This document and the associated file Notation_Guide.xlsx will serve as the official guide to notation used throughout the model. The goal is to make sure variables and parameters have intuitive and consistent notation.

The notation will be grouped into three tables: endogenous variables, exogenous variables, and parameters. Note that other versions of the variables used here will exist. Throughout, we will try to adhear to the following rules:

i. Use "tilde" (e.g., $\tilde{c}$) to denote "special" versions of variables (these may have various meanings depending upon variable)

ii. Use "hats" (e.g., $\hat{c}$) to denote stationarized variables

iii. Use "bars" over variables (e.g., $\bar{c}$) to denote steady-state values of variables

iv. The English alphabet is used for variables

v. The Greek alphabet is used for parameters

Note that we might think about what assigned letters we can use in our Python code and it there are conflicts with that. Because it would be nice for users of the code to be able to use the same notation as in the text.

# 1 Fixing Inconsistencies in the Notation

Also, we should use lower case letter for firm investment, capital stock, output. However, the lower case $i$ for firm investment conflicts with the $i$ for the immigration rate. We might also want to to use $i$ for the nominal interest rate at some point (e.g. using both $r$ and $i$ to differentiate between the before and after tax rates).

## 1.1 Handling notation for consumption

The model has individuals consuming multiple goods, $c_{i,j,s,t}$. Summing over $i$ for the individual's consumption we get $c_{j,s,t}$, consumption of the composite consumption good. The total consumption of this composite consumption good needs to be broken down into the component that is due to the minimum consumption requirements and that is "supernumerary" consumption. Right now, I've written these as $\bar{c}_{j,s}$ and $\tilde{c}_{j,s,t}$, respectively. However, this notation breaks the rules about the use of tildes and bars on variables that we might like to follow. Also, note that minimum consumption requirements are technically parameters, so maybe should be using a Greek letter rather than $\bar{c}_{j,s}$ for these.

Any ideas to fix this? One idea is to use Gothic c, $\mathfrak{c}$, for discretionary consumption. Can't think of a good Greek letter leftover for minimum consumption, though...

**Table 1:** Endogenous Variables in the OSPC Model

| Variable | Description |
|---|---|
| Household | |
| $U_{j,s,t}$ | Discounted lifetime utility for household of lifetime income group $j$, age $s$, at time $t$ |
| $\tilde{c}_{j,s,t}$ | Discretionary consumption of composite consumption good by household of lifetime income g |
| $c_{j,s,t}$ | Consumption of composite consumption good by household of lifetime income group $j$, age $s$, |
| $c_{i,j,s,t}$ | Consumption of consumption good $i$ by household of lifetime income group $j$, age $s$, at time |
| $n_{j,s,t}$ | Labor supply of household of lifetime income group $j$, age $s$, at time $t$ |
| $b_{j,s,t}$ | Asset holdings of household of lifetime income group $j$, age $s$, at time $t$ |
| $\tilde{p}_{s,t}$ | Price of discretionary composite consumption good for household of age $s$ |
| $p_{i,t}$ | Price of consumption good $i$ at time $t$ |
| $\Omega$ | Household beliefs about the distribution of assets |
| | |
| Firms | |
| $I^C_{m,t}$ | Investment by the representative firm of sector $C$ and industry $m$ at time $t$ |
| $EL^C_{m,t}$ | Effective labor demand by the representative firm of sector $C$ and industry $m$ at time $t$ |
| $X^C_{m,t}$ | Production output by the representative firm of sector $C$ and industry $m$ at time $t$ |
| $K^C_{m,t}$ | Capital stock of the representative firm of sector $C$ and industry $m$ at time $t$ |
| $EARN^C_{m,t}$ | Corporate earnings before deprec, corp taxes, and adjust costs, but after property taxes of the representative firm of sector $C$ and industry $m$ at time $t$ |
| $DIV^C_{m,t}$ | Dividend distributions by the representative firm of sector $C$ and industry $m$ at time $t$ |
| $TE^C_{m,t}$ | Total corporate income taxes by the representative firm of sector $C$ and industry $m$ at time $t$ |
| $\Phi^C_{m,t}$ | Investment adjustment costs borne by the representative firm of sector $C$ and industry $m$ at |
| $B^C_{m,t}$ | Debt outstanding for the representative firm of sector $C$ and industry $m$ at time $t$ |
| $K^{\tau C}_{m,t}$ | Tax basis of capital for the representative firm of sector $C$ and industry $m$ at time $t$ |
| $VN^C_{m,t}$ | New equity issues by the representative firm of sector $C$ and industry $m$ at time $t$ |
| $V^C_{m,t}$ | Value of the representative firm of sector $C$ and industry $m$ at time $t$ |
| $q^C_{m,t}$ | Marginal $q$ (change in firm value per dollar of investment) of the representative firm of sector $C$ and industry $m$ at time $t$ |
| $Q^C_{m,t}$ | Average $Q$ of the representative firm of sector $C$ and industry $m$ at time $t$ |
| $p^C_{m,t}$ | Price of output from sector $C$, industry $m$, at time $t$. |
| | |
| Government | |
| $D_t$ | Government debt at time $t$ |
| $T_H$ | Total government transfers at time $t$ |
| $G^{subs}_t$ | Government subsidies to the production of private goods at time t |
| $G^{emp}_t$ | Government expenditures on employment in the production of public goods at time $t$ |
| $I^G_t$ | Government investment at time $t$ |
| $K^G_t$ | Government capital stock at time $t$ |
| | |
| Aggregates and Prices | |
| $r_t$ | Rate of return on assets at time $t$ |
| $w_t$ | Wage rate at time $t$ |
| $BQ_{j,t}$ | Aggregate bequests to households of lifetime income group $j$ at time $t$ |
| $T_{j,s,t}$ | Total taxes paid by households of lifetime income group $j$, age $s$, at time $t$ |
| $Y_t$ | Total economic output at time $t$ |
| $\Gamma_t$ | Distribution of assets at time $t$ |
| $B_t$ | Aggregate assets held by households at time $t$ |
| $C_t$ | Aggregate consumption by households at time $t$ |
| $I_t$ | Aggregate investment by firms and government at time $t$ |
| $G_t$ | Total government expenditures (on public goods and subsidies to private goods) |
| $K_t$ | The total capital stock held by firms and government at time $t$ |

