Diamond-Zodrow based model of supply side of economy

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Abstract

The note outlines the components of the supply-side of the OLG tax model

The Diamond-Zodrow model on which we will base the supply-side of our model is composed of four sectors, each producing a unique good that household have preferences over. These four sectors are the corporate goods producing sector, the non-corporate goods producing sector, the owner-occupied housing sector (home-owners are modeled as firms), and the rental housing sector. I just describe the corporate sector below, although all are very similar. After walking through the corporate model, I outline the differences between the corporate sector and the other three sectors of the production economy.

Each sector is composed of a representative firm that maximizes firm value (which is equivalent to discounted, after-tax profits). Firms are competitive and there is no uncertainty in the model.

1 Model Components: Variables and Parameters

1.1 Variables

There are six unique state variables, all of which are exogenous. Note that r_s is directly determined by i_s and the tax rate on interest income.

Table 1: State Variables

Variable	Description
K_s^C	Capital stock at the beginning of period s
$K_{s-1}^{\tau C}$	Tax basis of the capital stock at the beginning of period s
B_s^C	Debt at the beginning of period s
p_s^C	Price of corporate good in period s
w_s	Wage rate in period s
i_s	Nominal interest rate in period s
r_s	After tax nominal rate of return in period s

Note in $K_{s-1}^{\tau C}$ the $s-1$ subscript - depreciation in period s will be based on this and investment in period s.

There are 14 control variables, although all of these are trivial after the determination of I_s^C and EL_s^C.

*Thanks here.

Table 2: Control Variables

Variable	Description	Identifying Equation
I_s^C	Firm investment in period s	2.20
EL_s^C	Firm effective labor demand in period s	2.1
X_s^C	Corp goods produced	2.8
K_{s+1}^C	Firm's capital stock at the end of period s (beginning of period $s+1$)	2.10
$EARN_s^C$	Corp earnbefore deprec, corp taxes, and adjust costs, but after property taxes in period s	2.7
DIV_s^C	Corporate dividends in period s	2.14
TE_s^C	Total corporate income taxes in period s	2.12
Φ_s^C	Investment adjustment costs in period s	2.11
B_{s+1}^C	Corp debt at the end of period s (start of period $s+1$)	2.9
$K_s^{\tau C}$	Tax basis of capital under corp income tax at end of period s (beginning of $s+1$)	2.13
VN_s^C	New equity issued by the corp sector in period s	2.6
V_s^C	Firm value in period s	2.22
q_s^C	Marginal q (change in firm value per dollar of investment)	2.21
Q_s^C	Average Q	2.22

1.2 Parameters

The model has 19 parameters. Of these, 7 relate to the firm's production function, 2 to economic growth, 2 to firm financial policy, and 10 to tax policy.

2 Necessary equations

To solve the model, we want to get the optimal choices of labor and investment demand by firms. Labor demand is determined through an intratemporal trade off between the costs and benefits of labor. The necessary condition for the optimal choice of labor is:

$$p_s^C \frac{\partial F(K_s^C, EL_s^C)}{\partial EL_s^C} = w_s \tag{2.1}$$

Investment is more complicated, as it presents an intertemporal tradeoff between the costs of investment today and the benefits of a higher capital stock tomorrow. Once we have investment, all other endogenous variables follow from various accounting identities and assumptions on financial policies.

To derive the necessary conditions for investment, we need to first solve for the value of the firm as a function of the state variables (noted above) and the choice of investment. We do this by substituting in the various accounting identities to our equation for firm value.

Begin with the asset market equilibrium condition that the after-tax returns on all assets must be equalized if households simultaneously hold equity and bonds (and there is no aggregate uncertainty). The after-tax, nominal return on holding bonds is:

$$r_s = (1 - \tau_s^i) i_s, \tag{2.2}$$

Where i_s is the nominal interest rate on bonds. Thus the return on holding corporate equity must equal r_s in equilibrium:

$$r_s = (1 - \tau_s^i) i_s = \frac{(1 - \tau_s^d) DIV_s^C + (1 - \tau_s^g)(V_{s+1}^C - V_s^C - VN_s^C)}{V_s^C}, \tag{2.3}$$

