Reg. No. :

## Question Paper Code : 30234

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

First Semester

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours                                             Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1.  If two eigen values of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are equal to 1 each, find the

    eigen value of $A^{-1}$.

2.  Write the uses of Cayley-Hamilton Theorem.

3.  If $y = x \log\left(\dfrac{x-1}{x+1}\right)$, then find $\dfrac{dy}{dx}$.

4.  Find the point of inflection of $f(x) = x^3 - 9x^2 + 7x - 6$.

5.  Write Euler's theorem on homogeneous functions.

6.  If $x = r\cos\theta$, $y = r\sin\theta$, find $\dfrac{\partial(x, y)}{\partial(r, \theta)}$.

7.  Evaluate $\int \theta \cos\theta \, d\theta$ using integration by parts.

8.  Find the value of $\int_{0}^{\pi/2} \sin^6 x \, dx$.

9.  Evaluate $\int_{0}^{1}\int_{0}^{x} dy \, dx$.

10. Transform the double integral $\int_{0}^{2}\int_{y}^{2} \dfrac{x \, dx \, dy}{x^2 + y^2}$ into polar coordinates.

11. (a) (i) Find the eigen values and eigen vectors of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$. (8)

(ii) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $2x_1 x_2 - 2x_2 x_3 + 2x_3 x_1$ into the canonical form and hence find its nature. (16)

12. (a) (i) Find the values of $a$ and $b$ that make $f$ continuous on $(-\infty, \infty)$ if

$$f(x) = \begin{cases} \dfrac{x^3 - 8}{x - 2}, & \text{if } x < 2 \\ ax^2 - bx + 3, & \text{if } 2 \le x < 3 \\ 2x - a + b, & \text{if } x \ge 3 \end{cases}$$

(8)

(ii) Find $\dfrac{dy}{dx}$ if $y = x^2 e^{2x} (x^2 + 1)^4$. (4)

(iii) If $x^y = y^x$, Prove that $\dfrac{dy}{dx} = \dfrac{y(y - x \log y)}{x(x - y \log x)}$ using implicit differentiation. (4)

Or

(b) (i) Show that $\sin x (1 + \cos x)$ is maximum when $x = \pi/3$. (6)

(ii) A window has the form of a rectangle surmounted by a semicircle. If the perimeter is 40 ft., find its dimensions so that greatest amount of light may be admitted. (10)

13. (a) (i) Given the transformations $u = e^x \cos y$ and $v = e^x \sin y$ and that $f$ is a function of $u$ and $v$ and also of $x$ and $y$, prove that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right).$$

(8)

(ii) Expand $e^x \log(1 + y)$ in powers of $x$ and $y$ up to terms of third degree. (8)

Or

(b) (i) Examine for extreme values of $f(x, y) x^4 + y^4 - 2x^2 + 4xy - 2y^2$. (8)

(ii) A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box, that requires the least material for its construction. (8)

14. (a) (i) Evaluate $\int \dfrac{3x+1}{(x-1)^2 (x+3)} dx$ by applying partial fraction on the integrand; (6)

(ii) Evaluate $\int\limits_0^{\pi/2} \log \sin x \, dx$ and hence find the value of $\int\limits_0^1 \dfrac{\sin^{-1} x}{x} dx$. (10)

Or

(b) (i) Evaluate $\int \dfrac{\sqrt{9-x^2}}{x^2} dx$ using trigonometric substitution. (6)

(ii) Determine whether the integral $\int\limits_1^\infty \dfrac{1}{x} dx$ is convergent or divergent. (4)

(iii) Find the volume of the reel shaped solid formed by the revolution about the y-axis, of the part of the parabola $y^2 = 4ax$ cut off by its latusrectum. (6)

15. (a) (i) Find the area between the curves $y^2 = 4x$ and $x^2 = 4y$. (8)

(ii) Change the order of integration in $\int\limits_0^\infty \int\limits_0^y ye^{-y^2/x} dx \, dy$ and then evaluate it. (8)

Or

(b) (i) Find the volume of the sphere of radius '$a$'. (8)

(ii) Find the moment of inertia of the area bounded by the curve $r^2 = a^2 \cos 2\theta$ about its axis. (8)

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