

# 除法原理

1.

$$\begin{array}{r} x^2 + (m+1) \\ x^2-1 \overline{) x^4 + mx^2 - nx + b} \\ \underline{x^4 - x^2} \phantom{+ b} \\ (m+1)x^2 - nx + b \\ \underline{(m+1)x^2 + 0 - (m+1)} \\ 0 \end{array}$$

$$\Rightarrow \begin{cases} n=0 \\ m+1=-b \end{cases} \Rightarrow (m, n) = \underline{(-7, 0)} \#$$

2.

$f(x) = f(x)$  除以  $(x-3)$  的餘數.

(建議可用分離係數法)

$$\begin{array}{r} 1250x^5 + \dots \\ x-3 \overline{) 1250x^6 - 2790x^5 - 3125x^4 + 707x^3 + 100x^2 - 43x - 12} \\ \underline{1250x^6 - 3750x^5} \phantom{+ b} \\ \dots \end{array}$$

(or)

$$\begin{array}{r} 1250 - 2790 - 3125 + 707 + 100 - 43 - 12 \\ \hline 3750 \quad 2880 \quad -735 \quad -84 \quad 48 \quad 9 \quad 3 \\ 1250 \quad 960 \quad -245 \quad -28 \quad 16 \quad 3 \quad \underline{-3} \end{array}$$

Ans: -3 #

3.

(a) 过  $(1, 3)$ ,  $(2, 5)$ ,  $(3, 9)$

(b.)

$f(x) = (x-1)(x-2)(x-3)Q(x) + ax^2 + bx + c$ , 代 (a.) 中的三點

$$\begin{cases} 3 = a + b + c \\ 5 = 4a + 2b + c \\ 9 = 9a + 3b + c \end{cases} \Rightarrow (a, b, c) = \underline{(1, -1, 3)} \#$$

4.

$$f(x) = g(x)(x-6) + (-98) \Rightarrow f(7) = g(7)(7-6) + (-98)$$

$$\Rightarrow g(7) = f(7) + 98$$

給  $f(x)$ , 問  $f(7)$   $\Rightarrow$  同第 2 題解法  $\Rightarrow$

$$\begin{array}{r} 1 - 8 + 9 - 15 + 100 \\ \hline 7 \quad -7 \quad 14 \quad -7 \quad 7 \\ \hline 1 \quad -1 \quad 2 \quad -1 \quad \underline{93} \end{array}$$

$$g(7) = f(7) + 98 = 93 + 98 = \underline{191} \#$$

$$5. \quad f(x) = (x^2 - 5x + 4)Q(x) + (x+2) = (x-4)(x-1)Q(x) + (x+2) \\ = (x^2 - 5x + 6)Q'(x) + (3x+4) = (x-3)(x-2)Q'(x) + (3x+4) \Rightarrow \begin{cases} f(1) = 3 \\ f(4) = 6 \\ f(2) = 10 \\ f(3) = 13 \end{cases}$$

(a.)  $f(1) = 3$

(b.)  $f(3) = 13$

(c.)  $f(x) = (x^2 - 4x + 3)g(x) + (ax+b) = (x-3)(x-1)g(x) + (ax+b)$ , 代  $f(1) = 3, f(3) = 13$

$$\Rightarrow \begin{cases} a+b=3 \\ 3a+b=13 \end{cases} \Rightarrow a=5, b=-2 \Rightarrow \text{餘式 } ax+b = \underline{5x-2} \#$$

(d.)  $f(x) = (x-4)(x-2)g(x) + (ax+b)$ , 代  $f(2) = 10, f(4) = 6 \Rightarrow ax+b = \underline{-2x+14} \#$

6.  $f(x) = (3x-2)Q(x) + 4$

(a.)  $f(x) = (x - \frac{2}{3}) \underbrace{[3Q(x)]}_{\text{新商}} + 4$

(b.)  $f(\frac{x}{3}) = (x-2)Q(\frac{x}{3}) + 4 \\ \Rightarrow f(\frac{x}{3}) = (\frac{x}{3}-1)[Q(\frac{x}{3}) \cdot 2] + 4$