Table 3: Model Parameters

Parameter	Description
Production Function	
γ_C	Capital weighting in CES production function
ϵ_C	Elasticity of substitution of capital for labor in CES production function
δ^C	Rate of economic depreciation on capital stock in the corporate sector
β^C	Scaling parameter for quadratic investment adjustment costs
μ_C	Steady-state investment rate
Economic Growth	
n	Rate of population growth (exogenous)
g	Rate of productivity growth (exogenous)
Financial Policy	
ζ^C	Fraction of earnings paid out in dividends
b^C	Debt/Capital ratio
Tax Policy	
τ_s^b	Corporate business income tax rate
$\delta_s^{\tau C}$	Rate of tax depreciation on corporate capital
τ_s^{pC}	Property tax rate on corporate capital
τ_s^i	Individual income tax rate on interest income
τ_s^g	Individual income tax rate on capital gains
f_e	Dummy variable for full expensing of investment
f_i	Dummy variable for deductibility of corporate interest paid
f_p	Dummy variable for deductibility of repayment of principle on loans
f_b	Dummy variable for inclusion of proceeds of loan in corp income tax base
f_d	Dummy variable for deductibility of depreciation expenses

where the first part of the numerator is the dividend returns from holding shares of the corporation and the second part are the capital gains returns from holding corporate equity, which are diluted by the issuance of new shares, VN_s^C.

We can rearrange this equation 2.3 to solve for V_{s+1}^C:

$$
\begin{aligned}
V_{s+1}^C &= \frac{V_s^C(1-\tau_s^i)i_s - (1-\tau_s^d)DIV_s^C}{(1-\tau_s^g)} + V_s^C + VN_s^C \\
&= V_s^C \underbrace{\left(1 + \frac{(1-\tau_s^i)i_s}{(1-\tau_s^g)}\right)}_{\text{Let this be } 1+\theta_s} + VN_s^C - \frac{(1-\tau_s^d)}{(1-\tau_s^g)}DIV_s^C
\end{aligned}
\tag{2.4}
$$

Now we can solve this for V_s^c by repeatedly substituting for V_{s+1}^C and applying the transversality condition ($\lim_{T\to\infty}\prod_{t=1}^{T}(1+\theta_t)V_T^C=0$):

$$
\begin{aligned}
V_s^C &= \frac{V_{s+1}^C}{(1+\theta_s)} - \frac{VN_s^C}{(1+\theta_s)} + \frac{\left(\frac{1-\tau_s^d}{1-\tau_s^g}\right)DIV_s^C}{(1+\theta_s)} \\
\Longrightarrow V_s^C &= \frac{V_{s+2}^C}{(1+\theta_s)(1+\theta_{s+1})} - \frac{VN_{s+1}^C}{(1+\theta_s)(1+\theta_{s+1})} + \frac{\left(\frac{1-\tau_s^d}{1-\tau_s^g}\right)DIV_{s+1}^C}{(1+\theta_s)(1+\theta_{s+1})} - \frac{VN_s^C}{(1+\theta_s)} + \frac{\left(\frac{1-\tau_s^d}{1-\tau_s^g}\right)DIV_s^C}{(1+\theta_s)} \\
\Longrightarrow V_s^C &= \frac{V_{s+3}^C}{(1+\theta_s)(1+\theta_{s+1})(1+\theta_{s+2})} - \frac{VN_{s+2}^C}{(1+\theta_s)(1+\theta_{s+1})(1+\theta_{s+2})} + \frac{\left(\frac{1-\tau_s^d}{1-\tau_s^g}\right)DIV_{s+2}^C}{(1+\theta_s)(1+\theta_{s+1})(1+\theta_{s+2})} \\
&\quad - \frac{VN_{s+1}^C}{(1+\theta_s)(1+\theta_{s+1})} + \frac{\left(\frac{1-\tau_s^d}{1-\tau_s^g}\right)DIV_{s+1}^C}{(1+\theta_s)(1+\theta_{s+1})} - \frac{VN_s^C}{(1+\theta_s)} + \frac{\left(\frac{1-\tau_s^d}{1-\tau_s^g}\right)DIV_s^C}{(1+\theta_s)} \\
&\text{and so on...} \\
\Longrightarrow V_s^C &= \underbrace{\prod_{\nu=s}^{\infty}\left(\frac{1}{1+\theta\nu}\right)V_\infty^C}_{=0 \text{ by transversality condition}} - \sum_{u=s}^{\infty}\prod_{\nu=s}^{u}\left(\frac{1}{1+\theta\nu}\right)\left[VN_u^C - \left(\frac{1-\tau_s^d}{1-\tau_s^g}\right)DIV_u^C\right] \\
\Longrightarrow V_s^C &= \sum_{u=s}^{\infty}\prod_{\nu=s}^{u}\left(\frac{1}{1+\theta\nu}\right)\left[\left(\frac{1-\tau_s^d}{1-\tau_s^g}\right)DIV_u^C - VN_u^C\right]
\end{aligned}
\tag{2.5}
$$

