



# **Cinemática das Colisões em Altas Energias**

**Sandro Fonseca de Souza**

# Sumário

- Motivações
- Transformações de Lorentz
- Sistemas de referência para processos de colisão em FAE
- Variáveis cinemáticas
- Variáveis de Mandelstam
- Seção de Choque
- Espaço de Fase
- Resultados da Seção de Choque Exp.
- Luminosidade x Beyond LHC- FCC collider

# Bibliografia Sugerida

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- R. Hagedorn - *Relativistic Kinematics: A guide to the Kinematic problems of High Energy Physics*
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# Motivações

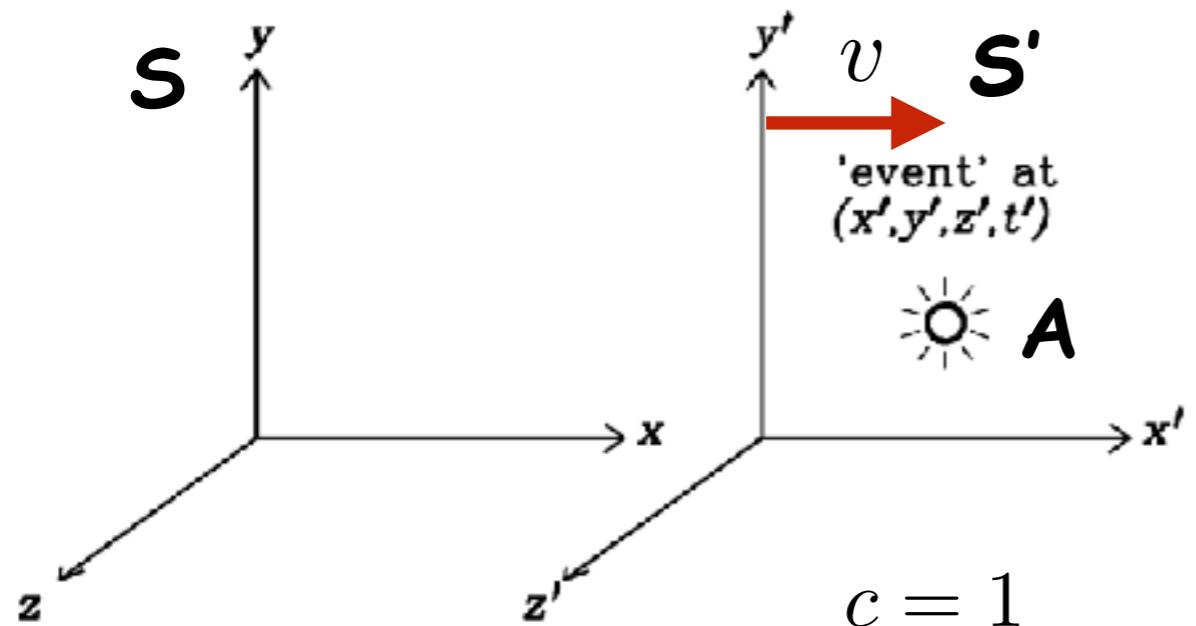
- Introdução dos princípios básicos, aplicações práticas e métodos conhecidos dos aspectos da FAE que são baseados puramente na cinemática.
- Cinemática pode ser definida como “a geometria do movimento”
- Cinemática relativística é uma aplicação da relatividade especial para reações com partículas elementares.

# Motivações

- Do ponto de vista da puramente cinemático, partículas são completamente caracterizados por suas energias e momentum (ex. seus quadrimomentum  $p$ );
- As reações de partículas observáveis são por tanto os decaimentos ou colisões;
- Os números quânticos internos são irrelevantes para a cinemática das partículas elementares.

# Transformações de Lorentz

$$v_x = |v| = v$$



- Considerando um ponto A no espaço tempo onde:

- S pode ser descrito  $(x, y, z, t)$
- e S' (em movimento) pode ser descrito  $(x', y', z', t')$

**(1)** Considerando que o sistema S' se move com uma velocidade constante  $v$  ao longo do eixo x

$$S \Rightarrow S'$$

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx) \end{aligned}$$

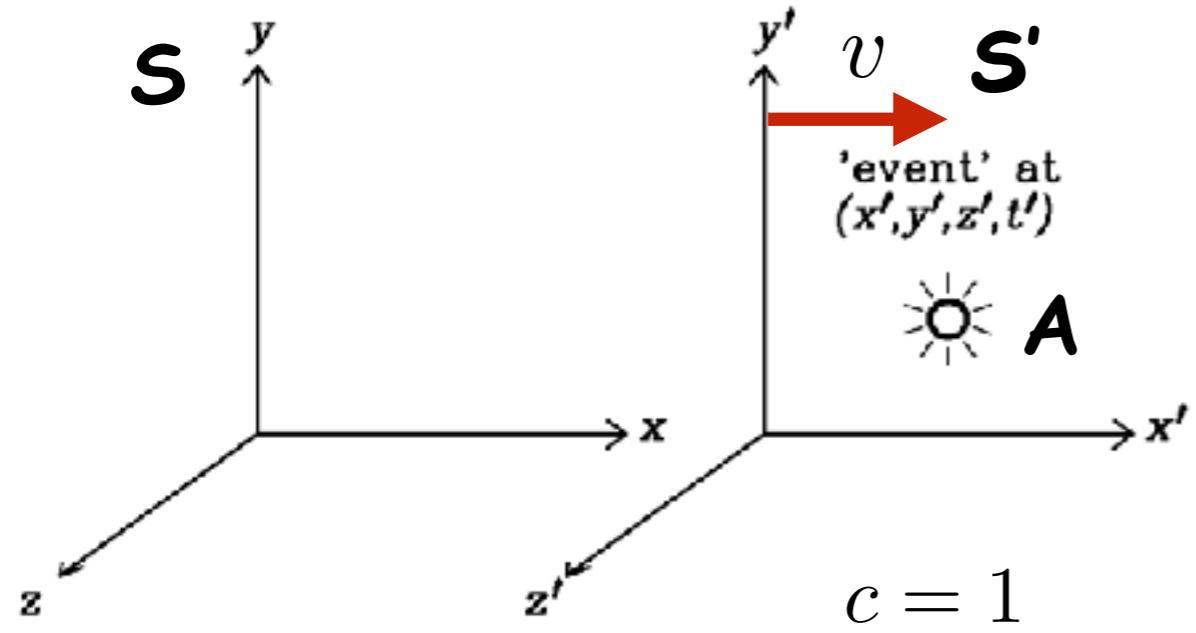
$$S' \Rightarrow S$$

$$\begin{aligned} x &= \gamma(x' + vt) \\ y &= y' \\ z &= z' \\ t &= \gamma(t' + vt) \end{aligned}$$

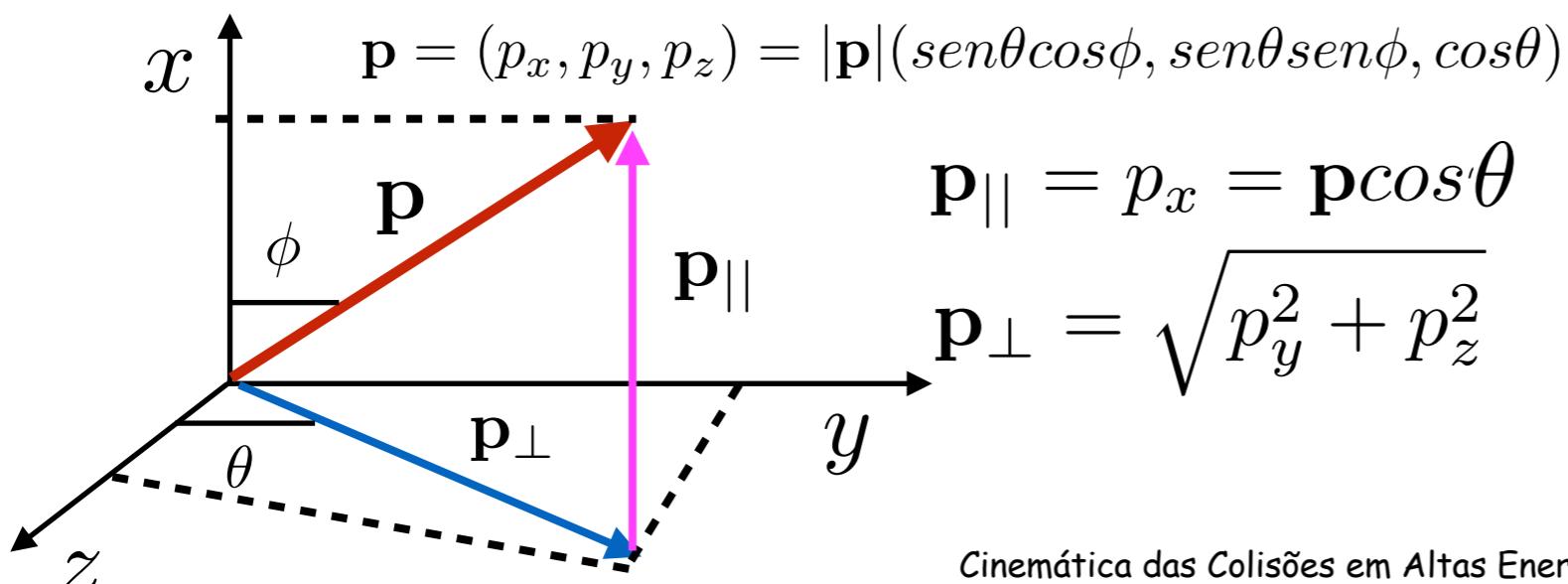
fator de Lorentz  $\gamma = \frac{1}{\sqrt{1 - v^2}}$

# Transformações de Lorentz

$$v_x = |v| = v$$



**fator de Lorentz**  $\gamma = \frac{1}{\sqrt{1 - v^2}}$



- Considerando o quadri-momentum
  - ▶ **S** pode ser descrito  $\mathbf{p} \equiv (\mathbf{E}, \mathbf{p}) = (\mathbf{E}, p_x, p_y, p_z)$
  - ▶ e **S'** (em movimento) pode ser:  $\mathbf{p}' \equiv (\mathbf{E}', \mathbf{p}') = (\mathbf{E}, p'_x, p'_y, p'_z)$
  - ▶ As transformações de Lorentz para o quadri-momentum são:

$$S \Rightarrow S'$$

$$\begin{aligned} p'_x &= \gamma(p_x - vE) \\ p'_y &= p_y \\ p'_z &= p_z \\ E' &= \gamma(E - vp_x) \end{aligned}$$

# Noções e Convenções



Unidades Naturais

$$c = \hbar = 1$$

Exercício 0 : Quando um píon decai um  
dois fótons, qual a energia do fóton?

Coordenadas Espaço-Tempo  
Vetor contravariante

$$x^\nu = (x^0, x^1, x^2, x^3) = (t, \vec{x}) = (t, x, y, z)$$

Momentum e Energia Relativística

$$p = \gamma \beta m \quad E = \gamma m \quad \text{m = massa de repouso}$$

Vetor quadrimomentum

$$p^\mu = (p^0, p^1, p^2, p^3) = (E, \vec{p}) = (E, \vec{p}_T, p_z) = (E, p_x, p_y, p_z)$$

Momentum escalar de dois quadrvetores a e b

$$a.b = a^0 b^0 - \vec{a} \cdot \vec{b}$$

Relação entre energia e momentum

$$E^2 = p^2 + m^2 \quad p^2 = p^\mu p_\mu = E^2 - |\vec{p}|^2 = m^2$$

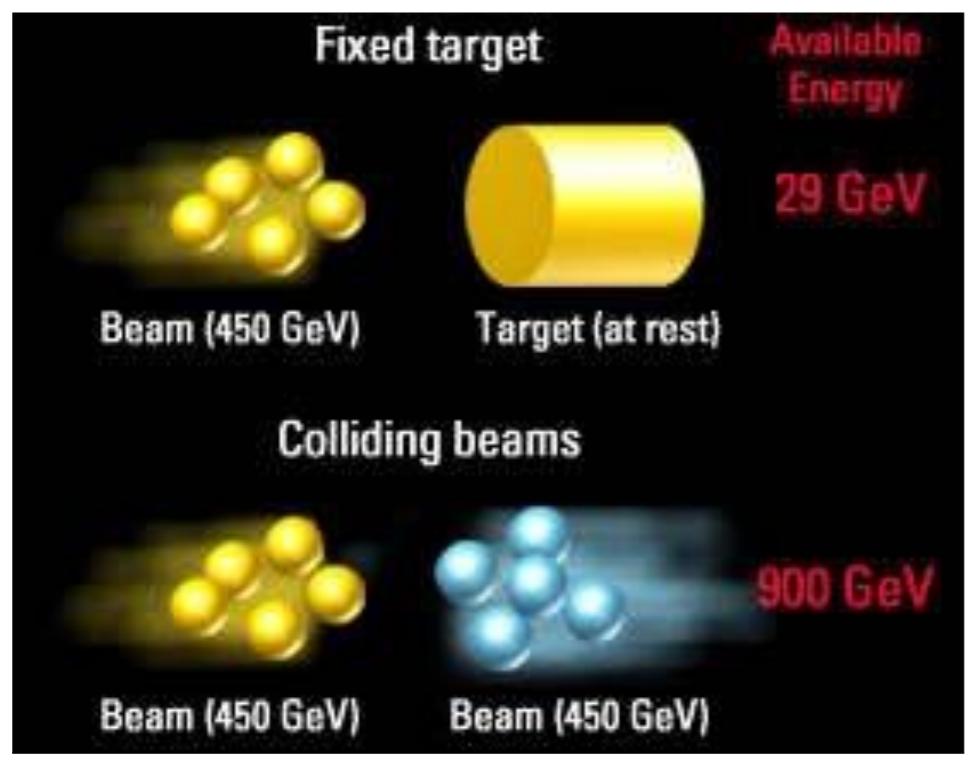
Velocidade da partícula:

$$\beta = \frac{\vec{p}}{E} \quad \gamma = (1 - \beta^2)^{-1/2} = E/m$$

# Sistemas de coordenadas

Consideremos a colisão de duas partículas de quadrimomentum

$$(E_a, \vec{p}_a) \quad (E_b, \vec{p}_b)$$



Na descrição destas colisões, dois sistemas de referência são usualmente utilizados:

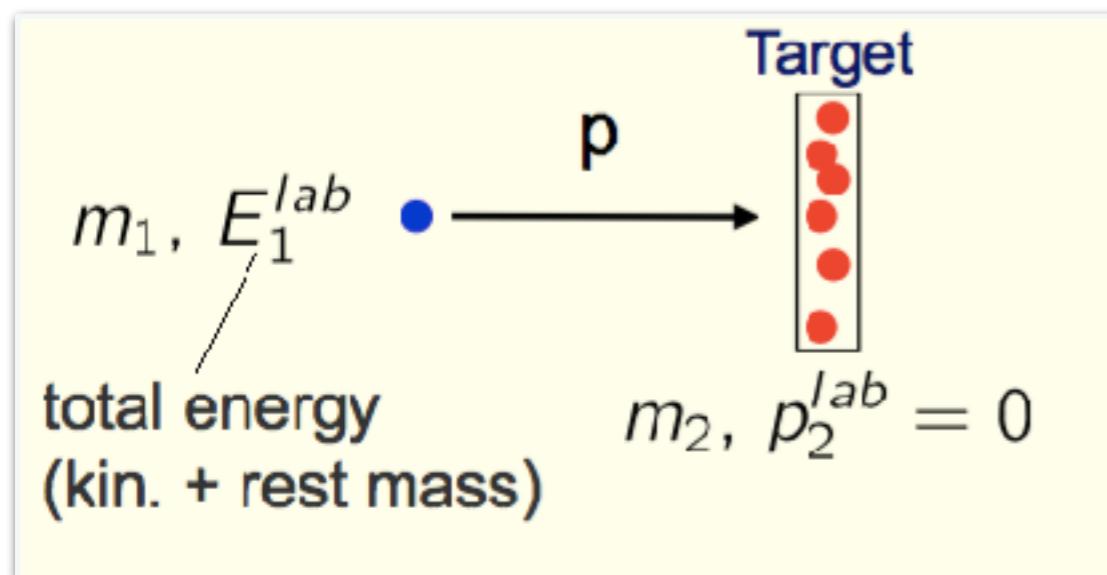
- **Sistema de Centro de Massa (CM):** é o sistema onde:

$$\vec{p}_a + \vec{p}_b = 0$$

- **Sistema de Laboratório (LAB):** é o sistema no qual são feitas as medidas.
  - Em experimentos de alvo fixo este sistema coincide com o sistema do alvo, onde uma das partículas encontra-se em repouso (e.g. b):
$$\vec{p}_b = 0$$
  - Nos experimentos de anéis de colisão, onde feixes de partículas idênticas colidem em direções opostas, este sistema coincide com o CM.

# Sistemas de coordenadas

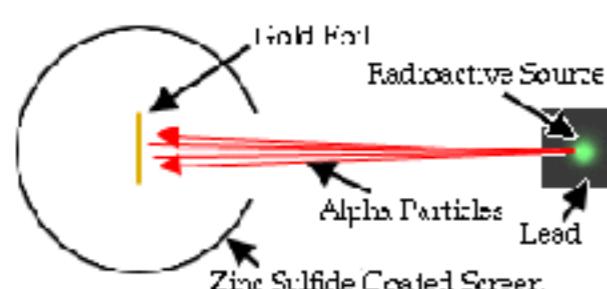
## Sistema de Laboratório (LAB)



A energia total da colisão como sendo:

$$E_T = \sqrt{s} = \sqrt{m_1^2 + m_2^2 + 2E_1^{lab}m_2}$$

**Exercício 1:** Prove a equação acima.



**Exercício 2:** Considerando

$$E_1^{lab} \gg m_1, m_2$$

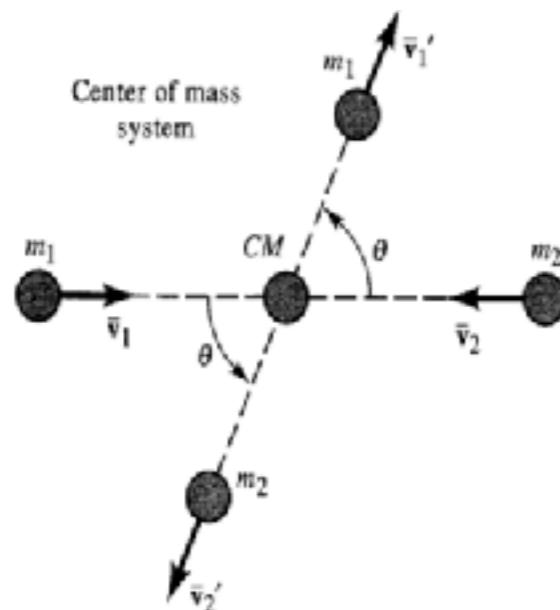
Prove esta aproximação;

$$E_T \approx \sqrt{s} \approx \sqrt{2E_1^{lab}m_2}$$

$$\beta = \frac{p}{E}$$

# Sistemas de coordenadas

## Sistema de Centro de Massa (CM)



A energia total da colisão como sendo:

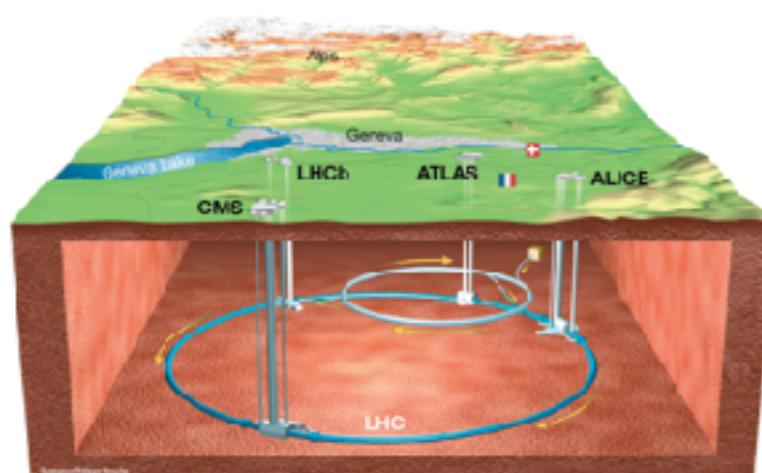
$$E_T = \sqrt{s} = \sqrt{m_1^2 + m_2^2 + 2E_1 E_2 (1 - \beta_1 \beta_2 \cos\theta)}$$

Exercício 3: Prove a equação acima.

Collider:

$$m_1, E_1^{lab} \quad m_2, E_2^{lab}$$

The diagram shows two particles, represented by blue dots, moving towards each other along a horizontal axis. Above the axis, the labels  $m_1, E_1^{lab}$  and  $m_2, E_2^{lab}$  are shown, indicating the mass and energy of the particles in the laboratory frame. Below the axis, two arrows pointing towards each other represent the momenta  $p_1$  and  $p_2$  of the particles.



Exercício 4: Considerando

Prove esta aproximação;

$$\boxed{\begin{array}{l} p_1 = -p_2 \\ m_1 = m_2 \end{array}}$$

$$E_T = \sqrt{s} = 2E_1$$

$$\beta = \frac{p}{E}$$

# Sistemas de coordenadas

## Exercício 5: Sistema de Centro de Massa (CM)

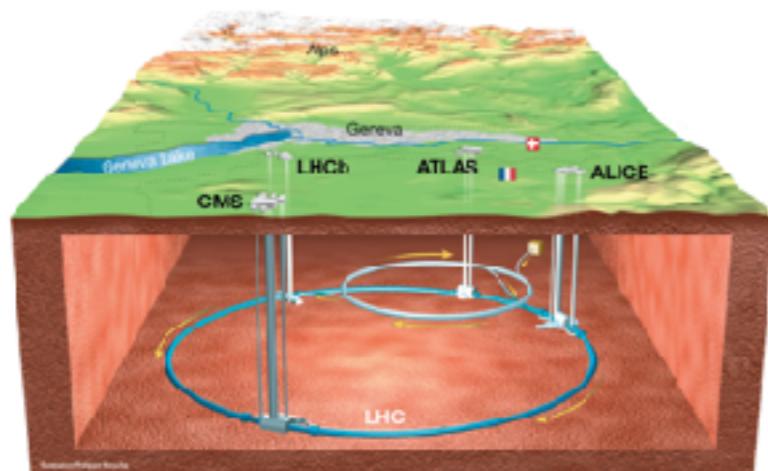
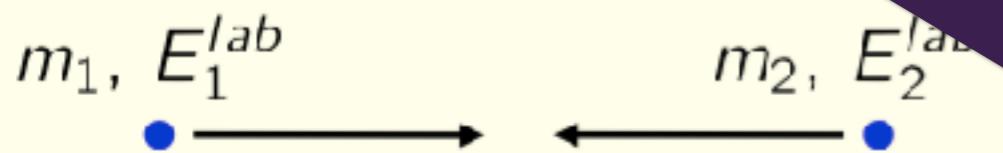
A energia total da colisão como sendo:

$$\sqrt{m_1^2 + m_2^2 + 2E_1 E_2 (1 - \beta_1 \beta_2 \cos\theta)}$$

Center of mass system



Collider:



$$E_T -$$

$$p_2 - m_2$$

Exercício 5: Um feixe de prótons com momentum de 100 GeV atinge um alvo fixo de hidrogênio. Qual é a energia de centro de massa para está interação? Qual seria a energia do feixe necessário para atingir a mesma energia do LHC? Quais os colisores assimétricos usados atualmente e por que não usar um colisor mais potente?

