# Алгоритм Баумана-Велша

Рассмотрим корпус из словаря состоящего из {ABBA, BAB}, где $c(ABBA) = 10$, $c(BAB) = 20$.

## Исходные данные

Возьмем произвольные матрицы переходов и состояний.

Таблица 1: Переходы

| - | $s_1$ | $s_2$ |
|---|---|---|
| $s_1$ | 0.3 | 0.7 |
| $s_2$ | 0.1 | 0.9 |

Таблица 2: Состояния

| - | $Pr(A)$ | $Pr(B)$ |
|---|---|---|
| $s_1$ | 0.4 | 0.6 |
| $s_2$ | 0.5 | 0.5 |

## Прямой проход

$\alpha(y, 1, s) = start(s) * out(s, A_1)$, $\alpha(y, j + 1, s) = \Sigma_{t \in S}\, \alpha(y, j, t) * go(t, s) * out(s, A_{j+1})$

Таблица 3: Начальная вероятность

| - | $Pr$ |
|---|---|
| $s_1$ | 0.85 |
| $s_2$ | 0.15 |

ABBA

$\alpha(A, 1, s_1) = 0.85 * 0.4 = 0.34$

$\alpha(A, 1, s_2) = 0.15 * 0.5 = 0.08$

$\alpha(AB, 2, s_1) = 0.34 * 0.3 * 0.6 + 0.08 * 0.1 * 0.6 = 0.066$

$\alpha(AB, 2, s_2) = 0.34 * 0.7 * 0.5 + 0.08 * 0.9 * 0.5 = 0.155$

$\alpha(AB, 3, s_1) = 0.66 * 0.3 * 0.6 + 0.155 * 0.1 * 0.6 = 0.021$

$\alpha(AB, 3, s_2) = 0.66 * 0.7 * 0.5 + 0.155 * 0.9 * 0.5 = 0.093$

$\alpha(AB, 4, s_1) = 0.021 * 0.3 * 0.6 + 0.093 * 0.1 * 0.6 = 0.006$

$\alpha(AB, 4, s_2) = 0.021 * 0.7 * 0.5 + 0.093 * 0.9 * 0.5 = 0.049$

Общая: $0.006 + 0.049 = 0.055$

BAB

$\alpha(B, 1, s_1) = 0.85 * 0.6 = 0.51$

$\alpha(B, 1, s_2) = 0.15 * 0.5 = 0.08$

$\alpha(BA, 2, s_1) = 0.51 * 0.3 * 0.4 + 0.08 * 0.1 * 0.4 = 0.064$

$\alpha(BA, 2, s_2) = 0.51 * 0.7 * 0.5 + 0.08 * 0.9 * 0.5 = 0.214$

$\alpha(BAB, 3, s_1) = 0.064 * 0.3 * 0.6 + 0.214 * 0.1 * 0.6 = 0.024$

$\alpha(BAB, 3, s_2) = 0.064 * 0.7 * 0.5 + 0.214 * 0.9 * 0.5 = 0.119$

Общая: $0.024 + 0.119 = 0.143$

## Оценка максимального правдоподобия модели $h_1$

$L(c, h_1) = Pr(ABBA)^{c(ABBA)} + Pr(BAB)^{c(BAB)}$, $\log L(c, h_1) = 10 * \log 0.055 + 20 * \log 0.143 = -67.90$

## Обратный проход

$\beta(y, n, s) = 1$, $\beta(y, j, s) = \sum_{t \in S} go(s, t) * out(t, A_{j+1}) * \beta(y, j + 1, t)$