Now we have firm value as a functions discounted, after-tax value of dividends, less the discounted value of new shares issuance, which dilutes the value of the shares held at time s. We continue working towards writing firm value as a function of the state variable and the choice of investment and labor demand. First, we solve for VN_u^C. New shares issued in period u are given by the cash flow identity:

$$EARN_u^C + BN_u^C + VN_u^C = DIV_u^C + I_u^C(1+\Phi_u^C) + TE_u^C, \tag{2.6}$$

where $EARN_u^C$ are earnings before depreciation, corporate income taxes, and adjustment costs, but after property taxes; BN_u^C are new bond issues, I_u^C is investment, Phi_u^C are adjustment costs, and TE_u^C are total corporate income taxes (all in period u). These variable are determined as follows:

Earnings are determined by an accounting identity and the corporate production function.

$$EARN_u^C = p_u^C X_u^C - w_u EL_u^C - i_u B_u^C - \tau_u^{PC} K_u^C, \tag{2.7}$$

where output, X_u^C, is determined by a constant elasticity of substitution production function:

$$F(K_u^C, EL_u^C) = X_u^C = \left[(\gamma_C)^{1/\epsilon_C}(K_u^C)^{(\epsilon^C-1)/\epsilon_C} + (1-\gamma_C)^{1/\epsilon_C}(EL_u^C)^{(\epsilon_C-1)/\epsilon_C}\right]^{(\epsilon_C/(\epsilon_C-1))} \tag{2.8}$$

New debt issues are solved for by the assumption of a constant debt-to-capital ratio (and the law of motion for the capital stock):

$$BN_u^C = B_{u+1}^C - B_u^C \text{ and } B_u^C = b^C K_u^C \text{ by assumption} \tag{2.9}$$

The law of motion of the capital stock is given by:

$$K_{u+1}^C = (1-\delta^C)K_u^C + I_u^C \tag{2.10}$$

Adjustment costs are assumed to be a quadratic function of deviations from the steady-state investment rate:

$$\Phi_u^C = \frac{p_u^C\left(\frac{\beta^C}{2}\right)\left(\frac{I_u^C}{K_u^C} - \mu^C\right)^2}{\left(\frac{I_u^C}{K_u^C}\right)} \tag{2.11}$$

Corporate income taxes:

$$\begin{aligned} TE_u^C =& \tau_u^b\left[p_u^C X_u^C - w_u EL_u^C - f_e I_u^C - \Phi_u^c I_u^C - f_i i_u B_u^C - f_p \delta^C b^C K_u^C + f_b b^C I_u^C - f_d \delta^{\tau C} K_u^{\tau C} - \tau_u^{pC} K_u^C\right] \\ &+ \underbrace{\tau_u^{icC} I_u^C}_{\text{not in DZ (2013), added to account for investment tax credits as policy}}, \end{aligned} \tag{2.12}$$

Note that we are assuming that investment may or may not be deductible (depending upon the dummy variable f_e), but that investment adjustment costs are always deductible (i.e., they are not preceded by f_e). Under a pre-pay consumption tax system, investments are not deductible. Whether or not adjustment costs are deductible under a pre-pay consumption tax depends upon what you think these costs derive from. For example, if adjustment costs are from retraining employees to use new equipment, then these costs may be deductible under a consumption tax system (pre or post-pay) because they would likely be in the form of wage/labor costs. It's not clear how best to handle this and I believe the notion in Zodrow and Diamond (2013) is inconsistent on this point.
where the tax basis of the capital stock evolves according to:

$$K_{u+1}^{\tau C} = (1-\delta^{\tau C})(K_u^{\tau C} + (1-f_e)I_u^C) \tag{2.13}$$

Note how we form the law of motion for the tax basis. Zodrow and Diamond (2013) do not specify this, but the above formulation accounts for the fact that investment in year t receives a depreciation deduction in year t.[1] We can think about modifying this so that you get no deduction in the year the investment is made, which may or may not be more consistent with the "time to build" built into the law of motion for the physical capital stock.

