Tax Multipliers in a DSGE Model*

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Abstract

This paper analyzes the stabilization properties of tax cuts in a DSGE model.

Consumption taxes have long been favored by tax economists for their relative efficiency vis-a-vis taxes on capital and labor. This paper attempts to argue that consumption taxes should also be favored by macroeconomists for their stabilization properties. Cuts to consumption taxes can quickly and effectively stimulate aggregate demand. The following analysis studies the quantitative impact of tax cuts in a dynamic schocastic general equilibrium (DSGE) model. In addition to consumption taxes, the model nests other tax policies, such as investment tax credits and changes to expensing and depreciation policies. These policies target investment by lowering the after-tax cost of capital. Historically, such instruments have been used by the U.S. to provide counter-cyclical fiscal policy (e.g. the Economic Recovery Act of 1981 increased the ability of businesses to take advantage of the investment tax credit). Despite their use, such policies have not been studied in a DSGE environment.

The goal of this paper is to study specific tax policies in a DSGE environment. In a sense, bring in the tax policies favored by tax economists into a model used by macroeconomists. Recently, a number of researchers have used DSGE models to study the government spending multiplier (e.g., Christiano, Eichenbaum and Rebelo (2010)), but less work has been done on tax multipliers. Those who provide insights on tax multipliers (e.g., Zubairy (2010), Kumhof, Coenen, Muir, Freedman, Mursula, Erceg, Furceri, Lalonde, Linde, Mourougane, Roberts, Laxton, de Resende, Roeger, Snudden, Trabandt and in 't Veld (2010)) focus on multipliers associated with broad taxes on capital and labor income. Such tax cuts are generally not the most effective counter-cyclical policies. We are the first to provide quantitative insights into the the size of the multipliers associated with tax cuts that directly target consumption and investment.

Two recent, but related, events have spurred interest in consumption taxes; the 2007-2009 recession and the large federal budget deficits. In a 2008 *Financial Times* aricle Kotlikoff and Leamer (2008) argue that a national sales tax holiday would be an especially effective response to the dip in aggregate demand experienced during the 2007-2009 financial crisis. While possible, it is difficult to implement such a policy without a national sales tax. Recent debate about the fiscal sustainability of the federal government has brough much attention to value added taxes (VAT) as a way to increase revenue with minimal efficiency

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costs. We hope to show that a tax on consumption has a second positive attribute - a non-zero tax on consumption allows policy makers important flexibility for macro stabilization policies.

Maybe cite some of the micro evidence of effects of sales tax holidays...

To cite: - Romer and Romer (AER?) find tax multiplier of 3 with reduced form estimates - More on multipliers: http://www.frbsf.org/publications/economics/letter/2009/el2009-20.html - Nov 24, 2008 - UK cuts VAT from 17.5 to 15% to respond to downturn - find more press on this. And compare model results to actuals. Note that 15% is the lowest rate allowed by EU law. VAT then raised to 20% for a few years to cut budget deficits (part of 2010 budget plan). - Note this for what is the Frisch labor elasticity: http://www.econ.umn.edu/ vr0j/ec8503-10/daolufrisch.pdf

Stress:

- lower dwl due to consumption tax than other forms of tax
- faster impact of consumption tax cut
- consumption beats spending because people get to choose what to spend it on (though we can't easily model this)
- current focus on the VAT, which is a consumption tax. Most of talk is about less distortionary way to raise revenue, but it would also allow to a good fiscal stabilizer.

Other novelties:

- build model do can handle other tax cuts like bonus depreciation (something simple so don't have to take account of vintage) and investment tax credits- see what the multiplier is for these in short and long run
- Be best if could have heterogenous firms who vary in productivity and have price stickiness. Build in corporate income tax, div tax, cap gains tax in more realistic detail (now all lumped in tax on capital).

1 Environment

1.1 Households

Representative household.

HH maximizes:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}, l^s_{i,t}, g_{i,t}) \tag{1.1}$$

We'll assume a per-period utility of the form:

$$u(c_{i,t}, l^s_{i,t}, g_{i,t}) = u^b_t \left(\frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \theta \frac{(l^s_{i,t})^{1+\varphi}}{1+\varphi} + \chi^g \frac{g_t^{1-\sigma_g}}{1-\sigma_g} \right) \tag{1.2}$$

Where u^b_t is a shock to the agent's impatience, c is consumption of private good and services, g consumption of public goods and services, and l^s is labor supply. The parameter σ is the coefficient of relative risk aversion for consumption of private goods and services, and the parameter σ_g is the analogue for consumption of public goods and services. The parameter φ is the inverse of the Frisch elasticity of labor supply. θ and χ^g give the utility weight places on leisure and public goods consumption relative to private goods consumption.

$$log(u_t^b) = \rho_b log(u_{t-1}^b) + \varepsilon_t^b \tag{1.3}$$

where $\varepsilon_t^b \sim N(0, \sigma_b)$.

Households choose consumption, investment, and labor supply, subject to the real budget constraint:

$$c_{i,t}(1+\tau_t^c) + i_{i,t} + b_{i,t} =$$
$$(1-\tau_t^k)r_t^k v_{i,t} k_{i,t-1} + \tau_t^k \delta_t^{tau} k_{i,t-1}^{\tau} + \tau_t^k i_{i,t} e_t^{\tau} + \tau_t^{ic} i_{i,t} + (1-\tau_t^l) w_t l_{i,t}^s + \frac{(1+(r_t(1-\tau_t^i)))b_{i,t-1}}{\pi_t} + (1-\tau_t^d) d_{i,t} + x \tag{1.4}$$

Where all variables pre-determined at time t have subscripts strictly less than time t. c represents consumption, i investment, b government bond holdings, π inflation ($\pi = \frac{P_t}{Pt-1}$), l labor supply, k capital stock, x government transfers, d dividends, w the nominal wage rate, v intensity of capital utilization. Taxes τ^i, τ^l, τ^k, τ^d are taxes on interest income, labor income, capital income, and dividend income. τ_c is the tax rate on consumption. r is the nominal interest rate on government bonds and r^k is the rental rate on capital. k^τ is the tax basis for the household's capital stock, δ^τ is the rate of depreciation for tax purposes, e^τ is the rate of expensing for tax purposes, and τ^{ic} is the investment tax credit.

The law of motion of the household's capital stock is:

$$k_{i,t} = (1-\delta(v_t))k_{i,t-1} + i_{i,t}\left[1 - S\left(\frac{i_{i,t}}{i_{i,t-1}}\right)\right] \tag{1.5}$$

I'll assume that the investment adjustment cost function takes the form $S\left(\frac{i_{i,t}}{i_{i,t-1}}\right) = \frac{\kappa}{2}\left(\frac{u_t^i i_{i,t}}{i_{i,t-1}} - 1\right)^2$. Here, κ is the adjustment cost parameter and u_t^i is a investment specific efficiency shock. Such a functional form is common in the literature (e.g., Smets and Wouters (2003), Christiano et al. (2010), Zubairy (2010), et al) and has the nice properties that in the steady state $S = 0$, $S' = 0$ and $S'' > 0$, and that in a deterministic steady state, the adjustment costs are zero.

$$log(u_t^i) = \rho_i log(u_{t-1}^i) + \varepsilon_t^i \tag{1.6}$$

where $\varepsilon_t^i \sim N(0, \sigma_i)$.

