Theoretical Framework of Random Walks and Their Boundaries with 2D Game Applications

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CS226 | Mathematical Modeling Applications
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Date: 02 December 2024

Introduction to Random Walks

Random walks are a cornerstone concept in statistical physics, mathematics, and computer science, providing a framework to describe a sequence of steps taken in a random manner. This concept models diverse phenomena, such as diffusion, market fluctuations, and search algorithms. Continuous Time Random Walks (CTRWs) extend the basic principle by incorporating probabilistic waiting times between successive steps, making them powerful tools for studying transport processes across various domains.

In their work, Continuous Time Random Walks in Finite Domains and General Boundary Conditions, van Milligen, Calvo, and Sánchez (2008) focus on CTRWs within finite domains. They explore the influence of boundary conditions, such as reflecting and absorbing boundaries, on system behavior. These boundary conditions are significant for practical applications because they model real-world constraints on random movements.

Summary of Key Concepts

1. **Finite Domains and Boundary Conditions**
   - Reflecting boundaries redirect random walkers back into the domain, ensuring particles remain within the specified space.
   - Absorbing boundaries remove particles upon contact, effectively terminating their trajectory.
   - Mixed boundaries combine these behaviors, allowing for customizable interactions with edges.
2. **Mapping Finite Domains to Infinite Domains** Finite-domain CTRWs can be reformulated into infinite domains with modified step size distributions, enabling the application of techniques developed for infinite domains.
3. **Fractional Differential Operators** Fractional calculus allows for modeling non-local transport processes within CTRWs, illustrating how microscopic randomness projects onto macroscopic diffusion equations.

Random Walks in 2D Games

Random walks and their boundary conditions find natural analogies in gaming, where characters or entities move within confined spaces.

The Snake Game as a Representation of Random Walk Boundaries

In the classic Snake game, the snake moves step by step within a grid-based 2D domain. Its behavior can model boundary interactions:

- **Boundary Types**:
  - Reflecting boundaries occur when walls bounce the snake back into the play area.
  - Absorbing boundaries terminate the game when the snake collides with a wall.
  - Periodic boundaries allow the snake to exit one side of the grid and reappear on the opposite side.
- **Analytical Insights**:
  - Snake movement can be modeled as a constrained random walk, particularly when user input is randomized.
  - Each boundary condition reflects a distinct mathematical framework for controlling random walk behavior at the edges.

Reflecting boundaries simulate scenarios where entities remain within a domain. Mathematically, this requires adjusting the probability distribution to ensure the walker is redirected back into the domain without being lost.

Application of Theory

CTRWs provide a robust framework for designing experiments, such as:

1. Simulating random walkers on a constrained 2D grid with various boundary conditions.
2. Comparing these models with game mechanics to analyze the impact of edge interactions on gameplay or user experience.

By bridging the theoretical understanding of CTRWs with practical examples like the Snake game, this work illustrates the versatility of these mathematical tools in real-world applications. Analogies such as this enhance the understanding of random walks while inspiring innovation in game design and other applied fields.

References

van Milligen, B. P., Calvo, I., & Sánchez, R. (2008). Continuous time random walks in finite domains and general boundary conditions: Some formal considerations. *Journal of Physics A: Mathematical and Theoretical, 41*(21), 215004. https://doi.org/10.1088/1751-8113/41/21/215004