Dividends are determined by the assumption that dividends are a constant fraction of pre-tax earnings.

$$DIV_u^C = \zeta^C(EARN_u^C - TE_u^C - p_u^C\delta^C K_u^C) \tag{2.14}$$

[1]What is actually used is a "half year rule", where you deduct the value of investment based on the assumption that it was put in place half-way through the year (so you get one half the annual deprecation rate on this new investment).

We will return to how investment, I_u^C, is determined, but first let us write the value of the firm as function of the states and the control variables I_s^C and EL_s^C. Substituting Equations 2.6 - 2.14 into Equation 2.5 (and letting $\Omega_u^C = 1 - \zeta^C + \zeta^C\left(\frac{1-\tau_u^d}{1-\tau_u^g}\right) = \left[\zeta^C(1-\tau_u^d) + (1-\zeta^C)(1-\tau_u^g)\right]/(1-\tau_u^g)$) we get:

$$
\begin{aligned}
V_s^C = &\sum_{u=s}^{\infty}\prod_{\nu=s}^{u}\left(\frac{1}{1+\theta\nu}\right)(1-\tau_u^b)\Omega_u^C(p_u^C X_u^C - w_s EL_s^C) \\
&- K_s^C\left\{(1-\tau_u^b)\Omega_u^C\tau_u^{pC} + (1-f_i\tau_u^i)i_u\Omega_u^C b^C - \delta^C(p_u^C - b^C - \Omega_u^C(p_u^C - f_p\tau_u^b b^C))\right\} \\
&- I_u^C\left\{1 - b^C + \Omega_u^C f_b\tau_u^b b^C - \Omega_u^C f_e\tau_u^b + (1-\Omega_u^C\tau_u^b)\Phi_u^C\right\} \\
&- \Omega_u^C f_d\tau_u^b\delta^{\tau C}K_u^{\tau C}
\end{aligned}
\tag{2.15}
$$

Note that $K_u^{\tau C}$ tracks depreciation deductions in in all periods $u = s, ..., \infty$. Future depreciation deductions on the tax basis of the capital stock in existence at time u do not affect investment decisions at time u (or forward) since the tax basis is predetermined.[2] However, future depreciation deductions for investments made at time u do affect investment decisions (since they lower the after-tax cost of investments). Therefore it's useful to distinguish between old and new capital.

The time u value of future depreciation deductions on the capital stock existing at the beginning of period u, $K_{u-1}^{\tau C}$, is given by:

$$
\begin{aligned}
f_d Z_u^C K_{u-1}^{\tau C} &= \sum_{j=u}^{\infty}\prod_{\nu=u}^{j}\left(\frac{1}{1+\theta_\nu)}\right) f_d\Omega_j^c\tau_j^b\delta^{\tau C}(1-\delta^{\tau C})^{j-u}K_u^{\tau C} \\
&= f_d K_{u-1}^{\tau C}\underbrace{\sum_{j=u}^{\infty}\prod_{\nu=u}^{j}\left(\frac{1}{1+\theta_\nu)}\right) f_d\Omega_j^c\tau_j^b\delta^{\tau C}(1-\delta^{\tau C})^{j-u}}_{Z_u^C}
\end{aligned}
\tag{2.16}
$$

We can derive the time u value of future depreciation deductions on investments made at time u, $I_u^{\tau C}$, similarly. These are given by $f_d(1-f_e)Z_u^C I_u^C$.

Thus we can rewrite Equation 2.15 as:

$$
\begin{aligned}
V_s^C = &\sum_{u=s}^{\infty}\prod_{\nu=s}^{u}\left(\frac{1}{1+\theta\nu}\right)(1-\tau_u^b)\Omega_u^C(p_u^C X_u^C - w_s EL_s^C) \\
&- K_s^C\left\{(1-\tau_u^b)\Omega_u^C\tau_u^{pC} + (1-f_i\tau_u^i)i_u\Omega_u^C b^C - \delta^C(p_u^C - b^C - \Omega_u^C(p_u^C - f_p\tau_u^b b^C))\right\} \\
&- I_u^C\left\{1 - b^C + \Omega_u^C f_b\tau_u^b b^C - \Omega_u^C f_e\tau_u^b - f_d(1-f_e)Z_u^C + (1-\Omega_u^C\tau_u^b)\Phi_u^C\right\} \\
&+ f_d Z_s^C K_{s-1}^{\tau C}
\end{aligned}
$$

