A Theory of Power-Law Distributions in Financial Market Fluctuations

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Hill's scaling is only approximate. Experimental simulations revealed that, for fixed integration times, the main effect of changing γ was to influence the final average values of the semimajor axes of captured moons with respect to the planet; in contrast, the final inclination distributions were quite robust to the precise value of this parameter. Thus we chose γ heuristically so that the captured moons ended up roughly in the observed semimajor axis ranges; we set $\gamma = 7 \times 10^{-5}$ in units scaled for Jupiter, and $\gamma = 2 \times 10^{-4}$ in units scaled for Saturn.

Integrations were performed for both Jupiter and Saturn for a maximum of 10,000 years for each test particle. Integrations were stopped, as explained in the text, if test particles crossed the orbit of Callisto (at Jupiter) or Titan (at Saturn), or if they left the Hill sphere. The simulations reported in Fig. 4 were stopped when 50 moons had been captured, but computations in which several thousand moons were captured produce similar results. We have also performed parallel simulations in the elliptic restricted three-body problem for Jupiter and Saturn, and also used different forms of dissipation—for example, nebular gas-drag, $\mathbf{F}_{\rm drag} = -\gamma|\mathbf{v}|\mathbf{v}$ (ref. 11). All of these variations produced comparable results.

Kapitza averaging

The relative stability of prograde and retrograde orbits in 2D can be understood qualitatively by Taylor expansion of the solar part of the CRTBP hamiltonian, followed by Kapitza averaging in plane polar coordinates over the angle φ conjugate to h_z. This is similar to the analogous problem of ionization (escape) of an electron from a hydrogen atom in a rotating field[29,30]. As in the atomic problem, this strategy produces an effective potential whose saddle point is higher for one sense of angular momentum: in this case, the retrograde orbits, which are therefore more stable than the prograde orbits. Further, using methods similar to those in ref. 23, it is possible to show that h_z is the lowest-order term in an approximate 'third-integral' valid inside the Hill sphere.

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A theory of power-law distributions in financial market fluctuations

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Insights into the dynamics of a complex system are often gained by focusing on large fluctuations. For the financial system, huge databases now exist that facilitate the analysis of large fluctuations and the characterization of their statistical behaviour[1,2]. Power laws appear to describe histograms of relevant financial fluctuations, such as fluctuations in stock price, trading volume and the number of trades[3–10]. Surprisingly, the exponents that characterize these power laws are similar for different types and sizes of markets, for different market trends and even for different countries—suggesting that a generic theoretical basis may underlie these phenomena. Here we propose a model, based on a plausible set of assumptions, which provides an explanation for these empirical power laws. Our model is based on the hypothesis that large movements in stock market activity arise from the trades of large participants. Starting from an empirical characterization of the size distribution of those large market participants (mutual funds), we show that the power laws observed in financial data arise when the trading behaviour is performed in an optimal way. Our model additionally explains certain striking empirical regularities that describe the relationship between large fluctuations in prices, trading volume and the number of trades.

Define p_t as the price of a given stock and the stock price 'return' r_t as the change of the logarithm of stock price in a given time interval Δt, $r_t \equiv \ln p_t - \ln p_{t-\Delta t}$. The probability that a return has an absolute value larger than x is found empirically to be (see Fig. 1)[4,8]:

$$P(|r_t| > x) \sim x^{-\zeta_r} \quad (1)$$

with $\zeta_r \approx 3$. Empirical studies also show that the distribution of trading volume V_t obeys a similar power law[9]:

$$P(V_t > x) \sim x^{-\zeta_V} \quad (2)$$

with $\zeta_V \approx 1.5$, while the number of trades N_t obeys[10]:

$$P(N_t > x) \sim x^{-\zeta_N} \quad (3)$$

with $\zeta_N \approx 3.4$.

The 'inverse cubic law' of equation (1) is rather 'universal', holding over as many as 80 standard deviations for some stock markets, with Δt ranging from one minute to one month, across different sizes of stocks, different time periods, and also for different stock market indices[4,8]. Moreover, the most extreme events—including the 1929 and 1987 market crashes—conform to equation (1),

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demonstrating that crashes do not appear to be outliers of the distribution. We test the universality of equations (2) and (3) by analysing the 35 million transactions of the 30 largest stocks on the Paris Bourse over the 5-yr period 1994–1999. Our analysis shows that the power laws (2) and (3) obtained for US stocks also hold for a distinctly different market, consistent with the possibility that equations (2) and (3) are as universal as equation (1).

