

## 2

# Elements of a Computable General Equilibrium Model

In this chapter, we deconstruct the computable general equilibrium model and describe its core elements. These include sets, endogenous and exogenous variables, exogenous parameters, behavioral and identity equations, and model closure. We describe prices, price normalization and the numeraire. We explain how the CGE model runs, including the processes of calibrating and solving the model, and carrying out an experiment.

A computable general equilibrium (CGE) model is a system of mathematical equations that describes an economy as a whole, and the interactions among its parts. A model this comprehensive is more complex than the bicycle industry model that we built in Chapter 1, but it need not be a “black box.” In this chapter, we deconstruct the CGE model and describe its core elements. We show that a CGE model and the simple bicycle model share many features, such as exogenous and endogenous variables, market-clearing constraints, and identity and behavioral equations. We describe how the price of a single commodity changes as the product moves along the supply chain from producers to consumers. We explain the practice of normalizing prices and the role of the price numeraire. We introduce model closure, which is the decision about which variables are exogenous and which are endogenous. We also describe how the CGE model runs by explaining the sequence of model calibration, model solution, and model experiment.

In this chapter, our objective is to introduce, at a general level, the model’s elements and mechanics. Even so, for many students, it may suffice to skim this chapter and return to it as needed as your modeling skills progress. For now, we also set aside any consideration of the economic theory that governs behavior in the model. Here, we do not consider how the model describes the motivations behind producers’ decisions about how much to produce or consumers’ decisions about how much to buy, or a nation’s choice between consumption of its domestic production and imported goods. Of course, the economic properties of a CGE model are its real heart and soul but they also

present a much broader area of study; most of the other chapters in this book address this study.

### Sets

A CGE model starts by introducing sets. **Sets** are the domain over which parameters, variables, and equations are subsequently defined. For example, we can define set  $i$  as industries, which in the 3x3 U.S. database consists of agriculture, manufacturing, and services. If “QO” is output, then we can define a variable  $QO_i$ , which is the output quantity defined over the set  $i$ . That is,  $QO_i$  is a vector with three elements. It includes the output of agriculture, output of manufacturing, and output of services. To refer to only one element in set  $i$ , for example, the quantity of agricultural output, we express the variable as  $QO_{\text{“agriculture”}}$ , where one element of set  $i$ , in this case agriculture, is identified in quotes.

Similarly, we might define a different variable, PS, over the same set  $i$ , where PS is the producer price. If our equation refers to  $PS_i$ , then we are referring to the producer prices of agriculture, manufacturing, and services. To refer to the price of services alone, we would identify the set element in quotes, as  $PS_{\text{“services”}}$ .

Different variables in the CGE model can have different set domains. For example, our model might also include a set  $f$  that contains two factors of production – labor and capital. In that case, we could define variable  $PF_f$  as the price of factor  $f$ . The variable is a vector with two elements – labor wage and capital rent. Variables may also have more than one domain. For example, variable  $QF_{f,i}$  is the quantity of factor  $f$  employed in the production of good  $i$ . The variable is a matrix, with  $f$  rows and  $i$  columns.

In multi-country CGE models, set notation related to bilateral trade usually follows the convention that the first country name is the source country and the second country name is the destination country, i.e., variable  $QM_{i,r,s}$ , describes QM quantity of commodity  $i$  imported from country  $r$  by country  $s$ . For example,  $QM_{\text{“agriculture”},\text{“USA”},\text{“ROW”}}$  refers to imports of agriculture from the United States by the rest-of-world region. It is equal to  $QE_{\text{“agriculture”},\text{“USA”},\text{“ROW”}}$ , which is the quantity of agricultural goods exported from the United States to the rest-of-world region.

### Endogenous Variables

**Endogenous variables** have values that are determined as solutions to the equations in the model, similar to the equilibrium price and quantity of bicycles in our simple partial equilibrium model of Chapter 1. Examples of endogenous variables in CGE models are prices and quantities of goods that

**Text Box 2.1. Math Refresher – Working with Percent Changes**

CGE model results are usually reported as the percent change from initial, or base, values. The following are three useful mathematical formulae for working with percent change data:

1. **Percent change in a variable** is the new value minus the base value, divided by the base value, multiplied by 100.

*Example:* If the labor supply,  $L$ , increases from a base value of 4 million to 6 million, then:

$$\text{Percent increase in } L = (6 - 4)/4 = 0.5 * 100 = 50$$

2. **Percent change in the product of two variables** is the sum of their percent changes.

*Example:*  $GDP = P * Q$ , where  $P$  is the price and  $Q$  is the quantity of all goods in the economy. If  $P$  increases 4 percent but  $Q$  decreases 2 percent, then:

$$\text{Percent change in } GDP = 4 + (-2) = 2$$

3. **Percent change in the quotient of two variables** is the dividend (numerator) minus the divisor (denominator).

*Example:* Per capita GDP is  $GDP/N$ , where  $N$  is population. If  $GDP$  grows 1 percent and  $N$  grows 2 percent, then:

$$\text{Percent change in } GDP/N = 1 - 2 = -1$$

are produced and consumed, prices and quantities of imports and exports, tax revenue, and aggregate savings.

When describing CGE model results, our notational convention in this book is to describe the level of a variable (e.g., the quantity of a good produced or its price) in upper or lower case letters and to denote the percent change in a variable in lower case italics. For example:

Variable  $QO_{\text{mfg}}$  = quantity of manufacturing output

Variable  $qo_{\text{mfg}}$  = percent change in quantity of manufacturing output

A CGE model usually has the same number of endogenous variables as independent equations. This is a necessary (although not a sufficient) condition to ensure that the model has a unique equilibrium solution.

### Exogenous Variables

**Exogenous variables** have fixed values that do not change when the model is solved. For example, if the labor supply is assumed to be an exogenous

variable, then the labor supply will remain at its base quantity, both before and after a model experiment.

### Model Closure

Modelers decide which variables are exogenous and which are endogenous. These decisions are called ***model closure***. An example of a closure decision is the modeler's choice between (1) assuming that the economy's labor supply is exogenous, and an endogenous wage adjusts until national labor supply and demand are equal, or (2) assuming that the economywide wage is exogenous, and an endogenous labor supply adjusts until national labor supply and demand are equal.

To illustrate the important concept of model closure, assume that we are studying the effects of a decline in the demand for computers, which causes the computer industry's demand for workers to fall. If we assume the nation's total labor supply is exogenous (i.e., fixed at its initial level), then economywide wages will fall until all laid-off computer workers are reemployed in other industries. However, if the closure instead defines the economywide wage as exogenous (and fixed at its initial level), then the loss of jobs in the computer industry may cause national unemployment. Because a change in the size of a country's labor force changes the productive capacity of its economy, its gross domestic product (GDP) will decline more in a CGE model that allows unemployment than in a model whose closure fixes the national labor supply.

Because the choice of closure can affect model results in significant ways, modelers try to choose closures that best describe the economy they are studying. CGE models usually have a section of model code that lists model closure decisions. In the Global Trade Analysis Project (GTAP) model, for example, one of the tabbed windows on the model's front page is titled "Closure." The closure page lists all of the exogenous variables, and the remainder is endogenous.

### Exogenous Parameters

CGE models include ***exogenous parameters*** that, like exogenous variables, have constant values. CGE models contain three types of exogenous parameters: tax and tariff rates, elasticities of supply and demand, and the shift and share coefficients used in supply and demand equations.

