Dyanmic General Equilibrim Tax Scoring Model *

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Abstract

This paper ...

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1 Introduction

2 Details of the Macro Model

2.1 Deomgraphics

A measure $\omega_{1,t}$ of individuals with heterogeneous working ability $e \in \mathcal{E} \subset \mathbb{R}_{++}$ is born in each period t and live for $E+S$ periods, with $S \geq 4$.[1] The population of age-s individuals in any period t is $\omega_{s,t}$. Households are termed "youth" and out of the market during ages $1 \leq s \leq E$. The households enter the workforce and economy in period $E+1$ and remain in the workforce until they unexpectedly die or live until age $s = E+S$.[2] The population of agents of each age in each period $\omega_{s,t}$ evolves according to the following function,

$$\begin{aligned} \omega_{1,t+1} &= \sum_{s=1}^{E+S} f_s \omega_{s,t} \quad \forall t \\ \omega_{s+1,t+1} &= (1 + i_s - \rho_s)\omega_{s,t} \quad \forall t \quad \text{and} \quad 1 \leq s \leq E+S-1 \end{aligned} \tag{1}$$

where $f_s \geq 0$ is an age-specific fertility rate, i_s is an age-specific immigration rate, ρ_s is an age specific mortality hazard rate,[3] and $1 + i_s - \rho_s$ is constrained to be nonnegative. The total population in the economy N_t at any period is simply the sum of individuals in the economy, the population growth rate in any period t from the previous period $t-1$ is $g_{n,t}$, $\tilde{N}_t$ is the working age population, and $\tilde{g}_{n,t}$ is the working age population growth rate in any period t from the previous period $t-1$.

$$N_t \equiv \sum_{s=1}^{E+S} \omega_{s,t} \quad \forall t \tag{2}$$

[1]Theoretically, the model exposition of the model works without loss of generality for $S \geq 3$. However, because we are calibrating the ages outside of the economy to be one-fourth of S (e.g., ages 21 to 100 in the economy, and ages 1 to 20 outside of the economy), we need S to be at least 4.

[2]We model the population with households age $s \leq E$ outside of the workforce and economy in order most closely match the empirical population dynamics.

[3]The parameter ρ_s is the probability that a household of age s dies before age $s+1$.

$$g_{n,t+1} \equiv \frac{N_{t+1}}{N_t} - 1 \quad \forall t \tag{3}$$

$$\tilde{N}_t \equiv \sum_{s=E+1}^{E+S} \omega_{s,t} \quad \forall t \tag{4}$$

$$\tilde{g}_{n,t+1} \equiv \frac{\tilde{N}_{t+1}}{\tilde{N}_t} - 1 \quad \forall t \tag{5}$$

2.2 Households

The consumer's maximization problem is:

$$\begin{aligned}
&U_{j,s,t} = \sum_{u=0}^{E+S-s} \beta^u \left[\prod_{v=s-1}^{s+u-1} (1-\rho_v)\right] u\left(c_{j,s+u,t+u}, n_{j,s+u,t+u}, b_{j,s+u+1,t+u+1}\right) \\
&\text{where} \quad \rho_{s-1} = 0 \\
&\text{and} \quad u\left(c_{j,s,t}, n_{j,s,t}, b_{j,s+1,t+1}\right) = \frac{\left(c_{j,s,t}\right)^{1-\sigma} - 1}{1-\sigma} ... \\
&\qquad + e^{g_y t(1-\sigma)} \chi^n_s \left(b\left[1 - \left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1}{\upsilon}} + k\right) + \rho_s \chi^b \frac{\left(b_{j,s+1,t+1}\right)^{1-\sigma} - 1}{1-\sigma} \\
&\text{and} \quad c_{j,s,t} = \prod_{i=1}^{I} \left(c_{i,j,s,t} - \bar{c}_{i,s}\right)^{\alpha_i}; \quad \sum_{i=1}^{I} \alpha_i = 1 \\
&\qquad \forall j,t \quad \text{and } E+1 \leq s \leq E+S
\end{aligned} \tag{6}$$