A energia total da colisão como sendo:

$$\sqrt{m_1^2 + m_2^2 + 2E_1 E_2 (1 - \beta_1 \beta_2 \cos\theta)}$$

Colisão acima.

# Sistemas de coordenadas

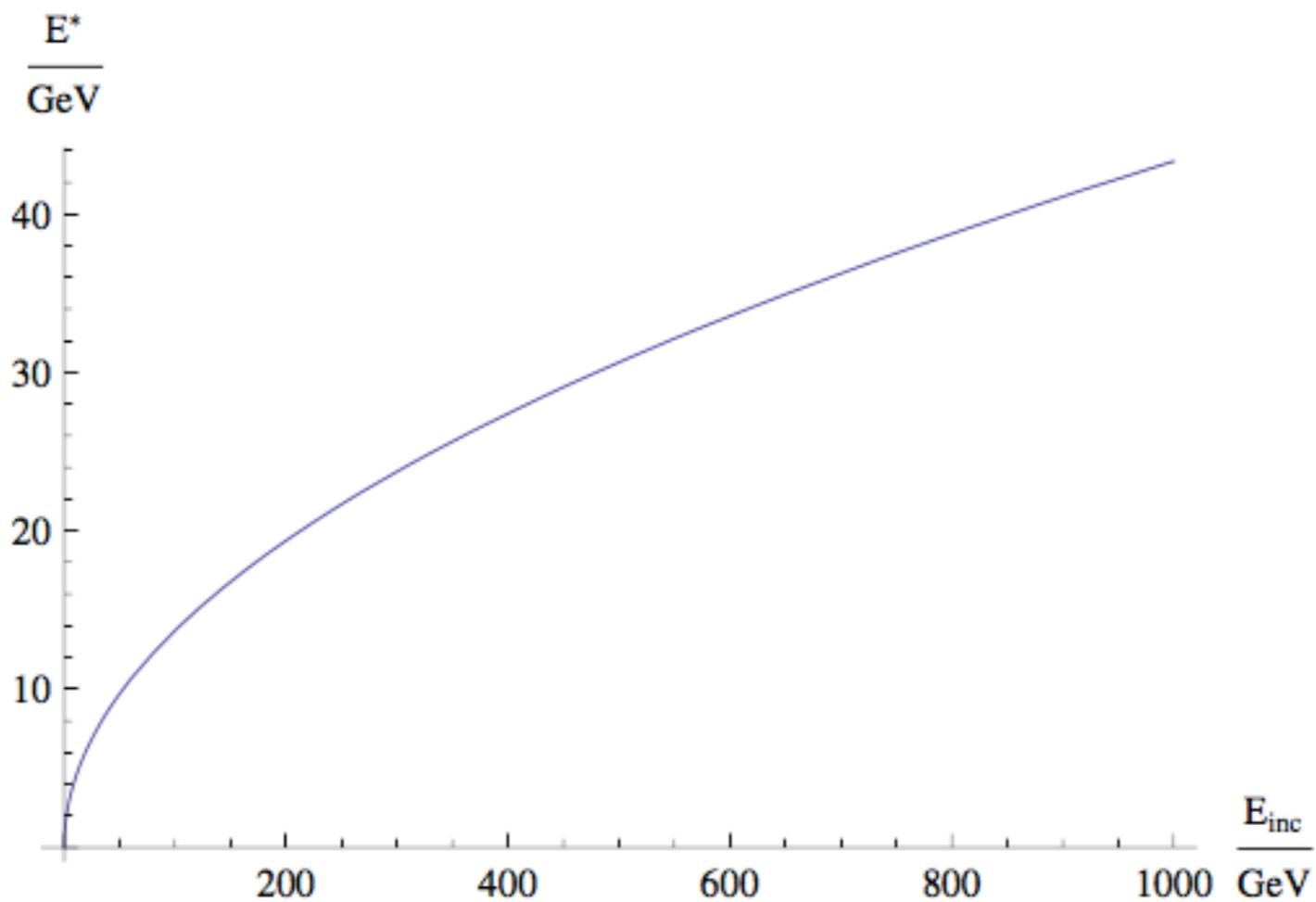


Figure 4.3: *Center of mass energy of the colliding beam for a fixed target experiment.* The energy increases with the square root of the beam energy.

# Comparando Colisões

Apresentamos na tabela abaixo os valores da energia total ( $E_T$ ) e do momento ( $p_a^*$ ) no CM para a colisão de **feixes de elétrons** ( $m_a = m_e \cong 0$ ) e **prótons** ( $m_a = m_p \cong 0.938$  GeV) colidindo com um **próton fixo** em função do momento do feixe ( $p_a^{lab}$ ).

$p_a^{lab}$ (GeV)	$E_T$ (GeV)		$p_a^*$ (GeV)	
	ep	pp	ep	pp
1	1.66	2.08	0.57	0.45
10	4.43	4.54	2.12	2.07
100	13.73	13.76	6.83	6.82
500	30.65	30.66	15.3	15.3
1000	43.33	43.34	21.7	21.6
10000	137.0	137.0	68.5	68.5

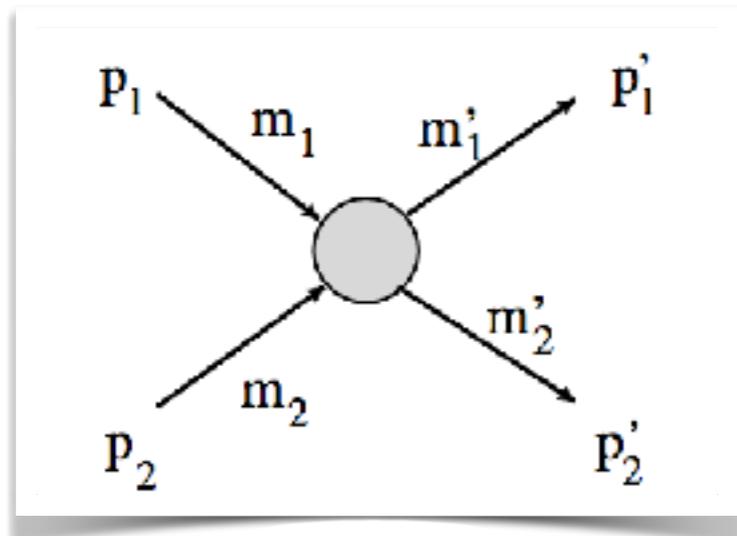
# Comparando Colisões

Apresentamos na tabela abaixo os valores da energia total ( $E_T$ ) e do momento ( $p_a^*$ ) no CM para a colisão de **feixes de elétrons** ( $m_a = m_e \cong 0$ ) e **prótons** ( $m_a = m_p \cong 0.938$  GeV) colidindo com um alvo fixo em função do momento do feixe ( $p_a^{lab}$ ).

$p_a^{lab}$ (GeV)	$E_T$ (GeV)	Colisão com alvo fixo	
		ep	pp
1	4.54	0.57	0.45
10	43.33	2.12	2.07
30	137.0	6.83	6.82
100	30.65	15.3	15.3
1000	43.33	21.7	21.6
10000	137.0	68.5	68.5

Exercício 5 a: Encontre a energia de centro de massa para o experimento de colisão de partículas cujo o feixe de prótons tem uma energia de 3,5 TeV.

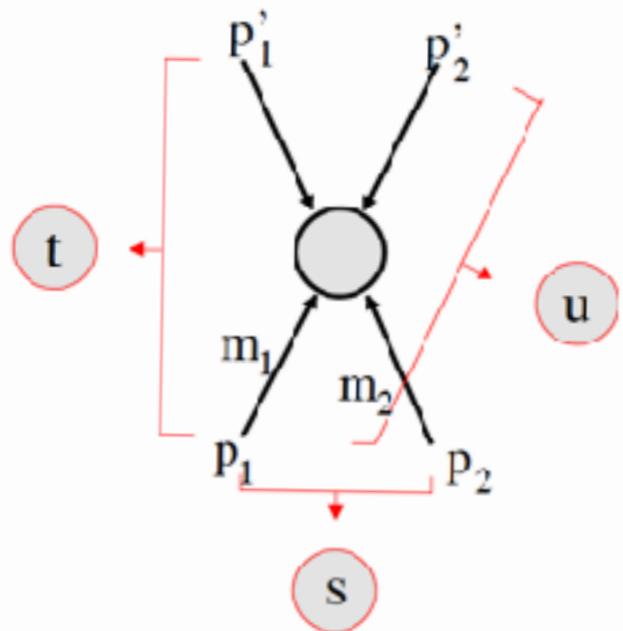
# Variáveis de Mandelstam



Em Física de Altas Energias seção de choque e razão de decaimentos são descritos por variáveis cinemáticas que são invariantes relativísticos. Nos decaimentos de dois corpos existem de fato quatro invariantes disponíveis desde que a energia e momentum seja conservada de somente dois deles para definir a cinemática do evento.

# Variáveis de Mandelstam

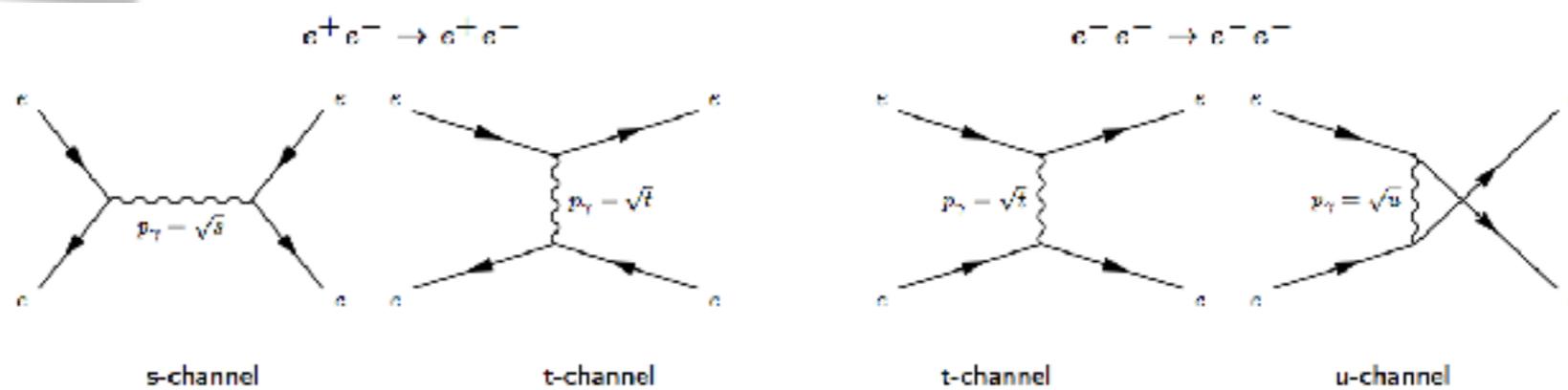
As variáveis de Mandelstam são invariantes de Lorentz em decaimentos de tipo 2->2



$$s = (p_1 + p_2)^2 = (p'_1 + p'_2)^2 = -(p_1 + p_2)(p'_1 + p'_2)$$

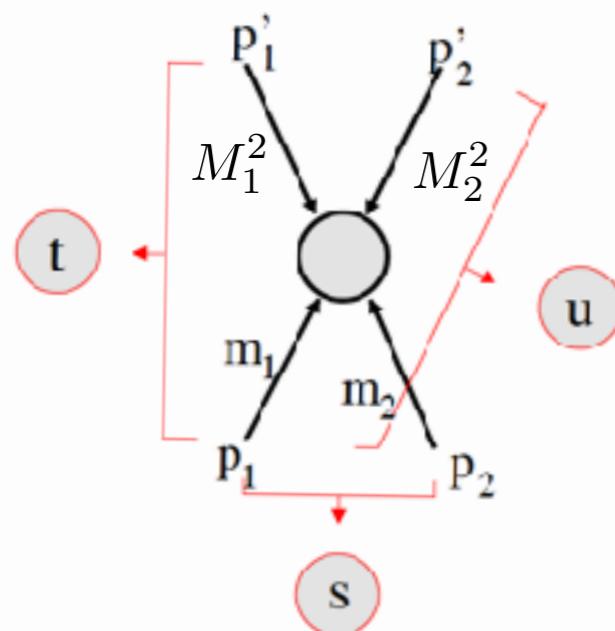
$$t = (p_1 + p'_1)^2 = (p_2 + p'_2)^2 = -(p_1 + p'_1)(p_2 + p'_2)$$

$$u = (p_1 + p'_2)^2 = (p_2 + p'_1)^2 = -(p_1 + p'_2)(p_2 + p'_1)$$



# Variáveis de Mandelstam

As variáveis de Mandelstam são invariantes de Lorentz em decaimentos de tipo 2->2



$$s = (p_1 + p_2)^2 = (p'_1 + p'_2)^2 = -(p_1 + p_2)(p'_1 + p'_2)$$

$$t = (p_1 + p'_1)^2 = (p_2 + p'_2)^2 = -(p_1 + p'_1)(p_2 + p'_2)$$

$$u = (p_1 + p'_2)^2 = (p_2 + p'_1)^2 = -(p_1 + p'_2)(p_2 + p'_1)$$

Exercício 6 : Em espalhamento elástico do tipo: A+A =A+A, quais são as variáveis de Mandelstam?

As variáveis de Mandelstam devem satisfazer a seguinte equação:

$$s + t + u = m_1^2 + m_2^2 + M_1^2 + M_2^2$$

Exercício 7: Prove a relação acima.

# Regiões do Espaço de Fase

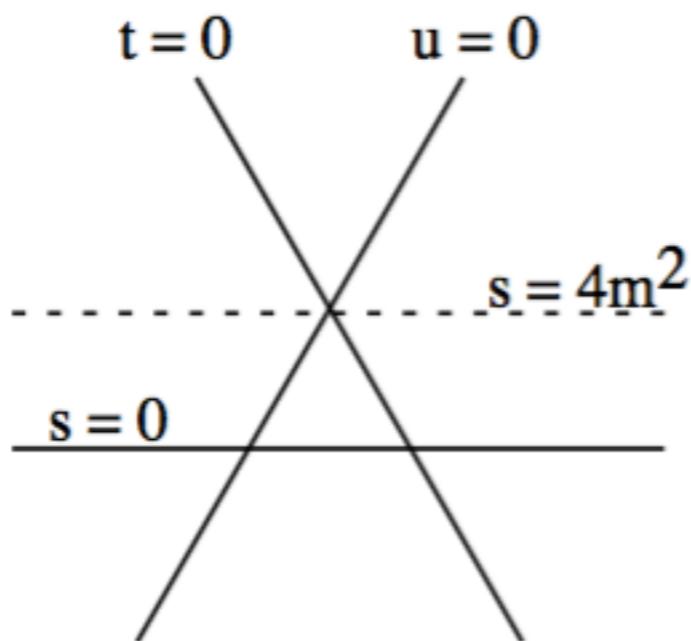
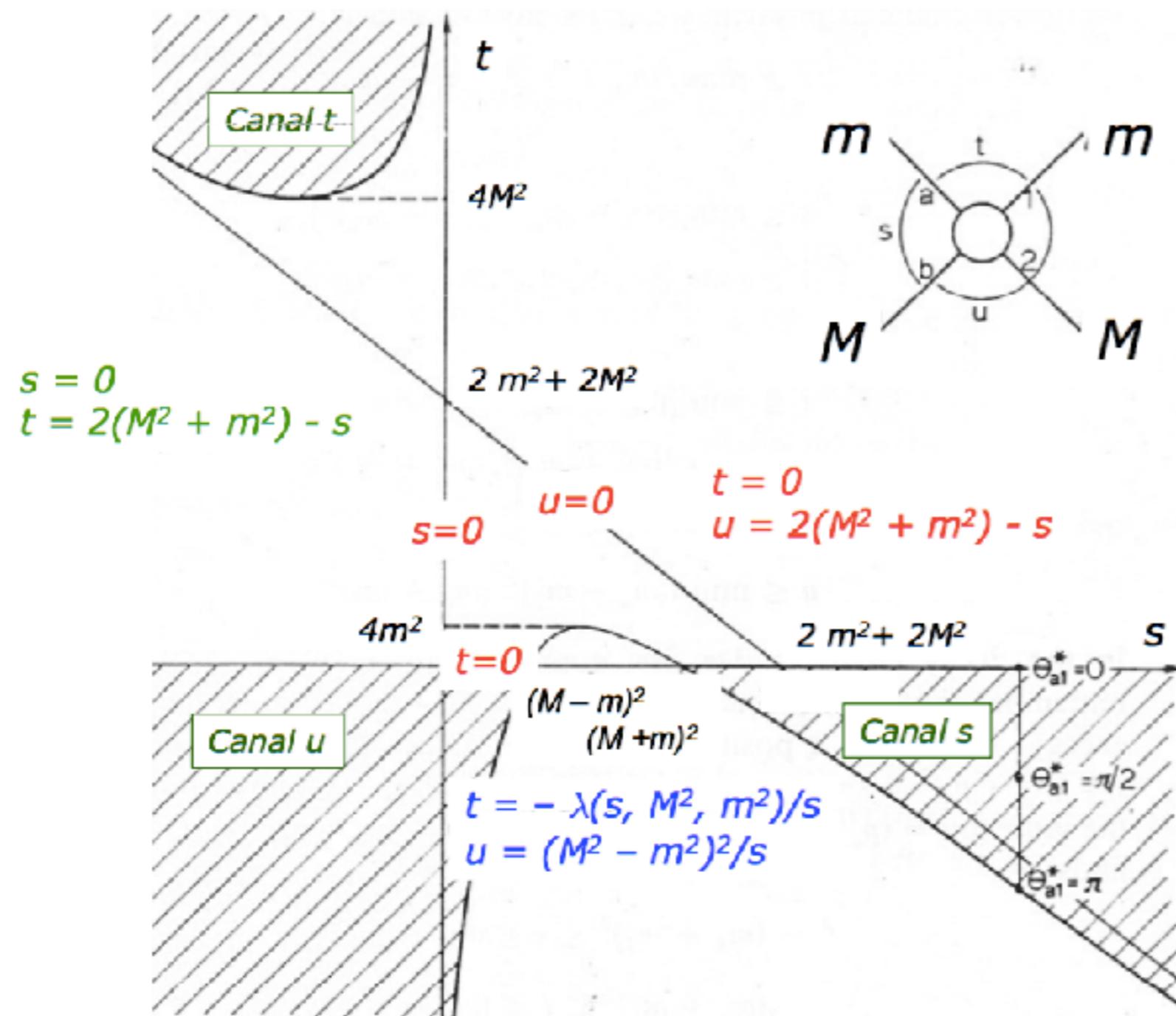


Figure 2: The figure shows the valid regions of phase space allowed by energy and momentum conservation.