### Tax and Tariff Rates

Tax and tariff rates are typically calculated by the CGE model from the model's base data. For example, a CGE model database reports the value of

imports in world prices and the amount of tariff revenue that is paid to the government. The model calculates the exogenous parameter – the import tariff rate – as:

$$\begin{aligned} &\text{Value of tariff revenue/Value of imports in world prices} * 100 \\ &= \text{Import tariff rate.} \end{aligned}$$

If the tariff revenue is \$10, and the world price of the import (including freight and other trade costs) is \$100, then consumers pay \$110 and the model calculates a tariff rate of 10 percent.

Modelers can change tax and tariff rates as a model experiment to analyze “what if” scenarios. For instance, the modeler may want to know what would happen in the economy if the government reduces the import tariff rate. As an experiment, the modeler lowers the tariff and re-solves the CGE model to find the resulting prices and the new quantities that are demanded and supplied.

### ***Elasticity Parameters***

***Elasticities*** are exogenous parameters in a CGE model that describe the responsiveness of supply and demand to changes in relative prices and income. The magnitudes of model results stem directly from the size of the elasticities assumed in the model. For example, suppose that the National Chefs’ Association has asked you to study the possible effects of economic growth on the demand for restaurant meals. If consumer demand for restaurant meals is assumed to be very responsive to income changes (so the income elasticity of demand parameter is high), then even a small increase in income will lead to a relatively large increase in the demand for restaurant services. However, if the income elasticity is assumed to be low, then even large economic growth will have only a small effect on the quantity of demand for restaurant services.

Because of the importance of elasticities to model results, modelers try to choose parameters based on a careful review of relevant econometric studies that estimate supply and demand elasticities. Often, however, the links between a CGE model’s parameter requirements and the elasticities available in the literature are weak. Econometric studies may include different categories of commodities from those in the CGE model; they may be estimated using different functional forms; and the estimates themselves may be statistically weak. For these reasons, many CGE modelers also carry out “sensitivity analysis” of their model results to alternative sizes of elasticities. First, they run their model experiment with their assumed elasticity parameter. Next, they repeatedly change the values of one or more elasticities and rerun both the model and the experiment. They then

compare the new experiment results with the results of the first experiment to determine whether their findings hold true across a reasonable range of elasticity values.<sup>1</sup>

The types of elasticities used in CGE models vary because they depend on the types of production and utility functions assumed in the model. Some elasticities may not be the types that you are familiar with from your microeconomics studies. In the following two sections, we describe the supply and demand elasticity parameters used in many CGE models and show how each influences the slope or shift in supply or demand curves. A CGE model generally utilizes some, but not all, of these parameters.

### *Supply Elasticity Parameters*

**Factor Substitution Elasticity.** This parameter,  $\sigma_{VA}$ , relates to demand for factors of production, e.g., labor,  $L$ , and capital,  $K$ . It describes the flexibility of a production technology to allow changes in the quantity ratios of factors used in the production of a given level of output as relative factor prices change. For example, the parameter describes the ease with which producers in an industry can hire more labor and use less capital when the wage falls relative to the price of machinery and equipment.

The elasticity – one for each industry  $i$  in the model – describes the percent change in the quantity ratio of factor inputs given a percent change in their inverse price ratio:

$$\frac{\% \text{ change } \frac{L_i}{K_i}}{\% \text{ change } \frac{r}{w}}$$

where  $L_i$  and  $K_i$  are labor and capital employed in industry  $i$ , and  $r$  and  $w$  are the economy-wide capital rent and wage. The parameter's value ranges from zero to infinity. For example, an 0.5 percent factor substitution elasticity means that a 2 percent increase in capital rents relative to wages will lead to a 1 percent increase in the ratio of labor to capital quantities in the production process. As the parameter value approaches infinity, labor and capital become perfect substitutes. One worker can always be substituted for the same amount of capital with no reduction in the level of output. When the parameter is zero, the factors are complements, and producers must use a fixed ratio of capital and labor, regardless of changes in wages compared to rents.

<sup>1</sup> In Model Exercise 8, you will use an automated utility developed for the GTAP model by Arndt and Pearson (1998) to systematically analyze the sensitivity of model results to alternative values of elasticity parameters. This utility considerably simplifies this reiterative process of sensitivity analysis.

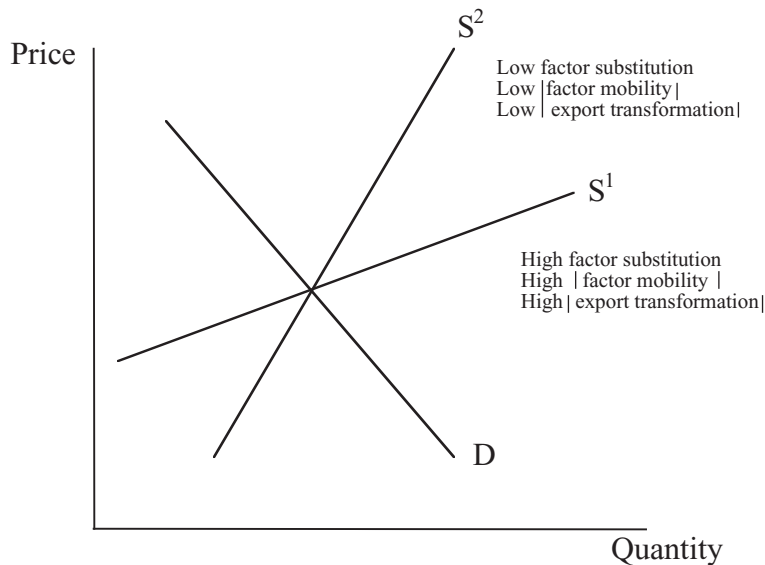


Figure 2.1. Effects of supply elasticity parameters on the slope of the supply curve

Producers who can more readily substitute among factors have a more elastic industry supply curve, such as curve  $S^1$  in Figure 2.1, where the axes represent output quantity and output price. When this industry increases its output, producers can keep the costs of production low by switching to lower cost factor inputs. For example, an industry with a flexible technology (a high factor substitution elasticity) can become more mechanized if its expansion causes wages to increase by more than capital rents. An industry with a more rigid technology, and a low factor substitution elasticity, is described in Figure 2.1 by the less elastic, and steeper, supply curve,  $S^2$ .

**Factor Mobility Elasticity.** This elasticity parameter,  $\sigma_F$ , relates to factor supply. It describes the ease with which a factor moves across industries in response to changing industry wages or rents. For example, it describes the willingness of a worker to move to another industry if it offers higher wages than his current job.

One elasticity is defined for each factor in the CGE model. It governs the percent change in the share of the national factor supply employed in industry  $i$  given a percent change in the economy-wide average factor price relative to the industry wage,  $w_i$ , or rent,  $r_i$ . For example, the labor mobility elasticity describes the share of the labor force employed in industry  $i$  as a function of its wage relative to the economy-wide wage:

$$\frac{\% \text{ change } \frac{L_i}{L}}{\% \text{ change } \frac{w}{w_i}}$$

The parameter value can range between zero (factors cannot move between sectors) and negative one (factors move proportionately to a change in relative factor prices). The lower range restriction of negative one reflects that the factor supply function, in those CGE models that explicitly include one, is used to describe relatively inflexible factor movements. As an example, an elasticity of minus 0.5 percent means that a 2 percent increase in the wage in the computer industry relative to the average wage, results in a 1 percent increase in the share of the labor force employed in computers.

When an industry employs factors that move sluggishly (with low absolute values of the mobility elasticity), its supply curve becomes relatively steep, like  $S^2$  in Figure 2.1, where the axes represent output quantity and output price. This is because its wage and rental costs must rise sharply to attract the additional factors needed to increase production. The more mobile factors are and the larger the parameter's absolute value, the more elastic is the industry's supply curve, such as  $S^1$  in Figure 2.1.

