKK per total amount of GJ saved". We denote this c^u.

In short, each technology reduces the use of some energy input, which might be partially offset by the increase of another energy input, at a cost per GJ saved, c^u. Currently our catalog does not include technologies which imply zero net savings of energy, i.e. purely input-mix changing technologies. This means that for all technologies, it holds that

$$\%\Delta b^u_{eii} < 0 \qquad \wedge \qquad -\%\Delta b^u_{eii} > \%\Delta b^u_{eij} \geq 0 \tag{8}$$

such that even if a technology increases the use of some energy input, this increase is never so large as to match the drop in the energy input that the technology displaces. We could easily accommodate cases where this was not true, as long as the cost is then not measured as "per GJ saved", since this could be zero or even negative.[10]

[5] We use a technology catalog supplied by the Danish Energy Agency. See the companion note, Beck and Ladefoged (Forthcoming), for details on the catalog.

[6] We do not consider technologies which reduce more than one energy input.

[7] In the data, this is a combination of a coverage potential, saying how large a fraction of the sector's current energy usage can be covered by the new technology, as well as an actual savings potential, saying how much of that energy can be saved.

[8] The catalog currently does not include technologies which imply zero net savings of energy, i.e. purely input-mix changing technologies. The model could easily accommodate these though.

[9] The catalog currently does not include any technologies which imply an increase of more than one energy input. The model could easily accommodate these though.

[10] For example, consider a technology which reduces oil by 100 GJ and increases electricity by 100 GJ, implying zero net savings. The costs of this technology measured in DKK per GJ saved are not defined, because no GJs are

3.2 Calibration

The following section describes how we calibrate the technology parameters, θ^u_{eij} and θ^u_{eik}, using $\%\Delta b^u_{eii}$, $\%\Delta b^u_{eij}$ and c^u. Although we use the word calibrate, in many cases formal calibration will not be necessary, because the modelling setup allows us to use key variables from the technology catalog directly as parameters. For technology catalogs constructed in other ways, formal calibration might be necessary more often.

As described in section 3.1 above, each technology involves some quantity changes as well as a cost measured in "DKK per total amount of GJ saved".

3.2.1 Energy input quantity changes

A technology will reduce one input and potentially increase another. As explained above, the size of these changes are defined in the technology catalog. As explained in section 2, reductions of an energy input i always happen in nest i, implying that $\%\Delta b^u_{eij}$ is negative if and only if $i = j$, and so to be explicit about that, we denote these $\%\Delta b^u_{eii}$ also in the following. Since the $\%\Delta b^u_{eii}$ are measured as percentages of the initial energy input use, it must be such that our model is consistent with this percentage change in the energy input;

$$\frac{\hat{b}_{eii} - \bar{b}_{eii}}{\bar{b}_{eii}} = \%\Delta b^u_{eii} \tag{9}$$

where bars denote initial values and hats denote values with the technology installed. This equation simply says that e.g. the percentage change in oil usage from when no technologies are installed ($\bar{b}_{eii}$) to when technology u is installed ($\hat{b}_{eii}$) must match the percentage change in energy usage stated in the catalog, $\%\Delta b^u_{eii}$.

For calibration purposes, we impose two main assumptions:

Assumption 1. When calibrating a particular technology u, all other technologies v are inactive, $f^v = 0\,\forall\, v \neq u$.

Assumption 2. When calibrating a particular technology u, the amount of any effective energy input is unchanged, $\hat{B}_{ei} = \bar{B}_{ei}\ \ \forall\, e, i$.[11]

The first assumption could be relaxed and the second is a simplification which is in line with the principles upon which most engineer-created technology catalogs are built, ignoring reoptimization due to changed costs. From equation (2) we see that this assumption corresponds to ignoring the price effects of technology adoption, a natural assumption for calibration purposes.

saved. Hence, to accommodate such a technology, the technology's cost would have to be measured differently. This would in turn alter the specific equation used to calibrate the investment in technology capital.