Variable	Definition	Value in CM Frame
$s$	the center of mass energy	$s = (E_A + E_B)^2$
$t$	the 4-momentum transfer of $e^+$ or $e^-$	$-2p^2(1 - \cos \theta)$
$u$	the 4-momentum difference between $e^+$ and $e^-$	$-2p^2(1 + \cos \theta)$

# Regiões do Espaço de Fase



# Rapidez

$$\begin{pmatrix} E' \\ p'_\parallel \end{pmatrix} = \begin{pmatrix} \gamma_s & -\gamma_s \beta_s \\ -\gamma_s \beta_s & \gamma_s \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix}, \quad p'_\perp = p_\perp$$

**Transformação de Lorentz na direção do eixo Z. Exercício 8 : Mostre esta transformação.**

Para introduzirmos o conceito de rapidez vamos escrever a transformação de Lorentz como:

$$\begin{pmatrix} E' \\ p'_\parallel \end{pmatrix} = \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix}, \quad p'_\perp = p_\perp$$

onde definimos o parâmetro  $y$  chamado de rapidez através de:

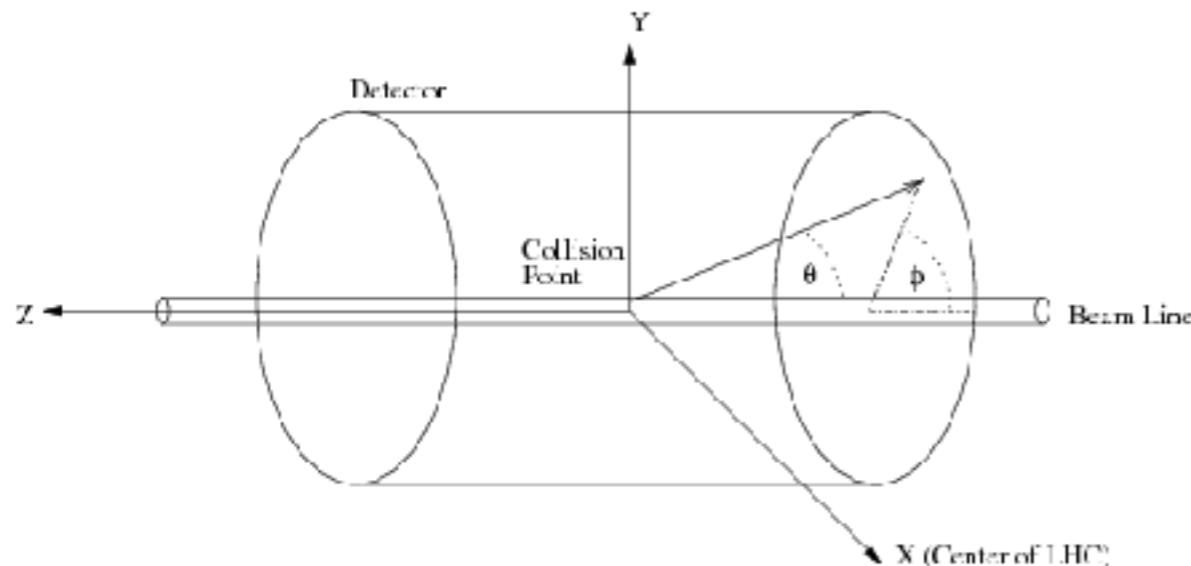
$$\beta_s = \tanh y, \quad \gamma_s = \cosh y, \quad \gamma_s \beta_s = \sinh y$$

Lembrando que:

$$\sinh^{-1}(z) = \ln(z + \sqrt{z^2 + 1}), \quad \cosh^{-1}(z) = \ln(z + \sqrt{z^2 - 1}), \quad \text{e} \quad \tanh^{-1}(z) = \frac{1}{2} \ln\left(\frac{1+z}{1-z}\right)$$

Temos:

$$\begin{aligned} y &= \cosh^{-1}(\gamma) = \ln(\gamma + \gamma \beta_s) \\ &= \tanh^{-1}(\beta_s) = \frac{1}{2} \ln\left(\frac{1 + \beta_s}{1 - \beta_s}\right) \end{aligned}$$



$$\begin{pmatrix} E' \\ p'_\parallel \end{pmatrix} = \begin{pmatrix} \gamma_s & -\gamma_s \beta_s \\ -\gamma_s \beta_s & \gamma_s \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix}, \quad p'_\perp = p_\perp$$

**Transformação de Lorentz na direção do eixo Z. Exercício 8 (a) : Mostre esta transformação abaixo.**

Para introduzirmos o conceito de rapidez vamos escrever a transformação de Lorentz como:

$$\begin{pmatrix} E' \\ p'_\parallel \end{pmatrix} = \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix}, \quad p'_\perp = p_\perp$$

onde definimos o parâmetro  $y$  chamado de rapidez através de:

$$\beta_s = \tanh y, \quad \gamma_s = \cosh y, \quad \gamma_s \beta_s = \sinh y$$

Lembrando que:

$$\sinh^{-1}(z) = \ln(z + \sqrt{z^2 + 1}), \quad \cosh^{-1}(z) = \ln(z + \sqrt{z^2 - 1}), \quad \text{e} \quad \tanh^{-1}(z) = \frac{1}{2} \ln\left(\frac{1+z}{1-z}\right)$$

Temos:

$$\begin{aligned} y &= \cosh^{-1}(\gamma) = \ln(\gamma + \gamma \beta_s) \\ &= \tanh^{-1}(\beta_s) = \frac{1}{2} \ln\left(\frac{1+\beta_s}{1-\beta_s}\right) \end{aligned}$$

# Rapidez

Supondo a partícula parada em  $S'$ , temos  $\beta = \beta_s$  e, como:

$$E \pm p_{\parallel} = m\gamma(1 \pm \beta)$$

podemos escrever a rapidez de uma partícula como:

$$y = \frac{1}{2} \ln \left( \frac{E + p_{\parallel}}{E - p_{\parallel}} \right)$$

Enquanto a velocidade varia no intervalo  $-1 \leq \beta \leq +1$  a rapidez varia entre  $-\infty \leq y \leq +\infty$ .

Pode-se também definir a massa transversal por  $M_{\perp}^2 = p_{\perp}^2 + m^2$  e escrever

$$y = \ln \left( \frac{E + p_{\parallel}}{M_{\perp}} \right)$$

Quando executamos duas transformações de Lorentz paralelas consecutivas de parâmetros  $\beta_1$  e  $\beta_2$  o resultado combinado é expresso pelos parâmetros:

$$\begin{aligned}\beta_3 &= \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2} \\ \gamma_3 &= \gamma_1\gamma_2(1 + \beta_1\beta_2)\end{aligned}$$

# Rapidez

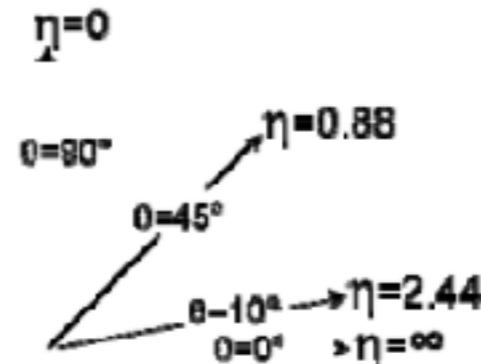
Desta forma, em termos da rapidez, temos:

$$\begin{aligned}\beta_3 &= \tanh y_3 = \frac{\tanh y_1 + \tanh y_2}{1 + \tanh y_1 \tanh y_2} \\ &\equiv \tanh(y_1 + y_2)\end{aligned}$$

ou seja, a rapidez é aditiva sob transformações de Lorentz paralelas.

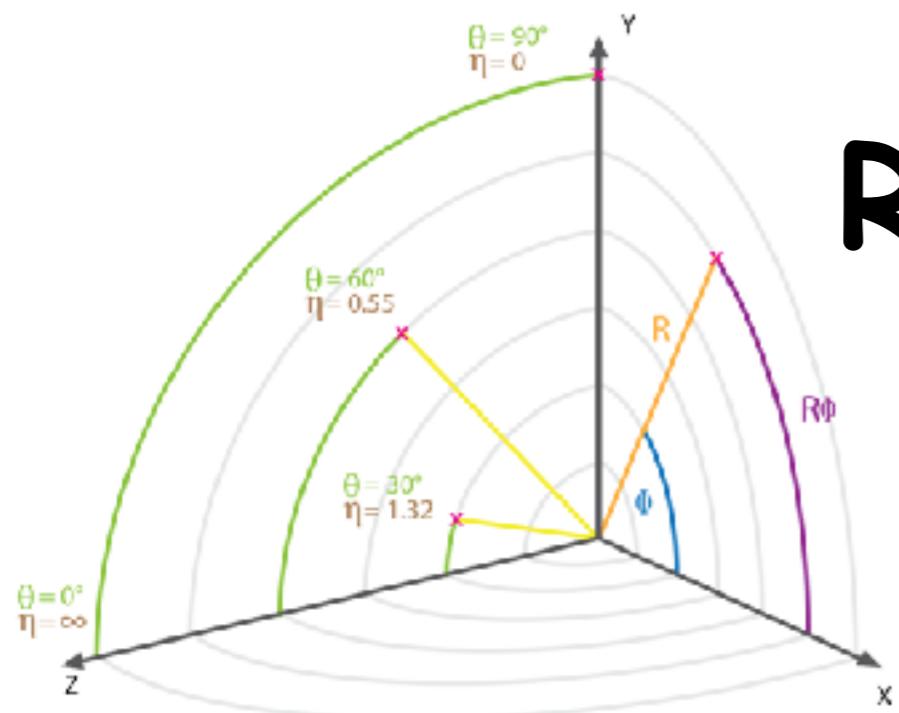
No limite de altas energias, a rapidez é chamada de **pseudo-rapidez** ( $\eta$ ) e possui uma relação simples com o ângulo de espalhamento  $\theta$ :

$$\eta = \frac{1}{2} \ln \left( \frac{1 + \cos \theta}{1 - \cos \theta} \right) = -\ln \tan \theta/2$$

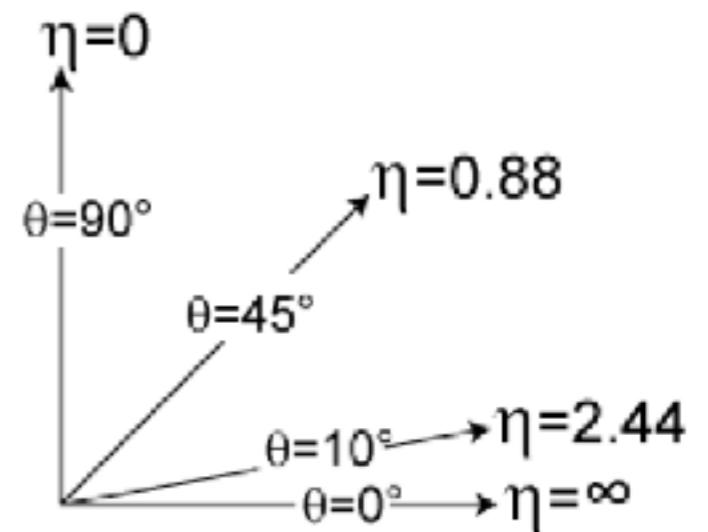
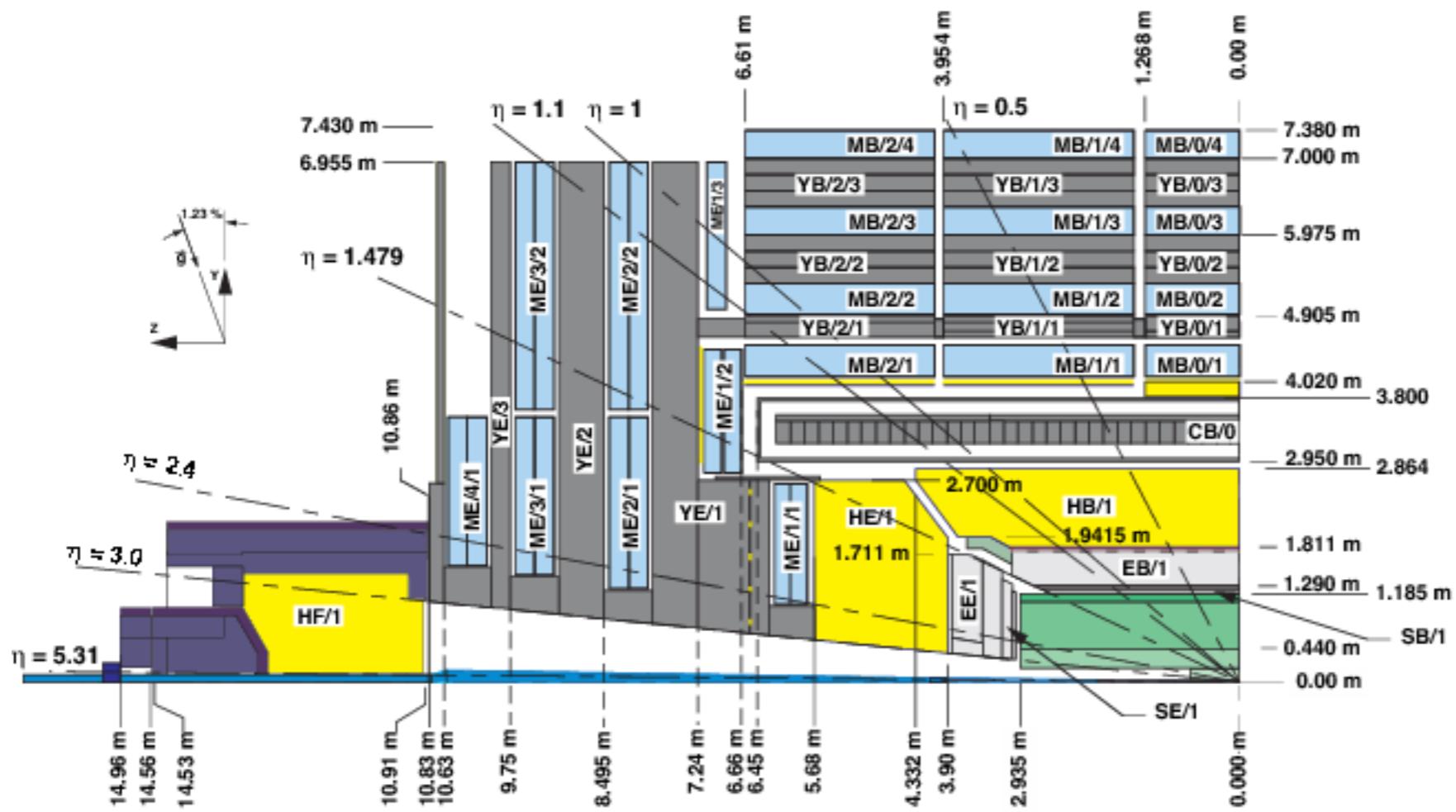
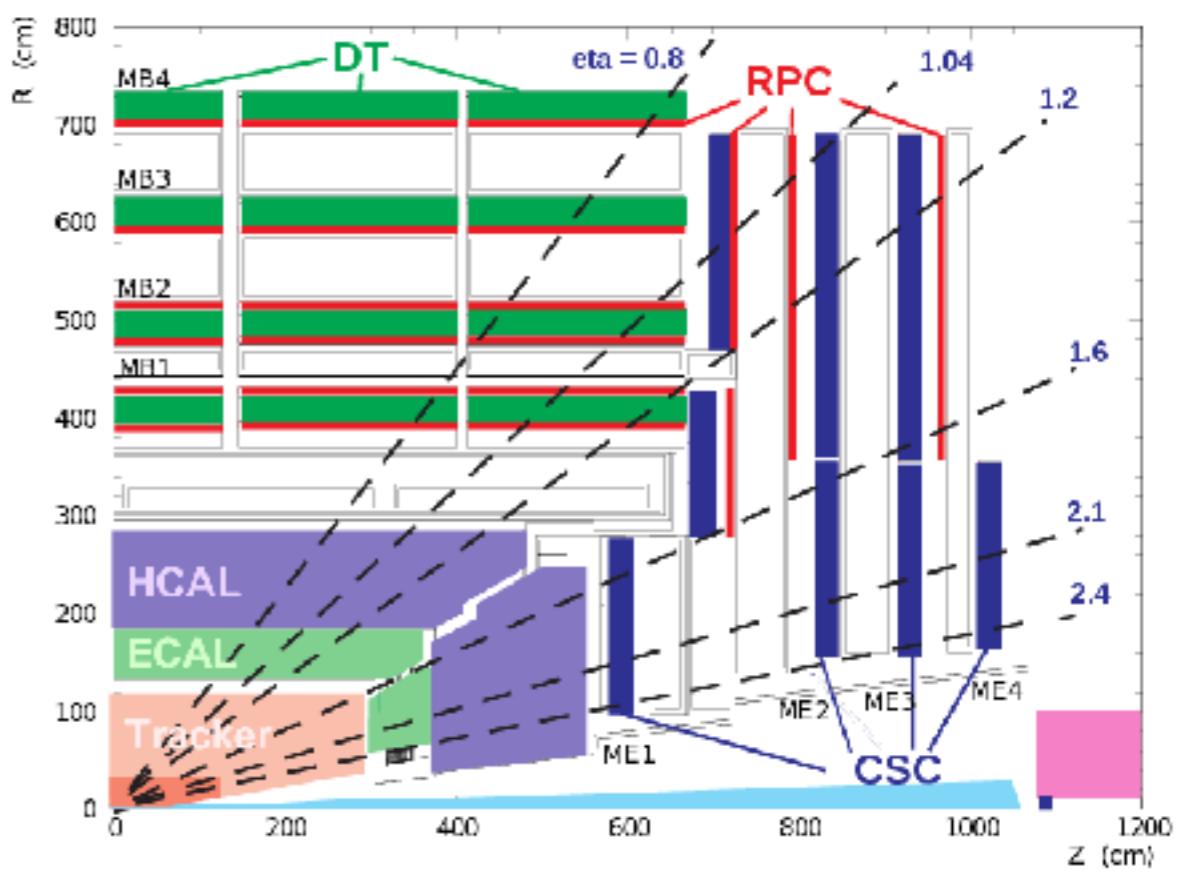


Note que a definição de  $\eta$  depende apenas de  $\theta$  e, portanto, ela é útil como uma aproximação para a rapidez quando o momento ou a massa de uma partícula são desconhecidos.

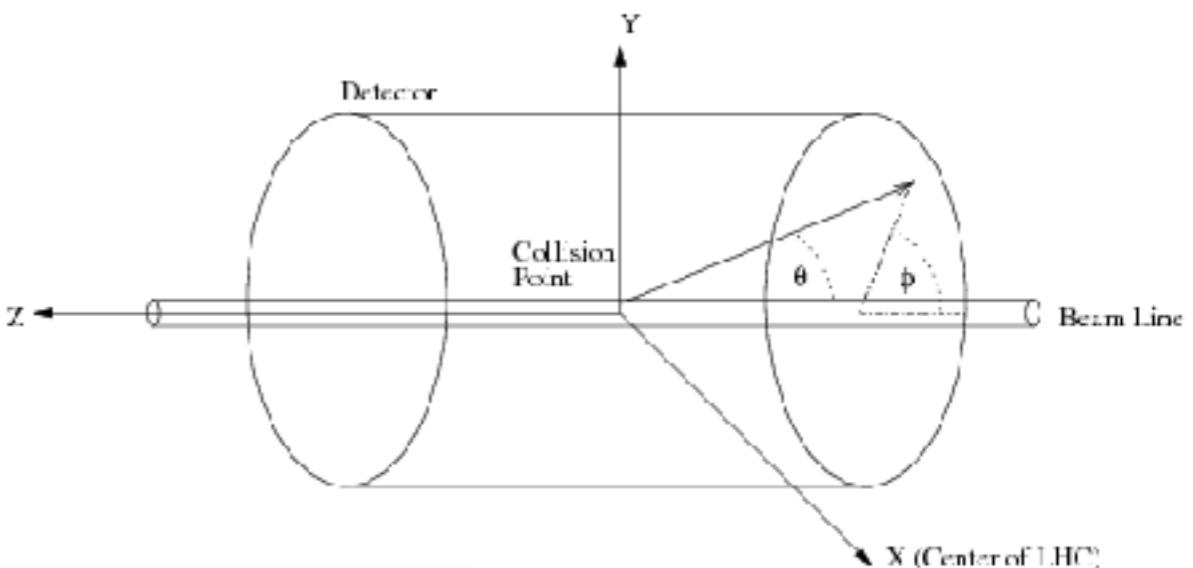
Ela é usada para parametrizar a cobertura dos detectores, localizando por exemplo as células de um calorímetro no plano  $(\eta, \phi)$ .



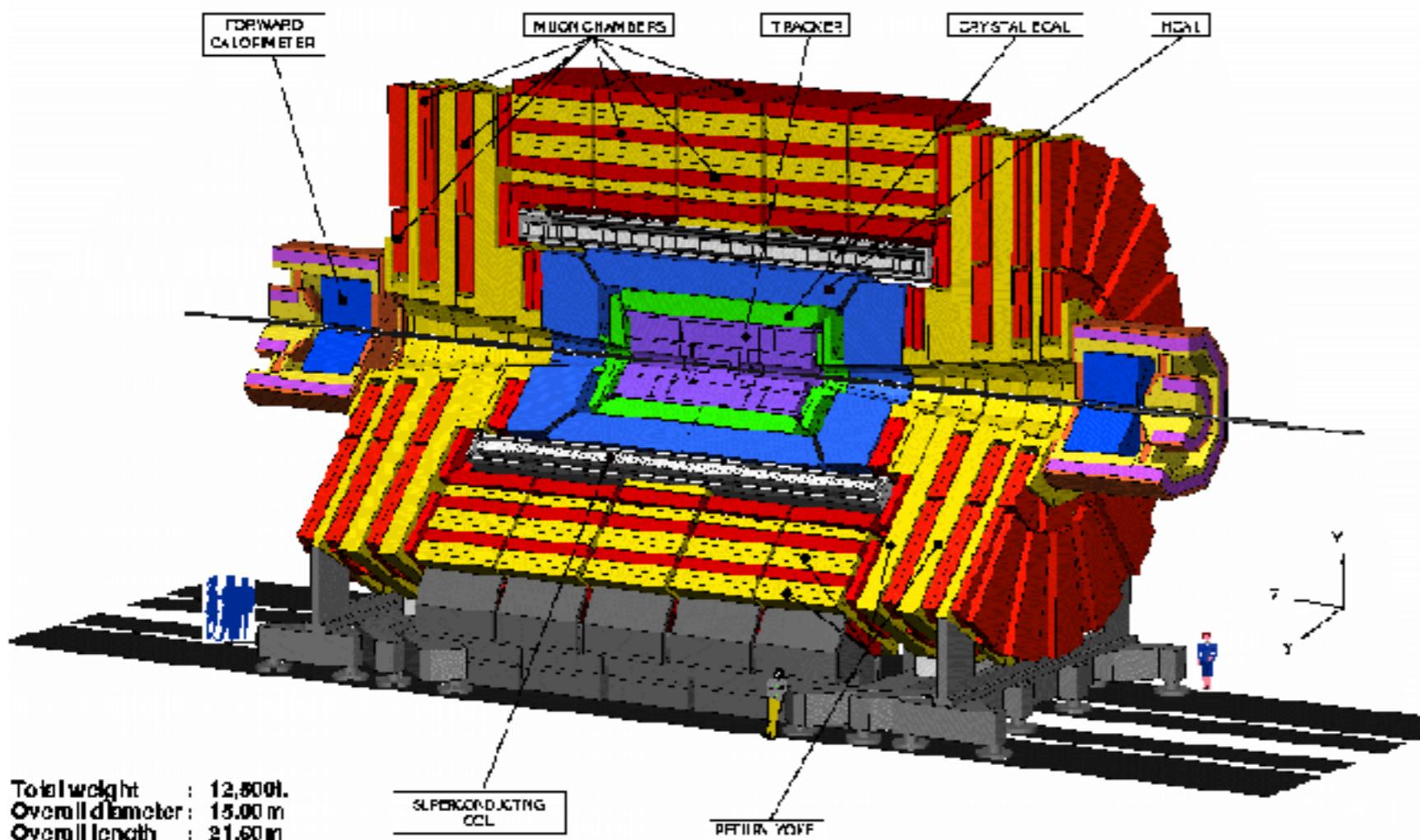
# Rapidez



# Rapidez

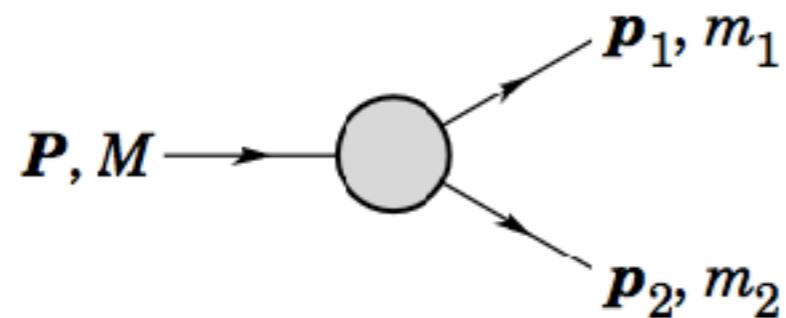


**CMS**  
A Compact Solenoidal Detector for LHC



# Decaimento de dois corpos

**Exercício 8 (b): Mostre em detalhes que o decaimento de dois corpos pode ser descrito por:**



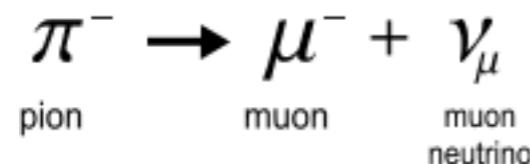
**Figure 43.1:** Definitions of variables for two-body decays.

