Basics of Neural Network Programming

Binary Classification

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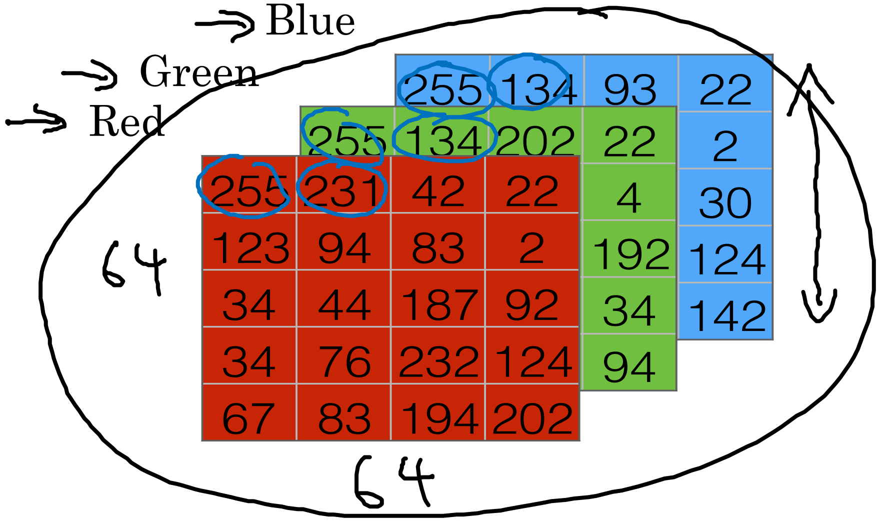
Binary Classification

64

64

$\longrightarrow$ 1 (cat) vs 0 (non cat)

y



$$x = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ \vdots \\ 255 \\ 134 \\ \vdots \end{bmatrix}$$

$$64 \times 64 \times 3 = 12288$$

$$n = n_x = 12288$$

$$x \longrightarrow y$$

Notation

$(x, y) \qquad x \in \mathbb{R}^{n_x}, \; y \in \{0, 1\}$

m training examples : $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$

$M = M_{train} \qquad M_{test}$ = #test examples.

$$X = \underbrace{\begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}}_{m} \Big\updownarrow n_x \qquad \cancel{\begin{bmatrix} — x^{(1)T} — \\ \vdots \\ — x^{(m)T} — \end{bmatrix}}$$

$$X \in \mathbb{R}^{n_x \times m} \qquad X.\text{shape} = (n_x, m)$$

$$Y = [y^{(1)} \; y^{(2)} \; \ldots, y^{(m)}]$$

$$Y \in \mathbb{R}^{1 \times m}$$

$$Y.\text{shape} = (1, m)$$

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Logistic Regression

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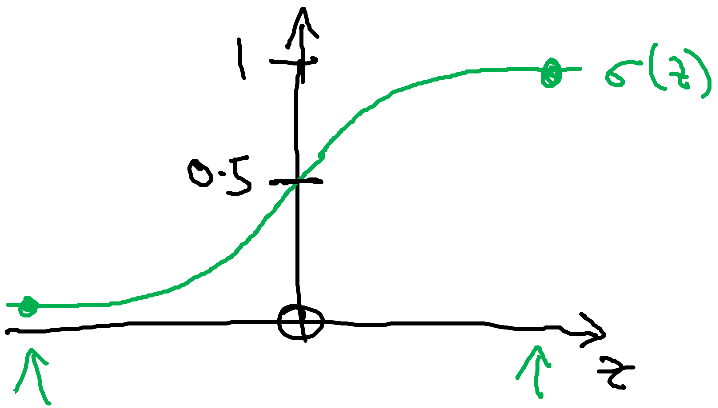
Logistic Regression

