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Question Paper Code : 21279

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023

Third Semester

Civil Engineering

MA 3351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to all branches)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Obtain the partial differential equations by eliminating arbitrary constants a and b from $(x + a)^2 + (y - b)^2 = z$.
2. Solve $(D^2 - 4DD' + 3D'^2)z = 0$.
3. State the condition for a function $f(x)$ to be expressed as a Fourier series.
4. If $f(x) = x^2 + x$ is expressed as a Fourier series in the interval $(-2, 2)$ to which value this series converges at $x = 2$.
5. Classify the partial differential equation $u_{xx} - y^4 u_{yy} = 2y^3 u_y$.
6. Write down the various solutions of one dimensional wave equation.
7. State the convolution theorem of Fourier Transform.
8. Prove that $F[f(x - a)] = e^{isa} F(s)$.
9. Find the Z transform of 1.
10. State initial and final value theorem of Z transforms.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the general solution of $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. (8)
 (ii) Solve the partial differential equation $[D^2 + 3DD' + 2D'^2]z = x + y$. (8)

Or

- (b) (i) Obtain the singular solution of the partial differential equation.
 $(pq - p - q)(z - px - qy) = pq$. (8)
 (ii) Solve the partial differential equation
 $[D^2 + 2DD' + D'^2]z = x^2y + e^{x-y}$. (8)
12. (a) (i) Expand the Fourier series for $f(x) = x(2\pi - x)$ in $(0, 2\pi)$ and hence deduce that the sum of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$. (10)
 (ii) Expand the function $f(x) = x$, $0 < x < \pi$ in the Fourier sine series. (6)

Or

- (b) (i) Obtain the function $f(x) = \sin x$, $0 < x < \pi$ in Fourier cosine series. (8)
 (ii) Determine the first two harmonic of the Fourier series for the following values (8)

$x:$	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
$y:$	1.98	1.30	1.05	1.30	-0.88	-0.25

13. (a) A string is tightly stretched and its ends are fastened at the two points $x = 0$ and $x = 2l$. The mid-point of the string is displaced transversely through a small distance b and the string is released from rest in that position. Obtain an expression for the displacement of the string at any subsequent time.

Or

- (b) The ends A and B of a rod 30 cm long have their temperature kept at 20°C . and 80°C respectively until steady state condition prevail. The temperature at each end is then suddenly reduced to 60°C and that of A is raised 40°C . Find the temperature distribution in the rod after time t .

14. (a) (i) Find the Fourier sine and cosine transform of e^{-ax} and hence deduce the inversion formula. (8)

- (ii) Find the Fourier integral of the function $f(t) = \begin{cases} e^{at}, & t < 0 \\ e^{-at}, & t > 0 \end{cases}$. (8)

Or

- (b) Obtain the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases}$ Hence deduce that

$$\int_0^\infty \left(\frac{\sin t}{t} \right)^2 dt = \frac{\pi}{2}, \quad \text{Using Parseval's identity find the value of}$$

$$\int_0^\infty \left(\frac{\sin t}{t} \right)^4 dt.$$

15. (a) (i) Find the Z -transforms of $\cos\left(\frac{n\pi}{2}\right)$ and $\frac{2n+3}{(n+1)(n+2)}$. (8)

- (ii) Obtain the inverse Z -transforms of $\frac{z^2 + 2z}{z^2 + 2z + 5}$. (8)

Or

- (b) (i) Find the inverse Z -transforms of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. (8)

- (ii) Solve the difference equation $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0, y_1 = 1$ by using Z transforms. (8)