[2] Note that if there were financial frictions (e.g. a borrowing constraint or costly external finance), then investment would be dependent on cash flow and would then be affected by changes in the value of deductions for the existing capital basis.

$$(2.17)$$

The Lagrangian to the firm's problem at time s can be written as:

$$\begin{aligned}
\mathcal{L}_s = \max_{\{I_u^c, K_{u+1}^C\}_{u=s}^{\infty}} \sum_{u=s}^{\infty} \prod_{\nu=s}^{u} \left(\frac{1}{1+\theta\nu}\right) & (1-\tau_u^b)\Omega_u^C(p_u^C X_u^C - w_s EL_s^C) \\
& - K_s^C \left\{(1-\tau_u^b)\Omega_u^C \tau_u^{pC} + (1-f_i\tau_u^i)i_u\Omega_u^C b^C - \delta^C(p_u^C - b^C - \Omega_u^C(p_u^C - f_p\tau_u^b b^C))\right\} \\
& - I_u^C \left\{1 - b^C + \Omega_u^C f_b\tau_u^b b^C - \Omega_u^C f_e\tau_u^b - f_d(1-f_e)Z_u^C + (1-\Omega_u^C\tau_u^b)\Phi_u^C\right\} \\
& + f_d Z_s^C K_{s-1}^{\tau C} + q_u^C((1-\delta^C)K_u^C + I_u^C - K_{u+1}^C)
\end{aligned} \tag{2.18}$$

The FOCs w.r.t. investment are given by:

$$\begin{aligned}
& \frac{\partial \mathcal{L}_s}{\partial I_u^C} = -\left\{1 - b^C + \Omega_u^C f_b\tau_u^b b^C - \Omega_u^C f_e\tau_u^b - f_d(1-f_e)Z_u^C + (1-\Omega_u^C\tau_u^b)\Phi_u^C\right\} - I_u^C(1-\Omega_u^C\tau_u^b)\frac{\partial \Phi_u^C}{\partial I_u^C} + q_u^C = 0 \\
& \Longrightarrow q_u^C = 1 - b^C + \Omega_u^C f_b\tau_u^b b^C - \Omega_u^C f_e\tau_u^b - f_d(1-f_e)Z_u^C + (1-\Omega_u^C\tau_u^b)\Phi_u^C + I_u^C(1-\Omega_u^C\tau_u^b)\frac{\partial \Phi_u^C}{\partial I_u^C} \\
& \Longrightarrow q_u^C = 1 - b^C - \Omega_u^C\tau_u^b(f_e - f_b b^C) - f_d(1-f_e)Z_u^C + (1-\Omega_u^C\tau_u^b)\Phi_u^C + I_u^C(1-\Omega_u^C\tau_u^b)\frac{\partial \Phi_u^C}{\partial I_u^C}
\end{aligned} \tag{2.19}$$

$$q_u^C = 1 - b^C - \Omega_u^C\tau_u^b(f_e - f_b b^C) - f_d(1-f_e)Z_u^C + (1-\Omega_u^C\tau_u^b)\Phi_u^C + I_u^C(1-\Omega_u^C\tau_u^b)\frac{\partial \Phi_u^C}{\partial I_u^C} \tag{2.20}$$

q_u^C is Tobin's q or the marginal change in firm value for a dollar of investment (which is to say it's the shadow price of investment). The FOC for investment says that the firm invests until the marginal benefit (the LHS of Equation 2.20) is equal to the marginal cost of investment (the RHS of Equation 2.20). The cost of investment in the absence of taxes and frictions is equal to 1 (the first term on the RHS of Equation 2.20) since investment goods are the numeraire. The second term reflects the reduction in the cost of debt due to debt financing. The third term on the RHS of Equation 2.20 is the change in the cost of capital due to debt being included or excluded from corporate income taxes. The fourth term reflects the reduction in the cost of debt due to depreciation deductions. The last term reflects the costs of capital that are due to adjustment costs (net of the expensing of adjustment costs for tax purposes).

