Q1)

Ans) mean $= \theta_1$, variance $= \theta_2$

$$f(x_i)(x_i, \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_i)^2}{2\theta_2}}$$

Likelihood fn :-

$$L = f(x_1, \theta) \cdot f(x_2, \theta) \cdots f(x_n, \theta)$$

$$= \frac{1}{(\sqrt{2\pi\theta_2})^n} e^{-\frac{\Sigma(x_i - \theta_1)^2}{2\theta_2}}$$

$$\log L = -\frac{n}{2}\log 2\pi - \frac{n}{2}\log\theta_2 - \frac{\Sigma(x_i - \theta_i)^2}{2\theta_2}$$

$$\frac{\partial \log L}{\partial \theta_1} = -\frac{1}{2\theta_2} \cdot 2\,\Sigma(x_i - \theta_1) = 0$$

$$\Rightarrow \Sigma(x_i - \theta_1) = 0$$

$$\Rightarrow \boxed{\frac{\Sigma x_i}{n} = \theta_1}$$

$$\frac{\partial \log L}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{\Sigma(x_i - \theta_1)^2}{2\theta_2^2} = 0$$

$$\Rightarrow -n\theta_2 + \Sigma(x_i - \theta_1)^2 = 0$$

$$\Rightarrow \boxed{\theta_2 = \frac{\Sigma(x_i - \theta_1)^2}{n}}$$

Q2)

Ans) $B(m, \theta) = {}^nC_m \, \theta^m (1-\theta)^{n-m}$; $m = 0, 1, 2 \cdots n$

$$L(\theta) = \prod_{i=1}^{n} {}^nC_{m_i} \theta^{m_i} (1-\theta)^{n-m_i}$$

$$L(\theta) = \theta^{\Sigma m_i} (1-\theta)^{\Sigma(n-m_i)} \prod_{i=1}^{n} {}^nC_{m_i}$$

$$L(\theta) = k\, \theta^{\Sigma m_i} (1-\theta)^{n^2 - \Sigma m_i}$$

taking log :-

$$\ln L(\theta) = \ln k + \Sigma m_i \ln \theta + (n^2 - \Sigma m_i) \ln(1-\theta)$$

$$\frac{\delta \ln L(\theta)}{\delta \theta} = 0 + \frac{\Sigma m_i}{\theta} + \frac{(n^2 - \Sigma m_i)}{-(1-\theta)} = 0$$

$$\Rightarrow \quad \frac{\Sigma m_i}{\theta} = \frac{n^2 - \Sigma m_i}{(1-\theta)}$$

$$\therefore (1-\theta) \Sigma m_i = \theta (n^2 - \Sigma m_i)$$

$$\Rightarrow \quad \Sigma m_i = \theta n^2$$

$$\therefore \boxed{\theta = \frac{\Sigma m_i}{n^2}}$$