

Consumers Problem with Many Goods

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The consumer's maximization problem is:

$$\begin{aligned}
 U_{j,s,t} &= \sum_{u=0}^{E+S-s} \beta^u \left[\prod_{v=s-1}^{s+u-1} (1 - \rho_v) \right] u(c_{j,s+u,t+u}, n_{j,s+u,t+u}, b_{j,s+u+1,t+u+1}) \\
 \text{where } \rho_{s-1} &= 0 \\
 \text{and } u(c_{j,s,t}, n_{j,s,t}, b_{j,s+1,t+1}) &= \frac{(c_{j,s,t})^{1-\sigma} - 1}{1-\sigma} \dots \\
 &\quad + e^{g_y t(1-\sigma)} \chi_s^n \left(b \left[1 - \left(\frac{n_{j,s,t}}{\tilde{l}} \right)^v \right]^{\frac{1}{v}} + k \right) + \rho_s \chi^b \frac{(b_{j,s+1,t+1})^{1-\sigma} - 1}{1-\sigma} \\
 \text{and } c_{j,s,t} &= \prod_{i=1}^I (c_{i,j,s,t} - \bar{c}_{i,j,s})^{\alpha_i}; \quad \sum_{i=1}^I \alpha_i = 1 \\
 &\quad \forall j, t \quad \text{and } E+1 \leq s \leq E+S
 \end{aligned} \tag{1}$$

They maximize subject to the following budget constraint.

$$\begin{aligned}
 \sum_{i=1}^I p_{i,t} c_{i,j,s,t} + b_{j,s+1,t+1} &\leq (1+r_t) b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \frac{BQ_{j,t}}{\lambda_j \tilde{N}_t} - T_{j,s,t} \\
 \text{where } b_{j,s,1} &= 0 \\
 \text{for } E+1 \leq s \leq E+S \quad \forall j, t
 \end{aligned} \tag{2}$$

We set up a Lagrangian and solve by taking derivatives with respect to $\{c_{i,j,s,t}, n_{j,s,t+u}, b_{j,s,t+1}\}$ for all i, j, s and t .

$$\begin{aligned}
 \frac{\partial U}{\partial c_{i,j,s+u,t+u}} &= \beta^u \left[\prod_{v=s-1}^{s+u-1} (1 - \rho_v) \right] \left[\prod_{i=1}^I (c_{i,j,s,t} - \bar{c}_{i,j,s})^{\alpha_i} \right]^{-\sigma} \alpha_i (c_{i,j,s,t} - \bar{c}_{i,j,s})^{\alpha_i-1} \\
 &\quad - \lambda_{t+u} \left(p_{i,t+u} + \frac{\partial T_{j,s+u,t+u}}{\partial c_{i,j,s+u,t+u}} \right) = 0
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 \frac{\partial U}{\partial n_{j,s+u,t+u}} &= \beta^u \left[\prod_{v=s-1}^{s+u-1} (1 - \rho_v) \right] e^{g_y(t+u)(1-\sigma)} \chi_s^n \left(\frac{b}{\tilde{l}} \right) \left(\frac{n_{j,s+u,t+u}}{\tilde{l}} \right)^{v-1} \left[1 - \left(\frac{n_{j,s+u,t+u}}{\tilde{l}} \right) \right]^{\frac{1-v}{v}} \\
 &\quad - \lambda_{t+u} \left(w_{+u} e_{j,s+u} - \frac{\partial T_{j,s+u,t+u}}{\partial n_{j,s+u,t+u}} \right) = 0
 \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{\partial U}{\partial b_{j,s+u+1,t+u+1}} &= \beta^u \left[\prod_{v=s-1}^{s+u-1} (1 - \rho_v) \right] \rho_s \chi^b(b_{j,s+U+1,t+U+1})^{-\sigma} \\ &\quad - \lambda_{t+u} - \lambda_{t+u+1} \left(1 + r_{t+u+1} \right) - \frac{\partial T_{j,s+U+1,t+U+1}}{\partial b_{j,s+U+1,t+u+1}} = 0 \end{aligned} \quad (5)$$