[11] The catalog was constructed by measuring the energy savings potential of each technology under the assumption that output is fixed. We mimic that by imposing $\hat{B}_{ei} = \bar{B}_{ei}$ when calibrating. There are two ways in which this could potentially be untrue in model simulations: If μ^B_{ei} changed or the relative price/cost of B_{ei} changed. The first of these is ruled out in our modelling framework, as μ^B_{ei} is kept fixed throughout. Therefore our assumption of $\hat{B}_{ei} = \bar{B}_{ei}$ essentially comes down to assuming that we can calibrate technologies to only incorporate the direct effects of the technology, and not the indirect effects induced by reoptimization following changes to the unit costs of different B_{ei}, which would also imply output changes. This is a natural assumption to make for most technology catalogs.

It is also consistent with an unaltered output quantity (holding inputs B_{ei} and above fixed, i.e. before reoptimization), $\Delta Y = 0$, because their corresponding share-parameters, the μ^B_{ei}, stay unchanged. We simply think of the assumption $\hat{B}_{ei} = \bar{B}_{ei}$ as saying that the firm still does the same activities as before adopting the technology, but now it uses a different set of "machines" and energy inputs. So effective oil usage reflects a set of oil-using activities, and the firm still does these activities, but has partly replaced the machines handling these activities, and some of these machines might run on e.g. electricity rather than oil.

With these calibration assumptions, inserting the expressions for b_{eii} and μ^b_{eii}, i.e. equations (5) and (6) into equation (9) on the left hand side gives

$$\begin{aligned}
\frac{\hat{b}_{eii} - \bar{b}_{eii}}{\bar{b}_{eii}} &= \%\Delta b^u_{eii} & \Leftrightarrow \\
\frac{\hat{\mu}^b_{eii}\hat{B}_{ei} - \bar{\mu}^b_{eii}\bar{B}_{ei}}{\bar{\mu}^b_{eii}\bar{B}_{ei}} &= \%\Delta b^u_{eii} & \Leftrightarrow \\
\frac{\hat{\mu}^b_{eii} - \bar{\mu}^b_{eii}}{\bar{\mu}^b_{eii}} &= \%\Delta b^u_{eii} & \Leftrightarrow \\
\frac{1 + \theta^u_{eii} - 1}{1} &= \%\Delta b^u_{eii} & \Leftrightarrow \\
\theta^u_{eii} &= \%\Delta b^u_{eii}. & (10)
\end{aligned}$$

This shows that setting our technology parameter θ^u_{eii} exactly equal to the percentage reduction stated in the technology catalog ensures that the model mimics the energy input reduction stated in the catalog. Of course, when simulating the model, the actual quantity changes will be different, because they incorporate the optimal economic response of the representative firm to reduced costs (which installing a technology implies).

Suppose then that technology u also increases energy input j where $i \neq j$. This increase is measured in the technology catalog as a fraction of initial use of energy input i. To be consistent with the catalog, our model must therefore be consistent with the expression

$$\frac{\hat{b}_{eij} - \bar{b}_{eij}}{\bar{b}_{eii}} = \%\Delta b^u_{eij} \tag{11}$$

Again, we calibrate using $\hat{B}_{ei} = \bar{B}_{ei}$ and that no other technologies are installed. Doing so, using equations (5) and (6) in equation (11) on the left hand side gives

$$\begin{aligned}
\frac{\hat{b}_{eij} - \bar{b}_{eij}}{\bar{b}_{eii}} &= \%\Delta b^u_{eij} & \Leftrightarrow \\
\frac{\hat{\mu}^b_{eij}\hat{B}_{ei} - \bar{\mu}^b_{eij}\bar{B}_{ei}}{\bar{\mu}^b_{eii}\bar{B}_{ei}} &= \%\Delta b^u_{eij} & \Leftrightarrow \\
\frac{\hat{\mu}^b_{eij} - \bar{\mu}^b_{eij}}{\bar{\mu}^b_{eii}} &= \%\Delta b^u_{eij} & \Leftrightarrow \\
\frac{\theta^u_{eij} - 0}{1} &= \%\Delta b^u_{eij} & \Leftrightarrow \\
\theta^u_{eij} &= \%\Delta b^u_{eij}. & (12)
\end{aligned}$$

This shows that setting our technology parameter θ^u_{eij} exactly equal to the percentage increase stated in the technology catalog ensures that the model is consistent with the catalog.

