Taxation, early environments and growth

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Abstract

This paper analyses the impact on growth of a policy reform which modifies the tax on labour income, developing a general equilibrium OLG growth model where human capital is formed both via formal schooling and via early environments. Parents can affect the early environments where children grow up via child care, that is parental time and child care expenditure. On the one hand, a change in the tax rate affects the returns to education and it therefore impinges on the decision to invest in formal schooling: this is the standard effect studied in the literature. On the other hand, it influences the time parents devote to child care and the amount of child care expenditure: this is the new channel we identify. The computational experiment we perform shows that the introduction of early environments in the human capital accumulation process weakens, in most cases, the negative relationship between taxation and growth stemming from the formal schooling channel. Depending on the degree of complementarity between the inputs in the human capital production function, there can be instances where the relationship between taxation and growth becomes positive.

KEYWORDS: taxation, growth, human capital production function, child care, labour supply
JEL Classification: J22, J26

1 Introduction

This paper analyses the impact on growth of a policy reform which modifies the tax on labour income, using a general equilibrium OLG growth model where human capital is formed both via formal schooling and via the early environments where children grow up.

The importance of human capital as a growth promoting factor has long been recognised by the economic literature. More recently, the analysis of how early environments are shaped and of how important they are in affecting the individual's skill formation process has received increasing attention (see Section 2). However, no paper, as far as we are aware, combines the two views and analyses the relationship between the development of early environments, the accumulation of human capital and growth. Indeed, early environments are generally captured in macroeconomic models by introducing the human capital of a parent into the human capital production function of the child. This intergenerational link is however modelled as an externality and the early environments where children grow up are a by-product of parental activity and not the result of a conscious and voluntary choice. This is not satisfactory for at least two reasons: on the one side, one does not have a framework where to investigate how alternative policies targeted at the early stage of the life cycle skill formation process affect growth; on the other, the existing research focusing on the growth impact of public policies, namely of taxation, ignores an important channel - that of early investments -, through which policy variables may affect human capital accumulation and growth. This paper tries to correct for this second drawback.[1]

We develop a three-period general equilibrium OLG growth model where formal schooling and early environments enter the production function of human capital as complements. We model the influence of parents on early environments by assuming that they can dedicate their own time to child rearing and they can buy goods and services which may impinge on the child development (e.g books or toys, day care centres' services, pre-school programs, baby-sitting). We refer to these activities as child care. In this framework we assess the impact on growth of a policy reform which modifies the tax on labour income. On the one hand, such a reform affects the returns to education and it therefore impinges on the decision to invest in formal schooling: this is the standard effect studied in the literature. On the other hand, it influences the time parents devote to child care and the amount of child care expenditure: this is the new channel we identify.

To explore these effects, we first derive the analytical solution of the model for a benchmark case, obtained imposing some restrictions: this analysis allows us to identify the different forces at work in our set-up. We show that the introduction of early environments in the human

[1] The first issue is addressed in our companion paper Casarico and Sommacal (2007).

capital production function can either reinforce, weaken or overturn the negative relationship between taxation and growth stemming from the formal schooling channel. To understand what the most relevant case is, we then calibrate and solve numerically the model. We finally relax the restrictions adopted in the benchmark case and we perform a sensitivity analysis on two parameters affecting the human capital production function: the elasticity of substitution between formal schooling and early environments and the elasticity of substitution between child care expenditures and parental time. We find that a reduction in taxation always determines an increase in formal schooling and, in most cases, a deterioration of the process of acquisition of skills taking place in childhood. As net wages go up, people work more and dedicate less time to child care: this reduction is not compensated by the increase in the amount of child care expenditure. As an implication, the introduction of early environments weakens the negative relationship between taxation and growth. There can also be instances where the relationship between taxation on labour income and growth becomes positive.

The paper is organized as follows: in Section 2 we provide a brief review of the related literature. In Section 3 we describe the building blocks of the model and define the intertemporal equilibrium and the balanced growth path. In Section 4 we derive the analytical solution of the model for a benchmark case; we then perform a computational experiment and we conduct a sensitivity analysis with respect to some crucial parameters. Section 5 concludes and indicates avenues for further research. Technical details and the proofs of the Propositions are relegated to Appendix A and B.

2 Related literature

Our analysis sits at the juncture of three strands in the economic literature.

