5

Supply in a CGE Model

In this chapter, we examine the supply side of an economy as represented in computable general equilibrium (CGE) models. The production data in the Social Accounting Matrix (SAM) depict the production process, in which firms combine intermediate inputs with factors of production to produce goods and services. We use these data to calculate input-output coefficients, and forward and backward linkages. CGE models break down the production technology into parts, depicting how subprocesses are nested within the overall production process. Within each nest, behavioral equations describe producers' efficiency-maximizing input demands and output levels, subject to their production technology. Export transformation functions, used in some CGE models, describe the allocation of production between domestic and export markets.

In 2009, the United States government offered financial assistance to its auto manufacturers to help them survive a deep recession and a freefall in consumer demand for cars. The bailout was controversial in part because the government seemed to be choosing to support a particular manufacturing industry. The government response was that the aid package not only helped save the jobs of autoworkers but also preserved jobs in the many industries that supply parts to the automakers and that sell and service autos. This part of the U.S. economic stimulus program built on the idea that an injection of support into one part of the economy would move in a circular flow to the rest of the economy, starting with the strong interindustry linkages between automakers and other manufacturing and service sectors.

In this chapter and the next, we explore the supply side of the economy as represented in a CGE model, emphasizing the linkages among industries through their demands for intermediate inputs and their competition for the factors of production. We start with an examination of the production data in the SAM. The activity column accounts of the SAM describe the inputs used in industries' production processes and the activity row accounts describe the use of industries' outputs as inputs for other industries. In the CGE model, producers are assumed to maximize their efficiency, subject to the flexibility

Table 5.1. *Production Inputs in the U.S. 3x3 Micro SAM ($U.S. Billions)*

SAM entry	Activities: Agric.	Activities: Mfg.	Activities: Services	Definition	
Commodities – total	212	3,411	5,574	Intermediate inputs	
Agric. imports	2	160	6		
Mfg. imports	12	393	216		
Services–imports	1	16	149		
Agric.–domestic	24	229	70		
Mfg.–domestic	66	1,533	1,141		
Services–domestic	109	1,079	3,991		
Factors–total	223	1,576	7,999	Factor payments	Value-added
Land	34	0	0		
Labor	68	1,109	5,667		
Capital	122	467	2,332		
Factor use taxes–total	1	182	926	Factor use taxes	
Land	−3	0	0		
Labor	5	166	850		
Capital	−1	15	76		
Sales tax	−6	17	52	Sales taxes	
Production tax	4	42	423	Production tax	
Total	434	5,227	14,974	Gross value of output	

Note: Sales taxes rows in the SAM are aggregated into a single sales tax row.
Source: GTAP v.7.0 U.S. 3x3 database.

afforded by their technological production process, as they choose inputs and their levels of output. We describe technologies and producer behavior in detail in this chapter and conclude by describing how producers are assumed, in some CGE models, to allocate their output between domestic and export sales.

Production Data in a SAM

Production activities use inputs to produce goods and services. Inputs are of two types: ***intermediate inputs*** (such as electronic components for a television or computer) and the ***primary factor inputs*** (land, labor, and capital) that are necessary to turn these intermediate inputs into final products. The activity columns in a SAM report the value of all intermediate and factor inputs and any taxes paid (or subsidies received) in the production of industry output.

To illustrate, Table 5.1 presents the three production activity columns from the U.S. 3x3 SAM (omitting the rows with zeros). Each column of the table shows the expenditure by that industry on all of its intermediate and factor inputs and on taxes. According to Table 5.1, U.S. agricultural producers spend \$212 billion (with rounding) on intermediate inputs. These are composed of \$26 billion of agricultural commodities (\$2 billion are imported and \$24 billion are produced domestically), \$78 billion of imported and domestic manufactured inputs, and \$110 billion of imported and domestic services. Notice that the table also shows how each type of good is used as an input into the other industries. The production of manufacturing, for example, requires substantial amounts of agricultural inputs.

In addition to intermediate inputs, U.S. agricultural production requires \$223 billion of factor inputs, which include \$34 billion for land, \$68 billion for labor, and \$122 billion for capital services. On net, U.S. agricultural producers pay \$1 billion in taxes on their use of factors, which includes \$3 billion in subsidies on land and capital use (which have negative factor use taxes). Agricultural producers received an additional \$6 billion in subsidies to purchase intermediate inputs (a negative sales tax). Finally, because production taxes change producers' costs, the activity column also reports the production taxes paid (or subsidies received) by an industry. In agriculture, producers pay \$4 billion in production taxes.

The contributions of factors (and including all tax and subsidies) to increasing the value of the industry's finished goods is called the industry's "value-added." For example, farm labor adds value to the agricultural and other intermediate inputs that are used to produce farm products. In U.S. agriculture, value-added totals \$222 billion (i.e., $\$223 + \$1 - \$6 + \$4 = \$222$ billion) Value-added plus the \$212 billion value of intermediate inputs equals the gross value of output of U.S. agriculture of \$434 billion.

Input-Output Coefficients

The data reported in the activity columns of the SAM can be used to calculate a useful descriptive statistic called an ***input-output coefficient***. These coefficients describe the ratio of the quantities of intermediate and factor inputs per unit of output. They are calculated by dividing every cell of Table 5.1 by its column total – the gross value of output.[1]

In Table 5.2, we display the input-output coefficients based on the U.S. 3x3 SAM (omitting the tax rows of the SAM). For example, the input-output coefficients for the agriculture activity indicate that .03 units of imported manufactured inputs are required per unit of output, and .05 units of domestically produced agricultural inputs are required, and so on.

[1] The SAM reports value data so the input-output coefficients are value shares. But recall from Chapter 2 that if we normalize the data by assuming that it reports quantities per dollar, then we can interpret our input-output coefficients as ratios of input and output quantities.

Table 5.2. *U.S. Input-Output Coefficients*

	Production Activities		
	Agric.	Mfg.	Services
Intermediate inputs			
Agric.–imports	0.00	0.03	0.00
Mfg.–imports	0.03	0.08	0.01
Services–imports	0.00	0.00	0.01
Agric.–domestic	0.05	0.04	0.00
Mfg.–domestic	0.15	0.29	0.08
Services–domestic	0.25	0.21	0.27
Factor Inputs			
Land	0.08	0.00	0.00
Labor	0.16	0.21	0.38
Capital	0.28	0.09	0.16

Source: GTAP v.7.0 U.S. 3x3 database.

Input-output coefficients allow us to describe the ***intermediate input intensity*** or ***factor intensity*** of a production activity. A sector is "intensive" in the intermediate and factor inputs whose input-output coefficients are highest. For example, U.S. agriculture is capital-intensive because it uses more units of capital per unit of output than of land or labor. This knowledge can be useful if we want to design experiments or predict and interpret model results. For example, what if the U.S. government asks us to identify and study input subsidies that would most benefit farmers? Based on our input-output table, we could choose to focus our study on subsidies to services or capital inputs, because these are the inputs in which agricultural production is most intensive.

