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# Functional Forms and Parametrization
# of CGE Models

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## Abstract

This study focused on the choice of functional forms and their parametrization (estimation of free parameters and calibration of other parameters) in the context of CGE models. Various types of elasticities are defined, followed by a presentation of the functional forms most commonly used in these models and various econometric methods for estimating their free parameters. Following this presentation of the theoretical framework, we review parameter estimates used in the literature. This brief literature review was carried out to be used as a guideline for the choice of parameters for CGE models of developing countries.

**Keywords:** Trade liberalization, Poverty, elasticities, functional forms, calibration, Computable General Equilibrium (CGE) model

**JEL Classification:** C51, C81, C82, D58, E27

## Introduction

The construction of a Computable General Equilibrium (CGE) models is usually based on a social accounting matrix (SAM) that describes the initial state of the economy. The implementation of CGEMs relies on the principle of calibration, given that a model is actually characterized by various functional forms that illustrate consumption and production-related behaviors. Calibration therefore consists in determining the numerical values of the various parameters of functions compatible with the equilibrium of the initial SAM. In some cases, information contained in the SAM is inadequate for the calibration of all parameters. When forms such as the constant elasticity of substitution or linear expenditure system are selected, estimates of other parameters, such as the elasticity of substitution or the income elasticity, may be required for calibration. The values attributed to these parameters can be postulated or based on econometric estimations. Where such estimations are not available for the countries concerned, assumptions derived from literature on elasticities estimated for a country with similar characteristics can be applied. However, the choice of these free parameters is critical, given that it seriously affects the findings of the model.

The present study has a dual objective consisting, on the one hand, in defining the theoretical framework of parameters involved in most CGEMs and, on the other hand, in presenting a number of elasticity estimates available in the literature concerning developing countries. Section 2 of this chapter is devoted to the definition of demand elasticities. Section 3 presents the functional forms commonly used in CGE models and discuss methods for calibrating their non-free parameters. Section 4 deals with techniques for econometrically estimating the free parameters in these functions. We then turn our attention in section 5 to reviewing parameter estimates obtained econometrically or through personal judgment in a wide variety of developing countries, which we hope will serve as a reference for future models.

## Elasticities: Definitions

Economic theory distinguishes a number of elasticities, each measuring the percentage variation in one variable (e.g. consumer demand for bread) to a variation in another variable (e.g. the price of bread or household income). In this section, we present the notions of price elasticity, income elasticity and elasticity of substitution.

### *Own Price Elasticity*

Own price elasticity assesses the variation in the demand for a commodity that results from a variation in the price of the latter. Consumer demand is the result of the maximization of utility subject to a budget constraint. With the demand curve presenting a decreasing slope, the own price elasticity is negative. Considering that $C_i$ represents the quantity demanded of commodity *i* and $p_i$ is the price of the commodity, $\varepsilon_p$, the own price elasticity reads as follows:

$$\varepsilon_p = \frac{\partial C_i / C_i}{\partial p_i / p_i} = \frac{\partial C_i}{\partial p_i} \frac{p_i}{C_i}$$

Table 1 describes various special cases. In the two first cases, the demand function is represented by a straight line, which is horizontal when the price elasticity is infinite and vertical when the price elasticity is nil. In the event of an intermediary case where the absolute value of the price elasticity is unitary, the variation in demand will be proportionate to the price variation. On the other hand, when the absolute value of the elasticity is greater (less) than unity, a change in the price will result in a demand variation that is more (less) than proportional.

**Table 1: Characteristics of demand**

| Price elasticity value | Characteristics of demand |
|---|---|
| $\varepsilon_p = -\infty$ | Perfectly elastic |
| $\varepsilon_p = 0$ | Perfectly inelastic |
| $\varepsilon_p = -1$ | Unitary elasticity |
| $\varepsilon_p < -1$ | Elastic |
| $-1 < \varepsilon_p < 0$ | Inelastic |

### *Cross Price Elasticity*

For each pair of commodities *i* and *j*, cross price elasticity is defined as follows:

$$\varepsilon_{pc} = \frac{\partial C_i / C_i}{\partial p_j / p_j} = \frac{\partial C_i}{\partial p_j} \frac{p_j}{C_i}$$

Cross price elasticity of demand assesses the variation in the demand for commodity *i*, which results from the variation in the price of commodity j. Where the cross price elasticity is positive, the two commodities are said to be substitutes. On the contrary, where negative, an increase in the price of the commodity j will result in a drop in the demand of commodity *i*, and the two commodities are said to be complementary. Cross price elasticity is nil for commodities which are neither substitutes nor complements.

### *Income Elasticity*

The income elasticity assesses the variation in the demand for a commodity *i*, following a variation in income *r* referred to as:

$$\varepsilon_r = \frac{\partial C_i / C_i}{\partial r / r} = \frac{\partial C_i}{\partial r} \frac{r}{C_i}$$

According to the value of the income elasticity, we can distinguish three groups of commodities. Income elasticity is superior to one for luxury goods and inferior to one for normal goods. For inferior goods, the value of $\varepsilon_r$ is negative; any increase in income reduces the demand for the this commodity. Figure 1 illustrates the link between the demand (Y or ordinate axis) and the income (X or abscissa axis). The curves represented are known as Engel's curves.

**Figure 1: The Engel's curve**
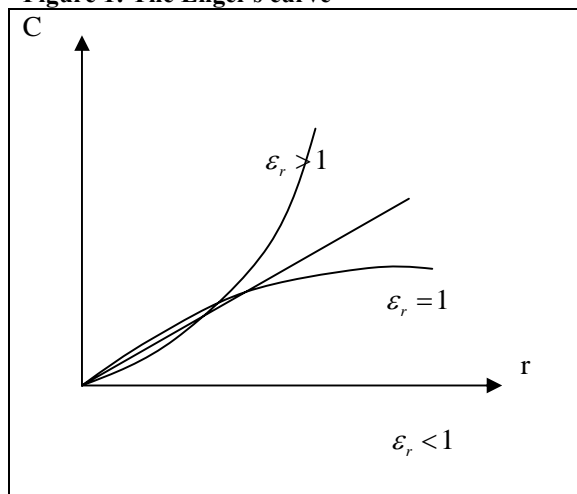


**Source**: Sadoulet and de Janvry, 1995

***Elasticity of Substitution or Transformation***

Unlike the preceding elasticities, which pertain only to demand, elasticities of substitution and transformation apply equally to supply decisions. On the demand side, the elasticity of substitution between two commodities, *i* and *j*, measures the variation in the relative demand for the two commodities resulting from a variation in their relative price. Considering the derivative of the log of these variables ($\partial \ln$), the elasticity of substitution reads as follows:

$$\varepsilon_s = \frac{\partial \ln\left(X_i / X_j\right)}{\partial \ln\left(p_j / p_i\right)} \qquad 0 \le \varepsilon_s < \infty$$

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If the elasticity of substitution is nil, the two commodities are considered to be perfect complements (Figure 2), whereas if it is infinite, the two commodities are considered to be perfect substitutes. When this elasticity is comprised between these two extremes, the products considered to be imperfect substitutes. The value of the elasticity of substitution determines the curvature of the indifference curve (rather than the slope) in the case of a utility function, and the isoquant in the case of a production function. Similar expressions are obtained in the case of the elasticity of substitution between factors in a production function or between destination markets in a CET function of export and local supply.