They maximize subject to the following budget constraint.

$$\begin{aligned}
&\sum_{i=1}^{I} p_{i,t} c_{i,j,s,t} + b_{j,s+1,t+1} \leq \left(1+r_t\right) b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \frac{BQ_{j,t}}{\lambda_j \tilde{N}_t} - T_{j,s,t} \\
&\qquad \text{where} \quad b_{j,s,1} = 0 \\
&\qquad \text{for} \quad E+1 \leq s \leq E+S \quad \forall j,t
\end{aligned} \tag{7}$$

We set up a Lagrangian and solve by taking derivatives with respect to $\{c_{i,j,s,t}, n_{j,s,t+u}, b_{j,s,t+1}\}$ for all i, j, s and t.

With respect to each consumption good i:

$$\begin{aligned}\frac{\partial U}{\partial c_{i,j,s+u,t+u}} = \beta^u \left[\prod_{v=s-1}^{s+u-1}(1-\rho_v)\right]\left[\prod_{i=1}^{I}(c_{i,j,s,t}-\bar{c}_{i,s})^{\alpha_i}\right]^{-\sigma} \alpha_i\left(c_{i,j,s,t}-\bar{c}_{i,s}\right)^{\alpha_i-1} \\ -\lambda_{t+u}\left(p_{i,t+u}+\frac{\partial T_{j,s+u,t+u}}{\partial c_{i,j,s+u,t+u}}\right) = 0\end{aligned} \tag{8}$$

With respect to labor:

$$\begin{aligned}\frac{\partial U}{\partial n_{j,s+u,t+u}} = \beta^u \left[\prod_{v=s-1}^{s+u-1}(1-\rho_v)\right] e^{g_y(t+u)(1-\sigma)}\chi_s^n\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{j,s+u,t+u}}{\tilde{l}}\right)^{v-1}\left[1-\left(\frac{n_{j,s+u,t+u}}{\tilde{l}}\right)\right]^{\frac{1-v}{v}} \\ -\lambda_{t+u}\left(w_{+u}e_{j,s+u}-\frac{\partial T_{j,s+u,t+u}}{\partial n_{j,s+u,t+u}}\right) = 0\end{aligned} \tag{9}$$

With respect to savings:

$$\begin{aligned}\frac{\partial U}{\partial b_{j,s+u+1,t+u+1}} = \beta^u \left[\prod_{v=s-1}^{s+u-1}(1-\rho_v)\right]\rho_s\chi^b\big(b_{j,s+U+1,t+U+1}\big)^{-\sigma} \\ -\lambda_{t+u}-\lambda_{t+u+1}\left(1+r_{t+u+1}-\frac{\partial T_{j,s+U+1,t+U+1}}{\partial b_{j,s+U+1,t+u+1}}\right) = 0\end{aligned} \tag{10}$$

We can solve each of these for λ_{t+u} to get the following.

$$\lambda_{t+u} = \frac{\beta^u\left[\prod_{v=s-1}^{s+u-1}(1-\rho_v)\right]\left[\prod_{i=1}^{I}(c_{i,j,s,t}-\bar{c}_{i,s})^{\alpha_i}\right]^{-\sigma}\alpha_i\left(c_{i,j,s,t}-\bar{c}_{i,s}\right)^{\alpha_i-1}}{p_{i,t+u}+\frac{\partial T_{j,s+u,t+u}}{\partial c_{i,j,s+u,t+u}}}$$

$$\lambda_{t+u} = \frac{\beta^u\left[\prod_{v=s-1}^{s+u-1}(1-\rho_v)\right]e^{g_y(t+u)(1-\sigma)}\chi_s^n\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{j,s+u,t+u}}{\tilde{l}}\right)^{v-1}\left[1-\left(\frac{n_{j,s+u,t+u}}{\tilde{l}}\right)\right]^{\frac{1-v}{v}}}{w_{+u}e_{j,s+u}-\frac{\partial T_{j,s+u,t+u}}{\partial n_{j,s+u,t+u}}}$$

$$\lambda_{t+u} = \beta^u \left[\prod_{v=s-1}^{s+u-1} (1-\rho_v)\right] \rho_s \chi^b \big(b_{j,s+U+1,t+U+1}\big)^{-\sigma} - \lambda_{t+u+1} \left(1 + r_{t+u+1} - \frac{\partial T_{j,s+U+1,t+U+1}}{\partial b_{j,s+U+1,t+u+1}}\right)$$