We can also use input-output coefficients to make scale-neutral comparisons of input intensities across industries and countries. For example, we could compare the capital input-output ratio between U.S. agriculture, .28, and U.S. services, .16. We can conclude that U.S. agriculture is more capital intensive than services because it has a higher capital-output ratio. The comparison is scale neutral because we can make this observation without confusing it with the observation that services, the largest sector in the U.S. economy, accounts for vastly more capital usage than agriculture.

Input-output coefficients in addition describe linkages among industries through their demands for intermediate inputs. ***Upstream*** industries are the activities that produce goods that are used as inputs into other ***downstream*** industries – as if products flowed downstream on a river from a producer toward the industries that use them as inputs. Auto parts suppliers, for example, are an upstream industry that produces parts used downstream by auto assembly industries. Based on the U.S. 3x3 SAM, services is the major upstream industry providing intermediate inputs into agriculture and manufacturing.

In a CGE model, intermediate input linkages are a potentially important channel through which a shock in one industry can affect the rest of the economy. For example, consider a shock that lowers the price of services. Given the input-output coefficients reported in Table 5.2, we can see that this shock will lower the input costs of all sectors in the U.S. economy, but particularly of services and agriculture, which use services inputs most intensively.

Input-output coefficients sometimes are used to identify the sectors that are most intensive in their total requirements for intermediate inputs. This information may be useful for policymakers who want to know which industry exerts the strongest "pull" on the rest of the economy through its demand for intermediate goods. A stimulus program or development plan that targets growth in that industry could be particularly effective in spurring economywide growth because its expansion will increase demand for products produced upstream. We can compare the intensities of sectors' demands for intermediate inputs by calculating a ***backward linkage index*** for each industry. The index is the sum of the input-output coefficients for all intermediate goods used in an industry. It is called a "backward" index because we are looking backward at where the inputs for this industry came from. For agriculture, we use the input-output coefficients in Table 5.2 to calculate the index as:

$$\text{Backward linkage index for agriculture} = .03 + .05 + .15 + .25 = 0.48.$$

This shows that agricultural output requires .48 units of intermediates per unit of final product. Verify that the backward index for services is .37. A comparison of the two index values indicates that, relative to its size, increased agricultural output exerts a greater pull on the economy as a whole than increased production of services.

A similar concept is the ***forward linkage index***. The index describes the role of industries in providing inputs into downstream industries. Lowering the cost of inputs exerts a "push" on other sectors of the economy by lowering their costs of production. The index is the share of an industry's output that is used as intermediate inputs by other industries. It is called a "forward" index because we are looking forward to where the inputs from this industry are used. The index is calculated using data from the SAM on demand for domestically produced intermediates and industry output. For example, the forward linkage index for agriculture, calculated from the values in domestic agriculture's row account in Table 5.1, is:

$$\text{Forward index for agriculture} = \$24 + \$229 + \$70/\$434 = 0.74.$$

Verify that the forward index for manufacturing is .52. A comparison of the two index values indicates that, relative to their size, increased agricultural output exerts a greater push force on the U.S. economy than an increase in manufacturing output.

These interindustry linkages often play an important role in explaining the results of economic shocks in a CGE model. However, as we will demonstrate in this chapter, a CGE model accounts for additional aspects of intermediate demand that are also important to consider. These include the relative size of each sector in the economy, the potential for imports to supplant domestic products in meeting demand for intermediates, and the ability of producers to substitute toward cheaper intermediate inputs in their production process.

Producer Behavior in a CGE Model

Behavioral equations in a CGE model govern producers' decisions about their input quantities and levels of output. In some models, producers are assumed to be cost-minimizers who choose the least-cost level of inputs for a given level of output, given input and product prices and technological feasibility. Other CGE models describe producers as profit maximizers who choose quantities of both inputs and output, given input and product prices and subject to technological feasibility. The two approaches are just two sides of the same coin; both describe producers as maximizing their efficiency. Our discussion in the following sections mostly describes a cost-minimizing producer.

In addition to maximizing their efficiency, other important assumptions about producers that are commonly made in standard CGE models are that markets are perfectly competitive. Individual producers cannot influence the market prices of outputs or inputs, and they sell their output at their cost of production, making zero profits (in the economic sense). In many standard CGE models, production is also assumed to exhibit constant returns to scale. Thus, an increase of the same proportions in all inputs leads to an increase in output of the same proportion.

Technology Tree and Nested Production Functions

Because a producer's economic decisions on input and output levels are constrained by the firm's physical production technology, let's first explore in some detail how technological processes are described in a standard CGE model, before we consider economic choices any further. Technology defines the physical production process by which intermediate inputs, such as rubber tires and engines, are transformed by machinery and workers into a final product, such as an auto. This physical relationship is depicted by a ***production function***. CGE models typically separate the production function into parts. In a diagram, it looks a lot like an upside down tree. The trunk of the ***technology tree*** describes the final assembly of a good or service.

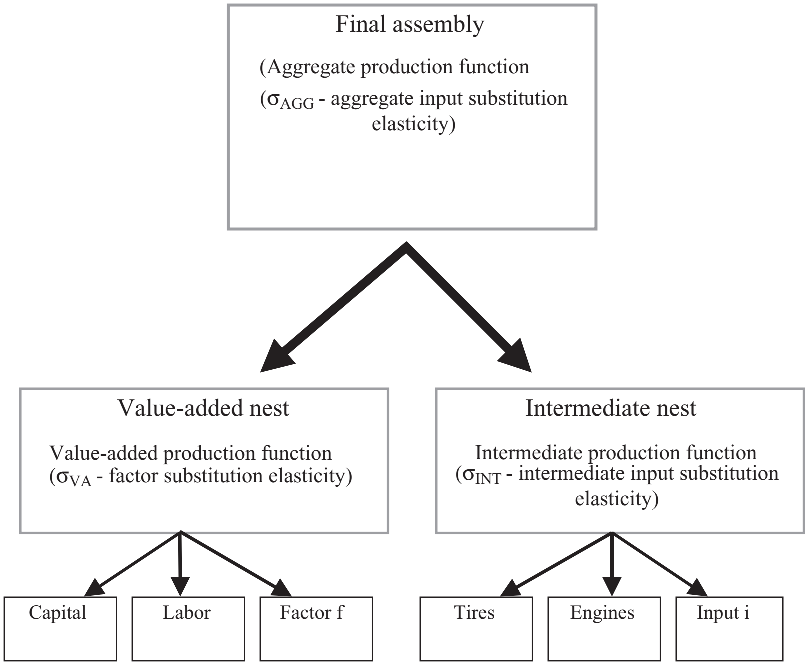

Figure 5.1. Technology tree for a nested production functions

Each tree branch is a subprocess with its own production function, or technology. The branches are called ***nested production functions*** because these smaller production processes are "nested" within the larger process of producing the final product. The twigs describe every input into the production process; each sprouts from the subprocess in which it is nested.

Figure 5.1 shows a technology tree that is typical of those assumed in standard CGE models. Notice how the figure shows two levels of the production process. At the bottom level are two nested production functions. The separate nests describe how the producer can combine labor and capital (and any other factors) into a value-added bundle that contains factor inputs, and how intermediate inputs, such as tires and engines, are combined to form an intermediate bundle. Moving above, an aggregate production function describes how the producer combines the value-added bundle with the intermediate bundle to make the final product, such as an auto.