**Figure 2: Substitutes and complements**



Substitutes                    Complements
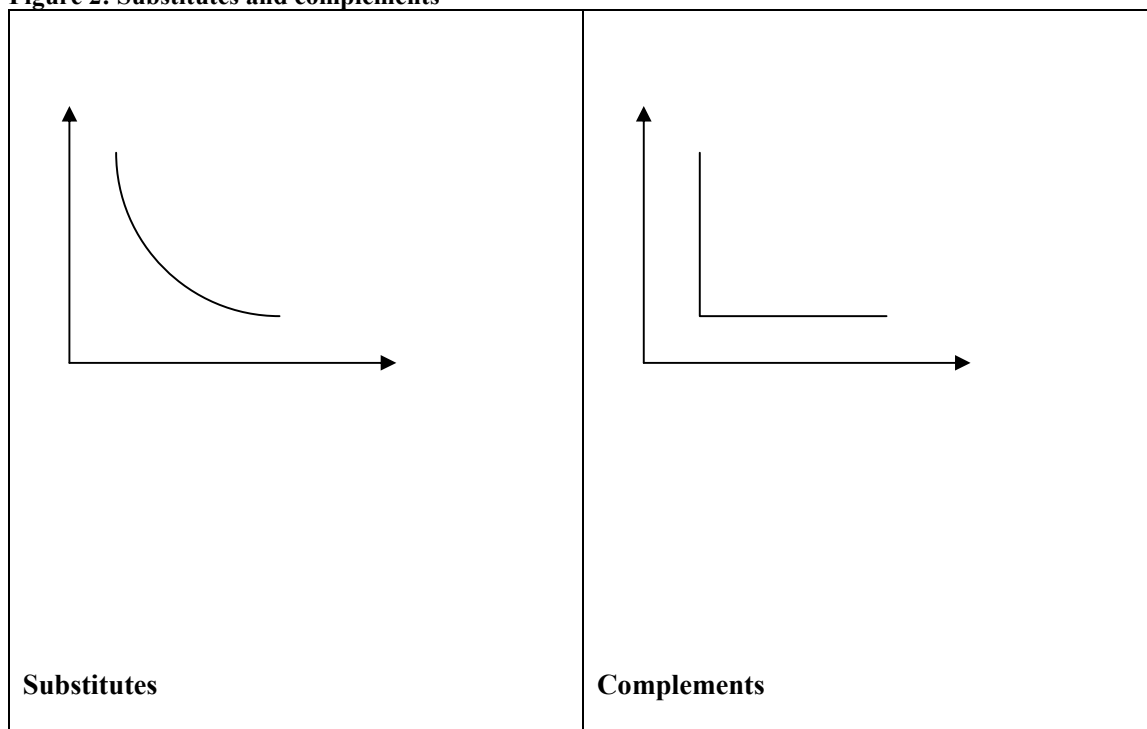
## Functional Forms in CGE Models

Various factors guide the choice of functional forms in CGE models. In general, the function chosen should be continuous and homogeneous of degree zero and result in a system of demand in conformity with the Walras Law (Shoven and Whalley, 1984). These conditions

are used to help ensure equilibrium and ease the analysis of variations in the prices resulting from economic policies. Besides, the choice of behavioral functions in the construction of CGE models depends on the characteristics of the sectors or products under study and consequently on the values of the various related elasticities. These restrictions require that the choice of functional forms be limited to functions such as the Cobb-Douglas (C-D) function, the constant elasticity of substitution function (CES) or the linear expenditure system (LES). More flexible functional forms such at the translog function may be used, but present a number of analytical difficulties.

Here, we present the functional forms most commonly used in CGE models. The calibration method1 is presented each time. Although these different functions may also be used on the production side, attention here is focused on the modeling of systems of demand.

### The Cobb-Douglas (C-D) Function

With a Cobb-Douglas utility function, the consumer's demand is obtained as the solution to the following maximization program:

$$Max\ U = \prod C_i^{\alpha_i}$$
$$s.t. \sum p_i C_i = R \ \ and \ \ \sum \alpha_i = 1$$

with $R$ - total income. The consumption of each commodity $i$ reads: $C_i = \dfrac{\alpha_i R}{p_i}$

Values for the various elasticities presented above can be derived from this demand function and provide information on the restrictions of the C-D function. Price and income elasticities, as well as the elasticity of substitution between each pair of goods, are all equal to

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1Hansen and Heckman (1996) discuss the empirical fundamentals of calibration. A review on the principles of calibration and its use in modeling is presented by Dawkins et al (2001).

one, whereas the cross price elasticity is nil. Despite these assumptions, which may be perceived as very strong and unrealistic, many authors resort to the C-D function given that it can be easily calibrated and does not require outside estimates of free parameters (Box 1).

**Box 1: The calibration of a Cobb-Douglas function**

> With a C-D utility function, the only unknown parameter is the budgetary share of the consumption of each commodity in the overall consumption. Considering the income, the consumption and prices provided by the SAM, the computation of the share of each good in overall consumption income (total expenditure) is a simple inversion of the demand equation: $\alpha_i = \dfrac{p_i C_i}{R}$ where the $p_i$ are normalized to one in the base year.

However, such restrictions are rarely observed in empirical estimates (Shoven and Whalley, 1992). In order to relax some of these restrictions, one may choose some more flexible functional forms, which we now explore.

### *The Constant Elasticity of Substitution (CES) Function*

The constant elasticity of substitution (CES) function allows for non-unitary, but constant, price elasticities and non-nil, but constant, substitution elasticites. The consumer's utility maximization program is as follows:

$$Max\ U = \sum \left[ \alpha_i C_i^{(\varepsilon-1)/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}$$
$$s.t. \sum p_i C_i = R \ and \ \sum \alpha_i = 1$$

The demand function for each commodity reads: $C_i = \dfrac{a_i^e R}{p_i^e \left[ \sum a_i^e p_i^{1-e} \right]}$

where $\alpha_i$ is the share parameter and $\rho$ the substitution parameter defined as follows:

$\varepsilon_s = \dfrac{1}{1+\rho}$, with $0 \prec \varepsilon_s \prec \infty$ and $-1 \prec \rho \prec \infty$ where $\varepsilon$ is the constant elasticity of substitution

between all pairs of commodities. Where $r \circledR ¥$ , $e_s \circledR 0$ the two goods are perfect complements and where $r = -1$, $e_s \circledR ¥$ , they are perfect substitutes.

Own price elasticity and income elasticity are both derived from the demand function $C_i$ :

$$e_p = -e_s - a_i^e (1 - e_s) p_i^{1-e_s} [\text{å} \ a_i^e p_i^{1-e_s}]^1 \quad \varepsilon_r = 1$$

When budgetary shares are low, own price elasticity is close to the negative of the elasticity of substitution. The CES function thus avoids the unit price elasticity constraint imposed by the C-D function. The income elasticity is unitary, as in the case of the C-D function. The CES function implies an identical elasticity of substitution for all pairs of commodities2.

Box 2 presents the CES function applied in various contexts. This function is the most commonly used function for modeling international trade in CGE models in order to capture the widely observed phenomenon of cross-hauling. The CES function can also represent imperfect substitution between factors of production in value added.

The C-D function presented in paragraph 0 stands as a special case of the CES function with an elasticity of substitution equal to one. Both functional forms impose unit income elasticity, an assumption that some people would not consider. Indeed, unit income elasticity implies that the budget shares of each good do not vary with the level of income. In order to go beyond this restriction, choosing the linear expenditure system may be helpful.