These then reduce to the following $I+1$ Euler equations for each j, s and t:

Marginal utility of consumption for each good i compared to the marginal utility of labor:

$$\frac{\left[\prod_{i=1}^{I} \left(c_{i,j,s,t} - \bar{c}_{i,s}\right)^{\alpha_i}\right]^{-\sigma} \alpha_i \left(c_{i,j,s,t} - \bar{c}_{i,s}\right)^{\alpha_i - 1}}{p_{i,t+u} + \frac{\partial T_{j,s+u,t+u}}{\partial c_{i,j,s+u,t+u}}} = \frac{e^{g_y(t+u)(1-\sigma)} \chi^n_s \left(\frac{b}{\tilde{l}}\right) \left(\frac{n_{j,s+u,t+u}}{\tilde{l}}\right)^{v-1} \left[1 - \left(\frac{n_{j,s+u,t+u}}{\tilde{l}}\right)\right]^{\frac{1-v}{v}}}{w_{t+u} e_{j,s+u} - \frac{\partial T_{j,s+u,t+u}}{\partial n_{j,s+u,t+u}}} \tag{11}$$

Intertemporal Euler equation for savings, including the utility effects of bequests:

$$\frac{e^{g_y(t+u)(1-\sigma)} \chi^n_s \left(\frac{b}{\tilde{l}}\right) \left(\frac{n_{j,s+u,t+u}}{\tilde{l}}\right)^{v-1} \left[1 - \left(\frac{n_{j,s+u,t+u}}{\tilde{l}}\right)\right]^{\frac{1-v}{v}}}{w_{t+u} e_{j,s+u} - \frac{\partial T_{j,s+u,t+u}}{\partial n_{j,s+u,t+u}}} = \rho_s \chi^b \big(b_{j,s+U+1,t+U+1}\big)^{-\sigma}$$
$$- \frac{\beta(1-\rho_{s+u}) e^{g_y(t+u+1)(1-\sigma)} \chi^n_s \left(\frac{b}{\tilde{l}}\right) \left(\frac{n_{j,s+u+1,t+u+1}}{\tilde{l}}\right)^{v-1} \left[1 - \left(\frac{n_{j,s+u+1,t+u+1}}{\tilde{l}}\right)\right]^{\frac{1-v}{v}}}{w_{t+u+1} e_{j,s+u+1} - \frac{\partial T_{j,s+u+1,t+u+1}}{\partial n_{j,s+u+1,t+u+1}}} \times$$
$$\left(1 + r_{t+u+1} - \frac{\partial T_{j,s+U+1,t+U+1}}{\partial b_{j,s+U+1,t+u+1}}\right) \tag{12}$$

An Euler equation that compares marginal utilities of two arbitrary goods (n & m) is given below.

$$\frac{\alpha_n \left(c_{n,j,s,t} - \bar{c}_{n,s}\right)^{\alpha_n - 1}}{p_{n,t+u} + \frac{\partial T_{j,s,t}}{\partial c_{n,j,s,t}}} = \frac{\alpha_m \left(c_{m,j,s,t} - \bar{c}_{m,s}\right)^{\alpha_m - 1}}{p_{m,t+u} + \frac{\partial T_{j,s,t}}{\partial c_{m,j,s,t}}} \tag{13}$$

We can use this equation for $m \in \{1, 2, ..., I\}$ solving for $c_{m,j,s,t} - \bar{c}_{m,s}$.