(c.)  $f(2x) = (6x-2)Q(2x) + 4 \\ = (3x-1)[2Q(2x)] + 4$

(d.)  $xf(x) = \underbrace{(3x-2) \cdot [xQ(x)]}_{\text{等次, 不合}} + 4x$

(e.)  $f(x^2) = (3x^2-2)Q(x^2) + 4$

$\Rightarrow$  故選 (b.) #

7.

$$f(x) = (x^2 - 2x - 2)Q(x) + (6x-7) \Rightarrow (x-1)f(x) = \underbrace{(x^2 - 2x - 2)[(x-1)Q(x)]}_{\text{等次, 沒除乾淨}} + (6x-7)(x-1)$$

$$\Rightarrow \begin{array}{r} x^2 - 2x - 2 \overline{) 6x^2 - 13x + 7} \\ \underline{6x^2 - 12x - 12} \phantom{7} \\ -x + 19 \phantom{7} \end{array} \Rightarrow \underline{-x + 19} \#$$

8. 看成  $f(x) = x^4 - 13x^3 + 25x^2 - 37x + 56$ , 且問  $f(11) = ?$

則同第2題解法

$$\begin{array}{r|rrrrr} 1 & -13 & +25 & -37 & +56 & \\ & 11 & -22 & +33 & -44 & \\ \hline 1 & -2 & 3 & -4 & 12 & \end{array}$$

即  $f(11) = 12 \#$

$$\begin{array}{r|l}
 -2 & +4 & -29 & +18 & 3 \\
 -6 & 24 & -15 & & \\
 \hline
 -2 & 8 & -5 & & \textcircled{3} \rightarrow d \\
 -6 & 6 & & & 3 \\
 \hline
 -2 & 2 & & & \textcircled{1} \rightarrow c \\
 -6 & & & & 3 \\
 \hline
 \textcircled{-2} & \textcircled{-4} & & & \rightarrow b \\
 \downarrow & & & & \\
 a & & & & 
 \end{array}$$

$$(1) (a, b, c, d) = (-2, -4, 1, 3) \quad \#$$

$$(2) \text{ 即 } c(x-3)+d = (x-3)+3 = x \quad \#$$

$$\begin{aligned}
 (3) & -2(-0.02)^3 - 4(-0.02)^2 + (-0.02) + 3 \\
 & = -0.0016 - 0.02 + 3 = 2.9784 \quad \#
 \end{aligned}$$

10. 令  $(x-1)=A$ , 則  $(x+2)=A+3$ , 此題同第9題

$$\begin{array}{r|l}
 1 & +2 & +3 & +15 & -3 \\
 -3 & 3 & -9 & & \\
 \hline
 1 & -1 & 6 & & \textcircled{6} \rightarrow d \\
 -3 & 12 & -3 & & \\
 \hline
 1 & -4 & 18 & & \rightarrow c \\
 -3 & -3 & & & \\
 \hline
 \textcircled{1} & \textcircled{-7} & & & \rightarrow b \\
 \downarrow & & & & \\
 a & & & & 
 \end{array}$$

$$(1) (a, b, c, d) = (1, -7, 18, 6) \quad \#$$

$$(2) 3(x-1)+15 = 3x+12 \quad \#$$

$$\begin{aligned}
 (3) & (-0.01)^3 - 7(-0.01)^2 + 18(-0.01) + 6 \\
 & = -0.0007 - 0.18 + 6 = 5.8193 \quad \#
 \end{aligned}$$

## 多項式圖形

1. 過  $(1,0)$ ,  $(2,0)$ , 且  $(2,0)$  處重根 (和  $x$  軸相切), 領導係數為負  $\Rightarrow$  右側往下  
則選 (B)

2. (A) 線:  $a>0, b>0$   
=次:  $a>0, b>0, c<0$

(B) 線:  $a>0, b<0$   
=次:  $a>0, b>0, c>0$  ( $\rightarrow \leftarrow$ )

(C) 線:  $a<0, b<0$   
=次:  $a<0, b<0, c>0$

(D) 線:  $a>0, b<0$   
=次:  $a<0, b<0, c>0$  ( $\rightarrow \leftarrow$ )

