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Project Report

Profit-Driven Fleet Optimization

Abstract

This report details the development and solution of a mathematical optimization model designed to address the complex fleet management challenges of a car rental company. The primary objective is to determine the optimal fleet size and the daily distribution of vehicles across four depots (Glasgow, Manchester, Birmingham, Plymouth) to maximize total weekly profit. The model is formulated as a Linear Program, capturing intricate operational dynamics including variable daily demand, multi-day rental periods, inter-depot transfers, and a vehicle damage and repair cycle with capacity-constrained repair facilities. The model was implemented in Python using the Gurobi solver to find a profit-maximizing "steady-state" solution. The optimal results prescribe a total fleet of 617 cars, which, when allocated according to the model's daily schedule, yields a maximum weekly profit of $121,160. The resulting operational plan provides a precise, data-driven strategy for enhancing logistical efficiency and profitability.

Introduction

Effective fleet management is a critical success factor in the car rental industry, which is characterized by fluctuating demand and complex logistical constraints. Companies must make strategic decisions about fleet size and tactical decisions about vehicle allocation to maximize revenue while controlling costs associated with ownership, transfers, and maintenance.

This project addresses this challenge by applying prescriptive analytics, specifically mathematical optimization, to create a deterministic, steady-state model. The model is based on a problem description from H. Paul Williams' *Model Building in Mathematical Programming* and is tailored to a company with four depots, one car type, and a six-day work week. It accounts for a variety of real-world factors, including:

Variable rental demand for each day and depot.
Different rental durations (1, 2, or 3 days).
The probability of cars being returned to different depots.
Costs for vehicle transfers, marginal rental costs, and weekly ownership costs.
A cycle for damaged vehicles, including transfer to one of two repair depots with limited daily repair capacity.

The ultimate goal is to provide a clear, actionable weekly plan that answers two fundamental business questions: How many cars should the company own? and Where should those cars be located each day?

Methodology

The problem was formulated as a Linear Programming (LP) model, which seeks to optimize a linear objective function subject to a set of linear constraints.

Model Components

Sets and Parameters:
Sets: The model is defined over sets of Depots, Days of the week, and RentDays (1, 2, or 3).
Parameters: Key business inputs were defined as parameters, including daily demand at each depot, cstTransfer between depots, pctRent for different durations, and capRepair at Manchester and Birmingham.

Decision Variables:
The core outputs of the model were defined as decision variables, including:
xOwned: The total number of cars in the fleet.
xRented(d, t): Number of cars rented out from depot d on day t.
xUndamaged(d, t) / xDamaged(d, t): Number of cars available at the start of day t.
xUDtransfer(d1, d2, t) / xDtransfer(d1, d2, t): Number of cars transferred between depots.
xRepaired(d, t): Number of cars repaired at depot d on day t.

Objective Function:
The objective is to maximize total weekly profit. This was formulated as:
Profit = (Total Rental Revenue + Damaged Car Fees) - (Total Transfer Costs + Total Ownership Costs)

Constraints:
The operational logic was encoded using a series of linear constraints:
Car Flow Balance: Separate constraints for undamaged and damaged cars ensure that the number of cars available at a depot at the start of a day is equal to the cars remaining from the previous day plus all incoming vehicles (returns,

transfers, and completed repairs). This is the core mass-balance logic of the model.
Repair Capacity: The number of cars repaired (xRepaired) at Manchester and Birmingham on any given day cannot exceed their respective capacities (12 and 20).
Demand: The number of cars rented out (xRented) cannot exceed the estimated demand for that location and day.
Total Fleet Size: A critical constraint connects the individual daily variables to the total fleet size (xOwned). This was achieved by choosing a single day (Wednesday) and equating xOwned to the sum of all cars in the system at that time (idle, rented, in transit, or under repair).

The model was implemented in Python using the gurobipy library and solved to find the optimal variable values.

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3. Results and Inference

The optimization model produced a clear and actionable operational plan.
Optimal High-Level Solution
Maximum Weekly Profit: The model projects a maximum achievable profit of $121,160.21.
Optimal Fleet Size: To achieve this profit, the company must own a total of 617 cars.

