Dyanmic General Equilibrim Tax Scoring with Micro Tax Simulations *

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Abstract

This paper ...

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1 Introduction

2 Details of the Macro Model

We use a model based initially on that from Evans and Phillips (2014) and incorporate many of the features of Zodrow and Diamond (2013) which we refer to hereafter as the DZ model.

2.1 Baseline Model

For our first baseline model we take Evans and Phillips (2014) and add a leisure-labor decision, while removing the switching of ability from period to period. Hence all workers remain the same type throughout their lifetime. Agents live for S periods and exogenously retire in period R. This is a perfect foresight model.

Housholds maximize utility as given in the equation below.

$$U_{ist} = \sum_{u=0}^{S-s} \beta^u u(c_{i,s+u,t+u}, \ell_{i,s+u,t+u}); \text{ where } u(c,\ell) = \frac{c^{1-\gamma}-1}{1-\gamma} - \eta \frac{\ell^{1-\xi}-1}{1-\xi} \tag{2.1}$$

U_{ist} is the remaining lifetime utility of a household with ability level i of age s in period t. c denotes consumption of goods and ℓ denotes labor supplied to the market.

The household faces the following set of budget constraints.

$$w_t \ell_{ist} n_i \geq c_{ist} + k_{i,s+1,t+1} \text{ for } s = 1, \forall i \tag{2.2}$$

$$w_t \ell_{ist} n_i + (1 + r_t - \delta) k_{ist} \geq c_{ist} + k_{i,s+1,t+1} \text{ for } 1 < s < S, \forall i \tag{2.3}$$

$$w_t \ell_{ist} n_i + (1 + r_t - \delta) k_{ist} \geq c_{ist} \text{ for } s = S, \forall i \tag{2.4}$$

k_{ist} is the holdings of capital by household of type i coming due in period t when the household is age s. w is the wage rate, r denotes the return on savings, n denotes the effective labor productivity of the houshold.

The Euler equations from this maximization problem are given below.

$$c_{ist}^{-\gamma} = \beta c_{i,s+1,t+1}^{-\gamma}(1 + r_{t+1} - \delta) \text{ for } 1 \leq s < S, \forall i \tag{2.5}$$

$$c_{ist}^{-\gamma} w_t = \eta \ell_{ist}^{-\xi}, \forall s, i \tag{2.6}$$

Firms produce using a Cobb-Douglas production function each period and maximize profits as shown below:

$$\Pi_t = K_t^{\alpha}(e^{gt} L_t)^{1-\alpha} - r_t K_t - w_t L_t \tag{2.7}$$

The profit maximizing conditions are:

$$r_t = \alpha K_t^{\alpha-1}(e^{gt} L_t)^{1-\alpha} \tag{2.8}$$

$$w_t = (1-\alpha) K_t^{\alpha} e^{(1-\alpha)gt} L_t^{-\alpha} \tag{2.9}$$

Market-clearing conditions require the following:

$$K_t = \sum_{s=2}^{S} \sum_{i=1}^{I} \phi_i k_{ist} \tag{2.10}$$

$$L_t = \sum_{s=1}^{S} \sum_{i=1}^{I} \phi_i \ell_{ist} \tag{2.11}$$

ϕ_i is the proportion of type i in the total population of workers.

This model can be simulated using either the TPI or AMF method described in Evans and Phillips (2014).

2.2 Adding Taxes on the Household

The social security payroll tax paid or benefit received is caclualted as follows.

$$T_{ist}^{P} = \begin{cases} \tau_P w_t \ell_{ist} n_i & \text{if } w_t \ell_{ist} n_i < \chi_P, s < R \\ \tau_P \chi_P & \text{if } w_t \ell_{ist} n_i \geq \chi_P, s < R \\ -\theta w_t n_i & \text{if } s \geq R \end{cases} \tag{2.12}$$

τ_P is the payroll tax rate and χ_P is the payroll tax ceiling.

