

Walk through of sample getGHA() calculation:

{'op':'predict'}

Adjusts the celestial sighting according to amospheric conditions

Given: observation 37d21.7
height 10
temperature 65
pressure 1010
horizon artificial

A. Calculate dip:

dip = horizon == natural? true: $\{(-0.97 * \sqrt{\text{height}})/60\}$ false: $\{0.00\}$
= $(-0.97 * 3.162) / 60$
= -0.051

B. Calculate refraction:

refraction = $(-0.00452 * \text{pressure}) / (273 + \text{convert_to_celsius}(\text{temperature}))/\tan(\text{altitude})$
= $(-0.005 * 1010) / (273 + 18.333) / 0.763$
= -4.565

C. Adjust observation:

altitude = observation + dip + refraction
= 37d21.7 + -0.051 + -4.565
= 37d17.4

D. Add adjusted altitude to dictionary:

{"altitude": "37d17.4"}

{'op':'predict'}

When navigation by the stars, we need to understand two things. First, the stars are relatively fixed in their positions, moving very slight amounts from year to year. Second, the earth rotates, giving the impression that the stars move across the sky. If we know the where we are on earth and the time of day, we can predict where each star will be. Looking at it another way, we can determine our position on earth by knowing where stars are at a specific time of day.

The challenge to navigation is knowing that where we are located on earth and where the night sky is pointing are both relative to a fixed reference point. We establish our earthly location relative to a point on the equator where the line running from the north pole to the south pole intersects Greenwich, England. We express our earthly position relative to how far east or west we are of this line and how far north or south we are of the equator. There are known as longitude and latitude respectively. We express how much the earth has rotated relative to the position of the earth at the time of the vernal equinox, the exact time in the spring when the sun is directly over the equator. In navigational lingo, this is referred to as the First Point of Aries, a term used by ancient astronomers and connoted by the symbol " γ ". It is the analog of the prime meridian in the celestial sphere. Stars positions are expressed relative to it.

Navigation requires that we know where the earth's prime meridian is rotated relative to Aries then we can determine the position of a star.

Given body Polaris
date 15-Mar-17
time 1:42:10

A. Find the angular displacement of the star relative to Aries.

1. Locate the observed body in the star table.

The star table lists the positions of the primary navigable stars.

Star	SHA	Dec
Polaris	316d41.3	89d20.1

2. Let latitude be the star's declination obtained from the table.

latitude = 89d20.1
Interpretation: If we sighted Polaris directly overhead, we would have to be at latitude 89d20.1.

3. Let SHA_{star} be the Sidereal Hour Angle obtained from the table.

shaStar = 316d41.3
Interpretation: This star is located 316d41.3 away from a specific reference point. In other words, we would see Polaris if we were to face the first point of Aries and rotate clockwise 316d41.3.

B. Calculate the Greenwich Hour Angle of Aries for the date and time of the observation.

1. Establish a reference angle based on a known Greenwich Hour Angle (GHA) for Aries.

We will be basing our calculations on how far the earth has rotated away from the vernal equinox (a.k.a., first point of Aries) at 00:00:00 on 1 January 2001.

Date	Time (UTC)	GHA _{Aries}
2001-01-01	3	100d42.6

2. Determine where the prime meridian is relative to Aries for the year of the observation

The earth rotates 360 degrees every 86,164.1 seconds, somewhat short of the $24 * 60 * 60 = 86400$ seconds we normally use. This means $\text{GHA}_{\text{Aries}}$ decreases by approximately 0d14.31667 each year. We offset each leap year by adding a day to the number of time the earth rotates.

a. Determine angular difference for each year

Reference Year = 2001
Observation Year = 2017
Difference = 16 years
Cumulative Progression = $16 * -0d14.31666667$ -3d49.1
= -3d49.1

b. Take into account leap years

Number of leap years after 2001 and before 2017: 4 leap days
Earth rotational period = 86164.1 seconds