## Exercício 9: Determine a energia e momentum para os seguinte decaimento de dois corpos

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M} ,$$

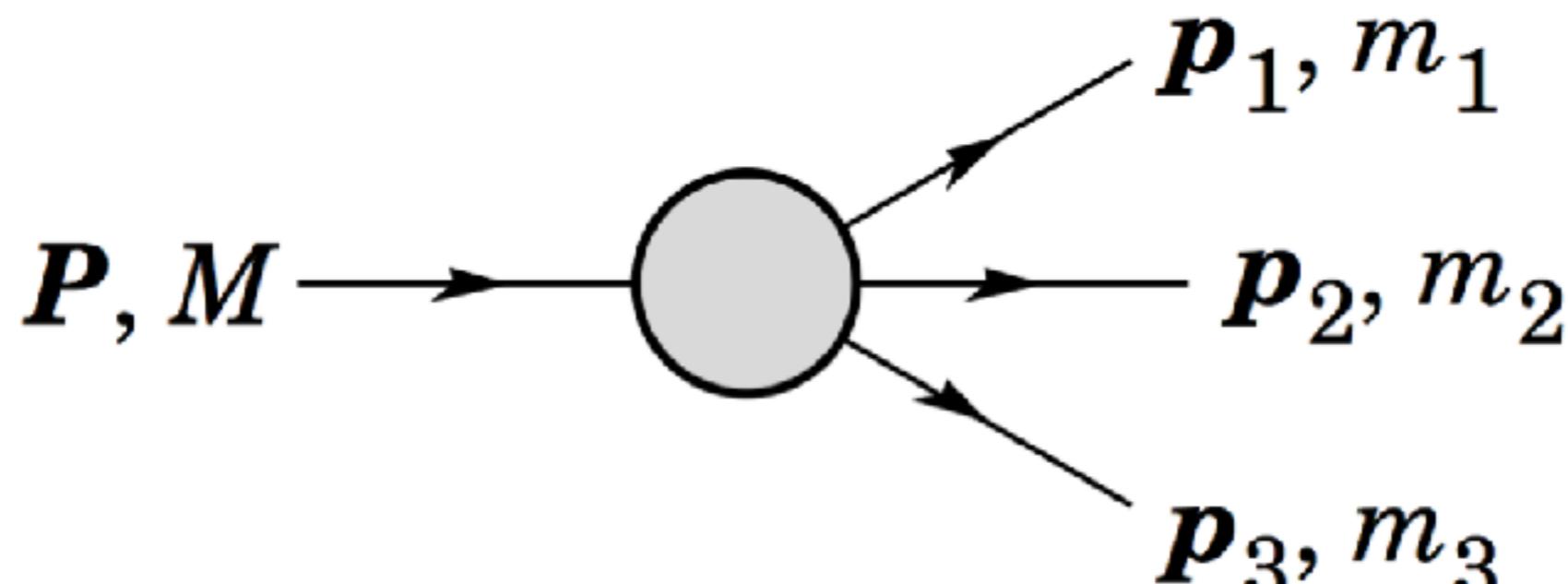
$$|p_1| = |p_2|$$

$$= \frac{[(M^2 - (m_1 + m_2)^2)(M^2 - (m_1 - m_2)^2)]^{1/2}}{2M} ,$$



# Decaimento de Três corpos

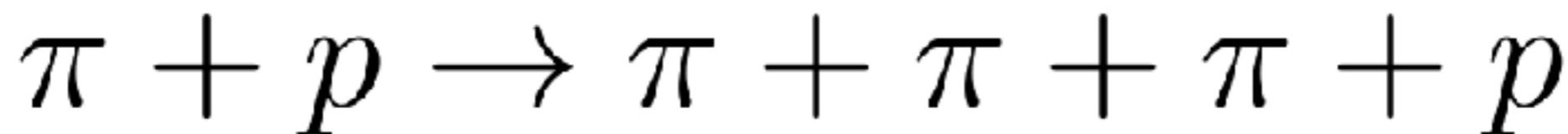
Exercício 10: Prove o decaimento de 3 corpos.



$$|\mathbf{p}_3| = \frac{[(M^2 - (m_{12} + m_3)^2) (M^2 - (m_{12} - m_3)^2)]^{1/2}}{2M}$$

# Decaimento de Três corpos

**Exercício 10 (a) Prove o decaimento abaixo**



$$\sqrt{s} \geq \sum_i m_i c^2$$

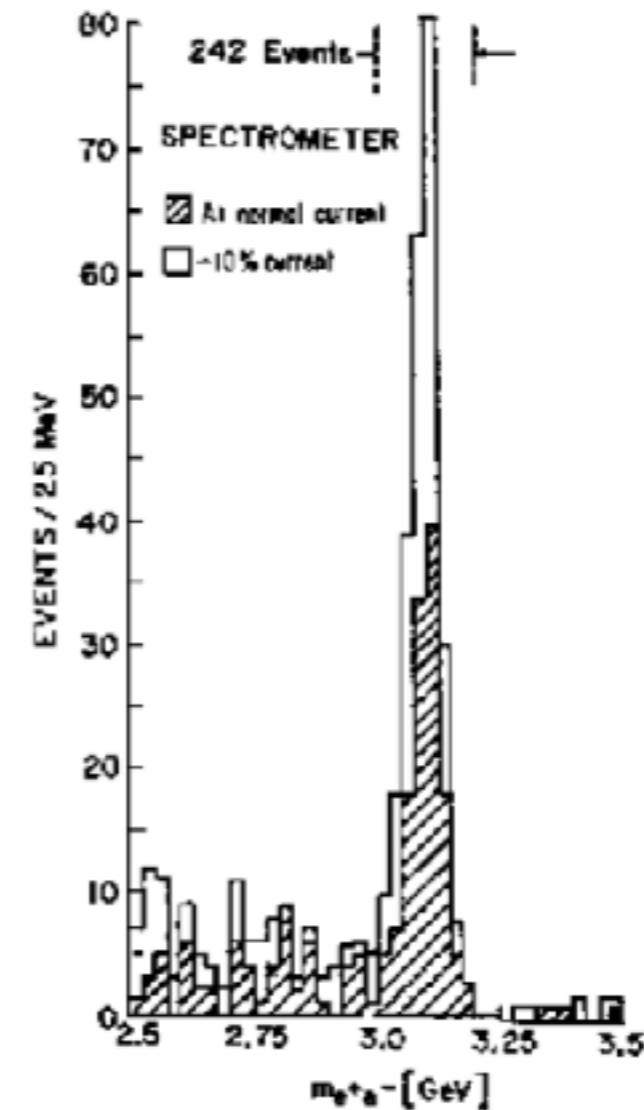
$$\begin{aligned} s &= (p_\pi + p_p)^2 c^2 = (E_\pi + m_p c^2)^2 - |\mathbf{p}_\pi|^2 \\ &= (m_\pi c^2)^2 + (m_p c^2)^2 + 2E_\pi(m_p c^2) \end{aligned}$$

$$E_\pi \geq \frac{(\sum_i m_i c^2)^2 - (m_\pi c^2)^2 - (m_p c^2)^2}{2m_p c^2} \simeq 500 \text{ MeV}$$

# Massa Invariante

**Exercício 11: Prove a equação abaixo**

$$M = \sqrt{\left(\sum E_i\right)^2 - \left(\sum \vec{p}_i\right)^2}$$



[https://en.wikipedia.org/wiki/Invariant\\_mass](https://en.wikipedia.org/wiki/Invariant_mass)

# Energia Faltante Transversa

- Se partículas invisíveis são criadas, apenas o seu momentum transversal pode ser limitado: falta de energia transversa.

$$E_T^{\text{miss}} = \sum p_T(i)$$

- Se uma partícula pesada é produzido e decai em duas partículas um dos quais é invisível, a massa da partícula principal pode ser restringida com a quantidade de massa transversa.

$$\begin{aligned} M_T^2 &\equiv [E_T(1) + E_T(2)]^2 - [\mathbf{p}_T(1) + \mathbf{p}_T(2)]^2 \\ &= m_1^2 + m_2^2 + 2[E_T(1)E_T(2) - \mathbf{p}_T(1) \cdot \mathbf{p}_T(2)] \end{aligned}$$

$$m_1 = m_2 = 0 \quad M_T^2 = 2|\mathbf{p}_T(1)||\mathbf{p}_T(2)|(1 - \cos \phi_{12})$$

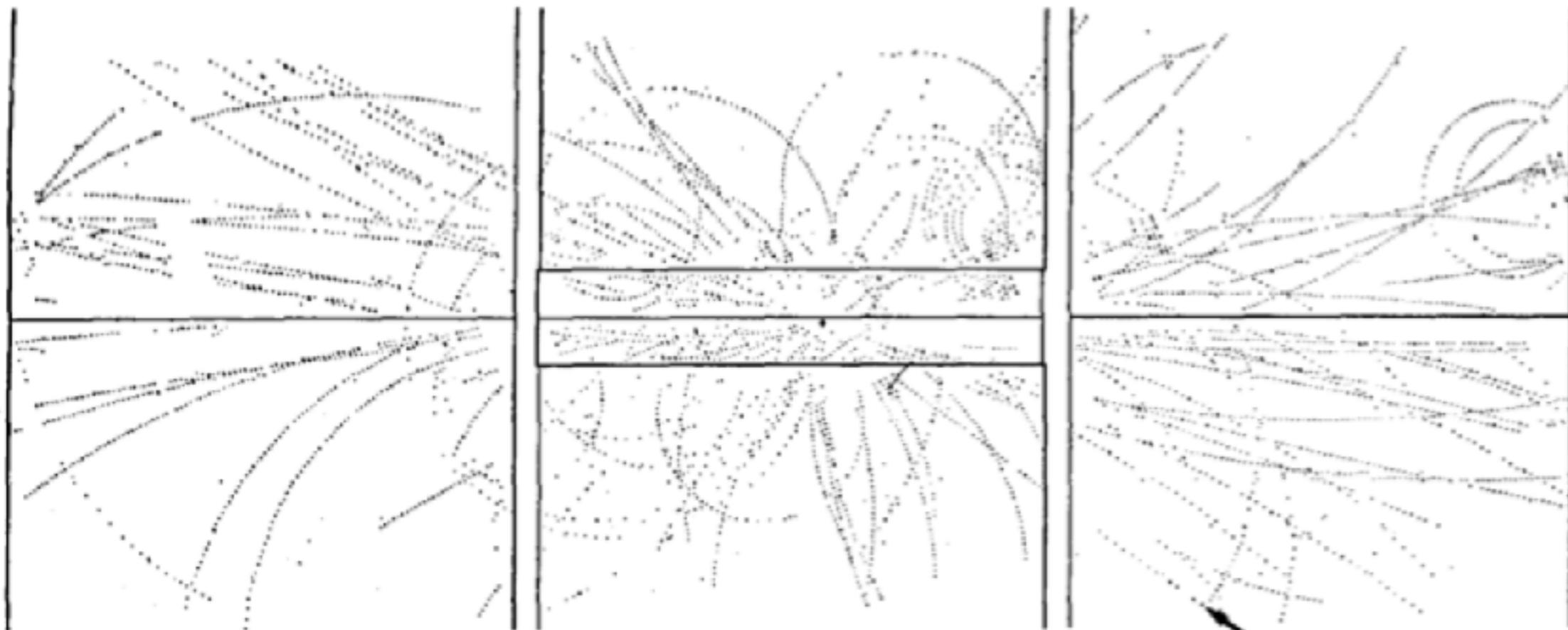
# Descoberta do $W \rightarrow e + \text{neutrino}$

Volume 122B, number 1

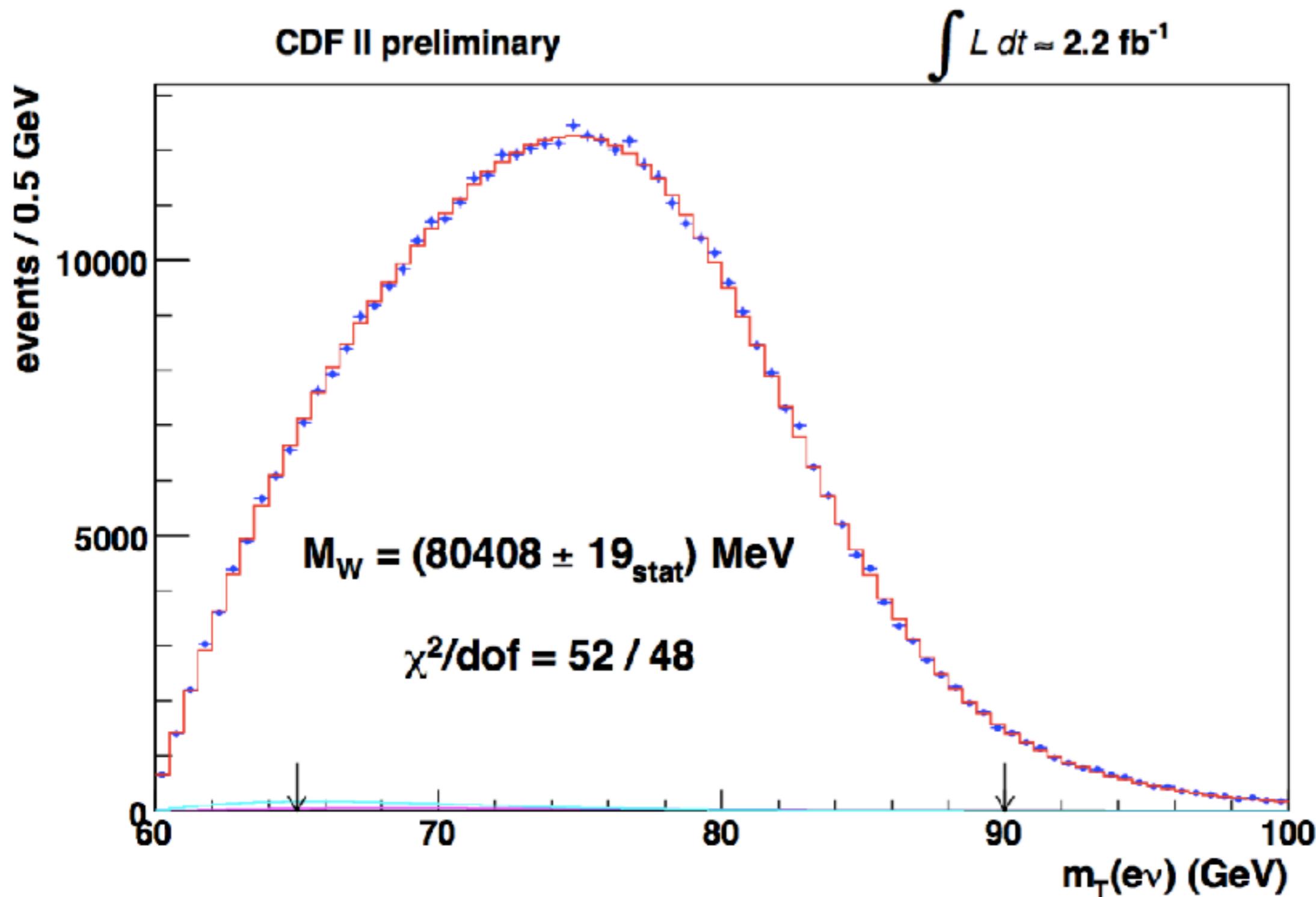
PHYSICS LETTERS

24 February 1983

**a**  
EVENT 2958. 1279.

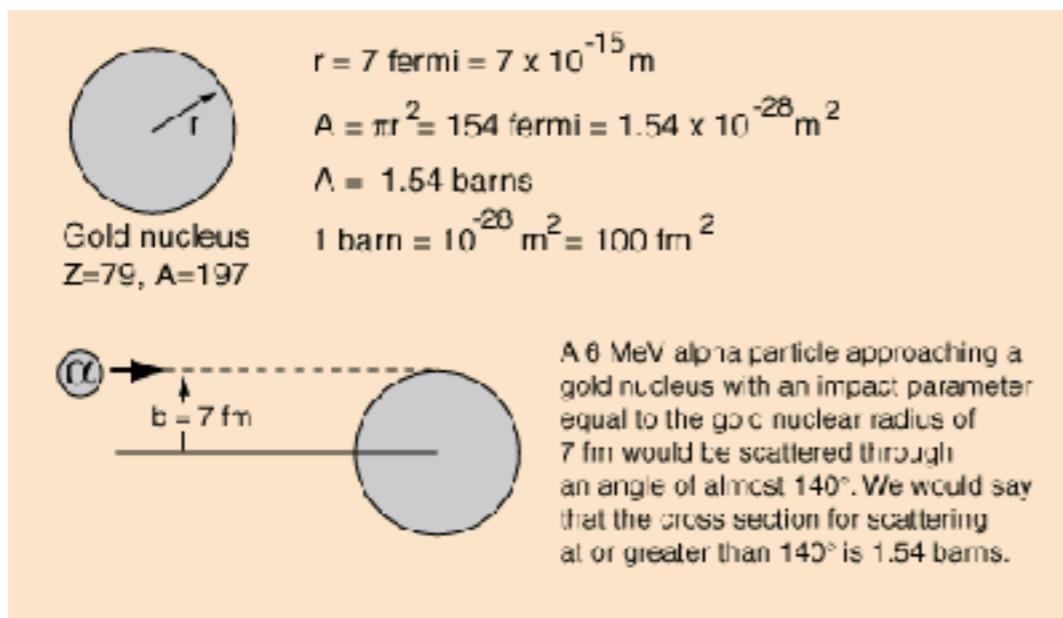


# Descoberta do $W \rightarrow e + \text{neutrino}$



# Seção de Choque

O conceito de seção de choque, como o nome sugere, corresponde a área efetiva de para uma colisão. A seção de choque de uma alvo esférico é definido por:



$$\sigma = \pi r^2$$

A unidade da seção de choque são dados em unidades de área, mas para espalhamento nuclear a área efetiva é da ordem da seção reta (seccional) do núcleo. Por exemplo, para um núcleo de ouro de número de massa  $A = 197$ , o raio determinado pelo raio do núcleo é da ordem de 7 fermis.

