$$u_{tt} - u_{xx} = 0, \quad x \in (a,b), t \in (0,T)$$

$$u(x,0) = f(x), \quad \forall x \in (a,b)$$

$$u_t(x,0) = g(x), \quad \forall x \in (a,b)$$

$$(*) \; u(a,t) = u(b,t) = 0, \quad \forall t \in (0,T)$$

<u>Ley de energía</u>

$$(**) \int_\Omega u_{tt} u_t - \underbrace{\int_\Omega u_{xx} u_t}_{} = 0$$

$$\int_\Omega u_{xx} u_t = -\int_\Omega u_x u_{xt} + \underbrace{u_x u_t \Big|_a^b}_{}$$

Usando $(*)$ $\Rightarrow$ $u(a,t) = u(b,t) = 0, \forall t$

$$u_t(a,t) = u_t(b,t) = 0$$

$$\Rightarrow u_x u_t \Big|_a^b = u_x(b,t)\, u_t(b,t) - u_x(a,t)\, u_t(a,t) = 0$$

Volviendo a $(**)$:

$$0 = \int_\Omega u_{tt} u_t - \int_\Omega u_{xx} u_t = \int_\Omega u_{tt} u_t + \int_\Omega u_x u_{xt}$$

$$= \frac{1}{2}\frac{d}{dt}\int_\Omega u_t^2 + \frac{1}{2}\frac{d}{dt}\int_\Omega u_x^2 = \frac{d}{dt} E(u)$$

$$E(u) := \frac{1}{2}\left[\int_\Omega u_t^2 + \int_\Omega u_x^2\right] \leftarrow \text{Energía}$$

Ley de energía: $\frac{d}{dt} E(u) = 0.$

<u>Condiciones Neumann homogéneas:</u>

Sustituimos $(*)$ por

$u_x(a,t) = u_x(b,t) = 0.$ $\{$

$$¿\,\partial_{x^+x^-} = \underbrace{\partial_{x^+}\,\partial_{x^-}}\,?$$

$$¡\,\delta_{x^+x^-} = \delta_{x^-}\,\delta_{x^+}\,!$$

Orden de convergencia:

$$\varepsilon(h) = \| u_{ex} - u(h) \|$$

$$\left.\begin{aligned} \varepsilon(h) &\approx C\,h^p \\ \varepsilon(h/2) &\approx C\left(\frac{h^p}{2}\right) \end{aligned}\right\} \quad \frac{\varepsilon(h)}{\varepsilon(h/2)} \approx 2^p$$

$$\log\left(\frac{\varepsilon(h)}{\varepsilon(h/2)}\right) \approx p\,\log(2) \Rightarrow$$

$$\Rightarrow p \approx \frac{\log\left(\varepsilon(h)/\varepsilon(h/2)\right)}{\log(2)} \qquad (p=2)$$