Reg. No. : | | | | | | | | | | | |

Question Paper Code : 70142

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Electronics and Telecommunication Engineering

MA 3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to: B.E. Electronics and Communication Engineering)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Using the axioms of probability, prove $P(A^c) = 1 - P(A)$.

2. Consider a random experiment of tossing a fair coin three times. If X denotes the number heads obtained find, $P(X < 2)$.

3. For a bi-variate random variable (XY), prove that if X and Y are independent, then every event $a < X \leq b$ is independent of the other event $c < X \leq d$.

4. Let the joint probability mass function of (X, Y) be given by $P_{xy}(x, y) = \begin{cases} k(x+y) & x = 1, 2, 3; y = 1.2 \\ 0, & otherwise \end{cases}$. Find the value of k.

5. Let $X_1, X_2 \ldots$ be independent Bernoulli random variables with $P(X_n = 1) = p$ and $P(X_n = 0) = q$ for all n. Describe the Bernoulli process.

6. Consider a Markov chain with two states and transition probability matrix $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find the stationary distribution of the chain.

7. Determine whether the vectors $u = (1,1,2), v = (1,0,1)$, and $w = (2,1,3)$ span the vector space R^3.

8. Is a set of all vectors of the form $(a,1,1)$, where a is real, a subspace of R³? Justify.

9. Find the kernel and range of the identity operator.

10. Show that the vectors $u = (-2,3,1,4)$ and $v = (1,2,0,-1)$ are orthogonal in R^4.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) A lot of 100 semiconductor chips contain 20 that are defective. Two are selected randomly, without replacement, from the lot.

(1) What is the probability that the first one selected is defective?

(2) What is the probability that the second one selected is defective given that the first one was defective?

(3) What is the probability that both are defective? (8)

(ii) A company producing electric relays has three manufacturing plants producing 50, 30, and 20 percent respectively of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05 and 0.01 respectively. If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2? (8)

Or

(b) (i) All manufactured devices and machines fail to work sooner or later. Suppose that the failure rate is constant and the time to failure (in hours) is an exponential random variable X with parameter λ. Measurements shows that the probability that the time to failure for computer memory chips in a given class exceeds 10^4 hours is e^{-1}. Find the value of λ and calculate the time X_0 such that the probability that the time to failure is less than X_0 is 0.05. (8)

(ii) A production line manufactures 1000 ohm resistors that have 10% tolerance. Let X denotes the resistance of a resistor. Assuming that X is a normal random variable with mean 1000 and variance 2500, find the probability that a resistor picked at random will be rejected. (8)

12. (a) Consider an experiment of drawing randomly three balls from an urn containing two red, three white, and four blue balls. Let (X, Y) be a bi-variate random variable where X and Y denote respectively the number of red and white balls chosen.

(i) Find the range of (X, Y).

(ii) Find the joint probability mass function of (X,Y).

(iii) Find the marginal probability function of X and Y.

(iv) Are X and Y independent? (16)

Or

14. (a) Determine whether the set of all pairs of real numbers (x, y) with the operations $(x, y) + (p, q) = (x + p + 1, y + q + 1)$ and $k(x, y) = (kx, ky)$ is a vector space or not. If not, list all the axioms that fail to hold. (16)

Or

(b) Determine the basis and the dimension of the homogeneous system $2x_1 + 2x_2 - x_3 + x_5 = 0; - x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0;$ $x_1 + x_2 - 2x_3 - x_5 = 0 \;\; x_3 + x_4 + x_5 = 0$. (16)

15. (a) (i) State and prove the dimension theorem for linear transformation. (8)

(ii) Let $T : R^2 \to R^3$ be a linear transformation defined by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} y \\ -5x + 13y \\ -7x + 16y \end{bmatrix}$. Find the matrix for the transformation T with respect to the bases $B = \{u_1, u_2\}$ for R^2 and $B_1 = \{v_1, v_2, v_3\}$ for R^3 where $u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. (8)

Or

(b) Find the orthogonal projection of the vector $u = (-3, -3, 8, 9)$ on the subspace of R^4 spanned by the vectors $v_1 = (3, 1, 0, 1), v_2 = (1, 2, 1, 1)$, $v_3 = (-1, 0, 2, -1)$. (16)