# Seção de Choque de Espalhamento

$$\sigma = \pi r^2$$

$$\sigma = \pi Z^2 \left( \frac{ke^2}{KE} \right)^2 \left( \frac{1 + \cos\theta}{1 - \cos\theta} \right)$$

$$\frac{R_s}{R_i} = \frac{N_A L \rho \sigma}{A \cdot 10^{-3} \text{ kg}}$$

$N_A$  = Avogadro's number

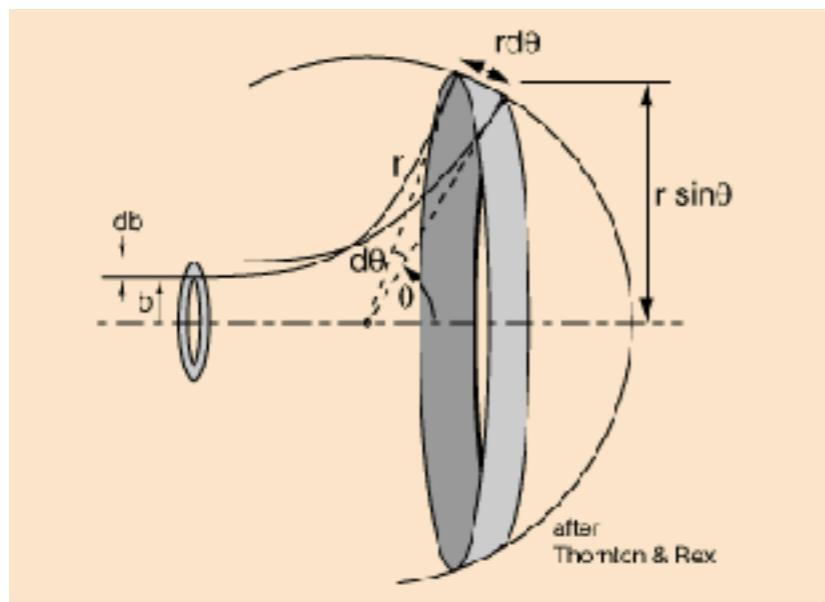
$L$  = target thickness

$A$  = mass number of target nuclei

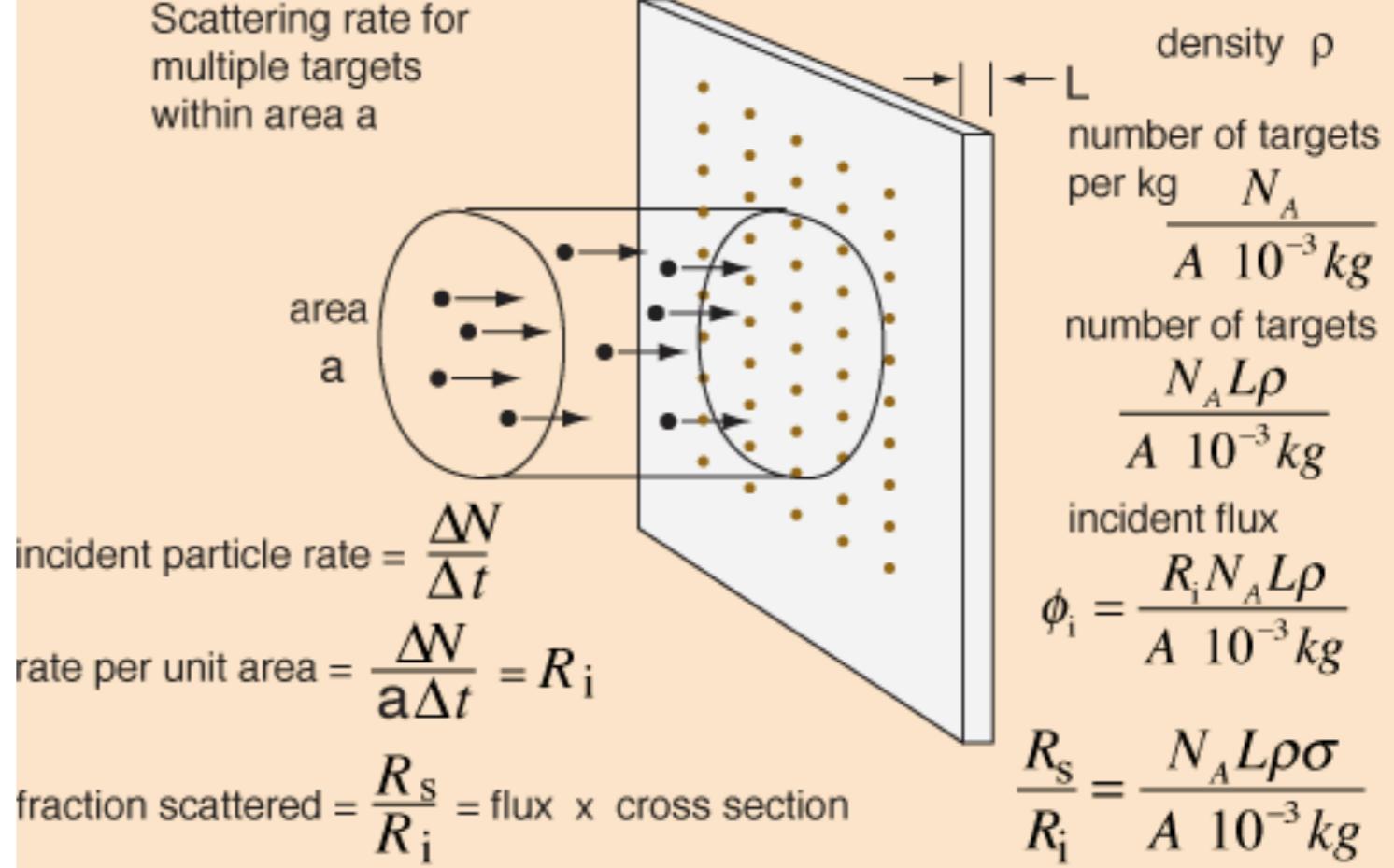
$\sigma$  = cross section

$\rho$  = density

## Seção de choque de Rutherford

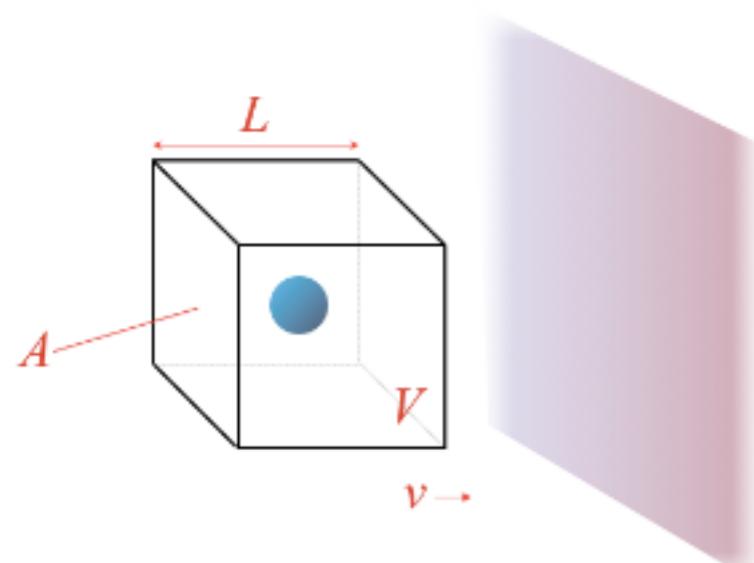


Scattering rate for multiple targets within area  $a$



# Seção de Choque em FAE

Seção de choque é uma expressão da probabilidade de ocorrência de uma transição. A origem do termo devida do espalhamento nuclear onde núcleos apresentam uma área finita de uma partícula incidente.



Hoje em dia a ideia de uma área física é apenas uma analogia (pense partículas pontuais!) Embora secção transversal mantém dimensões  $[L^2]$ . A unidade da seção de choque é barn ( $\sim 10^{-28} \text{ m}^2$ ).

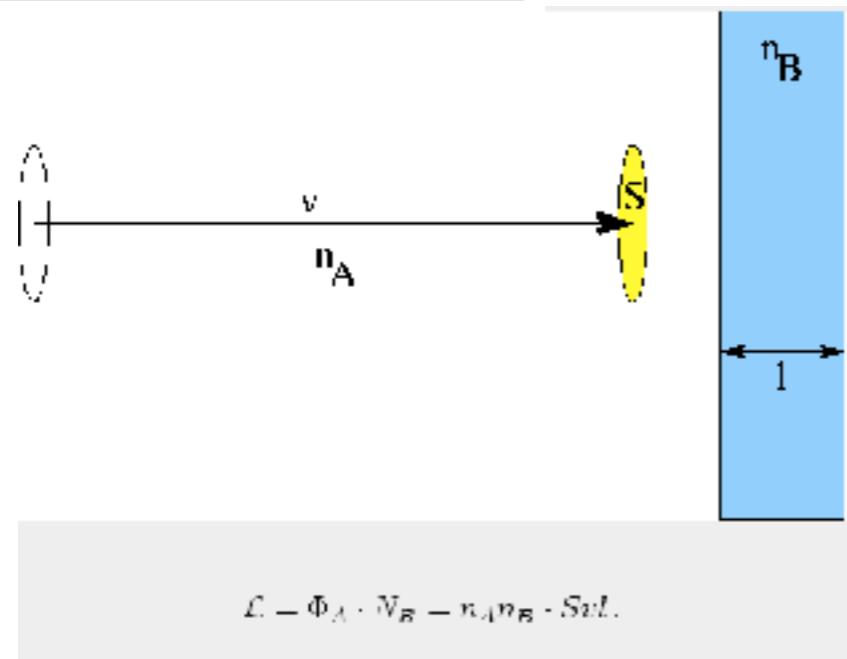
Imagine uma partícula em uma unidade de volume  $V$  que viaja através de objeto com uma velocidade  $v$ . O número de partículas que passam este alvo por unidade de tempo será  $v/L$ . O fluxo (número de partículas por unidade de área) é  $v/LA$  ou  $v/V$ . Definimos uma unidade de volume a ser aquela que contém uma partícula, em seguida, o fluxo simplifica apenas a velocidade  $v$

$$L = \frac{1}{\sigma} \frac{dN}{dt}.$$

**L = Luminosidade**

# Seção de Choque em FAE

Collider	Interaction	$L$ ( $\text{cm}^{-2} \cdot \text{s}^{-1}$ )
SPS	$p + \bar{p}$	$6.0 \times 10^{30}$
Tevatron <sup>[2]</sup>	$p + \bar{p}$	$4.0 \times 10^{32}$
HERA	proton + $e^+$	$4.0 \times 10^{31}$
LHC <sup>[3][4]</sup>	$p + p$	$1.2 \times 10^{34}$
LEP	$e^- + e^+$	$1.0 \times 10^{32}$
PEP	$e^- + e^+$	$3.0 \times 10^{33}$
KEKB <sup>[5]</sup>	$e^- + e^+$	$2.1 \times 10^{34}$

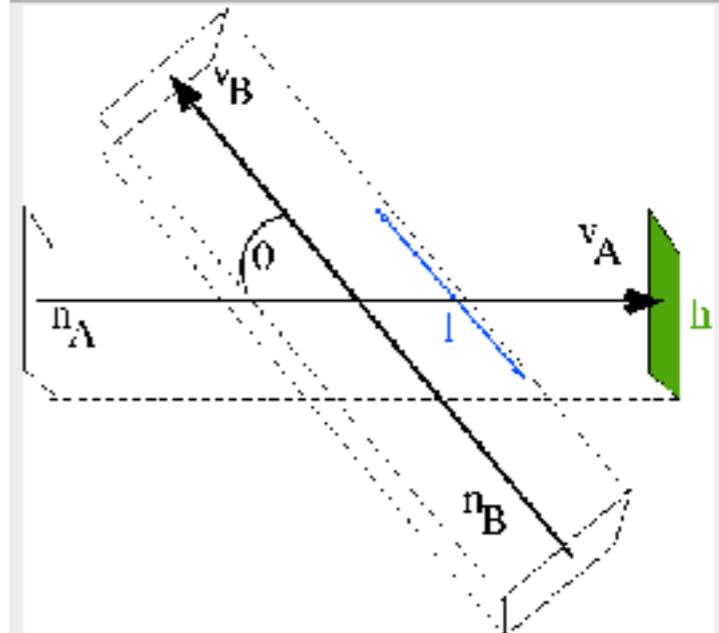


**Experimento de alvo fixo**

A razão de transição (razão de interação) é definida por:

$$\Gamma_{fi} = v \cdot \sigma$$

$$\sigma \cdot \mathcal{L} = \mathcal{R} .$$



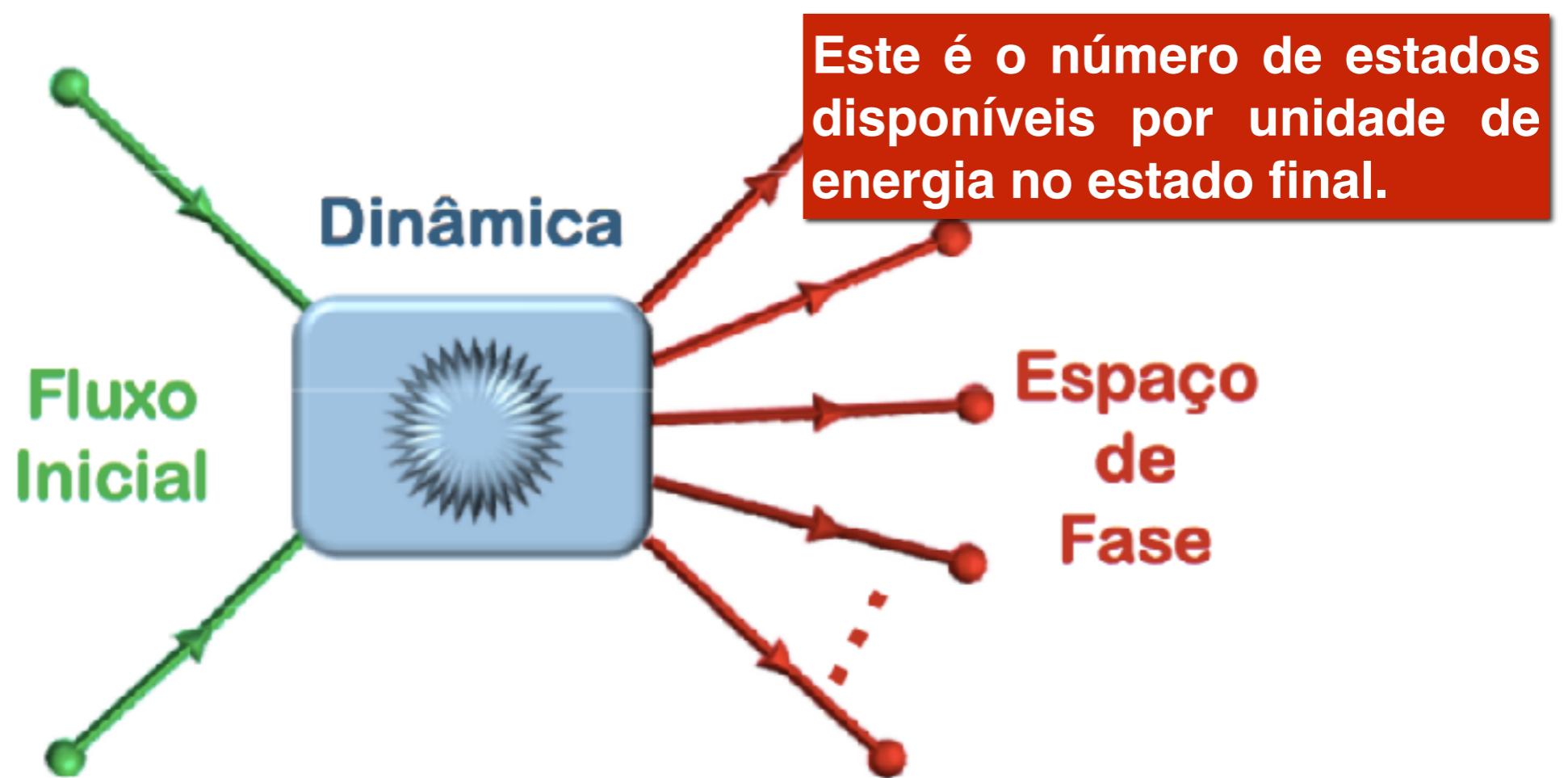
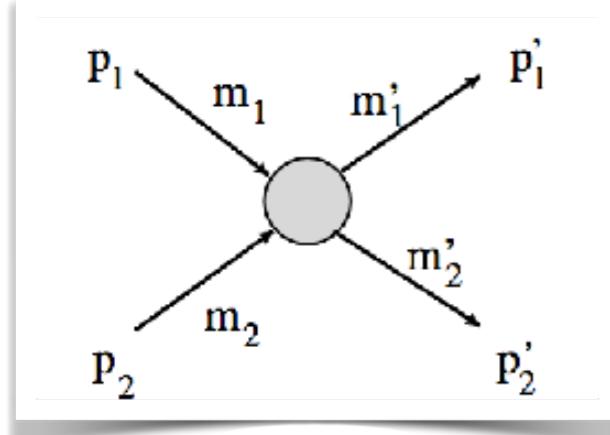
**Colisores de Partículas**

$$\mathcal{L} = \frac{\Phi_A \Phi_B}{hv_B \sin \theta} .$$

[https://en.wikipedia.org/wiki/Luminosity\\_\(scattering\\_theory\)](https://en.wikipedia.org/wiki/Luminosity_(scattering_theory))

<https://home.cern/cern-people/opinion/2011/03/luminosity-why-dont-we-just-say-collision-rate>

# Cinemática + Dinâmica



$$\frac{1}{|\vec{\beta}_a - \vec{\beta}_b|} \frac{1}{(2E_a)} \frac{1}{(2E_b)}$$

$$|\mathcal{M}|^2$$

$$\frac{S}{(2\pi)^{3n-4}} \prod_{i=1}^n \frac{d^3 p_i}{2E_i} \delta^4 \left( p_a + p_b - \sum_{i=1}^n p_n \right)$$

# Cinemática + Dinâmica



Este é o número de estados disponíveis por unidade de energia no estado final.

In quantum physics, Fermi's golden rule is a simple formula for the constant transition rate (probability of transition per unit time) from one energy eigenstate of a quantum system into other energy eigenstates in a continuum, effected by a perturbation. This rate is effectively constant.

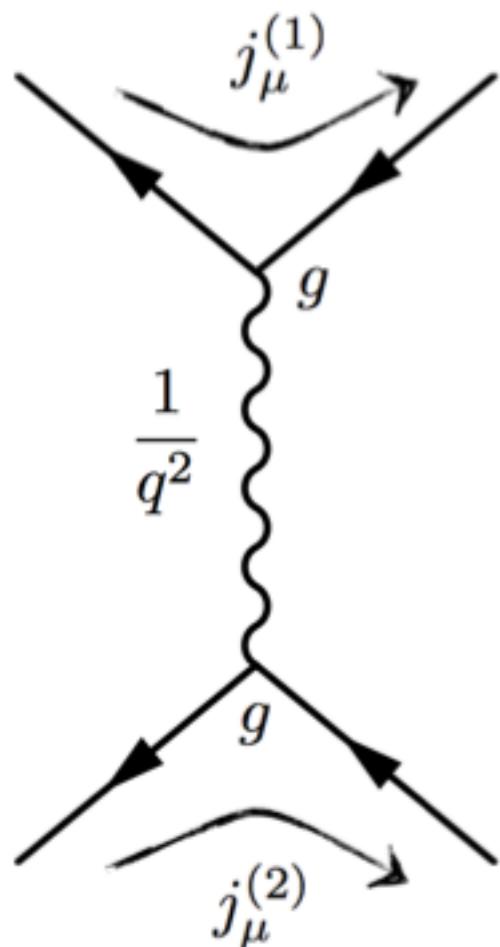


$$\frac{1}{|\vec{\beta}_a - \vec{\beta}_b|} \frac{1}{(2E_a)} \frac{1}{(2E_b)}$$

$$|\mathcal{M}|^2$$

$$\frac{S}{(2\pi)^{3n-4}} \prod_{i=1}^n \frac{d^3 p_i}{2E_i} \delta^4 \left( p_a + p_b - \sum_{i=1}^n p_n \right)$$

# Regra de ouro de Fermi



*transition probability*



*matrix element*



*energy density of final states*



$$W_{if} = \frac{2\pi}{\hbar} |M_{if}|^2 \frac{dN}{dE_f}$$

[ $t^I$ ]

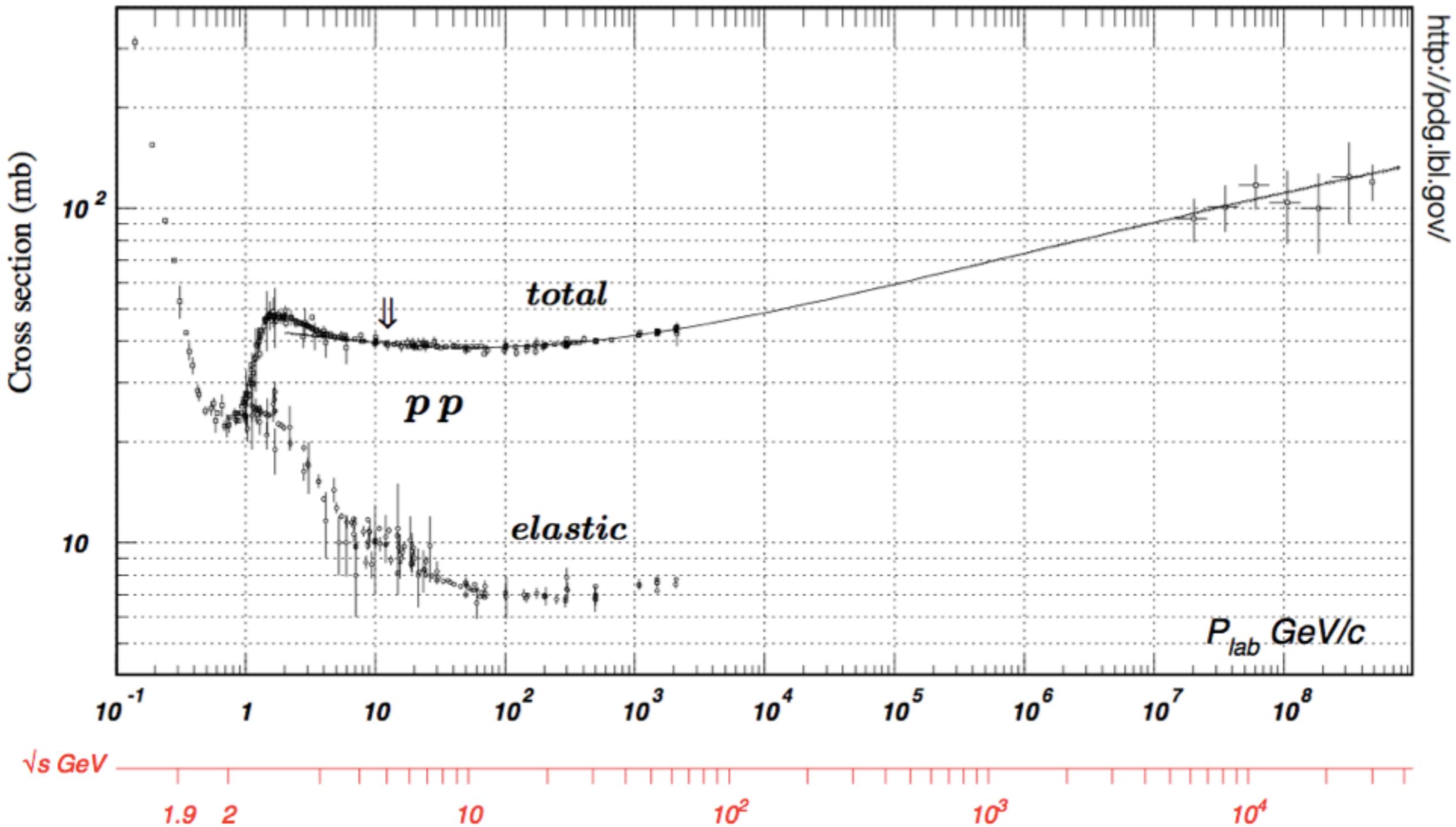
[ $E$ ]

[ $E^{-I}$ ]