Assume also that $\delta(v_t)$ takes the form: $\delta(v_{i,t}) = \delta_0 + \delta_1(v_{i,t} - 1) + \frac{\delta_2}{2}(v_{i,t} - 1)^2$. This means that in the deterministic steady state, we can calibrate δ such that $v_{i,t} = 1$, and $\delta(v) = \delta_0$ (see Traum and Yang (2010) on this point). The steady state will also imply the relationship between δ_0 and δ_1, namely $\delta_1 = \frac{1}{\beta} - 1 + \delta_0$.

The law of motion of the household's tax basis for it's capital stock is:

$$k_{i,t}^{\tau} = (1-\delta_t^{\tau})k_{i,t-1}^{\tau} + i_{i,t}(1-e_t^{\tau}) \tag{1.7}$$

Note that in general $\delta_t^\tau \neq \delta_0$, and typically tax depreciation is more accelerated than economic depreciation, lowering the user cost of capital.

1.2 Firms

1.2.1 Final Goods Producers

Final goods producers aggregate differentiated inputs to produce a homogenous output. The Dixit-Stiglitz aggregation function is given by;

$$Y_t = \left(\int_0^1 y_{i,t}^{\frac{1}{1+\eta_{p,t}}} di \right)^{1+\eta_{p,t}}, \tag{1.8}$$

where

$$log(\eta_{p,t}) = \rho_\eta log(\eta_{p,t-1}) + (1-\rho_\eta) log(\eta_p) + \varepsilon_t^\eta \tag{1.9}$$

is the stochastic price markup in the intermediate goods market. We assume that $\varepsilon_t^\eta \sim N(0, \sigma_\eta)$

Maximizing profits (or minimizing costs) subject to Equation 1.53 results in demand for intermediate input $y_{i,t}$ of:

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t} \right)^{-\frac{1+\eta_{p,t}}{\eta_{p,t}}} Y_t, \tag{1.10}$$

where the price of a unit of the final, homogenous output, P_t, is given by:

$$P_t = \left(\int_0^1 p_{i,t}^{-\frac{1}{\eta_{p,t}}} di \right)^{-\eta_{p,t}} \tag{1.11}$$

1.2.2 Working Through the Final Goods Producer's problem

Objective function:

$$\max_{y_{i,t}} P_t Y_t^d - \int_0^1 p_{i,t} y_{i,t} di \tag{1.12}$$

The FOC's are:

$$p_t(1+\eta_{p,t}) \left(\int_0^1 y_{i,t}^{\left(\frac{1}{1+\eta_{p,t}}\right)} di \right)^{\eta_{p,t}} \left(\frac{1}{1+\eta_{p,t}} \right) y_{i,t}^{\frac{1}{1+\eta_{p,t}}-1} - p_{i,t} = 0, \forall i \tag{1.13}$$

Dividing the FOCs for intermediate inputs i and j $\implies$

$$\frac{p_{i,t}}{p_{j,t}} = \left(\frac{y_{i,t}}{y_{j,t}} \right)^{\frac{1}{1+\eta_{p,t}}-1} = \left(\frac{y_{i,t}}{y_{j,t}} \right)^{\frac{-\eta_{p,t}}{1+\eta_{p,t}}} \tag{1.14}$$

Rearranging:

$$\begin{aligned} &\implies p_{i,t} = \left(\frac{y_{j,t}}{y_{i,t}} \right)^{\frac{\eta_{p,t}}{1+\eta_{p,t}}} p_{j,t} \\ &\implies p_{i,t} = y_{j,t}^{\frac{\eta_{p,t}}{1+\eta_{p,t}}} p_{j,t} y_{i,t}^{\frac{-\eta_{p,t}}{1+\eta_{p,t}}} \\ &\implies p_{i,t} y_{i,t} = p_{j,t} y_{j,t}^{\left(\frac{\eta_{p,t}}{1+\eta_{p,t}}\right)} y_{i,t}^{\frac{1}{1+\eta_{p,t}}} \end{aligned} \tag{1.15}$$

Then integrate this condition to yield:

$$\begin{aligned}\int_0^1 p_{i,t}y_{i,t}di &= p_{j,t}y_{j,t}^{\left(\frac{\eta_{p,t}}{1+\eta_{p,t}}\right)}\int_0^1 y_{i,t}^{\frac{1}{1+\eta_{p,t}}}di \\ &= p_{j,t}y_{j,t}^{\left(\frac{\eta_{p,t}}{1+\eta_{p,t}}\right)}(Y_t)^{\frac{1}{1+\eta_{p,t}}}\end{aligned} \tag{1.16}$$

The zero profit condition implies $P_tY_t = \int_0^1 p_{i,t}y_{i,t}di$. Plugging this into the above and we find:

$$\begin{aligned}&P_tY_t = p_{j,t}y_{j,t}^{\left(\frac{\eta_{p,t}}{1+\eta_{p,t}}\right)}(Y_t)^{\frac{1}{1+\eta_{p,t}}} \\ &\implies P_tY_t = p_{j,t}y_{j,t}^{\frac{\eta_{p,t}}{1+\eta_{p,t}}}(Y_t)^{\frac{1}{1+\eta_{p,t}}} \\ &\implies P_t = p_{j,t}y_{j,t}^{\frac{\eta_{p,t}}{1+\eta_{p,t}}}(Y_t)^{\frac{-\eta_{p,t}}{1+\eta_{p,t}}}\end{aligned} \tag{1.17}$$

Which implies the demand function (just rearranging terms):

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} Y_t, \forall i \tag{1.18}$$

To find P_t use the zero profit condition:

$$\begin{aligned}P_tY_t &= \int_0^1 p_{i,t}y_{i,t}^{\left(\frac{\eta_{p,t}}{1+\eta_{p,t}}\right)}(Y_t)^{\frac{1}{1+\eta_{p,t}}}di \\ \implies P_tY_t &= \int_0^1 p_{i,t}\left(\left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} Y_t\right)^{\left(\frac{\eta_{p,t}}{1+\eta_{p,t}}\right)}(Y_t)^{\frac{1}{1+\eta_{p,t}}}di \\ \implies P_tY_t &= \int_0^1 p_{i,t}\left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} Y_t di \\ \implies P_tY_t &= \int_0^1 p_{i,t}p_{i,t}^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} P_t^{\frac{(1+\eta_{p,t})}{\eta_{p,t}}} Y_t di \\ \implies P_tY_t &= \int_0^1 p_{i,t}^{\frac{-1}{\eta_{p,t}}} di\, Y_t P_t^{\frac{1+\eta_{p,t}}{\eta_{p,t}}} \\ \implies P_t &= \int_0^1 p_{i,t}^{\frac{-1}{\eta_{p,t}}} di\, P_t^{\frac{1+\eta_{p,t}}{\eta_{p,t}}} \\ \implies P_t^{\frac{-1}{\eta_{p,t}}} &= \int_0^1 p_{i,t}^{\frac{-1}{\eta_{p,t}}} di \\ \implies P_t &= \left(\int_0^1 p_{i,t}^{\frac{-1}{\eta_{p,t}}} di\right)^{-\eta_{p,t}}\end{aligned} \tag{1.19}$$

1.2.3 Intermediate Goods Producers

Each intermediate-goods producing firm has a monopoly on its heterogeneous output. Following Calvo (1983), firms can set prices optimally each period with probability $(1 - \omega_p)$. Firm's that cannot reset prices must rent labor and capital inputs to meet demand at last period's price, indexed to past inflation, $p_{i,t} = p_{i,t-1}\pi_{t-1}^{\chi_p}$. Each intermediate good's producer has the same production function and technology:

$$y_{i,t} = z_t \tilde{k}_{i,t}^{\alpha} l_{i,t}^{1-\alpha} - \Phi z_t, \tag{1.20}$$

where $\tilde{k}_{i,t} = v_t k_{i,t-1}$ are the effective units of capital rented from the households, $l_{i,t}$ is the effective labor rented from households, z_t is a serially correlated shock to firm productivity (which affect all intermediate-goods producers equally). Φ are fixed costs of production that are proportional to the technology shock.