3.3 Technology capital investment

As mentioned previously, all technologies in the catalog have a "cost per GJ saved" denoted c^u. The amount of GJ saved is for some technology u given by $-\sum_j^N \left(\hat{b}_{eij} - \bar{b}_{eij}\right)$. With the price of technology capital of p^k (usually set to 1 for calibration purposes), this means the model must ensure that

$$c^u = \frac{p^k(\hat{k}_{ei} - \bar{k}_{ei})}{-\sum_j^N \hat{b}_{eij} - \bar{b}_{eij}} \tag{13}$$

i.e. that the expenditures on technology capital relative to the amount of GJ saved are exactly c^u. Using equations (6) and (7), this can be rewritten into

$$\begin{aligned}
c^u &= \frac{p^k(\hat{k}_{ei} - \bar{k}_{ei})}{-\sum_j^N \hat{b}_{eij} - \bar{b}_{eij}} && \Leftrightarrow \\
c^u &= \frac{p^k \theta^u_{eik}}{-\sum_j^N \theta^u_{eij}} && \Leftrightarrow \\
\theta^u_{eik} &= \frac{c^u}{p^k}\left(-\sum_j^N \theta^u_{eij}\right)
\end{aligned} \tag{14}$$

where the last equality rearranges to isolate the variable we wish to calibrate, θ^u_{eik}. Note that with the current technology catalog all technologies imply a positive amount of net saving, i.e. that $\sum_j^N \theta^u_{eij}$ is always negative, meaning θ^u_{eik} is always positive. With this calibration procedure we utilize c^u to reveal how much capital a given technology requires. In model simulations these quantities are fixed at their calibrated values, such that the cost of technologies can change in model simulations to the extent that the price of capital changes. This is definitely a feature and not a bug, we wish the cost of adopting technologies to rise when capital gets more expensive. However so long as the price of capital is unchanged, the total costs to technologies will correspond to those stated in the catalog, irrespective of the number of technologies installed. This concludes our baseline methodology for calibrating the parameters of the model to the data in the technology catalog. The next section proposes possible methods for relaxing assumption 1.

4 Endogenous technology adoption

This section explains how we endogenize the firm's choice of which technologies to install, i.e. how we endogenize the f^u. The firm does so by solving a cost-minimization problem.

4.1 Cost-minimization problem

The firm chooses to invest in a given technology, i.e. set $f^u(\cdot) = 1$, if profits with the technology are larger than without. With constant returns to scale and hence, constant marginal cost, profits can be written as $\Pi = (P - C)Y$ where P denotes the output price, C denotes the constant average (equal to marginal) unit cost and Y denotes output. The firm is small relative to the market which is characterized by perfect competition, it takes output prices as given. Profits with the technology (indicated by ˆ) are higher than without (indicated without ˆ) if

$$\begin{aligned}
\hat{\Pi} &> \Pi && \Leftrightarrow \\
\left(P - \hat{C}\right)\hat{Y} &> (P - C)\,Y && \Leftrightarrow \\
\left(P - \hat{C}\right)\hat{Y} &> 0 && \Leftrightarrow \\
P &> \hat{C} && \Leftrightarrow \\
C &> \hat{C} && \qquad (15)
\end{aligned}$$

where the third and last step use that initially, profits are zero ($P = C$) in equilibrium when the market is perfectly competitive.[12] Hence, the optimal choice for the firm is to choose the combination of technologies which implies the lowest unit costs. In a partial equilibrium with fixed input prices, a change in technology would imply higher profits, consistent with equation (15). However, in general equilibrium, due to perfect competition and identical firms with constant returns to scale, the output price P will be brought down to $\hat{C}$ such that profits are again zero. The unit costs in equation (15) reflect the entire production structure of the firm, and are given by the standard expression in CES models that we also presented in equation (3);

$$C(\boldsymbol{P}, \boldsymbol{\mu}) \equiv \left(\sum_i \mu_i P_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \qquad (16)$$

where P_i is again defined in the same way using the inputs below it and $\boldsymbol{P}$ and $\boldsymbol{\mu}$ are the vector of price indexes and the vector of share-parameters, respectively. $C(\boldsymbol{P}, \boldsymbol{\mu})$ is the unit cost when the firm chooses all its inputs optimally given their prices. We wish to compare the realization of the function C when its inputs, the share-parameters and the prices, change. It can easily be shown that since each technology only affects the production structure in one particular nest,

[12] Rather than imposing that profits are zero initially, an alternative assumption is that the firm does not incorporate output quantity changes when choosing which technology to adopt, $\hat{Y} = Y$, which is essentially the same as saying we only look at cost-minimization.

we only need to compare the unit cost of this particular nest[13], in this case the nest of the B_{ei} where technology u works. The price index of B_{ei} combines the input prices from the lowest nests with the share-parameters which the technology can directly alter. We denote the former p^b_{ej} rather than p^b_{eij} because these pure input prices are the same irrespective of which effective input i they contribute to.