Here, we develop a model that demonstrates how trading by large market participants explains the above power laws. We begin by noting that large market participants have large price impacts[11–14]. To see why this is the case, observe that a typical stock has a turnover (fraction of shares exchanged) of approximately 50% a year, which implies a daily turnover of approximately 50%/250 = 0.2%—that is, on average 0.2% of outstanding shares change hands each day. The 30th-largest mutual fund owns about 0.1% of such a stock (Center for Research in Security Prices; http//gsbwww.uchicago.edu/research/crsp/). If the manager of such a fund sells its holdings of this stock, the sale will represent half of the daily turnover, and so will affect both the price and the total volume[15–17]. Such a theory where large individual participants move the market is consistent with the evidence that stock market movements are difficult to explain with changes in fundamental values[18].

Accordingly, we first perform an empirical analysis of the distribution of the largest market participants—mutual funds. We find, for each year of the period 1961–1999, that for the top 10% of distribution of the mutual funds, the market value of the managed assets S obeys the power law

$$P(S > x) \sim x^{-\zeta_S} \tag{4}$$

with $\zeta_S = 1.05 \pm 0.08$. Exponents of approximately one have also been found for the cumulative distributions city size[19] and firm sizes[20,21], and the origins of this 'Zipf' distribution are becoming better understood[22]. On the basis of the assumption that managers of large funds trade on their beliefs about the future direction of the market, and that they adjust their speed of trading to avoid moving the market too much, we will see that their trading activity leads to $\zeta_r = 3$ and $\zeta_V = 1.5$.

In order to proceed, we first present empirical evidence for the shape of the price impact, then propose an explanation for this shape, and finally show how the resulting trading behaviour generates power laws (1)–(3).

First, the price impact Δp of a trade of size V has been established to have an increasing and concave functional form that is similar for a large number of stocks[23,24]. We hypothesize that for large volumes V its functional form is:

$$r = \Delta p \simeq kV^{1/2} \tag{5}$$

for some constant k. So we investigate empirically the relation (Fig. 2):

$$E[r^2 \mid V] \sim V \tag{6}$$

which is supported by standard statistical tests. Because relation (5) implies $P(r > x) \sim P(kV^{1/2} > x) = P(V > x^2/k^2) \sim x^{-2\zeta_V}$, it follows that:

$$\zeta_r = 2\zeta_V \tag{7}$$

Thus, the power law of returns, equation (1), follows from the power law of volumes, equation (2), and the square-root form-price impact, equation (5). We next develop a framework for explaining equations (2) and (5).

We consider the behaviour of one stock whose original price is, say, one. The mutual fund manager who wishes to buy V shares offers a price increment Δp, so that the new price will become $1 + \Delta p$. Each seller i of size s_i who is offered a price increase Δp supplies the fund manager with q_i shares. Elementary considerations lead us to hypothesize $q_i \approx s_i \Delta p$ (see Supplementary Information). The number of sellers available after the fund manager has waited a time T is proportional to T. Thus after a time T, the fund manager can, on average, buy a quantity of shares equal to $kT\langle s\rangle\Delta p$ for some proportionality constant k. The search process stops (and the trades are executed simultaneously) when the desired quantity V is reached—that is, when $kT\langle s\rangle\Delta p = V$, so the time needed to find the shares is:

$$T = \frac{V}{\langle s\rangle k\Delta p} \sim \frac{V}{\Delta p} \tag{8}$$

Hence there is a trade-off between cost Δp and the time to execution T; if the fund manager desires to realize the trade in a short amount of time T, the manager must pay a large price impact $\Delta p \sim V/T$.

Let us consider the fund manager's decision problem. Managers trade on the assumption that a given stock is mispriced by an amount M, defined as the difference between the fair value of the stock and the traded price[14,25,26]. The manager wants to exploit this mispricing quickly, as he expects that the mispricing will be progressively corrected, that is, expects that the price will increase

