Question 3 of Assignment 3:

1- input dimensions: $6 \times 6 \times 1$
Kernel dimensions: $3 \times 3 \times 1$

number of parameters $= 3 \times 3 \times 1 + \underbrace{1}_{\text{for bias}} = 10$

2- with Stride = 1, we have:

output dimensions $= (h_x - h_f + 1) \times (w_x - w_f + 1) \times 1 =$ $4 \times 4 \times 1$

$$X = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 7 & 5 & 0 & 0 & 3 & 2 \\ 6 & 4 & 5 & 1 & 4 & 8 \\ 9 & 0 & 2 & 2 & 5 & 4 \\ 6 & 3 & 4 & 7 & 2 & 8 \\ 5 & 7 & 5 & 6 & 2 & 0 \\ 7 & 9 & 0 & 8 & 2 & 3 \end{bmatrix} \end{array}, \quad f = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$y_{1,1} = (7 \times 1) + (6 \times 2) + (9 \times 1) + (5 \times -2) + (2 \times -1) = 16$

$y_{1,2} = (5 \times 1) + (4 \times 2) + (1 \times -2) + (2 \times -1) = 9$

$y_{1,3} = (5 \times 2) + (2 \times 1) + (3 \times -1) + (4 \times -2) + (5 \times -1) = -4$

$y_{1,4} = (1 \times 2) + (2 \times 1) + (2 \times -1) + (8 \times -2) + (4 \times -1) = -18$

$y_{2,1} = (6 \times 1) + (9 \times 2) + (6 \times 1) + (5 \times -1) + (2 \times -2) + (4 \times -1) = 17$

$y_{2,2} = (4 \times 1) + (3 \times 1) + (1 \times -1) + (2 \times -2) + (7 \times -1) = -5$

$y_{2,3} = (5\times1)+(2\times2)+(4\times1)+(4\times-1)+(5\times-2)+(9\times-1) = -10$

$y_{2,4} = (1\times1)+(2\times2)+(7\times1)+(8\times-1)+(4\times-2)+(8\times-1) = -12$

$y_{3,1} = (9\times1)+(6\times2)+(5\times1)+(2\times-1)+(4\times-2)+(5\times-1) = 11$

$y_{3,2} = (3\times2)+(7\times1)+(2\times-1)+(7\times-2)+(6\times-1) = -9$

$y_{3,3} = (2\times1)+(4\times2)+(5\times1)+(5\times-1)+(9\times-2)+(9\times-1) = -17$

$y_{3,4} = (2\times1)+(7\times2)+(6\times1)+(4\times-1)+(8\times-2) = 2$

$y_{4,1} = (6\times1)+(5\times2)+(7\times1)+(4\times-1)+(5\times-2) = 9$

$y_{4,2} = (3\times1)+(7\times2)+(9\times1)+(7\times-1)+(6\times-2)+(8\times-1) = -1$

$y_{4,3} = (4\times1)+(5\times2)+(9\times-1)+(9\times-2)+(2\times-1) = -15$

$y_{4,4} = (7\times1)+(6\times2)+(8\times1)+(8\times-1)+(3\times-1) = 16$

$$y = \begin{bmatrix} 16 & 9 & -4 & -18 \\ 17 & -5 & -10 & -12 \\ 11 & -9 & -17 & 2 \\ 9 & -1 & -15 & 16 \end{bmatrix}$$

⇐ The output with $S=1$, $P=0$

3/ Max-pooling operation on the above ~~output~~ matrix :

By choosing a 2x2 filter and stride = 2, we have:

output dimensions $\Rightarrow \frac{D_x - D_f}{S} + 1 = \frac{4-2}{2} + 1 = 2 \Rightarrow$

y'

output dimensions = 2x2x1 → y': new output
y: input (previous output)

$$\Rightarrow y' = \begin{bmatrix} 17 & -4 \\ 11 & 16 \end{bmatrix}$$