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Estimating consumer demands across the development spectrum: maximum likelihood estimates of an implicit direct additivity model

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Abstract

This paper characterizes consumer demand patterns across the development spectrum using elasticity estimates from a demand system possessing non-linear Engel effects. Demands for six broadly defined goods are then projected under the assumption that per capita expenditure growth rates differ across the development spectrum. Such projections illustrate the extent to which non-linear income effects generate more plausible demand responses. Estimated marginal budget shares for food decline in a logistical manner and range from about 0.5 for Ethiopia to less than 0.05 for the USA. Engel elasticities for food range from 0.95 for Ethiopia to less than 0.1 for the USA. © 2002 Elsevier Science B.V. All rights reserved.

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In the next 30 years, differing rates of economic growth are projected to occur in different regions of the world. For example, between 1992 and 2020, real per capita GDP is projected to increase by 2.5% per annum in OECD countries versus 7% per annum in China (World Bank, 1997). While the causes of differential growth rates are important, the consequences have a number of far reaching implications. In particular, it is widely regarded that as real gross domestic product increases, so too does real expenditure.

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Expenditure growth is especially relevant in food and agricultural sectors, as Engel's Law implies that food's share of total expenditure declines as expenditure increases. A clear understanding of what may happen to food demand is necessary to appreciate the impact of economic growth on agriculturally related industries. Moreover, it is prudent that policy analysis designed to measure the economic impact of changes in the pattern of food demand be based on models that portray consumer behavior in a realistic manner.

In particular, across countries, consumer demand patterns reflect differences in prices, income and preferences. Given these differences, one would not expect to observe similar demand patterns across countries with widely varying per capita income levels, especially for necessity goods such as food and clothing. Moreover, as income grows, one would not expect changes in consumer demand patterns to be the same across countries of widely varying income levels. Nevertheless, some consumer demand models fail to account differences in consumer behavior across the development spectrum. For example, the Linear Expenditure System and Rotterdam models have constant marginal budget shares.[1] Consequently, in these systems an additional dollar of income in Ethiopia, for example, generates the same change in expenditure on a particular good as in the United States. For goods such as food, the constancy of marginal budget shares does not accord with observed data. Other functional forms possess marginal budget shares that vary linearly with expenditure (or the log of expenditure), but do not constrain budget shares to lie within the unit interval (e.g., Deaton and Muellbauer's, 1980 Almost Ideal Demand System, and Working's, 1943 model). All of the mentioned demand systems are in the class of rank two demand models (Lewbel, 1991, p.715). Rank two demand models offer limited Engel responses compared to rank three demand systems.[2] This puts into question the use of demand systems with less than rank three when considering demands when expenditure shows considerable variation. However, capturing realistic Engel responses often requires increasing the number of unknown parameters to be estimated, which is not necessarily desirable.

Rimmer and Powell (1992a,b, 1996) propose a new demand system that addresses the issue of flexible Engel responses, but with a parsimonious number of unknown parameters to estimate. This demand system, referred to as an implicit directly additive demand system (or AIDADS) has rank three (Rimmer and Powell, 1996, p.1614) and possesses fitted and marginal budget shares that vary non-linearly with real expenditure. Moreover, predicted budget shares from AIDADS are also restricted to the unit simplex by construction. Consequently, AIDADS is well suited to modeling demands where per capita income levels vary widely across the sample and to projecting consumer demands in situations were large expenditure growth may be encountered.

In this paper, the AIDADS model is estimated using a maximum likelihood framework and data from the 1985 International Comparisons Project. Demands for six broadly defined goods are then projected under the assumption that per capita income growth rates differ across the development spectrum. For food, such projections illustrate the extent to which

[1] The marginal budget share is "...the fraction of an additional dollar of expenditure spent on each good..." (Pollak and Wales, 1992, p.5).

[2] Gorman (1980) and Lewbel (1991) provide detailed discussion of demand system rank.

non-linear income effects generate more plausible demand responses. The paper is structured as follows. AIDADS is discussed in the next section. Implementation of AIDADS in a maximum likelihood framework is then presented. Since some parameters of AIDADS are bounded by inequality constraints, traditional sampling theory cannot be used for statistical inference. Bootstrap methods are used to construct standard deviations for the estimated parameters. The bootstrap methodology is discussed after the implementation framework. Results are then presented and discussed, followed by concluding comments.

1. An implicit, directly additive demand system

The simplest way to characterize AIDADS is as a generalization of the LES that allows for non-linear Engel curves while maintaining a parsimonious parameterization of consumer preferences. Following Hanoch (1975), Rimmer and Powell (1992a,b, 1996) assume the following defining equation for the AIDADS utility function:

$$\sum_{i=1}^{n} \frac{\alpha_i + \beta_i G(u)}{1 + G(u)} \ln\left(\frac{q_i - \gamma_i}{A\exp(u)}\right) = 1 \tag{1}$$

where q_i and u are consumption and utility levels, respectively, $q_i > \gamma_i \geq 0$, α_i, β_i, γ_i, and A are parameters, and $G(u) = \exp(u)$. Rimmer and Powell's choice of $G(u) = \exp(u)$ is rationalized as it:

> ...preserves the LES interpretation of γ as the subsistence bundle, and ensures that utility varies monotonically through the interval $(-\infty, \infty)$ as the consumption bundles...varies over (γ, ∞). (Rimmer and Powell, 1996, p.1615)

Thus, utility in AIDADS can be negative, moreover utility in the AIDADS model is a cardinal measure. Furthermore, the following parametric restrictions are used to ensure well-behaved demands $0 \leq \alpha_i, \beta_i \leq 1$ for all i, and $\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \beta_i = 1$. Solving the first order cost minimization conditions, AIDADS is written in budget share form as:

$$w_i = \frac{p_i \gamma_i}{y} + \frac{\alpha_i + \beta_i \exp(u)}{1 + \exp(u)} \left(1 - \frac{\boldsymbol{p}'\boldsymbol{\gamma}}{y}\right) \forall i \tag{2}$$

where w_i is the ith good's budget share, $\boldsymbol{p}$ is an n vector of prices with typical element $p_i \in \Re_{++}$, $\boldsymbol{\gamma}$ is an n vector with typical element γ_i, and y denotes expenditure. Note that if $\alpha_i = \beta_i$ for all i, then AIDADS simplifies to the LES.

