Numerieke Methoden Assignment 1

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1 Exercise 1

The function 'taylor(x)' is written to give an approximation of the $\sin(x)$ using the Taylor series around zero. The function exist of two functions, the first function 'taylor)(x)' gives the an approximation of the $\sin(x)$ around zero for a single number x, the seconds function 'vectortaylor(x)' gives the same approximation m-times for a vector x containing m elements. The function 'taylor(x)' takes a single real number. For this number x, the sine of x is calculated using the Taylor series around zero:

$$f(x) = \sum_{n=0}^{\infty} \frac{f(a)^{(n)}}{n!}(x)^n. \tag{1}$$

We look at the derivatives of sine and observe $\sin(a)^{(4k)} = sin(a), \sin(a)^{(1+4k)} = cos(a)$, $\sin(a)^{(2+4k)} = -\sin(a), \sin(a)^{(3+4k)} = -\cos(a)$. Since $a = 0$ and $\sin(0) = -\sin(0) = 0$, we observe the Taylor series for the $\sin(x)$ around zero can be written as follows: $f(x) = \sum_{n=0}^{\infty} \frac{sin(0)^{(n)}}{n!}(x)^n$.As written above, we would like to be able to give vectors of length m as an input to the function. The output of this function will be a vector y of equal length with every i^{th} element of this vector the sine calculated using the Taylor series around zero of the i^{th} element of the input vector. To make such a function, we write a function that takes a vector of length m as input. Using a for loop that calls the function taylor(x(i)) for every element of the input vector, this function vectortaylor(x) creates the desired output vector y.

The computer of course can't process these kind of series since the Taylor series has no end. This means the number of terms has to be chosen. We prefer high n, since the error of the Taylor series will only become too big for large x. On the other hand, if we use high n, the computer will give lots of rounding error since for example $n = 101, x = 36$ implies $n! = 101! \approx 10^{159}, x^n \approx 10^{157}$, numbers that by far exceed the maximal digits the computer can handle.

Using the function vectortaylor(x) we are able to plot a vector against the output of the the taylorfunction for each element of the vector. Figure 1 shows such a plot for a vector t with elements from 0 to 7. The Taylor series was calculated for $n = 11$ terms. We see for large x ($x > 5$) the Taylor series is not a good approximation for the sine. To have a good approximation for larger x the number of terms n in the Taylor series was changed to $n = 101$. This Taylor series was plotted for the vector t ith elements from 0 to 40, figure 2. We see the Taylor series is a good approximation for the sine for $x < 40$. After that we see, in contrary to 1, the Taylor series goes to infinity with a random line. This is due to the fact of rounding errors from the computer, as described above. The difference between the sine and the Taylor series of the plot in 2 is shown in 2.

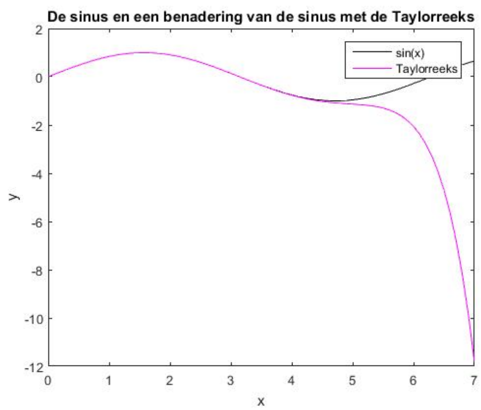

Figure 1

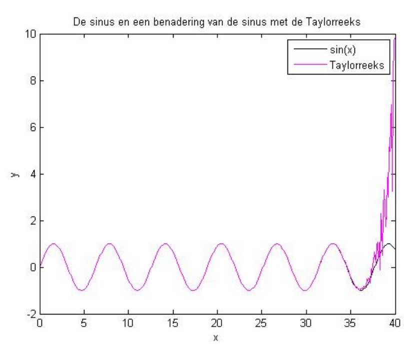

Figure 2

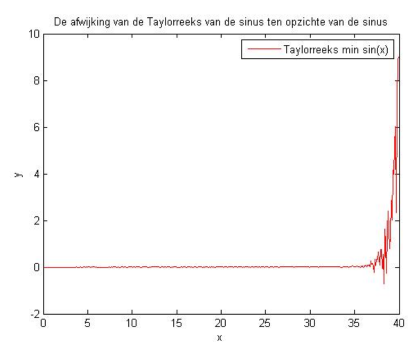

Figure 3

2 Exercise 2

A function was written to solve a second order differential equation with initial conditions as gives in the exercise. We take the determinant $D = \beta^2 - 4m\alpha$ and distinguish three cases for the parameters: $D > 0, D = 0, D < 0$. In the first case the solution will be of the form $x(t) = c_1 \exp{(r_1 t)} + c_2 \exp{(r_2 t)}$. The seconds case gives $r_1 = r_2$ and the solution will be of the form $x(t) = c_1 \exp{(r_1 t)} +$

$c_2 x \exp{(r_1 t)}$. In the last case of $D < 0$ gives complex solutions of the form $x(t) = c_1 \exp{(a + bit)} + c_2 \exp{(a - bit)}$. Using Eulers formula this gives $x(t) = c_1 \exp{(at)}\big(\cos(bt) + i\sin(b)\big) + c_2 \exp{(at)}\big(\cos(bt) - i\sin(b)\big)$. Analogous to exercise 1, there were written two functions: one Oosterhuis_assigment1_exercise2_1 that takes a single differential equation with parameters $m, \beta\alpha$ and initial conditions $x(0) = x_0, \frac{dx}{dt}(0) = v_0$. and one Oosterhuis_assigment1_exercise2_2that takes a vector of length n, with every i^{th} element being a differential equation. The function Oosterhuis_assigment1_exercise2_2 then returns a vector of length n with every i^{th} element being the solution of the i^{th} differential equation of the input vector.

