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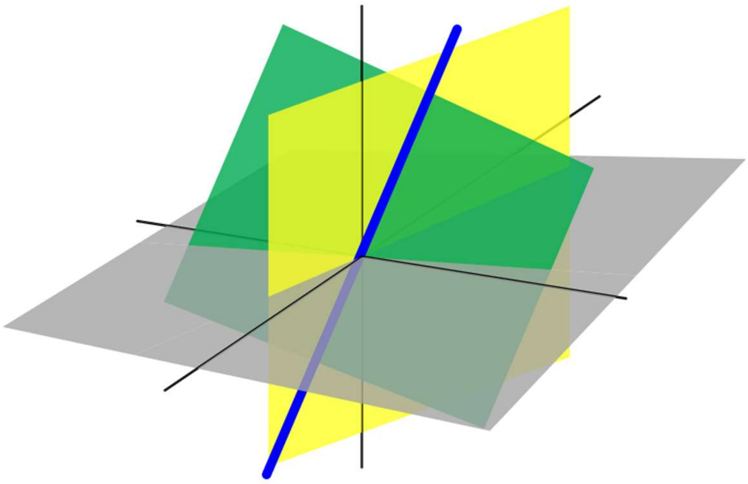
University of Delhi



Linear Algebra

Assignment for Paper Code 32355202

Submitted By

Sudipto Ghosh

College Roll No. 19/78003

BSc (Hons) Computer Science

Submitted To

Dr Kanika Sharma

Department of Mathematics

Sudipto Ghosh (19/78003)

ASSIGNMENT

Q1. If $\vec{x}$ and $\vec{y}$ are vectors in $\mathbb{R}^n$, then prove that $\|\vec{x}+\vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$

Ans. To prove $\|\vec{x}+\vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$, (given that $\vec{x}, \vec{y} \in \mathbb{R}^n$)

it is sufficient to show that $\|\vec{x}+\vec{y}\|^2 \leq (\|\vec{x}\| + \|\vec{y}\|)^2$

considering LHS of the inequality

$\Rightarrow \|\vec{x}+\vec{y}\|^2$

$\Rightarrow (\vec{x}+\vec{y})\cdot(\vec{x}+\vec{y})$ $\quad (\because \vec{a}\cdot\vec{a} = \|\vec{a}\|^2)$

$\Rightarrow (\vec{x}\cdot\vec{x}) + 2(\vec{x}\cdot\vec{y}) + (\vec{y}\cdot\vec{y})$

$\Rightarrow \|x\|^2 + 2(\vec{x}\cdot\vec{y}) + \|\vec{y}\|^2$

Now, according to Cauchy-Schwarz inequality,

$|\vec{x}\cdot\vec{y}| \leq \|\vec{x}\| \, \|\vec{y}\|$

Therefore,

$\|\vec{x}+\vec{y}\|^2 \leq \|\vec{x}\|^2 + 2\|\vec{x}\| \, \|\vec{y}\| + \|\vec{y}\|^2$

or, $\|\vec{x}+\vec{y}\|^2 \leq (\|\vec{x}\| + \|\vec{y}\|)^2$

Hence proved.

Q2. Solve the following system by Gauss Jordan method.

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 16 \\ 3x_1 + 2x_2 + x_4 = 16 \\ 2x_1 + 12x_3 - 5x_4 = 5 \end{cases}$$

Ans Given system of equations has the corresponding augmented matrix

$$\left[\begin{array}{cccc|c} 2 & 1 & 3 & 0 & 16 \\ 3 & 2 & 0 & 1 & 16 \\ 2 & 0 & 12 & -5 & 5 \end{array}\right]$$

Row reducing,

$$\Rightarrow \left[\begin{array}{cccc|c} 2 & 1 & 3 & 0 & 16 \\ 3 & 2 & 0 & 1 & 16 \\ 2 & 0 & 12 & -5 & 5 \end{array}\right] \quad R_1 \to R_1/2$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 3/2 & 0 & 8 \\ 3 & 2 & 0 & 1 & 16 \\ 2 & 0 & 12 & -5 & 5 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 3/2 & 0 & 8 \\ 0 & 1/2 & -9/2 & 1 & -8 \\ 0 & -1 & 9 & -5 & -11 \end{array}\right] \quad R_2 \to 2R_2$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 3/2 & 0 & 8 \\ 0 & 1 & -9 & 2 & -16 \\ 0 & -1 & 9 & -5 & -11 \end{array}\right] \quad R_3 \to R_3 + R_2$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 3/2 & 0 & 8 \\ 0 & 1 & -9 & 2 & -16 \\ 0 & 0 & 0 & -3 & -27 \end{array}\right] \quad R_3 \to -R_3/3$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 3/2 & 0 & 8 \\ 0 & 1 & -9 & 2 & -16 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right] \quad R_2 \to R_2 - 2R_3$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 1/2 & 3/2 & 0 & 8 \\ 0 & 1 & -9 & 0 & -34 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right] \quad R_1 \to R_1 - R_2/2$$