Earth clock period = 86400 seconds
Amount of daily rotation = $\text{abs}(360\text{d}0.00 - \text{rotation} / \text{clock} * 360\text{d}00.0)$
= $\text{abs}(360\text{d}0.00 - 86164.1 / -3\text{d}49.1 * 360\text{d}00.0)$
= 0d59.0
Leap progression = daily rotation * number of leap days
= 0d59.0 * 4
= 2d56.9

c. Calculate how far the prime meridian has rotated since the beginning of the observation year.
 $\text{GHA}_{\text{Aries}}_{\text{beginning of observation year}} = \text{GHA}_{\text{Aries}2001-01-01\ 00:00:00} + \text{Cum Progression} + \text{Leap Progression}$
= $100\text{d}42.6 + -3\text{d}49.1 + 2\text{d}56.9 =$
= 99d50.5

d. Calculate the angle of the earth's rotation since the beginning of the observation's year
Elapsed seconds since the beginning of 2017 = 6313330
Earth rotational period = 86164.1 seconds
Amount of rotation = $\text{total seconds} / \text{rotational period} * 360\text{d}00.0 =$ 26377d33.7
= 97d33.7

e. Calculate total
 $\text{GHA}_{\text{Aries}} = \text{GHA}_{\text{Aries}}_{\text{beginning of year}} + \text{rotation in observation year} =$
= $99\text{d}50.5 + 97\text{d}33.7$
= 197d24.1

C. Calculate the star's GHA

1. Let $\text{GHA}_{\text{observation}}$ be the GHA of Aries + SHA of the star

$\text{GHA}_{\text{observation}} = \text{GHA}_{\text{Aries}} + \text{SHA}_{\text{star}}$
= 197d24.1 + 316d41.3
= 514d5.4

2. Clean up $\text{GHA}_{\text{observation}}$ by mod'ing it to fall in [0,360) and round to nearest 0.1 arc minute

$\text{GHA}_{\text{observation}} = 154\text{d}5.4$

3. Add GHA and latitude to dictionary

`{"long": "154d5.4",`
`{"lat": "89d20.1"}`

`{'op':'correct'}`

The "correct" step of navigation entails determining how different our actual star sighting is from its predicted location.

Given: lat 89d20.1
long 154d5.4
altitude 37d17.4
assumedLat 35d59.7
assumedLong 74d35.3

A. Calculate the local hour angle of the navigator:

LHA = long + assumedLong
= 154d5.4 + 74d35.3
= 228d40.7

B. Calculate the angle by which to adjust the observed altitude to match the star observed from the assumed position:

$\text{intermediateDistance} = ((\sin(\text{lat}) * \sin(\text{assumedLat})) + (\cos(\text{lat}) * \cos(\text{assumedLat}) * \cos(\text{LHA})))$

 $\text{intermediateDistance} = ((\sin(\text{lat}) * \sin(\text{assumedLat})) + (\cos(\text{lat}) * \cos(\text{assumedLat}) * \cos(\text{LHA})))$
= $(1 * 0.588) + (0.012 * 0.809 * -0.66)$
= 0.581474856 radians

 $\text{correctedAltitude} = \arcsin(\text{intermediateDistance})$
= 0.620540351 radians
= 35d33.3

C. Calculate distance in arc-minutes (i.e., nautical miles) needed to move to make the observed and calculated star positions match. Round to nearest arc-minute

$\text{correctedDistance} = \text{altitude} - \text{correctedAltitude}$
= 37d17.4 - 35d33.3
= 1d44.1
= 104 arc-minutes

D. Determine the compass direction in which to correct the distance:

$\text{correctedAzimuth} = \arccos(\frac{(\sin(\text{lat}) - (\sin(\text{assumedLat}) * \text{intermediateDistance}))}{(\cos(\text{assumedLat}) * \cos(\arcsin(\text{intermediateDistance})))})$

= $\arccos((1 - (0.588 * 0.581)) / ((0.809 * 0.814)))$
= $\arccos(1)$
= 0.010714057 radians
= 0d36.8