AIDADS may look rather peculiar as it contains both expenditure and utility. In principal, one could substitute out for utility to obtain uncompensated demands that depend on prices and expenditure only. However, since the defining equation for AIDADS is implicit in utility, one cannot obtain an analytical solution for utility and make the necessary substitution. But, given the parameters of AIDADS, one could solve for utility numerically and make the needed substitution. Since utility depends on prices and expenditure, such numerical uncompensation results in a demand system that depends implicitly on prices and expenditure only. Viewed in this vein, the inclusion of utility in the AIDADS model is an intermediate step that introduces non-linearities in prices and expenditure.

When analyzing data spanning a wide range of expenditure levels, as we do in this paper, it is important that the chosen functional form satisfies regularity conditions of consumer theory. AIDADS satisfies these conditions over the price–expenditure space where consumers have strictly positive discretionary expenditure (i.e., $y - \boldsymbol{p}'\boldsymbol{\gamma} > 0$). In fact, in an unpublished research memorandum, McLaren et al. (1998) show that the AIDADS expenditure function is non-negative, continuous, homogenous of degree one in prices, non-decreasing in prices provided $\gamma_i \geq 0$ for all i, concave in prices, and non-decreasing in utility. Monotonicity of the expenditure function in utility is an important aspect of the regularity conditions associated with AIDADS. McLaren, Powell and Rimmer show the expenditure function is non-decreasing in utility if the term

$$\Xi = \left(\sum_{i}^{n} (\beta_i - \alpha_i)\ln(q_i - \gamma_i) - \frac{(1 + \exp(u))^2}{\exp(u)} \right)^{-1} \tag{3}$$

is strictly negative over the restricted price–expenditure space. In fact, the AIDADS expenditure function satisfies the regularity conditions over the restricted price–expenditure space if and only if $\Xi < 0$.

In all demand systems, the Engel elasticities, budget shares, and Allen–Uzawa partial elasticities of substitution are all that is required to compute price elasticities. Since elasticities can be used to characterize consumer demands along the development spectrum, it is important to know these values differ across countries with different per capita expenditure levels. The formula for the Engel elasticity is:

$$\eta_i = \frac{\Psi_i}{w_i} \tag{4}$$

where Ψ_i is the marginal budget share. For AIDADS, the marginal budget share is defined as:

$$\Psi_i = \frac{\alpha_i + \beta_i \exp(u)}{1 + \exp(u)} - (\beta_i - \alpha_i)\Xi. \tag{5}$$

Note that the $(\alpha_i + \beta_i \exp(u))/(1 + \exp(u))$ term in Eq. (5) behaves logistically in utility. By the monotonic mapping from expenditure to utility, $(\alpha_i + \beta_i \exp(u))/(1 + \exp(u))$ is also logistic in expenditure, moreover it is also contained between the values of α_i and β_i. Since Eq. (2) also behaves logistically in utility, the Engel elasticities will typically have non-linear properties with respect to expenditure changes.

A useful property of AIDADS is that estimates of α_i and β_i govern the asymptotic behavior of the marginal budget shares for AIDADS. If $\alpha_i > \beta_i$, the lower asymptote for the ith good's marginal budget share equals β_i, which is attained as expenditure grows without bound; the upper asymptote is α_i, which is attained when expenditure approaches discretionary expenditure from above. If $\alpha_i < \beta_i$, the ith good's marginal budget share has an upper asymptote of β_i, which is attained when expenditure grows without bound, while the lower asymptote of α_i is attained when expenditure approaches discretionary expenditure from above.

Another economically relevant measure obtained from AIDADS is the partial elasticity of substitution, given by:

$$\sigma_{ij} = \frac{(q_i - \gamma_i)(q_j - \gamma_j)}{q_i q_j}\left(\frac{y - \boldsymbol{p}'\gamma}{y}\right)\forall j \neq i \tag{6}$$

where σ_{ij} is the partial substitution elasticity and all other variables have been defined previously. Note that the own-partial substitution elasticity is recovered by the homogeneity property of demand. The limiting behavior of the cross partial elasticity of substitution is an important aspect of AIDADS. As expenditure approaches $\boldsymbol{p}'\gamma$ from above, σ_{ij} (for all $i \neq j$) approaches zero, in which case the substitution elasticities are consistent with Leontief preferences. As expenditure grows without bound from below, σ_{ij} (for all $i \neq j$) approaches unity, in which case the cross partial elasticity of substitution is consistent with Cobb–Douglas preferences. It is the implicit additive nature of AIDADS that allows for this variety of preference structures and for more general cases between the two specific cases.

While useful in itself, σ_{ij} is typically used to compute compensated and uncompensated price elasticities according to the following equations:

$$\eta'_{ij} = \sigma_{ij} w_j \tag{7}$$

$$\eta_{ij} = w_j(\sigma_{ij} - \eta_i) \tag{8}$$

where Eqs. (7) and (8) are the compensated and uncompensated price elasticity formulae, respectively. Given the additive nature of AIDADS preferences, compensated cross-price elasticities are expected to be positive, while uncompensated cross-price elasticities may be have positive or negative sign. Naturally, to compute all elasticities, estimates of AIDADS are needed. In the next section, the maximum likelihood framework used to estimate AIDADS is presented.

2. Estimation framework

Following Rimmer and Powell (1996) and Cranfield et al. (2000), AIDADS is estimated using maximum likelihood techniques. However, given the functional structure of AIDADS, in particular the fact that the unobservable level of utility is an argument in the demand function that cannot be eliminated, it is necessary to estimate utility with the other unknown parameters. While unconventional, it is important to recognize that estimation of any demand system satisfying integrability conditions provides sufficient information to compute the utility level association with a bundle of goods. In this regard, estimation of a Linear Expenditure System, for example, provides the parameters needed to compute utility levels. Moreover, since AIDADS was designed with applied general equilibrium in mind, the estimation of utility ought to be viewed an additional benefit, as such utility levels can be used when comparing alternative equilibria when conducting policy analysis. Nevertheless, estimation of a utility level for each expenditure level (i.e., for each observation) prohibits use of analytical methods in estimation. Rather, a maximum likelihood mathematical programming approach is adopted to allow for direct estimation of utility.

