

Step-by-Step Derivations of a Model for Dynamic General Equilibrium Tax Scoring with Micro Tax Simulations *

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Abstract

This paper ...

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1 Introduction

2 Details of the Macro Model

We use a model based initially on that from [Evans and Phillips \(2014\)](#) and incorporate many of the features of [Zodrow and Diamond \(2013\)](#) which we refer to hereafter as the DZ model.

2.1 Baseline Model - Model 1

For our first baseline model we take [Evans and Phillips \(2014\)](#) and add a leisure-labor decision, while removing the switching of ability from period to period. Hence all workers remain the same type throughout their lifetime. Agents live for S periods and exogenously retire in period R . This is a perfect foresight model. Both households and firms exist in a unit measure. All firms are identical, but households are distinguished by age and ability.

2.1.1 Households

Housholds maximize utility as given in the equation below.

$$U_{ist} = \sum_{u=0}^{S-s} \beta^u u(c_{i,s+u,t+u}, \ell_{i,s+u,t+u}); \text{ where } u(c, \ell) = \frac{c^{1-\gamma} - 1}{1-\gamma} - \eta \frac{\ell^{1-\xi} - 1}{1-\xi}$$

U_{ist} is the remaining lifetime utility of a household with ability level i of age s in period t . c denotes consumption of goods and ℓ denotes labor supplied to the market.

The household faces the following set of budget constraints.

$$w_t \ell_{ist} n_i \geq c_{ist} + k_{i,s+1,t+1} \text{ for } s = 1, \forall i \quad (2.1)$$

$$w_t \ell_{ist} n_i + (1 + r_t - \delta) k_{ist} \geq c_{ist} + k_{i,s+1,t+1} \text{ for } 1 < s < S, \forall i \quad (2.2)$$

$$w_t \ell_{ist} n_i + (1 + r_t - \delta) k_{ist} \geq c_{ist} \text{ for } s = S, \forall i \quad (2.3)$$

k_{ist} is the holdings of capital by household of type i coming due in period t when the

household is age s . w is the wage rate, r denotes the return on savings, n denotes the effective labor productivity of the household.

The Euler equations from this maximization problem are given below.

$$c_{ist}^{-\gamma} = \beta c_{i,s+1,t+1}^{-\gamma} (1 + r_{t+1} - \delta) \text{ for } 1 \leq s < S, \forall i \quad (2.4)$$

$$c_{ist}^{-\gamma} w_t = \eta \ell_{ist}^{-\xi}, \forall s, i \quad (2.5)$$

2.1.2 Firms

Firms produce using a Cobb-Douglas production function each period and maximize profits as shown below:

$$\Pi_t = K_t^\alpha (e^{gt} L_t)^{1-\alpha} - r_t K_t - w_t L_t$$

The profit maximizing conditions are:

$$r_t = \alpha K_t^{\alpha-1} (e^{gt} L_t)^{1-\alpha} \quad (2.6)$$

$$w_t = (1 - \alpha) K_t^\alpha e^{(1-\alpha)gt} L_t^{-\alpha} \quad (2.7)$$

2.1.3 Market Clearing

Market-clearing conditions require the following:

$$K_t = \sum_{s=2}^S \sum_{i=1}^I \phi_i k_{ist} \quad (2.8)$$

$$L_t = \sum_{s=1}^S \sum_{i=1}^I \phi_i \ell_{ist} \quad (2.9)$$

ϕ_i is the proportion of type i in the total population of workers.

2.1.4 Solution and Simulation

This model can be simulated using either the TPI or AMF method described in [Evans and Phillips \(2014\)](#).

Assuming there are I ability types and S cohorts alive in any period, equations (2.1) through (2.9) define a dynamic system of $4 + 3IS - S$ equations in the variables: $K_t, L_t, w_t, r_t, \{c_{ist}\}_{s=1}^S, \{\ell_{ist}\}_{s=1}^S$ and $\{k_{ist}\}_{s=2}^S$.