$$c_{m,j,s,t} - \bar{c}_{m,s} = \left[(c_{n,j,s,t} - \bar{c}_{n,s})^{1-\alpha_n} \frac{\Gamma_m}{\Gamma_n} \right]^{\frac{1}{1-\alpha_m}} \tag{14}$$
$$\Gamma_m \equiv \frac{\alpha_m}{p_{m,t} + \frac{\partial T_{j,s,t}}{\partial c_{m,j,s,t}}}$$

We can solve the household's problem fairly rapidly, if the values of $\frac{\partial T_{j,s,t}}{\partial c_{i,j,s,t}}$ are just constants, as they would be with a typical sales tax.

- First, given $\{p_{i,t}\}_{i=1}^{I}$ use Euler equation (11) to find the value for $c_{1,j,s,t}$.
- Then use equation (14) to get the rest of the $c_{i,j,s,t}$'s.

2.3 Production Goods Firms

2.3.1 Jason's Notes

Firms maximize firm value, which is the net present value of dividends less equity issuance:

$$V_t = \sum u = t^{\infty} \prod_{\nu=t} u \left(\frac{1}{1 + r_\nu} \right) DIV_u - VN_u, \tag{15}$$

where DIV_u are dividend distributions in period u and VN is new equity issuance in period u. The firm's cash flow constraint will give us the value of dividends distributed after investment and earnings (a function of capital and labor) are determined:

$$EARN_u + VN_u = DIV_u + I_u \tag{16}$$

Here, I_u is investment in capital in period u (where we have the price of capital normalized to 1). Earnings are defined as revenues from the sale of production goods less the price of variable inputs (i.e., labor):

$$EARN_u = p_u F(K_u, L_u) - w_u L_u \tag{17}$$

Plugging Equation 17 and the law of motion for the capital stock into Equation 16 yields:

$$pF(K_u, L_u) - w_u L_u + VN_u = DIV_u + K_{u+1} - (1-\delta)K_u \tag{18}$$

We can not find the Belman Equation for the firm's problem by solving for DIV from Equation 19 and substituting the result into Equation 15:

$$V(K; r, w) = pF(K, L) - wL - K' + (1-\delta)K + \frac{1}{1+r}V(K'; r', w') \tag{19}$$

The two FOCs are:

$$\frac{\partial V(K; r, w)}{\partial K'} : 1 = \frac{1}{1+r}\frac{\partial V(K'; r', w')}{\partial K'} \tag{20}$$

$$\frac{\partial V(K; r, w)}{\partial L} : w = \frac{\partial V(K; r, w)}{\partial L} \tag{21}$$

The envelope condition allows us to write 20 as:

$$\frac{\partial V(K; r, w)}{\partial K'} : 1 = \frac{1}{1+r}\left[\frac{\partial F(K', L')}{\partial K'} + 1 - \delta\right] \tag{22}$$

2.3.2 Parameterization

We will assume that the production function for the firm is a Constant Elasticity of Substitution (CES) function:

$$F(K, L) = \left(\gamma^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{23}$$

where ε is the elasticity of substation between capital and labor and γ is the share parameter for the production function (?).

Given this parameterization, our two FOCs become:

$$r + \delta = \left(\gamma^{\frac{1}{\varepsilon}} K'^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{\frac{1}{\varepsilon}} L'^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} \gamma^{\frac{1}{\varepsilon}} K'^{\frac{-1}{\varepsilon}} \tag{24}$$

(double check the timing on the interest rate - not sure if it should be the current period or one period ahead - depends upon the timing convention for our notation)

$$w = \left(\gamma^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}} (1-\gamma)^{\frac{1}{\varepsilon}} L^{\frac{-1}{\varepsilon}} \tag{25}$$