Given x, want $\hat{y} = P(y=1 \mid x)$

$x \in \mathbb{R}^{n_x}$ $\qquad 0 \le \hat{y} \le 1$

Parameters: $w \in \mathbb{R}^{n_x}$, $b \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$x_0 = 1$, $\quad x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$$\theta = \begin{bmatrix}\theta_0\\ \theta_1\\ \theta_2\\ \vdots\\ \theta_{n_x}\end{bmatrix} \begin{matrix} \} \; b \leftarrow \\ \left.\begin{matrix}\\ \\ \\ \\ \end{matrix}\right\} w \leftarrow \end{matrix}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$$\sigma(z) = \frac{1}{1+e^{-z}} \approx \frac{1}{1+\text{Bignum}} \approx 0$$

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Logistic Regression cost function

Logistic Regression cost function

$\rightarrow \hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \dfrac{1}{1+e^{-z^{(i)}}}$

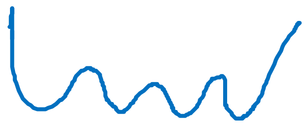
$z^{(i)} = w^T x^{(i)} + b$

Given $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

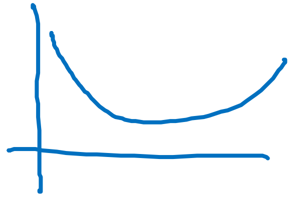
$x^{(i)}$
$y^{(i)}$
$z^{(i)}$

i-th example.

$\boxed{\text{Loss}}$ (error) function: $\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$



$$\boxed{\mathcal{L}(\hat{y}, y)} = -\left(\boxed{y \log \hat{y}} + (1-y)\log(1-\hat{y})\right) \leftarrow$$



If $y = 1$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y} \leftarrow$ Want $\log \hat{y}$ large, want $\hat{y}$ large.

If $y = 0$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y}) \leftarrow$ Want $\log 1-\hat{y}$ large .... want $\hat{y}$ small

$\boxed{\text{Cost}}$ function: $J(w, b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log \hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$

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Basics of Neural Network Programming

Gradient Descent

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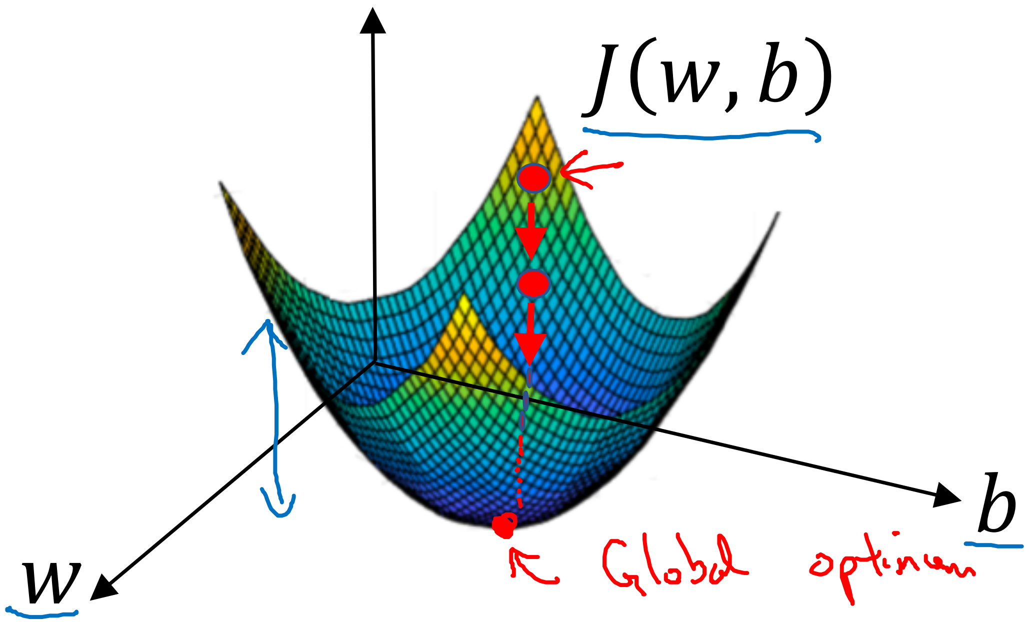
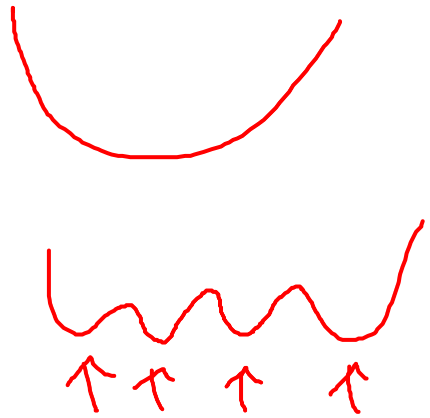
Gradient Descent