Following the formula in equation (16), consider the condition from equation (15) for the relevant B_{ei}:

$$P^B_{ei}\left(p^b_{e1},\ldots,p^b_{ej},\ldots,p^b_{eN},p^k\right) > P^B_{ei}\left(\hat{p}^b_{e1},\ldots,\hat{p}^b_{ej},\ldots,\hat{p}^b_{eN},\hat{p}^k\right) \tag{19}$$

which since the elasticity of substitution is zero when the aggregator is Leontief and the price index therefore has a simple linear form, is given by

$$\begin{aligned} P^B_{ei}\left(p^b_{e1},\ldots,p^b_{ej},\ldots,p^b_{eN},p^k\right) &> P^B_{ei}\left(\hat{p}^b_{e1},\ldots,\hat{p}^b_{ej},\ldots,\hat{p}^b_{eN},\hat{p}^k\right) && \Leftrightarrow \\ \mu^k_{ei}p^k + \sum_j^N \mu^b_{eij}p^b_{ej} &> \hat{\mu}^k_{ei}\hat{p}^k + \sum_j^N \hat{\mu}^b_{eij}\hat{p}^b_{ej}. \end{aligned} \tag{20}$$

Now, since the firm takes input prices of energy inputs as given, we have that $\hat{p}^b_{ej} = p^b_{ej}$ and $\hat{p}^k = p^k$, so we can further rewrite into

$$\begin{aligned} \underbrace{\left(\hat{\mu}^k_{ei} - \mu^k_{ei}\right)}_{\theta^u_{eik}} p^k + \sum_j^N \underbrace{\left(\hat{\mu}^b_{eij} - \mu^b_{eij}\right)}_{\theta^u_{eij}} p^b_{ej} &< 0 && \Leftrightarrow \\ \theta^u_{eik}p^k + \sum_j^N \theta^u_{eij}p^b_{ej} &< 0 \end{aligned} \tag{21}$$

which has a straightforward interpretation. The firm decides to invest in a given technology if the weighted sum of quantity changes, with relevant prices as weights, is negative. This inequality

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[13] Take an arbitrary price index

$$P = \left(\sum_{i=1} \mu_i P_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{17}$$

where the prices P_i are themselves price indexes from lower nests. Assume that $\sigma \in]0,1[$ (the case of $\sigma > 1$ leads to the same conclusion, but the inequality sign flips twice during the derivation). Now, assume that only one of these prices are affected (consistent with how each technology will only affect the price of one P^B_{ei}), say price P_1. Then, the condition of whether $\hat{P}$ is smaller than P becomes

$$\begin{aligned} \hat{P} &< P && \Leftrightarrow \\ \left(\mu_1\hat{P}_1^{1-\sigma} + \sum_{i=2}\mu_i P_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} &< \left(\sum_{i=1}\mu_i P_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}} && \Leftrightarrow \\ \mu_1\hat{P}_1^{1-\sigma} + \sum_{i=2}\mu_i P_i^{1-\sigma} &< \sum_{i=1}\mu_i P_i^{1-\sigma} && \Leftrightarrow \\ \mu_1\hat{P}_1^{1-\sigma} &< \mu_1 P_1^{1-\sigma} && \Leftrightarrow \\ \hat{P}_1 &< P_1. \end{aligned} \tag{18}$$

This shows that a sufficient condition for whether the overall price index drops is, if only one of the prices in it change, to look at this price change in isolation.

allows us to define the indicator function for whether the firm invests in technology u as

$$f^u(\boldsymbol{\theta}, \boldsymbol{p}) = \begin{cases} 1 & \text{if} \quad \theta^u_{eik} p^k + \sum_j^N \theta^u_{eij} p^b_{ej} < 0 \\ 0 & \text{otherwise} \end{cases} . \tag{22}$$

Neatly, this holds for an arbitrary combination of other technologies that the firm might already have invested in, because all technologies enter additively. This concludes our methodology for endogenizing technology adoption of the representative firm. In section 5 we consider specific extensions appropriate when one wants to implement the setup in a larger CGE model. Before that, we show a few model simulations in section 6.

