A Macroeconomic Model for Dynamic Scoring *

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Abstract

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1 Introduction

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2 Model

Model intro here.

2.1 Individual problem

A measure $1/S$ of individuals with heterogeneous working ability $e \in \mathcal{E} \subset \mathbb{R}_{++}$ is born in each period t and live for $S \geq 3$ periods. Their working ability evolves over their lifetime according to an age-dependent deterministic process. At birth, a fraction $1/J$ of the $1/S$ measure of new agents are randomly assigned to one of J ability types indexed by $j = 1, 2, ...J$. Once ability type is determined, that measure $1/(SJ)$ of individuals' ability evolves deterministically according to $e_j(s)$. We calibrate the matrix of lifetime ability paths $e_j(s)$ for all types j using CPS hourly wage by age distribution data.[1]

Individuals are endowed with a measure of time in each period t that they supply inelastically to the labor market. Let s represent the periods that an individual has been alive. The fixed labor supply in each period t by each age-s individual is denoted by $l(s)$.

At time t, all generation s agents with ability $e_j(s)$ know the real wage rate w_t and know the one-period real net interest rate $r_t - \delta$ on bond holdings $b_{j,s,t}$ that mature at the beginning of period t. For ease of notation, we subtract the depreciation rate δ from the net interest rate r_t to represent the fact that any depreciation is passed through directly from firms to households. In each period t, age-s agents with working ability e choose how much to consume $c_{j,s,t}$ and how much to save for the next period by loaning capital to firms in the form of a one-period bond $b_{j,s+1,t+1}$ in order to

[1] Appendix A-1 gives a detailed description of the calibration of the deterministic ability process by age s and type j, as well as alternative specifications and calibrations.

maximize expected lifetime utility of the following form,

$$U_{j,s,t} = \sum_{v=0}^{S-s} \beta^u u\left(c_{j,s+u,t+u}\right) \quad \text{where} \quad u\left(c_{j,s,t}\right) = \frac{\left(c_{j,s,t}\right)^{1-\sigma} - 1}{1-\sigma} \quad \forall j,s,t \tag{1}$$

where $u(c)$ is a constant relative risk aversion utility function, $\sigma \geq 1$ is the coefficient of relative risk aversion, and $\beta \in (0,1)$ is the agent's discount factor.

Because agents are born without any bonds maturing and because they purchase no bonds in the last period of life $s = S$, the per-period budget constraints for each agent normalized by the price of consumption are the following,

$$w_t e_j(s) l(s) \geq c_{j,s,t} + b_{j,s+1,t+1} \quad \text{for} \quad s = 1 \qquad \forall j,t \tag{2}$$

$$\left(1 + r_t - \delta\right) b_{j,s,t} + w_t e_j(s) l(s) \geq c_{j,s,t} + b_{j,s+1,t+1} \quad \text{for} \quad 2 \leq s \leq S-1 \quad \forall j,t \tag{3}$$

$$\left(1 + r_t - \delta\right) b_{j,s,t} + w_t e_j(s) l(s) \geq c_{j,s,t} \quad \text{for} \quad s = S \qquad \forall j,t \tag{4}$$

In addition to the budget constraints in each period, the utility function imposes nonnegative consumption through inifinite marginal utility. We allow the possibility for individual agents to borrow $b_{j,s,t} < 0$ for some j and s in period t. However, the borrowing must satisfy a series of individual feasibility constraints as well as a strict constraint that the aggregate capital stock $K_t > 0$ be positive in every period.[2]

We next describe the Euler equations that govern the choices of consumption $c_{j,s,t}$ and savings $b_{j,s+1,t+1}$ by household of age s and ability $e_j(s)$ in each period t. We work backward from the last period of life $s = S$. Because households do not save in the last period of life $b_{j,s+1,t+1} = 0$ due to our assumption of no bequest motive, the household's final-period maximization problem is given by the following.

$$\max_{c_{j,S,t}} \frac{\left(c_{j,S,t}\right)^{1-\sigma} - 1}{1-\sigma} \quad \text{s.t.} \quad \left(1 + r_t - \delta\right) b_{j,S,t} + w_t e_j(S) l(S) \geq c_{j,S,t} \quad \forall t \tag{5}$$

Because $u(c)$ is monotonically increasing in c, the $s = S$ problem (5) is simply to choose the maximum amount of consumption possible. The household trivially con-

[2] We describe these constraints in detail in Appendix A-2.

sumes all of its income in the last period of life.

$$c_{j,S,t} = (1 + r_t - \delta)\, b_{j,S,t} + w_t e_j(S) l(S) \quad \forall t \tag{6}$$

In general, maximizing (1) with respect to (2), (3), (4), and the implied individual and aggregate borrowing constraints gives the following set of $S-1$ intertemporal Euler equations.

$$\begin{aligned} (c_{j,s,t})^{-\sigma} &= \beta\,(1 + r_{t+1} - \delta)\,(c_{j,s+1,t+1})^{-\sigma} \\ &\text{for} \quad 1 \leq s \leq S-1, \quad \forall t \end{aligned} \tag{7}$$

Note from (3) that $c_{j,s,t}$ in (7) depends on the household's age s, his ability $e_j(s)$, and the initial wealth with which the household entered the period $b_{j,s,t}$.