**Table 2:** Exogenous Variables in the OSPC Model

| Variable | Description |
|---|---|
| Household | |
| $\omega_{s,t}$ | The measure of households of age $s$ at time $t$ |
| $N_t$ | Number of households at time $t$ |
| $\tilde{N}_t$ | Number of working age households at time $t$ |
| | |
| Initial Values | |
| $N_0$ | Population in initial year |
| $\omega_{s,0}$ | Fraction of initial population of age $s$ in initial model year |
| $D_0$ | Government debt in the model's initial period |
| $\Gamma_1$ | Initial distribution of assets |

**Table 3:** Parameters in the OSPC Model

| Parameter | Description |
|---|---|
| Demand Side | |
| Consumer Preferences | |
| $\sigma$ | Household coefficient of relative risk aversion on consumption and bequests |
| $\chi_b$ | Utility weight on the utility from bequests |
| $\beta$ | Household rate of time preference |
| $I$ | Number of consumption goods that go into composite good |
| $\alpha_{i,s}$ | Household sub-utility function share parameter on good $i$ for households of age $s$ |
| $\bar{c}_{i,s}$ | Household sub-utility function minimum amount of good $i$ for households of age $s$ |
| $Z$ | "Transition" matrix relating output of firms to consumption goods (dimensions are $M \times I$) |
| $\gamma_m$ | Share parameter for corporate output in industry $m$ |
| $\varepsilon_m$ | Elasticity of substitution of corporate for non-corporate output in industry $m$ |
| Labor Supply/Ability | |
| $e_{j,s}$ | Effective labor for household of lifetime income type $j$ at age $s$ |
| $J$ | Number of lifetime income groups for household |
| $\lambda_j$ | Fraction of population of lifetime income group $J$ |
| $\chi_{n,s}$ | Utility weight on the disutility of labor supply for households of age $s$ |
| $b$ | Scale parameter for elliptical function for disutility of labor |
| $k$ | Shift parameter for elliptical function for disutility of labor |
| $v$ | Frisch elasticity of labor supply (?) |
| $\tilde{l}$ | Time endowment |
| Population Dynamics | |
| $E$ | Number of years from birth to start of economic life |
| $S$ | Maximum length of economic life |
| $R$ | Age at which retirment benefits paid out |
| $f_s$ | Fertility rate at age $s$ |
| $i_s$ | Immigration rate of individuals of age $s$ |
| $\rho_s$ | Mortality rate for individuals of age $s$ |
| Individual Income Tax Code | |
| $A$ | Coefficient on the quadratic term in the individual income tax function |
| $B$ | Coefficient on the linear term in the individual income tax function |
| $C$ | Constant in the individual income tax function |
| $D$ | Level parameter in the individual income tax function |
| $F$ | Income factor in the individual income tax function |
| $\tau^P$ | Payroll tax rate |
| $\theta_j$ | Replacement rate for retirement benefits for those in lifetime income group $j$ |
| $\tau^{BQ}$ | Estate tax rate |
| Supply Side | |
| Production Function | |
| $\gamma^C_m$ | Capital weighting in CES production function in sector $C$ and production industry $m$ |
| $\epsilon^C_m$ | Elasticity of substitution of capital for labor in CES production function in sector $C$ and production industry $m$ |