The parameters of the model are $\alpha, \beta, \delta, \gamma, \xi, g, \{n_i\}$ and $\{\phi_i\}$

2.2 Adding Taxes on the Household - Model 2

2.2.1 Households

The social security payroll tax paid or benefit received is calculated as follows.

$$T_{ist}^P = \begin{cases} \tau_P w_t \ell_{ist} n_i & \text{if } w_t \ell_{ist} n_i < \chi_P, s < R \\ \tau_P \chi_P & \text{if } w_t \ell_{ist} n_i \geq \chi_P, s < R \\ -\theta w_t n_i & \text{if } s \geq R \end{cases} \quad (2.10)$$

τ_P is the payroll tax rate and χ_P is the payroll tax ceiling.

Income is $w_t \ell_{ist} n_i + (r_t - \delta)k_{ist}$. Define $D\{w_t \ell_{ist} n_i + (r_t - \delta)k_{ist}, \Omega\}$ as the exemptions and benefits claimed as a function of income and other variables, Ω . Adjusted gross income is $X_{ist} \equiv w_t \ell_{ist} n_i + (r_t - \delta)k_{ist} - D\{w_t \ell_{ist} n_i + (r_t - \delta)k_{ist}, \Omega_{ist}\} - \tau_\delta \delta k_{ist}$. The final term is a capital depreciation allowance at rate τ_δ . We have fit this D function to the data for 2011 using a polynomial function. Income tax paid is defined as follows.

$$T_{ist}^I = \begin{cases} 0 & \text{if } X_{ist} < \chi_1 \\ \tau_1(X_{ist} - \chi_1) & \text{if } \chi_1 \leq X_{ist} < \chi_2 \\ \tau_1 \chi_1 + \tau_2(X_{ist} - \chi_2) & \text{if } \chi_2 \leq X_{ist} < \chi_3 \\ \tau_1 \chi_1 + \tau_2(\chi_3 - \chi_2) + \tau_3(X_{ist} - \chi_3) & \text{if } \chi_3 \leq X_{ist} < \chi_4 \\ \vdots & \\ \tau_1 \chi_1 + \tau_2(\chi_3 - \chi_2) + \cdots + \tau_N(X_{ist} - \chi_N) & \text{if } \chi_N \leq X_{ist} \end{cases} \quad (2.11)$$

τ_i is the marginal tax rate in bracket i , the bend points between brackets are denoted χ_i .

The consumption tax rate is denoted τ_c

The household faces the following set of budget constraints.

$$c_{ist} = (1 - \tau_c) [w_t \ell_{ist} n_i - k_{i,s+1,t+1} - T_{ist}^p - T_{ist}^i] \quad (2.12)$$

$$c_{ist} = (1 - \tau_c) [w_t \ell_{ist} n_i + (1 + r_t - \delta)k_{ist} - k_{i,s+1,t+1} - T_{ist}^p - T_{ist}^i] \quad (2.13)$$

$$c_{ist} = (1 - \tau_c) [w_t \ell_{ist} n_i + (1 + r_t - \delta)k_{ist} - T_{ist}^p - T_{ist}^i] \quad (2.14)$$

2.2.2 Government

Government collects the following amounts of tax revenue each period.

$$R_t = \sum_s \sum_i \phi_i \left(T_{ist}^p + T_{ist}^i + \frac{\tau_c}{1-\tau_c} c_{ist} \right) \quad (2.15)$$

2.2.3 Solution and Simulation

The model now consists of $5 + 5IS - I$ equations with the addition of (2.10), (2.11) and (2.15), and the substitution of (2.12) - (2.14) for (2.1) - (2.3).

The variables are $K_t, L_t, w_t, r_t, \{c_{ist}\}_{s=1}^S, \{\ell_{ist}\}_{s=1}^S, \{k_{ist}\}_{s=2}^S, \{T_{ist}^p\}_{s=1}^S, \{T_{ist}^i\}_{s=1}^S$ and R_t .

