Reg. No. : | | | | | | | | | | | |

Question Paper Code : 51320

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024

Third Semester

Electrical and Electronics Engineering

MA 3303 – PROBABILITY AND COMPLEX FUNCTIONS

(Regulations 2021)

Time : Three hours Maximum : 100 marks

(Normal distribution table values to be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State Baye's theorem.

2. Find the moment generating function of a random variable whose probability density function is given by $f(x) = \begin{cases} 1/k, & for 0 < x < k \\ 0, & otherwise \end{cases}$

3. If the joint probability density function of the random variable (X,Y) is given by $f(x,y) = cx(x-y)$, $0 < x < 2$, $-x < y < x$. Find the value of c.

4. The life time of a certain brand of an electric bulb may be considered a random variable with mean 1200 h and standard deviation 20 h. Find the probability, using central limit theorem, that the average lifetime of 0 bulbs exceeds 1250h.

5. Test the analyticity of the function $w = \sin z$.

6. Find the fixed points of the bilinear transformation $w = 2 - \frac{2}{z}$.

7. Make use of Cauchy's integral formula to find $\int_C \frac{z}{z-2}\, dz$, where c is $|z| = 3$.

8. Identify the type of singularity of function $f(z) = e^{\frac{1}{(z-1)}}$.

9. Solve $(D^2+1)y = e^{-x}$.

10. Build the equation $x^2 y'' + xy' = x$ into linear differential equation with constants coefficients.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A bag contains 5 balls and is not known how many of the are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white? (8)

(ii) Find the probability density function of $Y = 8X^3$, given the random variable X with the density function $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$. (8)

Or

(b) (i) Out of 800 families with 4 children each, how many families would be expected to have. (8)

(1) 2 boys and 2 girls

(2) atleast 1 boy.

(ii) The time (in hour) to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. (8)

(1) What is the probability that the repair time exceeds 2h?

(2) What is the conditional probability that a repair takes at least 10 h given that its duration exceeds 9h?

12. (a) (i) The joint probability density function of a two-dimensional random variable (X,Y) is given by $f(x,y) = xy^2 + \frac{x^2}{8}$, $0 \le x \le 2$, $0 \le y \le 1$

Compute (8)

(1) $P(X > 1)$

(2) $P\left(Y < \frac{1}{2}\right)$

(ii) Find the coefficient of correlation between X and Y, using the following data: (8)

X:	1	3	5	7	8	10
Y:	8	12	15	17	18	20

Or

(b) (i) The probability density function (X,Y) is given by $f_{XY}(x,y) = x + y,\ 0 \le x,\ y \le 1$, find the probability density function of $U = XY$. (8)

(ii) Find the equations of the lines of regression from the following data : (8)

X:	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

13. (a) (i) If $f(z) = u + iv$, is analytic find $f(z)$ if $u = \dfrac{2\sin 2x}{(e^{2y} + e^{-2y} - 2\cos 2x)}$. (8)

(ii) Show that $u = x^2 - y^2$ and $v = \dfrac{-y}{x^2 + y^2}$ are harmonic but $u + iv$ is not regular. (8)

Or

(b) (i) Find the bilinear map which maps the points $z = 1,\ i,\ -1$ onto $w = i,\ 0,\ -i$. (8)

(ii) Find the image of $|z - 2i| = 2$, under the transformation $w = \dfrac{1}{z}$. (8)

14. (a) (i) Evaluate, using Cauchy's integral formula $\int_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$. Where C is $|z| = 3$. (8)

(ii) Expand $\cos z$ as a Taylor's series about the point $z = \dfrac{\pi}{4}$. (8)

Or

(b) (i) Evaluate, the residue of $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ at each of the poles. (8)

(ii) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{13 + 12\cos\theta}$. (8)

15. (a) (i) Solve the equation $\left(D^3 - D^2 - 6D\right)y = x^2 + 1$. (8)

(ii) Solve $\left(D^2 + a^2\right)y = \tan ax$ by method of variation of parameters. (8)

Or

(b) (i) Solve the equation $(x+2)^2 \frac{d^2y}{dx^2} - (x+2)\frac{dy}{dx} + y = 3x + 4$. (8)

(ii) Solve the simultaneous equation $\frac{dx}{dt} + y = e^t$, $x - \frac{dy}{dt} = t$. (8)