Income is $w_t \ell_{ist} n_i + (r_t - \delta) k_{ist}$. Define $D\{w\ell n + (r-\delta)b, \Omega\}$ as the exemptions and benefits claimed as a function of income and other variables, Ω. Adjusted gross income is $X_{ist} \equiv w_t \ell_{ist} n_i + (r_t - \delta) k_{ist} - D\{w_t \ell_{ist} n_i + (r_t - \delta) k_{ist}, \Omega_{ist}\} - \tau_\delta \delta k_{ist}$. The final term is a capital depreciation allowance at rate τ_δ. We have fit this D function to the data for 2011 using a polynomial function. Income tax paid is defined as follows.

$$T^I_{ist} = \begin{cases} 0 & \text{if } X_{ist} < \chi_1 \\ \tau_1 (X_{ist} - \chi_1) & \text{if } \chi_1 \leq X_{ist} < \chi_2 \\ \tau_1 \chi_1 + \tau_2 (X_{ist} - \chi_2) & \text{if } \chi_2 \leq X_{ist} < \chi_3 \\ \tau_1 \chi_1 + \tau_2 (\chi_3 - \chi_2) + \tau_3 (X_{ist} - \chi_3) & \text{if } \chi_3 \leq X_{ist} < \chi_4 \\ \vdots & \\ \tau_1 \chi_1 + \tau_2 (\chi_3 - \chi_2) + \cdots + \tau_N (X_{ist} - \chi_N) & \text{if } \chi_N \leq X_{ist} \end{cases} \tag{2.13}$$

τ_i is the marginal tax rate in bracket i, the bend points between brackets are denoted χ_i.

The consumption tax rate is denoted τ_c

The household faces the following set of budget constraints.

$$c_{ist} = (1 - \tau_c) \left[w_t \ell_{ist} n_i - k_{i,s+1,t+1} - T^p_{ist} - T^i_{ist} \right] \tag{2.14}$$

$$c_{ist} = (1 - \tau_c) \left[w_t \ell_{ist} n_i + (1 + r_t - \delta) k_{ist} - k_{i,s+1,t+1} - T^p_{ist} - T^i_{ist} \right] \tag{2.15}$$

$$c_{ist} = (1 - \tau_c) \left[w_t \ell_{ist} n_i + (1 + r_t - \delta) k_{ist} - T^p_{ist} - T^i_{ist} \right] \tag{2.16}$$

2.3 Adding Taxes on Firms

We nedd to allow firms to acquire capital by renting it as above, or by accumulating their own capital and paying dividends, or by issuing bonds. How do we determine the proportions?

Imagine a financial intermediary which takes deposits from households and invests these in bonds and equities. Each houshold puts deposits of b_{ist} in the intermediary. The intermediary pool these to get total funds of B_t. It then allowcates a fraction ξ_t

to bonds and the remainder to equities.

$$B_t = \xi_t \sum_i \sum_s b_{ist} \tag{2.17}$$

$$P_t E_t = (1 - \xi_t) \sum_i \sum_s b_{ist} \tag{2.18}$$

Next period the intermediary recieves principal and interest on the bonds and dividends on the equties. So that it's total assests are $B_t(1 + i_{t+1}) + (\pi_t + P_{t+1})E_t$. The intermediate must pay a tax on interest income (T^B), a tax on dividends (T^π), and a capital gains tax (T^E).

$$T_t^B = \tau_B i_t B_{t-1} \tag{2.19}$$

$$T_t^\pi = \tau_\pi \pi_t E_{t-1} \tag{2.20}$$

$$T_t^E = \tau_E (\tfrac{P_t}{P_{t-1}} - 1) E_{t-1} \tag{2.21}$$

The intermediary chooses ξ_t given it's deposits to maximize its return:

$$R_{t+1} = \xi_t i_{t+1}(1 - \tau_B) + (1 - \xi_t) \left[\pi_{t+1}(1 - \tau_\pi) + \left(\tfrac{P_{t+1}}{P_t} - 1 \right) (1 - \tau_E) \right] \tag{2.22}$$

The necessary condition for this maximization is:

$$i_{t+1}(1 - \tau_B) = \left[\pi_{t+1}(1 - \tau_\pi) + \left(\tfrac{P_{t+1}}{P_t} - 1 \right) (1 - \tau_E) \right] \tag{2.23}$$

We assume firms and households pay a percent quadratic capital adjustment cost of $\psi \left\{ \frac{K_t}{K_{t-1}} \right\} = \frac{\kappa}{2} \left(\frac{K_t}{K_{t-1}} \right)^2$.