**Export Transformation Elasticity.** This parameter,  $\sigma_E$ , relates to an industry's export supply. It describes the technological ability of an industry to transform its product between the varieties sold in the domestic and export markets. For example, it describes how easily automakers could shift production between models for the home market and models that are more popular in foreign markets.

For each industry  $i$ , the elasticity measures the percent change in the ratio of the export quantity,  $QE_i$ , to the quantity sold domestically,  $QD_i$ , given a percent change in the ratio of the domestic price,  $PD_i$  to the free on board (*fob*) export price,  $PE_i$ :

$$\frac{\% \text{ change } \frac{QE_i}{QD_i}}{\% \text{ change } \frac{PD_i}{PE_i}}$$

One export transformation elasticity is defined for each industry, with a value that ranges from zero to negative infinity. For example, a minus 0.8 percent parameter value means that a 2 percent increase in the domestic price relative to the export price will lead to a 1.6 percent decline in the quantity ratio of exports to domestic sales in producers' total output.

If the parameter has a low absolute value, then the resources used in the production of one variety are relatively difficult to transform into the production of the other variety. For example, to increase their production of exports, producers must shift toward greater use of relatively unsuitable inputs from the production of the domestic variety. This raises the cost of expanding export sales and therefore limits the export supply response. In



Figure 2.1, assuming that the axes represent export quantity and export price, the lower the absolute value of the export transformation elasticity, the less elastic (and steeper) is the industry's export supply curve such as  $S^2$  in Figure 2.1. When the export transformation parameter is high in absolute value, then producers can readily expand their export output with less upward push on their costs of production. Their export supply curve is therefore more elastic, such as  $S^1$ .

### *Demand Elasticity Parameters*

**Income Elasticity of Demand.** This elasticity parameter describes the effect of a change in income upon demand for a commodity. One parameter is defined for each consumption good  $i$  in the model. It measures the percent change in the quantity demanded,  $Q$ , given a percent change in income,  $Y$ :

$$\frac{\% \text{ change } Q_i}{\% \text{ change } Y}$$

Income elasticity parameter values between zero and one indicate necessity goods, such as food, for which demand grows by proportionately less than growth in income. Parameter values greater than one describe luxury goods, for which demand grows by proportionately more than growth in income. A unitary income elasticity assumes that consumer demand changes by the same proportion as a change in income.

In CGE models, goods are usually “*normal*”; that is, income elasticities are positive so that an increase in income leads to an increase in demand for a good. Not all CGE models allow the modeler to specify an income elasticity of demand. Often, the models assume utility functions in which the income elasticity of demand is “hardwired” to have a value of one. See Chapter 4 for a more complete discussion of this point. In Figure 2.2, a change in income could be shown as a shift in the demand curve, where the axes represent the quantity of the consumption good and its consumer price. The higher the income elasticity, the larger is the rightward (leftward) shift in the demand curve for any given increase (decrease) in income.

**Own- and Cross-Price Substitution Elasticities.** These parameters measure the responsiveness of consumer demand to changes in the price of commodities. The own-price elasticity measures the percent change in quantity demanded for good  $i$  given a percent change in its consumer price,  $P$ :

$$\frac{\% \text{ change } Q_i}{\% \text{ change } P_i}$$

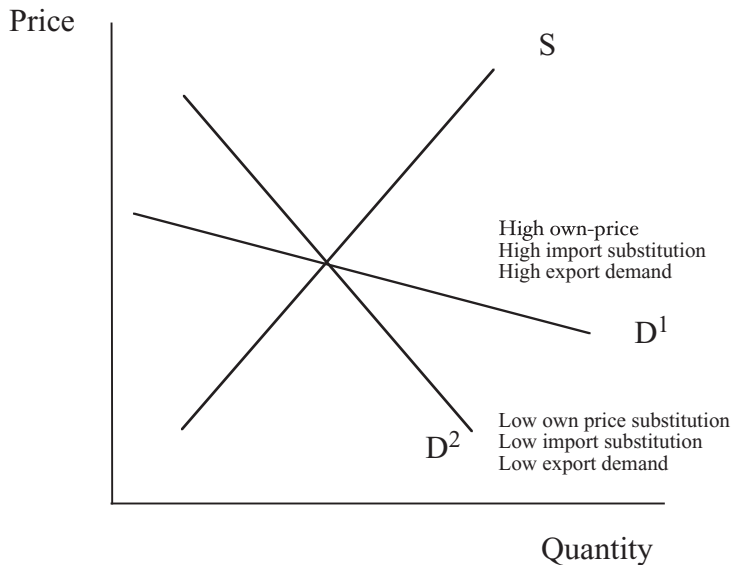


Figure 2.2. Effects of demand elasticity parameters on the slope of the demand curve

CGE models generally assume that the **Law of Demand** holds; that is, an increase in the price of a good causes the quantity demanded to fall, so the own-price elasticity of demand is negative. When consumer demand is price sensitive, the own-price parameter is large in absolute terms. The demand curve for good  $i$  is relatively elastic, such as curve  $D^1$  in Figure 2.2, where the axes describe the quantity demanded of good  $i$  and its consumer price. When the own-price elasticity parameter is low, then the demand curve becomes less elastic, such as  $D^2$ .

A cross-price elasticity of demand measures the percent change in demand for a good  $i$  given a percent change in the consumer price of another good,  $j$ :

$$\frac{\% \text{ change } Q_i}{\% \text{ change } P_j}$$

The cross-price elasticity is positive when the two goods are easily substituted (like brown sugar and dark brown sugar), negative when the goods are **complements** (like left shoes and right shoes), and zero when demand for one good is **independent** of the price of the other good. In Figure 2.2, a change in the price of a **substitute** good can be shown as a shift in the demand curve for consumption of good  $i$ . The larger the positive cross-price substitution elasticity parameter, the larger the rightward (leftward) shift in the demand curve as the price of the substitute good falls (rises).

**Import Substitution.** This elasticity parameter,  $\sigma_M$ , relates to consumer demand for imports. It describes consumers' willingness to shift between imported (QM) and domestically produced varieties in their consumption of commodity  $i$  as the relative price of domestic (PD) to imported (PM) varieties changes. For example, it describes a consumer's willingness to shift from an imported car to a domestic model when the relative price of the import rises.

The parameter is calculated as the percent change in the quantity ratio given a percent change in their inverse price ratio:

$$\frac{\% \text{ change } \frac{QM_i}{QD_i}}{\% \text{ change } \frac{PD_i}{PM_i}}$$

Its value may range between zero and infinity. For example, if the substitution elasticity is two, then a 1 percent increase in the price of the domestic relative to the imported variety will lead to a 2 percent increase in the ratio of the import relative to the domestic quantity for a given consumption level. Assume that the axes in Figure 2.2 describe quantities of imports and the import price. When the import substitution parameter has a low value, import demand is inelastic, shown as  $D^2$  in Figure 2.2. As the parameter value increases, the import demand curve becomes more elastic, such as  $D^1$ .

**Export Demand Elasticity.** Single-country CGE models describe the rest of the world's demand for a country's exports as a function of its export price. Usually, when its export price rises relative to the world price,  $\theta$ , the country's foreign sales will fall. An export demand elasticity parameter is defined for each exported commodity  $i$  in the CGE model. It describes the percent change in the share of country's exports, QE, in world trade, QW, given a percent change in the ratio between the average world price, PXW, and the exporter's price:

$$\frac{\% \text{ change } \frac{QE_i}{QW_i}}{\% \text{ change } \frac{PXW_i}{PE_i}}$$

An increase in the exporter's price relative to the world price causes its export quantity and world market share to decline. The larger the elasticity parameter value, the larger the decline in its exports as the country's relative export price increases. The export demand elasticity ranges from zero to

infinity. A parameter value that approaches infinity describes a small country, and a parameter value near zero describes a very large country in world markets. In Figure 2.2, if we assume that the axes represent the quantity of a country's export good and its export price, then a high value of the export demand elasticity parameter is shown as the very elastic demand curve,  $D^1$ , for a small country's exports. A low parameter value is described by the relatively inelastic export demand curve of a large country,  $D^2$ .