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2 The constant elasticity of substitution assumption in the case of a production function with several factors may be relaxed by using a more general functional form known as the homogenous or homothethic constant elasticity of substitution ratio (CRESH), Hanoch, G. (1971).

**Box 2: Implementation of the CES function**

The **Armington (1969) assumption** of imperfect substitutability between two products of different origins implies that total domestic demand $Q_i$ is a CES function:

$$Q_i = A_i \left[ \alpha_i M_i^{-\rho} + (1-\alpha_i) D_i^{-\rho} \right]^{-\frac{1}{\rho}}$$

with $D_i$ as demand for the locally manufactured good, $M_i$ as the demand for the imported imperfect substitute, $A_i$ a scale parameter and the elasticity of substitution given by: $\varepsilon_s = \dfrac{1}{1+\rho}$. The maximization problem is to minimize cost: $PQ_i Q_i = PD_i D_i + PM_i M_i$ subject to the Armington function. We obtain the relative demand for imported versus local goods as a function of their relative prices:

$$\frac{M_i}{D_i} = \left[ \frac{PD_i}{PM_i} \frac{\alpha_i}{1-\alpha_i} \right]^{\varepsilon_s}$$

Given price normalization, the volumes of demand for both domestic and imported products are directly provided by the SAM. The only parameters to be calibrated therefore are the share and scale parameters. For a given external estimate of the elasticity of substitution, the share parameter is easily computed by inverting the above import demand equation. The scale parameter is then obtained by simple inversion of the Armington function.

Similarly, **export supply** may be represented, depending on the destination, by a constant elasticity of transformation (CET) function that takes a form similar to that of the CES:

$$X_i = B_i \left[ \alpha_i E_i^{-\varphi} + (1-\alpha_i) D_i^{-\varphi} \right]^{-\frac{1}{\varphi}}$$

with $\varepsilon_t = \dfrac{1}{\varphi+1}$ as the elasticity of transformation, $-\infty \prec \varphi \prec -1$ and $-\infty \prec \varepsilon_t \prec 0$. Export supply resulting from the maximization of profits to the producers reads as follows:

$$\frac{D_i}{E_i} = \left[ \frac{PE_i}{PD_i} \frac{1-\alpha_i}{\alpha_i} \right]^{\varepsilon_t}$$

This same process may be applied for the calibration of the CET. For a given $\varepsilon_t$, by normalizing prices, distributive parts of the export offer function are derived.

Finally, we can capture imperfect substitution between **factors of production** with a CES value added function: $VA_i = CES(K_i, L_i)$

The relative demand for the two factors reads: $\dfrac{K_i}{L_i} = \left[ \dfrac{w_i}{r_i} \dfrac{\alpha_i}{1-\alpha_i} \right]^{\varepsilon_s}$

with $w_i$ and $r_i$ respectively representing the wage rate and the rate of return to capital. By normalizing these two rates, the elasticity of substitution $\alpha_i$ is deduced.

### The Linear Expenditure System (LES)

The Stone-Geary[3] function also known as the linear expenditure system or LES, does not assume unit income elasticity. This function can be expressed alternatively as a variant of the C-D function or the CES function (Shoven and Whalley, 1992). By introducing a term which represents minimal or subsistence consumption $C_{\min i}$ of each commodity *i* in a C-D function, the LES demand function is obtained as the result of the following utility maximization problem:

$$Max\ U = \prod \left( C_i - C_{\min i} \right)^{\alpha_i}$$
$$s.t. \sum p_i C_i = R \ and \ \sum \alpha_i = 1$$

The resulting demand for the consumption of commodity *i* is the sum of the minimal and discretionary components:

$$C_i = C_{\min i} + \frac{\alpha_i}{p_i} \left[ R - \sum p_j C_{\min j} \right]$$

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3 Stone (1954).

The term $R - \sum p_j C_{\min j}$ is known as supernumerary or residual income. It represents the income available after satisfaction of minimal consumption. With the introduction of the minimal consumption level, the Engel's curve ceases to be represented by a straight line passing through the origin and the income elasticities are no longer unitary:

$$\varepsilon_{ri} = \frac{\alpha_i R}{p_i C_i}$$

Own price elasticities are: $\varepsilon_{pi} = \frac{(1 - \alpha_i) C_{\min i}}{C_i} - 1$

One problem arising in using a LES function is the need for several free parameters in the calibration process, as explained in Box 3.

**Box 3: Calibration of a linear expenditure system**

The calibration of a LES function is not as easy as that of a C-D function or a CES function as minimal consumption levels must also be determined. Two methods can be adopted depending on the availability of estimates for each of the free parameters:

**Case 1: Estimates of income and price elasticities available**

In this case, we first calibrate the discretionary consumption budget shares:

$$\alpha_i = \frac{\varepsilon_r p_i C_i}{R}$$

where the initial levels of consumption of each good and overall income are observed and initial prices are normalized. Then, the minimal consumption levels can be calibrated from the price elasticity equation:

$$C_{\min i} = \frac{(1 + \varepsilon_p) C_i}{1 - \alpha_i}$$

*The Expanded Linear Expenditure System (ELES)*

The LES demand function may be expanded to take into account savings behavior in the utility function. Lluch (1973) introduces the Stone-Geary function into a program of inter-temporal utility maximization under a wealth constraint of households. Howe (1975) shows that the extended linear system of expenditure (ELES) is obtained simply from a static program meant to maximize the Stone-Geary's function with savings considered as a good whose minimal consumption level is nil.

By considering the LES function from the previous section and introducing savings ($S$), the ELES function is obtained in the following manner:

$$u = \tilde{O}(C_i - C_{\min i})^{a_i} S^b$$
$$U = \ln u = å \; a_i \ln(C_i - C_{\min i}) + b \ln S \qquad avec å \; a_i + b = 1$$

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4 De Melo and Tarr (1992) and Sadoulet and de Janvry (1995).

The consumer's welfare maximization program reads:

$$Max \ U = \sum \alpha_i \ln\left(C_i - C_{\min i}\right) + \beta \ln S$$
$$s.t. \ p_i C_i + S = R \ and \ \sum \alpha_i + \beta = 1$$

The resulting demand functions for consumption and savings read:

$$C_i = C_{\min i} + \frac{\alpha_i}{p_i}\left[R - \sum p_j C_{\min j}\right]$$

$$S = \beta\left(R - \sum p_j C_{\min j}\right)$$

Consumption demand is similar to the one obtained with the LES function. The following income and direct price elasticities are obtained based on the consumption demand equation:

$$\varepsilon_r = \frac{\alpha_i R}{p_i C_i}$$

$$\varepsilon_p = \frac{(1 - \alpha_i) C_{\min i}}{C_i} - 1$$

Calibration of this function is outlined in Box 4.