E. Add correctedDistance and correctedAzimuth to the dictionary:

`{"correctedDistance": "104",`
`{"correctedAzimuth": "0d36.8"}`

Walk through of sample getGHA() calculation:

{'op':'predict'}

Adjusts the celestial sighting according to atmospheric conditions

Given: observation 13d51.6
height 33
temperature 72
pressure 1010
horizon natural

A. Calculate dip:

dip = horizon == natural? true: $\{(-0.97 * \sqrt{\text{height}})/60\}$ false: $\{0.00\}$
= $(-0.97 * 5.745) / 60$
= -0.093

B. Calculate refraction:

refraction = $(-0.00452 * \text{pressure}) / (273 + \text{convert_to_celsius}(\text{temperature}))/\text{tangent}(\text{altitude})$
= $(-0.005 * 1010) / (273 + 22.222) / 0.247$
= -4.565

C. Adjust observation:

altitude = observation + dip + refraction
= 13d51.6 + -0.093 + -4.565
= 13d42.3

D. Add adjusted altitude to dictionary:

{"altitude": "13d42.3"}

{'op':'predict'}

When navigation by the stars, we need to understand two things. First, the stars are relatively fixed in their positions, moving very slight amounts from year to year. Second, the earth rotates, giving the impression that the stars move across the sky. If we know the where we are on earth and the time of day, we can predict where each star will be. Looking at it another way, we can determine our position on earth by knowing where stars are at a specific time of day.

The challenge to navigation is knowing that where we are located on earth and where the night sky is pointing are both relative to a fixed reference point. We establish our earthly location relative to a point on the equator where the line running from the north pole to the south pole intersects Greenwich, England. We express our earthly position relative to how far east or west we are of this line and how far north or south we are of the equator. There are known as longitude and latitude respectively. We express how much the earth has rotated relative to the position of the earth at the time of the vernal equinox, the exact time in the spring when the sun is directly over the equator. In navigational lingo, this is referred to as the First Point of Aries, a term used by ancient astronomers and connoted by the symbol "♈". It is the analog of the prime meridian in the celestial sphere. Stars positions are expressed relative to it.

Navigation requires that we know where the earth's prime meridian is rotated relative to Aries then we can determine the position of a star.

Given body Aldebaran
date 17-Jan-16
time 3:15:42

A. Find the angular displacement of the star relative to Aries.

1. Locate the observed body in the star table.

The star table lists the positions of the primary navigable stars.

Star	SHA	Dec
Aldebaran	290d47.1	16d32.3

2. Let latitude be the star's declination obtained from the table.

latitude = 16d32.3
Interpretation: If we sighted Aldebaran directly overhead, we would have to be at latitude 16d32.3.

3. Let SHA_{star} be the Sidereal Hour Angle obtained from the table.

shaStar = 290d47.1
Interpretation: This star is located 290d47.1 away from a specific reference point. In other words, we would see Aldebaran if we were to face the first point of Aries and rotate clockwise 290d47.1.

B. Calculate the Greenwich Hour Angle of Aries for the date and time of the observation.

1. Establish a reference angle based on a known Greenwich Hour Angle (GHA) for Aries.

We will be basing our calculations on how far the earth has rotated away from the vernal equinox (a.k.a., first point of Aries) at 00:00:00 on 1 January 2001.

Date	Time (UTC)	GHA _{Aries2001-01-01 00:00:00}
2001-01-01	3	100d42.6

2. Determine where the prime meridian is relative to Aries for the year of the observation

The earth rotates 360 degrees every 86,164.1 seconds, somewhat short of the $24 * 60 * 60 = 86400$ seconds we normally use. This means GHA_{Aries} decreases by approximately 0d14.31667 each year. We offset each leap year by adding a day to the number of time the earth rotates.