ABBA

$\beta(ABBA, 4, s_1) = 1$

$\beta(ABBA, 4, s_2) = 1$

$\beta(ABB, 3, s_1) = 0.3 * 0.4 * 1 + 0.7 * 0.5 * 1 = 0.47$

$\beta(ABB, 3, s_2) = 0.1 * 0.4 * 1 + 0.9 * 0.5 * 1 = 0.49$

$\beta(AB, 2, s_1) = 0.3 * 0.6 * 0.47 + 0.7 * 0.5 * 0.49 = 0.256$

$\beta(AB, 2, s_2) = 0.1 * 0.6 * 0.47 + 0.9 * 0.5 * 0.49 = 0.249$

$\beta(A, 1, s_1) = 0.3 * 0.6 * 0.256 + 0.7 * 0.5 * 0.249 = 0.133$

$\beta(A, 1, s_2) = 0.1 * 0.6 * 0.256 + 0.9 * 0.5 * 0.248 = 0.127$

BAB

$\beta(BAB, 3, s_1) = 1$

$\beta(BAB, 3, s_2) = 1$

$\beta(BA, 2, s_1) = 0.3 * 0.6 * 1 + 0.7 * 0.5 * 1 = 0.53$

$\beta(BA, 2, s_2) = 0.1 * 0.6 * 1 + 0.9 * 0.5 * 1 = 0.51$

$\beta(B, 1, s_1) = 0.3 * 0.4 * 0.53 + 0.7 * 0.5 * 0.51 = 0.24$

$\beta(B, 1, s_2) = 0.1 * 0.4 * 0.53 + 0.9 * 0.5 * 0.51 = 0.25$

## Подсчет $\gamma(y, j, s, t)$

Считаем вероятность того, что s — j символ, а t — (j+1) символ.

$\gamma(y, j, s, t) = \frac{\alpha(y,j,s) * go(s,t) * out(t, A_{j+1} * \beta(y,j+1,t))}{Pr_h(y)}$

$\gamma(ABBA, 1, s_2, s_1) = \frac{0.08*0.1*0.6*0.256}{0.05544} = 0.02217$

$\gamma(ABBA, 1, s_1, s_1) = \frac{0.34*0.3*0.6*0.256}{0.05544} = 0.28271$

$\gamma(ABBA, 1, s_1, s_2) = 0.534$

$\gamma(ABBA, 1, s_2, s_2) = 0.161$

$\gamma(ABBA, 2, s_1, s_1) = 0.101$

$\gamma(ABBA, 2, s_1, s_2) = 0.204$

$\gamma(ABBA, 2, s_2, s_1) = 0.0788$

$\gamma(ABBA, 2, s_2, s_2) = 0.616$

$\gamma(ABBA, 3, s_1, s_1) = 0.045$

$\gamma(ABBA, 3, s_1, s_2) = 0.134$

$\gamma(ABBA, 3, s_2, s_1) = 0.067$

$\gamma(ABBA, 3, s_2, s_2) = 0.753$

$\gamma(BAB, 1, s_1, s_1) = 0.231$

$\gamma(BAB, 1, s_1, s_2) = 0.651$

$\gamma(BAB, 1, s_2, s_1) = 0.012$

$\gamma(BAB, 1, s_2, s_2) = 0.131$

$\gamma(BAB, 2, s_1, s_1) = 0.083$

$\gamma(BAB, 2, s_1, s_2) = 0.161$

$\gamma(BAB, 2, s_2, s_1) = 0.092$

$\gamma(BAB, 2, s_2, s_2) = 0.690$

## Подсчет $\delta(y, j, s)$

$\delta(y, j, s) = \sum_{t \in S} \gamma(y, j, s, t)$, $\delta(y, n, s) = \frac{\alpha(y,n,s)}{Pr_h(y)}$

$\delta(ABBA, 1, s_1) = 0.8165$

$\delta(ABBA, 1, s_2) = 0.183$

$\delta(ABBA, 2, s_1) = 0.304$

$\delta(ABBA, 2, s_2) = 0.695$

$\delta(ABBA, 3, s_1) = 0179$

$\delta(ABBA, 3, s_2) = 0.821$

$\delta(ABBA, 4, s_1) = 0.113$

$\delta(ABBA, 4, s_2) = 0.887$

$\delta(BAB, 1, s_1) = 0.882$

$\delta(BAB, 1, s_2) = 0.143$

$\delta(BAB, 2, s_1) = 0.244$

$\delta(BAB, 2, s_2) = 0.782$

$\delta(BAB, 3, s_1) = 0.150$

$\delta(BAB, 3, s_2) = 0.850S$

## Пересчет параметров HMM

$Pr(s_1) = \frac{I}{I+J}$, $Pr(s_2) = \frac{J}{I+J}$

$I = \delta(ABBA, 1, s_1) * c(ABBA) + \delta(BAB, 1, s_1) * c(BAB) = 25.82$

$J = \delta(ABBA, 1, s_2) * c(ABBA) + \delta(BAB, 1, s_2) * c(BAB) = 4.704$

Таблица 4: Начальная вероятность $h_2$

| - | $Pr$ |
|---|---|
| $s_1$ | 0.846 |
| $s_2$ | 0.154 |

Таблица 5: Переходы $h_2$

| - | $s_1$ | $s_2$ |
|---|---|---|
| $s_1$ | 0.298 | 0.702 |
| $s_2$ | 0.106 | 0.894 |