I think you should just be able to substitute in these two firm FOCs for the static firm FOCs. The price out capital and consumption are the same since they are the same good - and we can normalize their price to 1. Since firms hold capital, we won't have a capital market cleaning condition (I don't think). But we will have an asset market clearing condition. This will be that $B_t = V_t$. We should be able to solve the infinite geometric series to get the value of $V(\bar{K})$:

$$\begin{aligned} V(\bar{K};\bar{r},\bar{w}) &= F(\bar{K},\bar{L}) - \bar{w}\bar{L} - \delta\bar{K} + \frac{1}{1+\bar{r}}V(\bar{K};\bar{r},\bar{w}) \\ \implies V(\bar{K};\bar{r},\bar{w}) &= \frac{F(\bar{K},\bar{L}) - \bar{w}\bar{L} - \delta\bar{K}}{\bar{r}}(1+\bar{r}) \end{aligned} \tag{26}$$

To solve for V_t outside of the SS, we'll have to use backwards induction. So, one period before the SS, we have:

$$V(K_{T-1};r_{T-1},w_{T-1}) = F(K_{T-1},L_{T-1}) - w_{T-1}L_{T-1} - \bar{K} + (1-\delta)K_{T-1} + \frac{1}{1+\bar{r}}V(\bar{K};\bar{r},\bar{w}) \tag{27}$$

(Again, check the timing for the interest rate).

One can then keep iterating backwards from the steady state to the initial period using the recursive relationship as described in Equation 27.

2.3.3 Kerk's Notes

Denote the stock of bonds issued by the firm as b^F. T^F denotes taxes on the firm.

Firm ι's maximization problem is:

$$
\begin{aligned}
& V_\iota(k^F_{\iota,t}, b^F_{\iota,t}, \Omega_t) \\
&= \max_{n^F_{\iota,t}, k^F_{\iota,t+1}, b^F_{\iota,t+1}} e^F_{\iota,t} + \frac{1}{1+r_{t+1}} V_\iota(k^F_{\iota,t+1}, b^F_{\iota,t+1}, \epsilon^F_{\iota,t}, \Omega_{t+1})
\end{aligned} \tag{28}
$$

$$
e^F_{\iota,t} = p_{\iota,t} y^F_{\iota,t} - w_t n^F_{\iota,t} + (1-\delta) k^F_{\iota,t} - (1+r_t) b^F_{\iota,t} - k^F_{\iota,t+1} + b^F_{\iota,t+1} - T^F_{\iota,t} \tag{29}
$$

$$
y^F_{\iota,t} = f(z_t, k^F_{\iota,t}, n^F_{\iota,t}) = z_t \left[\zeta (k^F_{\iota,t})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1}{\eta}} \tag{30}
$$

The first-order conditions are:

$$
\begin{aligned}
& p_{\iota,t} z_t \left[\zeta (k^F_{\iota,t})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1-\eta}{\eta}} (1-\zeta)(n^F_{\iota,t})^{\eta-1} - w_t - \frac{\partial T^F_{\iota,t}}{\partial n^F_{\iota,t}} = 0 \\
& -1 - \frac{\partial T^F_{\iota,t}}{\partial k^F_{\iota,t+1}} + \frac{1}{1+r_{t+1}} \frac{\partial V_\iota}{\partial k^F_\iota}(t+1) = 0 \\
& 1 - \frac{\partial T^F_{\iota,t}}{\partial b^F_{\iota,t+1}} + \frac{1}{1+r_{t+1}} \frac{\partial V_\iota}{\partial b^F_\iota}(t+1) = 0
\end{aligned}
$$

Envelope conditions are:

$$
\begin{aligned}
& \frac{\partial V_\iota}{\partial k^F_\iota}(t) = p_{\iota,t} z_t \left[\zeta (k^F_{\iota,t})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1-\eta}{\eta}} \zeta (k^F_{\iota,t})^{\eta-1} + 1 - \delta - \frac{\partial T^F_{\iota,t}}{\partial k^F_{\iota,t}} \\
& \frac{\partial V_\iota}{\partial b^F_\iota}(t) = -\left(1 + r_t + \frac{\partial T^F_{\iota,t}}{\partial b^F_{\iota,t}}\right)
\end{aligned}
$$