Recap: $\hat{y} = \sigma(w^T x + b)$, $\sigma(z) = \frac{1}{1+e^{-z}}$

$$J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\boxed{\hat{y}^{(i)}}, \boxed{y^{(i)}}) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log(1-\hat{y}^{(i)})$$

Want to find w, b that minimize $J(w,b)$





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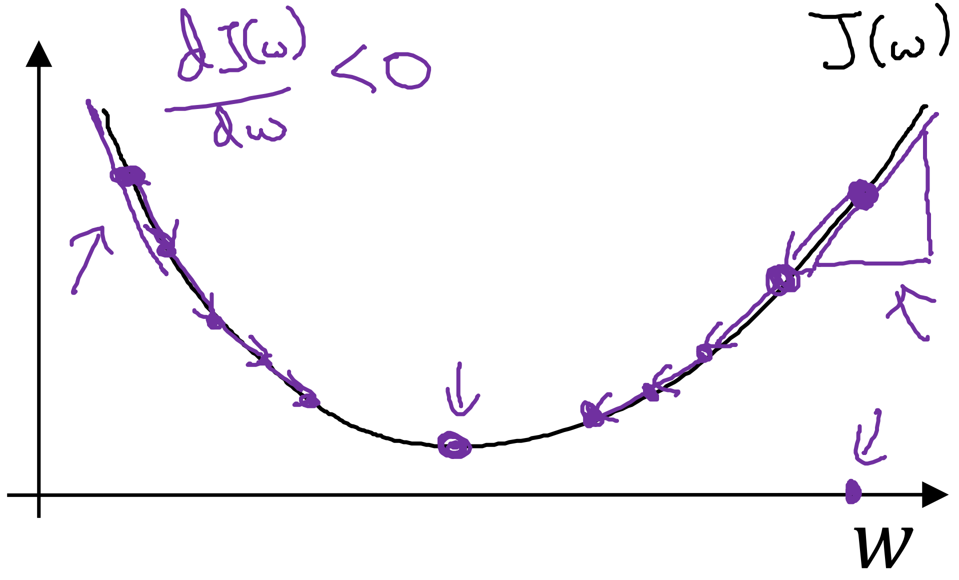
Gradient Descent



learning rate

Repeat {

$$w := w - \alpha \boxed{\frac{dJ(w)}{dw}}$$

}

"dw"

$$w := w - \alpha \, dw$$

$$\frac{dJ(w)}{dw} = ?$$

$J(w, b)$

$$w := w - \alpha \boxed{\frac{dJ(w,b)}{dw}} \qquad \boxed{\frac{\partial J(w,b)}{\partial w}} \to dw$$

$$b := b - \alpha \boxed{\frac{dJ(w,b)}{db}} \qquad \boxed{\frac{\partial J(w,b)}{\partial b}} \to db$$

