Task 1

As said, variable ξ has Poisson distribution, so:

$p_\xi(\xi = k) = \frac{\lambda^k}{k!}e^{-\lambda}$

Also variable η has a binomial distribution as far as η is number of successful outcomes of Bernoulli trials.

The conditional probability of η given $\xi = k$:

$p_{\eta|\xi}(x|\xi = k) = C_k^x p^x (1-p)^{k-x}$,

where $C_k^x = \frac{k!}{x!(k-x)!}$.

Marginal probability of η is:

$$p_\eta(\eta = x) = \sum_{k=x}^{\infty} P(\eta = x|\xi = k)$$

,

where $P(\eta = x|\xi = k)$ is joint probability. In our case it is:

$P(\eta = x|\xi = k) = C_k^x p^x (1-p)^{k-x} \frac{\lambda^k}{k!} e^{-\lambda}$

So:

$p_\eta(\eta = x) = \sum_{k=x}^{\infty} P(\eta = x|\xi = k) = \sum_{k=x}^{\infty} \frac{k!}{x!(k-x)!} p^x (1-p)^{k-x} \frac{\lambda^k}{k!} e^{-\lambda} = \frac{(\lambda p)^x e^{-\lambda}}{x!} \sum_{k=x}^{\infty} \frac{\lambda^{k-x}(1-p)^{k-x}}{(k-x)!} = \frac{(\lambda p)^x e^{-\lambda}}{x!} e^{\lambda(1-p)} = \frac{(\lambda p)^x e^{-\lambda p}}{x!}$

So, η has Poisson distribution with the parameter λp.

Task 2

Let's calculate the conditional probability that the application was checked by a kind reviewer with Bayes' rule. In our case we need this formula in such form: $P(A|B) = \frac{f_{B|A}(b)P(A)}{f_B(b)}$ where A is discrete and B is continuous. So we want to find probability $P(A_1|t = 10)$ where A_1 denote the event that the application was checked by a kind reviewer.

To answer the question, we first find $f_B(b)$ where $f_B(b)$ is a value of probability density function in the point $t = 10$. That can be done in the following way:

$f_B(b) = P(A_1)f_{B|A_1}(b) + P(A_2)f_{B|A_2}(b)$.

A_2 in the equation above denote the event hat the application was checked by a strict reviewer.

It's given that t_1 has normal distribution with parameters $\mu_1 = 30$, $\sigma_1 = 10$ and t_2 has normal distribution with parameters $\mu_2 = 20$, $\sigma_2 = 5$. So we can find $f_{B|A_1}$ and $f_{B|A_2}$ from pdf of normal distribution:

$f_{(t=10|A_1)} = \frac{1}{\sqrt{2\pi * 10^2}} e^{-\frac{(10-30)^2}{2*10^2}} \approx 0.04 * 0.14 = 0.0056$

$f_{(t=10|A_2)} = \frac{1}{\sqrt{2\pi * 5^2}} e^{-\frac{(10-20)^2}{2*5^2}} \approx 0.08 * 0.14 = 0.011$

So:

$P(A_1|t = 10) = \frac{f_{t=10|A_1}(t=10)P(A_1)}{f(t=10)} = \frac{0.0056*0.5}{0.0056*0.5+0.01*0.5} = 0.359$