### *Shift and Share Parameters*

**Shift parameters** and **share parameters** are exogenous values used in the supply and demand functions in a CGE model. As an example, consider the shift and share parameters in a Cobb-Douglas production function. This function is used in many CGE models to describe the production technology of an industry:

$$QO = A(K^\alpha L^{1-\alpha})$$

where  $QO$  is the output quantity. Parameter  $A$  is a shift parameter whose value is greater than zero and that describes the productivity of capital,  $K$ , and labor,  $L$ , in the production process. Parameter  $\alpha$  is a share parameter, ranging between zero and one. It measures the share of  $K$  in the total income received by labor and capital from their employment in the industry. Labor's income share parameter is  $1 - \alpha$ .

Parameter  $A$  is called a shift parameter because a change in its value causes the industry supply curve to shift to the right or the left. For example, if the shift parameter increases in value, perhaps from  $A = 5$  to  $A = 10$ , then factors are more productive, and the same quantity of  $K$  and  $L$  can produce a larger quantity of output. This change in the shift parameter is described by the rightward shift in the supply curve from  $S^1$  to  $S^2$  in Figure 2.3. CGE modelers can change the value of the shift parameter in the production function as a model experiment to describe changes in the productivity of one or more inputs.

Other share parameters in the production and consumption functions in a CGE model include the shares of commodities in consumers' total consumption; shares of imported and domestic varieties in the demand for commodities; and the shares of domestic and export sales in total industry output.

## **Model Calibration**

The **model calibration** procedure calculates the shift and share parameters used in the production and utility functions in the CGE model so that the

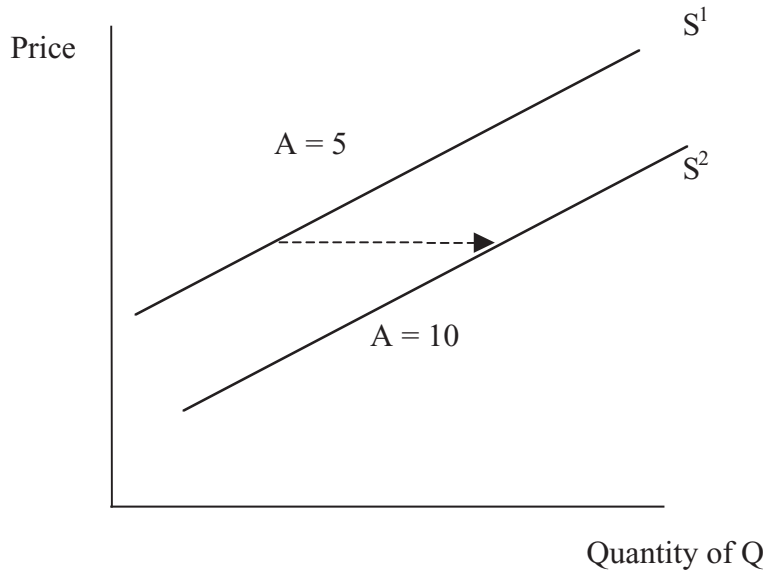


Figure 2.3. Effect of an increase in the shift parameter value on the supply curve

solutions to the equation replicate the initial equilibrium as reported in the base data. The calibrated model solution is then used as the benchmark equilibrium, against which the results of model experiments are compared. The inputs to the calibration process are the CGE model database, which describes an economy in an initial equilibrium; the model's production or utility functions (such as a Cobb-Douglas production function), and the elasticity parameters assumed by the modeler.

As an example of the calibration procedure, let's again consider a CGE model that assumes this Cobb-Douglas production function:<sup>2</sup>

$$QO = A (K^\alpha L^{1-\alpha})$$

Suppose the model database reports that the industry employs thirty units of capital,  $K$ , and seventy units of labor,  $L$ , with per unit wages and rents of one dollar each, at a total factor cost of \$100. The base-year output quantity,  $QO$ , is 109 units. The model calibration process first calculates the share parameters  $\alpha$  and  $1 - \alpha$ . The share of capital,  $\alpha$ , in total factor payments of \$100 is 0.3, and the income share of labor,  $1 - \alpha$ , is 0.7. With these share parameters, and the given values of  $QO$ ,  $K$ , and  $L$  from the model database, the calibration process then solves for  $A$ :

$$109 = A (30^{.3} 70^{.7})$$

<sup>2</sup> Note that the modeler does not need to specify any elasticities in the case of a Cobb-Douglas production function because these are implied by the properties of the function: the own-price elasticity of demand for each factor is negative one, and the cross-price elasticities of demand for capital and labor are zero.

whose value is two. You can also verify for yourself that the production function, with these calibrated shift and share parameters, reproduces the base year output of 109.

The calibrated shift and share parameters used in the model's production and utility functions always remain at their initial values, even though actual shares may later change as the result of model experiments. Modelers sometimes change the shift parameters used in production functions to analyze the effects of productivity shocks. Sometimes, too, modelers change calibrated share parameters as an experiment. Two interesting examples of this approach are Kuiper and van Tongeren (2006), summarized in Text Box 2.2, who change the import share parameters; and Nielson, Thierfelder, and Robinson (2001), summarized in Text Box 4.1, who change consumer budget share parameters.

### Equations

CGE models have behavioral and identity equations. **Behavioral equations** describe the economic behavior of producers, consumers, and other agents in the model based on microeconomic theory. You may recognize some of the behavioral supply and demand equations in the model from your economics coursework. For example, CGE models include a behavioral equation that describes how firms minimize the costs of inputs to produce a specific level of output, given input and output prices and subject to the technological constraints of their production process.

CGE models also include a utility function that describes the combinations of goods that consumers prefer. The choice of utility function, for example, Cobb-Douglas or Stone-Geary, depends on which best describes consumer preferences in the country under study. Given consumers' preferences, a behavioral equation describes how they choose quantities of goods that maximize their utility subject to the prices of goods and their budget. Additional behavioral equations in the CGE model explain the demand for imports and the supply of exports.

**Identity equations** define a variable as a mathematical function (sum, product, etc.) of other variables. Identity equations therefore hold true by definition. If the value of any one of the variables in the identity equation changes, then one or more of the other variables must also change in order to maintain the equivalence. For some equations, model closure is the choice made by the modeler as to which variable adjusts to maintain the identity.

Identity equations act as constraints in a CGE model to ensure that the model solves for a market-clearing set of prices at which quantities supplied and demanded are equal. The equations are similar to the market-clearing constraint in our bicycle model of Chapter 1, that  $Q_d = Q_s$ .

**Text Box 2.2. The Small Share Problem and the Armington Import Aggregation Function**

*“An Empirical Approach to the Small Initial Trade Share Problem in General Equilibrium Models.”* Kuiper and van Tongeren (2006).

**What is the research question?** CGE-based analyses of trade liberalization describe the effects of eliminating trade barriers on import quantities. The majority of these analyses assume an Armington import aggregation function. The “small share problem” is due to the scaling effect of the share parameter  $\alpha$  in the Armington import demand equation:

$$\frac{M}{Q} = \alpha \frac{P_M}{P_Q}^{-\rho}$$

where  $M$  is the import,  $Q$  is the composite commodity (the sum of imported and domestically produced varieties),  $P_M$  and  $P_Q$  are the prices of the import and the composite commodity, respectively, and parameter  $\rho$  is related to the import substitution elasticity parameter. Parameter  $\alpha$  is the initial quantity share of imports in the consumption of commodity  $Q$ . Its value is calculated during model calibration and does not change following a model experiment. Notice that if the initial import share is small, then even a large change in the relative price of the import, or a large increase in the size of the import substitution parameter, can result only in small changes in the import share of consumption. This scaling effect may lead to unrealistically small import quantity results in trade liberalization simulations that cause the import price to fall. Could a gravity model provide an empirical basis for changing the share parameters as part of trade liberalization experiment?