We assume z_t follows the following AR(1) process:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_t^z \quad \text{where} \quad \varepsilon_t^z \sim N(0, \sigma_z), \quad \text{and} \quad \rho_z \in [0,1) \tag{1.21}$$

There is no entry or exit.

The problem of intermediate-goods producers who can optimally reset their prices can be solved in two stages. In the first, the demand for factor inputs are found, taking the rental rate for capital, r_t^k, and the wage rate, w_t, as given. This stage yields the relative factor demands and the marginal cost of production. In the second stage, firm's take their marginal cost function as given, and choose the their, $p_{i,t}$ to maximize their expected, discounted profits.

The firms' per-period profit function takes the following form:

$$d_{i,t} = p_{i,t} y_{i,t} - r_t^k \tilde{k}_{i,t} - w_t l_{i,t} \quad \forall i, t \tag{1.22}$$

Thus, in the first stage, the firm solves:

$$\min_{\tilde{k}_{i,t}, l_{i,t}} \; w_t l_{i,t} + r_t \tilde{k}_{i,t} \tag{1.23}$$

subject to the supply curve:

$$y_{i,t} = \begin{cases} z_t \tilde{k}_{i,t}^{\alpha} l_{i,t}^{1-\alpha} - \Phi z_t, & \text{if } z_t \tilde{k}_{i,t}^{\alpha} l_{i,t}^{1-\alpha} > \Phi z_t. \\ 0, & \text{otherwise.} \end{cases} \tag{1.24}$$

FOC's:

$$\frac{\partial E}{\partial \tilde{k}_{i,t}} : \frac{\alpha p_{i,t} y_{i,t}}{\tilde{k}_{i,t}} - r_t^k = 0 \tag{1.25}$$

$$\frac{\partial E}{\partial l_{i,t}} : \frac{(1-\alpha) p_{i,t} y_{i,t}}{l_{i,t}} - w_t = 0 \tag{1.26}$$

Dividing Equation 1.25 by Equation 1.26 and rearranging some terms, one gets:

$$\frac{\tilde{k}_{i,t}}{l_{i,t}} = \left(\frac{\alpha}{1-\alpha}\right) \frac{w_t}{r_t^k} \tag{1.27}$$

Plugging this ratio into Equation 1.20 and setting it equal to 1, one can find the amount of labor needed to produce one unit of output in terms of w_t, rt^k, z_t, and the parameter α.

Rearranging 1.27 to put $\tilde{k}_{i,t}$ in terms of $l_{i,t}$, factor input prices, and parameters then writing the cost function $(r^k_{t+j}\tilde{k}_{i,t+j}+w_{t+j}l_{i,t+j})$ in terms of $l_{i,t}$, factor input prices, and parameters, one can substitute in the equation for $l_{i,t}$ This yields the marginal cost of production in real terms (this procedure is described in Fernandez-Villaverde and Rubio-Ramirez (2006)):

$$mc_t = \frac{(r^k_t)^\alpha w_t^{1-\alpha}}{z_t}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} \tag{1.28}$$

N.B., the marginal cost function, mc_t does not depend upon i. This is because all firms receive the same technology shock and face the same factor input prices.

The second stage, were firms optimally choose price given the optimal choice of factor inputs, can be setup as a profit maximization problem, taking mc_t as given. Assuming an interior solution, the problem can be written as:

$$V = \max_{p_{i,t}} E_t\left(\sum_{j=0}^{\infty}\beta^j\omega_p^j\frac{\lambda_{t+j}}{\lambda_t}\left[\left(\prod_{s=1}^{j}\pi^{\chi_p}_{t+s-1}\frac{p_{i,t}}{P_{t+j}} - mc_{t+j}\right)y_{i,t+j}\right]\right), \tag{1.29}$$

subject to:

$$y_{i,t+j} = \left(\prod_{s=1}^{j}\pi^{\chi_p}_{t+s-1}\frac{p_{i,t}}{P_{t+j}}\right)^{-\left(\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}\right)} Y_{t+j} \tag{1.30}$$

where Y_{t+j} is the demand for final goods in period $t+j$. N.B. there is no $p_{i,t+j}$ because prices are fixed at the value chosen in period t. Note also that the value of profits are weighted by the shadow value of income for the households (since all profits are returned to households).

Rewriting the second-stage problem gives:

$$\begin{aligned}
& V = \max_{p_{i,t}} E_t\left(\sum_{j=0}^{\infty}\beta^j\omega_p^j\frac{\lambda_{t+j}}{\lambda_t}\left[\left(\prod_{s=1}^{j}\pi^{\chi_p}_{t+s-1}\frac{p_{i,t}}{P_{t+j}} - mc_{t+j}\right)y_{i,t+j}\right]\right) \\
\implies & V = \max_{p_{i,t}} E_t\left(\sum_{j=0}^{\infty}\beta^j\omega_p^j\frac{\lambda_{t+j}}{\lambda_t}\left[\left(\prod_{s=1}^{j}\pi^{\chi_p}_{t+s-1}\frac{p_{i,t}}{P_{t+j}} - mc_{t+j}\right)\left(\prod_{s=1}^{j}\pi^{\chi_p}_{t+s-1}\frac{p_{i,t}}{P_{t+j}}\right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} Y_{t+j}\right]\right) \\
\implies & V = \max_{p_{i,t}} E_t\left(\sum_{j=0}^{\infty}\beta^j\omega_p^j\frac{\lambda_{t+j}}{\lambda_t}\left[\left(\prod_{s=1}^{j}\pi^{\chi_p}_{t+s-1}\frac{p_{i,t}}{P_{t+j}}\right)^{\frac{-1}{\eta_{p,t+j}}} - \left(\prod_{s=1}^{j}\pi^{\chi_p}_{t+s-1}\frac{p_{i,t}}{P_{t+j}}\right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} mc_{t+j}\right] Y_{t+j}\right) \\
\implies & V = \max_{p_{i,t}} E_t\left(\sum_{j=0}^{\infty}\beta^j\omega_p^j\frac{\lambda_{t+j}}{\lambda_t}\left[\left(\prod_{s=1}^{j}\pi^{\chi_p}_{t+s-1}\frac{p_{i,t}}{P_{t+j}}\right)^{\frac{-1}{\eta_{p,t+j}}} - \left(\prod_{s=1}^{j}\pi^{\chi_p}_{t+s-1}\frac{p_{i,t}}{P_{t+j}}\right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} mc_{t+j}\right] Y_{t+j}\right) \\
\implies & V = \max_{p_{i,t}} E_t\left(\sum_{j=0}^{\infty}\beta^j\omega_p^j\frac{\lambda_{t+j}}{\lambda_t}\left[\left(\prod_{s=1}^{j}\frac{\pi^{\chi_p}_{t+s-1}}{\pi_{t+s}}\frac{p_{i,t}}{P_t}\right)^{\frac{-1}{\eta_{p,t+j}}} - \left(\prod_{s=1}^{j}\frac{\pi^{\chi_p}_{t+s-1}}{\pi_{t+s}}\frac{p_{i,t}}{P_t}\right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} mc_{t+j}\right] Y_{t+j}\right)
\end{aligned}$$

(1.31)

In the above, $\pi_t = \frac{P_t}{P_{t-1}}$.