Euler equations are:

$$
p_{\iota,t} z_t \left[\zeta (k^F_{\iota,t})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1-\eta}{\eta}} (1-\zeta)(n^F_{\iota,t})^{\eta-1} = w_t + \frac{\partial T^F_{\iota,t+1}}{\partial n^F_{\iota,t+1}} \tag{31}
$$

$$
\begin{aligned}
& p_{\iota,t+1} z_{t+1} \left[\zeta (k^F_{\iota,t+1})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1-\eta}{\eta}} \zeta (k^F_{\iota,t+1})^{\eta-1} \\
&= \left(1 + \frac{\partial T^F_{\iota,t+1}}{\partial k^F_{\iota,t+1}}\right)(1+r_{t+1}) - 1 - \delta - \frac{\partial T^F_{\iota,t}}{\partial k^F_{\iota,t+1}}
\end{aligned} \tag{32}
$$

$$
r_{t+1} = -\frac{\frac{\partial T^F_{\iota,t+1}}{\partial b^F_{\iota,t+1}} + \frac{\partial T^F_{\iota,t}}{\partial b^F_{\iota,t+1}}}{\frac{\partial T^F_{\iota,t}}{\partial b^F_{\iota,t+1}}} \tag{33}
$$

Starting with a values for $k^F_{\iota,1}$, $b^F_{\iota,1}$, w_t, $p_{\iota,t}$ and z_t, we get $n^F_{\iota,t}$ from equation (31).

Equations (32) and (33) then give $k^F_{\iota,t+1}$ and $b^F_{\iota,t+1}$using the known value of r_{t+1}. This allows us to iteratively solve for labor hired, capital and outstanding debt for firm ι over time.

WE NEED TO ADD EQUITY SHARES CHOICE

2.4 Consumption Goods Firms

2.5 Government

Government will have four functions in our model:

i. The government runs a tax and social security system

   - The tax system will be input by the user and/or determined by the current tax law (the default unless the user supplies changes)

ii. The government makes transfers to households outside of the tax/social security system

iii. The government produces output that contributes to private consumption goods (e.g., education)

iv. The government purchases capital and hires labor to produced a non-rival public good (e.g., national defense)

2.5.1 Government budgeting

$$D_{t+1} + T^{\tau}_t = (1 + r_t)D_t + T^H_t + G^{subs}_t + G^{emp}_t + I^G_t \tag{34}$$

2.5.2 Rule for long-term fiscal stability

Let D_t denote the government's outstanding real debt. T_t is total tax revenue, T^H_t is total household transfers, G_t is government purchases of goods, L_t is the real value of purchases of labor services, and S_t is subsidies to government run firms.

$$D_{t+1} = D_t(1 + r_t) - T_t + T_t^H + G_t + L_t + S_t \tag{35}$$

Letting a carat denote the ratio of a variable to GDP, we can rewrite this as follows:

$$(1 + g_{Yt})\hat{D}_{t+1} = \hat{D}_t(1 + r_t) - \hat{T}_t + \hat{T}_t^H + \hat{G}_t + \hat{L}_t + \hat{S}_t \tag{36}$$

We need to adopt a government fiscal rule that determines how our residual expenditure $\hat{G}_t$ evolves over time.

One way is to adopt a balanced budget rule which keeps the debt-to-GDP ratio constant at it's initial value of $\hat{D}_0$.

$$\begin{aligned}(1 + g_{Yt})\hat{D}_0 &= \hat{D}_0(1 + r_t) - \hat{T}_t + \hat{T}_t^H + \hat{G}_t + \hat{L}_t + \hat{S}_t \\ \hat{G}_t &= \hat{D}_0(g_{Yt} - r_t) + \hat{T}_t - \hat{T}_t^H - \hat{L}_t - \hat{S}_t \end{aligned} \tag{37}$$

Another rule is to hold govenrment spending constant and let debt evolve as it will for several period. Then in period T impose fiscal austerity which forces $\hat{G}_t$ to adjust over time so that $\hat{D}_t$ goes to a steady value.

