PREFACE

I decided to write this book when I began looking for a textbook for a methods of applied mathematics course that I was to teach during the 1981–1982 academic year. There were many fine textbooks available—with some of the recent ones having been written by friends and colleagues for whom I have a great deal of respect. However, I felt that over the fifteen years that had elapsed since I left the Courant Institute, my research and point of view about the problems of interest to me had evolved in a manner unique to the combination of scientific experiences that have made up my career to date. It was my own point of view that I wanted to communicate in the course I was to teach. Certainly, the reader will find common ground here with other texts and references. However, it is my hope that I have communicated enough of the ideas that comprise my approach to direct and inverse scattering problems to have made this project worthwhile.

Much of my success in research is based on a fundamental education in *ray methods*, in particular, and *asymptotic methods*, in general, to which I was introduced at the Courant Institute. I take the point of view that an exact solution to a problem in wave phenomena is not an end in itself. Rather, it is the *asymptotic solution* that provides a means of interpretation and a basis for understanding. The exact solution, then, only provides a point of departure for obtaining a *meaningful* solution. This point of view can be seen in the contents of this book, where I have made relatively short shrift of exact solutions on the road to asymptotic techniques for wave problems.

The goal I was trying to reach appears in Chapters 8 and 9. In the former I discuss some asymptotic techniques for direct scattering problems. In the latter, I discuss the class of inverse problems in which one seeks to image reflecting surfaces from "backscattered" data. In order to reach this material in what I consider a one-year course, I have had to make some compromises with more traditional subject matter in a first course in applied mathematics. Certainly my choices are open to criticism. However, after having gone through this process myself, I retract all such casual criticism of others that I have ever made in the past; the authors who are my predecessors now have my profound respect for having made the hard choices that were necessary.

In those last two chapters, I have referenced material in all of the preceding chapters. I believe therefore that the material "hangs together," as I hoped it would.

Throughout the book, I have used the exercises to teach additional material. I recommend perusal of the exercises to even the most casual reader.

The student taking a course that uses this book should have a good basic education in classical analysis and differential equations. I make extensive use of complex function theory techniques, especially contour integration and analysis of singularities of functions of a complex variable. I also assume that the student has had a course in ordinary differential equations and one in linear algebra.

The prerequisite that is harder to define is "some experience in applied mathematics." By its very nature, applied mathematics is interdisciplinary. Unfortunately, it is not often practiced or taught in mathematics departments but as an adjunct in engineering, physics, and geophysics departments, often under titles that do not necessarily indicate to the uninitiated that the course is really one in applied mathematics. A course in electromagnetics using the text by Jackson is a perfect example. I can only suggest that the student who has taken courses in complementary disciplines or who has on-the-job experience with solutions to real-world problems has an advantage over the student lacking that background.

Richard Courant said[†] that the applied mathematician stands "in active and reciprocal relation" to the other sciences and to society. I am an adherent to that point of view. I have made some attempt in the text to draw on my experience in applications to give meaning to some of the text material. However, it is difficult to do much of that in the press of space on the mathematical material itself. It is my hope that each instructor using this textbook will have an equivalent store of experience to bring to the classroom or that the self-taught person has some resource of experience to provide a context for the material.

My objective in Chapter 1, on nonlinear first-order partial differential equations, is to discuss the eikonal equation—rays and wave front propagation. In particular, I wanted to present all of the anomalous cases that lead

[†] Constance Reid, "Courant in Göttingen and New York, The Story of an Improbable Mathematician," Springer-Verlag, New York, 1976.

to focusing, caustics, and various diffraction phenomena. My interest in these more exotic aspects of ray theory first arose from a research project at the Courant Institute with R. M. Lewis and D. Ludwig on smooth-body diffraction. I first wrote about this material from the point of view presented here in a set of lecture notes as part of the North British Symposium on Differential Equations in Dundee, Scotland, in 1972. It is my hope that this chapter has profited from the additional insight I have gained over the intervening nine years.

Chapter 2 contains some topics covering necessary machinery. I have tried to present coherent developments of distribution theory, (one- and multidimensional) stationary phase, along with a synthesis of all three of these subjects. In my opinion, stationary phase is the method of choice for analyzing Fourier representations for multidimensional wave fields.