**Box 4: Calibration of an ELES function**

This method was developed by Burniaux and Van der Mensbrugghe (1991). It is based on matrix computation and can be applied with the GAMS software. Discretionary budget shares are first calibrated from the income elasticity equation:

$$\alpha_i = \frac{\varepsilon_r p_i C_i}{R}$$

We then rewrite the consumption demand function in matrix form:

$$[C] = [IC_{\min}] + [AR] - [APC_{\min}]$$

$$[C] = [I - AP][C_{\min}] + [AR]$$

where:

$[I]$ :           Identity matrix (nxn)

$[A]$ :           Diagonal matrix of $\alpha_i$ terms

$[P]$ :           Transposed matrix of prices (normalized)

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| | |
|---|---|
| $[C]$ : | Total consumption matrix |

| |
|---|
| $[C_{\min}]$ :       Minimal consumption matrix |
| Through matrix inversion, we calibrate minimal consumption levels: |
| $$[C_{\min}] = [I - \mathrm{AP}]^{-1}[C - \mathrm{A}R]$$ |

### *The Almost Ideal Demand System (AIDS)*[5]

Proposed by Deaton and Muellbauer (1980), this system gives an approximation of the first order of any demand system and meets the conditions of the traditional axioms in consumer theory. It is easy to aggregate all the consumers, and Engel's curves are not necessarily straight. It can be simply estimated without non linear estimation techniques. Moreover, it makes it possible to test demand homogeneity and symmetry assumptions by using linear constraints on parameters. This system results from a set of preferences known as *PIGLOG*[6]. It is represented in the form of expenditure share functions $\omega_i$ :

$$\frac{p_i C_i}{R} \circ \quad w_i = a_i + \mathring{a} \quad g_{ij} \ln p_j + b_i \ln \frac{R}{P}$$

where $P$ is the price index. The change in prices is captured by the parameter $\gamma_{ij}$ and that of real expenditure by the parameter $\beta_i$. The AIDS system implies the following conditions:

(1)   $\sum \alpha_i = 1, \quad \sum_i \gamma_{ij} = 0, \quad \sum \beta_i = 0$

(2) $\sum \gamma_{ij} = 0$          (3) $\gamma_{ij} = \gamma_{ji}$

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5 Almost ideal demand system

6 Price independent generalized linear log, Deaton and Muellbauer (1980)

Condition (1) guarantees additivity, condition (2) ensures the homogeneity of the demand function and condition (3) ensures symmetry. From the demand function are derived the following price and income elasticities:

$$\varepsilon_p = -1 + \frac{\gamma_{ij}}{\omega_i} - \beta_i$$

$$\varepsilon_r = 1 + \frac{\beta_i}{\omega_i}$$

The calibration of the AIDS function also requires estimates of various parameters (Box 5).

**Box 5: Calibration of the AIDS system parameters**

The demand elasticities derived from this system read:

$$\varepsilon_p = -1 + \frac{\gamma_{ij}}{\omega_i} - \beta_i \qquad \text{and} \qquad \varepsilon_r = 1 + \frac{\beta_i}{\omega_i}$$

If estimates of these elasticities are available, then the other parameters are calibrated as:

$$\beta_i = (\varepsilon_r - 1)\omega_i$$

$$\gamma_{ij} = (\varepsilon_p + \beta_i + 1)\omega_i$$

hence $\qquad \alpha_i = \omega_i - \sum \gamma_{ij} \ln p_j - \beta_i \ln \frac{R}{P}$

## Econometric Estimates of Free Parameters

In this section, econometric methods used in the empirical estimations of various elasticities will be presented. These methods follow from the form of the demand function, which depends on the different functional forms selected. They are applicable both to the demand for consumer goods and the demand for production factors. As noted earlier, the C-D function is fully calibrated from the initial SAM and requires no free parameter estimates.

### The Constant Elasticity of Substitution (CES) Function

The calibration of the CES function requires an outside estimate of the elasticity of substitution. Three methods of estimation are presented. The first one is commonly applied to international trade and the last two are applied to the demand for production factors.

*Ordinary Least Squares (OLS) Method*

The first method for estimating the elasticity of substitution is based on the first order conditions from the consumer welfare maximization program. Applied to international trade, the elasticity of substitution between local goods and imported products may be obtained from the import demand function. This function is written without indices as follows:

$$\ln\left(\frac{M}{D}\right) = \varepsilon_s \ln\left(\frac{\alpha}{1-\alpha}\right) + \varepsilon_s \ln\left(\frac{PD}{PM}\right)$$

A linear regression model is obtained by adding a random error ($u$) and replacing $\varepsilon_s \ln\left(\frac{\alpha}{1-\alpha}\right)$ by a constant:

$$\ln\left(\frac{M}{D}\right)_t = a + \varepsilon_s \ln\left(\frac{PD}{PM}\right)_t + u_t$$

This equation can be used to estimate for each commodity in a CGE model by OLS, assuming that the time series data meets the usual conditions. In practice, a variable reflecting the overall level of economic activity, such as the gross domestic product (GDP), is generally introduced in order to take into account the effect of the pressure on demand.

$$\ln\left(\frac{M}{D}\right) = \alpha_0 + \varepsilon_s \ln\left(\frac{PD}{PM}\right) + \alpha_1 \ln(GDP) + u$$

where $M$ is the import volume index, $D$ is the volume index of domestically-consumed local goods, $PD$ is the ex-factory consumer price index for domestically-consumed local goods including sales taxes, $PM$ is the import price index including all tariffs and sales taxes.

A similar method can be adopted in the case of the producer's decision between export and domestic sales. We can estimate the constant elasticity of transformation between the commodities based on the following export supply equation:

$$\ln\left(\frac{E}{D}\right)_t = b + \varepsilon_t \ln\left(\frac{PE}{PD}\right)_t + v_t$$

Note that if the "small country" assumption is relaxed, a foreign export demand function will need to be introduced and its finite export demand price elasticity will need to be estimated

*Non-Linear Methods*

Other methods for estimating the elasticity of substitution have also been examined in production theory. Here we assume a stochastic CES value added function of capital and the labor:

$$VA_i = A_i\left[\alpha_i K_i^{-\rho} + (1-\alpha_i)L_i^{-\rho}\right]^{-\frac{1}{\rho}} + u_i$$

Several methods for estimating elasticities of substitution have been used in this context, of which two are explored here. The first method is based on the minimization of the squares of the error term $u_i$:

$$Min\sum_{i=1}^{n}\left(VA_i - A_i\left[\alpha_i K_i^{-\rho} + (1-\alpha_i)L_i^{-\rho}\right]^{-\frac{1}{\rho}}\right)^2$$

If the disturbance terms are multiplicative7, then the expression to minimize reads as follows:

7 I.e. $VA_i = A_i\left[\alpha_i K_i^{-\rho} + (1-\alpha_i)L_i^{-\rho}\right]^{-\frac{1}{\rho}} * u_i$

$$\sum_{i=1}^{n}\left(\ln VA_i - \ln A + \frac{1}{\rho}\ln\left[\alpha_i K_i^{-\rho} + (1-\alpha_i)L_i^{-\rho}\right]\right)^2$$

Non linear methods may be adopted using various econometric software packages.

*Taylor Approximation*

The second method is based on the approximation of the CES by the Taylor's series. If the value added function is written in the form:

$$\ln VA = \ln A - \frac{1}{\rho}f(\rho)$$

with $f(\rho) = \ln\left[\alpha_i K_i^{-\rho} + (1-\alpha_i)L_i^{-\rho}\right]$, Taylor's logic near $\rho = 0$ (which corresponds to $\varepsilon_s = 1$, $\varepsilon_s = \frac{1}{1+\rho}$) takes the following general form:

$$f(\rho) - f(0) = \rho f'(0) + \frac{1}{2}\rho^2 f''(0) + \dots$$

By neglecting the higher order terms, the equation to estimate reads as follows8:

$$\ln VA_i = \ln A_i + \alpha_i \ln K_i + (1-\alpha_i)\ln L_i - \frac{1}{2}\rho\alpha_i(1-\alpha_i)\left(\ln\frac{K_i}{L_i}\right)^2$$

The last term, relating to the capital-labor ratio, constitutes the difference with the log-linear regression of a C-D function. It implies that the substitution elasticity is different from one.