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ \end{array}$$
$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 6 & 0 & 25 \\ 0 & 1 & -9 & 0 & -34 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right]$$

In the reduced row echelon form of the augmented matrix,
$\Rightarrow x_3$ is free and we can assume $\underline{x_3 = c}$ where $c \in \mathbb{R}$ (arbitrary)

Now, $\begin{cases} x_1 = 25 - 6x_3 \\ x_2 = 9x_3 - 34 \\ x_4 = 9 \end{cases} \Rightarrow \begin{cases} \underline{x_1 = 25 - 6c} \\ \underline{x_2 = 9c - 34} \\ \underline{x_4 = 9} \end{cases}$

Therefore,

the solution set of the system is given by

$$\underline{\underline{\{(25-6c, 9c-34, c, 9) \mid c \in \mathbb{R}\}}}$$

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Q4 Is $X = [3 \quad 5]$ in the row space of $B = \begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$?

Ans We know that some X is in the row space of B iff X is a linear combination of the rows of B.

That is, if X is, infact, in the row space of B then,

$\Rightarrow X = [3 \quad 5] = c_1[2 \quad -4] + c_2[-1 \quad 2]$ for $c_1, c_2 \in \mathbb{R}$

$$\Rightarrow \begin{cases} 2c_1 - c_2 = 3 \\ -4c_1 + 2c_2 = 5 \end{cases}$$

we have the augmented matrix $[B^T | X]$,

$$\Rightarrow \left[\begin{array}{cc|c} 2 & -1 & 3 \\ -4 & 2 & 5 \end{array}\right]$$

Row reducing,

$$\Rightarrow \left[\begin{array}{cc|c} 2 & -1 & 3 \\ -4 & 2 & 5 \end{array}\right] \quad R_2 \to R_1 + 2R_1$$

$$\Rightarrow \left[\begin{array}{cc|c} 2 & -1 & 3 \\ 0 & 0 & 11 \end{array}\right] \quad R_1 \to R_1/2$$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & -1/2 & 3/2 \\ 0 & 0 & 11 \end{array}\right]$$

As the second row in rref has zeroes to the left of the augmentation bar but with last entry on the right ~~entry~~ as non-zero,

$\Rightarrow$ This system has no solution.

$\therefore$ X is not in the row space of B.

Q6. Solve for Eigenspace E_6 corresponding to eigenvalue $\lambda = 6$

$$A = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$$

Ans Eigenspace E_6 is spanned by fundamental eigenvector corresponding to $\lambda = 6$.

Finding fundamental eigenvector corresponding to $\lambda = 6$,
solving the homogenous system $(\lambda I_2 - A)X = 0$,
having the augmented matrix $[\lambda I_2 - A \mid 0]$
we have $\lambda I_2 - A = 6I_2 - A$,

$$\Rightarrow \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -6 & 51 \\ -2 & 17 \end{bmatrix}$$

So $[6I_2 - A \mid 0] = \left[\begin{array}{cc|c} -6 & 51 & 0 \\ -2 & 17 & 0 \end{array}\right]$

Row reducing,

$$\Rightarrow \left[\begin{array}{cc|c} -6 & 51 & 0 \\ -2 & 17 & 0 \end{array}\right] \quad R_2 \to R_2 - R_1/3$$

$$\Rightarrow \left[\begin{array}{cc|c} -6 & 51 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad R_1 \to R_1/(-6)$$

$$\Rightarrow \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & -17/2 & 0 \\ 0 & 0 & 0 \end{array}\right] \end{array}$$

From the rref, x_2 is free and suppose $x_2 = c$ for $c \in \mathbb{R}$,
then $x_1 = (17/2)c$

The solution set is $\{c[17/2, 1] \mid c \in \mathbb{R}\}$.
The fundamental eigenvector is $[17, 2]$.

$\therefore E_6 = \{c[17, 2] \mid c \in \mathbb{R}\} = \text{span}\left(\left\{\begin{bmatrix} 17 \\ 2 \end{bmatrix}\right\}\right)$

Sudipto Ghosh (19178003)