∂ "partial derivative"

d J

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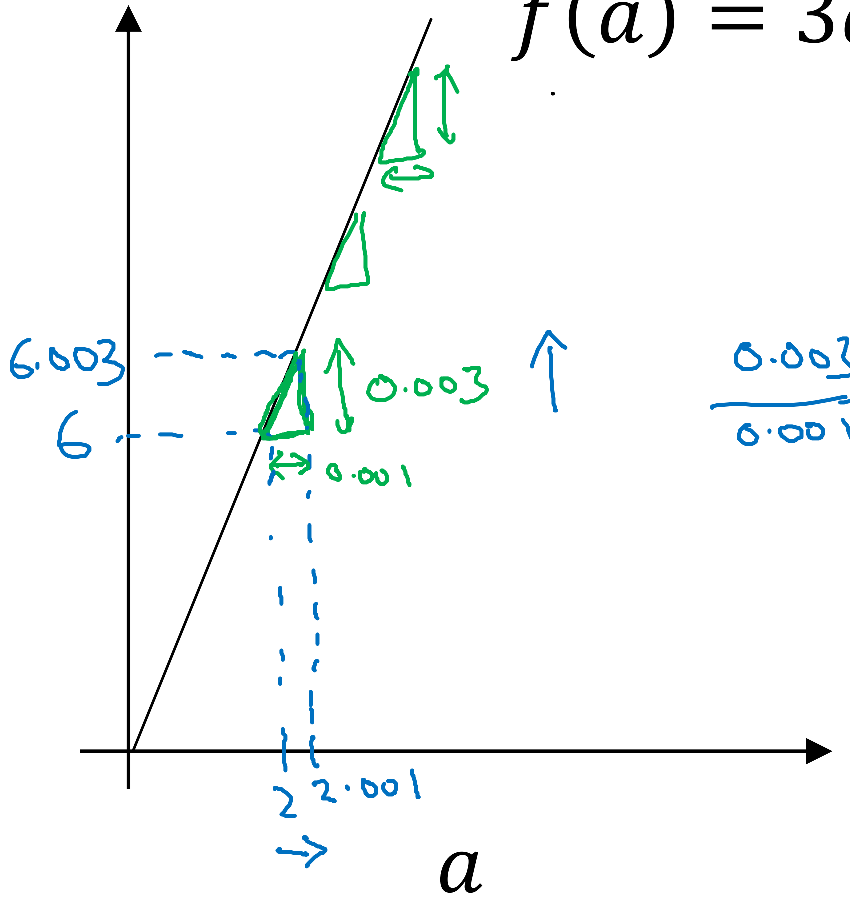
Basics of Neural Network Programming