Таблица 6: Состояния $h_2$

| - | $Pr(A)$ | $Pr(B)$ |
|---|---|---|
| $s_1$ | 0.357 | 0.643 |
| $s_2$ | 0.429 | 0.569 |

## Погнали заново

ABBA

$\alpha(A, 1, s_1) = 0.846 * 0.357 = 0.302$

$\alpha(A, 1, s_2) = 0.154 * 0.429 = 0.066$

$\alpha(AB, 2, s_1) = 0.302 * 0.298 * 0.643 + 0.066 * 0.106 * 0.643 = 0.062$

$\alpha(AB, 2, s_2) = 0.302 * 0.702 * 0.569 + 0.066 * 0.894 * 0.569 = 0.154$

$\alpha(ABB, 3, s_1) = 0.062 * 0.298 * 0.643 + 0.154 * 0.106 * 0.643 = 0.022$

$\alpha(ABB, 3, s_2) = 0.062 * 0.702 * 0.569 + 0.154 * 0.894 * 0.569 = 0.103$

$\alpha(ABBA, 4, s_1) = 0.022 * 0.298 * 0.357 + 0.103 * 0.106 * 0.357 = 0.006$

$\alpha(ABBA, 4, s_2) = 0.022 * 0.702 * 0.429 + 0.103 * 0.894 * 0.429 = 0.047$

Общая: $0.006 + 0.047 = 0.053$

BAB

$\alpha(B, 1, s_1) = 0.846 * 0.643 = 0.54$

$\alpha(B, 1, s_2) = 0.154 * 0.569 = 0.088$

$\alpha(BA, 2, s_1) = 0.54 * 0.298 * 0.357 + 0.088 * 0.106 * 0.357 = 0.061$

$\alpha(BA, 2, s_2) = 0.54 * 0.702 * 0.43 + 0.088 * 0.894 * 0.43 = 0.197$

$\alpha(BAB, 3, s_1) = 0.061 * 0.298 * 0.643 + 0.197 * 0.106 * 0.643 = 0.025$

$\alpha(BAB, 3, s_2) = 0.061 * 0.702 * 0.571 + 0.197 * 0.894 * 0.571 = 0.125$

Общая: $0.025 + 0.125 = 0.15$

$L(c, h_1) = Pr(ABBA)^{c(ABBA)} + Pr(BAB)^{c(BAB)}$, $\log L(c, h_1) = 10 * \log 0.053 + 20 * \log 0.15 = -67.63$