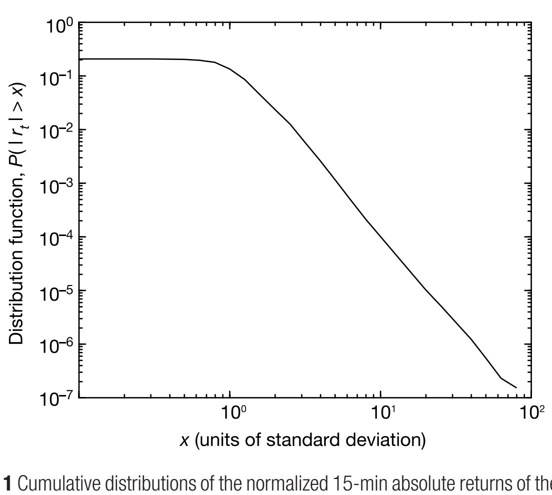

Figure 1 Cumulative distributions of the normalized 15-min absolute returns of the 1,000 largest companies in the 'Trades and Quotes' database for the 2-yr period 1994–1995. We define the normalized return as $r_{it} = (\tilde{r}_{it} - \bar{\tilde{r}}_i)/\sigma_i$, where $\bar{\tilde{r}}_i$ and σ_i are the mean and the standard deviation of the unnormalized return $\tilde{r}_{it}$ of stock i. We obtain $P(|r_t| > x) \sim x^{-\zeta_r}$ with $\zeta_r = 3.1 \pm 0.1$.

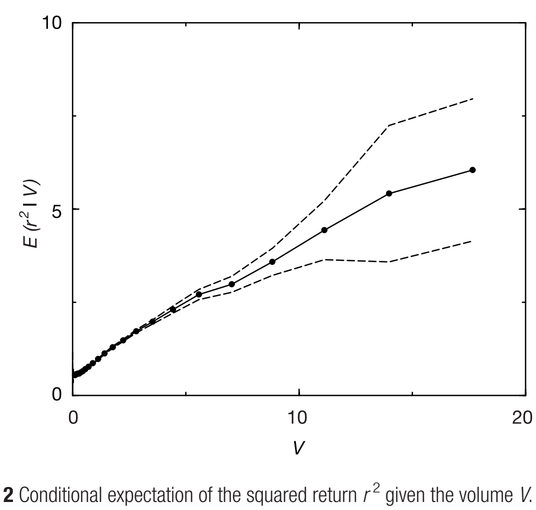

Figure 2 Conditional expectation of the squared return r^2 given the volume V. Here, the return is normalized as in Fig.1, and the volume is normalized as $V_{it} = \tilde{V}_{it}/\bar{\tilde{V}}_i$, where $\bar{\tilde{V}}_i$ is the average of the unnormalized volumes $\tilde{V}_{it}$ of stock i. The bands represent 95% confidence intervals. The theory predicts a relation $E[r^2 \mid V] = aV + b$, the 'square root' price impact of volume. Statistical tests reported in the Supplementary Information confirm this relation.

at a rate μ. Hence, after a delay of T, the remaining mispricing is only $M - \mu T$. The total profit per share B/V is the realized excess return $M - \mu T$ minus the price concession Δp, which gives:

$$B = V(M - \mu T - \Delta p) \qquad (9)$$

The fund manager's goal is thus to maximize B, the perceived dollar benefit from trading. The optimal price impact Δp maximizes B subject to equation (8), $T = aV/\Delta p$, that is, Δp maximizes $V(M - \mu aV/\Delta p - \Delta p)$, which gives equation (5).

The time to execution is $T \sim V/\Delta p \sim V^{1/2}$, and the number of 'chunks' in which the block is divided is $N \sim T \sim V^{1/2}$. These effects have been qualitatively documented in refs 11, 12, 23. The last relation gives:

$$\zeta_N = 2\zeta_V \qquad (10)$$

which in turn predicts $\zeta_N = 3$, a value that is approximately consistent with the empirical value of 3.4 (ref. 10).

Thus far, we have a theoretical framework for understanding the square-root price impact of trades, equation (5), which with equation (2) explains the cubic law of returns, equation (1). We now focus on understanding equation (2).

We show that returns and volumes are power-law distributed with tail exponents:

$$\zeta_r = 3 \text{ and } \zeta_V = 3/2 \qquad (11)$$

provided the following four conditions hold: (1) the power law exponent of mutual fund sizes is $\zeta_S = 1$ (Zipf's law); (2) the price impact follows the square root law, equation (5); (3) funds trade in typical volumes $V \approx S^{\delta}$ with $\delta > 0$; and (4) funds adjust trading frequency and/or volume so as to pay transactions costs in such a way that if we define $c(S)$ as:

$$c(S) \equiv \frac{\text{Annual amount lost by the fund in price impact}}{\text{Value } S \text{ of the assets under management}} \qquad (12)$$

then $c(S)$ is independent of S for large S.

The empirical validity of conditions (1) and (2) was shown above, while condition (3) is a weak, largely technical, assumption discussed in the Supplementary Information. Condition (4) means that funds in the upper tail of the distribution pay roughly similar annual price-impact costs; that is, $c(S)$ reaches an asymptote for large sizes. We interpret this as an evolutionary 'survival constraint'. Funds that would have a very large $c(S)$ would have small returns and would be eliminated from the market. The average return $r(S)$ of funds of size S is independent of S (ref. 27). Because both small and large funds have similarly low ability to outperform the market, $c(S)$ is also independent of S.