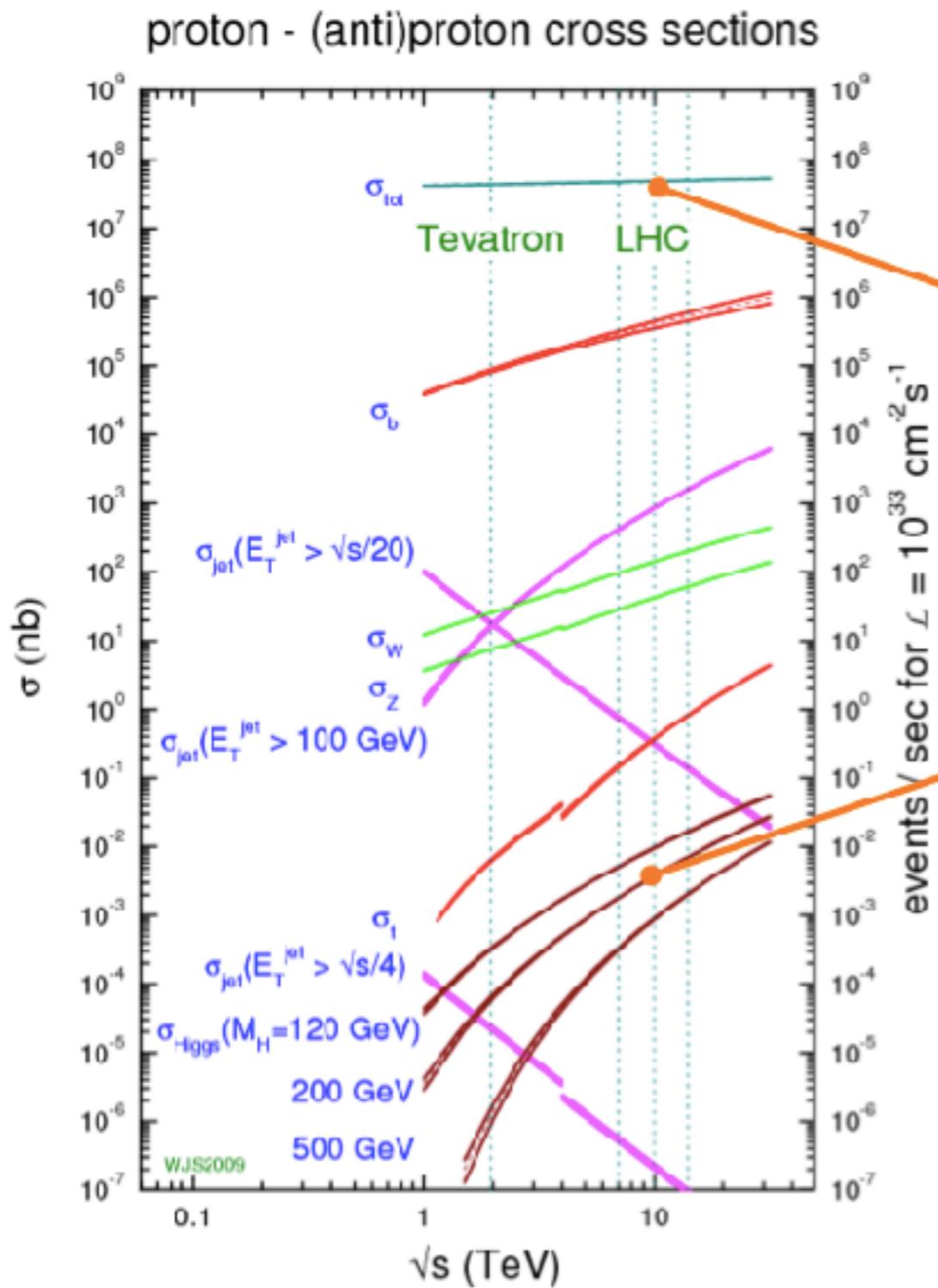
$$M_{if} = -i \int j_\mu^{(1)} \left( \frac{1}{q^2} \right) j_\mu^{(2)} d^4x$$

$$\sigma \sim |M_{if}|^2 \sim g^4 \left( \frac{1}{q^4} \right)$$

# Seção de choque pp



# Seção de choque no LHC



$10^8 \text{ events/s}$

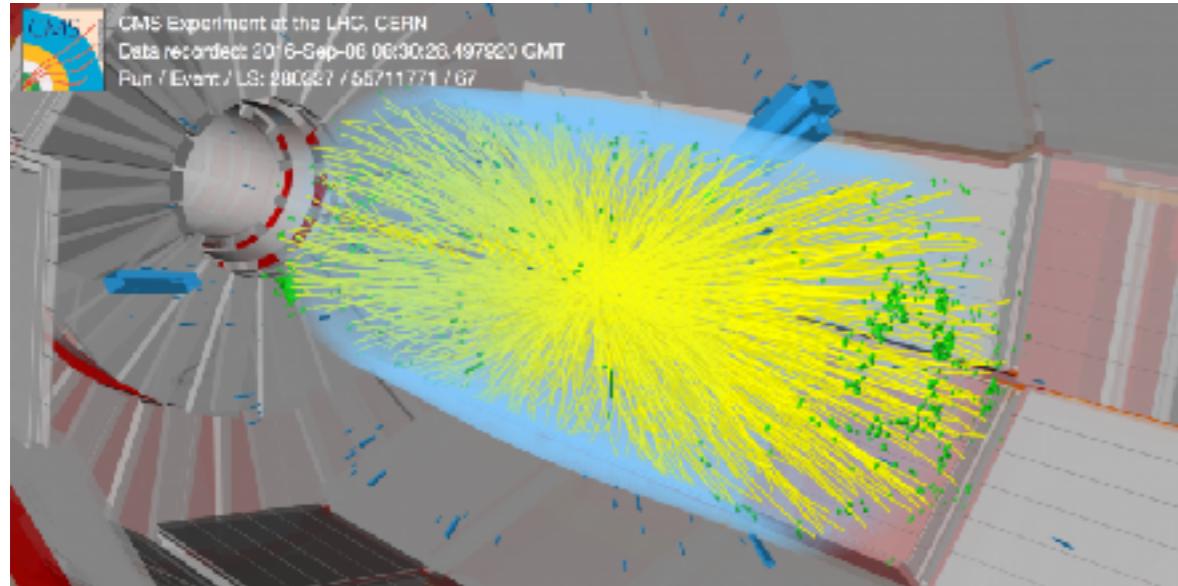
$\sim 10^{10}$

$10^{-2} \text{ events/s} \sim$   
 $10 \text{ events/min}$

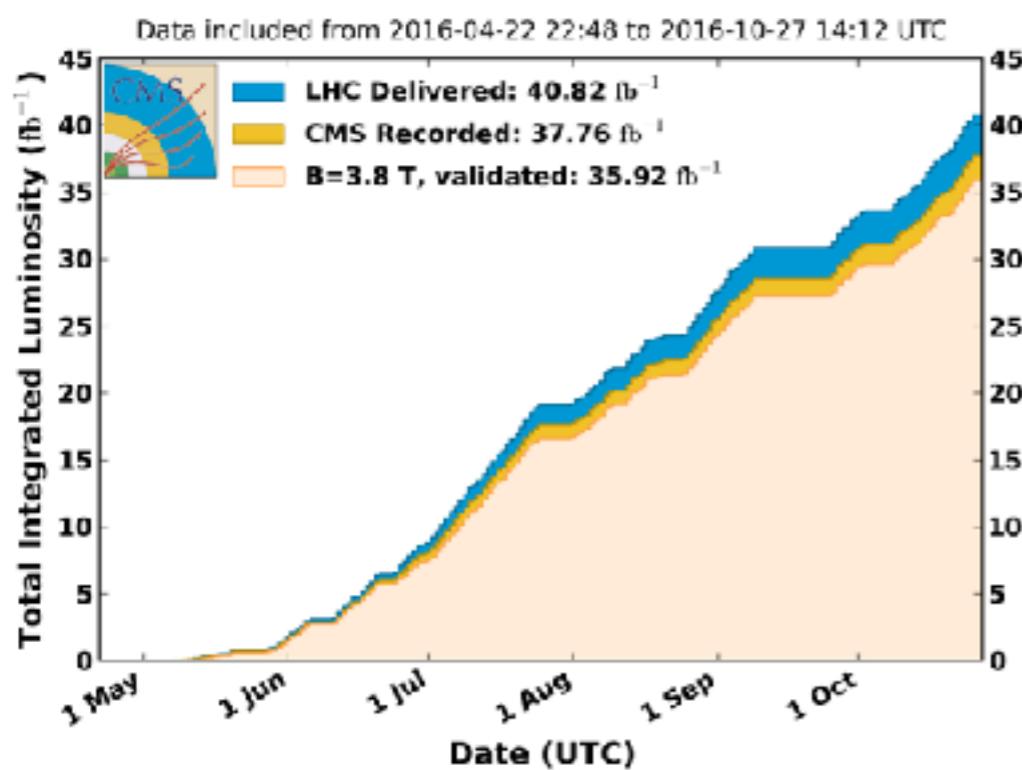
$[m_H \sim 120 \text{ GeV}]$

0.2%  $H \rightarrow \gamma\gamma$   
1.5%  $H \rightarrow ZZ$

# Luminosidade



CMS Integrated Luminosity, pp, 2016,  $\sqrt{s} = 13 \text{ TeV}$



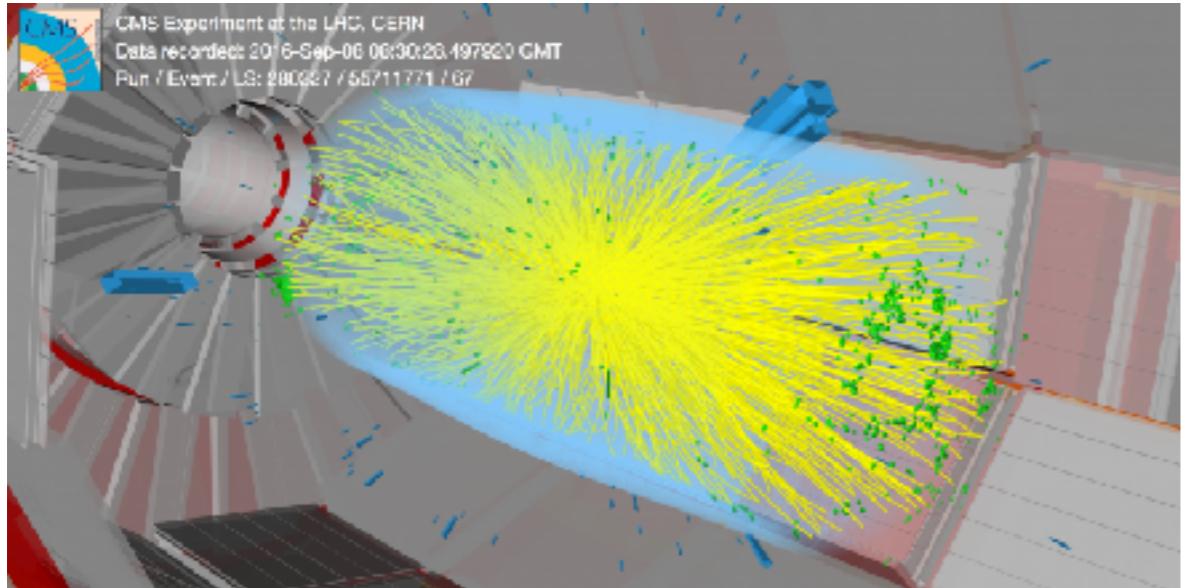
Luminosidade ( $L$ ) é um dos parâmetros mais importantes de um acelerador. É uma medida do número de colisões que podem ser produzidas em um detector por  $\text{cm}^2$  e por segundo.



Quanto maior for o valor de  $L$ , maior será o número de colisões.

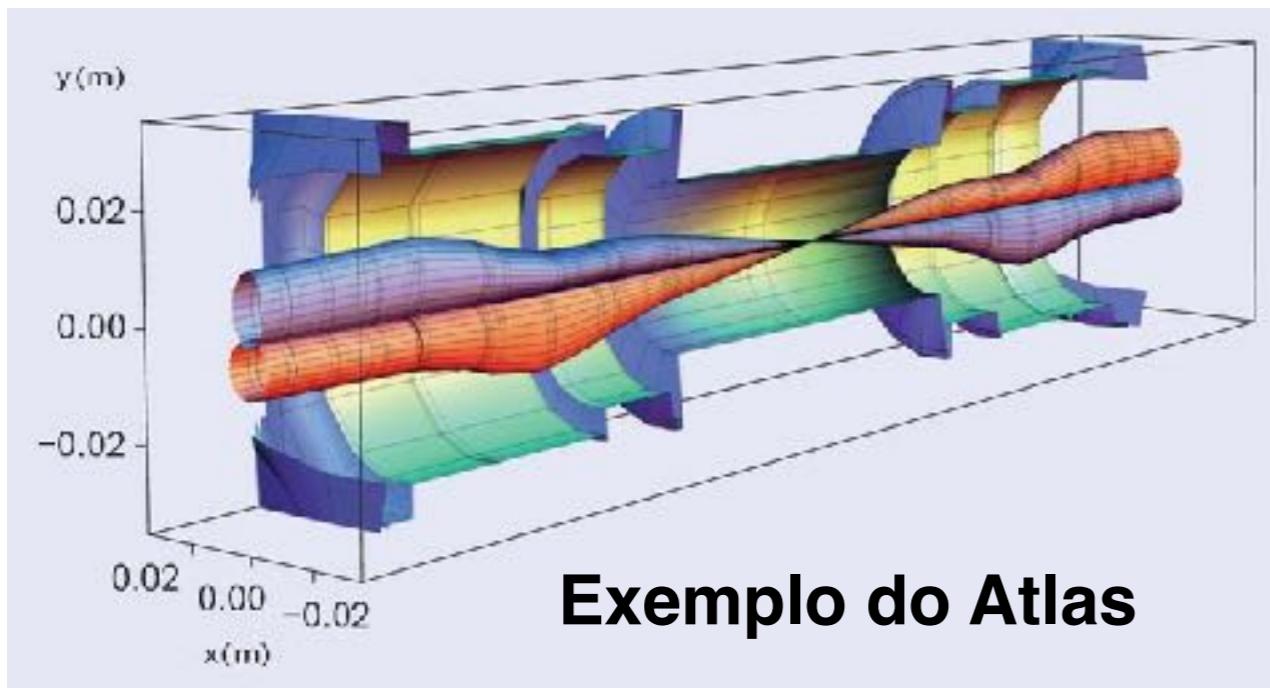
Para determinar o número de colisões, precisamos também considerar a seção de choque do processo

# Luminosidade



$L$  pode ser obtido semiqualitativamente a partir de:

- $N^2$ : número de prótons, porque cada partícula em um grupo pode colidir com qualquer um do grupo se aproximando de frente.
- $t$ : tempo entre pacotes
- $S_{\text{eff}}$ : seção efetiva de colisão que depende da seção transversal do feixe (“efetiva” porque o perfil da feixe não possui borda viva); a fórmula para isso é dada por:  $S_{\text{eff}} = 4 \cdot \pi \cdot \sigma^2$  com  $\sigma = 16$  micrões ou  $16 \cdot 10^{-4}$  cm (tamanho transversal do pacote no Ponto de Interação).
- Outro parâmetro a ser considerado é  $F$ , o fator geométrico de redução da luminosidade ( $\leq 1$ ), devido ao ângulo de cruzamento no ponto de interação (IP). Mas em 2011  $F \sim 0,95$ , portanto, pode ser considerado como 1.



Exemplo do Atlas

Envelopes do feixe na região de interação em torno do ponto 1 (ATLAS) mostrando como os tamanhos dos feixes são reduzidos nos últimos 60 m em cada lado do ponto de interação após o aperto. Observe a escala transversal diferente: o raio do tubo de feixe cortado é de apenas 18 mm no ponto de colisão. O feixe do sentido horário é azul e o feixe anti-horário é vermelho.

$$L \sim N^2 / (t \cdot S_{\text{eff}})$$

$$\text{Now, with } N^2 = (1,15 \cdot 10^{11})^2$$

$$t = 25 \cdot 10^{-9} \text{ s}, S_{\text{eff}} = 4 \cdot \pi (16 \cdot 10^{-4})^2 \text{ cm}^2$$

$$L \sim 10^{34} \text{ cm}^{-2} \cdot \text{s}^{-1}$$

<https://cerncourier.com/a/the-lhcs-first-long-run-2/>

# Luminosidade

Se usarmos a frequência de cruzamento dos pacotes ( $f$ , neste caso  $40 \times 10^6$ ) e  $S_{eff} = 4 \pi \sigma^2$ , podemos expressar a Luminosidade de uma forma mais conhecida:

$$L \sim f \cdot N^2 / (4 \cdot \pi \cdot \sigma^2)$$

E considerando o número diferente de prótons por cruzamento de feixes, e os componentes x e y para  $\sigma$  separadamente:

$$L = f \cdot N_1 N_2 / (4\pi \sigma_x \sigma_y)$$

Também podemos expressar a luminosidade ( $L$ ) em termos de  $\epsilon$  (emitância) e  $\beta$  (função de amplitude) como:

$$L = f \cdot N^2 / (4 \cdot \epsilon \cdot \beta^*)$$

Este valor,  $10^{34}$  cm $^{-2} \cdot s^{-1}$ , significa que os detectores do LHC podem produzir  $10^{34}$  colisões por segundo e por cm $^2$ . Uma vez que no LHC o valor de  $L$  é 100 vezes maior que o de LEP ou Tevatron.

# Luminosidade

## Beta function (accelerator physics)

From Wikipedia, the free encyclopedia

The [beta function in accelerator physics](#) is a function related to the transverse size of the particle beam at the location  $s$  along the nominal beam trajectory.

It is related to the transverse beam size as follows:<sup>[1]</sup>

$$\sigma(s) = \sqrt{\epsilon \cdot \beta(s)}$$

where

- $s$  is the location along the nominal beam trajectory
- the beam is assumed to have a Gaussian shape in the transverse direction
- $\sigma(s)$  is the width of this Gaussian
- $\epsilon$  is the RMS geometrical [beam emittance](#), which is normally constant along the trajectory when there is no acceleration

Typically, separate beta functions are used for two perpendicular directions in the plane transverse to the beam direction (e.g. horizontal and vertical directions).

The beta function is one of the [Twiss parameters](#) (also called [Courant-Snyder](#) functions).

### Beta star [edit]

The value of the beta function at an interaction point is referred to as **beta star**. The beta function is typically adjusted to have a local minimum at such points (in order to minimize the beam size and thus maximise the interaction rate). Assuming that this point is in a drift space, one can show that the evolution of the beta function around the minimum point is given by:

$$\beta(z) = \beta^* + \frac{z^2}{\beta^*}$$

where  $z$  is the distance along the nominal beam direction from the minimum point.

This implies that the smaller the beam size at the interaction point, the faster the rise of the beta function (and thus the beam size) when going away from the interaction point. In practice, the [aperture](#) of the beam line elements (e.g. focusing magnets) around the interaction point limit how small beta star can be made.

# Luminosidade

## Beam emittance

From Wikipedia, the free encyclopedia

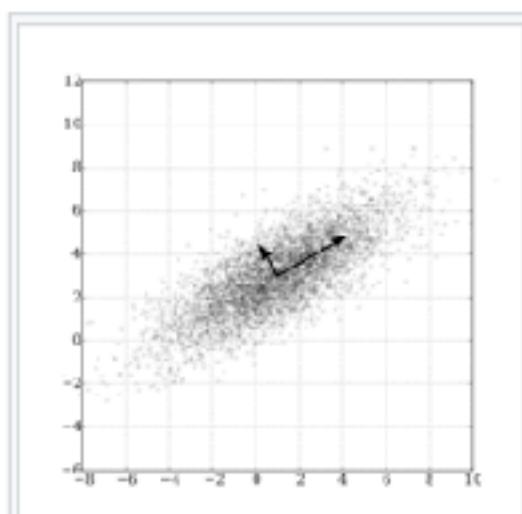
You have a new message (last change).

**Emittance** is a property of a [charged particle beam](#) in a [particle accelerator](#). It is a measure for the average spread of particle coordinates in [position-and-momentum phase space](#) and has the dimension of length (e.g., meters) or length times angle (meters times radians). As a particle beam propagates along magnets and other beam-manipulating components of an accelerator, the position spread may change, but in a way that does not change the emittance. If the distribution over phase space is represented as a cloud in a plot (see figure), emittance is the area of the cloud. A more exact definition handles the fuzzy borders of the cloud and the case of a cloud that does not have an elliptical shape.

A low-emittance particle beam is a beam where the particles are confined to a small distance and have nearly the same [momentum](#). A beam transport system will only allow particles that are close to its design momentum, and of course they have to fit through the beam pipe and magnets that make up the system. In a colliding beam accelerator, keeping the emittance small means that the likelihood of particle interactions will be greater resulting in higher [luminosity](#). In a [synchrotron light source](#), low emittance means that the resulting x-ray beam will be small, and result in higher [brightness](#).

**Contents** [hide]

- 1 [Definition](#)
- 2 [Emittance of electrons versus heavy particles](#)



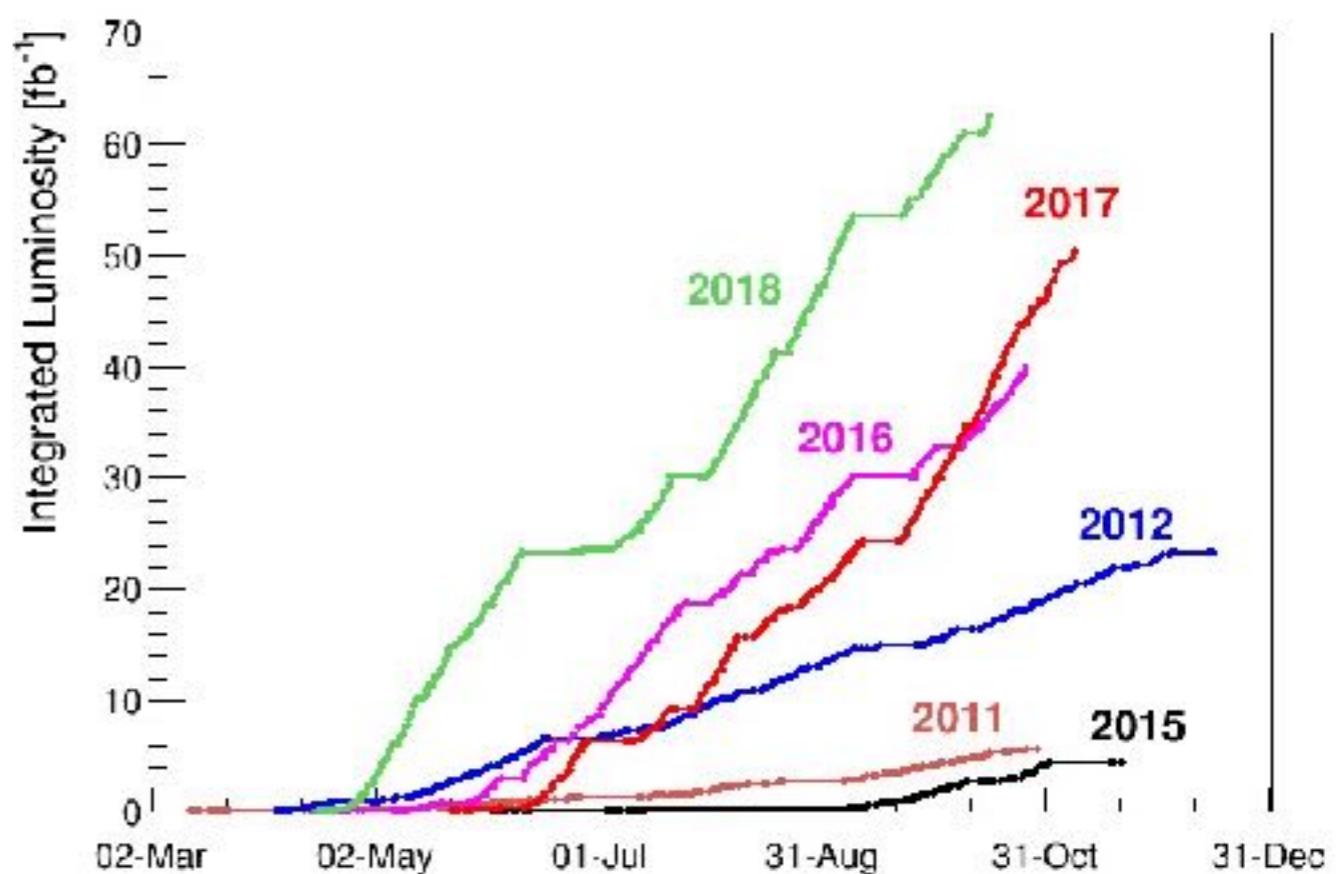
Samples of a [bivariate normal distribution](#), representing particles in phase space, with position horizontal and momentum vertical.