In the figures below some plotted solutions for single second order differential equations are shown. Figure 4 shows the solution for $\frac{d^2x}{dt^2}(t) + -4\frac{dx}{dt}(t) + 9x(t) = 0$ with initial conditions $x(0) = 0, \frac{dx}{dt}(t) = -8$ on the interval $[0, 20]$. It is clear this equation has a negative root, still it is possible to solve this system for real values. In figure 5 the solution for $\frac{d^2x}{dt^2}(t) + 11\frac{dx}{dt}(t) + 24x(t) = 0$ with initial conditions $x(0) = 0, \frac{dx}{dt}(t) = -7$ on the interval $[0, 3]$, since for larger t the solutions just stays zero. The solutions will go to zero or infinity (for $\beta < 0$), or will be periodic for $\beta = 0$m or will go to infinity (for $\beta > 0$. In figure 6 and figure 7 we see the solution for the same differential equation, only differing in the value for β. The periodic behavior of the solution in 7 is due tot he value of $\beta = 0$.

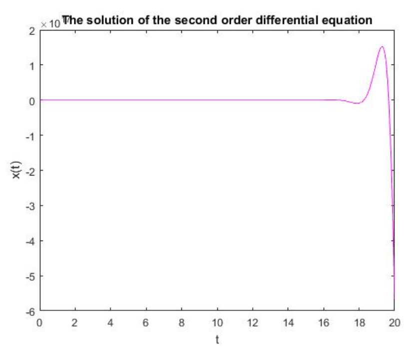

Figure 4: $\frac{d^2x}{dt^2}(t) + -4\frac{dx}{dt}(t) + 9x(t) = 0$, initial conditions $x(0) = 0, \frac{dx}{dt}(t) = -8$.

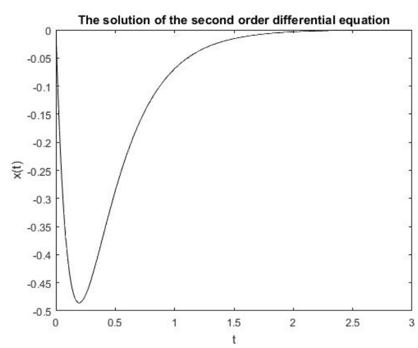

Figure 5: $\frac{d^2x}{dt^2}(t) + 11\frac{dx}{dt}(t) + 24x(t) = 0$, initial conditions $x(0) = 0, \frac{dx}{dt}(t) = -7$.

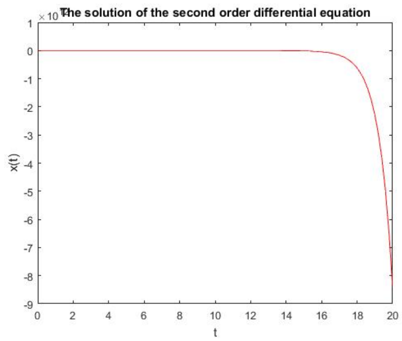

Figure 6: $16\frac{d^2x}{dt^2}(t) + -40\frac{dx}{dt}(t) + 25x(t) = 0$ with initial conditions $x(0) = 3, \frac{dx}{dt}(t) = -9/4$.

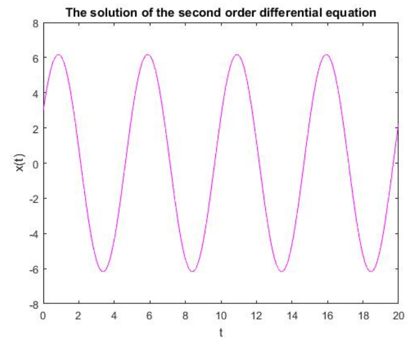

Figure 7: $16\frac{d^2x}{dt^2}(t) + 0\frac{dx}{dt}(t) + 25x(t) = 0$, initial conditions $x(0) = 3, \frac{dx}{dt}(t) = -9/4$.

3 Exercise 3

For this exercise a function that plots the logistic map $x_{n+1} = \lambda x_n(1 - x_n)$ for $x_i \in [0, 1], \lambda \in [0, 4]$ was written. The initial value for x_0 is choses at random, since after some iterations, say around 100, x_n doesn't depend on the value of x_0. In the plot in figure 8 the map is given for the whole interval of x_n and λ. It is shown the exciting stuff only starts to happen around $\lambda \approx 3, x_n \approx 0.7$. The function takes a minimum and maximum value for λ and a the number N, being the number of λ laying between λ_{min} and λ_{max} for which the sequence x_n should be calculated.

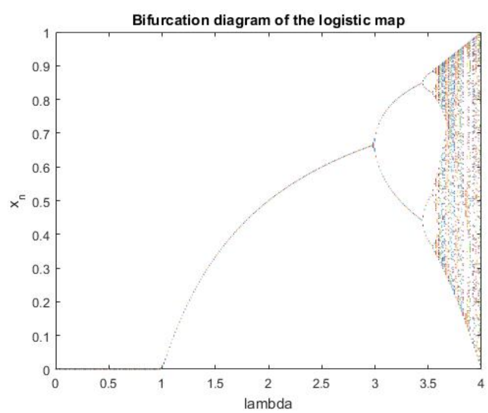

Figure 8: The logistic map