It is obvious that these estimation methods are also valid for the cases of a CES function representing the composite consumption of domestic goods and imports. As a matter of fact, various approaches exist in literature (Devarajan et al., 1999) and the success of each depends on data availability and quality.

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8 For a more detailed explication, refer to Wallis (1979) and Sadoulet and de Janvry (1995).

### *The Linear Expenditure System (LES)*

The use of a linear expenditure system entails the following elasticities:

$$\varepsilon_r = \frac{\alpha_i R}{p_i C_i}$$

$$\varepsilon_p = \frac{(1-\alpha_i)C_{\min i}}{C_i} - 1$$

Such elasticities are easy to compute from estimates of the parameters of the demand function. These parameters are estimated by taking into account the system of the LES equation demands to which we add random errors:

$$p_i C_i = p_i C_{\min i} + \alpha_i \Big[ R - \sum p_j C_{\min j} \Big] + u_i$$

This multivariate simultaneous equations model can be estimated using Seemingly Unrelated Regression Equations (SURE) or Full Information Maximum Likelihood (FIML) methods (Sadoulet and de Janvry, 1995). The objective is to simultaneously estimate all the equations, taking into account existing interdependencies due to the fact that the same variable (such as residual income) is present in all equations and errors in different equations are correlated.

Another way to obtain convergent estimates consists in proceeding by iteration. This procedure is based on the assumption that for a given $\alpha_i$, the LES equations are linear in $C_{\min i}$ and vice-versa. In fact, these equations can be written in the following forms:

$$p_i C_i - \alpha_i R = C_{\min i}(p_i - \alpha_i p_i) - \sum_j C_{\min j}(\alpha_j p_j) + u_i$$

$$p_i C_i - p_i C_{\min i} = \alpha_i \Big[ R - \sum p_j C_{\min j} \Big] + u_i$$

These two equations are linear in $C_{\min i}$ and $\alpha_i$, respectively. The iterative procedure used to converge involves two steps. We begin with a value for $\alpha_0$ and estimate $C_{\min i}$ using an OLS regression of the first system. With this value of $C_{\min i}$, $\alpha_i$ is then estimated with an OLS

regression of the second system. This iteration procedure is continued until stable values are obtained for both $\alpha_i$ and $C_{\min i}$. The demand functions are thus totally specified.

### *The Almost Ideal Demand System (AIDS)*

The demand elasticities for the AIDS model are computed on the basis of estimates of the parameters of its demand function. To do so, a stochastic element is introduced in the demand function to obtain the following simultaneous equations model (Sadoulet and de Janvry, 1995):

$$\omega_i = \alpha_i + \sum \gamma_{ij} \ln p_j + \beta_i \ln \frac{R}{P} + u_i$$

where $P$ is the consumer price index and $\omega_i$ is the budget share of commodity *i* in overall consumption expenditure. An OLS regression can be used to estimate this system on an equation by equation basis. However, since the equations are interrelated, it is preferable to use methods such as SURE or FIML. The endogenous variables are the budget shares, while prices and real income are exogenous.

## A Brief Review of Free Parameter Estimates for Developing Countries

There is no doubt that the choice of free parameters is an important element in CGE analysis of economic policies-related shocks. These parameters critically determine the magnitude of response to different exogenous shocks. A good example of a CGE model based on econometric estimates of all parameters is provided by Abdelkhalek and Dufour (1997) and (1998). In addition, they develop methods to construct confidence intervals for all endogenous variables in a CGE model given the variance of their parameter estimates.

However, most CGE models are based on fairly arbitrary estimates of these parameters. In fact, the difficulty in gathering data necessary for the econometric estimation of these parameters prompts modelers either to "borrow" these values from other studies conducted on

countries with similar characteristics, or to base them on their personal judgment as "guess-estimates". In some cases, the choice of the values of these elasticities is made on the basis of a "consensus" reached by researchers.

To explore the robustness of their results with respect to their parameter estimates, many modelers subsequently conduct sensitivity tests on their parameter estimates. Pagan and Shannon (1985) and Harrison and Vinod (1992) develop different methods for conducting sensitivity tests. Criticisms of these approaches by Jorgenson (1984) and McKitrick (1998) prompted some modelers to use more flexible functional forms. However this adds some analytical complexity[9].

The objective here is to provide a (non exhaustive) database of estimates for developing countries of the free parameters required for the most commonly used functional forms. To this effect, several sources of data have been assembled on trade elasticities and elasticities of substitution between labor and capital, as well as demand elasticities and Frisch parameters. Table 2 presents the sources that have been used for the purposes of this brief review.

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[9] Perroni and Rutherford (1998) compare different flexible forms that could be used in CGE models.

**Table 2: Sources and countries for elasticity data**

| | Country | Period | Elasticities | Table | Desegregation Sectors/Products |
|---|---|---|---|---|---|
| **Trade Elasticities** | | | | | |
| Dervis et al.(1982) | Turkey | n.a | Armington | T8.2 p263 | (19) |
| Sadoulet and Roland-Holst (1989) | Ecuador | 1965-87 | Armington, CET | TA.3.1, TA.3.4 | (30) |
| Devarajan et al.(1993) | Indonesia | n.a | Armington, CET | T3 p57 | (18) |
| - | Cameroon | n.a | Armington, CET | T5 p59 | (11) |
| Roland-Holst et al. (1994) | Mexico | n.a | Armington, CET | T2.7 p.67 | (26) |
| Abdelkhalek (1996) | Morocco | 1980-92 | Armington, CET | A-III pp.53-72 | (24) |
| Kapuscinski and Warr (1999) | Philippines | mid 70,late 80 | Armington | T4 p.21 | (33) |
| Lofgren (2001) | Egypt | n.a | Armington, CET | TA.5 p.46 | (9) |
| Arndt et al. (2001) | Mozambique | 1992-1996 | import, export | T2 p.26 | (6) |
| Tourinho et al. (2003) | Brazil | 1986-2001 | Armington | | (28) |
| **Elasticities between factors** | | | | | |
| Kemal, A. R. (1981) | Pakistan* | 1959-60,1969-70 | substitution(L,K) | T3 p.11 | (16) |
| Pohit et al.(2001) | India | 1988-89,1989-90 | substitution(L,K) | T6 p.73 | (23) |

**Demand Elasticities**

| | | | | | |
|---|---|---|---|---|---|
| Deaton (1989) | Java | n.a | income,price | T1 p. 203 | (11) |
| Sadoulet and De Janvry (1995) | India | 1951-68 | income,price | T2.2 p.48 | (4) |
| - | Ghana | 1953-70 | income,price | T2.2 p.48 | (3) |
| Ravelosoa et al.(1999) | Madagascar | 1993-94 | income,price | T8 p.22,T9 p.23 | (17) |
| Weerahewa and Nawaratna (2001) | Sri Lanka | 1969-72 | price | T2 p.5 | (8) |

* Comparison with India, Argentina and Bangladesh.