For each block trade $V(S)$ a fund of size S incurs a price impact proportional to $V\Delta p$ which, from condition (2), is $V^{3/2}$. If $F(S)$ is the fund's annual frequency of trading, then the annual loss in transactions costs is $F(S)\cdot V^{3/2}$, so:

$$c(S) = F(S)\cdot[V(S)]^{3/2}/S \qquad (13)$$

Condition (4) implies that either $V(S)$ or $F(S)$ will adjust in order to satisfy:

$$F(S) \sim S\cdot[V(S)]^{-3/2} \qquad (14)$$

Condition (1) implies that the probability density function for mutual funds of size S is $\rho(S) \sim S^{-2}$. Because condition (3) states that $V \sim S^{\delta} > x$, and because they trade with frequency given by $F(S)$ in equation (14):

$$P(V > x) \sim \int_{S^{\delta} > x} F(S)\rho(S)\,\mathrm{d}S \sim \int_{S > x^{1/\delta}} S^{1-3\delta/2} S^{-2}\,\mathrm{d}S \sim x^{-3/2} \qquad (15)$$

which leads to a power-law distribution of volumes with exponent

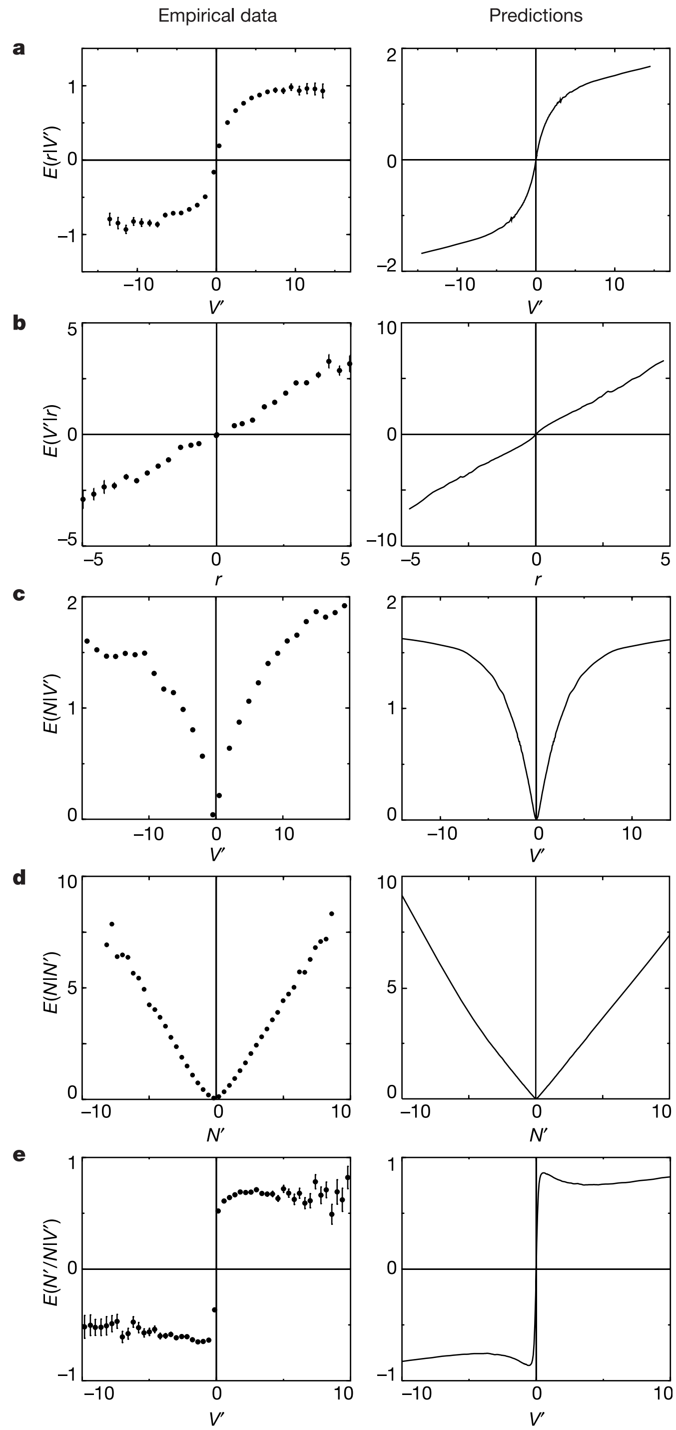

Figure 3 Conditional expectations for $E(r|V')$, $E(V'|r)$, $E(N|V')$, $E(N|N')$, and $E(N'/N|V')$. We form, for each interval $\Delta t = 15$ min, the quantities (1) r, the return; (2) V_B (or V_S), the number of shares exchanged in a buyer- (or seller-) initiated trade[28]; (3) N_B (or N_S), the number of buyer- (or seller-) initiated trades, $V' \equiv V_B - V_S$, and $N' \equiv N_B - N_S$. The left panels show the empirical values for the 116 most frequently traded stocks in the 'Trades and Quotes' database for the 2-yr period 1994–1995. Variables are normalized to unit variance after setting the mean to zero; for variables such as volume for which the variance does not exist, we have normalized by the first moment instead. The right panels show the model's predictions, which agree well with the empirical data.

$\zeta_V = 3/2$. Moreover, from equation (7), it follows that $\zeta_r = 3$. In addition, the above result does not depend on details of the trading strategy, such as the specific value of δ. (The Supplementary Information indicates a number of ways in which one can weaken the assumptions of independent and identical distributions made in this Letter.)

Although our model is mainly motivated by the regularities of returns, volume and number of trades taken separately, we also make predictions for the joint behaviour of those quantities. In a given time interval Δt, there will be J 'rounds' where a fund manager creates one or more trades. Each round j creates a volume V_j, a return $\pm V_j^{1/2}$ and a number of trades $V_j^{1/2}$. Then the total volume, number of trades, and returns, will be $V \equiv \sum_{j=1}^{J} V_j$, $N \equiv \sum_{j=1}^{J} V_j^{1/2}$ and $r \equiv \sum_{j=1}^{J} \varepsilon_j V_j^{1/2}$, with $\varepsilon_j = \pm 1$. As a measure of trade imbalance, we use N', the number of buyer-initiated trades minus the number of seller-initiated trades[28], and V', the number of shares exchanged in a buyer-initiated trade minus the number of shares exchanged in seller-initiated trades.