The first strand concerns how early environments in which children grow up are shaped and how important they are in affecting the individual's skill formation process (see Cuhna et al. 2005 for a review). This literature shows that skill formation is a dynamic process, characterized by strong complementarities between its different phases. As there are critical periods for the development of both cognitive and non cognitive abilities, later remediation for early deficits in the formation of some important abilities is difficult and costly. Some evidence suggests for example that the IQ can be affected by the environment in which the children live until the age of 10, but not later. Early investments not only have a direct impact on the level of human capital of an individual. As there is complementarity between investments at different stages, early investments make further investments more productive (skill begets skill). Carneiro and Heckman (2002, 2003), for instance, suggest that the most important factor explaining the positive relation between income and college enrollment in the US is not related to short term liquidity constraints that poor individuals may face, but to the fact that they lived in early environments which were unable to form the cognitive and noncognitive abilities required for success in school. This complementarity between formal schooling and skills acquired during childhood is also documented in Leibowitz (2003).[2] Family plays an important role in shaping the early environments in which children grow up. The importance of parental time vs. other types of child care in producing children abilities is analysed in a very recent empirical literature. Baker, Gruber, and Milligan (2005) for example find that the introduction of universal child care in Quebec at the end of the Nineties made children worse off in a variety of behavioural and health-related dimensions. Bernal and Keane (2006) analyse the effect of child care use by single mothers in the US on children development and find that it is on average negative, significant, and rather sizeable.

The second strand to which this paper is related refers to models on the growth impact of taxation when human capital is the engine of growth. We refer to Myles (2007) for a very recent and exhaustive review of this literature. These papers share the view that a change in income taxation may affect the incentive to invest in schooling, since it modifies the net wage and, in models with endogenous labour supply, it also alters the working time, feeding back into the growth rate.

Finally, this work is related to the (few) papers that explicitly insert child care choices in a dynamic general equilibrium model. Cardia and Ng (2003), Cardia and Michel (2004) and Belan, Wolff, and Messe (2007), are examples of OLG models in which child care (in the form of time and of child care services) affects the utility function of parents, as in the set-up we develop here.[3] However, in these papers child care is modelled as a home-produced consumption good: this excludes the role that child care plays in the children's skill formation process and in the individuals' labor market performance.

[2]See also the references quoted therein.

[3]In other models (usually those with endogenous fertility, e.g. de la Croix and Doepke 2003) child care does not directly affect the utility function, but it is viewed as a fixed cost in terms of time associated with the birth of a child.

3 The model

We develop an OLG model with intragenerational homogeneity and endogenous growth driven by human capital accumulation. The model is set up in discrete time, from 0 to infinity. Agents have perfect foresight on future variables. They live for three periods and they have one child in the second period of life (the population growth rate is zero and fertility is exogenous). The size of each generation is normalized to 1.

Early environments, together with formal schooling, affect an individual's human capital and thus her labor market performance. The young agent chooses the amount of schooling, while the altruistic parents take care of the early environments in which their children grow up. We model the influence of parents on early environments by assuming that they can dedicate their own time to child rearing and they can buy goods and services which may affect the child development (e.g books or toys, day care centers' services, pre-school programs, baby-sitting). We refer to these activities as child care.

In the first period of life, the child/the young receives child care and invests in formal schooling, borrowing on the capital market. In the second period of life the middle-aged/the parent pays back her loan and decides: how much to consume and save; how much time to devote to labor, leisure and child care; how much to spend on child care. In the third period of life, the old agent retires and consumes all her income.

In each period one physical good is produced using capital and labor measured in efficiency units. This good can be used for consumption, for investment in physical capital and for investment in human capital (both in the form of schooling expenditure and child care expenditure).[4]

3.1 Basic set-up

Preferences

Preferences of an agent born at t are described by the following utility function:

$$U_t = i_1 \log c^y_{t+1} + i_2 \log z_{t+1} + i_3 \log x_{t+1} + \theta \log c^o_{t+2} \tag{1}$$

where c^y_{t+1} and c^o_{t+2} denote respectively consumption when middle-aged and old as no con-

[4]The assumption that a single physical good can be used for consumption, investment in physical capital and investment in schooling is standard. Here, following the papers that introduce child care in the utility function (see Section 2), we assume that this good can also be used for investment in child care.

sumption takes place, by assumption, during childhood; z_{t+1} stands for leisure time; x_{t+1} indicates the early environment where a child born at time $t+1$ grows up; i_j (with $j = 1, 2, 3$ and $\sum_j i_j = 1$) are positive parameters determining the weight of consumption, leisure and altruism in the utility function; $0 < \theta < 1$ is the subjective discount factor.

The presence of early environments in the utility function captures the presence of descending intergenerational altruism, that is, altruism from parents to kids; in particular, we model altruism using warm-glow preferences. This is a common assumption in OLG models looking at parental financing of schooling (Kaganovich and Zilcha 1999; Glomm and Kaganovich 2003). Here, warm-glow preferences reflect the idea that joy of giving motives are prevalent when the focus is on the formation of early environments.