The FOCs w.r.t. capital one period ahead are given by:

$$\frac{\partial \mathcal{L}_s}{\partial K^C_{u+1}} = \prod_{\nu=s}^{u}\left(\frac{1}{1+\theta\nu}\right)[-q^C_u]$$
$$+\prod_{\nu=s}^{u+1}\left(\frac{1}{1+\theta\nu}\right)\Bigg[(1-\delta^C)q^C_{u+1} + p^C_{u+1}\frac{\partial X^C_{u+1}}{\partial K^C_{u+1}} - \{(1-\tau^b_{u+1})\Omega^C_{u+1}\tau^{pC}_{u+1} + (1-f_i\tau^i_{u+1})i_{u+1}\Omega^C_{u+1}b^C - \delta^C(p^C_{u+1} -$$
$$\implies q^C_u = \left(\frac{1}{1+\theta_{u+1}}\right)$$
$$\Bigg[(1-\delta^C)q^C_{u+1} + p^C_{u+1}\frac{\partial X^C_{u+1}}{\partial K^C_{u+1}} - \{(1-\tau^b_{u+1})\Omega^C_{u+1}\tau^{pC}_{u+1} + (1-f_i\tau^i_{u+1})i_u\Omega^C_{u+1}b^C - \delta^C(p^C_{u+1} - b^C - \Omega^C_{u+1}(p^C_{u+1} -$$
$$(2.21)$$

We should be able to solve for I^C_u, K^C_{u+1}, and q^C_u with Equations 2.10, 2.20, and 2.21. We can then use q^C_u to solve for V^C_u as we show next.

As Hayashi (1982) shows, with a constant returns to scale production function and quadratic adjustment costs, we can device that marginal q is equal to average q. Note that in our case, we must make an adjustment for the value of depreciation deductions on the tax basis of the capital stock. Relation between marginal q, q^C_u, and average q, Q^C_u:

$$q^C_u = \frac{[V^C_u - f_d Z^C_u K^{\tau C}_{u-1}]}{K^C_u} \text{ and } Q^C_u = \frac{V^C_u}{K^C_u} \tag{2.22}$$

3 Solving the model

We'll solve the model in two steps. First, we solve for the steady state prices and allocations. Next, we iterate backwards solving for prices and allocations along the transition path to the steady state.

3.1 Solving for the steady state

On the supply side (with one sector), we have to solve for the factor prices, $\bar{i}$ and $barw$ (the price of output $\bar{p}^C$ is normalized to one), the shadow price of capital, $\bar{q}^C$, and the allocations $\bar{EL}^C$, $\bar{K}^C$, $\bar{I}^C$. From these all the other variables follow trivially.

Start by solving for the steady-states of Equations 2.20 and 2.21. Equation 2.20 becomes:

$$\bar{q}^C = \underbrace{1 - b^C - \bar{\Omega}^C\bar{\tau}^b(f_e - f_b b^C) - f_d(1-f_e)\bar{Z}^C}_{\text{function of only parameters}} \tag{3.1}$$

This yields the solution to $\bar{q}^C$.

Next, consider the steady-state of Equation 2.21:

$$\bar{q}^C = \frac{1}{1+\bar{\theta}}\left[(1-\delta^C)\bar{q}^C + \frac{\partial \bar{X}^C}{\partial \bar{K}^C} - \{(1-\bar{\tau}^b)\bar{\Omega}^C\bar{\tau}^{pC} + (1-f_i\bar{\tau}^i)\bar{i}\bar{\Omega}^C b^C - \delta^C(1-b^C-\bar{\Omega}^C(1-f_p\bar{\tau}^b b^C))\}\right] \tag{3.2}$$

We can rearrange this and solve for the steady-state marginal product of capital in sector C:

$$\frac{\partial \bar{X}^C}{\partial \bar{K}^C} = (\bar{\theta}+\delta^C)\bar{q}^C + (1-\bar{\tau}^b)\bar{\Omega}^C\bar{\tau}^{pC} + (1-f_i\bar{\tau}^i)\bar{i}\bar{\Omega}^C b^C - \delta^C(1-b^C-\bar{\Omega}^C(1-f_p\bar{\tau}^b b^C)) \quad (3.3)$$

Notice that given Equation 3.1, the RHS to the above equation is function of parameters and the steady state nominal interest rate, $\bar{i}$. The LHS of the equation is a function of $\bar{K}^C$ and $\bar{EL}^C$.

