Modeling Bequests

August 21, 2014

Kerk's formulation of lifetime utility, rewritten:

$$U_{j,s,t} = \sum_{u=0}^{S-s} \beta^u \prod_{v=0}^{u} e^{1-p_{j,v+s+E}} u(c_{j,s+u,t+u}, n_{j,s+u,t+u}) + \beta^{S-s} \prod_{v=s}^{S} e^{1-p_{j,v+E}} \chi_b \left(\frac{(b_{j,S+1,t+S-s+1})^{1-\sigma} - 1}{1-\sigma} \right) \tag{0.1}$$

where the per period utility flow is given by:

$$u(c_{j,s,t}, n_{j,s,t}) = \frac{(c_{j,s,t} - 1)^{1-\sigma}}{1-\sigma} + \chi_n e^{g_y t(1-\sigma)} \frac{(\tilde{l} - n_{j,s,t})^{1-\eta}}{1-\eta} \tag{0.2}$$

The Lagrangian is thus:

$$\begin{aligned}
\mathcal{L}_{j,s,t} = \sum_{u=0}^{S-s} \Bigg(& \beta^u \prod_{v=0}^{u} e^{1-p_{j,v+s+E}} u(c_{j,s+u,t+u}, n_{j,s+u,t+u}) + \\
& \lambda_{j,s+u,t+u} \big((1 - \tau^c_{j,t+u})[(1 + r_{t+u}) b_{j,s+u,t+u} + w_{t+u} e_{j,s+u} n_{j,s+u,t+u} - b_{j,s+u+1,t+u+1} - \\
& T^P_{j,s+u,t+u} - T^I_{j,s+u,t+u} + B_{j,s+u,t+u}] - c_{i,s+u,t+u} \big) \Bigg) + \beta^{S-s} \prod_{v=s}^{S} e^{1-p_{j,v+E}} \chi_b \left(\frac{(b_{j,S+1,t+S-s+1})^{1-\sigma} - 1}{1-\sigma} \right)
\end{aligned} \tag{0.3}$$

Note: I don't know what T^I and T^P are. I'm going to assume T^P are taxes paid and that they are a function of capital and labor income (separately). I'll just assume that T^I is not a function of income for now.

Note: I indexed the consumption tax by year and type, allowing it to vary over time and by ability type (for example, because different groups of people consume different groups of goods - like low income disproportionately consume goods like alcohol and tobacco products which have high excise taxes).

The FOCs are thus:

$$\frac{\partial \mathcal{L}_{j,s,t}}{\partial c_{j,s+u,t+u}} \implies c^{-\sigma}_{j,s+u,t+u} = \lambda_{j,s+u,t+u}, \quad \forall s, t, u \tag{0.4}$$

$$\frac{\partial \mathcal{L}_{j,s,t}}{\partial n_{j,s+u,t+u}} \implies \chi_n e^{g_y(t+u)(1-\sigma)}(\tilde{l} - n_{j,s+u,t+u})^{-\eta} = \lambda_{j,s+u,t+u}\left[(1-\tau^c_{j,t+u})w_{t+u}e_{j,s+u}\left(1 - \frac{\partial T^P_{j,s+u,t+u}}{\partial y^l}\right)\right] \quad , \quad \forall s,t,u \tag{0.5}$$

Do I need $e^{g_y t(1-\sigma)}$ or $e^{g_y(t+u)(1-\sigma)}$ in the above? I guess the latter should be there, but I don't yet understand this (I get the idea, but haven't worked out the math).

Note: $\frac{\partial T^P_{j,s+u,t+u}}{\partial y^l}$ is the marginal tax rate w.r.t. labor income.

$$\begin{aligned} \frac{\partial \mathcal{L}_{j,s,t}}{\partial b_{j,s+u+1,t+u+1}} \implies & \lambda_{j,s+u,t+u}(1-\tau^c_{j,t+u}) = \beta e^{1-p_{j,s+u+1+E}}(1-\tau^c_{j,t+u+1}) \\ & \times \left[\lambda_{j,s+u+1,t+u+1}\left(1 + \left(1 - \frac{\partial T^P_{j,s+u+1,t+u+1}}{\partial y^c}\right)r_{t+u}\right)\right] \\ & , \quad \forall s,t,u \text{ except for } s+u=S \end{aligned} \tag{0.6}$$

Note: $\frac{\partial T^P_{j,s+u,t+u}}{\partial y^c}$ is the marginal tax rate w.r.t. capital income.

For bequests we have:

$$\frac{\partial \mathcal{L}_{j,S,t+S-s+1}}{\partial b_{j,S,t+S-s+1}} \implies \lambda_{j,S,t+S-s}(1-\tau^c_{j,t+S-s}) = \chi_b(b_{j,S+1,t+S-s})^{-\sigma} \tag{0.7}$$

Can we calibrate the weight of bequests using consumption/savings data of the very old? It seems that our model would suggest that χ^b is a function of the ratio of consumption to savings when 80 years old, right?