The parameters of the model are $\alpha, \beta, \delta, \gamma, \xi, g, \{n_i\}, \{\phi_i\}, \tau_p, \chi_p, \{\tau_n\}_{n=1}^N, \{\chi_n\}_{n=1}^N$ and τ_c .

2.3 Adding Taxes on Firms - Model 3

We allow firms to acquire capital by renting it as above, or by accumulating their own capital and paying dividends, or by issuing bonds.

We assume both firms and households pay a percent quadratic capital adjustment cost of $\psi(K_{t+1}) = \frac{\kappa}{2} (K_{t+1} - K_t)^2$. Both households and firms receive a depreciation allowance at the rate τ_δ and an investment credit at the rate $\tau_{\Delta k}$

2.3.1 Households

In addition to capital (k_{ist}), households now also hold bonds in the amount b_{ist} and equities in the amount q_{ist} . Interest income and dividends are taxed as regular income, but capital gains are taxed separately (T^q).

The household's problem can be written in the following recursive form:

$$V_s^h(k_{ist}, b_{ist}, q_{ist}) = \max_{k_{t+1}, b_{t+1}, q_{t+1}, \ell_t} \frac{c_{ist}^{1-\gamma} - 1}{1-\gamma} - \eta \frac{\ell_{ist}^{1-\xi} - 1}{1-\xi} + \beta V_{s+1}^h(k_{i,s+1,t+1}, b_{i,s+1,t+1}, q_{i,s+1,t+1})$$

The typical household budget constraint is:

$$c_{ist} = (1 - \tau_c) \left[\begin{array}{c} w_t \ell_{ist} n_i + (1 + r_t - \delta) k_{ist} + (1 + i_t) b_{ist} + p_t q_{ist} \\ - \frac{\kappa}{2} (k_{i,s+1,t+1} - k_{ist})^2 - k_{i,s+1,t+1} - b_{i,s+1,t+1} - p_t q_{i,s+1,t+1} \\ - T_{ist}^p - T_{ist}^i - T_{ist}^q + \tau_\delta \delta k_{ist} + \tau_{\Delta k} (k_{i,s+1,t+1} - k_{ist}) \end{array} \right] \quad (2.16)$$

Income subject to taxation (I_{ist}) and AGI (X_{ist}) are:

$$I_{ist} = w_t \ell_{ist} n_i + (r_t - \delta) k_{ist} + i_t b_{ist} + \pi_t q_{ist} \quad (2.17)$$

$$X_{ist} = I_{ist} - D\{I_{ist}, \Omega_{ist}\} - \tau_\delta \delta k_{ist} \quad (2.18)$$

With the following tax code formulas.

$$T_{ist}^P = \begin{cases} \tau_P w_t \ell_{ist} n_i & \text{if } w_t \ell_{ist} n_i < \chi_P, s < R \\ \tau_P \chi_P & \text{if } w_t \ell_{ist} n_i \geq \chi_P, s < R \\ -\theta w_t n_i & \text{if } s \geq R \end{cases} \quad (2.19)$$

$$T_{ist}^I = \begin{cases} 0 & \text{if } X_{ist} < \chi_1 \\ \tau_1(X_{ist} - \chi_1) & \text{if } \chi_1 \leq X_{ist} < \chi_2 \\ \tau_1 \chi_1 + \tau_2(X_{ist} - \chi_2) & \text{if } \chi_2 \leq X_{ist} < \chi_3 \\ \tau_1 \chi_1 + \tau_2(\chi_3 - \chi_2) + \tau_3(X_{ist} - \chi_3) & \text{if } \chi_3 \leq X_{ist} < \chi_4 \\ \vdots & \\ \tau_1 \chi_1 + \tau_2(\chi_3 - \chi_2) + \cdots + \tau_N(X_{ist} - \chi_N) & \text{if } \chi_N \leq X_{ist} \end{cases} \quad (2.20)$$

$$T_{ist}^q = \tau_q \left(\frac{p_t}{p_{t-1}} - 1 \right) q_{ist} \quad (2.21)$$