ABBA

$\beta(ABBA, 4, s_1) = 1$

$\beta(ABBA, 4, s_2) = 1$

$\beta(ABB, 3, s_1) = 0.292 * 0.357 * 1 + 0.702 * 0.429 * 1 = 0.405$

$\beta(ABB, 3, s_2) = 0.106 * 0.357 * 1 + 0.894 * 0.429 * 1 = 0.421$

$\beta(AB, 2, s_1) = 0.292 * 0.643 * 0.405 + 0.702 * 0.57 * 0.421 = 0.245$

$\beta(AB, 2, s_2) = 0.106 * 0.643 * 0.405 + 0.894 * 0.57 * 0.421 = 0.242$

$\beta(A, 1, s_1) = 0.292 * 0.643 * 0.245 + 0.702 * 0.57 * 0.242 = 0.143$

$\beta(A, 1, s_2) = 0.106 * 0.643 * 0.245 + 0.894 * 0.57 * 0.242 = 0.140$

BAB

$\beta(BAB, 3, s_1) = 1$

$\beta(BAB, 3, s_2) = 1$

$\beta(BA, 2, s_1) = 0.298 * 0.643 * 1 + 0.702 * 0.57 * 1 = 0.59$

$\beta(BA, 2, s_2) = 0.102 * 0.643 * 1 + 0.894 * 0.57 * 1 = 0.58$

$\beta(B, 1, s_1) = 0.298 * 0.357 * 0.59 + 0.702 * 0.429 * 0.58 = 0.24$

$\beta(B, 1, s_2) = 0.1 * 0.357 * 0.59 + 0.894 * 0.429 * 0.58 = 0.24$

$\gamma(ABBA, 1, s_2, s_1) = \frac{0.066*0.102*0.643*0.245}{0.053} = 0.0200$

$\gamma(ABBA, 1, s_1, s_1) = \frac{0.302*0.298*0.643*0.245}{0.053} = 0.2675$

$\gamma(ABBA, 1, s_1, s_2) = \frac{0.302*0.702*0.57*0.242}{0.053} = 0.55$

$\gamma(ABBA, 1, s_2, s_2) = \frac{0.066*0.898*0.57*0.242}{0.053} = 0.154$

$\gamma(ABBA, 2, s_1, s_1) = \frac{0.062*0.298*0.643*0.405}{0.053} = 0.090$

$\gamma(ABBA, 2, s_1, s_2) = \frac{0.062*0.702*0.57*0.421}{0.053} = 0.197$

$\gamma(ABBA, 2, s_2, s_1) = \frac{0.154*0.106*0.643*0.405}{0.053} = 0.080$

$\gamma(ABBA, 2, s_2, s_2) = \frac{0.154*0.894*0.57*0.421}{0.053} = 0.623$

$\gamma(ABBA, 3, s_1, s_1) = \frac{0.022*0.298*0.357*1}{0.053} = 0.044$

$\gamma(ABBA, 3, s_1, s_2) = \frac{0.022*0.702*0.429*1}{0.053} = 0.125$

$\gamma(ABBA, 3, s_2, s_1) = \frac{0.103*0.106*0.357*1}{0.053} = 0.074$

$\gamma(ABBA, 3, s_2, s_2) = \frac{0.103*0.894*0.429*1}{0.053} = 0.745$

$\gamma(BAB, 1, s_1, s_1) = \frac{0.54*0.298*0.357*0.59}{0.147} = 0.230$

$\gamma(BAB, 1, s_1, s_2) = \frac{0.54*0.702*0.429*0.58}{0.147} = 0.641$

$\gamma(BAB, 1, s_2, s_1) = \frac{0.088*0.106*0.357*0.59}{0.147} = 0.013$

$\gamma(BAB, 1, s_2, s_2) = \frac{0.088*0.894*0.429*0.58}{0.147} = 0.133$

$\gamma(BAB, 2, s_1, s_1) = \frac{0.061*0.298*0.643*1}{0.053} = 0.080$

$\gamma(BAB, 2, s_1, s_2) = \frac{0.061*0.702*0.57*1}{0.053} = 0.166$

$\gamma(BAB, 2, s_2, s_1) = \frac{0.197*0.106*0.643*1}{0.053} = 0.091$

$\gamma(BAB, 2, s_2, s_2) = \frac{0.197*0.894*0.57*1}{0.053} = 0.683$

$\delta(ABBA, 1, s_1) = 0.8175$

$\delta(ABBA, 1, s_2) = 0.174$

$\delta(ABBA, 2, s_1) = 0.287$

$\delta(ABBA, 2, s_2) = 0.703$

$\delta(ABBA, 3, s_1) = 0.165$

$\delta(ABBA, 3, s_2) = 0.811$

$\delta(ABBA, 4, s_1) = 0.113$

$\delta(ABBA, 4, s_2) = 0.886$

$\delta(BAB, 1, s_1) = 0.871$

$\delta(BAB, 1, s_2) = 0.146$

$\delta(BAB, 2, s_1) = 0.246$

$\delta(BAB, 2, s_2) = 0.774$

$\delta(BAB, 3, s_1) = 0.170$

$\delta(BAB, 3, s_2) = 0.850$

$I = \delta(ABBA, 1, s_1) * c(ABBA) + \delta(BAB, 1, s_1) * c(BAB) = 25.595$

$J = \delta(ABBA, 1, s_2) * c(ABBA) + \delta(BAB, 1, s_2) * c(BAB) = 4.66$

Таблица 7: Начальная вероятность $h_3$

| - | $Pr$ |
|---|---|
| $s_1$ | 0.842 |
| $s_2$ | 0.158 |

Таблица 8: Переходы $h_3$

| - | $s_1$ | $s_2$ |
|---|---|---|
| $s_1$ | 0.288 | 0.712 |
| $s_2$ | 0.109 | 0.891 |

Таблица 9: Состояния $h_3$

| - | $Pr(A)$ | $Pr(B)$ |
|---|---|---|
| $s_1$ | 0.363 | 0.637 |
| $s_2$ | 0.424 | 0.576 |