This document outlines how we can us the first order conditions of the household to estimate the utility weights on the disutility of work and the warm glow bequest.

1 Estimating the utility weight on the disutility of work, χ^n_s

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The household first order condition for the choice of hours worked yields:

$$
\begin{aligned}
&(c_{j,s,t})^{-\sigma}\left(w_t e_{j,s} - \frac{\partial T_{j,s,t}}{\partial n_{j,s,t}}\right) = e^{g_y t(1-\sigma)}\chi^n_s\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{v-1}\left[1-\left(\frac{n_{j,s,t}}{\tilde{l}}\right)\right]^{\frac{1-v}{v}} \\
&\qquad\qquad\qquad\qquad\qquad\qquad \forall j,t, \quad \text{and} \quad E+1\leq s\leq E+S \\
&\text{where} \quad c_{j,s,t} = (1+r_t)\,b_{j,s,t} + w_t e_{j,s} n_{j,s,t} + \frac{BQ_{j,t}}{\lambda_j \tilde{N}_t} - b_{j,s+1,t+1} - T_{j,s,t} \\
&\text{and} \quad \frac{\partial T_{j,s,t}}{\partial n_{j,s,t}} = w_t e_{j,s}\left[\tau^I\big(F\hat{a}_{j,s,t}\big) + \frac{F\hat{a}_{j,s,t}CD\big[2A(F\hat{a}_{j,s,t})+B\big]}{\big[A(F\hat{a}_{j,s,t})^2+B(F\hat{a}_{j,s,t})+C\big]^2} + \tau^P\right]
\end{aligned}
\tag{1}
$$

To simplify notation a bit, let $w_t e_{j,s} = \tilde{w}_{j,s,t}$, which is defined as the hourly earnings of household of ability type j, age s, at time t. Further, we can write derivative of the tax function, $\frac{\partial T_{j,s,t}}{\partial n_{j,s,t}}$ as $\tau^l(y_{j,s,t})\tilde{w}_{j,s,t}$, where $\tau^l(y_{j,s,t})$ is the marginal tax rate on labor income for an individual with taxable income $y_{j,s,t}$. Now we can write the FOC as:

$$
(c_{j,s,t})^{-\sigma}\left(\tilde{w}_{j,s,t}(1-\tau^l(y_{j,s,t})\right) = e^{g_y t(1-\sigma)}\chi^n_s\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{v-1}\left[1-\left(\frac{n_{j,s,t}}{\tilde{l}}\right)\right]^{\frac{1-v}{v}} \tag{2}
$$

This problem is deterministic, but we can assume that their is some noise in the data, thus the data analog to the model FOC is:

$$
(c_{j,s,t})^{-\sigma}\left(\tilde{w}_{j,s,t}(1-\tau^l(y_{j,s,t})\right) - e^{g_y t(1-\sigma)}\chi^n_s\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{v-1}\left[1-\left(\frac{n_{j,s,t}}{\tilde{l}}\right)\right]^{\frac{1-v}{v}} = \varepsilon_{j,s,t} \tag{3}
$$

And the moment condition for a GMM estimator for each χ^n_s would be:

$$
\begin{aligned}
&\sum_J\sum_T\left[(c_{j,s,t})^{-\sigma}\left(\tilde{w}_{j,s,t}(1-\tau^l(y_{j,s,t})\right) - e^{g_y t(1-\sigma)}\chi^n_s\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{j,s,t}}{\tilde{l}}\right)^{v-1}\left[1-\left(\frac{n_{j,s,t}}{\tilde{l}}\right)\right]^{\frac{1-v}{v}}\right] = \\
&\qquad\qquad \sum_J\sum_T \varepsilon_{j,s,t} = 0
\end{aligned}
\tag{4}
$$

Note that the above equation for each s - and we can have more moment conditions if we wish to have an over-identified model. We may also think about estimating the parameters of the ellipse via this method.