To estimate AIDADS, residual terms are appended to each equation of the demand system. The residuals, denoted as v_{it} for the ith equation in the tth observation ($t = 1,\ldots,T$), can be expressed as an n vector, $\tilde{\boldsymbol{v}}_t$. It is assumed that the $\tilde{\boldsymbol{v}}_t$ are distributed independently across observations as a multivariate normal with expectation $\mathsf{E}[\tilde{v}_t] = 0$ and a finite covariance matrix given by:

$$\mathsf{E}[\tilde{\boldsymbol{v}}_t \tilde{\boldsymbol{v}}_s'] = \begin{cases} \tilde{\boldsymbol{\Sigma}} & t = s \\ 0 & t \neq s. \end{cases} \tag{9}$$

By the adding up property of demands, $\sum_{i=1}^{n} v_{it} = 0$ for all t, and so $n \times n$ the matrix $\tilde{\boldsymbol{\Sigma}}$ is singular. Dropping the last equation from each observation allows one to define Σ, an $(n-1) \times (n-1)$ covariance matrix, in terms of the $n-1$ vector $\boldsymbol{v}_t$. Upon concentration, and ignoring terms that are independent of the unknown parameters, the log-likelihood function can be written as $-0.5\ln|\hat{\Sigma}|$, where $\hat{\Sigma}$ is the estimate of Σ with typical element $\hat{\Sigma}_{ij} = T^{-1} \sum_{t=1}^{T} v_{it} v_{jt}$ for all $i \neq n$, $j \neq n$. Since maximizing $-0.5\ln|\hat{\Sigma}|$ is equivalent to minimizing $0.5\ln|\hat{\Sigma}|$, the latter term is minimized with respect to the unknown parameters of the problem.

The determinant in the objective function to be minimized is computationally expensive to evaluate when the number of goods is large. However, evaluation of the objective function is simplified by noting that for any positive definite matrix, $\mathbf{X}$, the following is true: $\mathbf{X} = \mathbf{R}^t\mathbf{R}$ where $\mathbf{R}$ is an upper triangular matrix of conformable dimension. Assuming $\hat{\Sigma}$ has full row rank, then $\hat{\Sigma} = \mathbf{R}^t\mathbf{R}$, and $0.5\ln|\hat{\Sigma}|$ can be expressed as:

$$0.5\ln \prod_{i=1}^{n-1} r_{ii}^2 \tag{10}$$

where r_{ii} are the diagonal elements of $\mathbf{R}$.

The relationship between the residuals and elements of $\mathbf{R}$ can be defined by noting that each element of $\hat{\Sigma}$ must be the same regardless of whether it is computed as $T^{-1} \sum_{t=1}^{T} v_{it} v_{kt}$ or based on the matrix decomposition of $\hat{\Sigma}$. Accordingly, the following must be true:

$$T^{-1} \sum_{t=1}^{T} v_{it} v_{jt} = \sum_{k=1}^{n-1} r_{ki} r_{kj} \forall i \neq n, j \neq n. \tag{11}$$

Upper triangularity of $\mathbf{R}$ is imposed with the following restriction: $r_{kl} = 0$ for all $k > l$.

Since the parameters of AIDADS are the key unknown variables in this study, the AIDADS model is included as a constraint during estimation. For this purpose, the following share-based demand system is included:

$$\hat{w}_{it} = \frac{\gamma_i p_{it}}{y_t} + \frac{\alpha_i + \beta_i \exp(u_t)}{1 + \exp(u_t)} \left(1 - \frac{\boldsymbol{p}_t{}'\gamma}{y_t}\right) \forall i,t \tag{12}$$

where $\hat{w}_{it}$ is the fitted budget share for the ith good in the tth observation. In addition, parametric restrictions on α_i and β_i (i.e., $0 \le \alpha_i, \beta_i \le 1$ for all i, and $\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \beta_i = 1$) are used to ensure the predicted budget shares satisfy the adding up property of demand and regularity of the predicted budget shares.

The implicit defining equation for AIDADS is also included as a constraint during estimation. This constraint helps identify the unobserved level of utility, u_t, which is included as a right-hand side variable in the demand system, and facilitates the numerical uncompensation of AIDADS. To ensure well-behaved demands, it must be true that $q_{it} > \gamma_i$ for all i. This suggests that upper bounds be placed on the subsistence parameters. To ensure estimates of γ_i do not occur at their respective upper bounds, each consumption-based demand equation is substituted into the corresponding q_{it} term in the implicit utility function (see Eq. (1)). This turns the direct (implicit) utility function for AIDADS into an indirect (implicit) utility function:

$$\sum_{i=1}^{n} \frac{\alpha_i + \beta_i \exp(u_t)}{1 + \exp(u_t)} \ln\left(\frac{1}{p_{it}} \frac{\alpha_i + \beta_i \exp(u_t)}{1 + \exp(u_t)} (y_t - \boldsymbol{p}_t{}'\gamma)\right) - u_t = \kappa \quad \forall t. \tag{13}$$

where $\kappa = 1 + \ln(A)$. This indirect utility function is used to help identify u_t rather than the direct utility function.

To prevent the estimation procedure from attempting mathematically impossible operations, and to ensure the properties of demand are satisfied, the term in the logarithm operator in Eq. (13) must be positive. As $(\alpha_i + \beta_i \exp(u_t))/(1 + \exp(u_t))$ is bounded between zero and one and $p_i \in \Re_{++}$, discretionary expenditure must be positive. Consequently, the following constraint is also included:

$$0.99 y_t \geq \boldsymbol{p}_t{}'\gamma \quad \forall t. \tag{14}$$

Finally, the residuals are defined using:

$$v_{it} = w_{it} - \hat{w}_{it} \quad \forall i,t. \tag{15}$$

Since AIDADS is a non-linear model, and the constraint set has non-linear equality and linear inequality constraints, using starting values that are at least feasible, and preferably close to optimal, helps reduce the computational burden of finding an optimal solution. In addition, appropriate choice of upper and lower bounds on the parameters, fitted budget shares, utility levels, and error terms helps to reduce the space over which the solution algorithm searches for an optimal solution. Table 1 shows the lower and upper bounds for all unknown variables. However, the precise choice of bounds depends on the data used. In particular, upper bounds on α_i and β_i are data dependent. If one of these upper bounds is active, it should be relaxed until it is no longer binding, or reaches a level dictated by theory to be an active constraint. Since the bounds on γ_i are particularly important, they merit some explanation. Non-negativity γ_i is necessary for discretionary expenditure not to exceed the actual level of expenditure, while upper bounds on γ_i are chosen so as not to be active in the optimal solution. Consequently, estimates of γ_i *may* exceed the minimum *observed* level of consumption, but will be strictly less than fitted consumption (i.e., $\hat{q}_{it} > \hat{\gamma}_i$

Table 1
Lower and upper bounds, and starting values

Choice variable	Lower bound	Upper bound	Starting value
α	0	0.8	$\overline{w}_i$[a]
β	0	0.8	$\overline{w}_i$[a]
γ	0	$1.65\times\min_t\{q_{it}\}$	$0.25\times\min_t\{q_{it}\}$
κ	$-\infty$	∞	1
u	-12	20	$\sum_{i=1}^{n} \alpha_i \ln(q_{it}-\gamma_i)-1$
w	0.001	0.99	Solve Eq. (12)
v	-1	1	Solve Eq. (15)

[a] $\overline{w}_i$ denotes the sample mean of the ith good's budget share.

for all i and t). Since the regularity conditions for AIDADS depend, in part, on the relationship between consumption and the estimates of γ_i, it is vital that the *fitted* model be consistent with these conditions. In conjunction with Eq. (14), the choice of upper bounds for γ_i helps to ensure that fitted consumption values are greater than the estimates of γ_i. Lower bounds on α_i, β_i, and γ_i are consistent with the theoretical restrictions of the model and are not changed if binding. Bounds on $\hat{w}_{it}$ help to limit the search to an expected range of values, but only if active are relaxed until the offending bound is inactive or reaches a theoretical limit. Bounds on the utility levels and error terms are somewhat arbitrary, and if active are relaxed until not active. Finally, $\kappa\in\Re$, which implies $A\in\Re_{++}$.

