1 Estimation equation

The simplest form of the LES demand equation in share form that we would like to estimate is

$$w_{it} = \omega^i(z_t, \theta) = \frac{p_{it}b_i}{\mu_t} + \alpha_i \left(1 - \frac{\sum p_{kt}b_k}{\mu_t}\right) + u_{it} \tag{1.1}$$

where t indicates time, w_{it} is the budget share of good i, z_t is a vector of explanatory variables, θ are the coefficients p_{it} is the price of good i, μ_t is the income, b_{it} is the subsistence level of consumption of good i and u_{it} in a stochastic error term. The equation is estimated in share form to reduce the likelihood of heteroscedasticity, which is argued by Pollak and Wales (1992). A proposed extension is to let the minimum consumption level depend on the consumption of the past, which can be interpreted as a simple form of 'habit formation.

$$b_{it} = b_i^* + \beta_i x_{i,t-1}, \tag{1.2}$$

where b_i^* can be interpreted as the 'physiological necessity' and b_{it} as the psychological necessity, where x_{it} is consumption of good i. This is a simple 'habit formation' model. Inserting habit formation into the simple LES equation gives the estimation equation:

$$w_{it} = \omega^i(z_t, \theta) = \frac{p_{it}(b_i^* + \beta_i x_{i,t-1})}{\mu_t} + \alpha_i \left(1 - \frac{\sum_k (p_{kt}\beta_k x_{k,t-1})}{\mu_t}\right) + u_{it} \tag{1.3}$$

It must hold for every consumption good i that

$$\sum w_{it} = \sum \omega^i(z_t, \theta) = 1 \tag{1.4}$$

which also implies that $\sum_i \alpha_i = 1$ and $\sum u_{it} = 0$ for each t, thus the u_{it}'s cannot be independent.

Due to this restriction, according to Pollak and Wales (1992), one (arbitrarily chosen) good can be dropped, such that the vector of error terms is

$$u_t = [u_{1t}, u_{2t}, ..., u_{n-1,t}], \qquad u_t \sim N(0, \Omega), \tag{1.5}$$

and a likelihood function can be written:

$$L(\theta, \Omega) = -\frac{(n-1)T}{2}\log 2\pi - \frac{T}{2}\log|\Omega| - \frac{1}{2}\sum_{t=1}^{T} u_t'\Omega^{-1}u_t \tag{1.6}$$

This likelihood function should be maximized with respect to θ and Ω. The question is just: How do we do that in practice?

Bibliography

Pollak, R. A. and Wales, T. J. (1992). *Demand system specification and estimation.* Oxford University Press on Demand.