Derivatives

Intuition about derivatives

$f(a) = 3a$



$\frac{0.003}{0.001}$ $\frac{\text{height}}{\text{width}}$

→ $a = 2$ $f(a) = 6$

$a = 2.001$ $f(a) = 6.003$

slope (derivative) of $f(a)$ at $a=2$ is 3

→ $a = 5$ $f(a) = 15$

$a = 5.001$ $f(a) = 15.003$

slope at $a=5$ is also 3

$$\frac{d f(a)}{da} = 3 = \frac{d}{da} f(a)$$

0.001
0.00000001
0.0000000001

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More derivatives examples

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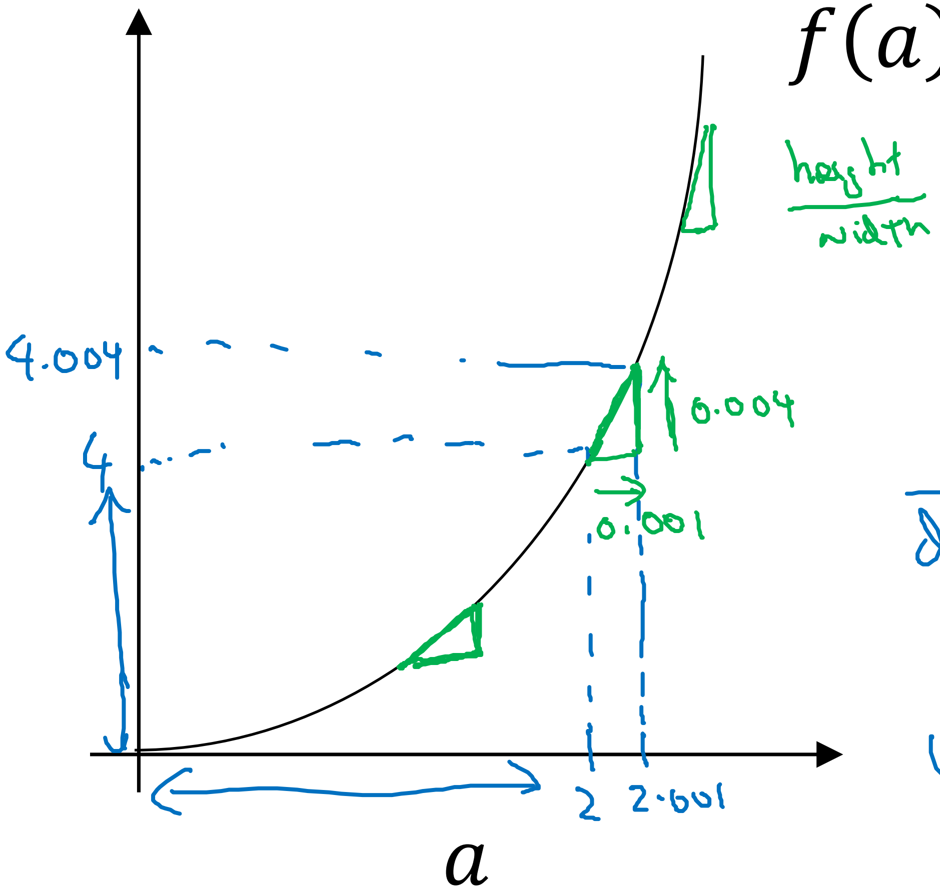
Intuition about derivatives

0.001
0.00000.....01

$f(a) = a^2$



$$\frac{d}{da} a^2 = 2a$$

0.001

$(2a) \times 0.001$

$a = 2$ $\quad f(a) = 4$

$a = 2.001$ $\quad f(a) \approx 4.004$

$(4.004\boxed{001})$

slope (derivative) of $f(a)$ at $a = 2$ is 4.

$$\boxed{\frac{d}{da} f(a) = 4} \text{ when } \boxed{a = 2}.$$

$a = 5$ $\quad f(a) = 25$

$a = 5.001$ $\quad f(a) \approx 25.010$

$$\boxed{\frac{d}{da} f(a) = 10} \text{ when } \boxed{a = 5}$$

$$\frac{d}{da} f(a) = \frac{d}{da} a^2 = \boxed{2a}$$

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More derivative examples

$f(a) = a^2$

$\frac{d}{da} f(a) = \underbrace{2a}_{4}$

$a = 2 \qquad f(a) = 4$

$a = 2.001 \qquad f(a) \approx 4.004$

$f(a) = a^3$

$\frac{d}{da} f(a) = \underbrace{3a^2}_{3 \times 2^2 = 12}$

$a = 2 \qquad f(a) = 8$

$a = \underline{2.001} \qquad f(a) \approx 8.\underline{012}$

$f(a) = \log_e(a)$

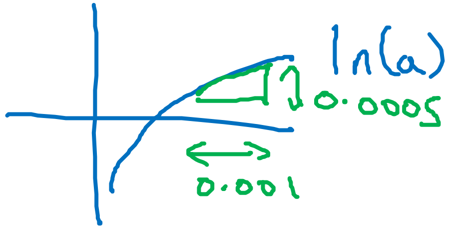
$\ln(a)$

$\frac{d}{da} f(a) = \frac{1}{a}$

$a = 2 \qquad f(a) \approx 0.69315$

$a = 2.\underline{001} \qquad \underline{f(a)} \approx 0.69365$

$0.0005 \qquad \underline{0.0005}$



$\frac{d}{da} f(a) = \boxed{\frac{1}{2}}$

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Basics of Neural Network Programming