(E) 線:  $a<0, b>0$   
=次:  $a>0, b>0, c<0$  ( $\rightarrow \leftarrow$ )  $\Rightarrow$  (A)(C)  $\#$

3. (A)  $a > 0$  (開口方向向上)

(D)  $b^2 - 4ac > 0$  (和  $x$  軸 2 交點)

(B)  $b < 0$  ( $-\frac{b}{2a} > 0, a > 0, \rightarrow b < 0$ )

(E)  $9a - 3b + c > 0$  ( $x = -3$  時, 函數值為正)

(C)  $c > 0$  (和  $y$  軸交於正)

$\Rightarrow (A)(C) \#$



4. (1)  $y = a(x-1)^2 + (1+a) - a$

$= a(x-1)^2 + 1$

則頂點  $= (1, 1)$ , 最小值  $= 1 \#$

(2) 2 點代入

$\begin{cases} a+1=b \\ a+1=c \end{cases} \Rightarrow b-c=0 \#$

5.  $-2a^2 + 4a + 2019 = -2(a+4)^2 + 4(a+4) + 2019$

(或用對稱看)

$\Rightarrow 0 = -16a - 32 + 16 \Rightarrow a = -1 \#$

6. 對稱中心即  $(h, k)$

$f(x) = 4(x^3 - 9x^2 + 27x - 27) - 2x + 13 = 4(x-3)^3 - 2(x-3) + 7$

$(p, h, k) = (-2, 3, 7), (h, k) = (3, 7) \#$

7.  $f(x) = a(x-1)^3 + p(x-1) + (-9) = ax^3 - 3x^2 + bx + 2$

$\Rightarrow \begin{cases} -3ax^2 = -3x^2 \\ (a+p)x = bx \\ -a-p-9 = 2 \end{cases} \Rightarrow \begin{cases} a=1 \\ p=-12 \\ b=-11 \end{cases}$

$-20 = (c-1)^3 - 12(c-1) - 9$

$-11 = (c-1)(c^2 - 2c - 11)$

$1 \times -11 = (2-1)(2^2 - 2 \times 2 - 11) \Rightarrow c=2 \#$

8.  $(3, b), (5, b)$  對稱於  $(4, b)$ , 又  $\equiv$  次函數僅一對稱中心  $\Rightarrow$  所求  $= (4, b) \#$

(or)

$f(x) = [(x-4)+1](x-4)[(x-4)-1] + b = (x-4)^3 - (x-4) + b \#$

9.

$$f(x) \text{ 在 } p \text{ 处 } \Rightarrow x \rightarrow x-p \quad y \rightarrow y-q$$

$$\Rightarrow f(x)' = 2(x-p)^3 - 3(x-p) + q = g(x) = 2x^3 - 6x^2 + 3x + 5$$

$$\Rightarrow 2(x-p)^3 - 3(x-p) + q = 2(x-1)^3 - 3(x-1) + 4 \Rightarrow (p, q) = (1, 4) \#$$

10.

$$(x-2)^2 + a(x-2) + b-3 \text{ 过 } (2,0), (4,0), \text{ 对称轴 } x=3$$

$$\Rightarrow \begin{cases} b-3=0 \Rightarrow b=3 \\ 4+2a+b-3=0 \Rightarrow a=-2 \end{cases}$$

$$\Rightarrow (a, b) = (-2, 3) \#$$

$$(2) \text{ 最小值即 } x=3 \text{ 时, 值} = (3-2)^2 - 2(3-2) + 3-3 = 1-2 = -1 \#$$

### 多项式不等式

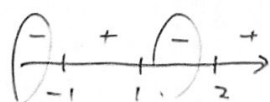
$$1. (1) \underbrace{(x-2)^3}_{(x-2)^2 \text{ 恒正}} \underbrace{(x-1)^2}_{\text{恒正}} \underbrace{(2x^2+2x+1)}_{2>0, 2^2-4 \times 2 \times 1 < 0 \Rightarrow \text{恒正}} < 0 \Leftrightarrow \underline{x-2 < 0} \Rightarrow \underline{x < 2} \#$$

$$(2) \frac{(x^2-4)(x+2)}{(x^2-1)} \leq 0 \Rightarrow (x^2-4)(x+2)(x^2-1) \leq 0 \Rightarrow (x-2)(x+2)^2(x+1)(x-1) \leq 0$$