A nested production function is a useful approach when the technologies of the component processes are substantially different. For example, an automaker may find that it is easy to substitute between workers and mechanized assembly equipment within the value-added bundle but that it is difficult to substitute more tires for one less steering wheel within the intermediate bundle. Nested production functions allow the modeler to describe realistically the different ways that subsets of inputs are combined with each other during the production process.

An additional advantage of nesting is that the selection of input combinations within each nested process is independent of the contents of other nests. This assumption about their separability simplifies the database and the solution of a CGE model considerably. Instead of making pair wise decisions among all inputs, the producer is instead assumed to make one decision about the contents of the intermediate bundle, a separate decision about the contents of the value-added bundle, and another decision about the ratio of the intermediate and value-added bundles in the final product. Changing the ratios of inputs within the intermediate bundle will not influence the ratios of inputs within the value-added bundle.

The specific type of production function, such as a Cobb-Douglas or Constant Elasticity of Substitution, that is assumed in each nest and for the final assembly, is determined by the modeler. A standard approach in CGE models is to assume functions that allow some substitution among factors of production in the value-added nest, but fixed input-output ratios in the intermediate nest and between the valued added and intermediate bundles. Later in this chapter, we describe in more detail the different types of production functions and their assumptions about input substitutability.

Sometimes, modelers choose to add additional nests to the production technology. CGE-based analyses of energy use and climate change, for example, usually add one or more levels of nesting to the value-added nest. Although the specific nesting structure varies across models, in general, climate models include nests that describe the substitution possibilities between labor, capital, and a bundle of energy inputs. Additional nests then describe substitution possibilities among different types of energy within an energy bundle, such as coal, oil, or gas. An advantage of adding nests is that it allows the modeler to describe subsets of inputs as complements, instead of substitutes, within the production process. Technical Appendix 5.1 provides a more detailed discussion of nesting in CGE models of climate change.

Intermediate Input Demand

Now we are ready to study the producer's economic decisions, focusing on one nest at a time. We start with the demand for intermediates, which has the simplest technology. This is because CGE modelers typically assume that intermediate inputs are used in fixed proportions to produce the bundle of intermediate goods. This means that, for any given input bundle, the producer has no ability to substitute more of one intermediate input for another.[2]

[2] This treatment is widely used in CGE models. However, some models provide the modeler with the flexibility to define a nonzero elasticity of substitution between intermediate inputs. In this case, the

Text Box 5.1. Climate Change, Emissions Taxes, and Trade in the CIM-EARTH Model
"Trade and Carbon Taxes" (Elliott, et al. 2010b).

What is the research question? Climate change is a function of global CO_2 emissions and the most efficient strategy to control them is to impose a uniform carbon tax wherever emissions occur. However, this approach presents a free-riding problem because nations have an incentive to not comply while gaining the benefits of reduced emissions elsewhere. How will carbon tax policies perform, given international trade, if countries adopt different carbon emissions tax rates?

What is the CGE model innovation? The researchers use CIM-EARTH, an open-source, recursive-dynamic, global CGE model with the GTAP v.7.0 database. The model places energy in the value-added nest, and extends that nest to describe substitution possibilities among energy sources in the production of goods and services.

What is the model experiment? The authors define four scenarios: (i) the baseline time path is business as usual, with no carbon tax; (ii) a carbon tax is applied uniformly across the globe; (iii) a carbon tax is applied to emissions only in Kyoto Protocol Annex B countries (who have pledged to cut emissions); and (iv) a carbon tax is applied to Annex B countries in combination with complete border tax adjustments that rebate their emissions taxes on exported goods and impose tariffs on emissions embodied in their imported goods. Carbon taxes in the last three scenarios range from $4 to $48 per ton of CO_2.

What are the key findings? A carbon tax applied worldwide at a uniform rate of $48 per ton of CO_2 reduces emissions by 40 percent from 2020 levels. Increasing tax rates yield ever smaller reductions in emissions because the least-costly carbon-reducing steps are taken first. A carbon tax imposed only in Annex B countries generates little more than one-third of the emission reduction achieved with a uniform, global tax, due in part to substantial "carbon leakage" as production shifts to nontaxing countries. With full import and export border tax adjustments, carbon leakage is halted.

For example, the production of an auto requires a bundle of intermediate inputs like rubber tires, engines, and mirrors. Furthermore, these inputs are ordinarily used in a fixed ratio. For each auto, the intermediate bundle must include four tires, one engine, and three mirrors. If the producer wishes to make another auto, he needs another bundle of auto parts – adding another wheel without an additional engine and so on, would not increase the number of intermediate bundles. This technology is often called a ***Leontief fixed proportions production function***. It is named after Wassily Leontief, an economist well known for his work on interindustry linkages in an economy.

technology in the intermediates nest is similar to that in the value-added nest, described in the next section.

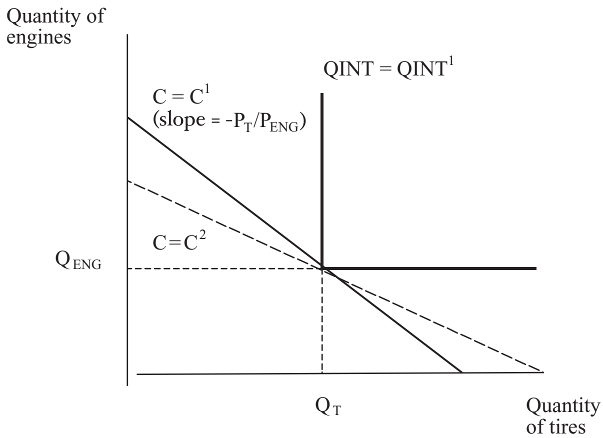

Figure 5.2. Nested production function – intermediate input demand

This type of intermediate production function offers a reasonable description of many intermediate production activities.

A Leontief production function is depicted graphically as an L-shaped curve, QINT, in Figure 5.2. The curve is an ***isoquant*** that shows all combinations of two inputs – in this case, tires and engines – that can be used to produce bundles of intermediate car parts of quantity $QINT^1$. The further an isoquant lies from the origin, the higher is the number of intermediate bundles that it represents. You can see from the isoquant's L-shape that increasing the amount of either tires or engines without increasing the quantity of the other input will not change the quantity of intermediate input bundles from level $QINT^1$.

The straight line in Figure 5.2, C, is an ***isocost*** line. It shows all combinations of engines and tires that cost the same total amount. The closer an isocost line lies to the origin, the lower is the total cost or outlay on tires and engines. The slope of an isocost line describes the ratio of input prices – in this case, the ratio of the tire price to the engine price, $-P_T/P_{ENG}$. The producer minimizes the cost of producing the input bundle $QINT^1$ when he operates at a point of tangency between the $QINT^1$ isoquant and the lowest attainable isocost line, which is C^1 in Figure 5.2, using the input bundle Q_{ENG} and Q_T.