As above, τ_i is the marginal tax rate in bracket i , the bend points between brackets are denoted χ_i , and the consumption tax rate is denoted τ_c

The first-order conditions from the household's problem are:

$$\begin{aligned} & c_{ist}^{-\gamma} [(1 - \tau_c)(-1 + \tau_\delta \delta + \tau_{\Delta k} - \kappa |k_{i,s+1,t+1} - k_{ist}|)] \\ & + \beta \frac{\partial V_{s+1}^h(k_{i,s+1,t+1}, b_{i,s+1,t+1}, q_{i,s+1,t+1})}{\partial k_{i,s+1,t+1}} = 0 \\ & c_{ist}^{-\gamma} [(1 - \tau_c)(-1)] + \beta \frac{\partial V_{s+1}^h(k_{i,s+1,t+1}, b_{i,s+1,t+1}, q_{i,s+1,t+1})}{\partial b_{i,s+1,t+1}} = 0 \\ & c_{ist}^{-\gamma} [(1 - \tau_c)(-p_t)] + \beta \frac{\partial V_{s+1}^h(k_{i,s+1,t+1}, b_{i,s+1,t+1}, q_{i,s+1,t+1})}{\partial q_{i,s+1,t+1}} = 0 \end{aligned}$$

The envelope conditions are:

$$\begin{aligned}
& \frac{\partial V_{s+1}^h(k_{i,s+1,t+1}, b_{i,s+1,t+1}, q_{i,s+1,t+1})}{\partial k_{i,s+1,t+1}} \\
&= c_{ist}^{-\gamma} (1 - \tau_c) [1 + (1 - \tau_j)r_t + (\tau_\delta - 1)\delta + \tau_{\Delta k}] \\
& \frac{\partial V_{s+1}^h(k_{i,s+1,t+1}, b_{i,s+1,t+1}, q_{i,s+1,t+1})}{\partial b_{i,s+1,t+1}} \\
&= c_{ist}^{-\gamma} (1 - \tau_c) [1 + (1 - \tau_j)i_t] \\
& \frac{\partial V_{s+1}^h(k_{i,s+1,t+1}, b_{i,s+1,t+1}, q_{i,s+1,t+1})}{\partial q_{i,s+1,t+1}} \\
&= c_{ist}^{-\gamma} (1 - \tau_c) [p_t - \tau_q(p_t - p_{t-1} + (1 - \tau_j)\pi_t)]
\end{aligned}$$

The Euler equations are:

$$c_{ist}^{-\gamma} [(1 - \tau_c) (-1 + \tau_\delta \delta + \tau_{\Delta k} - \kappa |k_{i,s+1,t+1} - k_{ist}|)] \quad (2.22)$$

$$= \beta c_{i,s+1,t+1}^{-\gamma} (1 - \tau_c) [1 + (1 - \tau_j)r_{t+1} + (\tau_\delta - 1)\delta + \tau_{\Delta k}] \text{ for } E \leq s < S, \forall i$$

$$c_{ist}^{-\gamma} = \beta c_{i,s+1,t+1}^{-\gamma} [1 + (1 - \tau_j)i_{t+1}] \text{ for } 1 \leq s < S, \forall i \quad (2.23)$$

$$c_{ist}^{-\gamma} = \beta c_{i,s+1,t+1}^{-\gamma} \left[1 + (1 - \tau_q) \left(\frac{p_{t+1}}{p_t} - 1 \right) + (1 - \tau_j) \frac{\pi_t}{p_t} \right] \text{ for } 1 \leq s < S, \forall i \quad (2.24)$$

$$c_{ist}^{-\gamma} w_t (1 - \tau_j - \tau_p) = \eta \ell_{ist}^{-\xi}, \forall s, i \quad (2.25)$$