In summary, Eq. (10) is minimized with respect to the parameters of AIDADS (α_i, β_i, γ_i, κ, u_t), the fitted budget shares ($\hat{w}_{it}$), and terms used to define the residuals (v_{it}, and r_{ij}), subject to constraints represented by Eqs. (11)–(15), parametric restrictions on α_i and β_i, and bounds on the choice variables. This mathematical programming problem is implemented in the General Algebraic Modeling System (GAMS) Release 2.50 and solved using the MINOS solver. In the next section, issues related to inference in AIDADS and the bootstrap methodology are discussed.

3. Bootstrapping AIDADS

Once the parameter estimates are obtained for the AIDADS model, evaluation of each parameter's statistical significance becomes the next logical step. In the context of unrestricted maximum likelihood estimation, the ML estimator is consistent, asymptotically efficient, and asymptotically normal distributed with mean vector $\boldsymbol{\zeta}$ and covariance matrix equal to the Cramer–Rao lower bound for consistent estimators. However, existence of bounded parameter spaces makes it difficult, even impossible, to use traditional sampling theory for inference. Therefore, the bootstrap methodology is used to obtain standard deviations for the parameter and elasticity estimates of AIDADS. The basic idea of the bootstrap, which is due to Efron (1979, 1982), is to generate a large number of pseudo-data sets by sampling with replacement from the empirical distribution of the residuals estimated using the actual data set. The "resampled" data sets are then used to compute statistics of interest. Since each resampled data set will have a corresponding set of statistics, an empirical distribution for each statistic can be constructed.

The bootstrap methodology begins by estimating AIDADS with the framework presented in the previous section, saving each observation's fitted budget shares and residuals to the $(n-1)$ vectors $\hat{\mathbf{w}}_t$ and $\hat{\mathbf{v}}_t$, respectively. Then, divide the unit interval into T equi-distance units (one for each observation), with lower and upper bounds defined by $\underline{d}_t$ and $\bar{d}_t$, respectively, where $\underline{d}_1=0$, $\bar{d}_t=\underline{d}_{t+1}$ and $\bar{d}_T=1$. For each observation select a random variable, uni_t, from a univariate uniform distribution over the unit interval. For the tth observation, if $\underline{d}_s<uni_t\leq\bar{d}_s$ add the sth observation's error vector, $\hat{\mathbf{v}}_s$, to the tth observation's fitted budget share vector $\hat{\mathbf{w}}_t$ to generate a new $(n-1)$ vector of budget shares for the tth observation, $\tilde{\mathbf{w}}_t$. Next, compute the nth pseudo-budget share for each observation using the adding up property of demand $\sum_i \tilde{w}_{it}=1$. If any element of $\tilde{\mathbf{w}}_t$ is non-positive, make a new random draw for that observation until all elements of $\tilde{\mathbf{w}}_t$ are in the unit interval. Finally estimate AIDADS using $\tilde{\mathbf{w}}_t$ and the original price and expenditure data. Repeat these steps a total of B times.

In order to have inactive upper bounds on the estimates of γ_i, the original upper bounds are doubled in the bootstrap simulations. However, the remaining lower and upper bounds are unchanged, while the starting values are computed in the manner described earlier but with the pseudo-data. A total of $B=100$ bootstrap replications are drawn, which appears to be a sufficient number of replications to estimate standard deviations (Efron and Tibshirani, 1986; Efron, 1987).

In each bootstrap replication, estimates of the AIDADS parameters are obtained, as are fitted budget shares, utility levels, and residuals. To express the bootstrap estimates of AIDADS in a compact manner, the mean, standard deviation and root mean squared error (RMSE) of the bootstrap estimates are reported. The standard deviation describes the dispersion of the bootstrap estimates about their estimated means, while the RMSE measures the dispersion of the bootstrap estimates around the maximum likelihood estimates used to generate the pseudo-data. Likewise, the mean, standard deviation, and RMSE for the fitted budget shares, marginal budget shares, and all elasticity measures can be computed across bootstrap replications.

4. Data

AIDADS is estimated using a cross-section sample of countries from the 1985 International Comparisons Project (ICP).[3] These data are useful in analyzing international demand patterns since they are provided in identical units (i.e., international dollars) and facilitate comparison of prices and quantities for disaggregate commodities across countries. ICP data sets have been compiled for the years 1970, 1973, 1975, 1980, 1985, 1990 and 1995. However, at the time of writing this study, the 1985 data set was the most up-to-date publicly available release.

[3] Others have used previous ICP data sets for analyzing international demand patterns. For example, Kravis et al. (1982), Theil and Clements (1987), and Rimmer and Powell (1992a) all used the 1975 ICP data set, Theil et al. (1989) used the 1970, 1973, 1975 and 1980 releases, while Wang (1996) used the 1985 data set used in this study.

The 1985 data set consists of 64 countries, ranging from Ethiopia, with real per capita consumption of \$159 (1985 International Dollars) to the USA, with real per capita consumption of \$8881 (1985 International Dollars). The raw data reports final consumption of 113 goods and services. For tractability, and in keeping with the additive nature of AIDADS, these 113 goods and services are aggregated in to six broad aggregate goods. These broad aggregates are food (FOOD), beverages and tobacco (BEVRTOBC), clothing and footwear (CLTHFOOT), gross rent and fuel (RENTFUEL), household furnishings and operations (HFURNOPS), and other expenditure (OTHEREXP). This is the same aggregation as used by Rimmer and Powell (1992a) and Wang (1996).

Budget shares are constructed by dividing nominal expenditure on each aggregate good by total nominal expenditure for that good. The price of each good equals the ratio of total nominal expenditure for that good to total real expenditure for the same good. Total nominal expenditure per capita (stated in hundreds of international dollars) serves as the expenditure term in AIDADS.