Human capital production function

Human capital h_{t+1} is generated according to the following technology:

$$h_{t+1} = q\left[\lambda(e_t)^\rho + (1-\lambda)(x_t)^\rho\right]^{\frac{1}{\rho}} \tag{2}$$

where e_t denotes expenditures for formal schooling and where x_t are early environments. $q > 0$ is a parameter; $0 < \lambda < 1$ is a parameter determining the relative importance of formal schooling and child care in the production of human capital; $\rho \leq 1$ is a parameter governing the elasticity of substitution between formal schooling and child care $\zeta_\rho = \frac{1}{1-\rho}$.[5]

The above production function captures the idea that, depending on the degree of complementarity/substitutability between e_t and x_t, early investments via child care can have permanent effects on educational outcomes and that early additions to a child's human capital may enhance the return of schooling investments.

Child care

Early environments x_t are developed according to the following production function:

$$x_t = \left[\sigma(\varphi_t)^\nu + (1-\sigma)(n_t h_t)^\nu\right]^{\frac{1}{\nu}} \tag{3}$$

where φ_t indicates child care expenditure; n_t is the time parents devote to child rearing; $0 < \sigma < 1$ is a parameter determining the relative importance of child care expenditure and family time in the production of early environments; $\nu \leq 1$ is a parameter governing the elasticity of substitution between φ_t and n_t, $\zeta_\nu = \frac{1}{1-\nu}$.[6] As equation (3) suggests, child care

[5] If $\rho \to 1$ then $\zeta_\rho \to \infty$, i.e. e_t and x_t are perfect substitutes; if $\rho \to 0$ then $\zeta_\rho \to 1$, i.e. we have the Cobb-Douglas case; if $\rho \to -\infty$ then $\zeta_\rho \to 0$, i.e. e_t and x_t are perfect complements.

[6] If $\nu \to 1$ then $\zeta_\nu \to \infty$, i.e. φ_t and $n_t h_t$ are perfect substitutes; if $\nu \to 0$ then $\zeta_\nu \to 1$, i.e. we have the Cobb-Douglas case; if $\nu \to -\infty$ then $\zeta_\nu \to 0$, i.e. φ_t and $n_t h_t$ are perfect complements.

outcomes depend on the productivity of the time devoted to it, that is, they depend on the human capital of the providers.

Government's budget constraints

We want to analyse the impact on growth of a policy reform which changes proportional taxation on labour income. In order to focus on the cost side of distortions, we assume that tax proceeds are returned as lump-sum transfers to the very same generation. Namely, we exclude intergenerational transfers and the possibility to use tax revenues to finance policies which may affect human capital accumulation.[7]

The government budget constraints at $t+1$ is the following:

$$\tau_{t+1} w_{t+1} h_{t+1} l_{t+1} = T_{t+1} \tag{4}$$

where τ_{t+1} is the tax rate, T_{t+1} is the lump-sum transfer ; l_{t+1} is the labor supply of the middle-aged; w_{t+1} is the wage paid to them; τ_{t+1} is the exogenous policy variable and T_{t+1} is endogenously determined to guarantee the equilibrium in the budget constraint.

Individual budget constraints

A child born at time t decides the amount of resources e_t to devote to formal schooling. We assume that she borrows at the interest rate r_{t+1} on the capital market and pays back her loan in the second period.

The time and budget constraints are:

$$l_{t+1} + z_{t+1} + n_{t+1} = 1 \tag{5}$$

$$c_{t+1}^{y} = w_{t+1} h_{t+1} l_{t+1} (1 - \tau_{t+1}) - s_{t+1} + T_{t+1} - \varphi_{t+1} - (1 + r_{t+1}) e_t \tag{6}$$

$$c_{t+2}^{o} = (1 + r_{t+2}) s_{t+1} \tag{7}$$

where s_{t+1} are savings and where all the other variables have the same meaning as elucidated before.

Production function

Output y_t is produced according to the following technology:

$$y_t = K_t^{\delta} L_t^{1-\delta} \tag{8}$$

[7]The assumption that taxes are redistributed to individuals as lump-sum transfers is for instance used in Prescott (2004). As it will become apparent later on, if taxes financed expenditure programs which enhance human capital accumulation, our results would be strengthened.

where K_t is the capital stock, $L_t = l_t h_t$ is the labor supply in efficiency units, and $0 < \delta < 1$ is the share of capital income in output.