I think we can use the following to identify the SS values of the variables of interest:

1. $\bar{i}$ will be determined by the SS of the household's Euler equations (I think this can be done as described in the HH sol'n method)
2. $\bar{w}$ will be determined by the SS of the household's FOCs for labor supply ((I think this can be done as described in the HH sol'n method)
3. $\bar{q}^C$ is determined by Equation 3.1
4. $\bar{EL}^C$ is determined by the SS version of Equation 2.1, plus $\bar{w}$
5. $\bar{K}^C$ is determined by Equation 3.3 and $\bar{i}$
6. $\bar{I}^C$ is then solved for using the steady state law of motion for capital $\implies \bar{I}^C = \delta^C\bar{K}^C$

In solving for $\bar{EL}^C$ and $\bar{K}^C$, note that we'll have use the MPK and the MPL simultaneously. Given our production function, we have:

$$\frac{\partial X_u^C}{\partial K_u^C} = \left[(\gamma_C)^{1/\epsilon_C}(K_u^C)^{(\epsilon_C-1)/\epsilon_C} + (1-\gamma_C)^{1/\epsilon_C}(EL_u^C)^{(\epsilon_C-1)/\epsilon_C}\right]^{1/(\epsilon_C-1)}(\gamma_C)^{1/\epsilon_C}(K_u^C)^{-1/\epsilon_C} \quad (3.4)$$

and

$$\frac{\partial X_u^C}{\partial EL_u^C} = \left[(\gamma_C)^{1/\epsilon_C}(K_u^C)^{(\epsilon_C-1)/\epsilon_C} + (1-\gamma_C)^{1/\epsilon_C}(EL_u^C)^{(\epsilon_C-1)/\epsilon_C}\right]^{1/(\epsilon_C-1)}(1-\gamma_C)^{1/\epsilon_C}(EL_u^C)^{-1/\epsilon_C} \quad (3.5)$$

We know that, at an optimum, the marginal revenue product of labor equals the wage rate, and the marginal revenue product of capital equals a function of the interest rate, marginal q, and the model parameters. Call this function $g(i_u, q_u^C, q_{u-1}^C, \Theta)$. We thus have $p_u^C\frac{\partial X_u^C}{\partial EL_u^C} = w_u$ and $p_u^C\frac{\partial X_u^C}{\partial K_u^C} = g(i_u, q_u^C, q_{u-1}^C, \Theta)$. Dividing these two equations, we have:

$$\begin{aligned}
\frac{\frac{\partial X_u^C}{\partial K_u^C}}{\frac{\partial X_u^C}{\partial EL_u^C}} &= \frac{(\gamma_C)^{1/\epsilon_C}(K_u^C)^{-1/\epsilon_C}}{(1-\gamma_C)^{1/\epsilon_C}(EL_u^C)^{-1/\epsilon_C}} - = \frac{g(i_u, q_u^C, q_{u-1}^C, \Theta)}{w_u} \\
\implies \frac{K_u^C}{EL_u^C} &= \frac{(1-\gamma_C)}{\gamma_C}\left(\frac{w_u}{g(i_u, q_u^C, q_{u-1}^C, \Theta)}\right)^{\epsilon_C}
\end{aligned} \quad (3.6)$$

We can use the SS version of Equation 3.6 to solve for capital as function of labor (and $\bar{q}, \bar{i}, \bar{w}$), and then use that in the SS version of Equation 3.5 to solve for labor as s function of $\bar{q}, \bar{i}, \bar{w}$. We then go back to the SS version of Equation 3.6 to get the SS choice of capital as a function of $\bar{q}, \bar{i}, \bar{w}$.

All of the above will work for each sector in a model with any number of sectors (though care has to be taken to include the prices of output and capital in those other sectors, since only one sector's output can be the numeraire).

3.2 Solving for the transition path

I believe we can just use the Euler equations to go backwards in time, from the SS back along the transition path to $t = 0$. Assume period T is the SS, The solution would look like the following:

1. Use Equation 2.21 to solve for the for q^C_{T-1} since we have the solution to the RHS of the equation after we've solved for the SS.
2. Use the law of motion for capital to find: $K^C_{T-1} = \frac{K^C_T - I^C_{T-1}}{(1-\delta^C)} = \frac{\bar{K}^C - I^C_{T-1}}{(1-\delta^C)}$
3. Use Equation 2.20 and the value of q^C_{T-1} to find I^C_{T-1} (and K^C_{T-1} given the law of motion relationship.
4. Given w_{T-1} we can use the FOC for labor demand to find EL^C_{T-1}
5. Given i_{T-1} we can use Equation 2.21 to solve for q^C_{T-2}
6. We then repeat the above steps until we work back to $t = 0$.