*Trade Elasticities*

By trade elasticities, we refer to the Armington elasticities of substitution between imported and local goods (Armington elasticities), as well as CET elasticities of transformation between exports and local sales (CET elasticities). Some researchers have econometrically estimated these trade elasticities for developing countries, as illustrated by studies in the following four countries: Ecuador, Morocco, the Philippines and Brazil. These estimates are made in the context of a CGE model, except in the case of Brazil, and described in the first sub-section below. Note that the choice of sectors vary from one study to another, which explains the many empty cells in this table. We then present a number of studies in which trade elasticities are based on personal judgment. Finally, to avoid problems of data availability, Arndt et al (2001) recently implemented the entropy approach in computing these elasticities for Mozambique.

*Econometric Estimations*

**Sadoulet and Roland-Holst (1989),** in constructing a CGE model for Ecuador and in order to better capture the response of the economy to the variations in the terms of trade and exchange rates, conduct econometric estimations of Armington and CET elasticities for 30 sectors. Estimations are based on data obtained from the Ecuadorian National Accounting Sources covering the 1965-1987 period. These authors have paid particular attention to the modeling of imports as Ecuador experienced serious imports restrictions during the 1982-1987 period. Armington elasticities vary from 0.20 (Tobacco and Wood sectors) to 1.80 (Livestock, Forestry and Fishing). CET elasticities were estimated for most sectors. For the remaining sectors, they were set equal to those in other sectors. CET elasticities vary from 0.36 (Basic Minerals) to 2.79 (Milling). These values are all presented in Table 3.

**Kapuscinski and Warr (1999)** estimate Armington elasticities for 33 sectors in the context of a CGE model. Estimations are based on data obtained from the Philippines National Statistics Office and other institutions. For most of the goods, such data cover the period

running from the mid-1970s to the late 1980s. The econometric techniques used here draw from three models, namely the Ordinary Least Squares (OLS), Partial Adjustment Model (PAM) and the Error Correction Models (ECM). For their CGE model, the authors selected the estimates obtained through the ECM method. Estimated Armington elasticities, which are presented in Table 3, vary from 0.2 for "Metal products" to 4.1 for "Sugar".

**Abdelkhalek (1996)** estimated Armington and CET elasticities in a CGE model constructed by the OECD for the Moroccan economy. The estimations are based on data obtained from various ministries for the 1980-1992 period[1]. Abdelkhalek selected 24 sectors and, for each sector, different specifications (7 for imports and 6 for exports) were explored using OLS regressions. Armington elasticities vary from 0.19 for "Rice" to 3.44 for "Textiles" and. The author stresses that, in several cases, import demand is only weakly influenced by prices given that imports serve structurally as a complement to local production so as to meet domestic demand. CET elasticities are significantly negative for most sectors, except "Petroleum", "Non-metallic mineral products" and "Electrical equipment". In order to explain these exceptions, the author suggests that import restrictions and problems of access to foreign markets are responsible for the fact that export shares do not necessarily follow price fluctuations. Table 3 presents the estimates obtained with the author's preferred specification.

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[1] Ministry of Foreign Trade, Direction of Statistics, Ministry of Commerce, Industry and Crafts, and the Trade Office.

**Table 3: Econometrically estimated trade elasticities**

| | Ecuador | | Philippines | Morocco | | Brazil |
|---|---|---|---|---|---|---|
| | Sadoulet/R-H | | Kapuscinski | Abdelkhalek | | Tourinho |
| | | Exports | | | Exports | |
| | Armington | (CET) | Armington | Armington | (CET) | Armington |
| **AGGREGATE*** | **0.27** | **12.69** | **0.73** | **0.26** | **0.89** | **N.A** |
| **PRIMARY** | | | **3.71** | | | |
| Corn | | | | 3.17 | | |
| Maize | | | | 0.81 | | |
| Rice | | | 1.03 | | | |
| Banana and other fruits and nuts | | | 0.72 | | | |
| Vegetables | 0.43 | 0.56 | | | | |
| Other agriculture | 1.80 | | | | | |
| Livestock | | | 0.33 | | | |
| Hogs | | | 1.39 | | | |
| Chicken and poultry products | | | 1.32 | | | |
| Other livestock | 1.80 | | 1.06 | | | |
| Fishing | 1.80 | 0.87 | 0.82 | | | |
| Forestry | 0.24 | | 0.65 | | 0.87** | 0.82 |
| Oil, coal and gas | | | | 0.68 | | |
| Metal ore mining | | | | 1.19 | | |
| Non-metallic mineral | 1.32 | 1.92 | 1.11 | | | |
| Other mining | | | | | | |
| **INDUSTRY** | | | **0.61** | | | |

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| | | | | | | |
|---|---|---|---|---|---|---|
| Rice and corn milling | 0.30 | 2.37 | 4.10 | | | |
| Sugar | | | 0.75 | | | |
| Milk and dairy products | 1.62 | 2.50 | | | | 3.47 |
| Meat and fish | | | 1.37 | | | 2.22 |
| Oils and fats | 1.10 | 2.79 | 0.72 | | | |
| Milling | | | 0.11 | 1.31 | | 0.96 |
| Other food | 0.69 | | 0.32 | 0.49 | | |
| Beverage | 0.20 | | 0.32 | 0.49 | | |
| Tobacco | 0.83 | 1.48 | 0.65 | 3.44 | | 1.82 |
| Textiles and knitting mills | | | 0.24 | 0.54 | | 1.72 |
| Garments | 0.83 | 1.48 | 0.24 | 0.79 | | |
| Leather | | | | | | 0.57 |
| Footwear | 0.20 | 2.49 | | 1.05 | | 2.73 |
| Wood | 0.17 | 0.40 | 0.60 | 0.88 | | 0.54 |
| Paper | | | 0.55 | | | 0.23 |
| Coal and petroleum products | 0.67 | 0.64 | | 0.45 | | 1.24 |
| Chemicals | | | | | | 0.52 |
| Pharmaceutical and medical industries | | | | | | 0.56 |
| Fertilizers and other chemical industries | | | | 0.48 | | 1.18 |
| Rubber | | | | 0.48 | | 1.12 |
| Plastic | | | 0.58 | 1.30 | 0.29 | 0.76 |
| Non metallic minerals products | 0.46 | 0.36 | | 1.60 | | |
| Basic metals | | | 0.24 | 0.95 | | 0.22 |
| Metal products & non-electrical machinery | 0.94 | 0.64 | 1.76 | 0.57 | | 1.78 |
| Machinery | | | | | | 0.23 |
| Electronic equipment | | | | 0.53 | 0.03 | 0.16 |
| Electrical equipment | | | 2.00 | 1.05 | | 4.95 |
| Transport equipment | 0.94 | 0.64 | 1.04 | 1.03 | | 2.46 |
| Other manufacturing | | | | | | |
| **SERVICES** | **0.75** | **2.50** | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| Trade | 0.96 | 2.50 | | | | |
| Transport and communication | 0.75 | 1.00 | | | | |
| Financial services | 0.30 | 1.00 | | | | |
| Other services | 0.27 | 12.69 | 0.73 | 0.26 | 0.89 | N.A |

**Note:** Built from sources in Table 2. * import/export taken from Devarajan et al. (1999). ** only petroleum

**Tourinho et al. (2003)** estimate Armington elasticities for 28 Brazilian industries using quarterly data collected during the 1986-2001 period by the "Fundação Centro de Estudos de Comércio Exterior" and the "Fundação Getúlio Vargas". The authors obtained statistically significant estimates for 25 sectors that vary from 0.16 to 4.95.

*Personal Judgment*

This section reviews elasticities estimated on the basis of personal judgment, rather than econometric analysis, in the context of various CGE models implemented in developing countries. Omission has been made of studies in which the authors arbitrarily set the same values for CES and CET elasticities in all sectors.