Computation Graph

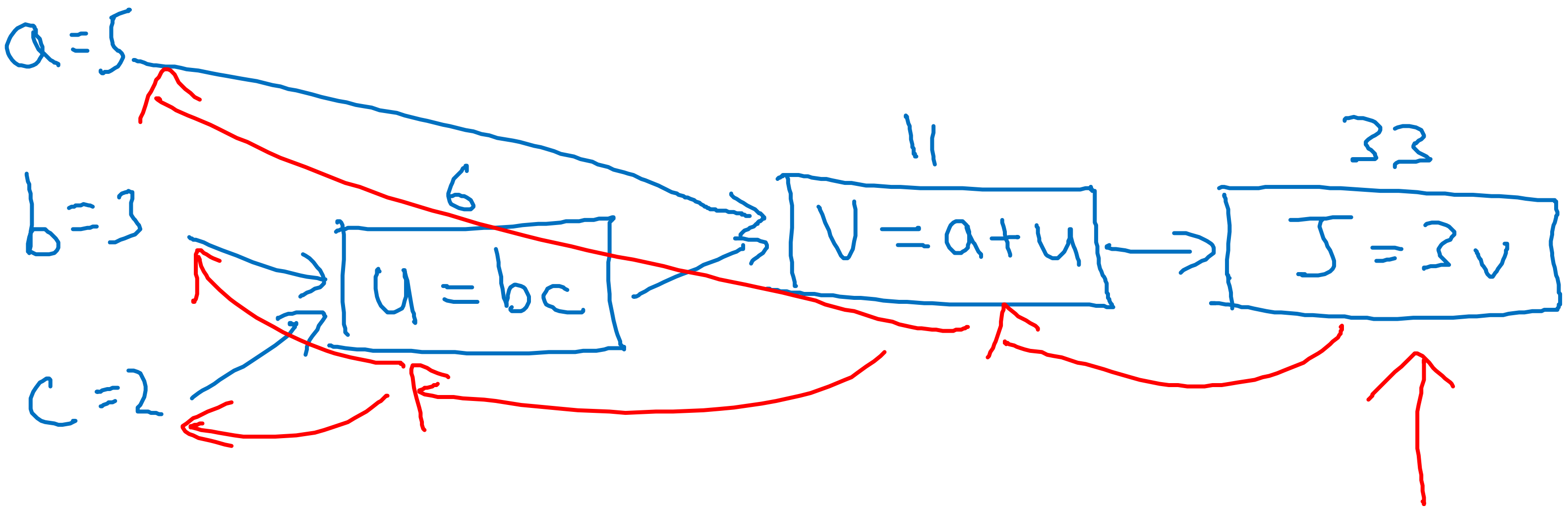
Computation Graph

$$J(a,b,c) = \underbrace{3(\underbrace{a+\underbrace{bc}_{u}}_{v})}_{J} = 3(5+3\times 2) = 33$$

$u = bc$

$v = a + u$

$J = 3v$



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Basics of Neural Network Programming