5 CGE Integration

This section explains a couple of extensions that one can make in order to make our model framework applicable for use in most CGE models. We use this in our implementation of the model as a part of GreenREFORM.

5.1 Smoothing of f^u

The indicator functions f^u imply that the firm solves U discrete choice problems simultaneously. To avoid actually implementing discrete choice problems, we smooth these functions f^u using the CDF of the normal distribution ($\Phi(\cdot)$) with the standard deviation determining to what degree f^u is smoothed. Alternative smoothers such as a log-normal or logit can also be used. This means that what we actually implement when solving the model is

$$f^u = \Phi\left(\frac{-\left(c^u\theta^u_{eik} + \sum_j^N \theta^u_{eij}p^b_{ej}\right)}{\sigma}\right) \in]0,1[\tag{23}$$

where σ determines the extend to which the f^u is actually smoothed.

6 Model simulations

6.1 Simulation experiments

To illustrate that the model works as intended, we present 3 stylized simulation experiments, in figures 2 to 4 below. All experiments are based on constructed data for illustration purposes only. They all include the smoothed versions of the indicator functions as described in section 5.1.

Figure 2 presents the simple experiment of letting the price of straw increase exogenously. Each point on the graphs reflects one independent model simulation. The cutoff for when the technology should optimally be adopted lines up well with the timing of the actual installation. The figure is most easily read from right to left: As the price of straw gets sufficiently low, the firm installs a technology which increases its dependency on and use of straw. This increased dependency is reflected in the increased gradient in the graph with B. The third panel shows that the smoothing of f^u implies that the installation does not happen exactly at the cutoff, but instead gradually in a small interval around it. Figure 3 should also be read from right to left and shows the model solution when the cost of the only straw-reducing technology changes from above 1 (the price of straw is 1 throughout) to below 1. An interesting takeaway is the realistic "rebound" effect: The pure usage of straw, b, decreases around the cutoff for when the technology becomes cost-reducing. However, as this technology gets increasingly cheaper, the price of effective straw decreases further, increasing the optimal quantity of effective straw, B. This in turn increases the optimal quantity of pure straw usage, b, creating a rebound effect. Hence, the percentage reduction in b from, say $c = 1.5$ to $c = 0.5$, is less in absolute terms than the technology prescribes (-0.42). This shows how the model reflects both the *direct* technology channel as well as the *indirect* price channel.

Figure 4 tells a similar story as figure 2, but with three technologies being gradually adopted at different thresholds.[14] Here, it is the price of biooil which increases.

[14]Note that the technologies here are constructed for illustrative purposes.