$$(x^2-1 \neq 0) \quad (x^2-1 \neq 0), (x=2)$$

$$\Rightarrow (x+1)(x-1)(x-2) \leq 0$$

$$(x^2-1 \neq 0, x=2)$$

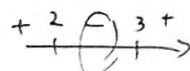


$$\rightarrow \begin{cases} 1 \leq x \leq 2 \\ x \leq -1 \end{cases} \text{ 且 } \begin{cases} x^2-1 \neq 0 \Rightarrow x \neq \pm 1 \\ x=2 \end{cases}$$

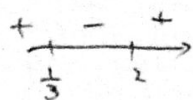
$$\Rightarrow \begin{cases} 1 < x \leq 2 \\ x < -1 \end{cases} \#$$

$$(3) \cancel{(x+1)^8} \cancel{(x^2+x+1)} (x-2)(x-3) \cancel{(x-4)^4} < 0$$

$$= (x-2)(x-3) < 0 \Rightarrow 2 < x < 3 \#$$



$$2. \frac{1}{3} < x < 2 \Rightarrow (x-2)(x-\frac{1}{3}) < 0 \Leftrightarrow (ax^2+bx-2) > 0$$



異號  $\Rightarrow a < 0$  (才能變號)

$$(x-2)(x-\frac{1}{3}) < 0 \Leftrightarrow (x^2 + \frac{b}{a}x - \frac{2}{a}) < 0$$

$$\text{比較} \Rightarrow a = -3, b = 7 \Rightarrow (a, b) = (-3, 7) \#$$

3. (同2.)  $-2 \leq x \leq 1$

$\begin{array}{c} + & - & 2 & (-) & + \\ \hline & & & & \end{array} \Rightarrow (x-1)(x+2) \leq 0 \Leftrightarrow (ax^4 - 2x^3 + bx + 2) \leq 0, (a > 0)$

表示  $(x-1)(x+2) (= \pi \text{ 式}) = (ax^4 - 2x^3 + bx + 2)$ , 表示  $ax^4 - 2x^3 + bx + 2$  被  $(x-1), (x+2)$  整除

$\Rightarrow \begin{array}{r} a(-2-a) + (3a+2) \\ 1 + 1 - 2 \quad \begin{array}{r} a \quad -2 \quad +0 \quad +b \quad +2 \\ a \quad +a \quad -2a \end{array} \\ \hline (-2-a) \quad 2a \quad +b \\ -2-a \quad -2-a \quad +4+2a \\ \hline 3a+2 \quad b-4-2a \quad 2 \\ 3a+2 \quad 3a+2 \quad -6a-4 \\ \hline 0 \end{array}$

$\Rightarrow \begin{cases} -6a-4=2 \\ b-4-2a=3a+2 \end{cases} \Rightarrow \begin{cases} a=-1 \\ b=1 \end{cases}$

但  $a > 0$ , 所以矛盾  
此題無解

(or)  $ax^4 - 2x^3 + bx + 2$ , 代  $x=1, x=-2$  會  $= 0$

$\Rightarrow \begin{cases} a+b=0 \\ 16a+1b-2b+2=0 \end{cases} \Rightarrow \begin{cases} a=-b \\ -16b-2b=-2-1b \Rightarrow b=1 \end{cases} \Rightarrow \begin{cases} a=-1 \\ b=1 \end{cases}$

4. 恆正  $\Rightarrow k > 0$   
 $2^2 - 4 \cdot k \cdot k < 0 \Rightarrow \begin{cases} k > 0 \\ 4 < 4k^2 \end{cases} \Rightarrow \begin{cases} k > 0 \\ k > 1, k < -1 \end{cases} \Rightarrow \boxed{k > 1} \#$

5. 無實數解 = 和  $x$  軸無交點  $\Rightarrow b^2 - 4ac < 0 \Rightarrow (-2k)^2 - 4(k+2)(2k-3) < 0$

$\Rightarrow 4k^2 - 8k^2 - 4k + 24 < 0 \Rightarrow 4k^2 + 4k - 24 > 0 \Rightarrow \begin{cases} k > 2 \\ k < -3 \end{cases} \#$