The typical household budget constraint is:

$$c_{ist} = (1 - \tau_c) \begin{bmatrix} w_t \ell_{ist} n_i + (1 + r_t - \delta) k_{ist} + (1 + R_t) b_{ist} \\ -k_{i,s+1,t+1} \psi \left\{ \frac{k_{i,s+1,t+1}}{k_{ist}} \right\} - b_{i,s+1,t+1} - T_{ist}^p - T_{ist}^i \end{bmatrix} \tag{2.24}$$

where income is now $w_t \ell_{ist} n_i + (r_t - \delta) k_{ist} + R_t b_{ist}$.

The firm's intertemporal profits are now:

$$\Pi_t = \sum_{u=0}^{\infty} d_{ut}\pi_{t+u} \tag{2.25}$$

$$d_{ut} \equiv \begin{cases} 1 & \text{if } u = 0 \\ \prod_{j=1}^{u} \frac{1}{1+i_{t+u+j}} & \text{otherwise} \end{cases} \tag{2.26}$$

$$\pi_t = \begin{array}{c} (K_t + H_t)^{\alpha}(e^{gt}L_t)^{1-\alpha} - r_t K_t - w_t L_t - (1+i_t)B_t \\ +B_{t+1} + (1-\delta)H_t - H_{t+1}\psi\left\{\frac{H_{t+1}}{H_t}\right\} \end{array} \tag{2.27}$$

FOCs with respect to K_t, L_t, H_{t+1} and B_{t+1} are:

$$r_t = \alpha(K_t + H_t)^{\alpha-1}(e^{gt}L_t)^{1-\alpha} \tag{2.28}$$

$$w_t = (1-\alpha)(K_t + H_t)^{-\alpha}e^{(1-\alpha)gt}L_t^{-\alpha} \tag{2.29}$$

$$1 + r_t - \delta = (1 + i_t)\kappa\left|\frac{H_{t+1}}{H_t}\right| \tag{2.30}$$

$$1 + i_t = 1 + i_t \tag{2.31}$$

3 Incorporating Feedbacks with Micro Tax Simulations

Follow this algorthim:

- Period 1
  - Use current IRS public use sample.
  - Run the following within-period routine
    * Do the static tax analysis of this sample, save the results
    * Summarize the public use sample by aggregating into bins over age and earnings ability
    * Use this as a starting point for the dynamic macro model
    * Get values for fundamental interest rates and effective wages for next period

- Period 2
  - Age the public use data demographically by one year.
  - Let wages and interest rates rise by the amounts predicted in the macro model.
  - Rerun the within-period routine
- Iterate over periods until end of forecast period is reached.

4 Calibration

4.1 Tax Bend Points

We use IRS data which summarizes individual tax returns for 2011 by 19 income categories and 4 filing statuses. For each filing status we fit the mapping from reported income into adjusted gross income (AGI) using a sufficiently high-order polynomial. We then use this function to solve for the income level which corresponds to each of the five bend points in the tax code for each filing type.

Table 1: AGI and Income Bend Points

AGI Bend Points

Tax rate	Married Joint	Married Separate	Head of Household	Single
10%	17,400	8700	12,400	8700
15%	70,700	35,350	47,350	35,350
25%	142,700	71,350	122,300	85,650
28%	217,450	108,725	198,050	178,650
33%	388,350	194,175	388,350	388,350

Corresponding Reported Income Bendpoints

Tax rate	Married Joint	Married Separate	Head of Household	Single
0%	5850	91	756	1435
10%	22,932	8591	12,911	9956
15%	75,181	34,592	47,023	36,021
25%	145,866	69,768	120,200	85,244
28%	219,162	106,245	194,176	176,270
33%	386,798	189,674	380,043	381,524

We then fit a bivariate probabililty density function over income and filing type from the data. For each bendpoint we calculate the probability density at that bendpoint and use these as weights in a weighted average over filing types to generate an aggregate bendpoint.

Table 2: Aggregated Bend Points

Tax rate	Bend Point
0%	2889
10%	15,116
15%	52,580
25%	114,552
28%	196,201
33%	380,657

5 Conclusion

TECHNICAL APPENDIX

References

Evans, Richard W. and Kerk L. Phillips, "Linearization about the Current State: A Computational Method for Approximating Nonlinear Policy Functions during Simulation," Technical Report, Brigham Young University Department of Economics 2014.

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