2.3.2 Firms

The firm's intertemporal profits are now:

$$\Pi_t = \sum_{u=0}^{\infty} d_{ut} \pi_{t+u}$$

where

$$d_{ut} \equiv \begin{cases} 1 & \text{if } u = 0 \\ \prod_{j=1}^u \frac{1}{1+i_{t+j}} & \text{otherwise} \end{cases}$$

We can write its problem as a dynamic program.

$$V^f(B_t, H_t) = \max \pi_t + \frac{1}{1+i_{t+1}} V^f(B_{t+1}, H_{t+1})$$

Profits are defined as:

$$\pi_t = (1 - \tau_f) \left[\begin{aligned} &(K_t + H_t)^\alpha (e^{gt} L_t)^{1-\alpha} - r_t K_t - w_t L_t - (1 + i_t) B_t \\ &+ B_{t+1} + (1 - \delta) H_t - H_{t+1} - H_{t+1} \psi \left\{ \frac{H_{t+1}}{H_t} \right\} \\ &+ \tau_\delta \delta H_t + \tau_{\Delta k} (H_{t+1} - H_t) \end{aligned} \right] \quad (2.26)$$

We also have the following constraint which indicates that new bonds are used to finance either capital rentals or expansion of the capital stock:

$$B_{t+1} = K_t + H_{t+1} - \frac{\kappa}{2} (H_{t+1} - H_t)^2 - \tau_{\Delta k} (H_{t+1} - H_t)$$

FOCs with respect to K_t, L_t and H_{t+1} are:

$$\begin{aligned} \alpha (K_t + H_t)^{\alpha-1} (e^{gt} L_t)^{1-\alpha} - r_t - 1 + \frac{1}{1+i_{t+1}} V_B^F(B_{t+1}, H_{t+1}) &= 0 \\ (1 - \alpha) (K_t + H_t)^{-\alpha} e^{(1-\alpha)gt} L_t^{-\alpha} - w_t &= 0 \\ 1 - 1 - \kappa |H_{t+1} - H_t| - \tau_{\Delta k} + \frac{1}{1+i_{t+1}} V_H^F(B_{t+1}, H_{t+1}) &= 0 \end{aligned}$$

Envelope conditions for H_t and B_t are:

$$\begin{aligned} V_B^F(B_t, H_t) &= -(1 + i_{t+1}) \\ V_H^F(B_t, H_t) &= (K_t + H_t)^{\alpha-1} (e^{gt} L_t)^{1-\alpha} + 1 - \delta \end{aligned}$$

Combining the above gives:

$$r_t = (K_t + H_t)^{\alpha-1} (e^{gt} L_t)^{1-\alpha} \quad (2.27)$$

$$w_t = (1 - \alpha) (K_t + H_t)^{-\alpha} e^{(1-\alpha)gt} L_t^{-\alpha} \quad (2.28)$$

$$1 + r_{t+1} - \delta = (1 + i_{t+1}) (\kappa |H_{t+1} - H_t| + \tau_{\Delta k}) \quad (2.29)$$

2.3.3 Market Clearing

Market-clearing conditions which determine r_t, w_t, i_t and p_t are:

$$K_t = \sum_{s=2}^S \sum_{i=1}^I \phi_i k_{ist} \quad (2.30)$$

$$L_t = \sum_{s=1}^S \sum_{i=1}^I \phi_i \ell_{ist} \quad (2.31)$$

$$B_t = \sum_{s=1}^S \sum_{i=1}^I \phi_i b_{ist} \quad (2.32)$$

$$1 = \sum_{s=1}^S \sum_{i=1}^I \phi_i q_{ist} \quad (2.33)$$

2.3.4 Government

Government collects the following amounts of tax revenue each period.

