

Instructions

1. Attempt all questions.

Q.I) Solve the following:

[2]

(a) $3x(xy - 2)dx + (x^3 + 2y)dy = 0$

(b) $(2 \cos y + 4x^2)dx - x \sin y dy = 0$

Q.II) Find a homogeneous linear second order ordinary differential equation whose solution is the set of all straight lines in the xy-plane.

[2]

Q.III) State whether the following differential equations are linear or non-linear, justify and solve:

[2]

(a) $xy' + 2y = \frac{e^{3x}}{x}, x > 0$ with $y(1) = 1 + \frac{e^3}{3}$.

(b) $x^2y \frac{dy}{dx} - xy^2 = 1$.

Q.IV) If x^2 and 1 are solutions of $yy'' - xy' = 0$ then so is any linear combination of these. State true or false and justify.

[2]

Q.V) Find a linear ordinary differential equation for which the functions $e^{-x} \cos 2x$ and $e^{-x} \sin 2x$ are linearly independent solutions.

[2]

Q.VI) A tank contains 200 liters of mixture of water and fertilizer (in the ratio 10 to 1) which is diluted by adding a similar mixture (in the ratio 19 to 1) at a rate of 2 liters per minute. The mixture in the tank is kept uniform by stirring and the solution is sprayed on plants at the same rate. What is the amount of fertilizer in the tank after 5 minutes?

[2]

Q.VII) Prove that the family of curves $x^2 = 4c(y + c)$ is its own orthogonal trajectory. In other words prove that it is self orthogonal.

[2]