The important property of a Leontief production function for CGE modelers to remember is that when relative input prices change, there is no change in the lowest-cost ratio of inputs for any level of QINT. Adding more of just one of the inputs would increase costs without increasing the number of intermediate bundles produced because the inputs must be used in fixed proportions. For example, assume that the price of tires falls relative to the price of engines, shown by the isocost curve, C^2. The lowest cost bundle of

Table 5.3. *Changes in Intermediate Input Demand When Relative Input Prices Change, with a Fixed Quantity of Intermediate Input Bundles (% Change From Base)*

	Production Activity		
Intermediate Input	Agriculture	Manufacturing	Services
Agriculture	0.00	0.00	0.00
Manufacturing	0.00	0.00	0.00
Services	0.00	0.00	0.00

Note: Because we assume that σ_{AGG} is zero, the change in demand for input i by activity j, remaining on the original isoquant, is approximately $qf_i - qo_j$.
Source: GTAP model, GTAP v.7.0 U.S. 3x3 database.

tires and engines remains unchanged. Because the ratio of input quantities does not change when input price ratios change, we say that the ***elasticity of intermediate input substitution elasticity*** σ_{INT}, of a Leontief production function is zero.

We demonstrate how a Leontief intermediate production function determines input demands in a CGE model by carrying out an experiment that changes relative intermediate input prices. We use the Global Trade Analysis Project (GTAP) model and the U.S. 3x3 database to run an experiment that imposes differing domestic sales tax rates of 5 percent on agriculture, 10 percent on manufactures, and 2 percent on services. Results, reported in Table 5.3, demonstrate that when holding output constant (i.e., we remain on isoquant $QINT^1$), there is no change in the quantities or ratios of intermediate inputs as their relative prices change. Thus, given a Leontief technology, the original proportions remain the least-cost mix of intermediate inputs for a given level of intermediate input bundles. However, if the output quantity changes, say by 5 percent, then demand for each intermediate input will also change by the same proportion, 5 percent, leaving the intermediate input ratios unchanged.

Factor (Value-Added) Demand

CGE models specify a ***valued-added production function*** to describe the technology in the nest in which producers assemble their bundle of factors (i.e., the combination of labor, capital, and other factors used in the final assembly stage). Most CGE modelers assume that producers have some flexibility with regard to the composition of the value-added bundle. For example, although the assembly of an auto requires fixed proportions of four tires and one engine, the mix of capital and labor used to assemble the parts into an auto is somewhat variable. The assembly process can use a lot of

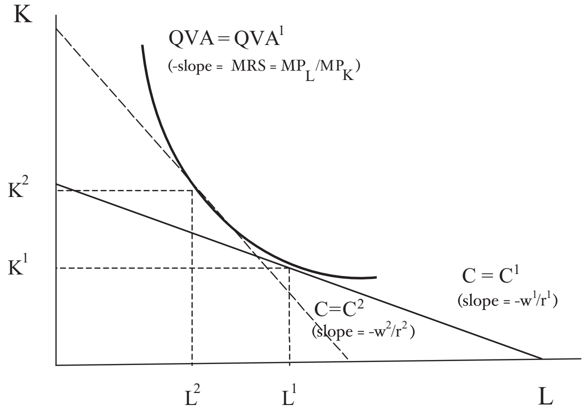

Figure 5.3. Nested production function – factor demand

manual labor and little machinery, or the process can be highly mechanized, depending on the relative cost of workers and equipment.

Figure 5.3 illustrates how producers choose the cost-minimizing factor ratio for a given quantity of value-added bundles. In the figure, an isoquant, QVA, describes the value-added production function. It depicts all technologically feasible combinations of two factors, capital, K, and labor, L, that can be used to produce the same value-added bundle, such as QVA^1. The negative of the slope at any point along the isoquant describes the marginal rate of substitution (MRS) between the two inputs. The MRS measures the amount by which capital could be reduced if the quantity of labor is increased by one unit, while keeping output constant. We can also express the MRS as the ratio of the marginal product of labor to the marginal product of capital, or MP_L/MP_K in the production of QVA.[3]

To visualize these concepts, assume that the producer described in Figure 5.3 moves downward along the isoquant, using less capital and more labor in the production of the value-added bundle. Notice that as the ratio of capital to workers declines, a smaller quantity of capital can be substituted for each additional worker to produce the same QVA. As an example, assume that the automaker moves downward on its isoquant, employing more labor and using less assembly equipment. As an increasing number of workers shares fewer assembly tools, each additional worker becomes a less productive input to the value added requirements of an auto, relative to an additional unit of equipment and tools. That is, as the K/L ratio falls, so does the inverse ratio of their marginal products, MP_L/MP_K.

[3] This is because, if d refers to a marginal change in quantity, then the slope at any point on the isoquant is $-dK/dL$, which is the rise over the run. Because the marginal product of L is dQVA/dL and of K is dQVA/dK, then the ratio $MP_L/MP_K = (dQVA/dL)/(dQVA/dK) = dK/dL$, which equals the MRS.

The isocost line, such as C^1 in Figure 5.3, describes all combinations of labor and capital that can be purchased for the same total cost. Its slope depicts the relative wage and capital rent at initial factor prices, w^1/r^1. The producer minimizes the cost of producing QVA^1 at the tangency between the isoquant and the lowest achievable isocost line, C^1, using input ratio L^1/K^1. At their tangency, the ratio of marginal products is equal to the price ratio: $MP_L/MP_K = w^1/r^1$. Rearranging (by multiplying both sides by MP_K/w^1), then $MP_L/w^1 = MP_K/r^1$. Input costs are minimized for a given QVA when the marginal product from an additional dollar spent on labor is equal to the marginal product from an additional dollar spent on capital inputs. If not, producers will spend more on the more productive factor input and less on the other input, until their marginal products per dollar spent are equalized.

Now consider how the cost-minimizing factor input ratio changes if there is an increase in wages relative to capital rents. The rise in the wage-rental price ratio is shown in Figure 5.3 by the dotted isocost line, C^2, with slope $-w^2/r^2$. As workers become relatively expensive, the producer can reduce costs by substituting them with machinery. In Figure 5.3, inputs L^2 and K^2 become the cost-minimizing ratio of capital to labor in the production of QVA^1.

The ***elasticity of factor substitution***, σ_{VA}, describes the relationship between changes in the capital-labor input ratio and the inverse ratio of their marginal products – that is, the curvature of the isoquant. When σ_{VA} is very large, the technology is flexible, and the isoquant becomes flatter. In this case, even large changes in factor intensities have little effect on factors' marginal products. Producers can therefore make large shifts in their capital-labor ratios to take advantage of changing relative factor prices without experiencing a sizeable change in either input's marginal product. For example, if wages fall relative to rents, an automaker could hire more labor and use far fewer tools, without causing labor productivity to decline relative to that of assembly equipment.