The discussion of the one-dimensional wave equation in Chapter 4 provides an opportunity to present some theoretical results about the wave equation in a relatively simple setting while also allowing—through the device of studying the vibrating string in great detail—an introduction to complex variable methods for analysis of Fourier representations, eigenfunction expansions, and the WKB method. It also provides an opportunity to use the method of stationary phase and to develop some simple ideas about superposition of waves and dispersion relations.

In Chapter 5, on the wave equation in higher dimensions, the ideas of domains of dependence and influence and energy conservation are introduced. It is apparent through the sparsity of discussion of exact solutions in this chapter that my interests and objectives in this book are to get on to the Helmholtz equation, where I can address asymptotic "high-frequency" solutions. I have gotten to that equation in Chapter 6. I have also taken some pains there to discuss uniqueness of solutions for the frequency variable in an upper or lower half plane. This again reflects my own interests in "causal" solutions to wave problems. Also in this chapter I have addressed the equivalence between the Sommerfeld radiation condition for the Helmholtz equation and the outward propagation of energy for the wave equation.

Chapter 7 discusses the method of steepest descents, which I have brought to bear with great success on many problems in wave theory. This method requires a strong grounding in certain aspects of complex function theory. However, the gain is well worth the pain!

The difficulty here and in the final two chapters was to filter from all of the ideas I would have liked to address those that would make up a complete introduction to the subject matter. The size of these last chapters attests to the difficulty I had in achieving that goal!

Finally, in Chapters 8 and 9, I have tried to give the reader a basic introduction to the methods that have dominated my research career and will probably continue to do so. To me there is a vitality and richness of experience associated with the implementation of these methods on problems in seismic exploration and nondestructive testing. In both areas, I have found that we with labels reading "applied mathematician" are late to the area; we have much catching up to do on methods that work and also have much to contribute to help make these methods work better.

A particular bias of mine will become apparent when the reader discovers that one-dimensional inverse problems are relegated to the exercises in Chapter 9. In my own research, I tend to examine only those one-dimensional inverse problems that arise as specializations of three-dimensional real-world inverse problems. I believe that the alternative—the development of sophisticated methods that do not extend from one to higher dimensions—teaches the student *and the researcher* the wrong lessons. A standard joke in our research group is that "only graduate students study the one-dimensional inverse problem!"

I owe a great debt of gratitude to the students in that first course—Mourad Lahlou, Dennis Nesser, Michael Shields, Marc Thuillard, Shelby Worley, and Bruce Zuver—for suffering through the first draft of much of this material. Often, these people would return to class after one week and explain to me how I *meant* to write the exercises. Shelby Worley and Mourad Lahlou were especially helpful in finding typos and errors.

I also recognize the important contribution to my growth as an applied mathematician made by the various contract monitors who have supported my research over the years: Stuart Brodsky at ONR and Milton E. Rose at DOE/ERDA, who supported my early efforts in inverse problems—work that was obviously primitive when viewed from the present perspective, ten years later; Hugo Bezdek and J. Michael McKisic at ONR, who supported our group's work in an ocean acoustics program even though we were mathematicians; Charles Holland at ONR, who has continued the support program begun under Stuart Brodsky. The encouragement, recognition, and financial support provided through these people have been essential elements in our continued success.

I owe a special debt of thanks to my colleagues Jack K. Cohen, Frank G. Hagin, and John A. DeSanto, all of whom have supported and encouraged me throughout this project and all of whom have taken up some of the slack in joint efforts caused by my distraction from those efforts during the writing of this book. In addition, Jack Cohen provided many suggestions to improve the exposition in Chapters 1–4.

I entered the entire text on a word processor, using the Interactive Corporation Unix System on a PDP/11. I used NROFF-based macros called -ms and -neqn. I am extremely grateful to Mourad Lahlou for writing a

filter to allow the commands of these files to be reinterpreted as print commands for an NEC Spinwriter. I created large amounts of the text off-line on my own Northstar Advantage Micro-Computer. I then transmitted the material to the mainframe with a smart terminal program. Near the end of this project, the University of Denver, where I had been printing, changed operating systems. The National Center for Atmospheric Research in Boulder allowed me time on their PDP running the Interactive Unix System. I shall be eternally grateful for their assistance, which was crucial to the timely completion of this project.

I shall probably never write by hand again!