The FOC for this problem (where the intermediate-goods producer is choosing $p_{i,t}$) is given by:

$$\frac{\partial V}{\partial p_{i,t}} : E_t \left(\sum_{j=0}^{\infty} \beta^j \omega_p^j \frac{\lambda_{t+j}}{\lambda_t} \left[\left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \frac{1}{P_t} \right)^{\frac{-1}{\eta_{p,t+j}}} \frac{-1}{\eta_{p,t}} p_{i,t}^{* \frac{-1}{\eta_{p,t}} - 1} - \right.\right.$$
$$\left.\left. \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \frac{1}{P_t} \right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} \left(\frac{-1-\eta_{p,t}}{\eta_{p,t}} \right) p_{i,t}^{* \frac{-1-\eta_{p,t}}{\eta_{p,t}} - 1} mc_{t+j} \right] Y_{t+j} \right) = 0$$
$$\implies \frac{\partial V}{\partial p_{i,t}} : E_t \left(\sum_{j=0}^{\infty} \beta^j \omega_p^j \frac{\lambda_{t+j}}{\lambda_t} \left[\frac{-1}{\eta_{p,t}} \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \frac{p_{i,t}^*}{P_t} \right)^{\frac{-1}{\eta_{p,t+j}}} p_{i,t}^{*\,-1} + \right.\right. \tag{1.32}$$
$$\left.\left. \left(\frac{1+\eta_{p,t}}{\eta_{p,t}} \right) \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \frac{p_{i,t}^*}{P_t} \right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} p_{i,t}^{*\,-1} mc_{t+j} \right] Y_{t+j} \right) = 0$$

Rearranging (and factoring out variables determined in period t from the expectations and summation operations):

$$\implies \frac{\partial V}{\partial p_{i,t}} : E_t \left(\sum_{j=0}^{\infty} \beta^j \omega_p^j \lambda_{t+j} \left[\frac{-1}{\eta_{p,t}} \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \right)^{\frac{-1}{\eta_{p,t+j}}} + \right.\right. \tag{1.33}$$
$$\left.\left. \left(\frac{1+\eta_{p,t}}{\eta_{p,t}} \right) \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} \frac{p_{i,t}^*}{P_t}^{-1} mc_{t+j} \right] Y_{t+j} \right) = 0$$

Now, multiply both sides by $\frac{p_{i,t}^*}{P_t}$ and get

$$\implies \frac{\partial V}{\partial p_{i,t}} : E_t \left(\sum_{j=0}^{\infty} \beta^j \omega_p^j \lambda_{t+j} \left[\frac{-1}{\eta_{p,t}} \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \right)^{\frac{-1}{\eta_{p,t+j}}} \frac{p_{i,t}^*}{P_t} + \right.\right. \tag{1.34}$$
$$\left.\left. \left(\frac{1+\eta_{p,t}}{\eta_{p,t}} \right) \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} mc_{t+j} \right] Y_{t+j} \right) = 0$$

$p_{i,t}^*$ solves this equation. We'll consider only symmetric equilibrium, thus write, $p_{i,t}^* = p_t^*$. Also, define two terms to help define this relationship recursively:

Which implies:

$$g_t^1 = E_t \sum_{j=0}^{\infty} \beta^j \omega_p^j \lambda_{t+j} \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} mc_{t+j} Y_{t+j} \tag{1.35}$$

and
Which implies:

$$g_t^2 = E_t \sum_{j=0}^{\infty} \beta^j \omega_p^j \lambda_{t+j} \left(\prod_{s=1}^{j} \frac{\pi_{t+s-1}^{\chi_p}}{\pi_{t+s}} \right)^{\frac{-1}{\eta_{p,t+j}}} \frac{p_{i,t}^*}{P_t} Y_{t+j} \tag{1.36}$$

With these, we can write the FOC as:

$$\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}} g_t^1 = \frac{1}{\eta_{p,t+j}} g_t^2 \tag{1.37}$$

We can then define these functions recursively:

$$g_t^1 = \lambda_t mc_t Y_t + \beta \omega_p E_t \left(\frac{\pi_t^{\chi_p}}{\pi_{t+1}} \right)^{-\frac{1+\eta_{p,t+j}}{\eta_{p,t+j}}} g_{t+1}^1 \tag{1.38}$$

$$g_t^2 = \lambda_t \pi_t^* Y_t + \beta \omega_p E_t \left(\frac{\pi_t^{\chi_p}}{\pi_{t+1}} \right)^{\frac{-1}{\eta_{p,t+j}}} \left(\frac{\pi_t^*}{\pi_{t+1}^*} \right) g_{t+1}^2 \tag{1.39}$$

where $\pi_t^* = p_t^* P_t$.
Calvo pricing implies:

$$P_t^{\frac{-1}{\eta_{p,t}}} = \omega_p \left(\pi_{t-1}^{\chi} \right)^{\frac{-1}{\eta_{p,t}}} P_{t-1}^{\frac{-1}{\eta_{p,t}}} + (1-\omega_p) p_t^{*\frac{-1}{\eta_{p,t}}} \tag{1.40}$$

You can derive the above from the definition of the Calvo model and the aggregate price index from the final goods producers problem.

Nominal profits from the intermediate-goods producers, $d_{i,t}$ are distributed as dividends to households.

1.3 Monetary Authority

The monetary authority follows a Taylor rule, targeting the nominal, after-tax interest rate (I'm taking this from Christiano et al. (2010), but feel free to modify to whatever- though I think it's important that we have the zero bound condition):

$$r_{t+1} = \max \left\{ \frac{\frac{1}{\beta}(1+\pi_t)^{\phi_1} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_2} \left(\frac{1+r_t}{1+\bar{r}}\right)^{\rho_r} - 1}{(1-\tau_t^i)}, 0 \right\} \tag{1.41}$$

Where $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ and $\phi_1 > 1$, $\phi_2 \in (0,1)$, and $\rho_r \in (0,1)$.

1.4 Government/Fiscal Authority

The government is described by a budget constraint and fiscal policy rules (like Taylor rules, but for fiscal policy). Some, like Zubairy (2010) and Traum and Yang (2010), have equations like Taylor rules for fiscal policy, the parameters of which they estimate using historical data. We will follow this. To then get multiplier effects, you just look at how outcomes vary when the policy rules are shocked.

1.5 Fiscal authority equations

Government budget constraint:

$$G_t + \frac{(1+r_t)B_{t-1}}{\pi_t} + X_t = T_t + B_t \tag{1.42}$$

The left-hand side are expenditures (G are expenditures on public goods, X on transfers), the right hand side are revenues from taxes, T, and bond sales.

Total tax revenue is given by:

$$T_t = \tau_t^l w_t L_t + \tau_t^k r_t^k \tilde{K}_t + \tau_t^d D_t + \tau_t^i r_t \frac{B_{t-1}}{\pi_t} + \tau_t^c C_t - \tau_t^k \delta_t^\tau K_{t-1}^\tau - \tau_t^k I_t e_t^\tau - \tau_t^{ic} I_t \tag{1.43}$$

Government spending and tax policy will be given by fiscal policy rules. These fiscal policy rules will react to the state of the economy and the amount of debt the government is carrying. Thus, they will stabilize government debt around it's steady state level. The amount of transfers, X_t, are thus determined as the residual in the government budget constraint.