a. Determine angular difference for each year

Reference Year = 2001
Observation Year = 2016
Difference = 15 years
Cumulative Progression = $15 * -0d14.31666667$
= -3d34.8

b. Take into account leap years

Number of leap years after 2001 and before 2016: 3 leap days
Earth rotational period = 86164.1 seconds
Earth clock period = 86400 seconds
Amount of daily rotation = $\text{abs}(360d0.00 - \text{rotation} / \text{clock} * 360d00.0)$
= $\text{abs}(360d0.00 - 86164.1 / * 360d00.0)$

= 0d59.0

Leap progression = daily rotation * number of leap days
 = 0d59.0 * 3
 = 2d56.9

c. Calculate how far the prime meridian has rotated since the beginning of the observation year.
 $GHA_{Aries \text{ beginning of observation year}} = GHA_{Aries2001-01-01 \ 00:00:00} + \text{Cum Progression} + \text{Leap Progression}$
 = 100d42.6 + 2d56.9 =
 = 100d4.8

d. Calculate the angle of the earth's rotation since the beginning of the observation's year
 Elapsed seconds since the beginning of 2016 = 1394142
 Earth rotational period = 86164.1 seconds
 Amount of rotation = total seconds / rotational period * 360d00.0 = 5824d49.7
 = 64d49.7

e. Calculate total
 $GHA_{Aries} = GHA_{Aries \text{ beginning of year}} + \text{rotation in observation year}$
 = 100d4.8 + 64d49.7 =
 = 164d54.5

C. Calculate the star's GHA

1. Let $GHA_{\text{observation}}$ be the GHA of Aries + SHA of the star

$GHA_{\text{observation}} = GHA_{Aries} + SHA_{\text{star}}$
 = 164d54.5 + 290d47.1
 = 455d41.6

2. Clean up $GHA_{\text{observation}}$ by mod'ing it to fall in [0,360) and round to nearest 0.1 arc minute

$GHA_{\text{observation}} = 95d41.6$

3. Add GHA and latitude to dictionary

`{"long": "95d41.6",
 {"lat": "16d32.3"}`

{'op':'correct'}

The "correct" step of navigation entails determining how different our actual star sighting is from its predicted location.

Given: lat 16d32.3
 long 95d41.6
 altitude 13d42.3
 assumedLat -53d38.4
 assumedLong 74d35.3

A. Calculate the local hour angle of the navigator:

LHA = long + assumedLong
 = 95d41.6 + 74d35.3
 = 170d16.9

B. Calculate the angle by which to adjust the observed altitude to match the star observed from the assumed position:

$\text{intermediateDistance} = ((\sin(\text{lat}) * \sin(\text{assumedLat})) + (\cos(\text{lat}) * \cos(\text{assumedLat}) * \cos(\text{LHA})))$

$\text{intermediateDistance} = ((\sin(\text{lat}) * \sin(\text{assumedLat})) + (\cos(\text{lat}) * \cos(\text{assumedLat}) * \cos(\text{LHA})))$
 = (0.285 * -0.805) + (0.959 * 0.593 * -0.986)
 = -0.789410565 radians

$\text{correctedAltitude} = \arcsin(\text{intermediateDistance})$
 = -0.909848202 radians
 = -52d7.8

C. Calculate distance in arc-minutes (i.e., nautical miles) needed to move to make the observed and calculated star positions match. Round to nearest arc-minute

$\text{correctedDistance} = \text{altitude} - \text{correctedAltitude}$
 = 13d42.3 - -52d7.8
 = 65d50.1
 = 3950 arc-minutes

D. Determine the compass direction in which to correct the distance:

$\text{correctedAzimuth} = \arccos\left(\frac{(\sin(\text{lat}) - (\sin(\text{assumedLat}) * \text{intermediateDistance}))}{(\cos(\text{assumedLat}) * \cos(\arcsin(\text{intermediateDistance})))}\right)$

= arccos((0.285 - (-0.805 * -0.789)) / (0.593 * 0.614))
 = arccos(-0.965)
 = 2.874829767 radians
 = 164d42.9

E. Add correctedDistance and correctedAzimuth to the dictionary:

`{"correctedDistance": "3950",
 {"correctedAzimuth": "164d42.9"}`