Reg. No. :

Question Paper Code : 70141

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Computer Science and Engineering

MA 3354 – DISCRETE MATHEMATICS

(Common to Computer and Communication Engineering/Artificial Intelligence and Data Science/Computer Science and Business Systems/Information Technology)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Prove that $(p \to (p \vee q))$ is a tautology.

2. Symbolize the statement "All men are mortal".

3. Among 200 people, how many of them were born on the same month?

4. Find the first four terms of the sequence defined by the recurrence relations and initial condition $a_n = a_{n-1}^2$, $a_1 = 2$.

5. Define the pseudograph with an example.

6. Draw the graph of the adjacency matrix $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

7. Define monoid.

8. Define the homomorphism of groups.

9. Draw Hasse diagram for $\leq$ relation on $\{0, 2, 5, 10, 11, 15\}$.

10. Define lattice.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $\neg(p \wedge (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent without using truth table. (8)

(ii) Using the indirect method, show that $p \to q,\ q \to r,\ \neg(p \wedge r)$ $p \vee r \Leftrightarrow r$. (8)

Or

(b) (i) Find a conjunctive normal form of $p \to [(p \to q) \wedge \neg(\neg q \vee \neg p)]$. (8)

(ii) Establish the validity of the following argument : "All integers are rational numbers. Some integers are powers of 2. Therefore, some rational numbers are powers of 2". (8)

12. (a) (i) In a class of 50 students, 20 students play football, and 16 students play hockey. It is found that 10 students play both the games. Find the number of students who play neither. (8)

(ii) Use generating functions to solve the recurrence relation $a_n - 2a_{n-1} - 3a_{n-2} = 0$, $n \geq 2$ with $a_0 = 3$, $a_1 = 1$. (8)

Or

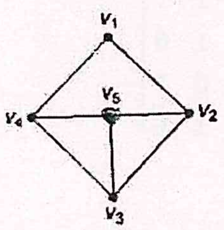
(b) (i) Use mathematical induction to prove that,

$1^2 + 3^2 + 5^2 + \ldots + (2n+1)^2 = \dfrac{(n+1)(2n+1)(2n+3)}{3}$, whenever n is positive integer. (8)

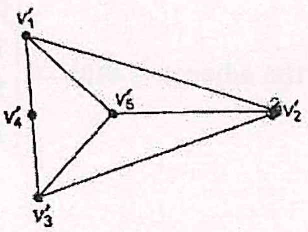
(ii) Solve the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ for $n \geq 2$, $a_0 = 16$, $a_1 = 80$. (8)

13. (a) (i) Show that the maximum number of edges in a simple graph with 'n' vertices is $\dfrac{n(n-1)}{2}$. (8)

(ii) Establish the isomorphism of the following pairs of graphs, by considering their adjacency matrices. (8)



(G_1)

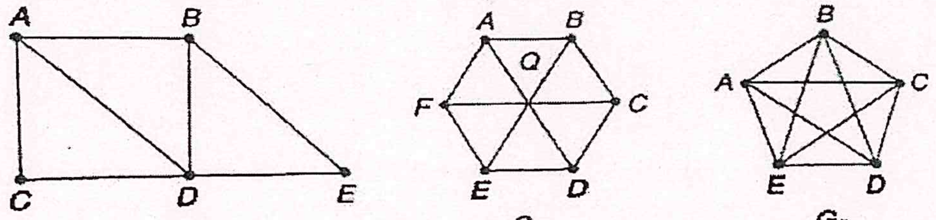


(G_2)

Or

(b) (i) In any graph G, prove that the total number of odd-degree vertices is even. (8)

(ii) Find an Euler path or an Euler circuit, if it exits in each of the following three graphs. If it does not exist, explain why? (8)



G_1 G_2 G_3

14. (a) (i) A subgroup H of a group G is a normal subgroup in G iff each left coset of H in G is equal to the right coset of H in G. (8)

(ii) Show that $(Z, *)$ is a group, where $*$ is defined by $a * b = a + b + 1$. (8)

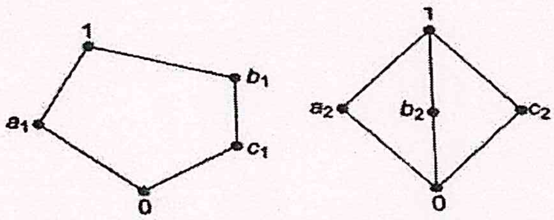
Or

(b) (i) Show that the group $G = \{1, -1, i, -i\}$ is cyclic and find its generators. (6)

(ii) Show that the set $z_4 = \{0, 1, 2, 3\}$ is a commutative ring with respect to the binary operations additive modulo $4(+_4)$ and multiplicative modulo $4(\times_4)$. (10)

15. (a) (i) Show that every chain is a distributive lattice. (8)

(ii) Verify whether the lattices given by the Hasse diagrams in following are distributive. (8)



(a) (b)

Or

(b) (i) State and prove De Morgan's law in any Boolean Algebra. (8)

(ii) If S_n is the set of all divisors of the positive integers 'n' and "aDb" if and only if 'a' divides 'b', prove that $\{S_{24}, D\}$ is a lattice. Find also all the sublattices of D_{24} that contain 5 or more elements. (8)