$$\hat{G}_t - \bar{G} = \rho_t(\hat{D}_t - \bar{D}); \quad \rho_t < 0 \tag{38}$$

Substituting this into (36) gives:

$$\begin{aligned}(1 + g_{Yt})\hat{D}_{t+1} &= \hat{D}_t(1 + r_t) - \hat{T}_t + \hat{T}_t^H + \rho_t(\hat{D}_t - \bar{D}) + \bar{G} + \hat{L}_t + \hat{S}_t \\ \hat{D}_{t+1} &= \frac{\hat{D}_t(1 + r_t) - \hat{T}_t + \hat{T}_t^H + \rho_t(\hat{D}_t - \bar{D}) + \bar{G} + \hat{L}_t + \hat{S}_t}{1 + g_{Yt}} \end{aligned} \tag{39}$$

Consider the steady state version of this.

$$\begin{aligned}(1+\bar{g}_Y)\bar{D} &= \bar{D}(1+\bar{r}) + \bar{T} - \bar{T}^H + \rho_t(\bar{D} - \bar{D}) + \bar{G} + \bar{L} + \bar{S}_t \\ \bar{G} &= \bar{D}(\bar{g}_Y - \bar{r}) + \bar{T} - \bar{T}^H - \bar{L} - \bar{S}\end{aligned} \tag{40}$$

This tells us the long-run value of government spending to GDP that will maintain the debt to GDP target.

In order for (39) to be a contraction mapping over $\hat{D}$ and thus converge to a steady state, we must put bounds on ρ_t. Rearranging (39) and using (40):

$$\begin{aligned}(1+g_{Yt})\hat{D}_{t+1} &= \hat{D}_t(1+r_t) - \hat{T}_t + \hat{T}_t^H + \rho_t(\hat{D}_t - \bar{D}) + \hat{L}_t + \hat{S}_t \\ &\quad + \bar{D}(\bar{g}_Y - \bar{r}) + \bar{T} - \bar{T}^H - \bar{L} - \bar{S} \\ (1+g_{Yt})\hat{D}_{t+1} &= \hat{D}_t(1+r_t) - \hat{T}_t + \hat{T}_t^H + \rho_t\hat{D}_t - \rho_t\bar{D} + \hat{L}_t + \hat{S}_t \\ &\quad + \bar{g}_Y\bar{D} - \bar{r}\bar{D} + \bar{T} - \bar{T}^H - \bar{L} - \bar{S} \\ (1+g_{Yt})\hat{D}_{t+1} &= \hat{D}_t(1+r_t) + \rho_t(\hat{D}_t - \bar{D}) + (\bar{g}_Y - \bar{r})\bar{D} \\ &\quad - (\hat{T}_t - \bar{T}) + (\hat{T}_t^H - \bar{T}^H) + (\hat{L}_t - \bar{L}) + (\hat{S}_t - \bar{S}) \\ \hat{D}_{t+1} - \bar{D} &= \hat{D}_t\frac{1+r_t}{1+g_{Yt}} + \frac{\rho_t}{1+g_{Yt}}(\hat{D}_t - \bar{D}) + \left(\frac{\bar{g}_Y - \bar{r}}{1+g_{Yt}} - 1\right)\bar{D} \\ &\quad + \frac{-(\hat{T}_t - \bar{T}) + (\hat{T}_t^H - \bar{T}^H) + (\hat{L}_t - \bar{L}) + (\hat{S}_t - \bar{S})}{1+g_{Yt}} \\ \hat{D}_{t+1} - \bar{D} &= \frac{1+r_t+\rho_t}{1+g_{Yt}}(\hat{D}_t - \bar{D}) \\ &\quad + \frac{-(\hat{T}_t - \bar{T}) + (\hat{T}_t^H - \bar{T}^H) + (\hat{L}_t - \bar{L}) + (\hat{S}_t - \bar{S})}{1+g_{Yt}}\end{aligned} \tag{41}$$