6.  $((a+1)x^2 + ax + 7) > (-x+5) \Rightarrow (a+1)x^2 + (a+1)x + 2 > 0$  恆成立

$\Rightarrow \begin{cases} a+1 > 0 \\ (a+1)^2 - 4(a+1) \cdot 2 < 0 \end{cases} \Rightarrow \begin{cases} a > -1 \\ a < 7 \end{cases} \Rightarrow -1 < a < 7 \#$

7.  $mx^2 + x + 4 \geq 3x + 2 \Rightarrow mx^2 - 2x + 2 \geq 0$   
恆正  $\begin{cases} m > 0 \\ (-2)^2 - 4 \cdot 2 \cdot m \leq 0 \end{cases} \Rightarrow \begin{cases} m > 0 \\ m \geq \frac{1}{2} \end{cases} \Rightarrow m \geq \frac{1}{2} \#$   
可以等於

8. (a.)  $-2 < t < 3 \Leftrightarrow (t-3)(t+2) < 0 \Rightarrow f(2t) = (2t-3)(2t+2) < 0$   
設  $= f(t) \Rightarrow -1 < t < \frac{3}{2} \#$   
 $\begin{array}{c} + & - & + \\ \hline & & \end{array} \quad \begin{array}{c} -1 & \frac{3}{2} \end{array}$

(b.)  $(x-2-3)(x-2+2) < 0 \Rightarrow (x-5) \cdot x < 0$   
 $\begin{array}{c} + & - & + \\ \hline 0 & 5 \end{array} \Rightarrow 0 < x < 5 \#$



# 回家練習

1.  $x^4 - 10x^3 + 11x^2 - 15x + 16$ , 且求  $f(9) = ?$

$$\begin{array}{r|rrrrr} 1 & -10 & +11 & -15 & +16 & \\ & 9 & -9 & 18 & 27 & 9 \\ \hline 1 & -1 & 2 & 3 & 43 & \end{array}$$

$$f(9) = 43 \neq$$

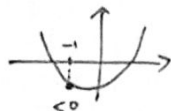
2.  $(\alpha, \beta)$  在  $f(x)$  上  $\Rightarrow (\alpha, \beta)$  對稱於  $(2, 4)$

(A) 不確定, (B)  $(-\alpha, -\beta)$  代入,  $-\beta = a(-\alpha-2)^2 + b(-\alpha-2) + 4$ , 無法由題目推來, 不確定

(C)  $\frac{4-\alpha+\alpha}{2} = 2, \frac{8-\beta+\beta}{2} = 4$ ,  $(2, 4)$  為  $(\alpha, \beta)$  和  $(4-\alpha, 8-\beta)$  的對稱中心, 可以選.

(D)  $f(x)$  過  $(2, 4)$  (E) 不確定  $\Rightarrow$  (C), (D)

3. (A)  $\underbrace{a > 0}, \underbrace{b > 0}, \underbrace{c < 0}$  (B)  $\underbrace{a-b+c < 0}$  (C)  $b^2 - 4ac > 0$  (和  $x$  軸交兩點)  
(開口方向) ( $\frac{b}{2a} < 0$ ) 軸截距 代  $x = -1$  看圖 又  $a \cdot c < 0$



$$\underbrace{b^2 - 4ac}_{>0} - \underbrace{ac}_{>0} > 0$$

(D) 看圖  $\frac{-b}{2a} > -1$

$\Rightarrow -b > 2a$  (乘的時候先確認  $a > 0$ )  
 $\Rightarrow b - 2a < 0$  (若  $a < 0$  要變號)

(E)  $x = -2$  時,  $(-2)^2 \cdot a - 2b + c > 0$  (看圖)

$$\Rightarrow 4a - 2b + c > 0$$

又  $x = 1$  時,  $a + b + c > 0$  (看圖)

$$\text{故 } (4a - 2b + c) + (a + b + c) > 0$$

$$\Rightarrow 5a - b + c > 0 \neq$$

$\Rightarrow$  (A)(C)(E)  $\neq$

4. 即  $f(-1) = (-1)^{100} - 99 - 5 = 1 - 99 - 5 = -103 \neq$

5.  $f(x) = (x-2)Q(x) + 3$   $\Rightarrow 2f(x) + g(x) = (2(x-2)Q(x) + 6) + ((x-2)Q'(x) + 5)$   
 $g(x) = (x-2)Q'(x) + 5$   
 $= (x-2) \underbrace{[2Q(x) + Q'(x)]}_{\text{新商}} + \underbrace{(6+5)}_{\text{餘}} \Rightarrow \text{餘式} = 11 \neq$   
餘式除式一次, 可以