CGE modelers usually express σ_{VA} in terms of factors' prices instead of their marginal products but the two concepts are equivalent. Parameter σ_{VA} is the percentage change in the quantity ratio of capital to labor given a percentage change in the wage relative to capital rents. In the limit, when σ_{VA} approaches infinity, factors are perfect substitutes in the production process, and the isoquant is a straight line. In this case, a decrease in one input can always be offset by a proportional increase in another input without affecting either input's marginal product. A change in relative factor prices will therefore lead to large changes in factor proportions. At the other extreme, a parameter value of zero describes a value-added isoquant with an L-shape. With this technology, capital and labor are Leontief complements that must be used in fixed proportions. A change in relative factor prices does not result in a change in the factor ratio. For most industries, substitutability is

Table 5.4. *Factor Substitution Effects of a 5 Percent Increase in the Labor Tax in Manufacturing, with Different Factor Substitution Elasticities, Holding Output Constant (% Change from Base)*

	Manufacturing Activity	
	Capital-labor Ratio ($qf_K - qf_L$)	Wage/rental Ratio ($pfe_L - pfe_K$)
Elasticity = 1.2	0.68	.55
Elasticity = 8.0	2.42	.30

Source: GTAP model, GTAP v.7.0 U.S. 3x3 database.

likely to fall between these two extremes. Balistreri, McDaniel, and Wong's (2003) review of the econometric literature on this parameter found a range of estimated values clustered around a value of one.

Many CGE models use a constant elasticity of substitution (CES) valued-added production function to describe the value-added production technology, similar to the CES utility function studied in Chapter 4. It derives its name from the fact that the factor substitution elasticity remains constant throughout an isoquant (i.e., at any given factor input ratio). CGE modelers are usually restricted to specifying one elasticity parameter for each industry that governs all pairwise substitutions among the factors of production in the model.

The most important thing to remember about a value-added production function is that the ratio of factor input quantities can change when the relative prices of inputs change. Note, too, that if we allow substitution of one primary factor for another in the production process, the input-output coefficients for the factors, shown in Table 5.2, also change. This is not the case for the input-output coefficients for intermediate inputs when their ratios are assumed to be fixed (the "Leontief fixed proportions").

To illustrate these value added concepts, we use the GTAP model and the U.S. 3x3 database to explore the effects on factor input ratios when the cost of labor increases relative to the cost of capital. Our experiment is a 5 percent increase in the tax on labor employed in the manufacturing activity. We compare the effects of the tax when we assume a low factor substitution elasticity in manufacturing of 1.2 versus a large value 8, holding the quantity of value added bundles constant (i.e., remaining on the same isoquant). You can visualize this experiment in Figure 5.3 by imagining that we are observing the producer substituting between the two inputs along (a) a highly curved isoquant in the case of the low substitution elasticity value, and (b) a flatter isoquant in the case of a high elasticity value.

Our model results illustrate that the larger the elasticity parameter value, the larger is the producers' shift toward capital as labor costs rise (Table 5.4). Notice, too, that wages do not rise as much relative to rents when the

Text Box 5.2. Climate Variability and Productivity in Ethiopia
"Impacts of Considering Climate Variability on Investment Decisions in Ethiopia." (Block, Strzepek, Rosegrant, and Diao, 2006).

What is the research question? Extreme interannual rainfall variability that causes droughts and floods is common in Ethiopia. A model that describes climate using mean climate conditions (a deterministic model) does not capture the effects of year-to-year changes and extreme weather events. Would a stochastic model that incorporates both annual variability and the probabilities of extreme weather events result in different and more realistic estimates of production and climate effects on an economy?

What is the CGE model innovation? The authors develop annual climate-yield factors (CYF) by crop and agricultural zone within Ethiopia. The calculation of CYF's uses data on crop sensitivity to water shortages and 100-years of monthly rainfall data by zone. It also includes a flood factor, which decreases the CYF if the year is significantly wet or if the probability of flooding is high. CYF factors are used as multipliers of the technological productivity parameter in the production functions in the CGE model, which is an extension of the IFPRI standard CGE model.

What is the experiment? The 100 years of CYF data are divided into nine 12-year time periods from 1900 to 2000. The authors explore four time path scenarios for the 2003–15 period: (i) a base scenario assumes historic, exogenous growth in endowments and productivity with no new policy initiatives, (ii) an irrigation scenario adds to the base case the government's plans for expanded irrigation acreage, (iii) an investment scenario adds to the base case a planned increase in government spending on infrastructure, and (iv) a combined scenario assumes both irrigation and investment plans are realized. Model results are stochastic in the sense that all four scenarios are run assuming nine alternative weather patterns, producing an ensemble of outcomes.

What are the key findings? In the deterministic model, use of mean climate conditions is adequate when modeling drought, but this approach significantly overestimates the country's welfare when there are floods, which not only reduce agricultural yields but also lead to longer-term damage to roads and infrastructure and sustained losses in output.

production technology is more flexible. This is because even a large increase in the capital-labor ratio causes only a small change in the productivity (and price) of each input.

Combining Intermediate Inputs and Factors

At the top level of the assembly process, the producer combines the bundle of intermediate inputs with the bundle of factors to produce the final output. This aggregate technology is described by a production function in which the two bundles can be substituted according to an ***aggregate input elasticity of substitution***, $\sigma_{AGG,}$ similar to the value-added production function. In practice, this final stage of production is usually depicted as a Leontief fixed proportions technology, with σ_{AGG} assumed to equal zero. For any level of output, Q, a fixed ratio of intermediate bundles and factor bundle is required. The addition of another bundle of intermediates without also adding a bundle of factors (or vice versa), will not increase output.

Input Prices and Level of Output

Until now, we have explained how the cost-minimizing producer can (or cannot) substitute among inputs as their relative prices change, to produce a given level of output, and we have remained on the same isoquant. However, in our general equilibrium framework, a change in input prices will usually lead to a change in output prices and in consumer demand. As a result, the level of output can change, too, whenever input prices change. The producer may shift to a higher output level, on an outer isoquant, or reduce his output, on an inner isoquant. These output changes will also affect the quantities of inputs required, although not their ratios.

First, let's consider in more detail how a change in the price of an input works through consumer demand in the CGE model to affect the level of output. Labor union concessions, for example, might lower wage costs for automakers. If their technology allows it, automakers will substitute more labor for less equipment in their production process at any given production level. The more that producers can substitute toward labor (i.e., the larger is the elasticity of factor substitution), the lower their production costs will become. As production costs fall, then in perfectly competitive markets, so will auto prices. This point is illustrated in Figure 5.4 as the downward shift in the supply curve from S^1 to S^2. The movement from equilibrium Point A to Point B shows that the same quantity of output is now produced at a lower cost. Depending on consumer preferences, lower auto prices will stimulate consumer demand, so auto production and auto prices will increase, as shown by the movement from Point B to Point C. Increased output, in turn, leads to an increase in demand by the same proportion for all inputs. That is, a 10 percent increase in auto output will lead to a 10 percent increase in demand for both autoworkers and assembly equipment, as well as all intermediate inputs.

