Reg. No. :

Question Paper Code : 51325

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Third Semester

Computer Science and Engineering

MA 3354 – DISCRETE MATHEMATICS

(Common to : Computer Science and Engineering (Artificial Intelligence and Machine Learning)/Computer Science and Engineering (Cyber Security)/ Computer and Communication Engineering/Artificial Intelligence and Data Science/ Computer Science and Business Systems/Information Technology)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that $p \to q$ and $\neg p \vee q$ are equivalent.

2. Translate the statement 'Every real number expect zero has a multiplicative inverse' into the statement involving nested quantifiers.

3. The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

4. State the pigeonhole principle.

5. Define degree of a vertex in an undirected graph.

6. Define path.

7. Is the set of integers under ordinary multiplication a group?

8. Prove that for each element a in a group G, there is a unique element b in G such that $ab = ba = e$.

9. Show that the 'greater than or equal' relation $(\geq)$ is a partial ordering on the set of integers.

10. Is the poset $(\mathbf{Z}^+, |)$ a lattice?

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) Find the principle disjunctive normal form of $(P \to Q) \wedge (P \rightleftarrows R)$. (8)

(ii) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. (8)

Or

(b) (i) Prove that if $n = ab$, where a and b are positive integers, then $a \le \sqrt{n}$ or $b \le \sqrt{n}$. (8)

(ii) Show that $(\forall x)[P(x) \vee Q(x)] \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$. (8)

12. (a) (i) Show that if n is an integer greater than 1, then n can be written as the product of primes. (8)

(ii) During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. (8)

Or

(b) (i) Find the number of 2-permutations with unlimited repetitions of $\{a, b, c, d\}$. (8)

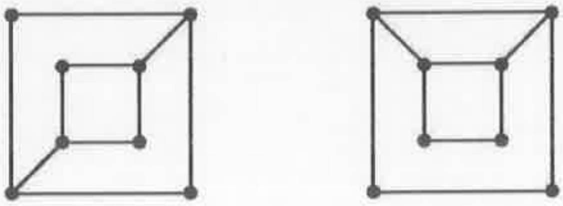
(ii) Solve $s(n+2) - 5s(n+1) + 6s(n) = 0$ for $n \ge 0$ with $s(0) = 1$, $s(1) = 1$. (8)

13. (a) (i) Prove that a simple connected graph is bipartite if and only if it contains no odd cycle. (8)

(ii) Draw the graph with the adjacency matrix $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ with respect to the ordering of a, b, c, d. (8)

Or

(b) (i) Check whether the graphs are isomorphic or not? (8)



G_1 G_2

(ii) Prove that, if G is a simple connected graph with $n \geq s$ vertices and $d(u) \geq \frac{n}{2}$, for all $u \in V(G)$, then G is Hamiltonian graph. (8)

14. (a) (i) Prove that the subgroup of a cyclic group must be cyclic subgroup. (8)

(ii) Prove that a subgroup H of G is normal in G if and only if $xHx^{-1} \subseteq H$ for all x in G. (8)

Or

(b) (i) State and prove Lagrange's theorem. (8)

(ii) Prove that the Kernal of a homomorphism f from the group $\langle G, * \rangle$ to a group $\langle H, \Delta \rangle$ is a normal subgroup of the group $\langle G, * \rangle$. (8)

15. (a) (i) Draw the Hasse diagram representing the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 8, 12\}$. (8)

(ii) Prove that every chain is a distributive lattice. (8)

Or

(b) (i) Find the values of the Boolean function represented by $F(x, y, z) = xy + \bar{z}$. (8)

(ii) Prove that the De Margon laws are valid in a Boolean Algebra. (8)