The fiscal policy rules are as follows (where "hats" denote percent deviations from the steady state values; i.e. $\hat{b}_{t-1} = \frac{B_{t-1}-\bar{B}}{\bar{B}}$):

$$\hat{g}_t = \rho_g \hat{g}_{t-1} - \rho_{gb}\hat{b}_{t-1} + \rho_{gy}\hat{y}_{t-1} + e_t^g \tag{1.44}$$

$$\hat{x}_t = \rho_x \hat{x}_{t-1} - \rho_{xb}\hat{b}_{t-1} + \rho_{xy}\hat{y}_{t-1} + e_t^x \tag{1.45}$$

$$\hat{\tau}_t^l = \rho_l \hat{\tau}_{t-1}^l + \rho_{lb}\hat{b}_{t-1} + \rho_{ly}\hat{y}_{t-1} + e_t^l \tag{1.46}$$

$$\hat{\tau}_t^k = \rho_k \hat{\tau}_{t-1}^k + \rho_{kb}\hat{b}_{t-1} + \rho_{ky}\hat{y}_{t-1} + e_t^k \tag{1.47}$$

$$\hat{\tau}_t^i = \rho_i^\tau \hat{\tau}_{t-1}^i + \rho_{ib}\hat{b}_{t-1} + \rho_{iy}\hat{y}_{t-1} + e_t^i \tag{1.48}$$

$$\hat{\tau}_t^d = \rho_d \hat{\tau}_{t-1}^d + \rho_{db}\hat{b}_{t-1} + \rho_{dy}\hat{y}_{t-1} + e_t^d \tag{1.49}$$

$$\hat{\tau}_t^c = \rho_c \hat{\tau}_{t-1}^c + \rho_{cb}\hat{b}_{t-1} + \rho_{cy}\hat{y}_{t-1} + e_t^c \tag{1.50}$$

The disturbances to these fiscal policy rules may have some correlation. Currently, I only have correlation between capital and labor taxes (as in Zubairy (2010)).

Namely:

$e_t^k = \phi_\tau \varepsilon_t^l + \varepsilon_t^k$

$e_t^l = \phi_\tau \varepsilon_t^k + \varepsilon_t^l$

Where, ϕ_τ is correlation between labor and capital tax shocks. The other variables are uncorrelated.
$e_t^d = \varepsilon_t^d$

$e_t^i = \varepsilon_t^i$

$e_t^c = \varepsilon_t^c$

$e_t^g = \varepsilon_t^g$

$e_t^x = \varepsilon_t^x$

Note that we assume $\varepsilon_t^s \sim i.i.d.\ N(0, \sigma_s)$ for $s \in \{l, k, d, c, i, g, x\}$ (except the notation for the interest tax is σ_i^τ and $\varepsilon_t^{tau,i}$).

For simplicity, we assume that the tax depreciation rate, investment expensing rate, and investment tax credits are constant. Thus, $\delta_t^\tau = \bar{\delta}^\tau$, $e_t^\tau = \bar{e}^\tau$, and $\tau_t^{ic} = \bar{\tau}^{ic}$, $\forall\ t$.

1.6 Aggregation

1.6.1 Aggregate factor inputs

Labor demand: $L_t^d = \int_0^1 l_{i,t} di$

Effective capital demand:
$\tilde{K}_t = \int_0^1 \tilde{k}_{i,t} di = \int_0^1 v_{i,t} k_{i,t} di$

And note that since all firms have the same capital-labor ratio, it must be the case that:
$\frac{\tilde{k}_{i,t}}{l_{i,t}} = \frac{\tilde{K}_t}{L_t}$

1.6.2 Aggregate Output

Supply from each i intermediate goods producer is:

$$y_{i,t}^s = max\{0, z_t \tilde{k}_{i,t}^\alpha l_{i,t}^{1-\alpha} - z_t \Phi\} \tag{1.51}$$

Demand from each i intermediate goods producer is:

$$y_{i,t}^d = \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} Y_t \tag{1.52}$$

In equilibrium, $y^s_{i,t} = y^d_{i,t}$ and in aggregate: $Y_t = \int_0^1 y^d_{i,t} di = \int_0^1 y^s_{i,t} di$. Thus we have:

$$\begin{aligned}
&\int_0^1 y^d_{i,t} di = \int_0^1 y^s_{i,t} di \\
&\implies \int_0^1 \left(\left(\frac{p_{i,t}}{P_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} Y_t \right) di = \int_0^1 \left(z_t \tilde{k}^{\alpha}_{i,t} l^{1-\alpha}_{i,t} - z_t \Phi \right) di \\
&\implies Y_t \underbrace{\int_0^1 \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} di}_{\nu^1_{p,t}} = \int_0^1 \left(z_t \left(\frac{\tilde{k}_{i,t}}{l_{i,t}} \right)^{\alpha} l_{i,t} \right) di - z_t \Phi \\
&\implies Y_t \nu^1_{p,t} = z_t \int_0^1 \left(\frac{\tilde{K}_t}{L_t} \right)^{\alpha} l_{i,t} di - z_t \Phi \qquad (1.53) \\
&\implies Y_t \nu^1_{p,t} = z_t \left(\frac{\tilde{K}_t}{L_t} \right)^{\alpha} \int_0^1 l_{i,t} di - z_t \Phi \\
&\implies Y_t \nu^1_{p,t} = z_t \left(\frac{\tilde{K}_t}{L_t} \right)^{\alpha} L_t - z_t \Phi \\
&\implies Y_t \nu^1_{p,t} = z_t \tilde{K}^{\alpha}_t L^{1-\alpha}_t - z_t \Phi \\
&\implies Y_t = \frac{z_t \tilde{K}^{\alpha}_{i,t} L^{1-\alpha}_{i,t} - z_t \Phi}{\nu^1_{p,t}}
\end{aligned}$$

Where,

$$\nu^1_{p,t} = \int_0^1 \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} di \qquad (1.54)$$

We can use the Calvo Pricing to define this recursively: $\nu^1_{p,t} = \omega_p \left(\frac{\pi^{\chi}_{t-1}}{\pi_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} \nu^1_{p,t-1} +$ $(1 - \omega_p \left(\frac{p^*_t}{P_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}}$

1.6.3 Aggregate Profits

Real profits for firm i in period t are given by:

$$d_{i,t} = \begin{cases} \left(\frac{p_{i,t}}{P_t} - mc_t \right) y_{i,t} - z_t \Phi, & \text{if } y_{i,t} > 0 \\ 0, & \text{otherwise} \end{cases} \qquad (1.55)$$

If profits are positive, we can rewrite $d_{i,t}$:

$$\begin{aligned}
d_{i,t} &= \left(\frac{p_{i,t}}{P_t} - mc_t \right) y_{i,t} - z_t \Phi \\
&= \left(\frac{p_{i,t}}{P_t} - mc_t \right) \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} Y_t - z_t \Phi \qquad (1.56) \\
&= Y_t \left[\left(\frac{p_{i,t}}{P_t} \right)^{\frac{-1}{\eta_{p,t}}} - mc_t \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} \right] - z_t \Phi
\end{aligned}$$

Then aggregating:

$$D_t = \int_0^1 d_{i,t} di = \int_0^1 \left(Y_t \left[\left(\frac{p_{i,t}}{P_t} \right)^{\frac{-1}{\eta_{p,t}}} - mc_t \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} \right] - z_t \Phi \right) di$$

$$= Y_t \underbrace{\int_0^1 \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-1}{\eta_{p,t}}} di}_{\nu_{p,t}^2} - mc_t Y_t \underbrace{\int_0^1 \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-(1+\eta_{p,t})}{\eta_{p,t}}} di}_{\nu_{p,t}^1} - z_t \Phi \tag{1.57}$$

$$= Y_t \left[\nu_{p,t}^2 - mc_t \nu_{p,t}^1 \right] - z_t \Phi$$

Where:

$$\nu_{p,t}^2 = \int_0^1 \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-1}{\eta_{p,t}}} di \tag{1.58}$$