In Figure 5.5, we show more generally how the effects of a change in one input price – in this case, a fall in capital rent – on demand for both factor inputs can be decomposed into ***substitution effects*** and ***output effects***. (The alert student will find similarities between this exposition and our discussion of income and substitution effects on consumer demand in Chapter 4.) In the

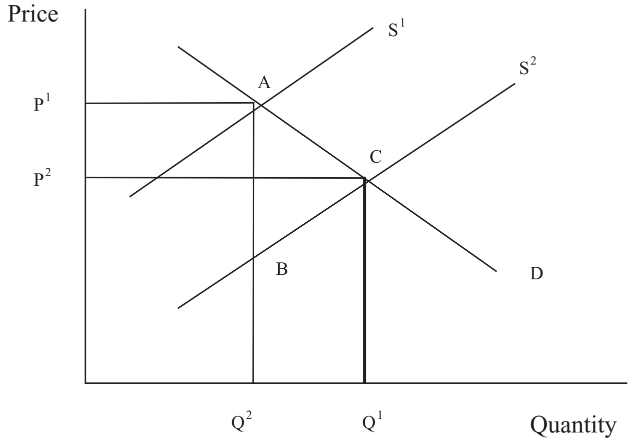

Figure 5.4. Input prices and level of output

figure, QO^1 is the initial level of output of QO, which is produced using the factor input ratio K^1/L^1. (You may notice that we have drawn the figure to show K and L as inputs into QO, instead of QVA, the value-added bundle. This is possible because we assume that the top of the nest requires a fixed proportion of value-added bundles in the production of QO.)

The slope of the isocost curve, C^1, describes the initial ratio of wages to rents, w/r^1. A fall in the price of capital is shown as isocost curve C^2, with slope $-w/r^2$. A decline in the cost of capital lowers the cost of production and leads to higher demand for the final product. Output increases to QO^2, using factor inputs quantities of K^3 and L^3.

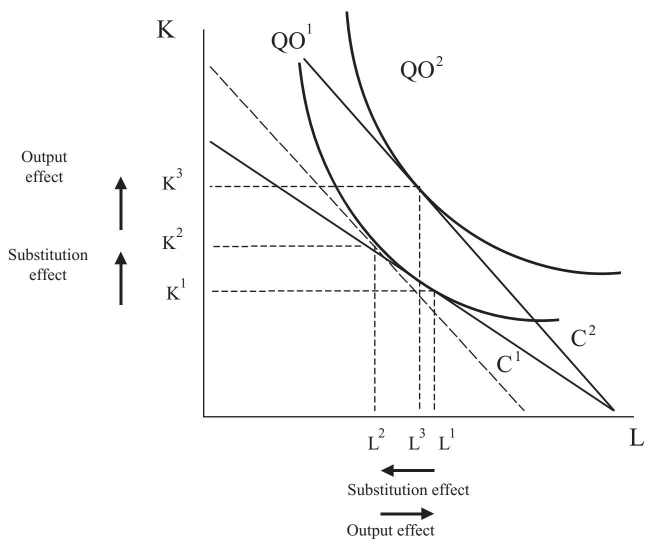

Figure 5.5. Input demand and output levels

Table 5.5. *Effects of a Fall in the Price of Capital Relative to Labor on Input Demand in U.S. Services Industry*

Input	Substitution Effect ($qf - qo$) or ($qfe - qo$)	Output Effect (qo)	% Change Input Demand (qf or qfe)
Intermediate inputs			
Agriculture	0.0	2.1	2.1
Manufacturing	0.0	2.1	2.1
Services	0.0	2.1	2.1
Factor inputs			
Capital	7.6	2.1	9.7
Labor	−2.7	2.1	−0.6

Note: The substitution effect is approximately $qfe - qo$.
Source: GTAP model, GTAP v.7.0 U.S. 3x3 database.

To measure the substitution effect, imagine that producers continue to produce QO^1 but purchase inputs at the new price ratio, shown as the dotted line drawn parallel to isocost curve C^2. The substitution effect measures the movement along the QO^1 isoquant to the tangency between the isoquant and the new isocost curve. As the relative price of capital falls, more capital and less labor are used in the production of QO^1. This change in the factor ratio, from L^1 and K^1 to L^2 and K^2, is the substitution effect. The movement from L^2 and K^2 to L^3 and K^3 is the output effect. It measures the change in factor demand due to the change in production quantity from QO^1 to QO^2, holding the factor prices constant at the new price ratio. The expansion of output leads to a proportionate increase in demand for both inputs.

To explore these concepts in a CGE model, we use the GTAP model with the U.S. 3x3 SAM to run an experiment in which capital rents fall relative to wages. The experiment assumes a 10 percent increase in the U.S. capital stock, which reduces economywide capital rents by 6.8 percent and increases U.S. wages by 0.2 percent. The percentage rise in the wage/rental ratio is therefore $0.2 - (-6.8) = 7.0$.

For brevity, we describe the results only for the U.S. services industry. The lower price of capital reduces the cost of the value-added bundle used in the production of services. In the new equilibrium, the consumer price of services declines by 1.6 percent, demand for domestically produced services increases 1.9 percent, and production of services rises 2.1 percent.

The effects on service's intermediate and factor input are reported in Table 5.5. The output effect increases demand for all intermediate and factor inputs by the same proportion as the change in services output, 2.1 percent. In the intermediate bundle, the substitution effects are zero because we assume a Leontief intermediate production technology with fixed input-output ratios.

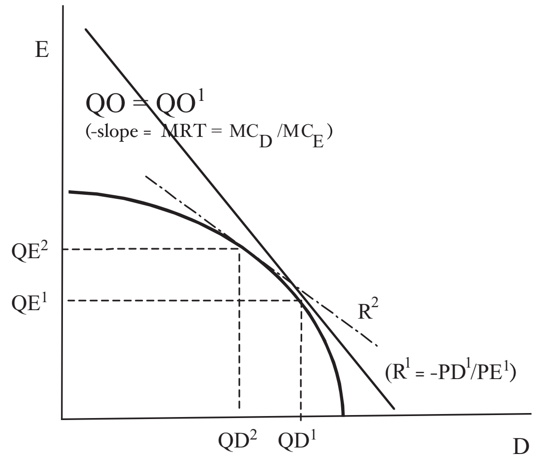

Figure 5.6. Export transformation function and a change in relative prices

In the value-added bundle, the substitution effect results from an increase in the wage-rental ratio, which causes the production of services to become more capital intensive. In total, the combined substitution and output effects stimulate service's demand for capital. In the case of labor, the negative substitution effect on labor demand outweighs the positive output effect and results in a decline in services' demand for labor.

Export Supply

In CGE models, an increase in price in the export market relative to that in the domestic market usually leads a producer to shift sales of his product toward exports, and vice versa. However, in some CGE models, the variety that is exported and the variety that is sold domestically are assumed to be two different goods, and the producer may not be able to readily transform his product line between them. Perhaps electric clocks require different electrical plugs when used in different countries, or a food item like beef may need to meet different consumer safety standards in each market. CGE models in which goods are differentiated by destination markets include an ***export transformation function*** to describe the technological flexibility of producers to transform their product between export and domestic sales.[4]

We depict the function as a ***product transformation curve***, shown in Figure 5.6. It shows all technologically possible combinations of the export, QE, and domestic, QD, varieties that can be produced from a given level of resources

[4] Mainly GAMS-based CGE models include export transformation functions. An early example is the single-country Cameroon model developed by Condon, et al. (1987). Others with this export treatment include the ERS-USDA CGE model (Robinson, et al. 1990), the IFPRI standard model (Lofgren, et al. 2002), the GLOBE model (McDonald, et al. 2007) and the TUG-CGE model (Thierfelder, 2009).

and that comprise the composite output quantity, QO. Perhaps QE and QD are European and American styles of the electric clocks, and QO is the total supply of electric clocks. The farther the transformation curve, QO, lies from the origin, the larger is the quantity of production of QO.