**What is the model innovation?** The researchers develop a gravity model to identify the role of trade barriers in bilateral trade flows. They use the gravity model to simulate trade liberalization and estimate changes in bilateral trade shares. Then, they modify their GTAP model to adjust the calibrated trade shares to those of the gravity model results as part of a trade liberalization experiment.

**What is the model experiment?** The authors eliminate global import tariffs and export subsidies (1) with and (2) without changes in import share parameters.

**What are the key findings?** The adjustments shift bilateral trade flows, causing some regions to gain larger shares of the world market following trade reform and other regions to lose market share, compared to a standard CGE model analysis. Adjusting the import share parameters does not change the size of global welfare effects by very much.

An example of a market-clearing identity equation from the GTAP model is this expression:

$$qo_f = \sum_j SHR_{f,j} qfe_{f,j}$$

The equation states that the percent change in the national supply,  $qo$ , of the mobile factor,  $f$ , must equal the weighted sum of percent changes in demand,  $qfe$ , for factor  $f$  by industry  $j$ , where  $SHR_{f,j}$  is each industry's share in national employment of factor  $f$ . That is, it imposes the constraint that aggregate supply must equal aggregate demand for each factor  $f$ .

### Macroclosure

CGE models include an identity equation that imposes the constraint that total savings is equal to total investment. Some multicountry models impose this constraint at the global level. Other single and multicountry models impose it at the national level. **Macroclosure** describes the modeler's decision about which of the two macroeconomic variables – savings or investment – will adjust to maintain the identity that savings equals investment.

Standard, static CGE models rely on an identity equation to model savings and investment because these behaviors are determined largely by macroeconomic forces, such as monetary policy and expectations about future economic conditions, that are outside the scope of a real CGE model.<sup>3</sup> Nevertheless, the models must account for them in some way because savings and investment are part of the circular flow of income and spending, with effects on the real economy. Investment affects the production side of the economy because investors buy capital equipment that is produced by industries. Savings affects the demand side of the economy because households and the government allocate some share of their disposable income to savings, which affects their demand for goods and services.

CGE models may differ in whether savings or investment is assumed to adjust to maintain the savings-investment identity. In some models, such as the default closure in the GTAP model, the savings rate (the percentage of income that is saved) is assumed to be exogenous and constant, so the quantity of savings changes whenever income changes. Investment spending then changes to accommodate the change in supply of savings. A model with this closure is called savings-driven, because changes in savings drive changes in investment. An advantage of this closure is that a nation's savings rate remains the same as the rate observed in the base year. This is appealing if we think that base year savings rates reveal the subjective preferences of a country's households and government.

In other CGE models, the aggregate value of investment is fixed at its initial level, and savings rates are assumed to adjust until savings are equal to

<sup>3</sup> For a more detailed discussion of macroclosure and savings and investment, see Lofgren, et al. (2002), Hertel and Tsigas (1997), Robinson (1991), and Dewatripont and Michel (1987). Shoven and Whalley (1984) discuss the effect of closure in predetermining model results.



investment spending. A model with this closure is called investment-driven. This closure is well suited for the study of countries in which governments use policies that influence savings rates to achieve targeted investment levels.

To demonstrate how this macroclosure decision can matter, assume that a country's income increases. In a savings-driven model, households save a fixed share of their income, so income growth will cause savings to increase and therefore investment spending to rise. In an investment-driven model, investment is fixed, so the supply of savings is also fixed. In this case, consumers will spend, rather than save their additional income. Because households and investors are likely to prefer different types of goods, the two alternative closures will lead to a different commodity composition of demand. The savings-driven model is likely to result in an increase in production of machinery and equipment, which is what investors prefer to buy. An investment-driven model is likely to result in an increased demand for consumer goods, like groceries, apparel, and consumer electronics.

Some CGE models, such as those in the Dervis, de Melo, and Robinson (1982) tradition, specify additional macroclosure rules to describe the current account balance and the government fiscal balance. These macroclosure decisions address components of national savings. The current account closure describes whether foreign savings inflows (the current account) are exogenous and the exchange rate is endogenous, or vice versa. An exogenous current account closure fixes the supply of foreign savings (the current account deficit or surplus) at its initial level and the exchange rate adjusts to maintain it, whereas a fixed exchange rate makes foreign savings endogenous. The government budget closure describes whether government savings (the federal deficit) is endogenous and government spending is fixed, or vice versa.

Modelers choose macroclosure rules that best describe the economy under study. The rules also offer researchers the flexibility to explore macroeconomic policy shocks in a CGE model, such as currency devaluation or pay-go federal budget rules. See, for example, Cattaneo, Hinojosa-Ojeda, and Robinson's (1999) methodical study of the effects of alternative macroeconomic policies in Costa Rica, which are simulated by running the same policy shock with different macroeconomic closures (Text Box 2.3).

### **Normalizing Prices**

The value of output of good X is the product of its prices times its quantity. For example, the value of production of apples is the product of their price (say, \$1.50 each) and the quantity of apples (ten), which is \$15. The database of a CGE model comprises only value flows. It reports the value of output of each good in the model, but not their quantities or prices. It reports the

**Text Box 2.3. Macro Closure and Structural Adjustment in Costa Rica**  
***“Costa Rica Trade Liberalization, Fiscal Imbalances, and Macroeconomic Policy: A Computable General Equilibrium Model.”*** (Cattaneo, Hinojosa-Ojeda, and Robinson, 1999).

***What is the research question?*** In the 1980s, Costa Rica signed structural adjustment agreements with the World Bank that included trade liberalization, elimination of producer and consumer subsidies, and other policy reforms. How might the broader reform program that Costa Rica must carry out temper the gains from the trade liberalization component?

***What is the CGE model innovation?*** The authors develop a multihousehold SAM for Costa Rica for 1991. Using the IFPRI standard CGE model, they vary macroclosure rules to describe alternative ways to implement structural adjustment commitments.

***What is the experiment?*** A single trade liberalization experiment, that removes all import tariffs and export taxes, is carried out under two alternative foreign savings closures: fixed foreign savings and an endogenous exchange rate versus a fixed exchange rate and endogenous foreign savings. Both scenarios are also conducted with three alternative closures for government savings: loss of trade tax revenue causes the government to run a deficit; and the government budget balance is fixed with trade tax revenue replaced by a corporate income tax or by a retail sales tax.

***What are the key findings?*** Trade liberalization generates efficiency gains for the economy as a whole, and changes in the distribution of income across households are small. However, there are trade-offs that the government must face to maximize these potential gains. The scenarios offer a blueprint for government policy, recommending reduced government expenditures and higher retail sales taxes to offset the significant loss of trade tax revenues.

value of factor inputs, such as labor, but not the number of workers who are employed or their wage rates. However, you will see that a CGE model reports the results of model experiments for both quantities and prices. For example, a new production subsidy may increase the quantity of X that is produced by 5 percent but cause its price to fall by 2 percent. How does a CGE model develop price and quantity data if its database contains only value data?

CGE models translate value data into price and quantity data by ***normalizing*** prices. This procedure converts most of the initial, or base, prices in the model into \$1 or one unit of the currency used in the model.<sup>4</sup>

<sup>4</sup> This practice is attributed to Arnold Harberger (1964), who normalized the prices and quantities of factors in a general equilibrium analysis of the U.S. income tax.