4 Features model has and those it is lacking

Table 4: Tax Distortions in DZ Model

Distortion	Accounted for in DZ model?
Amount of investment	Mostly accounted for
Entity form	Not directly accounted for.
Location of capital	Not in baseline model.
Type of investment (equip/structures/intangible)	No - just aggregate capital stock
Bias towards certain industries (because of type of capital or income risk)	No
Bias towards non-risky projects (due to tax loss asymmetry)	No
Bias towards non-risky businesses (due to tax loss asymmetry)	No
Double tax of profits (affects several distortions)	Yes, partially.
Dividend distribution policy	Not really.
Where recognize income (US or abroad)	No
Repatriate income (and when)	No

5 Roadmap for extensions

Possible order or model extensions:

1. Add industries/goods

Table 5: Tax Policy Instruments in DZ Model

Instrument	In DZ?	Large macro effects	Likely corp reform candidate
Corporate income tax top rate	Yes	Yes	Yes
Corporate income tax rate structure	No	No	Yes
Capital gains tax rate	Yes	Yes	
Dividend income tax rate	Yes	Yes	Yes
Depreciation rate structure	Yes	Yes	
Bonus deprecation/expensing	No	Yes	
Investment tax credits	No	Yes	Yes
General business credits	No	Prob not after above	Yes
Cap/Deny/Index for inflation interest expenses	No	Yes	Yes
Carry back/forward window	No	Maybe	
Inventory accounting rules (LIFO/FIFO)	No	Probably not	Yes
Repatriation holidays	No	Maybe	Yes
International tax system (Territorial vs Worldwide, deferral)	No	Yes	Yes
Consumption tax system (w/ pre and post pay)	Yes	Yes	No

  - Try to do this along the lines of Fullerton and Rogers (1993) with composite consumption and production goods. Important additions might include health care and a carbon intensive sector (e.g. utilities, transport). Important goods would be health services, large excise items (gasoline, alcohol, cigarettes), carbon intensive goods (e.g. utilities, transport), food
  - Maybe be costly to solve firm problem for many sectors. GE price vector may be large too, but Fullerton and Rogers (1993)) suggest that still just w and r (for each year) that need

2. Add types of capital
   - Try to do this along the lines of Fullerton and Rogers (1993) with composite capital in firm production function, but where capital can change type costlessly.
   - Not sure how/if this works in dynamic model where keep track of old capital.
3. Add profits via some markup
   - Want this so have supranormal returns, which are differentially affected by taxes.
   - See macro models of the markup, but want to just get markup that is a function of the elasticity of substitution.
4. Endogenize debt finance.
   - Have a cost to bankruptcy - use debt until this cost negates the tax advantage.
5. Endogenize payout policy
   - Want to at have dividends respond to dividend tax rate (e.g. by at least being done cash after investment made - since invest a function of div tax rate). This is easy. Harder I think is to fully endogenize so that firm considers the value of dividends to owners after tax vs the value of the dollar inside the firm.
6. Open economy
   - First thing to do here is to have some capital mobility in the model.

7. Have multiple types of labor (skilled/unskilled) in production function
   - Only if have endogenous human capital accumulation.
   - Would like to see how taxes on capital affect capital/labor mix to uncover distribution of incidence of taxes by industry and individuals.

Some features that might be interesting, but may not pass a cost-benefit test to adding into the model:

1. Stochastic profitability shocks
   - So can account for loss carry forward/back.
   - Would need to have idiosyncratic shocks over firms in an industry.
   - Think we'd have to solve the firm problem a lot more times and not sure how this interacts in GE with different type of capital etc.
2. Have a more serious model of income shifting by having some behavior where move profits offshore
   - Not sure how to do even in open economy model.
3. Endogenize entity choice.
   - Makes firm problem much harder.
   - Not sure how to calibrate elasticities, but Prisinzano and Pearce at OTA might have some estimates to help.
4. Pollution externalities
   - It'd be cool to be able to do some policy experiments with Pigouvian taxes like **?**

   .
5. Model evasion and avoidance
   - Perhaps just have some elasticity for reported income with respect to the marginal tax rate that scales actual income.
   - DeBacker, Heim, Tran, and Yuskavage can measure with audit data.

References

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