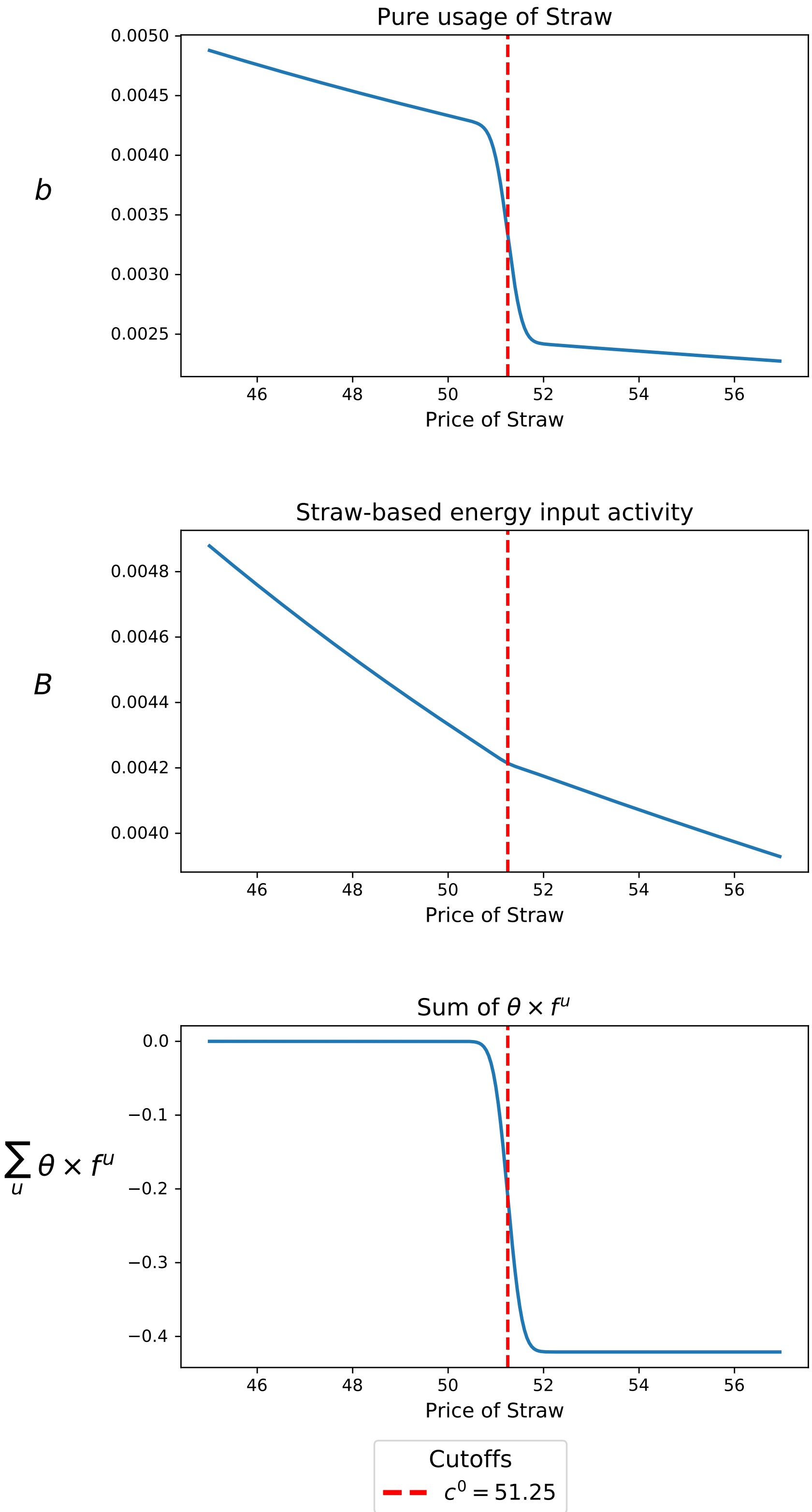

Figure 2: A gradual increase in the price of straw.