5. Results

Before discussing the results, note that during construction of the bootstrap data, 32,000 independent, pseudo-budget shares are generated.[4] Of these, 21 were less than zero, which equals less than half of 1% of the generated data. When a negative pseudo-budget share was computed, a new vector of pseudo-data is generated for that observation via a random draw. Since positivity of the pseudo-budget shares is violated in so few cases, little bias is expected to occur. In 26 of the 100 bootstrap replications, Ethiopia's utility level is active at its lower bound (recall Ethiopia is the lowest expenditure country and so such a low level of utility is not surprising). In these instances, the marginal value of the active bound is small (i.e., less than 10^{-6}). Experimentation with several replications where this bound is active indicated the objective function's value changed only in the sixth decimal place or lower, while the parameter values of AIDADS changed by a similar magnitude. Consequently, the activity of these bounds is expected to have little or no impact on the value of the choice variables in the optimal solution and therefore little or no impact on the bootstrapped estimates.

Table 2 shows the maximum likelihood and bootstrap summary statistics for the estimates of AIDADS. In general, the estimated equations fit the data in a manner similar to other studies (see, for example, Wang, 1996). The correlation coefficient (ρ) between actual and fitted budget shares was the highest for FOOD ($\rho = 0.885$) and OTHEREXP ($\rho = 0.840$), followed by RENTFUEL ($\rho = 0.566$), CLTHFOOT ($\rho = 0.380$), BEVRTOBC ($\rho = 0.195$), and finally HFURNOPS ($\rho = 0.045$). The poor performance of HFURNOPS may reflect the fact that this good contains durable goods. It has long been acknowledged that, in the context of static demand models, equations representing durable goods typically have a poor fit. This poor fit can be attributed to the fact that static demand models do not account for dynamic aspects of durable goods due to their long service life.

[4] Since there are of 64 observations, 5 independent budget shares in each observation, and 100 bootstrap replications, the total number of pseudo-data points is $5 \times 64 \times 100 = 32{,}000$.

Table 2
Maximum likelihood estimates of AIDADS, and bootstrap summary statistics

Parameter	Equation	Maximum likelihood estimates	Bootstrap summary statistics		
			Mean	Standard deviation[a]	RMSE[b]
α	FOOD	0.467	0.482	0.053	0.055
	BEVRTOBC	0.066	0.067	0.015	0.015
	CLTHFOOT	0.096	0.097	0.013	0.013
	RENTFUEL	0.083	0.079	0.021	0.021
	HFURNOPS	0.075	0.076	0.012	0.012
	OTHEREXP	0.212	0.198	0.049	0.051
β	FOOD	0.000	0.020	0.032	0.038
	BEVRTOBC	0.035	0.036	0.013	0.013
	CLTHFOOT	0.052	0.053	0.009	0.009
	RENTFUEL	0.231	0.227	0.020	0.021
	HFURNOPS	0.065	0.063	0.012	0.012
	OTHEREXP	0.617	0.601	0.034	0.037
γ[c]	FOOD	0.617	0.565	0.186	0.192
	BEVRTOBC	0.052	0.053	0.040	0.040
	CLTHFOOT	0.105	0.103	0.044	0.043
	RENTFUEL	0.091	0.088	0.028	0.028
	HFURNOPS	0.035	0.036	0.024	0.024
	OTHEREXP	0.265	0.282	0.155	0.155
κ		1.918	1.742	0.345	0.386

[a] Standard deviation is computed as $\left((B-1)^{-1}\sum_{b=1}^{B}(\hat{\xi}_b-\bar{\xi})^2\right)^{0.5}$ where $\hat{\xi}_b$ and $\bar{\xi}$ are the b-th bootstrap estimate of the parameter in that equation and its mean, respectively.

[b] Root Mean Square Error is computed as $\left((B-1)^{-1}\sum_{b=1}^{B}(\hat{\xi}_b-\xi^{\mathrm{ML}})^2\right)^{0.5}$ where ξ^{ML} is the maximum likelihood estimate of the parameter in equation.

[c] Since nominal per capita expenditure is expressed in hundreds of international dollars, the subsistence parameters ought to be multiplied by 100 for comparison to other studies.

The poor fit of the BEVRTOBC equation may be due to differences in societal norms and customs across the countries covered by the ICP data set. In particular, a number of countries have large segments of their population for which alcohol consumption is prohibited by religious (or social) reasons. Since BEVRTOBC includes alcoholic and non-alcoholic beverages, larger deviation of fitted budget shares from actual, when based on an equation estimated using the entire sample, is not surprising.

Nevertheless, the estimates are consistent with well-behaved preferences in the price–expenditure space where $y > \boldsymbol{p}'\boldsymbol{\gamma}$. Recall that regularity requires $\Xi < 0$ in the restricted price–expenditure space, which is satisfied at every point in the sample, with Ξ varying from -0.01 to -0.3. Finally, the log-likelihood function's value in the optimal solution is -17.19.

Except for $\hat{\beta}_{\mathrm{FOOD}}$, the maximum likelihood estimates of AIDADS are in the interior of their admissible ranges. The estimate of $\hat{\beta}_{\mathrm{FOOD}}$ is active at its lower bound of zero. Wang (1996) and Rimmer and Powell (1992a) reported identical values for $\hat{\beta}_{\mathrm{FOOD}}$. In general, the estimates of α_i and β_i reported in Table 2 are similar in value to those reported in Wang

(1996). However, estimates of α_i for FOOD, RENTFUEL, and OTHEREXP, and most estimates of γ_i differ from Wang's estimates by about one-third to one-half in absolute value. As well, most of the estimates differ from Rimmer and Powell (1992b), who used the 1975 ICP data set to estimate AIDADS. These differences are quite noticeable for the subsistence parameters, most of which Rimmer and Powell (1992b) report to be close to zero.

The means of the bootstrap estimates are similar in value to the estimates used to generate the bootstrap data. With a few exceptions, the standard deviations are well below the means of the bootstrap estimates. Since a standard deviation measures the spread of an estimate about its mean, a low value relative to the mean indicates a low measure of variability. Moreover, most RMSEs are less than the maximum likelihood estimates, which indicates little deviation of the bootstrap estimates from the maximum likelihood estimates. A notable exception occurs for the estimate of β_i for FOOD, which has a standard deviation that is larger than the mean of its bootstrap estimate and a RMSE that is larger than the maximum likelihood estimate. The latter result implies this parameter's value may be estimated with large variance and potential bias. In addition, standard deviations for the subsistence parameters indicate that these estimates also have greater variability compared to the estimates of α_i and β_i.