| $\delta^C_m$ | Rate of economic depreciation on capital stock in the corporate sector in sector $C$ and production industry $m$ |
| $\beta^C_m$ | Scaling parameter for quadratic investment adjustment costs in sector $C$ and production industry $m$ |
| $\mu^C_m$ | Steady-state investment rate in sector $C$ and production industry $m$ |
| Financial Policy | |
| $\varsigma^C_m$ | Dividend payout ratio in sector $C$ and industry $m$ |
| $b^C_m$ | Debt to capital ratio in sector $C$ and industry $m$ |
| Tax Policy | |
| $\tau^b_t$ | Corporate marginal income tax rate in year $t$ |
| $\delta^{\tau C}_{m,t}$ | Rate of tax depreciation capital in sector $C$, industry $m$, year $t$ |
| $\tau^{pC}_{m,t}$ | Property tax rate on capital in sector $C$, industry $m$, year $t$ |
| $\tau^i_t$ | Individual income tax rate on interest income in year $t$ |
| $\tau^g_t$ | Individual income tax rate on capital gains in year $t$ |
| $f_e$ | Dummy variable for full expensing of investment |
| $f_I$ | Dummy variable for deductibility of corporate interest paid |
| $f_p$ | Dummy variable for deductibility of repayment of principle on loans |
| $f_b$ | Dummy variable for inclusion of proceeds of loan in corp income tax base |
| $f_d$ | Dummy variable for deductibility of depreciation expenses |
| Population of firms | |
| $M$ | Number of industries |
| $\Omega$ | "Transition" matrix relating output of firms to the supply of new capital (dimesions are $M \times M$) |
| Government | |
| $\rho_t$ | Response rate of government spending on public goods to debt to GDP ratio |
| Economic Growth | |
| $g_{n,t}$ | Rate of population growth from period $t-1$ to period $t$ |
| $\tilde{g}_{n,t}$ | Rate of growth in working age population from period $t-1$ to period $t$ |
| $g_y$ | Rate of labor augmenting technological growth |
| Model Solution | |
| $T$ | Periods to read the SS in the TPI method |
| $\nu$ | Dampening parameter for TPI method |

**Table 4:** Possible Fixes to the Notation

| Letter | Variable/Parameter |
|---|---|
| $d$ | Dividends distributed by the firm |
| $l^d$ | Effective labor demand by firm |
| $x$ | Firm earnings |
| $y$ | Firm output |
| $T$ | Total entity level income taxes paid by firm (careful of conflict with indiv taxes) |
| $s$ | New equity issues? Does conflict with age, but is used by others (and s=shares) |
| $\upsilon$ | Replace v as labor supply elasticity |
| $\bar{q}$ | Average Q?? Does break rule for bar used for per capital variables |
| $\Pi$ | Use this instead of $Z$ for "transition" between productions output and consumption goods |
| $\Xi$ | Use this instead of $\Omega$ for "transition" between productions output and investment goods/c |
| $\phi$ | Use this instead of $\rho$ for gov't spending response to debt |
| $\eta$ | Use this instead of $\lambda$ for lifetime income distribution (saving $\lambda$ for Lagrangian multipliers) |
| $Z$ | Aggregate bequests?? Doesn't fit right, but would avoid two letters |
| $\kappa$ | Shift parameter in disutility of labor |
| $\psi_1 - \psi_4$ | Parameters of the income tax function |
| $\nu$ | Use instead of $\beta$ as scaling parameter in investment adjustment cost function |
| $\Delta$ | Use instead of $\nu$ as dampening parameter for TPI method |