3.2 Intertemporal equilibrium and balanced growth path

Taking as given aggregate past savings S_{-1} and the sequence of the exogenous policy parameter $\{\tau_t\}_0^\infty$, an intertemporal equilibrium is defined by a sequence $\{e_t, c_t^y, s_t, c_t^o, l_t, \varphi_t, z_t, n_t, h_t, w_t, r_t, K_t, T_t\}_0^\infty$ that satisfies: the agent's and the firms' maximization problem (characterized by the first order conditions reported in Appendix A); the production function for human capital (2), for child care (3) and for the final output (8); the government budget constraint (4); the clearing condition for the labor market, and the rule of accumulation of physical capital:

$$K_{t+1} = s_t \tag{9}$$

In Appendix A we stationarize the system of equations which define the intertemporal equilibrium, dividing all the equations by the human capital level. A balanced growth path (BGP) is, by definition, the steady state of such stationarized system and its existence requires the assumption that $\tau_t = \tau$ for all t. In other terms a BGP is as an intertemporal equilibrium such that $\{l_t, z_t, n_t, w_t, r_t\}$ are constant and $\{e_t, c_t^y, s_t, c_t^o, \varphi_t, h_t, K_t, T_t\}$ grow at a constant common rate:

$$g_{t+1} = g = \frac{h_{t+1}}{h_t} = q\left[\lambda(\widetilde{e})^\rho + (1-\lambda)(\widetilde{x})^\rho\right]^{\frac{1}{\rho}} \tag{10}$$

where $\widetilde{e} = \frac{e_t}{h_t}$ and $\widetilde{x} = \frac{x_t}{h_t}$. A change in $\widetilde{e}$ ($\widetilde{x}$) measures the change in formal schooling (early environments) as a ratio of the human capital of the preceding generation. For the sake of exposition and with a slight abuse of language, in the paper we refer to changes in $\widetilde{e}$ and $\widetilde{x}$ as changes (improvements or deteriorations) in formal schooling and in early environments.

4 Solution and policy experiment

In our analysis, we focus on the balanced growth path.[8] According to equation (10), the growth rate is a function of $\widetilde{e}$ and of $\widetilde{x}$. Intuitively, taxation affects, on the one hand, the returns to education because it alters both the net wage and the working time: this is the standard effect studied in the literature. On the other hand, it changes the time parents devote to child care

[8] Though the focus is not directly on individual utility and social welfare, we stress that in the long run the higher the growth rate, the higher the individual utility.

and it modifies the amount of child care expenditure, thus impinging on early environments: this is the new channel we identify. To explore these effects and investigate how they combine and affect the growth rate, we proceed in the following way. We first consider a benchmark case, obtained imposing some restrictions on the general model introduced above. Then we test the robustness of the results to the removal of these restrictions.

4.1 A benchmark case

In this benchmark case we impose two types of restrictions: first, factor prices are constant and thus they do not respond to changes in taxation. To this end we assume a small open economy with perfect capital mobility: r_{t+1} is fixed at the world level r. As a consequence, according to (19) and (20), $\frac{K_{t+1}}{L_{t+1}} = \frac{K}{L}$ and $w_{t+1} = w$, that is per capita capital and wages are constant over time. Second, we impose $\rho = 0$ and $\nu = 0$, which imply that in (2) and in (3) the productions function of, respectively, human capital and early environments become Cobb-Douglas.

Under these restrictions, we can solve the model analytically. The theoretical analysis of Section 4.1.1 identifies the main forces at work in our set-up. The sign of the effects, which depends on the value of the underlying parameters, is determined in the calibration and the simulation of Section 4.1.2.

4.1.1 Analytical solution

The effect of a change in labour income taxation τ on the growth rate g is:[9]

$$\frac{\partial g}{\partial \tau} = \frac{g}{1-\lambda}\left[\lambda\left(-\frac{1}{1-\tau} + \frac{\partial l}{\partial \tau}\frac{1}{l}\right) + (1-\lambda)\frac{\partial \widetilde{x}}{\partial \tau}\frac{1}{\widetilde{x}}\right] \tag{11}$$

The first term in square brackets captures the impact of taxation on formal schooling: this is the standard effect and it is always negative since $\frac{\partial l}{\partial \tau}\frac{1}{l} < 0$. An increase in labour income taxation deteriorates formal schooling as it reduces the net wage in efficiency units, and the labour supply. The second term stems from the presence of child care as an instrument to influence the children's early environments. We next study the sign and the magnitude of the effect of taxation on growth which operates through child care in order to investigate how it combines with the one operating through formal schooling.

[9] The relevant proofs of this Section are all relegated to Appendix B.