$\nu_{p,t}^2$ can further be written recursively: $\nu_{p,t}^2 = \omega_p \left(\frac{\pi_{t-1}^{\chi}}{\pi_t} \right)^{\frac{-1}{\eta_{p,t}}} \nu_{p,t-1}^2 + (1-\omega_p) \left(\frac{p_t^*}{P_t} \right)^{\frac{-1}{\eta_{p,t}}}$

1.6.4 Other aggregates:

Consumption: $C_t = \int_0^1 c_{i,t} di$

Investment: $I_t = \int_0^1 i_{i,t} di$

Labor Supply: $L_t^s = \int_0^1 l_{i,t}^s di$

Tax Depreciable Basis: $K_t^{tau} = \int_0^1 k_{i,t}^{tau} di$

Bond holdings : $B_t = \int_0^1 b_{i,t} di$

Transfers: $X_t = x_t di$ (Note, no i subscript because lump sum, so all get same)

Public Goods: $G_t = g_t di$ (Note, no i subscript because non-excludable, so all get same)

Prices: $P_t^{\frac{-1}{\eta_{p,t}}} = \omega_p \left(\pi_{t-1}^{\chi} \right)^{\frac{-1}{\eta_{p,t}}} P_{t-1}^{\frac{-1}{\eta_{p,t}}} + (1-\omega_p) {p_t^*}^{\frac{-1}{\eta_{p,t}}}$

1.7 Equilibrium

An equilibrium is where households and final good's producers optimize, taking prices as given. Intermediate goods producers optimize taking factor input prices as given. The government follows its fiscal policy rules and the monetary authority follows its Taylor rule. Markets clear and the resource constraint is satisfied.

1.7.1 Optimization

Firm optimization and fiscal and monetary rules of given above.

We can write the Lagrangian for the household as (dropping i subscripts since we'll assume a representative agent):

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \left[u_t^b \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \theta \frac{(l_t^s)^{1+\varphi}}{1+\varphi} + \chi^g \frac{g_t^{1-\sigma_g}}{1-\sigma_g} \right) + \right.$$
$$\lambda_t \left((1-\tau_t^k) r_t^k v_t k_{t-1} + \tau_t^k \delta_t^{tau} k_{t-1}^\tau + \tau_t^k i_t e_t^\tau + \tau_t^{ic} i_t + (1-\tau_t^l) w_t l_t^s + \frac{(1+(r_t(1-\tau_t^i)))b_{t-1}}{\pi_t} + (1-\tau_t^d) d_t \right.$$
$$\left. + x_t - c_t(1+\tau_t^c) - i_t - b_t) + q_t \left((1-\delta(v_t))k_{t-1} + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}} \right) \right] - k_t \right) \right] \tag{1.59}$$

Where λ_t and q_t are the multipliers on the household's budget constraint and the law of motion for the capital stock, respectively.

The necessary conditions of the household optimization problem are:

- HH FOC, consumption

$$\lambda_t * (1+\tau_t^c) = u_t^b c_t^{-\sigma} \tag{1.60}$$

- HH FOC, labor supply

$$(1-\tau_t^l) w_t \lambda_t = u_t^b \theta l_t^\varphi \tag{1.61}$$

- HH FOC, investment

$$\begin{aligned} \lambda_t(1-\tau_t^k e_t^\tau - \tau_t^{ic}) = & q_t \left(\left(1 - \left(\frac{\kappa}{2} \left(\frac{u_t^i i_t}{i_{t-1}} - 1 \right)^2 \right) \right) - \frac{u_t^i i_t}{i_{t-1}} \kappa \left(\frac{u_t^i i_t}{i_{t-1}} - 1 \right) \right) + \\ & \beta q_{t+1} i_{t+1} \kappa \left(\frac{u_{t+1}^i i_{t+1}}{i_t} - 1 \right) \frac{u_{t+1}^i i_{t+1}}{i_t^2} \end{aligned} \tag{1.62}$$

- HH FOC, capital

$$q_t = \beta q_{t+1} \left(1 - \delta_0 - \delta_1(v_{t+1}-1) - \frac{\delta_2}{2}(v_{t+1}-1)^2 \right) + \beta \lambda_{t+1}(1-\tau_{t+1}^k) v_{t+1} r_{t+1}^k \tag{1.63}$$

- HH FOC, capital utilization

$$r_t^k = \left((\frac{q_t}{(1-\tau_t^k) * \lambda_t} \right) (\delta_1 + \delta_2(v_t - 1)) \tag{1.64}$$

- HH FOC, capital utilization

$$\lambda_t = \beta \lambda_{t+1} \left(\frac{(1+r_{t+1})}{\pi_{t+1}} \right); \tag{1.65}$$

1.7.2 Market Clearing:

- Bond Market: Household demand for bonds equals gov't supply: $\int_0^1 b^d_{i,t} = B^s_t$
- Labor Market: Household labor supply equals intermediate producer labor demand: $L^s_t = L^d_t$
- Capital market: Household capital supply equals intermediate producer capital demand: $\int_0^1 v_{i,t}k_{i,t} = \tilde{K}_t$
- Goods market: Household and gov't consumption plus investment equals supply from the final goods producer (Resource Constraint):

$$Y_t = C_t + I_t + g_t \tag{1.66}$$

The money market then clears by Walras' Law.

1.7.3 Characterizing equations

Altogether, this model has 36 endogenous variables and 28 exogenous parameters. The 36 endogenous variables are characterized by the 37 equations listed in the sections below. By Walras' Law, one of the market clearing conditions is redundant. In the computation, we discard the resource constraint (**??**).

$$\begin{aligned}
\text{Endogenous variables:} \quad & \left\{c_t, \lambda_t, Q_t, l_t, i_t, b_t, v_t, k_t, k^\tau_t, \varepsilon^b_t\right\} \\
& \left\{Y_t, y^d_{i,t}, P_t, \lambda_{p,t}\right\} \\
& \left\{y_{i,t}, \tilde{k}_{i,t}, d_{i,t}, r^k_t, w_t, p_{i,t}, z_t\right\} \\
& \{G_t, X_t\} \\
& \{r_t\} \\
& \{p_t, d_t, x_t, g_t, k_{i,t}, l_{i,t}, C_t, L_t, B_t, K_t, K^\tau_t, D_t, I_t\} \\
\text{Exogenous parameters:} \quad & \left\{\tau^c_t, \tau^i_t, \tau^l_t, \tau^k_t, \tau^{ic}_t, \tau^d_t, \delta^\tau_t, e^\tau_t\right\} \\
& \{\beta, \zeta, \sigma\} \\
& \{\delta_0, \delta_1, \delta_2, \gamma\} \\
& \{\rho_\varepsilon, \mu_\varepsilon, \sigma_\varepsilon\} \\
& \{\rho_\lambda, \mu_\lambda, \sigma_\lambda\} \\
& \{\alpha, \theta\} \\
& \{\gamma_{x,t}\} \\
& \{\beta_r, \phi_1, \phi_2, \rho_r\}
\end{aligned}$$