6.  $a = -2$  (領導係數)

$$-2x^3 + bx^2 - 8x + 8 = (-2x^3 - 6bx^2 - 6b^2x - 2b^3) + (px + bp) + k$$

比較  $\Rightarrow \begin{cases} -6b = b \Rightarrow b = -1 \\ -8 = -6b^2 + p \Rightarrow p = -2 \\ 8 = -2b^3 + bp + k \Rightarrow k = 4 \end{cases} \Rightarrow (a, b, p, k) = (-2, -1, -2, 4)$ ; 對稱中心 =  $(-1, 4)$   $\neq$

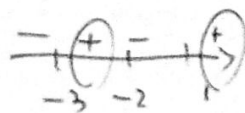
7. 設  $f(x) = ax^3 + bx^2 + cx + d$ , 代四點解聯立

or

設  $f(x) = a(x-1)(x+2)(x-k)$  1°  $f(3)=90, f(0)=-b$

$$\Rightarrow \begin{cases} 10(3-k) \cdot a = 90 \\ ak = -3 \end{cases} \Rightarrow k = -3, a = 1$$

$$\rightarrow \text{得 } f(x) = (x-1)(x+2)(x+3) \geq 0$$



$$\Rightarrow \underline{-3 \leq x \leq -2 \text{ or } x \geq 1} \quad \#$$

8.

$$\begin{array}{r|rrrr} 1 & -6 & +4 & +42 & \\ & -2 & 16 & -40 & -2 \\ \hline 1 & -8 & 20 & & \textcircled{2} \rightarrow d \\ & -2 & 20 & & -2 \\ \hline 1 & -10 & & & \textcircled{0} \rightarrow c \\ & -2 & & & -2 \\ \hline \textcircled{1} & \textcircled{-12} & & & \rightarrow b \\ & \downarrow & & & a \end{array}$$

$$(1) (a, b, c, d) = (1, -12, 0, 2) \quad \#$$

(2)  $x = -2$  附近  $\Rightarrow (x+2)$  沒錯 (代  $-2$  附近的值會很小)

$$\text{則取一次近似 } c(x+2) + d = 0 + 2 = 2 \quad \#$$

$$(3.) \underbrace{(0.01)^3}_{0} - 12(0.01)^2 + 0 + 2 = 2 - 0.0012 = 1.9988 \approx 1.999 \quad \#$$

9.

$$(1) -x^2 + 2x - 5 = -(x-1)^2 - 4 \Rightarrow \underline{\text{最大值} = -4} \quad \#$$

$$(2) \text{變數代換 } -x^2 + 2x - 5 = A \Rightarrow A^2 + 2A + 4 = (A+1)^2 + 3 \Rightarrow \text{最小值} = 3$$

不確定  $A$  可否為  $-1 \Rightarrow$  回算  $A = -1 \Rightarrow -x^2 + 2x - 5 = -1 \Rightarrow x^2 + 2x + 4 = 0$  (無解)

$A$  不可以是  $-1 \Rightarrow$  則取離  $-1$  最近的  $A$  值.