$$\begin{aligned} R_t = \sum_s \sum_i \phi_i \left[T_{ist}^p + T_{ist}^i + T_{ist}^q + \frac{\tau_c}{1-\tau_c} c_{ist} \right] - \tau_\delta (H_{t+1} + K_{t+1} - H_t - K_t) \\ - \tau_\delta \delta (H_t + K_t) \end{aligned} \quad (2.34)$$

2.3.5 Solution and Simulation

This version of the model has $9 + 9IS - 3I$ equations defined by replacing (2.12) - (2.14) with (2.16); replacing (2.4) & (2.5) with (2.22) - (2.25); adding (2.17), (2.18), (2.26); replacing (2.6) & (2.7) with (2.27) - (2.29); and replacing (2.15) with (2.34).

The variables are $K_t, L_t, w_t, r_t, \{c_{ist}\}_{s=1}^S, \{\ell_{ist}\}_{s=1}^S, \{k_{ist}\}_{s=2}^S, \{T_{ist}^p\}_{s=1}^S, \{T_{ist}^i\}_{s=1}^S, R_t, \{b_{ist}\}_{s=2}^S, \{q_{ist}\}_{s=2}^S, \{I_{ist}\}_{s=1}^S, \{X_{ist}\}_{s=1}^S, H_t, B_t, \pi_t$ and i_t .

The parameters of the model are $\alpha, \beta, \delta, \gamma, \xi, g, \{n_i\}, \{\phi_i\}, \tau_p, \chi_p, \{\tau_n\}_{n=1}^N, \{\chi_n\}_{n=1}^N, \tau_c, \tau_q, \tau_\delta, \tau \Delta k$ and κ .

2.4 Adding Demographics - Model 4

Our next step is to add demographic movements to the model. Rather than keeping the population distribution constant we allow it to evolve naturally over time.

We denote the number of households of type i age s in period t as N_{ist} . We assume that workers all become economically active at age E . For each age cohort a fraction ρ_s survive from period t into $t+1$. For each age cohort a fraction f_s of new children are added to next period's age 1 cohort. The distribution of these children over ability is assumed to be the previously described $\{\phi_i\}_{i=1}^I$. This gives us the following exogenous laws of motion for the population.

$$N_{i1,t+1} = \sum_s f_s N_{ist} \phi_i \quad (2.35)$$

$$N_{i,s+1,t+1} = N_{ist} \rho_s \quad (2.36)$$

Since workers become economically active at age E , rather than at age 1, we need to keep track of $I - E + 1$ cohorts' capital stocks, bond holdings and equity holdings. We assume that people in cohorts 1 through E do not work or consume (alternatively their consumption is included in that of their parents until age E).

We define the total working age population in period t as below.

$$N_t = \sum_{s=E}^S \sum_{i=1}^I N_{ist} \quad (2.37)$$

This model has unintentional bequests since households will die before age S . We assume their holdings of capital, bonds, and equity shares are distributed lump sum at the begining of the next peroid. The amounts of each of these lump-sum transfers is shown below.

$$T_t^k = \frac{1}{N_t} \sum_{s=E+1}^S \sum_{i=1}^I N_{ist} (1 - \rho_i) k_{i,s+1,t+1} \quad (2.38)$$

$$T_t^b = \frac{1}{N_t} \sum_{s=E+1}^S \sum_{i=1}^I N_{ist} (1 - \rho_i) b_{i,s+1,t+1} \quad (2.39)$$

$$T_t^q = \frac{1}{N_t} \sum_{s=E+1}^S \sum_{i=1}^I N_{ist} (1 - \rho_i) q_{i,s+1,t+1} \quad (2.40)$$

We add these lump sum transfers to the budget constraint and generate (2.41), which we substitute in place of (2.16)

$$c_{ist} = (1 - \tau_c) \left[\begin{aligned} &w_t \ell_{ist} n_i + (1 + r_t - \delta) k_{ist} + (1 + i_t) b_{ist} + p_t q_{ist} \\ &-\frac{\kappa}{2} (k_{i,s+1,t+1} - k_{ist})^2 - k_{i,s+1,t+1} - b_{i,s+1,t+1} - p_t q_{i,s+1,t+1} \\ &+ \tau_\delta \delta k_{ist} + \tau_{\Delta k} (k_{i,s+1,t+1} - k_{ist}) \\ &- T_{ist}^p - T_{ist}^i - T_{ist}^q + T_t^k + T_t^b + p_t T_t^q \end{aligned} \right] \quad (2.41)$$