This top-line result provides the core strategic answer: the ideal investment in the company's primary asset.
Detailed Operational Plan
The true value of the model lies in its detailed, day-by-day allocation plan for the entire fleet.

Optimal Daily Inventory of Undamaged Cars:
The following table shows the optimal number of undamaged cars that should be available at each depot at the start of each day.

	Day	Glasgow	Manchester	Birmingham	Plymouth
	Monday	68	98	146	41
	Tuesday	66	95	155	40
	Wednesday	70	100	123	43

	Day	Glasgow	Manchester	Birmingham	Plymouth
	Thursday	68	114	116	42
	Friday	70	102	124	43
	Saturday	67	95	158	40

Inference: The distribution is dynamic. Birmingham requires the largest inventory for most of the week, especially on Tuesday and Saturday, to meet its high demand. Manchester's inventory peaks on Monday and Thursday. This plan ensures cars are proactively positioned to meet demand rather than reactively responding to shortages.

Optimal Daily Inventory of Damaged Cars:

This table shows the optimal number of damaged cars held at each depot, awaiting transfer or repair.

	Day	Glasgow	Manchester	Birmingham	Plymouth
	Monday	8	12	20	6
	Tuesday	7	12	20	4
	Wednesday	8	13	20	5
	Thursday	8	12	21	5
	Friday	8	12	20	7
	Saturday	7	12	22	4

Inference: The model fully utilizes the repair capacity at Manchester (12/day) and Birmingham (20/day) on nearly all days. This indicates that repair capacity is a significant bottleneck in the system. The small floating inventory of damaged cars at non-repair depots (Glasgow, Plymouth) represents an optimal balance between the cost of immediate transfer and the need to get cars back into service.

Conclusion

This project successfully demonstrates the power of mathematical optimization to solve a complex, real-world business problem. By translating operational rules and financial data into a formal Linear Programming model, it was possible to move beyond simple forecasting and prescribe a profit-maximizing strategy.

Business Value:

The model's output is not just an analysis; it is a direct operational playbook. It provides a clear strategy that:
Maximizes Profit: By balancing all revenue and cost drivers simultaneously.
Optimizes Asset Utilization: By defining the exact fleet size needed, avoiding over-investment in idle cars or under-investment leading to lost revenue.
Improves Logistical Efficiency: By providing a precise schedule for vehicle inventory and transfers

Bottleneck Analysis through Shadow Prices

Beyond the optimal fleet distribution, the linear programming solution offers critical insights into the system's bottlenecks through the analysis of shadow prices (also known as dual values).

The shadow price of a constraint indicates how much the objective function—in this case, weekly profit—would improve if that constraint were relaxed by one unit. For this car rental problem, the most impactful constraints to analyze are the daily repair capacities at Manchester and Birmingham.

The model's output reveals a high shadow price on these constraints for most days, signifying that the repair facilities are operating at maximum capacity and are a primary limiting factor on overall profitability. For example, if the shadow price for Birmingham's repair capacity on a Wednesday is \$50, it means that adding one extra unit of repair capacity (i.e., being able to repair one more car that day) would increase the total weekly profit by \$50. This provides a direct, data-driven justification for capital investment.

The business can now quantitatively answer the question, "What is the return on investment for expanding our repair depots?" By comparing the cost of adding capacity to its shadow price, the company can prioritize investments that will have the greatest impact on the bottom line.

Strategic Planning with Sensitivity Analysis

While the current model provides a single optimal solution based on a fixed set of inputs, its true strategic value is unlocked through sensitivity analysis. This process involves systematically altering key parameters to understand how the optimal solution and total profit would change in response to real-world volatility. For instance, the model could be re-run with a 15% increase in transfer costs to simulate a rise in fuel prices, revealing how the optimal transfer strategy and fleet distribution would need to adapt. Similarly, by increasing the weekly ownership cost (cstOwn), the company could determine the breakeven point at which it becomes more profitable to operate a smaller fleet and accept a higher level of unmet demand. This analysis transforms the model from a static planning tool into a dynamic decision-support system. It allows management to test various "what-if" scenarios, quantify risk, and develop more robust, forward-looking strategies that are prepared for changing market conditions, such as demand surges or unexpected cost increases.