Derivatives with a Computation Graph

Computing derivatives

$\frac{dJ}{da}$ "da" = 3

$a = 5$

$b = 3$

$c = 2$

6

$u = bc$

11

$v = a + u$

33

$J = 3v$

$\frac{dJ}{dv}$ "dv" = 3

$\boxed{\frac{dJ}{dv} = \, ? = 3}$

$a \rightarrow v \rightarrow J$

$J = 3v$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$\boxed{f(a) = 3a}$

$\frac{df(a)}{da} = \frac{df}{da} = 3$

$\boxed{J = 3v}$

$\frac{dJ}{dv} = 3$

$\frac{dJ}{da} = 3 = \frac{dJ}{dv} \frac{dv}{da}$

3×1

$\boxed{\frac{dv}{da} = 1}$

$a = 5 \rightarrow 5.001$

$\rightarrow v = 11 \rightarrow 11.001$

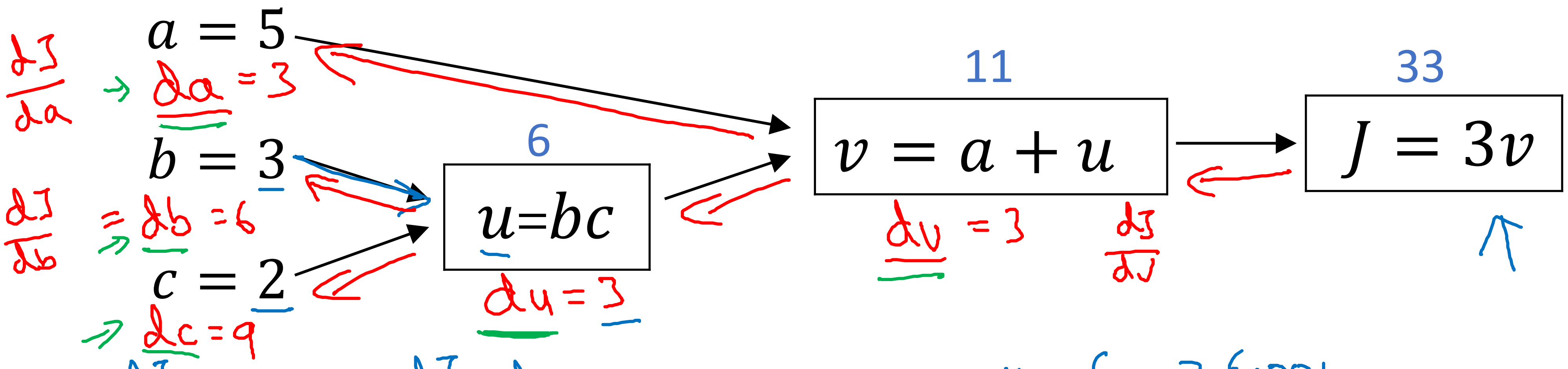
$J = 33 \rightarrow 33.003$

$\boxed{\frac{d \, \text{FinalOutputVar}}{d \, \text{var}}}$

dJdvar

"dvar"

Computing derivatives



$$\frac{dJ}{du} = 3 = \underbrace{\frac{dJ}{dv}}_{3} \cdot \underbrace{\frac{dv}{du}}_{1}$$

$$\frac{dJ}{db} = \underbrace{\boxed{\frac{dJ}{du}}}_{\to 3} \cdot \underbrace{\frac{du}{db}}_{=2} = 6$$

$$\frac{dJ}{da} = \underbrace{\boxed{\frac{dJ}{du}}}_{3} \cdot \underbrace{\frac{du}{da}}_{\times\, 3} = 9$$

$u = 6 \rightarrow 6.001$

$v = 11 \rightarrow 11.001$

$J = 33 \rightarrow 33.003$

$b = 3 \rightarrow 3.001$

$u = b \cdot c = 6 \rightarrow 6.002$

$c = 2$

$J = 33.006$

$.006$

$v = 11.002$

$J = 3v$

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Basics of Neural Network Programming