**Dervis et al. (1982),** for purposes of simulation of the reduction of tariff restrictions in Turkey using a CGE model, determine intervals of Armington elasticities as part of their sensitivity tests (Table 4). The superior limits of these intervals are simply equal to the inferior limit multiplied by three. The inferior limits vary between 0.25 and 2.00.

**Devarajan et al. (1993)**, using their 123 CGE model, analyze the impact of terms of trade shocks on the real exchange rate and the trade balance. They applied this model to Cameroon and Indonesia by choosing Armington and CET elasticities from the literature. In the case of Cameroon, they set Armington and CET elasticities equal in each sector with values varying between 0.4 and 1.5. For Indonesia, the two types of elasticities have different values comprised between 0.4 and 2.

**Roland-Holst et al. (1994)** build a CGE model for three countries (USA, Canada and Mexico) and 26 sectors in order to analyze the impact of integration in North America. For the Armington elasticities for Mexico between domestic goods, imports from the USA and Canada, and imports from the rest of the world, the authors base their estimates on a study by Sobarzo (1992) for the same countries. These elasticities vary from 0.46 for "Other manufacturing" to 2.25 for "Agriculture". CET elasticities, varying between 0.12 ("Garments") and 3.78 ("Agriculture"), are drawn from Reinert and Roland-Holst (1991), who estimated these parameters for the USA and applied them to both Canada and Mexico.

**Löfgren (2001)** simulates the impact of different development strategies on growth and poverty in Egypt using a recursive dynamic CGE model. Drawing from his literature review, Löfgren (1994) selects values for Armington elasticities that vary from 0.9 to 2.4. His CET elasticities are equal to 1.5, except for Agriculture (3).

**Table 4: Trade elasticities based on personal judgment**

| SECTORS | Turkey | Cameroon | Indonesia | | Mexico | | Egypt | |
|---|---|---|---|---|---|---|---|---|
| | Dervis | | Devarajan | | Roland-Holst | | Löfgren | |
| | Imports (CES) | CES/CET | Imports (CES) | Exports (CET) | Imports (CES) | Exports (CET) | Imports (CES) | Exports (CET) |
| **PRIMARY** | | | | | | | | |
| Agriculture | 2.00-6.00 | | 0.60 | 0.60 | 2.25 | 3.78 | 2.40 | 3.00 |
| Forestry | | 0.40 | | | 0.78 | 1.05 | | |
| Mining | 0.33-1.00 | | | | | | | |
| Petroleum | | | | | 0.58 | 0.89 | | |
| Other mining | | | 0.90 | 0.60 | | | | |
| **INDUSTRY** | | | | | | | | |
| Food | 0.75-2.25 | 1.25 | 0.90 | 1.20 | 1.00 | 0.75 | 0.90 | 1.50 |
| Beverages | | | 0.90 | 1.20 | 0.72 | 0.49 | | |
| Tobacco | | | | | 1.00 | 0.78 | | |
| Textiles | 0.75-2.25 | | 0.90 | 0.60 | 1.02 | 0.39 | 0.90 | 1.50 |
| Garments | 0.75-2.25 | | | | 0.80 | 0.12 | | |
| Leather | | | 0.90 | 0.60 | 1.06 | 1.16 | | |
| Wood | 0.75-2.25 | | 0.90 | 0.60 | | | | |
| Paper | 0.33-1.00 | | 0.90 | 2.00 | 0.73 | 0.42 | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Chemicals | 0.33-1.00 | | 0.60 | 0.50 | 0.70 | 0.36 | | |
| Petroleum products | 1.50-4.50 | | | | | | | |
| Rubber & plastics | 0.33-1.00 | | | | 0.76 | 0.27 | | |
| Non metallic minerals products | 0.33-1.00 | 0.75 | 0.60 | 2.00 | 0.82 | 0.21 | | |
| Basic metals | 0.33-1.00 | | 0.60 | 0.60 | 0.71 | 0.42 | | |
| Metallic products | 0.33-1.00 | | 0.60 | 0.60 | 0.59 | 0.54 | | |
| Non-electrical machinery | 0.75-2.25 | | 0.60 | 0.60 | 0.69 | 0.37 | | |
| Electrical machinery | 0.75-2.25 | | 0.60 | 0.60 | 0.70 | 0.31 | | |
| Transport equipment | 0.75-2.25 | | | | 0.67 | 1.01 | | |
| Other manufacturing | | | | | 0.46 | 0.41 | 0.90 | 1.50 |
| **SERVICES** | | | | | | | | |
| Transportation | | | 0.40 | 0.40 | 1.20 | 1.10 | 0.90 | 1.50 |
| Public administration | | | | | | | | 1.50 |
| Services | 0.75-2.25 | 0.40 | 0.40 | 0.40 | | | 0.90 | 1.50 |
| Trade | | | 0.40 | 0.40 | 1.20 | 1.10 | | |

**Note:** Built from sources in Table 2.

*Entropy methods*

As already mentioned, in many developing countries we lack sufficient time series data to econometrically estimate the various elasticities commonly used in CGE models. Such data, when they exist, are often unreliable and the length of the series is often very short. To address this problem, Arndt et al. (2001) have developed a new approach for the estimation of free parameters in CGE models. It is the maximum entropy approach, which is based on information theory and makes it possible to estimate parameters even with a limited amount of data. It is similar to the Jorgenson (1984)'s econometric approach, in that it uses past information, and the "validation and calibration"[10] approach, since the model can reproduce past events.

Table 5 presents the results obtained through the application, by Arndt et al (2001), of the maximum entropy method in Mozambique. The Armington elasticities are comprised between 0.57 and 5.54 and the CET elasticities vary from 0.33 to 2.84.

**Table 5: Trade elasticities for Mozambique**

|                | Armington | CET (export) |
|----------------|-----------|--------------|
| Food           | 5.54      | 0.72         |
| Cash crops     | 0.69      | 2.20         |
| Fish           |           | 0.74         |
| Processed Food | 0.57      | 0.33         |
| Manufactures   | 0.87      | 0.56         |
| Services       | 1.85      | 2.84         |

Source: Arndt et al. (2001), Table 2, p.26

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10 Devarajan and Robinson (2002) explain the difference between the maximum entropy approach, which may be used for estimating and updating SAMs (cross entropy), and the validation principle, which seeks to test the ability of the model to explain past events.

***Elasticities of Substitution between Labor and Capital***

In the case of the elasticities of substitution between labor and capital in the value added function, we present estimates from two econometric studies covering four developing countries : Pakistan, Argentina, Bangladesh and India (Table 6).

**Pohit et al. (2001)** conduct an econometric estimation of the elasticities of substitution between labor and capital for 23 sectors of the Indian economy in the context of a CGE analysis. This estimation is made for two distinct years (1988-89 and 1989-90), on the basis of data from the Annual Survey of Industries (ASI). The authors' objectives are to check if the values of the elasticities of substitution between capital and labor tend to change from one year to another and to test, for each sector, the C-D functional form as compared to the CES form. The elasticities vary from 0.58 to 2.26. These values presented in Table 6 appear in bold when the assumption of a C-D function between capital and labor is ruled out.

