Simplest model where firms own capital

Supply Side v2.0

Firms maximize firm value, which is the net present value of dividends less equity issuance:

$$V_t = \sum u = t^{\infty} \prod_{\nu=t} u \left(\frac{1}{1 + r_\nu} \right) DIV_u - VN_u, \tag{1}$$

where DIV_u are dividend distributions in period u and VN is new equity issuance in period u. The firm's cash flow constraint will give us the value of dividends distributed after investment and earnings (a function of capital and labor) are determined:

$$EARN_u + VN_u = DIV_u + I_u \tag{2}$$

Here, I_u is investment in capital in period u (where we have the price of capital normalized to 1). Earnings are defined as revenues from the sale of production goods less the price of variable inputs (i.e., labor):

$$EARN_u = p_u F(K_u, L_u) - w_u L_u \tag{3}$$

Plugging Equation 3 and the law of motion for the capital stock into Equation 2 yields:

$$pF(K_u, L_u) - w_u L_u + VN_u = DIV_u + K_{u+1} - (1 - \delta)K_u \tag{4}$$

We can not find the Belman Equation for the firm's problem by solving for DIV from Equation 5 and substituting the result into Equation 1:

$$V(K; r, w) = pF(K, L) - wL - K' + (1 - \delta)K + \frac{1}{1 + r} V(K'; r', w') \tag{5}$$

The two FOCs are:

$$\frac{\partial V(K; r, w)}{\partial K'} : 1 = \frac{1}{1 + r} \frac{\partial V(K'; r', w')}{\partial K'} \tag{6}$$

$$\frac{\partial V(K; r, w)}{\partial L} : w = \frac{\partial V(K; r, w)}{\partial L} \tag{7}$$

The envelope condition allows us to write 10 as:

$$\frac{\partial V(K; r, w)}{\partial K'} : 1 = \frac{1}{1 + r} \left[\frac{\partial F(K', L')}{\partial K'} + 1 - \delta \right] \tag{8}$$

Parameterization

We will assume that the production function for the firm is a Constant Elasticity of Substitution (CES) function:

$$F(K, L) = \left(\gamma^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon - 1}{\varepsilon}} + (1 - \gamma)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon - 1}}, \tag{9}$$

where ε is the elasticity of substation between capital and labor and γ is the share parameter for the production function (?).

Given this parameterization, our two FOCs become:

$$r + \delta = \left(\gamma^{\frac{1}{\varepsilon}} K'^{\frac{\varepsilon - 1}{\varepsilon}} + (1 - \gamma)^{\frac{1}{\varepsilon}} L'^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{1}{\varepsilon - 1}} \gamma^{\frac{1}{\varepsilon}} K'^{\frac{-1}{\varepsilon}} \tag{10}$$

(double check the timing on the interest rate - not sure if it should be the current period or one period ahead - depends upon the timing convention for our notation)

$$w = \left(\gamma^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon - 1}{\varepsilon}} + (1 - \gamma)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon - 1}{\varepsilon}} \right)^{\frac{1}{\varepsilon - 1}} (1 - \gamma)^{\frac{1}{\varepsilon}} L^{\frac{-1}{\varepsilon}} \tag{11}$$

I think you should just be able to substitute in these two firm FOCs for the static firm FOCs. The price out capital and consumption are the same since they are the same good - and we can normalize their price to 1. Since firms hold capital, we won't have a capital market cleaning condition (I don't think). But we will have an asset market clearing condition. This will be that $B_t = V_t$. We should be able to solve the infinite geometric series to get the value of $V(\bar{K})$:

$$\begin{aligned} V(\bar{K};\bar{r},\bar{w}) &= F(\bar{K},\bar{L}) - \bar{w}\bar{L} - \delta\bar{K} + \frac{1}{1+\bar{r}}V(\bar{K};\bar{r},\bar{w}) \\ \implies V(\bar{K};\bar{r},\bar{w}) &= \frac{F(\bar{K},\bar{L}) - \bar{w}\bar{L} - \delta\bar{K}}{\bar{r}}(1+\bar{r}) \end{aligned} \tag{12}$$

To solve for V_t outside of the SS, we'll have to use backwards induction. So, one period before the SS, we have:

$$V(K_{T-1};r_{T-1},w_{T-1}) = F(K_{T-1},L_{T-1}) - w_{T-1}L_{T-1} - \bar{K} + (1-\delta)K_{T-1} + \frac{1}{1+\bar{r}}V(\bar{K};\bar{r},\bar{w}) \tag{13}$$