There are 21-23 unknown variables after solving for the aggregates from the MC conditions and assuming a symmetric equilibrium for intermediate goods producers : $i, k, b, c, k^\tau, \lambda, Q, G, X, d, P, p_{i,t},$ These breakdown into the following groups:

- 4 choice variables (Household): i, v, l, c
- 4 prices: P, w, r, r^k

- 2 choice variables (gov't): g, x, reallly $2+n$ where n is the number tax instruments
- 1 choice variable (monetary authority): M
- 2 exogenous state: z, ε
- 5 endogenous state variable: k, b, d, k^{τ}, Y

Solve for these unknowns with the following equations:

- 8 FOCs, household $\implies i, v, l, \lambda, Q, b, c, ???$
- 3 from FOCs, intermediate goods producers $\implies r^k, w, p_{i,t}$ (These FOC's give demand for effective capital and labor. So together with the HH supply of these factors we can use our market clearing conditions to find the eq'm factor prices.)
- 1 from the profit function of intermediate goods producers $\implies d$
- 1 from FOCs, final goods producer $\implies Y$
- 1 from the zero profit condition for the final goods producer $\implies P$ (This together with the definition of Calvo pricing gives you the aggregate price index.)
- 2 Gov't budget constraint ($+n$ fiscal policy rules) and assumption of X being a certain fraction of spending $\implies B, X$, ($+n$ tax rates)
- 1 Resource constraint $\implies G$
- 2 laws of motion $\implies k^{\tau}, k$
- 3 exog shock process $\implies z, \varepsilon^b, \lambda_p$
- 1 Taylor rule for monetary authority $\implies r, M$ (There is the Taylor Rule plus the fact that $r = max(M, 0)$ that determines both these.)

So we have 21-23 unknowns and 21-23 equations. We should be able to plug this into Dynare and solve the model.

1.8 Estimation

Following many others (e.g., Traum and Yang (2010), Zubairy (2010), Smets and Wouters (2003)), we will use Bayesian estimation to estimate many parameters of the model. We use data from the UK to do the estimation. UK is probably a good bet since they have their own central bank and a VAT of about 20%.

- δ_0= annual depreciation rate (~10%), equals the aggregate SS investment rate ($\frac{\bar{I}}{\bar{K}}$).
- δ_1 =
- δ_2 =
- δ^{τ} = Not sure how set- varies by asset type and front loaded, but probably just set to 12-14%- something higher than econ depreciation

- τ^l = See Jones (2002) or just do statutory rate for median household or use tax calculator to get marginal rate for median household
- τ^i = assume = labor income tax
- τ^d = See Jones (2002) or just do statutory rate for median household or use tax calculator to get marginal rate for median household
- τ^k = See Jones (2002)
- τ^c = See Jones (2002)
- $\tau^{ic} = 0$, most years for most investments this is zero
- $e^\tau = 0$, in reality expensing for tax > 0 for many small businesses at most times, but we can set to zero
- $\gamma =$
- ζ = I think this is the Frisch elasticity of labor. Christiano et al. (2010) set to 0.29.
- σ = coeff of relative risk aversion. Often set to 2.
- $\chi^g =$
- $\sigma_g =$
- $\sigma_\varepsilon =$
- $\rho_\varepsilon =$
- $\sigma_\lambda =$
- λ_p = mean markup = $\frac{1+\lambda_p}{\lambda_p}$, Basu and Fernald (1995) give U.S. evidence of avg markup of 10-14% which implies $\lambda_p \sim 8$.
- β = set to match (real?) after-tax interest rate (given tax on interest income)
- α = capital share of output (30-35%)
- σ_z = Estimate log of production function using aggregate data (typical business cycle accounting) to get residual. Fit the residual to AR(1) to get σ_z and ρ_z.
- ρ_z = Estimate log of production function using aggregate data (typical business cycle accounting) to get residual. Fit the residual to AR(1) to get σ_z and ρ_z.
- θ = fraction of firms changing price
- ϕ_1 = estimate a log-log specification of the Taylor rule
- ϕ_2 = estimate a log-log specification of the Taylor rule
- ρ_r = estimate a log-log specification of the Taylor rule
- $\bar{L}$ = fraction of hours worked ($0.3 \sim \frac{8}{24}$, 0.5 corresponds to Frisch elasticity = 1??)

- $\frac{\bar{G}}{\bar{Y}}$ = historical average
- $\frac{\bar{B}}{\bar{Y}}$ = historial average
- $\frac{\bar{X}}{\bar{Y}}$ = historical average

TABLE: Estimated parameters (priors and posteriors from Bayesian Estimation)

1.9 Solving the model

We use Dynare to solve the model with a second order Taylor approximation.

2 Short Run Multipliers

1. FIGURES: impulse response functions for tax cuts (capital, income, consumption, investment) and gov't spending increase (% changes in output, emp, cons, inv, int rates, inflation) to policy change
2. TABLE: table with size of multipliers

3 Long Run Multipliers

FIGURE: graph of multiplier from each of fiscal policy measures over time

4 Sensitivity to Government Financing

FIGURE: graph of multiplier over time given how gov't financing temporary tax cuts (with higher levels of debt going forward or with paying back by increasing taxes). Maybe 3D graph with multiplier, years from cut, and year to pay back debt as axes.

5 Sensitivity to Monetary Policy

Talk about multipliers at and away from the zero-bound. What happens if hold interest rates constant?

6 Dead-weight Loss and Fiscal Policy

1. TABLE: table of DWL for each of policy responses
2. FIGURE: graph of DWL over time for policy response

Do some revenue neutral changes in tax policy and calculate welfare....

7 Sensitivity to Parameters

How do results change with change to key parameters...

8 Discussion

Note that hard to know preferences for gov't spending, but imagine that people know better what want, cutting taxes may be politically easier than increasing spending. Note other weaknesses of the modeling assumptions.

9 Conclusion

10 Questions:

1. Do those with models of the multiplier usually have transfers to households or gov't actually buying stuff?

2. Do I want to say anything about optimal consumption tax being non-zero (like want non-zero inflation) because can more easily do tax cut to respond to recession than could do negative tax rate (spending)?

3. Did Europe (or even US states) have any changes to consumption taxes in response to the recession? *A: I don't think so. In fact, I believe some states cancelled their sales tax holidays due to budget shortfalls.*

4. Do we want to have rules for fiscal policy as in Zubairy (2010)? Again, not sure why we need it if not trying to match data. Just need to estimate SS levels and propose levels that will revert to after aggregate shock.

11 Does anyone do this?

1. Kumhof et al. (2010) shows impulse response functions to a consumption tax cut. This paper seems to find that consumption taxes do well compared to other taxes and transfers, but don't do as well as gov't consumption and gov't investment

2. Leeper, Plante and Traum (2010) have consumption taxes, but don't allow them to vary with gov't debt

3. Zubairy (2010) doesn't not have consumption taxes, only capital and labor income taxes. Says few even have distortionary taxes in models of multipliers.

4. Traum and Yang (2010) allow for consumption taxes and allow them to vary. Not answering the same question as I want, but good to look at what they say about consumption taxes. In particular, how with cons taxes the consumer and producer price index will vary and what crowding out happens with consumption tax changes.

5. Coenen and Straub (2005) have consumption taxes, but don't use them as a policy instrument. They don't say anything interesting about consumption taxes.

6. Christiano et al. (2010) don't have consumption taxes and only focus on spending multipliers. I don't even think they have taxes. Fiscal and monetary policy rules look simple though- might want to take these.