Fig. 1 shows the estimated levels of utility in each country. Utility, which increases with expenditure, ranges from -4.5 for Ethiopia to about 1.2 for the United States. As expected, diminishing marginal utility is observed. The solid line in Fig. 1 shows utility levels with prices fixed at their sample means but allowing expenditure to vary. Holding prices fixed removes any noise that might be introduced via cross-country price differences, which better depicts diminishing marginal utility.

Since the relationship between α_i and β_i determines whether the good is a necessity or luxury, it is useful to note their values. Except for RENTFUEL and OTHEREXP, all estimates of α_i are greater than the corresponding estimates of β_i. In the context of

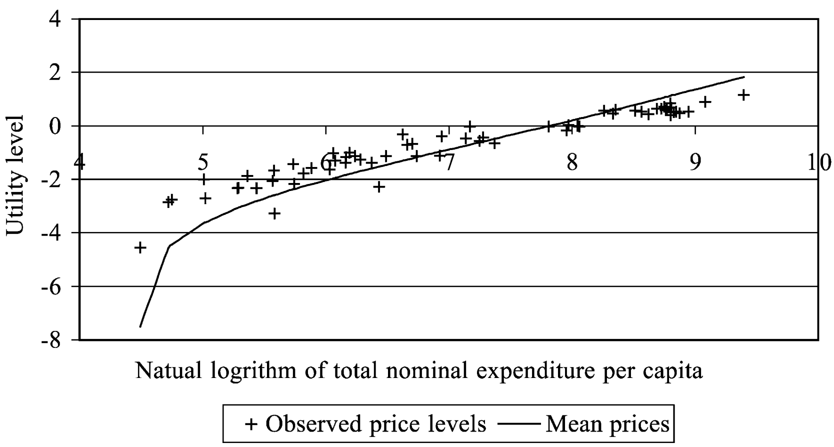

Fig. 1. Utility levels from AIDADS evaluated at the price levels and at sample means of the prices using maximum likelihood parameters estimates.

Table 3
Maximum likelihood estimates and bootstrap summary statistics of the fitted budget shares, marginal budget shares, and Engel elasticities for AIDADS evaluated at the sample means of the data[a]

Parameter	Good	Maximum likelihood estimate	Bootstrap summary statistics		
			Mean	Standard deviation	RMSE
MBS[b]	FOOD	0.104	0.106	0.018	0.018
	BEVRTOBC	0.042	0.042	0.008	0.008
	CLTHFOOT	0.062	0.061	0.006	0.006
	RENTFUEL	0.198	0.199	0.011	0.011
	HFURNOPS	0.067	0.066	0.008	0.008
	OTHEREXP	0.527	0.526	0.019	0.019
Share	FOOD	0.239	0.235	0.012	0.013
	BEVRTOBC	0.050	0.050	0.004	0.004
	CLTHFOOT	0.074	0.073	0.004	0.004
	RENTFUEL	0.156	0.158	0.006	0.006
	HFURNOPS	0.069	0.068	0.004	0.004
	OTHEREXP	0.411	0.416	0.011	0.012
Engel	FOOD	0.434	0.451	0.066	0.068
	BEVRTOBC	0.835	0.832	0.125	0.125
	CLTHFOOT	0.835	0.838	0.061	0.061
	RENTFUEL	1.269	1.256	0.051	0.053
	HFURNOPS	0.977	0.964	0.076	0.077
	OTHEREXP	1.281	1.266	0.036	0.039
u		0.036	0.218	0.348	0.392

[a] In each bootstrap replication, the marginal and fitted budget shares, and Engel elasticities are computed at the sample means of the data in that bootstrap replication. The bootstrap summary statistics are then computed across all replications.

[b] MBS denotes the marginal budget shares, share the fitted budget shares, Engel the Engel elasticities, and u the utility level.

AIDADS, this means RENTFUEL and OTHEREXP can be categorized as luxuries, while the remaining goods are categorized as necessities.

Table 3 shows the marginal budget shares (MBS), fitted budget shares (Share), Engel elasticities (Engel), and utility level (u) evaluated at the sample means.[5] The Engel elasticities echo the relative magnitudes of α_i and β_i, with FOOD, BEVRTOBC, CLTHFOOT, and HFURNOPS classified as necessities at the sample means, while RENTFUEL and OTHEREXP are categorized as luxuries. Furthermore, FOOD has the smallest Engel elasticity, while OTHEREXP is the most responsive to expenditure changes. Note that given its proximity to unity, the Engel elasticity for HFURNOPS is suspect as it could conceivably be greater than unity.

The means of the bootstrap estimates for the Engel elasticities are close to the actual estimates.[6] In addition, the standard deviations are small compared to the means, as are the RMSEs compared to the actual estimates. The latter two results suggest that Engel elasticities have low variance and do not show a great deal of bias relative to the maximum

[5] The utility level used in computing these measures is computed at the sample means of the data.

[6] These measures are computed at the means of the bootstrap data in each replication, and the bootstrap summary statistics are then computed across all bootstrap replications.