To estimate, we need data on consumption, c, and labor supply, n, by lifetime income group, age, and year. We can get this from the PSID - see http://www.federalreserve.gov/pubs/feds/2007/200716/200716pap.pdf for a document outlining the measurement of consumption from the PSID. We can find $\tau^l(y_{j,s,t})$ by running the PSID observation through a tax calculator (e.g. the OSPC calculator). The remaining parameters are calibrated elsewhere.

2 Estimating the utility weight on the warm glow bequest motive, χ^b_j

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The household first order condition for the choice of savings yields:

$$
\begin{split}
(c_{j,s,t})^{-\sigma} = \rho_s\chi^b_j\big(b_{j,s+1,t+1}\big)^{-\sigma} + \beta(1-\rho_s)(c_{j,s+1,t+1})^{-\sigma}\left[(1+r_{t+1}) - \frac{\partial T_{j,s+1,t+1}}{\partial b_{j,s+1,t+1}}\right] \\
\forall j,t, \quad \text{and} \quad E+1 \leq s \leq E+S-1 \\
\text{where} \quad \frac{\partial T_{j,s+1,t+1}}{\partial b_{j,s+1,t+1}} = ... \\
r_{t+1}\left(\tau^I(F\hat{a}_{j,s+1,t+1}) + \frac{F\hat{a}_{j,s+1,t+1}CD\left[2A(F\hat{a}_{j,s+1,t+1})+B\right]}{\left[A(F\hat{a}_{j,s+1,t+1})^2 + B(F\hat{a}_{j,s+1,t+1}) + C\right]^2}\right)... \\
\tau^W(\hat{b}_{j,s+1,t+1}) + \frac{\hat{b}_{j,s+1,t+1}PHM}{\left(H\hat{b}_{j,s+1,t+1}+M\right)^2}
\end{split}
\tag{5}
$$

We can write derivative of the tax function, $\frac{\partial T_{j,s,t}}{\partial b_{j,s,t}}$ as $\tau^b(y_{j,s,t})r_{t+1}$, where $\tau^b(y_{j,s,t})$ is the marginal tax rate on capital income for an individual with taxable income $y_{j,s,t}$. Now we can write the FOC as:

$$
(c_{j,s,t})^{-\sigma} = \rho_s\chi^b_j\big(b_{j,s+1,t+1}\big)^{-\sigma} + \beta(1-\rho_s)(c_{j,s+1,t+1})^{-\sigma}\left[(1+(1-\tau^b_{j,s+1,t+1})r_{t+1})\right] \tag{6}
$$

This problem is deterministic, but we can assume that their is some noise in the data, thus the data analog to the model FOC is:

$$
(c_{j,s,t})^{-\sigma} - \rho_s\chi^b_j\big(b_{j,s+1,t+1}\big)^{-\sigma} + \beta(1-\rho_s)(c_{j,s+1,t+1})^{-\sigma}\left[(1+(1-\tau^b_{j,s+1,t+1})r_{t+1})\right] = \varepsilon_{j,s,t} \tag{7}
$$

And the moment condition for a GMM estimator for each χ^b_j would be:

$$\sum_S \sum_T \left[((c_{j,s,t})^{-\sigma} - \rho_s \chi^b_j \big(b_{j,s+1,t+1}\big)^{-\sigma} + \beta(1-\rho_s)(c_{j,s+1,t+1})^{-\sigma} \left[(1 + (1 - \tau^b_{j,s+1,t+1} r_{t+1}) \right] \right] = \sum_S \sum_T \varepsilon_{j,s,t} = 0 \tag{8}$$

Note that the above equation for each j - and we can have more moment conditions if we wish to have an over-identified model.

To estimate, we need data on consumption, c, and wealth, b, by lifetime income group, age, and year. We can get consumption from the PSID, as noted above. We can also get wealth from the PSID, at least for the years 1984-2005 - see http://www.brookings.edu/ /media/research/files/papers/2009/2/saving-wealth-bosworth/02_saving_wealth_bosworth.pdf for a document outlining the measurement of wealth from the PSID. We can find $\tau^b(y_{j,s,t})$ by running the PSID observation through a tax calculator (e.g. the OSPC calculator). The remaining parameters are calibrated elsewhere.