12 Links

- http://www.econbrowser.com/archives/2010/03/policy_in_dsges.html
- Kumhof et al. (2010) paper: http://www.imf.org/external/pubs/ft/wp/2010/wp1073.pdf
- http://delong.typepad.com/sdj/2009/07/cracking-chistiano-eichenbaum-and-rebelos-big-mu html
- http://www.econbrowser.com/archives/multipliers/index.html
- Traum and Yang (2010) paper: http://www.nber.org/papers/w15160

13 MORE QUESTIONS:

1. Why do Christiano et al. (2010) have a subsidy to correct for monopoly of int goods producers? I don't see others with this.
2. What is capital tax equivalent to? See what Zubairy (2010) estimates from, but it's like a corp income tax, cap gains tax, div tax rolled into one.
3. Do Multiplier of: gov't spending (baseline to compare), cut in: cap tax, labor tax, cons tax, invest tax credit/bonus deprec/accelerated expensing
4. Can I do bonus deprec with my model- or do I need to keep track of vintage/basis since not partial expensing?

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TECHNICAL APPENDIX

T-1 Derivation of monopolistic competition intermediate goods demand and aggregate price Equations

The profit maximization approach to solving for each intermediate good demand equation $y_{i,t}$ and the aggregate price equation P_t is the following.

$$\max_{y_{i,t}} P_t Y_t^d - \int_0^1 p_{i,t} y_{i,t} di \tag{T.1.1}$$

The FOC's are:

$$p_t(1+\lambda_{p,t}) \left(\int_0^1 y_{i,t}^{\left(\frac{1}{1+\lambda_{p,t}}\right)} di \right)^{\lambda_{p,t}} \left(\frac{1}{1+\lambda_{p,t}} \right) y_{i,t}^{\frac{1}{1+\lambda_{p,t}}-1} - p_{i,t} = 0, \forall i \tag{T.1.2}$$

Dividing the FOCs for intermediate inputs i and j $\implies$

$$\frac{p_{i,t}}{p_{j,t}} = \left(\frac{y_{i,t}}{y_{j,t}} \right)^{\frac{1}{1+\lambda_{p,t}}-1} = \left(\frac{y_{i,t}}{y_{j,t}} \right)^{\frac{-\lambda_{p,t}}{1+\lambda_{p,t}}} \tag{T.1.3}$$

Rearranging:

$$\begin{aligned}
&\implies p_{i,t} = \left(\frac{y_{j,t}}{y_{i,t}} \right)^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} p_{j,t} \\
&\implies p_{i,t} = y_{j,t}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} p_{j,t} y_{i,t}^{\frac{-\lambda_{p,t}}{1+\lambda_{p,t}}} \\
&\implies p_{i,t} y_{i,t} = p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} y_{i,t}^{\frac{1}{1+\lambda_{p,t}}}
\end{aligned} \tag{T.1.4}$$

Then integrate this condition to yield:

$$\begin{aligned}
\int_0^1 p_{i,t} y_{i,t} di &= p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} \int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di \\
&= p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}}
\end{aligned} \tag{T.1.5}$$

The zero profit condition implies $P_t Y_t = \int_0^1 p_{i,t} y_{i,t} di$. Plugging this into the above and we find:

$$\begin{aligned}
&P_t Y_t = p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} \\
&\implies P_t Y_t = p_{j,t} y_{j,t}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} \\
&\implies P_t = p_{j,t} y_{j,t}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} (Y_t)^{\frac{-\lambda_{p,t}}{1+\lambda_{p,t}}}
\end{aligned} \tag{T.1.6}$$

Which implies the demand function (just rearranging terms):

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t, \forall i \tag{T.1.7}$$

To find P_t use the zero profit condition:

$$\begin{aligned}
P_t Y_t &= \int_0^1 p_{i,t} y_{i,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} di \\
\Longrightarrow P_t Y_t &= \int_0^1 p_{i,t} \left(\left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t\right)^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} di \\
\Longrightarrow P_t Y_t &= \int_0^1 p_{i,t} \left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t di \\
\Longrightarrow P_t Y_t &= \int_0^1 p_{i,t} p_{i,t}^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} P_t^{\frac{(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t di \qquad \text{(T.1.8)} \\
\Longrightarrow P_t Y_t &= \int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di \, Y_t P_t^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \\
\Longrightarrow P_t &= \int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di \, P_t^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \\
\Longrightarrow P_t^{\frac{-1}{\lambda_{p,t}}} &= \int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di \\
\Longrightarrow P_t &= \left(\int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di\right)^{-\lambda_{p,t}}
\end{aligned}$$

The cost minimization approach to solving for each intermediate good demand equation $y_{i,t}$ and the aggregate price equation P_t is the following.

$$\min_{y_{i,t}} \int_0^1 p_{i,t} y_{i,t} di \quad \text{s.t.} \quad Y_t \leq \left(\int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di\right)^{1+\lambda_{p,t}} \tag{T.1.9}$$

The Lagrangian for this minimization problem is the following,

$$\mathcal{L} = \int_0^1 p_{i,t} y_{i,t} di + \lambda_y \left[Y_t - \left(\int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di\right)^{1+\lambda_{p,t}}\right] \tag{T.1.10}$$

in which the multiplier λ_y has the interpretation of being the marginal cost of an extra unit of aggregated output. That is, λ_y is the price of aggregate output P_t.

$$\mathcal{L} = \int_0^1 p_{i,t} y_{i,t} di + P_t \left[Y_t - \left(\int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di\right)^{1+\lambda_{p,t}}\right] \tag{T.1.11}$$

We need to finish this....

T-2 Derivation of real marginal costs

Dividing the period profits equation (**??**) by aggregate prices P_t gives the following real total costs function.

$$tc_t = \frac{r_t^k}{P_t}\tilde{k}_{i,t} + \frac{w_t}{P_t}l_{i,t} \tag{T.2.1}$$

Substituting in the expression for $\tilde{k}_{i,t}$ from the capital-labor ratio equation (**??**) gives real total costs in terms of α, w_t, P_t, and $l_{i,t}$.

$$tc_t = \frac{1}{1-\alpha}\left(\frac{w_t}{P_t}\right)l_{i,t} \tag{T.2.2}$$

Because the intermediate goods production exhibits constant returns to scale, we can find the real marginal cost mc_t by finding the amount of labor necessary to produce 1 unit of output, and substituting that value into the real total costs equation (T.2.2).

$$z_t\tilde{k}_{i,t}^{\alpha}l_{i,t}^{1-\alpha} = 1 \tag{T.2.3}$$

Next, substitute in the equation for capital $\tilde{k}_{i,t}$ in terms of labor from the capital-labor ratio equation (**??**).

$$z_t\left(\frac{\alpha}{1-\alpha}\left[\frac{w_t}{r_t^k}\right]l_{i,t}\right)^{\alpha}l_{i,t}^{1-\alpha} = 1 \quad \Rightarrow \quad l_{i,t} = \frac{1}{z_t}\left(\frac{\alpha}{1-\alpha}\left[\frac{w_t}{r_t^k}\right]\right)^{-\alpha} \tag{T.2.4}$$

Lastly, substitute the amount of labor necessary to produce one unit of output from (T.2.4) into the expression for real total costs (T.2.2) to get real marginal costs.

$$mc_t = \frac{1}{1-\alpha}\left(\frac{w_t}{P_t}\right)\frac{1}{z_t}\left(\frac{\alpha}{1-\alpha}\left[\frac{w_t}{r_t^k}\right]\right)^{-\alpha} = \frac{w_t^{1-\alpha}(r_t^k)^{\alpha}}{z_tP_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{T.2.5}$$