# Luminosidade Integrada

A integral da luminosidade fornecida ao longo do tempo é chamada de luminosidade integrada. É uma medida do tamanho dos dados coletados e é um valor importante para caracterizar o desempenho de um acelerador.

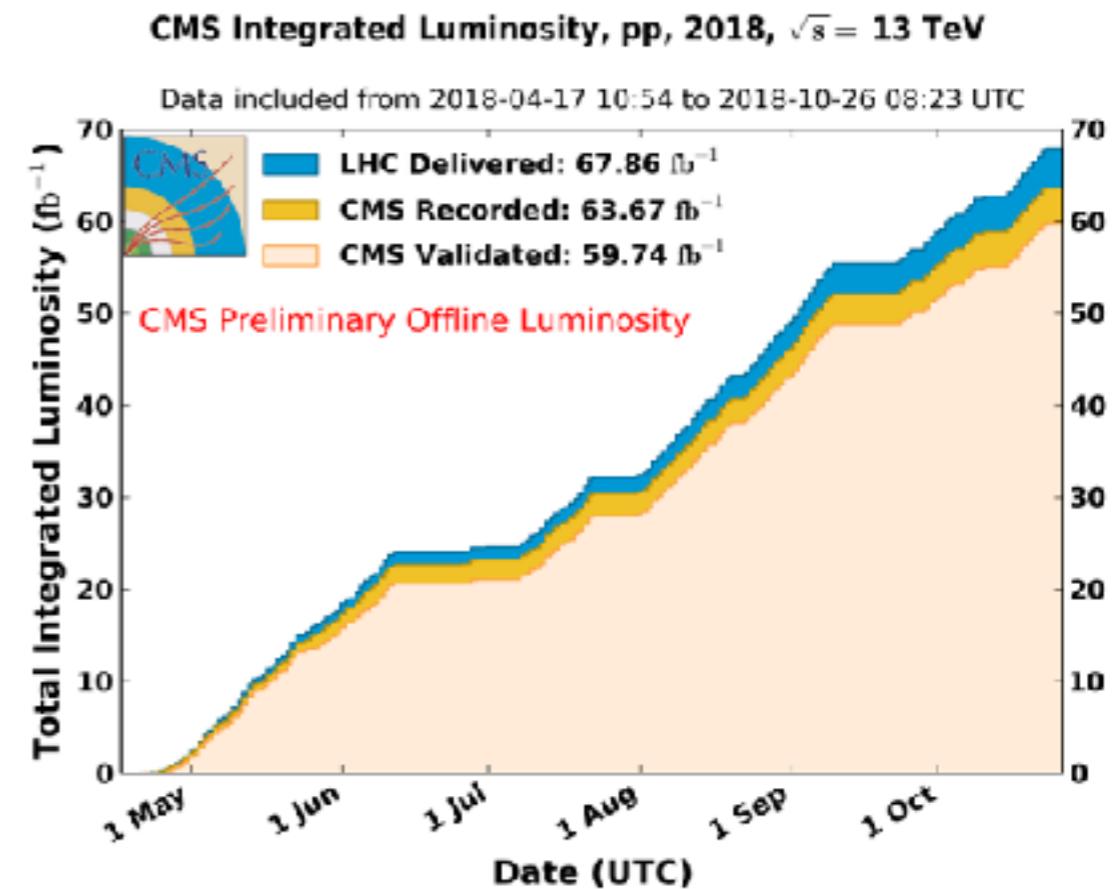
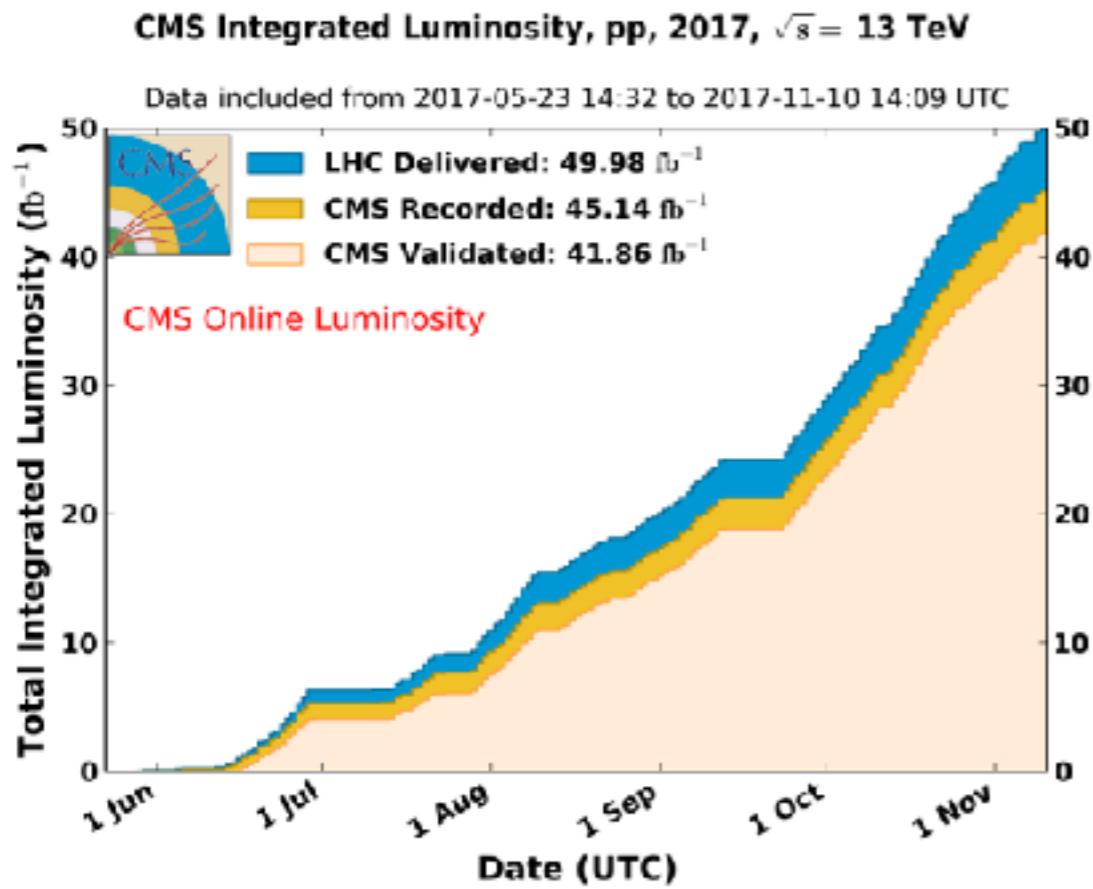
$$L = \int_a^b L dt,$$

A luminosidade integrada fornecida aos experimentos ATLAS e CMS durante diferentes execuções do LHC. A corrida de 2018 produziu 65 femtobarns inversos de dados, o que é 16 pontos a mais do que em 2017.

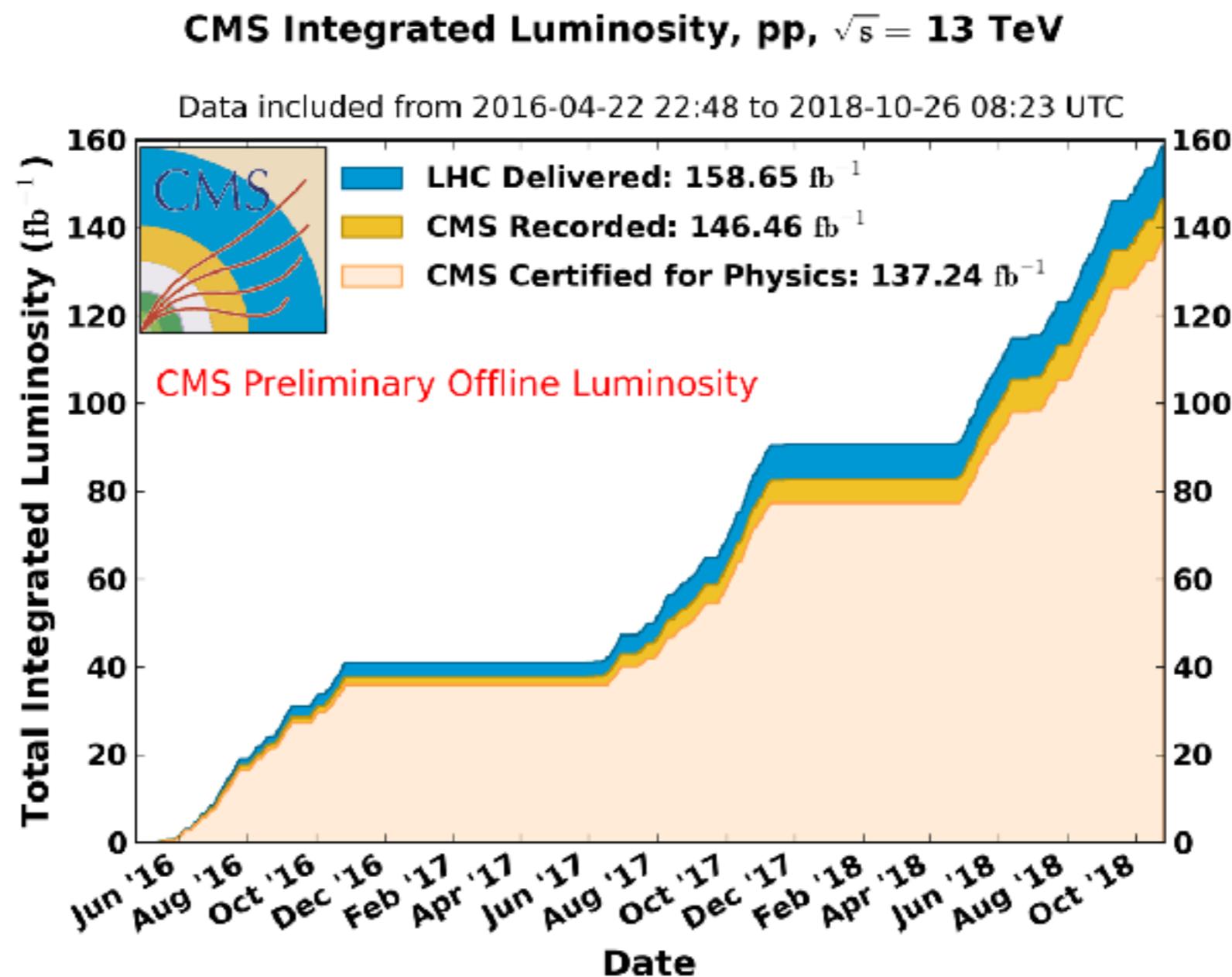


Normalmente, é expresso como o inverso da seção de choque (ou seja, 1 / nb ou nb⁻¹ - nanobarn⁻¹; 1 / pb ou pb⁻¹ - picobarn⁻¹; 1 / fb ou 1fb⁻¹ - femtobarn⁻¹).

# Luminosidade Integrada



# Luminosidade Integrada- Run 2



The plot shows the cumulative curves for the luminosity delivered by LHC (azule), recorded by CMS (orange) and certified as good for physics analysis during stable beams (light orange).  
The luminosity validated for physics analysis corresponds to data recorded with all detectors and reconstructed physics objects showing good performance.

# Beyond LHC- FCC collider

Voir en français

## Particle physicists update strategy for the future of the field in Europe

The CERN Council today announced that it has updated the strategy that will guide the future of particle physics in Europe

19 JUNE, 2020



(Image: CERN)

Following almost two years of discussion and deliberation, the CERN Council today announced that it has updated [the strategy that will guide the future of particle physics in Europe](#) within the global particle-physics landscape. Presented during the open part of the Council's meeting, held remotely due to the ongoing COVID-19 pandemic, the recommendations highlight the scientific impact of particle physics, as well as its technological, societal and human capital.

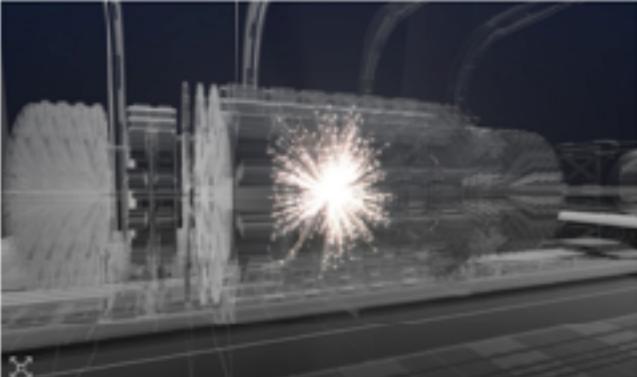
By probing ever-higher energy and thus smaller distance scales, particle physics has made discoveries that have transformed the scientific understanding of the world. Nevertheless, many of the mysteries about the universe, such as the nature of dark matter, and the [preponderance of matter over antimatter](#), are still to be explored. The 2020 update of the [European Strategy for Particle Physics](#) proposes a vision for both the near- and the long-term future of the field, which maintains Europe's leading role in addressing the outstanding questions in particle physics and in the innovative technologies being developed within the field.

## projects and facilities

PRESENTS AND FACILITIES - NEWS

CERN approves further work on Future Circular Collider – but delays final decision

19 Jun 2020 Michael Banks



Grand designs: the CERN council have endorsed a plan to for the coming decade in particle physics that includes further design work on the Future Circular Collider. (Courtesy: CERN)

The CERN Council has today approved [an update to the European Strategy for Particle Physics](#) that recommends further work on a huge 100 km collider – dubbed the [Future Circular Collider \(FCC\)](#) – that would be built in Geneva. But with no formal decision having been made to go ahead with the FCC, the strategy also calls for Europe to look a Japanese-led linear collider if it receives the go-ahead from the Japanese government.

The report, [released this morning](#), sets out a plan for the future of particle physics in Europe to the mid-2020s and beyond. It especially concerns planning the next collider that would succeed the [Large Hadron Collider](#), which first switched on in 2008. The 27 km-circumference LHC has been smashing protons together at energies up to 13 TeV in the hunt for new particles and in 2012 physicists announced they had discovered the Higgs boson with a mass of 126 GeV.

The LHC is currently undergoing [a major £1.1bn "high luminosity" upgrade](#) – dubbed HL-LHC

# Beyond LHC- FCC collider

Voir en français

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19 JUNE, 2020



(Image: CERN)

Following almost two years of discussion and deliberation, the CERN Council has approved an update to the European Strategy for Particle Physics. Presented during the open part of the Council's meeting on 19 June, the recommendations highlight the scientific, technological, societal and human capital.

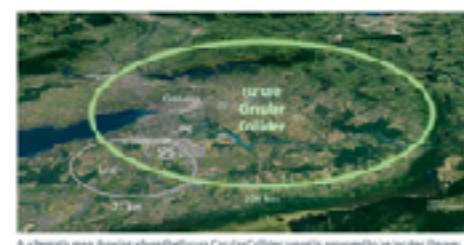
By probing ever-higher energy and thus smaller distance scales, particle physics has transformed the scientific understanding of the world. Nevertheless, many questions remain, such as the nature of dark matter, and the preponderance of matter over antimatter. The 2020 update of the European Strategy for Particle Physics proposes a new direction for the field, which maintains Europe's leading role in addressing these questions and in the innovative technologies being developed within the field.



The Future Circular Collider Study (FCC) is developing designs for a higher performance particle collider to extend the research currently being conducted at the Large Hadron Collider (LHC), whose life reaches the end of its lifespan.

The goal of the FCC is to greatly push the energy and intensity frontiers of particle colliders, with the aim of reaching collision energies of 100 TeV, in the search for new physics.

The FCC Study, involving CERN, linear accelerators and more than 150 universities, research institutes and industrial partners from all over the world, will elaborate on different possibilities for circular colliders, new detector facilities, the associated infrastructure, cost estimates, global implementation scenarios, as well as appropriate international governance structures.



A schematic map showing where the Future Circular Collider would be located (Image: CERN).

The FCC will consider scenarios for three different types of particle collisions: nuclear (proton-proton and heavy ion) collisions, like in the LHC; electron-positron collisions, as in the former LEP; and proton-electron collisions.

Scientists are currently conducting physics and detector studies for each option. In parallel, dedicated teams of experts are performing in-depth analyses of infrastructure, operations management, and the key technologies required.



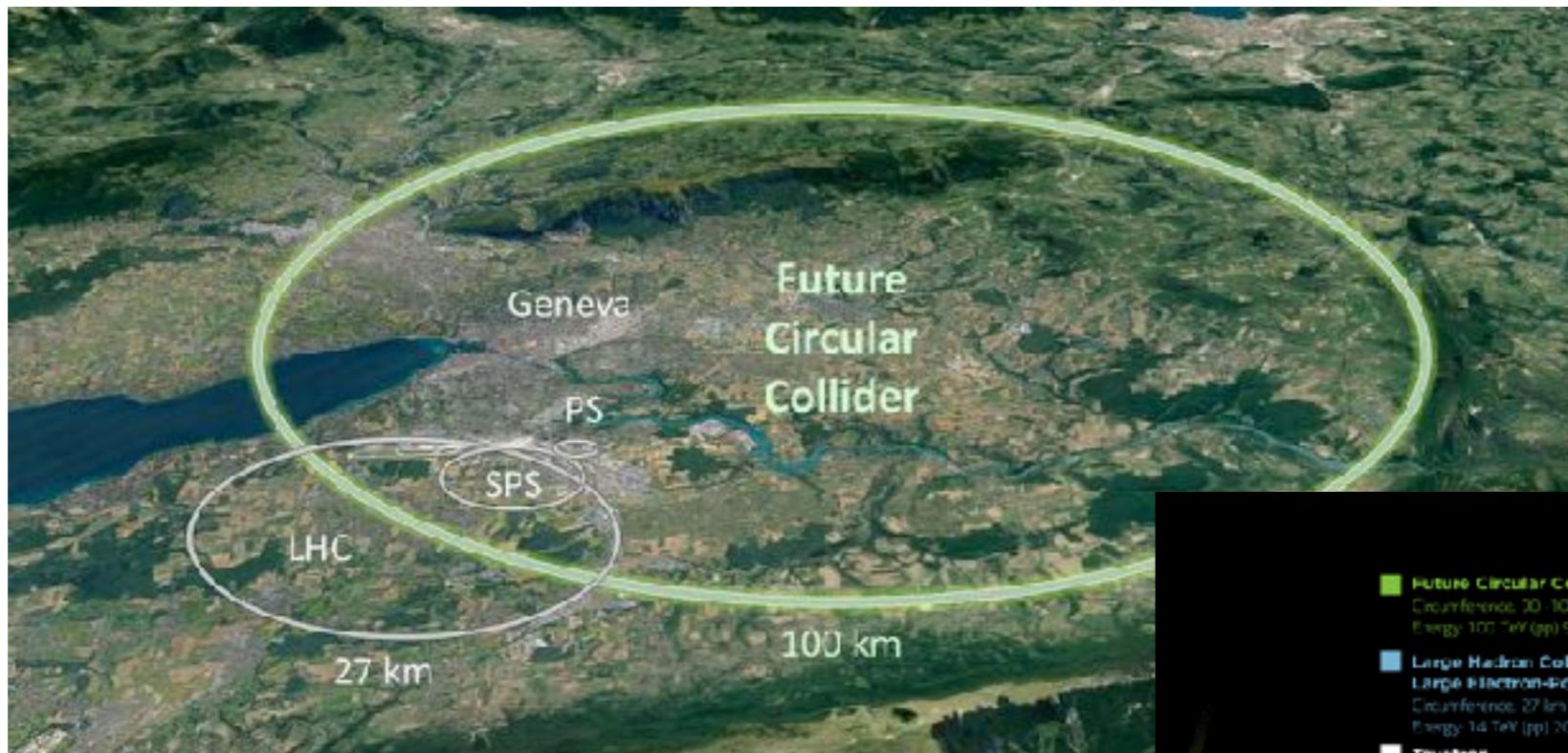
Further work on Future Circular Collider – but how?