Table 2.1. *Normalizing the Price and Quantity of Apples in a CGE Model*

	Base Values for Apples			50% Increase in Apple Quantity		
	Price	Quantity	Value	Price	Quantity	Value
Actual market data	.5	6	3	.5	9	4.5
Normalized data	1	3	3	1	4.5	4.5

Quantities of goods and of factors of production (e.g., labor and capital) are then interpreted as the quantity per \$1 or unit of currency.

Let's use a simple example of apples to show how prices are normalized. According to the actual market data reported in Table 2.1, apples cost fifty cents each and the initial quantity demanded is six, so the value of apples sold in the market is \$3. In a CGE model database, we know only the value of apples sales, which is \$3. By normalizing prices, we describe the apple price as \$1 and the quantity as the unit quantity per dollar, which is three. That is, each quantity unit of apples in the model is two actual apples.

Normalizing prices does not affect our results. To illustrate this point, consider what happens if the sales quantity of apples increases by 50 percent. If we use actual market data, then the value of sales increases to \$4.50 (nine apples times fifty cents). When we use the normalized data, we get the same answer. The apple quantity rises 50 percent, from three to 4.5 units of apples, and 4.5 apples times \$1 equals \$4.50.

The practice of normalizing data considerably reduces the information needed to build a CGE model database without losing the capability of the CGE model to generate results for prices, quantities, and values. This approach also means that most, but not all, prices in a CGE model have an initial value of one. Some prices in the CGE model are adjusted to include taxes or subsidies and these initial prices do not equal one. An example is the domestic consumer price of imports. If the normalized world import price is \$1 and the import tariff is 10 percent, then the initial domestic consumer price of imports in the CGE model is \$1.10.

### Price Linkages

If you purchase a shirt in China for \$14 that is imported from Brazil, you probably realize that the Brazilian company that manufactured the shirt does not receive \$14 for it. The difference between the price that you pay in China and that received by the producer includes any export taxes that the Brazilian firm paid to its government, the costs of transporting the shirt between Brazil and China, and any import tariffs and sales taxes that you

Table 2.2. *Prices in a Multicountry CGE Model*

Type of Price	Defined Over Sets	Definition
Producer price ( <i>ps</i> )	<i>i, r</i>	Cost of production, includes production tax or subsidy
Consumer price ( <i>pp</i> )	<i>i, r</i>	Producer price plus sales tax (domestic variety) and bilateral <i>cif</i> import price plus import tariff and sales tax (import variety)
Bilateral import price ( <i>pcif</i> )	<i>i, r, s</i>	Exporter's bilateral export price plus <i>cif</i> trade margins, excluding tariff.
Bilateral export price ( <i>pfob</i> )	<i>i, r, s</i>	Exporter's domestic producer price plus export tax
World import price ( <i>pim</i> )	<i>i, r</i>	Trade-weighted sum of bilateral <i>cif</i> import prices in country <i>r</i>
World export price ( <i>piw</i> )	<i>i, r</i>	Trade-weighted sum of bilateral <i>fob</i> export prices in country <i>r</i>
Global price ( <i>pxw</i> )	<i>i</i>	Trade-weighted sum of all countries' bilateral export prices

Notes: *i* is the set of commodities, *r* is the exporting country and *s* is the importing country. The corresponding variable names used in the GTAP model (in percent change terms) are listed in the first column, in parentheses.

paid to your own (Chinese) government. We omit discussion of the costs of wholesale and retail services incurred in bringing the shirt from the port to your department store.

A CGE model reports several prices for a single commodity, such as a shirt, because it tracks goods and prices all along the supply chain between producers and consumers (Table 2.2). The **producer price** is the sales price received by the producer. In a competitive market, it is equal to the cost of production, inclusive of any taxes or subsidies entailed in the production process. The Brazilian shirt, for example, may cost \$8 to manufacture, so the Brazilian producer price is \$8. Some Brazilian shirts are sold in the domestic market. In this case, Brazilian consumers pay \$8 plus any sales tax that the Brazilian government imposes. If we assume that the sales tax is \$2 per shirt, then the **consumer price** for a Brazilian-made shirt in Brazil is \$10.

Figure 2.4 describes prices for goods that are traded internationally. In this example, Brazil's **bilateral *fob* export price** to China is a “free on board” value. An ***fob*** value is the value of the export good when placed on board the ship at the Brazilian port of departure. It is the producer price plus any export taxes or subsidies on its sales to China. In Figure 2.4, the Brazilian

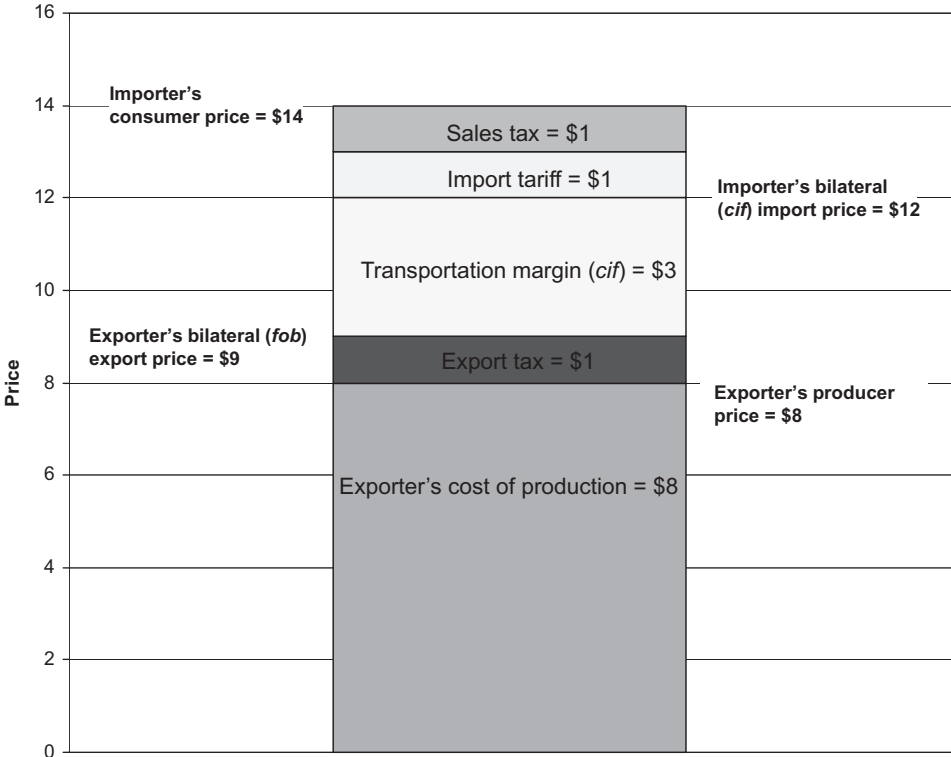


Figure 2.4. Price linkages in international trade

producer pays \$1 per shirt in export taxes on sales to China, so its bilateral *fob* export price is \$9 per shirt.

Imports incur insurance and freight charges, also called trade margin costs, to move goods from the exporter's port to that of the importer. Suppose that the trade margin cost for shipping the shirt to China totals \$3. China's ***bilateral cif import price*** (i.e., cost plus insurance and freight) for a shirt from Brazil, is therefore \$12. The *consumer price* of the Brazilian shirt in China is its bilateral *cif* import price plus any import tariffs and sales taxes imposed by the Chinese government. Assuming that China has a tariff of \$1 on the Brazilian shirt and imposes a \$1 sales tax on its consumers, then the consumer price of the Brazilian shirt in China totals \$14. Notice that the consumer price of the Brazilian shirt is higher in China (\$14) than in Brazil (\$10); the difference is because of trade margin costs, import tariffs, and the difference in sales taxes in the two countries.