The most obvious difference between this function and the value-added production function that we have already studied is that the export transformation curve is drawn concave to the origin, while isoquants are convex. As we will show, its concave shape means that an increase in the price of QD or QE *increases* its use in the production of QO, whereas with the convex isoquant, an increase in an input price *decreases* its use.

The export transformation curve is otherwise similar in many respects to the value-added isoquant. The negative of its slope at any point describes the ***marginal rate of transformation*** (MRT), which measures the producer's ability to substitute QE for QD in the production of a given level of QO. We can also express the MRT as the ratio of the marginal costs of QD and QE in the production of QO, or MC_D/MC_E.

You can visualize why the two expressions for the curve's slope are equivalent by imagining a point on the curve in Figure 5.5 at which production is almost entirely specialized in exports. As the producer shifts toward domestic sales, the value of MRT becomes larger, because more units of QE must be given up for each additional unit of QD that is produced. This is because the inputs that are most productive when used in QD, and the least productive when used in QE, are the first to be shifted as QD output increases. As output of QD expands further, it draws in less and less productive inputs and QE retains only its most productive inputs. Therefore, the marginal cost of producing QD rises and the marginal cost of producing QE falls as production shifts toward QD.

The line in the figure, R^1, with slope $-PD^1/PE^1$, is an ***isorevenue*** line, where PD is the sales price of the good in the domestic market and PE is the *fob* export sales price. The isorevenue line shows all combinations of QE and QD that generate the same amount of producer revenue from the sale of QO. The further this line from the origin, the higher is producer revenue.

The producer's problem is to choose the ratio of export and domestic varieties for a given QO that maximizes his revenue – shown by the achievement of the highest attainable isorevenue line on any given product transformation curve. In Figure 5.6, revenue from output QO^1 is maximized at output ratio QE^1/QD^1. At this point, the transformation curve and the isorevenue line are tangent and $MC_D/MC_E = PD^1/PE^1$. Rearranging (by multiplying both sides by MC_E/PD^1) revenue is maximized where $MC_D/PD^1 = MC_E/PE^1$. That is, each additional dollar of revenue from QE and QD incurs the same marginal cost. If not, producers will produce more of the variety whose marginal cost is lower, and less of the variety whose marginal cost is higher, relative to its price.

Assume that the relative price of exports increases, as shown by the dotted line R^2 in Figure 5.6. The revenue-maximizing producer will increase the ratio of exports to domestic sales in output QO^1, to ratio QE^2/QD^2. The size of this quantity response depends on the curvature of the transformation curve, which is defined by the ***export transformation elasticity***, σ_E. The parameter defines the percentage change in the ratio of exports to domestic goods given a percentage change in the ratio of the domestic to the export sales price. It has the varieties are perfect substitutes in the composition of Q, then the transformation parameter has a negative value that approaches minus infinity and transformation curve becomes linear. In this case, a small change in the price ratio will result in a large change in the product mix.

CGE models that describe export transformation generally assume a constant elasticity of transformation (CET) function to describe the producer's decision-making.[5] The CET function derives its name from the fact that the export transformation elasticity is constant throughout the product transformation curve, and at any level of QO.

We illustrate the properties of an export transformation function in a CGE model by running an experiment that increases the world export price of U.S. manufactured goods by 5 percent. We use the U.S. 3x3 database in the TUG-CGE model, a single country model developed by Thierfelder (2009) that contains a CET export transformation function.[6] We compare the effects of the price shock on the quantity ratio of exports to domestic goods, using two different values of the export transformation elasticity parameter. As the parameter becomes larger and the transformation technology is more flexible, a 5 percent increase in the world export price elicits a larger export supply response from U.S. manufacturers (Table 5.6). Notice, too, that total output increases more when producers are relatively flexible in shifting toward export opportunities. Because the inputs are relatively suitable for use in the production of either variety, the marginal cost of producing additional exports does not rise as fast as in the low-elasticity case.

Summary

In this chapter, we examine production data in the SAM and producer behavior in the CGE model. Data in the SAM describe each industry's production

[5] See Powell and Gruen (1968) for a detailed presentation on the CET function.

[6] World export and import prices are assumed to be exogenous variables in this single-country CGE model.

Table 5.6. *Effects of a 5 Percent Increase in the World Export Price of U.S. Manufactured Exports on the Production of Exported and Domestic Varieties (% Change from Base)*

	Export Transformation Elasticity	
U.S. Manufacturing	0.8	4.0
Export/domestic price ratio	6.0	5.5
Export/domestic production ratio	4.9	24.5
Total manufacturing output	2.7	6.2

Source: TUG-CGE, GTAP v.7.0 U.S. 3x3 database.

technology, reporting its use of intermediate and factor inputs and any taxes paid or subsidies received. We use the SAM's production data to calculate input-output coefficients that describe the units of intermediate and factor inputs required per unit of output. Input-output coefficients are useful for characterizing production activities' intermediate factor-intensities, comparing input intensities across industries, and describing inter-industry linkages from upstream to downstream industries. We also use data in the SAM to calculate indices of forward and backward linkages among industries, another measure of industry interdependence.

CGE models break down the production technology into subprocesses that, when diagrammed, look like an upside-down tree. The trunk is the assembly of the final good; its branches are the subprocesses that are nested within the overall production process; and its twigs are the inputs used in each subprocess. Each subprocess and final assembly has its own production technology, cost-minimization equation, and input substitution elasticity parameter. In the intermediates' nest, producers decide on the cost-minimizing levels of intermediate inputs, and in the value-added nest, producers choose the cost-minimizing levels of factor inputs. Some CGE models include export transformation functions, which describe how producers allocate their output between exports and sales in the domestic market.

Key Terms

Backward linkage index
Downstream industries
Elasticity of export transformation
Elasticity of factor substitution
Elasticity of intermediate input substitition
Export transformation function

 https://doi.org/10.1017/CBO9780511975004.006

Factor intensity
Forward linkage index
Input-output coefficient
Isocost
Isoquant
Isorevenue
Leontief fixed-proportion production function
Nested production function
Output effect
Product transformation curve
Substitution effect
Technology tree
Upstream industries
Value-added production function

PRACTICE AND REVIEW

1. Use the U.S. SAM (in the Appendix), to describe the production technology of the U.S. services sector:

   Total intermediate inputs ______________________________
   Total factor payments ________________________________
   Total tax (and subsidy) _______________________________
   Value-added ____
   Gross value of output ________________________________

2. Data in exercise Table 5.7 describe the inputs purchased by manufacturing and services for their production process. Calculate the input-output coefficients for the two industries and report them in the table. Answer the following questions:
   a. In which factor is the production of manufacturing most intensive?
   b. In which factor is the production of services most intensive?
   c. Which industry is more labor intensive?
   d. Describe the upstream and downstream role of manufacturing.
3. Assume that you are CEO of a small firm. The introduction of a universal health insurance program has eliminated your health premium payments and lowered your cost per worker. Use a graph that describes your cost-minimizing choice of capital and labor shares in the value- added bundle, explain how the new program will change the labor-capital ratio in your production process, for a fixed level of value-added.
4. Consider the following results reported in Table 5.8, from a model with a nested production function. Can you infer from the results the percentage change in the industry's production, the possible types of production functions used in the each nest, and the likely change in relative factor prices that accounts for these results?