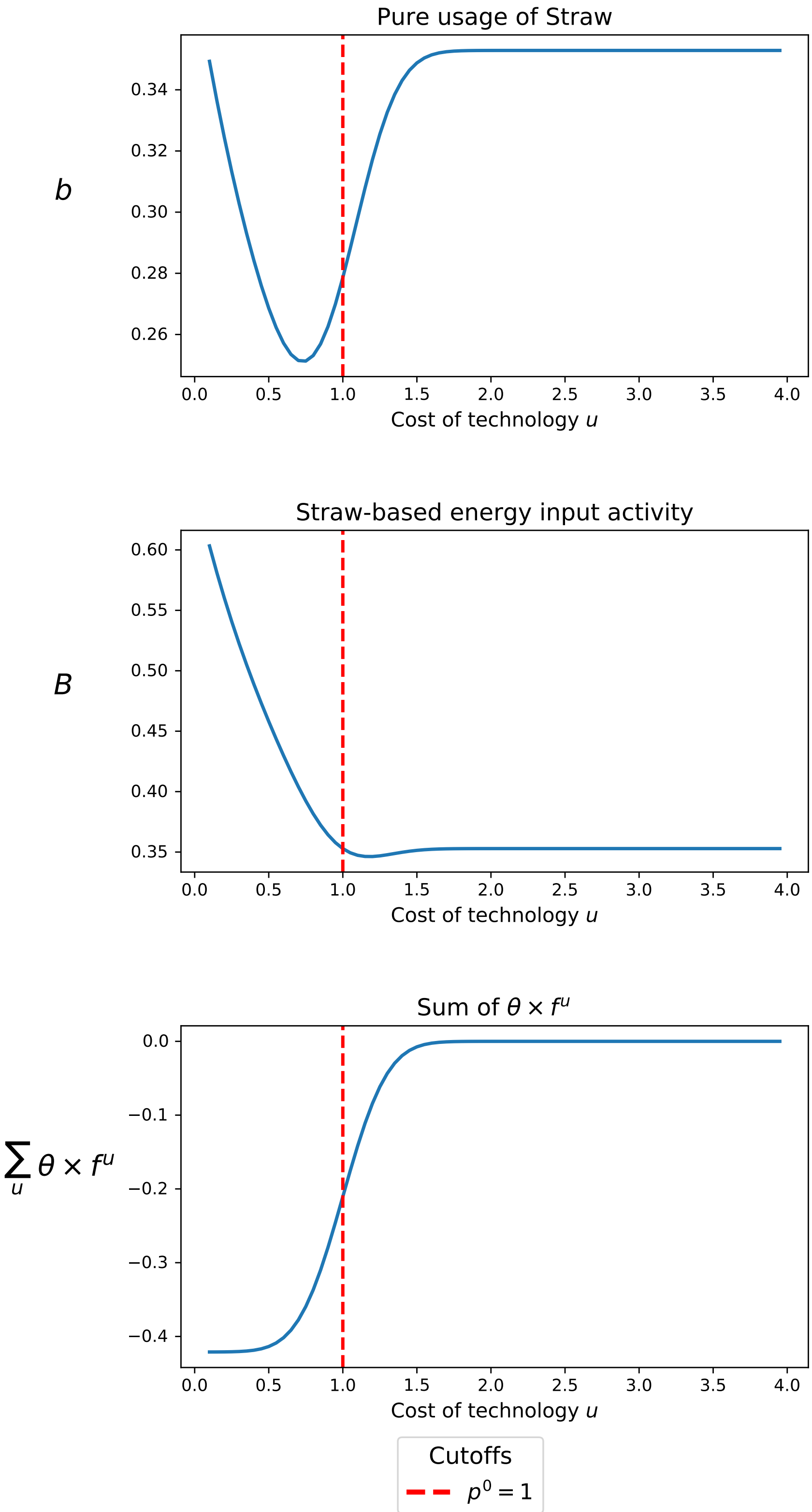

Figure 3: A gradual decrease in the cost of the straw-reducing technology.