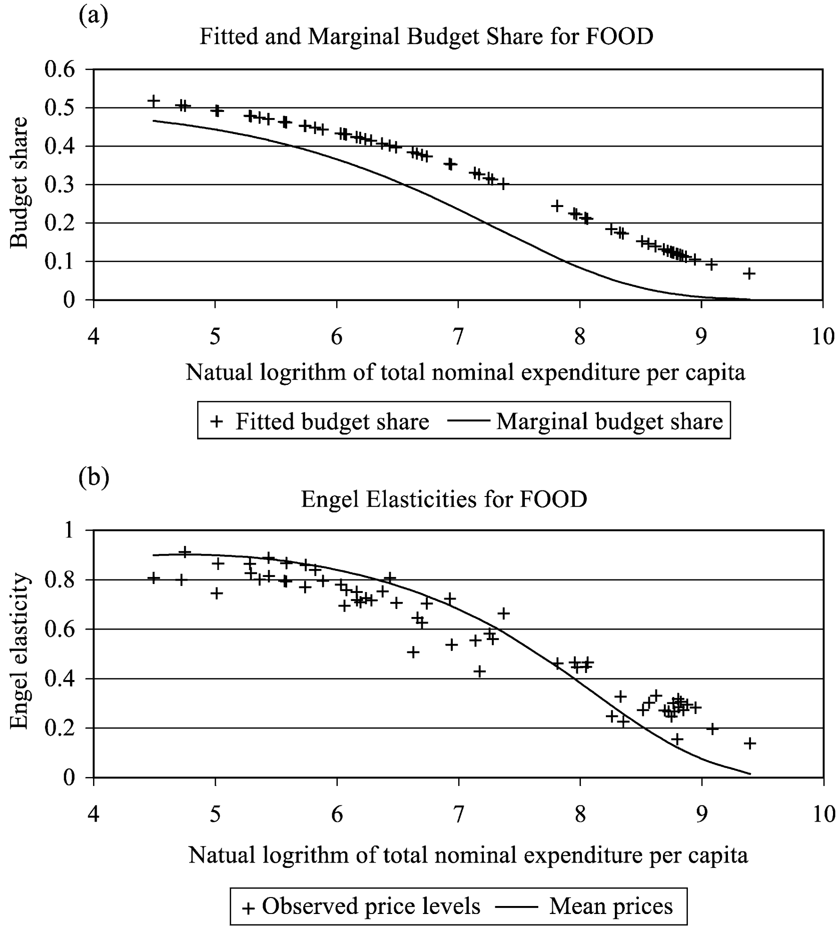

Fig. 2. Fitted and marginal budget shares for AIDADS evaluated at the sample means of the prices, and Engel elasticities for AIDADS evaluated at the price levels and the sample means of prices.

likelihood estimates. The utility levels based on the bootstrapped parameter estimates have large standard deviations and RMSE compared to the maximum likelihood estimate. Thus, the utility levels do not appear to have as low a variability and bias as the estimates of α_i and β_i, and the Engel elasticities. Such occurrence reflects less precise estimates of γ_i in the bootstrap simulations.

Recall that estimates of α_i and β_i govern the asymptotic behavior of the marginal budget shares for AIDADS. To illustrate the potential non-linearities embodied in AIDADS, Fig. 2a shows the marginal and fitted budget shares for FOOD, evaluated at the sample means of the prices, but allowing per capita expenditure to vary across countries.[7] Given the

[7] While the marginal budget shares are potentially non-linear in *real* expenditure, they are also potentially non-linear in nominal expenditure when prices are fixed at a common value across the sample, for example, the sample means (Rimmer and Powell, 1996, p. 1616).

interpretation of α_i and β_i in the limit, one can observe that FOOD's marginal budget share has a value of about 0.5 at low per capita expenditure levels, and a value of about 0 at high per capita expenditure levels. This means that as per capita expenditure grows without bound, FOOD's share of each additional dollar of expenditure is almost zero, while FOOD accounts for about half of each additional dollar of expenditure at the subsistence level of consumption.

Engel elasticities for FOOD, evaluated at each country's price level, and at the sample means of the prices, are plotted in Fig. 2b. The trend in FOOD's Engel elasticity reflects aspects of Fig. 2a, where fitted and marginal budget shares fall at different rates as expenditure grows. In general, the fitted budget shares for AIDADS approach the marginal budget shares as expenditure grows without bound (Rimmer and Powell, 1996, pp. 1615–1616). Consequently, the Engel elasticity for FOOD should, in principal, approach unity. However, the fact that FOOD's Engel elasticities do not approach unity as expenditure grows reflects the limiting behavior of the marginal budget shares. In particular, FOOD's marginal budget share approaches zero as expenditure grows without bound. However, the marginal budget share approaches zero at a faster rate than the fitted budget share. Consequently, the Engel elasticity for FOOD approaches zero in the limit. Note, however, that FOOD's Engel elasticity is less than unity across all expenditure levels. Thus, Engel's Law holds for the definition of the food good used in this study and when demands are modeled using the AIDADS model.

Table 4
Allen–Uzawa partial elasticities of substitution, compensated and uncompensated price elasticities for AIDADS, evaluated using the maximum likelihood estimates for AIDADS and the sample means of the data

Parameter[a]	Good	Maximum likelihood estimate	Bootstrap summary statistics		
			Mean	Standard deviation	RMSE
σ_{ii}	FOOD	– 2.985	– 3.080	0.210	0.229
	BEVRTOBC	– 18.210	– 18.474	1.605	1.619
	CLTHFOOT	– 11.965	– 12.187	0.624	0.660
	RENTFUEL	– 5.284	– 5.124	0.232	0.241
	HFURNOPS	– 13.326	– 13.458	0.796	0.803
	OTHEREXP	– 1.388	– 1.366	0.066	0.069
η_{ii}'	FOOD	– 0.714	– 0.720	0.017	0.018
	BEVRTOBC	– 0.918	– 0.918	0.024	0.024
	CLTHFOOT	– 0.889	– 0.889	0.015	0.015
	RENTFUEL	– 0.825	– 0.824	0.007	0.007
	HFURNOPS	– 0.915	– 0.915	0.010	0.010
	OTHEREXP	– 0.571	– 0.567	0.012	0.013
η_{ii}	FOOD	– 0.817	– 0.826	0.016	0.018
	BEVRTOBC	– 0.961	– 0.960	0.021	0.021
	CLTHFOOT	– 0.951	– 0.951	0.015	0.014
	RENTFUEL	– 1.023	– 1.022	0.009	0.009
	HFURNOPS	– 0.982	– 0.981	0.011	0.010
	OTHEREXP	– 1.098	– 1.093	0.016	0.017

[a] σ_{ii} denotes the own-price Allen–Uzawa partial elasticities of substitution, η'_{ii} the own-price compensated price elasticities and η_{ii} the own-price uncompensated price elasticities.

Table 4 shows the own Allen–Uzawa partial elasticities of substitution, and compensated and uncompensated own-price elasticities, all evaluated at the sample means of the data. All own partial substitution elasticities have the expected sign. While not reported, note that the cross-partial substitution elasticities were close to unity, which means that, at the sample means, curvature of the indifference surfaces for AIDADS is similar to those of the Cobb–Douglas functional form. The limiting behavior of AIDADS at high expenditure levels is that of Cobb–Douglas preferences. As such, one would naturally expect such an observation at high expenditure levels, but not necessarily at the sample means. In addition, the means of the bootstrapped estimates of the own-partial substitution elasticities are of similar magnitude to the actual estimates. Moreover, the standard deviation and RMSE are below the means of the bootstrapped and actual maximum likelihood estimates, respectively, suggesting low variability and little bias in the estimates.

All compensated own-price elasticities are negative and inelastic. While not reported, all compensated cross-price elasticities are positive—thus indicating a net substitute relationship. Since $\sigma_{ij}>0$ for all $i\neq j$ and $w_j>0$ for all j, and $\eta'_{ij}=\sigma_{ij}w_j$, the net substitute relationship between goods is expected. Among the compensated own-price elasticities, OTHEREXP is the most inelastic, followed by FOOD, while BEVRTOBC is the least inelastic. As with the substitution elasticities, the bootstrap summary statistics indicate the compensated own-price elasticities were estimated with low variance and little bias.