Multi-country CGE models with bilateral trade flows report bilateral *fob* export and *cif* import prices for every commodity traded between every pair of trading partners in the world. Tracking bilateral export and import prices allows the modeler to take into account that taxes, tariffs, and trade margin costs may diverge among trade partners. It also allows the modeler to take into account that many products are differentiated by country of origin. For

example, French consumers may think that oranges from Spain are different than oranges from Israel. As a result, France may import oranges from both Israel and Spain. There also can also be two-way trade in the same product. For example, Spain may export oranges to Israel and Israel may export oranges to Spain.

A country's **world export price** and **world import price** are the trade-weighted sum of its bilateral *fob* export and bilateral *cif* import prices for each good. Brazil's world export price of shirts, for example, is the trade-weighted sum of its bilateral *fob* export prices of shirts sold to China, the United States, and all other countries to which it exports. A **trade weight** is the share of a destination country in an exporter's total export quantity, or the share of a source country in an importer's total import quantity.

As an example, let's calculate the world export price of Brazil's shirts. Suppose China accounts for 75 percent of Brazil's total quantity of shirt exports, at a Brazilian bilateral export price of \$10 per shirt (this includes Brazil's export tax). Suppose the United States accounts for the remaining 25 percent of shirt exports, but there is no Brazilian export tax on its sales to the United States. Brazil's bilateral export price to the United States is therefore only \$8 per shirt. Brazil's world export price for shirts is calculated as:

China	$.75 * \$10 = \$7.50$
United States	$.25 * \$8 = \$2.00$
World price	$\$7.50 + \$2.00 = \$9.50$

Brazil's world import price for each good would be calculated in a similar fashion; this time the trade weights would be the quantity shares of each source country in Brazil's total imports of a commodity.

Finally, the **global price** of a good, such as shirts, is the trade-weighted sum of all of the bilateral, *fob* export prices of all countries in the world. The weights are the shares of each bilateral trade flow in the total quantity of global trade in that good.

## Numeraire

A CGE model describes only relative prices. To express all prices in relative terms, the modeler chooses one price variable in the CGE model to remain fixed at its initial level. This price serves as the model's **numeraire**, a benchmark of value against which the changes in all other prices can be measured (see Text Box 2.4).

As an example, consider a model with three goods: agriculture, services, and manufacturing. The producer prices of manufactured goods and services could be measured in terms of – or relative to – the price of the agricultural good, which we have selected to be the numeraire. Initially, the producer

**Text Box 2.4. The Numeraire and Walras' Law**

CGE modelers can be more confident that their model has a feasible and unique solution if it is “square;” that is, if the number of variables and equations in the model are equal. When we fix one price to serve as the numeraire, we are dropping one variable from our model. Are we therefore causing the number of variables to be one fewer than the number of equations? The answer is no, and it rests on Walras' Law.

Leon Walras was a 19th century economist who studied the interconnectedness among all markets in an economy. He focused in particular on the problem of whether a set of prices exists at which the quantity supplied is equal to the quantity demanded in every market simultaneously. His theoretical, general equilibrium model was much like the standard, “Walrasian” CGE model that we are studying. They share the features that: (1) producers are profit-maximizers who sell their goods in perfectly competitive markets at zero economic profit; (2) consumers are utility-maximizers who spend all of the income they receive from their production and sale of goods; and (3) prices adjust until demand for each commodity is equal to its supply. Based on these assumptions and market-clearing constraints, Walras' Law states that, for the economy as a whole, the aggregate value of excess supply in the economy must be matched by the aggregate value of excess demand. This is essentially because producers plan to sell that value of goods that will enable them to afford their desired purchases. A shortfall in their actual sales (excess supply) therefore results in an equal shortfall between their actual and desired consumption (excess demand).

An implication of Walras' Law is that equilibrium in the last market follows from the supply-demand balance in all other markets. As a result, the equations in his model were not all independent. One equation was redundant and had to be dropped – but this meant his model had one more variable than the number of equations. Walras' solution was to fix one price in the model to serve as numeraire, making his model “square” once again. He could now solve for the market-clearing set of relative prices.

To make their models square, CGE modelers, too, usually drop one equation and fix one price variable to serve as numeraire. Any equation can be dropped without influencing results if the model is homogenous of degree zero in prices (as they usually are). In practice, modelers usually omit the macroeconomic market-clearing equation that defines aggregate savings ( $S$ ) to be equal to aggregate investment ( $I$ ). As an alternative, some modelers fix a numeraire but keep the redundant equation and add an additional variable called “Walras,” i.e.,  $S = I + \text{Walras}$ . If all markets in the CGE model are in equilibrium, then the Walras variable's value will equal zero. Such a variable can be useful to the modeler as a way to check that all markets are in equilibrium in the base data and model solutions.



prices of all three goods are \$1, because they have been normalized. Let's assume that after a model shock, the producer price of the numeraire (agriculture) remains at \$1 (it must, because it is the numeraire) but the producer price of the manufactured good has doubled; the relative producer price of manufactures is now  $2/1 = 2$ .

Because the exchange ratios of all goods are specified relative to the numeraire, you can also compare the prices of non-numeraire goods – in this case, the price of manufactured goods relative to services. Assume that the price of services increased only 20 percent, then its relative price in terms of agriculture is  $1.20/1 = 1.20$ . The price of services (1.20) has fallen relative to manufacturing (2).

You can choose any price in the CGE model to be the numeraire. Your choice of numeraire has no impact on real, or quantity, variables that result from an experiment. Some modelers define the numeraire to be the consumer price index (CPI), which is calculated as the weighted sum of initial consumer prices, where the weights are each good's base budget share in the consumption basket. Other modelers select a producer price index or an index of the prices of domestically produced, nontraded goods. In the GTAP model, the default numeraire is an index of global wages and rents for labor, capital, and other factors.

### Structure of a CGE Model

The programming code of a CGE model can be lengthy, so it is a common practice to organize it into a small number of blocks that accomplish different tasks.<sup>5</sup> Although this organization can vary among models, the structure of most CGE models and the steps required to run the model and an experiment are similar to those described in Figure 2.5.

A CGE model often opens with one or more blocks of code whose task is to introduce and define each of the sets, exogenous and endogenous variables, and exogenous parameters used in the model. The modeler must define each of these elements in the model code before the model can recognize and use them.

For example, model code may define an endogenous variable, the quantity of imports of commodity  $i$ , as:

$$QM_i = \text{imports of commodity } i$$

<sup>5</sup> Models tend to get more complex as their analytical capabilities are enhanced. Two examples of relatively simple CGE models are the Cameroon model, developed by Condon, et al. (1987), and the ERS/USDA model developed by Robinson, et al. (1990). Both can be downloaded from the GAMS model library at [www.gams.com](http://www.gams.com). Students can run the models by downloading a demonstration version of GAMS software.