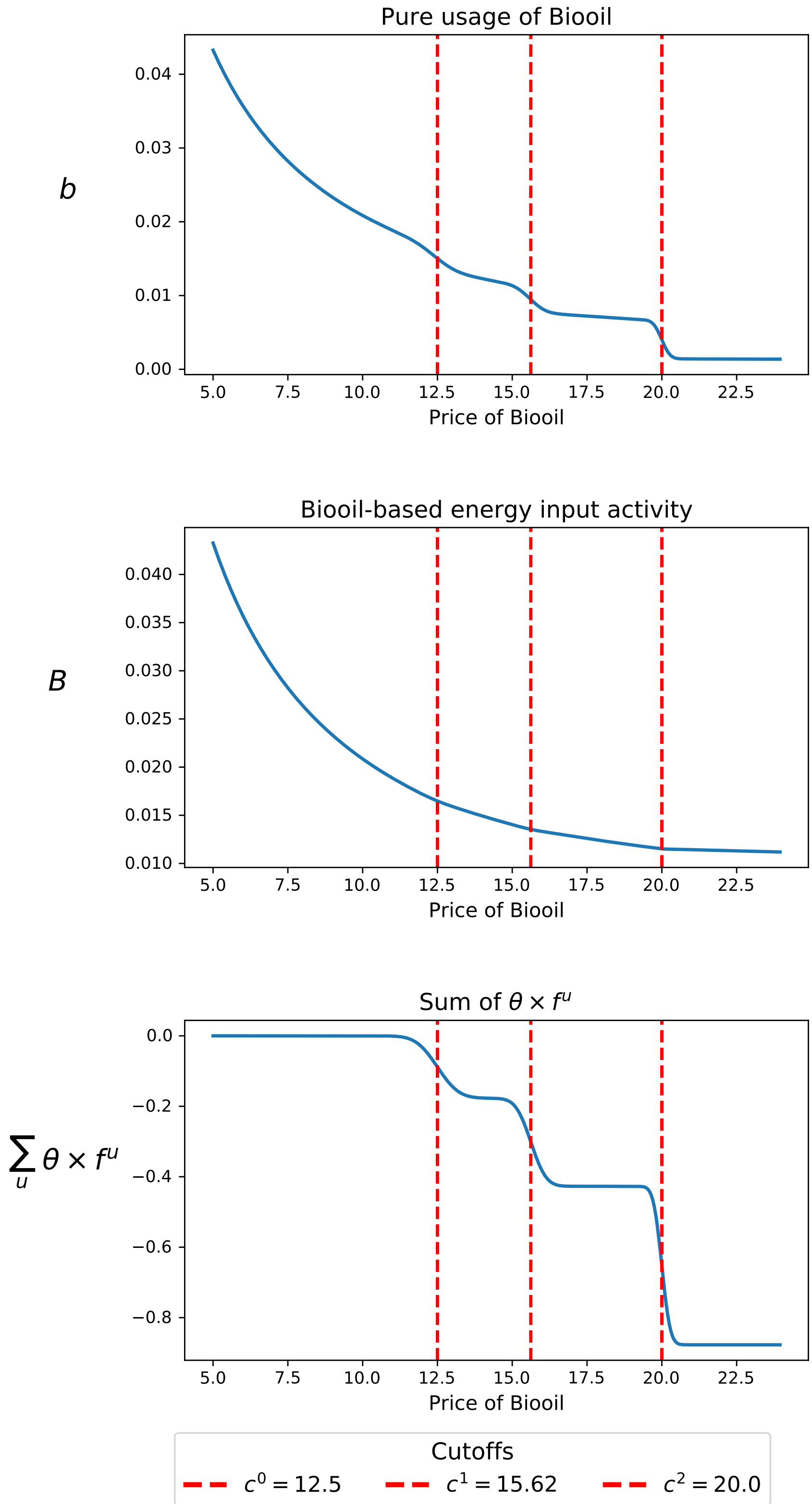

Figure 4: A gradual increase in the price of biooil.

A Appendix

A.1 Notation

The notation is as follows (see also figure 1 for a guiding visualization):

- KE is the aggregate of capital and energy.
- E is the aggregate of the $\mathcal{E}$ energy purposes, indexed by e, so $e = 1, \ldots, \mathcal{E}$.
- Each E_e is a CES aggregate of N "energy input-based activities", e.g. oil-based activities. These are denoted by B_{ei} and indexed by i, so $i = 1, \ldots, N$. The CES production function is characterized by an elasticity of substitution given by σ_e. Each B_{ei} has a share parameter given by μ^B_{ei}.
- Below each B_{ei}, N "pure energy inputs" combine, together with "technology capital", through a Leontief aggregator. Each of the N pure inputs here are indexed by j and use lower case b rather than B, i.e. they are denoted b_{eij}. technology capital is denoted k_{ei} and does not have a subscript j since there is only one type of capital for each nest i.
- We generally denote price indexes/unit costs using P and pure/actual prices using p. The unit cost of E_e, i.e. its price index, is denoted P^E_e. The "price" of B_{ei} is denoted P^B_{ei}. The (actual) price of b_{eij} is denoted p^b_{ej} since it does not vary with i. Finally, the price of technology capital, k_{ei}, is denoted p^k, and is equal to the price of regular capital, p^K.
- There are a total of U technologies, indexed by u. Technologies are characterized by a set of θ^u_{eij} and θ^u_{eik}, as well as their cost, c^u and an indicator function, f^u, reflecting whether the firms have installed that particular technology. Sometimes we use hats above symbols to indicate the value of a variable with some technology u installed and bars in the absence of technology u.
- Technologies can only affect the share parameters of b_{eij} and k_{ei} and these are denoted μ^b_{eij} and μ^k_{ei} respectively, with their baseline (no-technology installed values) given by $\bar{\mu}^b_{eij}$ and $\bar{\mu}^k_{ei}$. A technology can only affect one B_{ei} nest.