2.2 Firm problem

A unit measure of identical, perfectly competitive firms exist in this economy. The representative firm is characterized by the following Cobb-Douglas production technology,

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha} \quad \forall t \tag{8}$$

where A is the fixed technology process and $\alpha \in (0,1)$ and L_t is measured in efficiency units of labor. The interest rate r_t in the cost function is a gross real interest rate because depreciation is paid by the households. The real profit function of the firm is the following.

$$\text{Real Profits} = A K_t^{\alpha} L_t^{1-\alpha} - r_t K_t - w_t L_t \tag{9}$$

Profit maximization results in the real wage w_t and the real rental rate of capital r_t being determined by the marginal products of labor and capital, respectively.

$$w_t = (1-\alpha)\frac{Y_t}{L_t} \quad \forall t \tag{10}$$

$$r_t = \alpha \frac{Y_t}{K_t} \quad \forall t \tag{11}$$

2.3 Market clearing and equilibrium

Labor market clearing requires that aggregate labor demand L_t measured in efficiency units equal the sum of individual efficiency labor supplied $e_j sl(s)$. The supply side of market clearing in the labor market is trivial because household labor is supplied inelastically. Capital market clearing requires that aggregate capital demand K_t equal the sum of capital investment by households $b_{j,s,t}$. Aggregate consumption C_t is defined in (14), and investment is defined by the standard $Y = C + I$ constraint as shown in (15).

$$L_t = \frac{1}{SJ}\sum_{s=1}^{S}\sum_{j=1}^{J} e_j(s)l(s) \qquad \forall t \tag{12}$$

$$K_t = \frac{1}{SJ}\sum_{s=1}^{S}\sum_{j=1}^{J} b_{j,s,t} \qquad \forall t \tag{13}$$

$$C_t \equiv \frac{1}{SJ}\sum_{s=1}^{S}\sum_{j=1}^{J} c_{j,s,t} \qquad \forall t \tag{14}$$

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \quad \forall t \tag{15}$$

The steady-state equilibrium for this economy is defined as follows.

Definition 1 (**Steady-state equilibrium**)**.** A non-autarkic steady-state equilibrium in the overlapping generations model with S-period lived agents and heterogeneous ability $e_j(s)$ is defined as a constant distribution of capital

$$\Gamma_t = \bar{\Gamma} = \bar{\gamma}(s,e,b) \quad \forall t,$$

a savings decision rule given beliefs $b' = \phi(s,e,b|\Omega)$, consumption allocations $c_{s,t}$ for all s and t, aggregate firm production Y_t, aggregate labor demand N_t, aggregate capital demand K_t, real wage w_t, and real interest rate r_t for all t such that the following conditions hold:

i. households optimize according to (**??**), (**??**) and (**??**),

ii. firms optimize according to (**??**) and (**??**),

iii. markets clear according to (**??**), (**??**), and (**??**),

iv. and the steady-state distribution $\Gamma_t = \bar{\Gamma}$ is induced by the policy rule $b' = \phi(s,e,b|\Omega)$.

APPENDIX

A-1 Calibration of ability process

Put description of ability process $e_j(s)$ calibration here. Make sure to include careful description of what the data are and where they came from.

A-2 Constraints on individual borrowing

As described in Section 2.1, individuals are allowed to borrow $b_{j,s,t}$ for some j and s in period t. However, two constraints must hold. First, the individual must be able to pay back the balance with interest r_{t+1} in the next period without driving consumption in the next period $c_{j,s+1,t+1}$ to be nonpositive. Let $\bar{b}_{j,s,t}$ be the minimum value of savings in a period.

$$b_{j,s,t} \geq \bar{b}_{j,s,t} \quad \forall j, s, t \tag{A.2.1}$$

Rearranging the bugdet constraints in (2), (3), and (4) and using backward induction gives the following expressions for $\bar{b}_{j,s,t}$,

$$\begin{aligned} \bar{b}_{j,S,t} &= \frac{\varepsilon - w_t e_j(S)l(S)}{1 + r_t - \delta} \\ \bar{b}_{j,S-1,t-1} &= \frac{\varepsilon + \bar{b}_{j,S,t} - w_{t-1}e_j(S-1)l(S-1)}{1 + r_{t-1} - \delta} \\ &\vdots \\ \bar{b}_{j,2,t-S+2} &= \frac{\varepsilon + \bar{b}_{j,3,t-S+3} - w_{t-S+2}e_j(2)l(2)}{1 + r_{t-S+2} - \delta} \end{aligned} \tag{A.2.2}$$

In addition to the individual borrowing constraint (A.2.1), a strict aggregate borrowing constraint must be met. That is, the aggregate capital stock must be strictly positive.

$$K_t > 0 \quad \forall t \tag{A.2.3}$$

References

TECHNICAL APPENDIX

T-1 Structures to add to the model and order

i. Put depreciation on the firm side

ii. Endogenize labor

iii. Make sure bond holdings are correct

iv. Add demographics

v. Add household tax structures

vi. Add firm structures