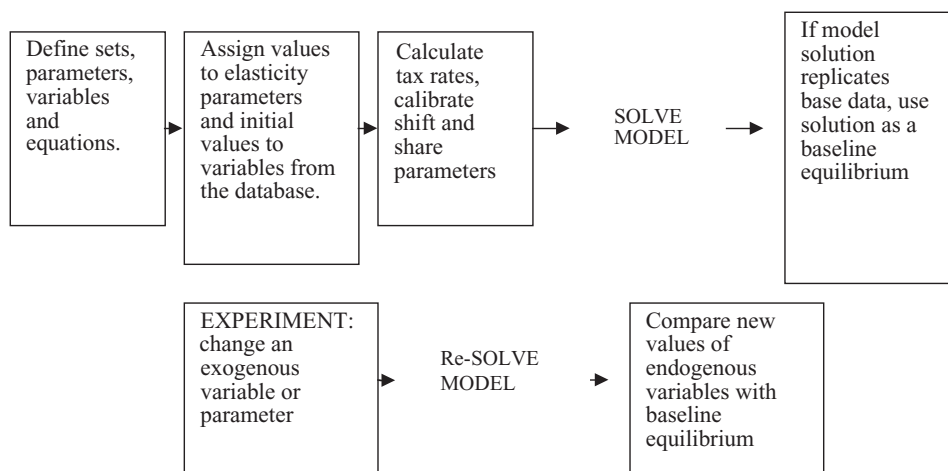


Figure 2.5. Structure of a CGE model and experiment

Once the model code defines the variable  $QM_i$ , all subsequent model code, such as equations, can recognize it. If an equation or other types of model commands refer to a set, parameter, or variable that has not yet been defined, the model will fail to solve.

Next, a CGE model has programming code whose task is to assign initial values to variables from the model database and to define elasticity parameter values. For instance, now that  $QM_c$  has been defined, it can be assigned its value from the database, such as:

$$QM_i = 552$$

Once sets, parameters, and variables have been defined and values have been assigned, the model can calculate tax rates and calibrate the shift and share parameters. CGE model equations are now numerical equations, similar to our bicycle model, which can be solved to find the equilibrium values of prices and quantities. This calibrated model solution should exactly replicate the original database. For example, we expect the model's solution value for the import quantity of good  $i$ ,  $QM_i$ , to be 552, which was its initial value in the database. If the CGE model can reproduce the initial data, then the economist can be confident that the model is capable of explaining the prices and quantities observed in the initial equilibrium. The calibrated model solution then becomes the baseline against which experiment results are compared.

Now the modeler is ready to carry out an experiment. An experiment involves changing the value of at least one of the exogenous parameters or variables, such as the import tariff on agricultural imports. This change in the economy – a “shock” – is a controlled experiment in which the only change in the economy is the value of the exogenous parameter or variable, as specified in the experiment. The modeler re-solves the model, which recalculates new equilibrium values for all endogenous variables. The new solution values for

the endogenous variables are compared with the baseline solution values from the calibrated model. The resulting changes in variables' values, such as a 5 percent decline in the quantity of imports compared to the base value, describe the effects of the economic shock on the economy.

### Summary

In this chapter, we described the elements of standard CGE models, focusing only on their mechanics and leaving the study of their economic behavior for Chapters 4–8. For many students, this chapter can serve as a practical reference guide that you can return to as your modeling skills progress and questions arise.

CGE models of all types share many common features. They include behavioral equations that describe the behavior of producers and consumers, identity equations that impose market-clearing constraints, and macroclosure rules that govern the savings and investment balance. CGE models follow the convention of normalizing prices so that the value data in the model database can be used to describe both prices and quantities. CGE models report several prices for a single commodity because the models track prices at all points in the supply chain that links producers and consumers. All prices in the model are relative and expressed in terms of the numeraire. In most CGE models, the program code first defines the names of the sets, endogenous and exogenous variables, and exogenous parameters used in its equations. Next, the model assigns numerical values from the database to all variables and defines elasticity parameter values. Blocks of equations then describe the model's economic behavior. The calibration procedure utilizes model equations, the initial database, and elasticities to solve for shift and share parameter values that yield a model solution that replicates the initial base data. This calibrated model solution becomes the baseline equilibrium against which the results of experiments are compared.

### Key Terms

Behavioral equation  
Bilateral *fob* export price  
Bilateral *cif* import price  
Calibration  
Complement  
Consumer price  
Cost, insurance, freight (*cif*)  
Cross-price elasticity of demand  
Elasticity  
Endogenous variables  
Exogenous parameters

Exogenous variables  
 Export demand elasticity  
 Export transformation elasticity  
 Factor mobility elasticity  
 Factor substitution elasticity  
 Free on board (*fob*)  
 Global price  
 Identity equation  
 Import substitution elasticity  
 Income elasticity of demand  
 Independent good  
 Law of Demand  
 Macroclosure  
 Model closure  
 Normal good  
 Normalized price  
 Numeraire  
 Own-price elasticity  
 Producer price  
 Set  
 Substitute  
 Trade weight  
 World export price  
 World import price

## PRACTICE AND REVIEW

1. Assume a set of consumer goods  $i$  with three elements: agriculture, manufacturing, and services. If  $P$  is the consumer price, use set notation to express these variables:

Consumer price for set  $i$  \_\_\_\_\_  
 Consumer price of manufactures \_\_\_\_\_

2. If  $Q_M$  is import quantity, define  $Q_M$  (“AGR”, “USA”, “Brazil”):  
 \_\_\_\_\_
3. Review the role of supply elasticities in a demand shock.
  - a. Draw a graph of the supply and demand for one good. Label the supply curve  $S^1$  and the demand curve  $D^1$ . Label the axes and the initial equilibrium.
  - b. Draw a second supply curve that shows the industry with a more elastic supply, that has the same equilibrium as  $S^1$  and  $D^1$ . Label the second supply curve  $S^2$ .
  - c. Assume that an income tax cut increases disposable income and consumer demand. Draw a new demand curve, labeled  $D^2$ , and label the two new equilibria along  $S^1$  and  $S^2$ .
  - d. In a paragraph, (1) explain the difference between the two market equilibria and (2) identify the elasticity parameters in a CGE model that can cause  $S^2$  to be more elastic than  $S^1$ .

Table 2.3. *Normalized Prices and Quantities of Apples*

	Base Values			50% Change in Quantity			
	Price	Quantity	Value	Price	Quantity	Value	% Change in Value
Actual	4	6		4	9		
Normalized	1			1			

4. Review the role of demand elasticities in a supply shock.
  - a. Draw a graph of the supply and demand for one good. Label the supply curve  $S^1$  and the demand curve  $D^1$ . Label the axes and the initial equilibrium.
  - b. Draw a second demand curve that shows the consumer with a less elastic demand curve, that has the same equilibrium as  $S^1$  and  $D^1$ . Label it  $D^2$ .
  - c. Assume a supply shock, such as favorable weather, that increases the supply of a good. Draw the new supply curve, labeled  $S^2$ , and label the two new equilibria along  $D^1$  and  $D^2$ .
  - d. In a paragraph, (1) explain the difference between the two market equilibria and (2) identify the elasticity parameters in a CGE model that can cause  $D^2$  to be more elastic than  $D^1$ .
5. Normalize prices.  
 Assume that the apple sales quantity has increased by 50 percent. Calculate the percent change in the value of apple sales in the first row of Table 2.3. Next, normalize apple prices and quantities and calculate the percent change in value of sales. Demonstrate that this result is the same for both actual and normalized data.
6. Calculate a trade-weighted world import price.  
 Use the data in Table 2.4 to calculate the U.S. world import price for corn:
  - a. Calculate the bilateral U.S. *cif*, import price for its corn imports from each trade partner.
  - b. Calculate the trade-weighted, bilateral *cif* import price U.S. imports from each trade partner.
  - c. Calculate the U.S. world import price as the sum of the trade-weighted bilateral *cif* import prices.

Table 2.4. *Calculating the U.S. World Import Price of Corn*

	France	Germany	South Africa
Exporter's market share of U.S. corn imports	50	25	25
Exporter bilateral (fob) export price	\$1.25	\$0.85	\$1.90
Trade margin	\$0.25	\$0.15	\$0.10
U.S. bilateral import price			
Trade-weighted bilateral import price			