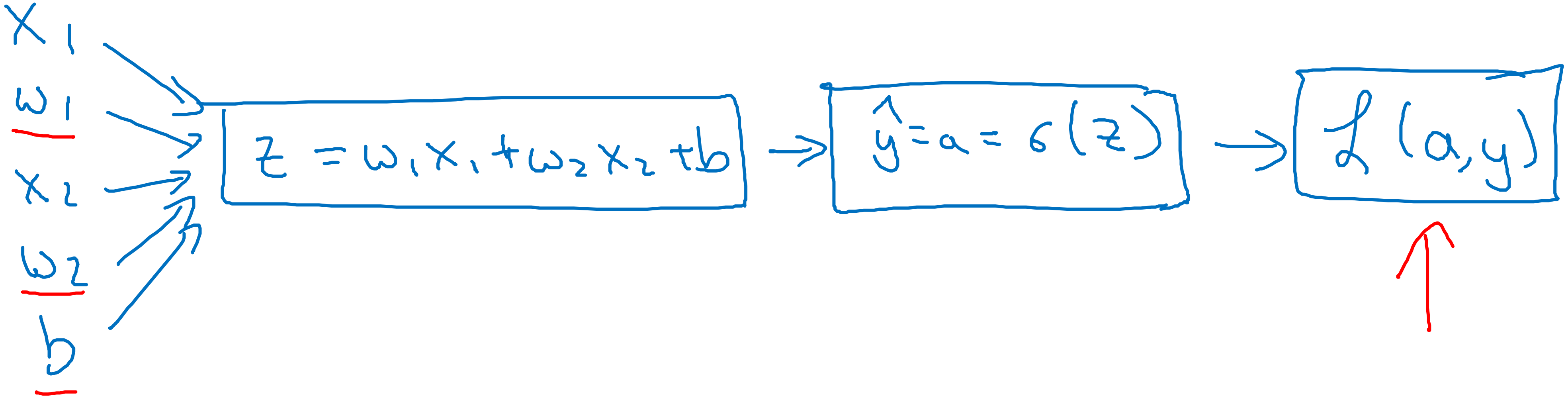
Logistic Regression Gradient descent

Logistic regression recap

$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$



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Logistic regression derivatives

x_1, w_1, x_2, w_2, b $\rightarrow$ $\boxed{z = w_1x_1 + w_2x_2 + b}$ $\rightarrow$ $\boxed{a = \sigma(z)}$ $\rightarrow$ $\boxed{\mathcal{L}(a, y)}$

$$dz = \frac{d\mathcal{L}}{dz} = \frac{d\mathcal{L}(a,y)}{dz}$$

$$= a - y$$

$$= \frac{d\mathcal{L}}{da} \cdot \frac{da}{dz}$$

$$a(1-a)$$

$$\text{"}da\text{"} = \frac{d\mathcal{L}(a,y)}{da} = -\frac{y}{a} + \frac{1-y}{1-a}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \text{"}dw_1\text{"} = x_1 \cdot dz. \qquad dw_2 = x_2 \cdot dz. \qquad db = dz.$$

$$w_1 := w_1 - \alpha\, dw_1$$

$$w_2 := w_2 - \alpha\, dw_2$$

$$b := b - \alpha\, db.$$

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Basics of Neural Network Programming

Gradient descent on m examples

Logistic regression on m examples

$$\underline{J(w,b)} = \underbrace{\frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)})}$$

$(x^{(i)}, y^{(i)})$

$\rightarrow a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$

$\underline{dw_1^{(i)}}, \underline{dw_2^{(i)}}, \underline{db^{(i)}}$

$$\underline{\frac{\partial}{\partial w_1} J(w,b)} = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\frac{\partial}{\partial w_1}\mathcal{L}(a^{(i)}, y^{(i)})}_{\underline{dw_1^{(i)}}\ -\ (x^{(i)},\, y^{(i)})}$$

Logistic regression on m examples

$J=0;\ dw_1=0;\ dw_2=0;\ db=0$

For $i=1$ to m

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J += -[y^{(i)} \log a^{(i)} + (1-y^{(i)}) \log(1-a^{(i)})]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$

$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$

$db \mathrel{+}= dz^{(i)}$

$n=2$

dw_3
$\vdots$
dw_n

$J /= m$

$dw_1 /= m;\ dw_2 /= m;\ db /= m.$

$dw_1 = \frac{\partial J}{\partial w_1}$

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db$

Vectorization

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