We need $\frac{\hat{D}_{t+1}-\bar{D}}{\hat{D}_t-\bar{D}} < 1$ for stability. Equation (41) gives:

$$\begin{aligned}
\frac{\hat{D}_{t+1}-\bar{D}}{\hat{D}_t-\bar{D}} = \frac{1+r_t+\rho_t}{1+g_{Yt}} + \frac{-(\hat{T}_t-\bar{T})+(\hat{T}^H_t-\bar{T}^H)+(\hat{L}_t-\bar{L})+(\hat{S}_t-\bar{S})}{(1+g_{Yt})(\hat{D}_t-\bar{D})} &< 1 \\
\frac{1+r_t+\rho_t}{1+g_{Yt}} &< \frac{(\hat{T}_t-\bar{T})-(\hat{T}^H_t-\bar{T}^H)-(\hat{L}_t-\bar{L})-(\hat{S}_t-\bar{S})}{(1+g_{Yt})(\hat{D}_t-\bar{D})} \\
\rho_t &< (1+r_t)\frac{(\hat{T}_t-\bar{T})-(\hat{T}^H_t-\bar{T}^H)-(\hat{L}_t-\bar{L})-(\hat{S}_t-\bar{S})}{\hat{D}_t-\bar{D}} \quad (42)
\end{aligned}$$

2.5.3 Transfer system

We'll need to estimate this. Probably following **?**. Or perhaps the micro simulation model calculates some of these. Or ideally we get something like **?**.

2.5.4 Government production of private goods

2.5.5 Government production of public goods

2.5.6 Steps for adding government to the dynamic model

i. 1 firm

ii. 1 firm + gov't

iii. 2 firms + gov't

iv. tax 2 firms + gov't

v. N firms with taxes + gov't

2.6 Market Clearing

2.7 Solution and Simulation

3 Incorporating Feedbacks with Micro Tax Simulations

Follow this algorthim:

- Period 1
  - Use current IRS public use sample.
  - Run the following within-period routine
    * Do the static tax analysis of this sample, save the results
    * Summarize the public use sample by aggregating into bins over age and earnings ability
    * Use this as a starting point for the dynamic macro model
    * Get values for fundamental interest rates and effective wages for next period
- Period 2
  - Age the public use data demographically by one year.
  - Let wages and interest rates rise by the amounts predicted in the macro model.
  - Rerun the within-period routine
- Iterate over periods until end of forecast period is reached.

4 Calibration

4.1 Tax Bend Points

We use IRS data which summarizes individual tax returns for 2011 by 19 income categories and 4 filing statuses. For each filing status we fit the mapping from reported income into adjusted gross income (AGI) using a sufficiently high-order polynomial.

We then use this function to solve for the income level which corresponds to each of the five bend points in the tax code for each filing type.

Table 1: AGI and Income Bend Points

AGI Bend Points				
Tax rate	Married Joint	Married Separate	Head of Household	Single
10%	17,400	8700	12,400	8700
15%	70,700	35,350	47,350	35,350
25%	142,700	71,350	122,300	85,650
28%	217,450	108,725	198,050	178,650
33%	388,350	194,175	388,350	388,350
Corresponding Reported Income Bendpoints				
Tax rate	Married Joint	Married Separate	Head of Household	Single
0%	5850	91	756	1435
10%	22,932	8591	12,911	9956
15%	75,181	34,592	47,023	36,021
25%	145,866	69,768	120,200	85,244
28%	219,162	106,245	194,176	176,270
33%	386,798	189,674	380,043	381,524

We then fit a bivariate probabililty density function over income and filing type from the data. For each bendpoint we calculate the probability density at that bendpoint and use these as weights in a weighted average over filing types to generate an aggregate bendpoint.

Table 2: Aggregated Bend Points

Tax rate	Bend Point
0%	2889
10%	15,116
15%	52,580
25%	114,552
28%	196,201
33%	380,657

5 Conclusion

APPENDIX

References