根據 (1) 所算  $A = -x^2 + 2x - 5$  最大  $= -4$

即  $-4$  是離  $-1$  最近的可行  $A$  值

$$\text{故 (2) 之最小值} = (-4)^2 + 2(-4) + 4 = 16 - 8 + 4 = 12 \quad \#$$

$$10. x^2 + (4-8m)x + (15m^2 - 2m - 4) > 2x + 3$$

$$\Rightarrow x^2 + (2-8m)x + (15m^2 - 2m - 7) > 0 \quad \text{恒正} \Rightarrow \begin{cases} 1 > 0 \quad (\#2) \\ (2-8m)^2 - 4(15m^2 - 2m - 7) < 0 \end{cases}$$

$$\Rightarrow 4 - 32m + 64m^2 - 60m^2 + 8m + 28 < 0 \Rightarrow 4m^2 - 24m + 32 < 0 \Rightarrow m^2 - 6m + 8 < 0$$

$$\Rightarrow (m-4)(m-2) < 0$$

$$\Rightarrow \underline{2 < m < 4} \quad \#$$



11. (1)  $(x-1)(x-3)(x-2)^2(x^2-x+5) < 0 \Rightarrow 1 < x < 3$  #

$\downarrow$   
 $1 > 2, 1 - 4.5 < 0 \Rightarrow$  恆正

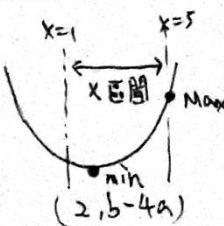
(2)  $(x-1)(x-3)(x-2)^2(x^2-x+5) \leq 0 \Rightarrow \begin{cases} (x-1)(x-3) \leq 0 \\ x=2 \end{cases} \Rightarrow 1 \leq x \leq 3$  #  
 消掉, 但要補上  $(x=2)$   
 $\downarrow$   
 同時有滿足  $x=2$  的可能

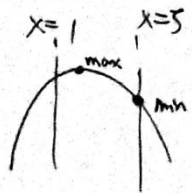
12. 同前第9題 (多項式圖形)

13.  $ax^2+2ax \leq 2ax^2+4a$   
 $\Rightarrow ax^2-2ax+4a \geq 0 \Rightarrow \begin{cases} a > 0 \\ 4a^2-4 \cdot 4a \cdot a \leq 0 \end{cases} \Rightarrow \begin{cases} a > 0 \\ a(a-4) \leq 0 \end{cases} \Rightarrow \begin{cases} a > 0 \\ 0 \leq a \leq 4 \end{cases}$   
 $\Rightarrow 0 < a \leq 4$  #  
 可以等於

14.  $(x-1)f(x) = (x^2-2x+2)Q(x) + \underline{(3x+4)}$   
 推  $f(x) = (x^2-2x+2)Q'(x) + \underline{(ax+b)}$   
 $\Rightarrow (x-1)f(x) = (x^2-2x+2)[(x-1)Q(x)] + (ax+b)(x-1)$   
 等次, 不行.  
 $\Rightarrow x^2-2x+2 \overline{ax^2+(b-a)x-b} \Rightarrow \begin{cases} b-a+2a=3 \\ -b-2a=4 \end{cases} \Rightarrow \begin{cases} a=-7 \\ b=10 \end{cases} \Rightarrow ax+b = -7x+10$  #  
 $\underline{3x+4}$

15.  $ax^2-4ax+b = a(x-2)^2+b-4a$ ; 題目若沒提  $a < 0$ , 要自己討論.

If  $(a > 0)$    
 $\Rightarrow \text{Min} = b-4a = -6 \Rightarrow b = -6+4a$   
 $\Rightarrow \text{Max} = 25a - 20a + b = 12$   
 $\Rightarrow 25a - 20a + 4a - 6 = 0$   
 $\Rightarrow 9a - 6 = 0$   
 $\Rightarrow a = \frac{2}{3}$  #

If  $(a < 0)$    
 $\Rightarrow \text{Max}: b-4a = 12 \Rightarrow b = 12+4a$   
 $\Rightarrow \text{Min}: 25a - 20a + b = -6$   
 $\Rightarrow 25a - 20a + 12 + 4a = -6$   
 $\Rightarrow 25a - 16a + 18 = 0$   
 $\Rightarrow 9a + 18 = 0 \Rightarrow \boxed{a = -2}$   
 $\boxed{b = 4}$  #

16.  $f(x-3) = 2(x-3)^4 + 3(x-3)^3 + 5(x-3)^2 + 9(x-3) + 2 = (x-1)Q(x) + r$   
 $x=1 \Rightarrow 2 \cdot (-2)^4 + 3(-2)^3 + 5(-2)^2 + 9(-2) + 2 = r$   
 $\Rightarrow r = 12$  #