We can solve for λ_{t+u} :

$$\begin{aligned} \lambda_{t+u} &= \frac{\beta^u \left[\prod_{v=s-1}^{s+u-1} (1 - \rho_v) \right] \left[\prod_{i=1}^I (c_{i,j,s,t} - \bar{c}_{i,j,s})^{\alpha_i} \right]^{-\sigma} \alpha_i (c_{i,j,s,t} - \bar{c}_{i,j,s})^{\alpha_i-1}}{p_{i,t+u} + \frac{\partial T_{j,s+u,t+u}}{\partial c_{i,j,s+u,t+u}}} \\ \lambda_{t+u} &= \frac{\beta^u \left[\prod_{v=s-1}^{s+u-1} (1 - \rho_v) \right] e^{g_y(t+u)(1-\sigma)} \chi_s^n \left(\frac{b}{l} \right) \left(\frac{n_{j,s+u,t+u}}{l} \right)^{v-1} \left[1 - \left(\frac{n_{j,s+u,t+u}}{l} \right) \right]^{\frac{1-v}{v}}}{w_{t+u} e_{j,s+u} - \frac{\partial T_{j,s+u,t+u}}{\partial n_{j,s+u,t+u}}} \\ \lambda_{t+u} &= \beta^u \left[\prod_{v=s-1}^{s+u-1} (1 - \rho_v) \right] \rho_s \chi^b(b_{j,s+U+1,t+U+1})^{-\sigma} - \lambda_{t+u+1} \left(1 + r_{t+u+1} \right) - \frac{\partial T_{j,s+U+1,t+U+1}}{\partial b_{j,s+U+1,t+u+1}} \end{aligned}$$

These then reduce to the following $I + 1$ Euler equations for each j, s and t :

$$\begin{aligned} &\frac{\left[\prod_{i=1}^I (c_{i,j,s,t} - \bar{c}_{i,j,s})^{\alpha_i} \right]^{-\sigma} \alpha_i (c_{i,j,s,t} - \bar{c}_{i,j,s})^{\alpha_i-1}}{p_{i,t+u} + \frac{\partial T_{j,s+u,t+u}}{\partial c_{i,j,s+u,t+u}}} \\ &= \frac{e^{g_y(t+u)(1-\sigma)} \chi_s^n \left(\frac{b}{l} \right) \left(\frac{n_{j,s+u,t+u}}{l} \right)^{v-1} \left[1 - \left(\frac{n_{j,s+u,t+u}}{l} \right) \right]^{\frac{1-v}{v}}}{w_{t+u} e_{j,s+u} - \frac{\partial T_{j,s+u,t+u}}{\partial n_{j,s+u,t+u}}} \end{aligned} \quad (6)$$

$$\begin{aligned} &\frac{e^{g_y(t+u)(1-\sigma)} \chi_s^n \left(\frac{b}{l} \right) \left(\frac{n_{j,s+u,t+u}}{l} \right)^{v-1} \left[1 - \left(\frac{n_{j,s+u,t+u}}{l} \right) \right]^{\frac{1-v}{v}}}{w_{t+u} e_{j,s+u} - \frac{\partial T_{j,s+u,t+u}}{\partial n_{j,s+u,t+u}}} = \rho_s \chi^b(b_{j,s+U+1,t+U+1})^{-\sigma} \\ &- \frac{\beta(1 - \rho_{s+u}) e^{g_y(t+u+1)(1-\sigma)} \chi_s^n \left(\frac{b}{l} \right) \left(\frac{n_{j,s+u+1,t+u+1}}{l} \right)^{v-1} \left[1 - \left(\frac{n_{j,s+u+1,t+u+1}}{l} \right) \right]^{\frac{1-v}{v}}}{w_{t+u+1} e_{j,s+u+1} - \frac{\partial T_{j,s+u+1,t+u+1}}{\partial n_{j,s+u+1,t+u+1}}} \times \\ &\quad \left(1 + r_{t+u+1} \right) - \frac{\partial T_{j,s+U+1,t+U+1}}{\partial b_{j,s+U+1,t+u+1}} \end{aligned} \quad (7)$$