Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2024.

Third Semester

Electrical and Electronics Engineering

MA 3303 – PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are the axioms of probability?

2. Give the moment generating function of Binomial and Poisson distributions.

3. Suppose X an Y are independent continuous random variables. Show that correlation between X and Y is zero.

4. State the central limit theorem for probability of random variables.

5. Is the function $f(z) = 2iz + 6\overline{z}$ analytic?

6. What is conformal mapping?

7. Identify the singular point and the type of singularity in $f(z) = \dfrac{1 - \cos z}{z}$.

8. State Cauchy's Residue theorem.

9. Solve the differential equation $(D^2 + 5D + 6)y = 0$.

10. Give the general form of Legendre's linear equation.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A printer manufacturer obtained the following probabilities from a database of test results. Printer failures are associated with three types of problems: hardware, software, and others, with probabilities 0.1, 0.6, and 0.3 respectively. The probability of a printer failure given a hardware problem is 0.9, given a software problem is 0.2 and given any other type of problem is 0.5. If a customer enters the manufacturer's website to diagnose a printer failure, using Baye's theorem determine what is the most likely cause of the problem? (8)

(ii) The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour.

(1) What is the probability that there are exactly 5 calls in one hour? (2)

(2) What is the probability that there are 3 or less calls in one hour? (3)

(3) What is the probability that there are exactly 15 calls in two hours? (3)

Or

(b) (i) The distance between major cracks in a highway follows an exponential distribution with a mean of five miles.

(1) What is the probability that there are two major cracks in a 10 -mile stretch of the highway? (2)

(2) What is the standard deviation of the distance between major cracks? (2)

(3) What is the probability that the first major crack occurs between 12 and 15 miles of the start of inspection? (2)

(4) Given that there are no cracks in the first five miles inspected, what is the probability that there are no major cracks in the next 10 miles inspected? (2)

(ii) The line width of semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.

(1) What is the probability that a line width is greater than 0.62 micrometer? (3)

(2) What is the probability that a line width is between 0.47 and 0.63 micrometer? (3)

(3) The line width of 90% of samples is below what value? (2)

12. (a) Determine the value of c that makes the function $f(x, y) = c(x + y)$ a joint probability function over the nine points with $x = 1, 2, 3$ and $y = 1, 2, 3$ (16)

(i) Determine $P(X < 2, Y < 2)$

(ii) Determine correlation for the joint probability mass function $f(x, y)$.

Or

(b) A study on the deflection of particle board from stress levels of relative humidity gave the following measurements.

X: Stress level (%)	54	57	61	65	68	72	75	80
Y : Deflection (mm)	16.4	18.6	14.3	15.1	13.5	11.6	11.1	12.5

(i) Construct a linear regression for thé given data. (8)

(ii) Find the deflection when stress level is 70%. (2)

(iii) What is the estimate of σ^2? (6)

13. (a) (i) State and prove Cauchy-Riemann equation for analytic functions in Cartesian coordinates. (8)

(ii) Verify that $u(x, y) = x^3 - 3xy^2$ is harmonic in the whole complex plane. Also find the conjugate harmonic and hence construct an analytic function $f(z) = u + iv$. (8)

Or

(b) (i) Describe the transformation $w = z + c$ and $w = cz$, under conformal mapping. (8)

(ii) Find the linear fractional transformation that maps –1, i, 1 in the z-plane onto 0, i, ∞ in the w-plane. Also find the images of x-axis and y-axis. (8)

14. (a) (i) State and prove Cauchy's integral theorem. (8)

(ii) Evaluate the integral $\oint_c \frac{1 - 4z + 6z^2}{\left(z + \frac{1}{4}\right)(3z - 1)} dz$, where $C : |z| = 1$. (8)

Or

(b) (i) Find the Taylor and Laurent series expansion of $f(z) = \frac{-2z + 3}{z^2 - 3z + 2}$ with centre 0. (8)

(ii) Evaluate the integral $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z - 1)^2 (z - 2)} dz$, where c is the circle $|z| = 3$. (8)

15. (a) (i) Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = x + e^x \cos x$. (8)

(ii) By method of variation of parameter, solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$. (8)

Or

(b) (i) Solve the simultaneous equation $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$. (8)

(ii) Solve the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$, by method of undetermined coefficients, assuming the particular solution as $y = a_1x^2 + a_2x + a_3 + a_4e^{-x}$. (8)