Table 5.7. *Input-Output Coefficients Exercise*

	Inputs Into Production		Input-Output Coefficients	
	Manufacturing	Services	Mfg.	Services
Labor	12	12		
Capital	8	18		
Manufacturing	10	50		
Services	20	20		
Gross value of output	50	100		

Technical Appendix 5.1: Inputs as Substitutes or Complements – Energy Nesting in Climate Models

The production functions used in CGE models describe inputs as substitutes or Leontief complements in the production process. However, in some cases, it may be more realistic to describe some inputs as true complements in the sense that an increase in one input price causes demand for the other input to fall. The presence of complementary inputs is especially important in the analysis of climate change. Climate change modelers often want to represent some degree of substitutability between capital and energy, yet characterize them as overall complements, at least in the short run. Capital-energy substitution assumptions are important because the estimated costs of reducing carbon emissions are lower the more flexible are production technologies.

CGE models used for climate change analysis typically move energy from the intermediate bundle into the value-added (VA) nest. Some models, including the example we study in this appendix, combine capital and energy into a composite bundle, KE, that is combined with labor in the VA nest, as illustrated in Figure 5.7. The modeler then adds a nest to describe how capital and energy are combined to produce the KE bundle. Other models combine capital and labor into a bundle, KL, that is combined with E. Both model

Table 5.8. *Effects of a Change in Factor Price on An Industry's Input Demand*

Input	Substitution Effect	Output Effect	% Change in Input Demand
Agriculture	0	3.5	3.5
Manufacturing	0	3.5	3.5
Services	0	3.5	3.5
Capital	–4	3.5	–0.5
Labor	6	3.5	9.5

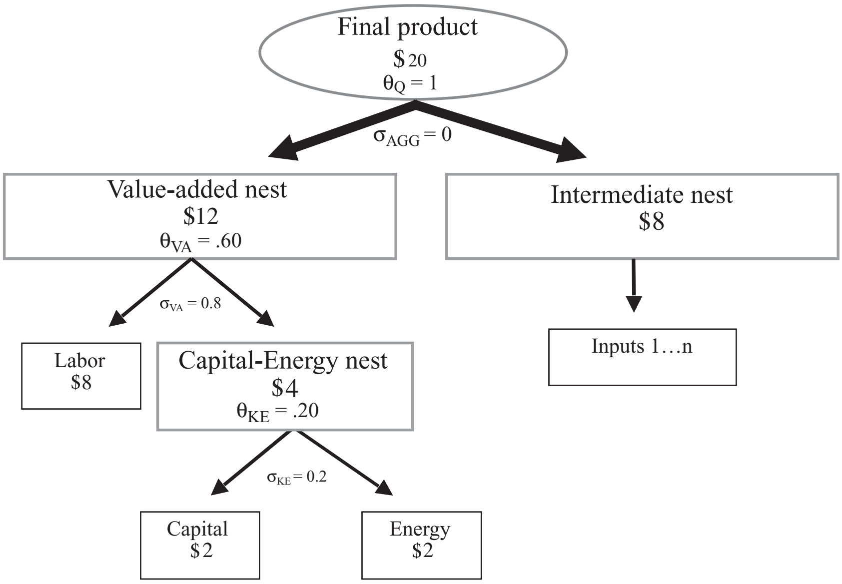

Figure 5.7. Technology tree with a KE-L nest

types usually add additional nests to describe substitution among energy types which, for brevity, we do not discuss.

Adding a KE nest to the value-added production function is a technique that allows modelers to describe K and E as overall complements while still allowing for a realistic amount of KE substitution. Suppose the price of energy rises. Within the KE nest, the quantity of energy demanded will fall and demand for capital will rise, to the extent that capital can be substituted for energy. Substitutability within the KE nest is likely to be quite low. For example, most machinery needs a certain amount of electricity to run properly. If the unit cost of the KE bundle still rises, then the producer will shift toward labor and away from the KE bundle in the higher level, VA nest. As demand for the KE bundle falls, demand for both capital and energy will fall by the same proportion. If the within-KE substitution effect dominates, then an increase in the energy price will cause demand for capital to rise – K and E are overall substitutes. If the VA substitution effect dominates, and the rise in the energy price causes demand for capital to fall – then K and E are overall complements.

Keller (1980) developed a formula to calculate the overall substitution parameter for nested inputs like capital and energy, σ^*_{KE}. His formula defines the parameter as a function of all three substitution effects – within KE, σ_{KE}; within VA, σ_{VA}; and at the top level of aggregation, σ_{AGG} – and of each nest's share in the total cost of the final product. Table 5.9 demonstrates how the overall substitution parameter is calculated, using the data shown in

Table 5.9. *Within-Nest and Overall Capital-Energy Substitution Parameters*

	Substitution Parameter			Share in Total Cost of Production			Overall K-E Substitution
	KE nest (σ_{KE})	VA nest (σ_{VA})	VA-Intermediate (top) nest (σ_{AGG})	KE (θ_{KE})	VA (θ_{VA})	Q (θ_Q)	σ^*_{KE}
Formula: $\sigma_{KE}\ (\theta_{KE}{}^{-1}) - \sigma_{VA}\ (\theta_{KE}{}^{-1} - \theta_{VA}{}^{-1}) - \sigma_{AGG}(\theta_{VA}{}^{-1} - \theta_Q{}^{-1}) = \sigma^*_{KE}$							
Base case	0.2	0.8	0	0.2	0.6	1	–1.66
High KE cost share	0.2	0.8	0	0.5	0.6	1	0.13
High KE substitution	0.9	0.8	0	0.2	0.6	1	1.83

Figure 5.7.[7] In this example, the cost share of the KE bundle, $\theta_{KE,}$ is \$4/\$20 = 0.20, and the KE substitution parameter is 0.2. The cost share of the VA bundle, θ_{VA}, is \$12/\$20 = 0.6 and the L-KE substitution parameter is 0.8. The elasticity parameter, σ_{AGG}, between VA and intermediate inputs, is zero. The cost share of the final product itself, θ_Q, is one.

Using Keller's formula, capital and energy inputs are overall complements, with an overall substitution elasticity parameter of minus 1.66. As illustrations, a change in the cost shares that gives more weight to the within-KE process causes its substitution effect to dominate, so capital and energy become overall substitutes, with a parameter value of .13. A change in relative elasticities, making capital and energy more substitutable in the KE nest, also causes the two inputs to become overall substitutes, with a parameter value of 1.83.

[7] For a more general statement of this formula for any number of nesting levels, see Keller (1980) and McDougall (2009).