The CERN Council has endorsed a plan to for the coming decade in particle physics that includes the Future Circular Collider. (Courtesy: CERN)

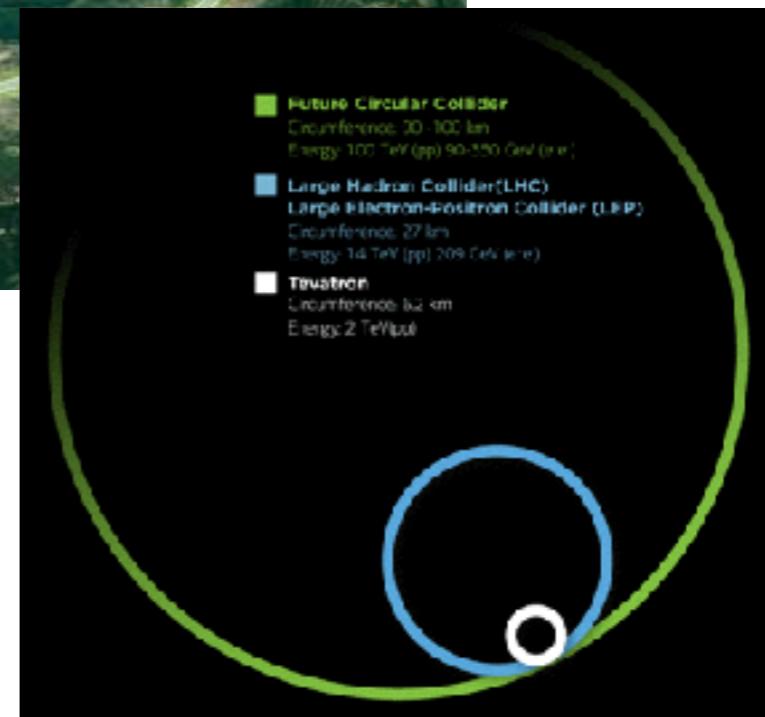
by approved an update to the European Strategy for Particle Physics work on a huge 100 km collider – dubbed the Future Circular Collider – will be built in Geneva. But with no formal decision having been made, the strategy also calls for Europe to look a Japanese-led linear collider ahead from the Japanese government.

erning, sets out a plan for the future of particle physics in Europe and it especially concerns planning the next collider that would follow the Large Hadron Collider, which first switched on in 2008. The 27 km-circumference machine together at energies up to 13 TeV in the hunt for new particles announced they had discovered the Higgs boson with a mass of 125 GeV.

going a major £1.1bn "high luminosity" upgrade – dubbed HL-LHC

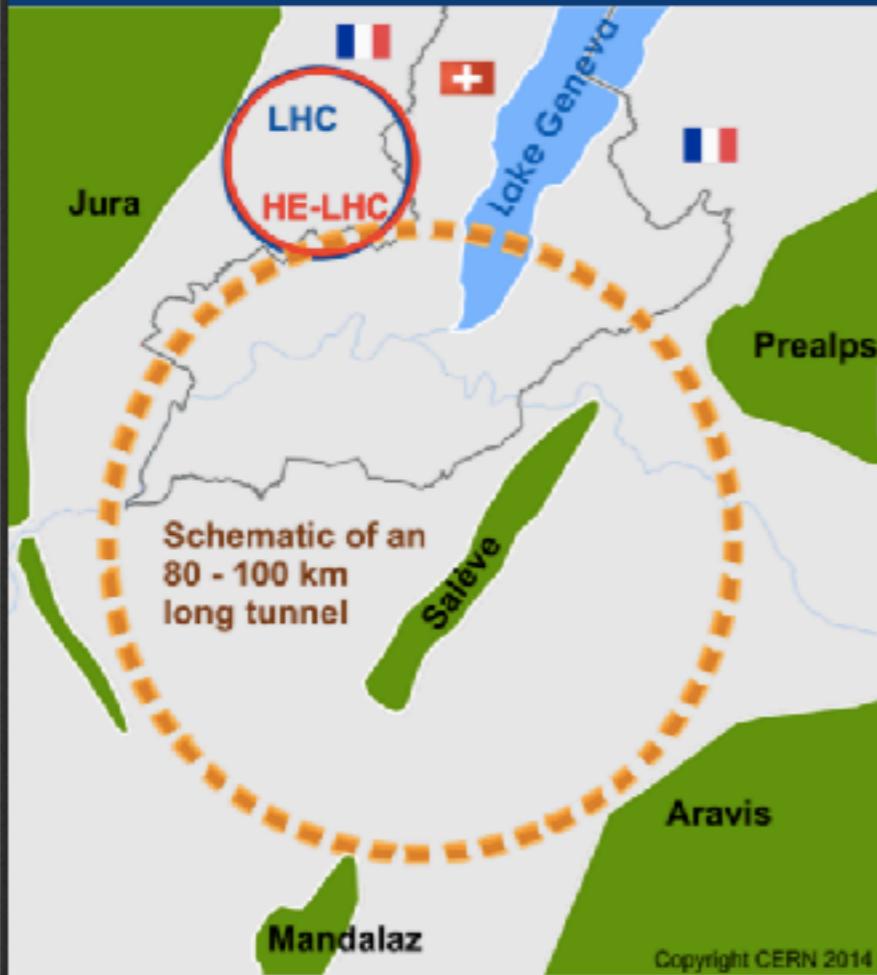


<https://www.youtube.com/watch?v=ctDgU-mHs3I>



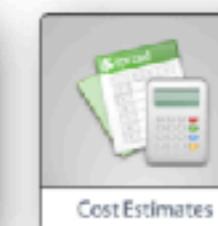
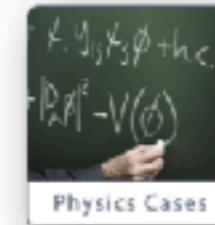


# Future Circular Collider (FCC) Study



International FCC collaboration (CERN as host lab) to study:

- **$p\bar{p}$ -collider (FCC-*hh*)**  
→ main emphasis, defining infrastructure requirements
- **$\sim 16 \text{ T} \Rightarrow 100 \text{ TeV } p\bar{p}$  in 100 km**
- **~100 km tunnel infrastructure** in Geneva area, site specific
- **$e^+e^-$  collider (FCC-*ee*),** as potential first step
- **HE-LHC with FCC-*hh* technology**
- **$p-e$  (FCC-*he*) option, IP integration,  $e^-$  from ERL**



# Proposta da exercício

Trabalho em grupo para entregar em 30 dias na forma de apresentação.

O que é o FCC-ee e FCC-hh e quais as limitações e performance de operação comparado com os aceleradores anteriores e em operação atualmente?

## FCC-ee accelerator parameters, performance and limitations

Mike Korzynos<sup>a</sup> on behalf of the FCC-ee study

<sup>a</sup>University of Geneva, Switzerland

### Abstract

CERN has recently launched the Future Circular Collider (FCC) study to deal with post-LHC possible programme. The emphasis of the study is on a 100 TeV proton-100 km new ring in the Geneva region. An electron machine will also be considered first step (FCC-ee). The study benefits from earlier work done in the context of TLI parameter table, to serve as the basis for the work to be done. The study aims to publish around 2018. The recent discovery of a light Higgs boson has opened up more Higgs factories around the world. FCC-ee is capable of very high luminosities in spectrum from 90 to 350 GeV. This allows the very precise study of the Z, W and quark, allowing for meaningful precision tests of the closure of the Standard Model.

Keywords: e+e- collider, circular, Higgs factory, toraZ

## THPHEU

Proceedings of ICHEP2014, Beijing, China

### FCC-ee OVERVIEW

F. Zimmermann, M. Benedikt, H. Burkhardt, F. Cerutti, A. Ferrari, J. Gutekunst, B. Haerter, B. Helmer, E. Jensen, R. Kersevan, P. Lebrun, R. Marin, A. Merzagore, J. Osborne, Y. Papaphilippou, D. Schulte, R. Tomas, J. Wenninger, CERN, Geneva, Switzerland; A. Blondel, M. Korzynos, U. Geneva, Switzerland; M. Boscolo, INFN Frascati, Italy; L. Laris, ESS, Lund, Sweden; K. Funakawa, K. Ohmi, K. Oide, KEK, Tsukuba, Japan; S. White, ESRF, Grenoble, France; A. Bogomysagov, I. Koup, B. Levichev, N. Muzhiev, S. Nikitin, D. Shatilov, BINP Novosibirsk, Russia; U. Wienands, SLAC, Stanford, USA; E. Gianfatici, FNAL, Batavia, USA; L. Medina, U. Guanajuato, Mexico

### Abstract

posed circular e+e- collider installed. Delivering high luminosity in four center-of-mass energies ranging from 90 GeV (W threshold) and 240 GeV to 350 GeV (J/psi physics). The FCC-ee part of the global Future Circular Collider which regards the FCC-ee at a step towards a 100-TeV version. All sharing the same tunnel report the FCC-ee design status.

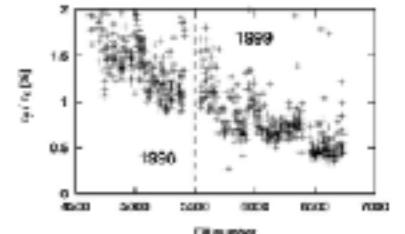


Figure 2: Vertical-to-horizonal resonance ratio at LEP in 1998 and 1999 [1]. The decrease reflects both changes in the damping particle numbers and improved steering [2].

### INTRODUCTION

30 ring colliders have been suggested. Many more e+e storage rings have been constructed, with ever increasing. In short, storage rings and represent a well understood technology their design performance EP was the highest energy lepton in maximum c.m. energy reached synchronization radiation power rate

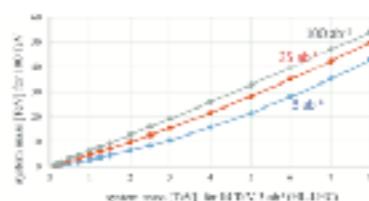


Figure 1: Relative energy reach of a 100-TeV pp collider, FCC-hh, with an integrated luminosity of  $3 \text{ ab}^{-1}$ ,  $25 \text{ ab}^{-1}$ , or  $100 \text{ ab}^{-1}$  compared with the case of the 14-TeV HL-LHC at  $3 \text{ ab}^{-1}$ . Shown is the perturbative evolution for an equal number of events, assuming that acceptance and efficiency remain the same. Total luminosity and associated energy reach were computed using the tool of Ref. [6].

In this paper we discuss the necessary and sufficient values of integrated luminosity for future higher energy hadron colliders, such as the FCC-hh, leading at both physics arguments and technical constraints.

### PHYSICS REQUIREMENTS

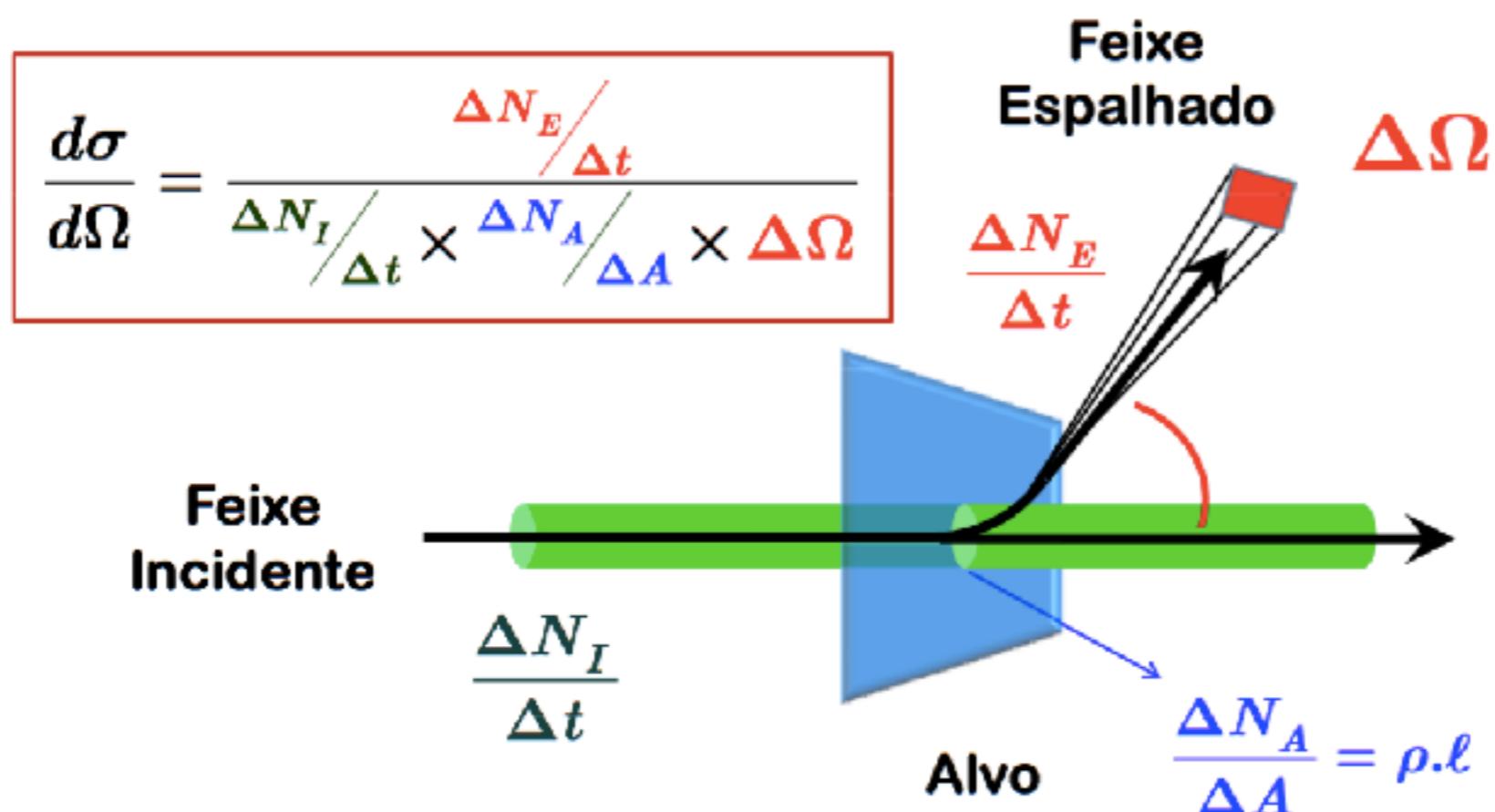
For example, the total number of events at 100 TeV with the HL-FCC would be equal to the one found at an energy of about 16 TeV with the FCC-hh of  $3 \text{ ab}^{-1}$ , about 10 TeV at  $25 \text{ ab}^{-1}$ , and 12 TeV at  $100 \text{ ab}^{-1}$ , i.e. an additional increase 300% in integrated luminosity raises the energy reach by nearly 20%.

<https://fcc.web.cern.ch/Pages/default.aspx>

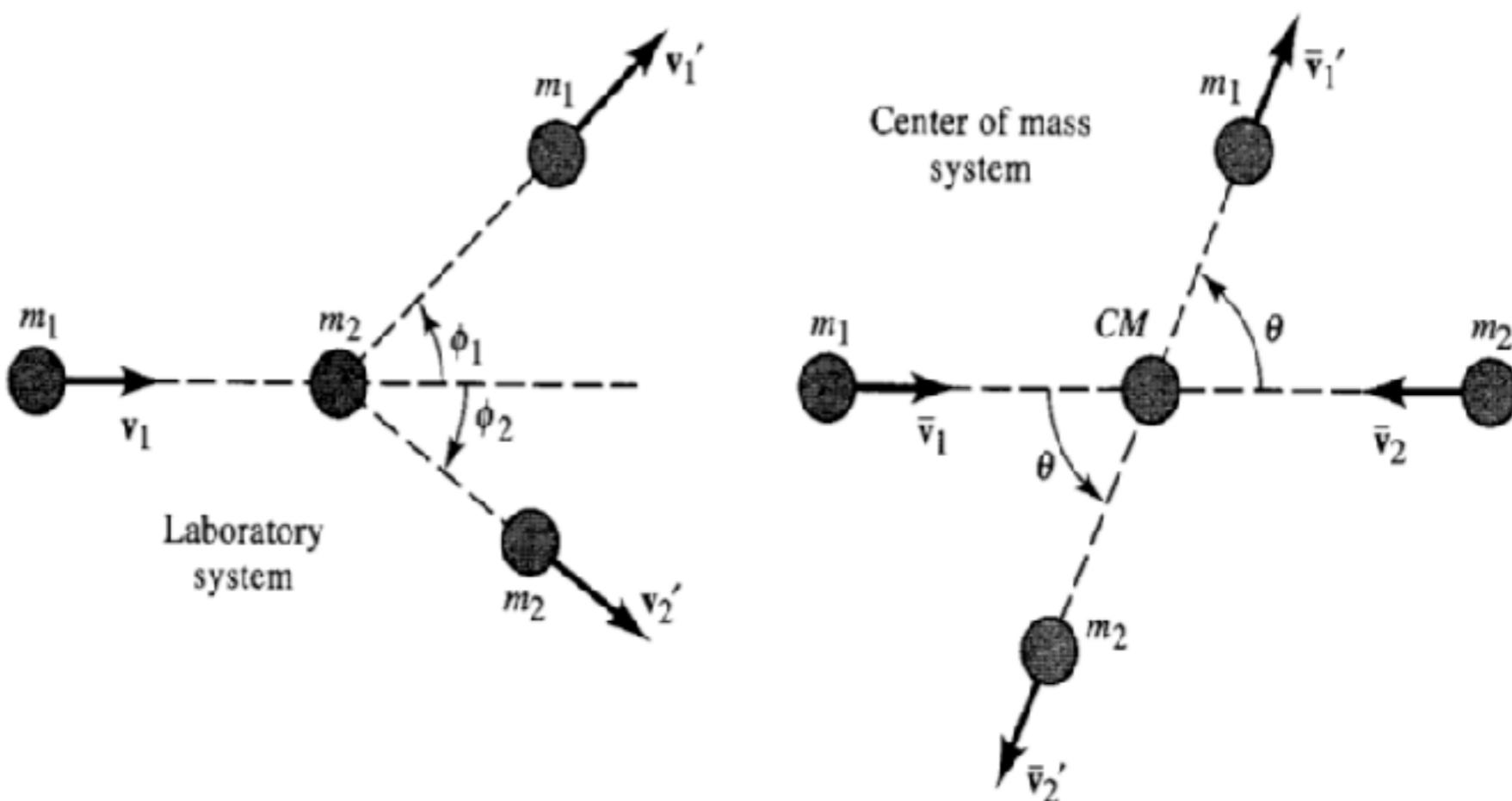
# **Backup slides**

# Seção de Choque

Seção de choque diferencial é a probabilidade de se observar uma partícula espalhada em um dado estado quântico por unidade de ângulo sólido, quando o alvo é atingido pelo fluxo de uma partícula por unidade de área.

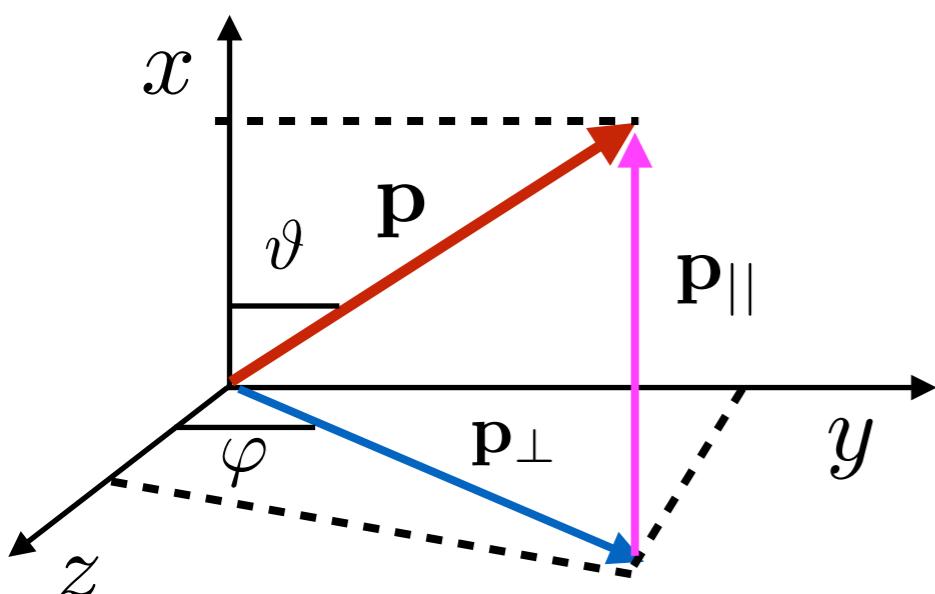
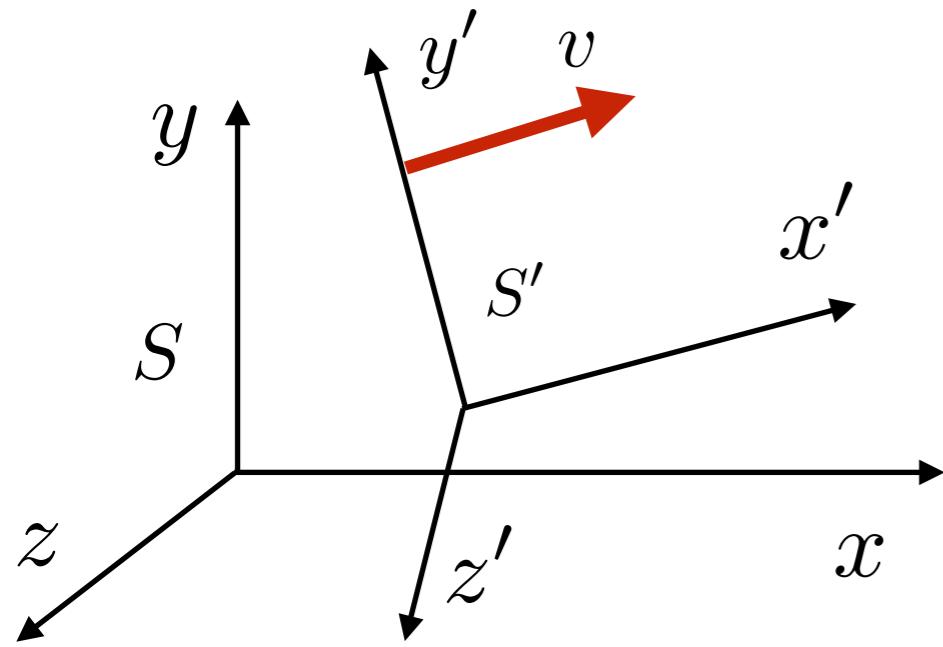


# Sistemas de coordenadas



**Figure 7.6.1** Comparison of laboratory and center of mass coordinates.

# Transformações de Lorentz



$$\mathbf{p}_{||} = p_x = \mathbf{p} \cos \vartheta$$

$$\mathbf{p}_{\perp} = \sqrt{p_y^2 + p_z^2}$$