We also add additional discounting to the household's problem and rewrite (2.22) - (2.24) as below:

$$c_{ist}^{-\gamma} [(1 - \tau_c) (-1 + \tau_\delta \delta + \tau_{\Delta k} - \kappa |k_{i,s+1,t+1} - k_{ist}|)] \quad (2.42)$$

$$= \beta \rho_s c_{i,s+1,t+1}^{-\gamma} (1 - \tau_c) [1 + (1 - \tau_j) r_{t+1} + (\tau_\delta - 1) \delta + \tau_{\Delta k}] \text{ for } E \leq s < S, \forall i$$

$$c_{ist}^{-\gamma} = \beta \rho_s c_{i,s+1,t+1}^{-\gamma} [1 + (1 - \tau_j) i_{t+1}] \text{ for } E \leq s < S, \forall i \quad (2.43)$$

$$c_{ist}^{-\gamma} = \beta \rho_s c_{i,s+1,t+1}^{-\gamma} \left[1 + (1 - \tau_q) \left(\frac{p_{t+1}}{p_t} - 1 \right) + (1 - \tau_j) \frac{\pi_t}{p_t} \right] \text{ for } E \leq s < S, \forall i \quad (2.44)$$

Market-clearing conditions (2.30) - (2.31) are replaced with the following.

$$K_t = \sum_{s=E+1}^S \sum_{i=1}^I N_{ist} k_{ist} \quad (2.45)$$

$$L_t = \sum_{s=E}^S \sum_{i=1}^I N_{ist} \ell_{ist} \quad (2.46)$$

$$B_t = \sum_{s=E+1}^S \sum_{i=1}^I N_{ist} b_{ist} \quad (2.47)$$

$$1 = \sum_{s=E+1}^S \sum_{i=1}^I N_{ist} q_{ist} \quad (2.48)$$

We have added $4 + IS$ new variables to the model: T_t^k , T_t^q , T_t^q , N_t and $\{N_{ist}\}_{s=1}^S$.

We have also eliminated $9(E - 1)$ variables: $\{c_{ist}\}_{s=E-1}^S$, $\{\ell_{ist}\}_{s=E-1}^S$, $\{k_{ist}\}_{s=E}^S$, $\{T_{ist}^p\}_{s=E-1}^S$, $\{T_{ist}^i\}_{s=E-1}^S$, $\{b_{ist}\}_{s=E}^S$, $\{q_{ist}\}_{s=E}^S$, $\{I_{ist}\}_{s=E-1}^S$ and $\{X_{ist}\}_{s=E-1}^S$.

This gives a total of $13 + 9(S - E + 1)I - 3I + IS$ variables.

We have also added the following parameters: $\{\rho_s\}_{s=1}^S$ and $\{f_s\}_{s=E}^S$

*** TO DO ***

- Add immigration - average into the cohort/type bin
- Let mortality and fertility vary by ability type as well as age.
- Allow for trends in these parameters

2.5 Adding Open Economy Interest Rate Responsiveness - Model 5

2.6 Adding Multiple Industries - Model 6

2.7 Adding Ability Switching - Model 7

3 Conclusion

TECHNICAL APPENDIX

References

- Evans, Richard W. and Kerk L. Phillips**, “Linearization about the Current State: A Computational Method for Approximating Nonlinear Policy Functions during Simulation,” Technical Report, Brigham Young University Department of Economics 2014.
- Zodrow, George R. and John W. Diamond**, *Handbook of CGE Modeling - Vol. I*, North Holland,