All uncompensated own-price elasticities are negative, with those for RENTFUEL and OTHEREXP being greater than unity in absolute value. Thus, the latter two goods may be characterized as having elastic uncompensated own-price responses, while FOOD, BEVRTOBC, CLTHFOOT, and HFURNOPS have inelastic uncompensated own-price responses. As with the compensated own-price elasticities, the standard deviations and RMSE values suggest the uncompensated own-price elasticities have low variance and little bias.

6. Growth rates for food demand across the development spectrum

The previous section's results are cast in economic significance when one recognizes the role of non-linear Engel responses when per capita expenditure levels increase, but at different rates across the development spectrum. To see an example of this role, results from the previous section are used to simulate the AIDADS model assuming that per capita expenditure growth drives changes in per capita demands in a selection of countries spanning the development spectrum. Here, per capita expenditure growth is computed using growth rates from Cranfield et al. (1998). Since these growth rates are temporally differentiated from 1985 to 1995 and then 1995 to 2020, demands are first predicted in the year 1995 (based on the 1985–1995 growth rates) and then in 2020 (based on the 1995–2020 growth rates). Focus is placed on the food good, as it holds a unique place in the consumption bundle when expenditure changes, and on growth rates for FOOD demand between the years 1995 and 2020. Simulations are performed for countries representative of positions along the development and expenditure spectrum, namely Ethiopia, Senegal, South Korea, France, and the USA.

To obtain a measure of the precision with which FOOD demand growth rates are estimated, AIDADS is simulated using the bootstrapped parameter estimates. For each of the six selected countries, AIDADS is simulated 100 times, once for each set of bootstrapped parameter estimates, using the respective county's 1985 price vector and projected expenditure level in 2020. Therefore, it is the variation in the bootstrapped parameter estimates of AIDADS that introduces variation in the growth rates.

Table 5 shows the summary statistics for the bootstrapped growth rates for budget shares and total expenditure on the six goods over the period 1995 to 2020. Focusing on FOOD budget shares, we see that mean growth rates range from 0.01% per annum in Ethiopia to − 2.8% per annum in Korea and that except for Ethiopia, all of the mean growth rates for FOOD's budget share are negative. However, the standard deviation for the bootstrapped growth rates in Ethiopia is about twice as large as the mean. In contrast, the standard deviation for the remaining countries are small compared to their respective means. Consequently, despite anticipated expenditure growth in Ethiopia over the period 1995 to 2020, little change is expected in FOOD's share of total expenditure. This result relates directly to Fig. 2a, where the curve for the fitted budget share is rather shallow across low per capita expenditure levels. As this is the range over which Ethiopia's per capita expenditure levels is projected to cover from 1995 to 2020, it should come as no surprise that such an imprecise estimate of the growth rate for FOOD is measured.

While FOOD's budget share is expected to change little in Ethiopia, total expenditure on the food good is expected to grow. The lower panel in Table 5 shows the mean growth rates for total expenditure on the six goods, over the period 1995 to 2020. These values reflect the role of population growth in determining expenditure on the various goods. Clearly, countries with lower per capita expenditure levels, which are those with higher

Table 5
Bootstrapped growth rates in demand for all goods in selected countries[a]

	FOOD	BEVRTOBC	CLTHFOOT	RENTFUEL	HFUNROPS	OTHEREXP
Mean budget share growth rates[b]						
Ethiopia	0.011 (0.028)	0.444 (0.096)	1.602 (0.070)	− 1.120 (0.067)	− 0.042 (0.086)	0.260 (0.080)
Pakistan	− 0.257 (0.013)	− 0.369 (0.032)	0.098 (0.026)	0.801 (0.023)	0.176 (0.025)	0.032 (0.016)
Senegal	− 0.207 (0.013)	− 0.306 (0.042)	0.127 (0.025)	0.595 (0.022)	0.223 (0.026)	− 0.003 (0.016)
Korea	− 2.787 (0.062)	− 0.597 (0.081)	− 0.250 (0.040)	0.721 (0.025)	0.436 (0.049)	0.658 (0.015)
France	− 1.590 (0.063)	− 0.042 (0.082)	0.203 (0.040)	0.117 (0.025)	0.660 (0.050)	0.233 (0.015)
USA	− 1.031 (0.085)	0.302 (0.096)	0.505 (0.047)	− 0.190 (0.027)	0.837 (0.056)	0.061 (0.017)
Mean expenditure growth rates						
Ethiopia	3.361	3.809	5.006	2.193	3.306	3.619
Pakistan	2.947	2.833	3.314	4.040	3.395	3.246
Senegal	3.073	2.971	3.418	3.902	3.518	3.285
Korea	1.307	3.589	3.950	4.962	4.665	4.897
France	0.608	2.191	2.442	2.354	2.908	2.472
USA	1.007	2.368	2.575	1.866	2.914	2.122

[a] Growth rates are computed for the period 1995 to 2020.

[b] Values in the cell are means of the bootstrapped growth rates; values in parentheses are standard deviations of the bootstrapped growth rates.

population growth rates, population plays a much more important role in determining food expenditure growth.

7. Conclusions

This paper characterizes consumer demand patterns across the development spectrum using elasticity estimates from an implicit, directly additive demand system (AIDADS). AIDADS possesses fitted and marginal budget shares that are non-linear in real expenditure. Consequently, it is well suited to modeling demands where per capita income levels vary widely across the sample, and to projecting consumer demands in situations where large expenditure growth may be encountered. Such is the situation considered in this paper.

Estimated marginal budget shares for the food good decline in a non-linear manner and range from about 0.5 for Ethiopia to less than 0.05 for the USA. A similar pattern is noted for fitted budget shares for food. Moreover, Engel (income) elasticities for food range from 0.95 for Ethiopia to less than 0.1 for the USA. As with the marginal budget shares, Engel elasticities fall in a non-linear manner. Note also that marginal and fitted budget shares and income elasticities follow a decreasing logistical shape. Thus, for countries in the middle of the development spectrum, like Korea, food's share of each additional unit of per capita income falls by a greater amount than in countries at the extremes of the development spectrum (e.g., Ethiopia and the USA).

Results indicate that the fitted model is consistent with Engel's Law. Moreover, given the estimated pattern of food demand across the development spectrum, one would expect that as per capita income levels in developing countries approach those of wealthier countries, food demand patterns in the former would resemble those in the latter. Finally, given the structure of AIDADS, results from this study provide more plausible measures of how food demand will adjust to per capita income growth compared to other demand models.

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