We next focus on equal-time relationships between V, N, and V' (ref. 24), using data from the 'Trades and Quotes' data base (New York Stock Exchange; http://www.nyse.com). These equal-time relationships are found to be universal across the large set of stocks analysed in ref. 24. Figure 3a shows that the prices impact function $E(r\,|\,V')$ produced by the model matches data. We observe that $J \gg 1$ (aggregation over several trades) flattens the shape of the price impact versus V. We study a variant of Fig. 3a in Fig. 3b, which plots $E(V'\,|\,r)$. Surprisingly, the shape is now roughly linear, a feature predicted by the model. The cause of the linearity is, again, the aggregation over several trades. Figure 3c, $E(N|V')$, tests the model prediction that periods with large volume imbalance V' are periods where a large number N of trades are made. One sees that the data display a relationship that is similar to that predicted by the model. Figures 3a–c support the view that large returns and large numbers of trades go together with large volume imbalances V'.

It is an important feature of the model that large trades beget more trades. Indeed, in our model:

$$|N'| \sim N \tag{16}$$

for large N and is dominated by one large fund manager who desires to trade a volume V_j, and creates a number of orders $V_j^{1/2}$, so that N_j, N, $|N_j'|$ and $|N'|$ have the same order of magnitude, $V_j^{1/2}$. Relation (16) means that most trades have the same sign, that is, move the price in the same direction—with the sign of the trade of the large fund manager. Equation (16) is indeed consistent with the empirical data shown in Fig. 3d. This contrasts with a simple alternative model where each desire to trade would create only one trade, as in a competitive market. In this alternative model we would have $N' = \sum_{i=1}^{N} \varepsilon_i$, where $\epsilon_i = \pm 1$, leading to $|N'| \sim N^{1/2}$ in the tail events or $E[N\,|\,N'] \sim N'^2$ in contrast to the data in Fig. 3d. Figure 3e supports the view that in periods of high volume imbalance V', most trades change the price in the same direction. Indeed, the data and the model exhibit a similar sharp transition of N'/N as V' changes sign. □

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Measurement of the displacement field of dislocations to 0.03 Å by electron microscopy

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Defects and their associated long-range strain fields are of considerable importance in many areas of materials science[1,2]. For example, a major challenge facing the semiconductor industry is to understand the influence of defects on device operation, a task made difficult by the fact that their interactions with charge carriers can occur far from defect cores, where the influence of the defect is subtle and difficult to quantify[3,4]. The accurate measurement of strain around defects would therefore allow more detailed understanding of how strain fields affect small structures—in particular their electronic, mechanical and chemi-