**Kemal (1981)** estimates the elasticities of substitution between labor and capital for 16 manufacturing sectors in Pakistan, using data from the "Census of Manufacturing Industries" that covers the period from 1956-1960 to 1969-1970. They use a CES production function and a variable elasticity function of substitution (VES) that takes into consideration capital intensity. The author compares the elasticity values obtained through by OLS regressions in Pakistan, Argentina, Bangladesh and India. Given that Pohit's et al. (2001)'s estimations of these elasticities for India are more recent, Table 6 only presents the values for Pakistan, Argentina and Bangladesh. Kemal concludes that values of elasticities of substitution are generally low in developing countries and reflect the lack of local technological development.

**Table 6: Elasticities of substitution between labor and capital**

| | India Pohit et al. | | Pakistan | Argentina | Bangladesh |
|---|---|---|---|---|---|
| | 1988-89 | 1989-90 | | Kemal | |
| Food manufacturing | | | 0.09 | 0.28 | 0.37 |
| Tobacco processing | | | 1.72 | 0.22 | 0.6 |
| Textiles | **0.58*** | | 0.52 | 0.26 | 0.34 |
| Wearing Apparel | 1.02 | | | | |
| Leather Prod. | 1.07 | | 0.56 | 1.00 | 0.64 |
| Wood Prod | 0.79 | **0.62** | | | |
| Paper | 0.91 | 1.23 | 0.05 | 0.21 | 0.48 |
| Print and publishing | | 0.95 | 2.66 | 0.87 | 0.50 |
| Chemicals | | 1.40 | 0.29 | 0.03 | 0.32 |
| Petrol and related product | 1.62 | **2.26** | | | |
| Rubber products | 1.20 | | 0.79 | 0.16 | 0.36 |
| Non metal Mineral Prod | 0.90 | 1.09 | | | 0.54 |
| Glass and Glass Prod. | 0.65 | 0.93 | | | |
| Iron and Steel | **0.56** | | | | |
| Non ferrous Metals | 0.96 | 1.50 | | | |
| Metal Products | 0.62 | 0.79 | 0.21 | | |
| Non electrical machinery | 0.89 | **0.64** | 0.81 | 0.10 | 0.53 |
| Electrical machinery | | **0.56** | | | |
| Transport Equipment | | 1.08 | | 0.05 | 0.38 |
| Misc. Manufacturing | 0.59 | 0.80 | 1.37 | | |
| Trans, Stor, Commu. | 0.88 | 0.91 | | | |

**Source**: Pohit et al. (2001) Table 6 p.73 and Kemal (1981) Table 3 p.11.

*: The values in bold correspond to cases where the C-D function assumption has been ruled out.

### Demand Elasticities

We complete this review with an examination of estimates of demand elasticities in the literature. Estimates of Frisch parameters, drawn from Hertel et al. (1997), are presented in Table 7.

**Table 7: Frisch parameters**

|                              | Frisch parameter |
| ---------------------------- | ---------------- |
| Indonesia                    | -5.42            |
| Philippines                  | -5.08            |
| India                        | -7.57            |
| Mexico                       | -2.94            |
| Brazil                       | -3.34            |
| Middle East and North Africa | -3.54            |
| Sub-Saharan Africa           | -5.85            |

Source: Hertel et al. (1997)

However, in the case of developing countries, most studies focus primarily on income and price elasticities of demand for food products (Table 8).

**Adelman and Timmer (1980)[11]** estimate the price and income elasticities in Sri Lanka for eight food products and three categories of households classified on the basis of income levels. The data are obtained from the 1969-1970 "Socio-economic Survey of Sri Lanka" and the 1973 "Survey of Consumer Finance". Only price elasticities, which are highest in the case of "bread and Meat", are presented.

**Deaton (1989)** estimate price and income elasticities for Java using data obtained from the "Central Bureau of Statistics, Indonesia" on 11 food products. The author presents the income elasticities related to two demand components: quantity and quality. These two components have been merged in order to obtain income elasticities of demand[12]. Price elasticities are lower for basic commodities. Income elasticities are higher than one for products such as wheat, fruit, meat and fish, which are regarded as luxury goods.

11 Reported by Weerahewa and Nawaratna (2001).

12 Deaton (1989) p.202-203 and Sadoulet and de Janvry (1995) p. 49.

**Sadoulet and de Janvry (1995)** present estimates of elasticities of the demand for several countries and commodities. Table 8 presents the values of these elasticities for India and Ghana as drawn from Scandizzo and Bruce (1980) and Sullivan et al. (1988).

**Ravelosoa et al (1999)** estimate the price and income elasticities of demand for 17 categories of commodities and six types of Malagasy households. The estimation is essentially based on an AIDS model and data drawn from the Permanent Household Survey conducted by the Institut National de la Statistique (INSTAT) during the one year period running from March 1993 to April 1994. Income elasticities vary from 1.55 for "Breeding" to 0.47 for "Rice", which is a staple food. As for price elasticities, the most sensitive products include "Fishing" (–1.1), followed by "non-food products" (–0.95) and "Fruit" (-0.91).

## Conclusion

This study focused on the choice of functional forms and their parametrization (estimation of free parameters and calibration of other parameters) in the context of CGE models. Various types of elasticities are defined, followed by a presentation of the functional forms most commonly used in these models and various econometric methods for estimating their free parameters. Following this presentation of the theoretical framework, we review parameter estimates used in the literature. This brief literature review was carried out to be used as a guideline for the choice of parameters for CGE models of developing countries.

**Table 8: Demand elasticities from econometric estimation**

| | India | | Ghana | | Sri Lanka | Java | | Madagascar | |
|---|---|---|---|---|---|---|---|---|---|
| | Sadoulet and de Janvry | | | | Adelman | Deaton | | | |
| | income | price | income | price | price | income | price | income | price |
| Food grains | 0.49 | -0.34 | | | | | | | |
| Total cereals | 0.79 | -0.50 | 0.71 | -2.32 | | | | | |
| Rice | 0.94 | -0.75 | 0.71 | -1.25 | -0.29 | 0.52 | -0.42 | 0.47 | -0.77 |
| Wheat | 1.06 | -0.22 | | | | 1.67 | -0.69 | | |
| Maize | | | | | | 0.09 | -0.82 | | -0.53 |
| Cassava | | | 0.82 | -0.64 | | 0.16 | -0.33 | | -0.30 |
| Other tubercle | | | | | | | | | -0.53 |
| Coffee | | | | | | | | 0.53 | |
| Roots | | | | | | 0.88 | -0.95 | | |
| Industrial crop | | | | | | | | | -0.87 |
| Vegetables | | | | | | 0.63 | -1.11 | 1.18 | -0.63 |
| Pulses | | | | | -0.71 | 0.89 | -0.95 | 0.85 | -0,40 |
| Fruit | | | | | | 1.46 | -0.95 | 0.59 | -0.91 |
| Coconut | | | | | -1,00 | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Breeding | | | | | | 1.55 | -0.65 |
| Meat | | | -1.83 | 2.39 | -1.09 | | |
| Oils | | | | | | 1.50 | -0.85 |
| Fishing | | | | | | 1.00 | -1.10 |
| Fresh fish | | | | 1.30 | -0.76 | | |
| Dried fish | | | | 0.63 | -0.24 | | |
| Bread | | | -1.14 | | | 1.49 | -0.73 |
| Beverages | | | | | | 1.41 | -0.57 |
| Tobacco | | | | | | 1.24 | -0.73 |
| Other foods | | | | | | 1.20 | -0.62 |
| Non food | | | | | | 1.41 | -0.95 |

**Note:** Build from sources in Table 2

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