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**Question Paper Code : 21272**

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

- Find the eigenvalues of  $A^{-1}$  and  $A^2$  if  $A = \begin{pmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{pmatrix}$ .
- State Cayley-Hamilton theorem.
- Sketch the graph of the function  $f(x) = \begin{cases} x^2 & \text{if } -2 \leq x \leq 0 \\ 2-x & \text{if } 0 < x \leq 2 \end{cases}$ .
- The equation of motion of a particle is given by  $s = 2t^3 - 5t^2 + 3t + 4$  where  $s$  is measured in meters and  $t$  in seconds. Find the velocity and acceleration as functions of time.
- If  $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ , find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ .
- Write any two properties of Jacobians.
- Evaluate  $\int_0^{\frac{\pi}{2}} \sin^9 x \, dx$ .
- Prove that the integral  $\int_1^{\infty} \frac{1}{x} \, dx$  is divergent.

9. Evaluate  $\int_1^2 \int_1^3 xy^2 dx dy$ .

10. Find the area of a circle  $x^2 + y^2 = a^2$  using polar coordinates in double integrals.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of  $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$ . (8)

(ii) Using Cayley-Hamilton theorem, find  $A^{-1}$  if  $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ . (8)

Or

(b) Reduce the quadratic form  $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$  into the canonical form and hence find its rank, index, signature and nature. (16)

12. (a) (i) Let  $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x \leq 3 \\ (x - 3)^2 & \text{if } x > 3 \end{cases}$ . Evaluate each of the following limits, if they exist.

(1)  $\lim_{x \rightarrow 0^-} f(x)$

(2)  $\lim_{x \rightarrow 0^+} f(x)$

(3)  $\lim_{x \rightarrow 3^-} f(x)$

(4)  $\lim_{x \rightarrow 3^+} f(x)$

(5)  $\lim_{x \rightarrow 0} f(x)$

(6)  $\lim_{x \rightarrow 3} f(x)$

Also, find where  $f(x)$  is continuous. (8)

(ii) Find the  $n^{\text{th}}$  derivative of  $f(x) = xe^x$ . (4)

(iii) Differentiate  $F(t) = \frac{t^2}{\sqrt{t^3 + 1}}$ . (4)

Or

(b) (i) Use logarithmic differentiation to differentiate  $y = \frac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}$ . (8)

(ii) Discuss the curve  $f(x) = x^4 - 4x^3$  for points of inflection, and local maxima and minima. (8)

13. (a) (i) Given the transformations  $u = e^x \cos y$  and  $v = e^x \sin y$  and that  $f$  is a function of  $u$  and  $v$  and also of  $x$  and  $y$ , prove that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2) \left( \frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2} \right) \quad (8)$$

(ii) Expand  $e^x \cos y$  in a series of powers of  $x$  and  $y$  as far as the terms of the third degree. (8)

Or

(b) (i) Examine for extreme values of  $f(x, y) = x^3 + y^3 - 12x - 3y + 20$ . (8)

(ii) A rectangular box, open at the top is constructed so as to have a volume of 108 cubic meters. Find the dimensions of the box that requires the least material for its construction. (8)

14. (a) (i) Find a reduction formula for  $\int e^{ax} \sin^n x \, dx$ . (8)

(ii) Integrate the following :  $\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} \, dx$ . (8)

Or

(b) (i) Evaluate  $\int \sqrt{\frac{1-x}{1+x}} \, dx$ . (8)

(ii) Find the centre of mass of a semicircular plate of radius  $r$ . (8)

15. (a) (i) Change the order of integration in  $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} xy \, dy \, dx$  and then evaluate it. (8)

(ii) Find the area enclosed by the curves  $y = 2x - x^2$  and  $x - y = 0$ . (8)

Or

(b) (i) Find the volume of the tetrahedron bounded by the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$  and  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ . (8)

(ii) Find the moment of inertia of a hollow sphere about a diameter, given that its internal and external radii are 4 meters and 5 meters respectively. (8)