The last term in (11) can be written as:

$$\frac{\partial \widetilde{x}}{\partial \tau}\frac{1}{\widetilde{x}} = \sigma \frac{\partial \widetilde{\varphi}}{\partial \tau}\frac{1}{\widetilde{\varphi}} + (1-\sigma)\frac{\partial n}{\partial \tau}\frac{1}{n} \tag{12}$$

where $\widetilde{\varphi} = \frac{\varphi_t}{h_t}$. Two effects are at work: a first effect, captured by the term $\frac{\partial n}{\partial \tau}\frac{1}{n}$, refers to the change in the time parents devote to child care. This can be shown to be positive. A second effect is captured by the relative change in $\widetilde{\varphi}$, which can be written as:

$$\frac{\partial \widetilde{\varphi}}{\partial \tau}\frac{1}{\widetilde{\varphi}} = \frac{\lambda}{[1-(1-\tau)\lambda]} + \frac{\partial l}{\partial \tau}\frac{1}{l} \tag{13}$$

Equation (13) can be positive or negative as there are two conflicting forces. First: when taxes go up, the investment in formal schooling (as a share of h_t) is reduced and thus more resources are available for child care expenditure. Second, the labour supply goes down, reducing disposable income: this has a negative impact on child care expenditure.

Thus, when τ increases, $\widetilde{e}$ and l decrease and n rises. The following Propositions provide necessary and sufficient conditions to determine the signs of (13), (12) and (11), which respectively represent the changes of $\widetilde{\varphi}$, $\widetilde{x}$, and g.

Proposition 1 $\frac{\partial \widetilde{\varphi}}{\partial \tau}\frac{1}{\widetilde{\varphi}} \gtreqless 0$ *if and only if:*

$$\Gamma_1 = \lambda i_1 (1-\tau)^2 - [1-(1-\tau)\lambda]^2 \frac{i_1 [i_2 + (1-\sigma) i_3]}{\theta + i_1 + \sigma i_3} \gtreqless 0.$$

Proposition 2 $\frac{\partial \widetilde{x}}{\partial \tau}\frac{1}{\widetilde{x}} \gtreqless 0$ *if and only if:*

$$\Gamma_2 = (1-\sigma) i_1 (1-\tau) + \sigma \Gamma_1 \gtreqless 0.$$

Proposition 3 $\frac{\partial g}{\partial \tau} \gtreqless 0$ *if and only if*

$$\Gamma_3 = -\lambda [1-(1-\tau)\lambda] \left\{ i_1(1-\tau) + \frac{[2-(1-\tau)\lambda]\, i_1 [i_2 + (1-\sigma) i_3]}{\theta + i_1 + \sigma i_3} \right\} + (1-\lambda)\Gamma_2 \gtreqless 0.$$

Proposition 1 focuses on the impact of taxation on child care expenditure. Proposition 2 offers a condition to establish the impact of taxation on early environments. When $\frac{\partial \widetilde{x}}{\partial \tau}\frac{1}{\widetilde{x}} < (>) 0$, an increase in taxation worsens (improves) early environments, therefore strengthening (weakening) the negative effects via formal schooling. Proposition 3 tells for what parameter values the effect of taxation on growth is positive (negative). When $\frac{\partial \widetilde{x}}{\partial \tau}\frac{1}{\widetilde{x}} > 0$, that is when $\Gamma_2 > 0$, there can be instances where $\frac{\partial g}{\partial \tau} > 0$: the effect of taxation on child care more than offset the effect on formal schooling, delivering a positive relationship between taxation and the growth rate.

It is possible to show that the higher τ is, the more likely it is that: $\frac{\partial \widetilde{\varphi}}{\partial \tau}\frac{1}{\widetilde{\varphi}} < 0$, $\frac{\partial \widetilde{x}}{\partial \tau}\frac{1}{\widetilde{x}} < 0$, $\frac{\partial g}{\partial \tau} < 0$. When τ tends to 1, these inequalities are guaranteed. Thus for very high values of the tax rate, its reduction always improves early environments and growth.

Summarising, the introduction of early environments in the human capital production function and the presence of child care as a tool to affect them offer new channels through which taxation affects growth. The theoretical analysis allows us to identify the different forces at work. Child care can either reinforce, weaken or overturn the negative relationship between taxation and growth stemming from the formal schooling channel. Through the calibration of the model, we next investigate which of the different cases illustrated here obtains.

4.1.2 Numerical analysis

To quantitatively explore the effects of taxation, we focus on a specific policy experiment: namely, we reduce by 10% the tax rate on labour income.[10] We calibrate the model and we solve it numerically, using the package Dynare of Juillard (1996).[11]

Parameterization and Calibration Each period has a length of 25 years. The tax rate τ on the labor income of the young is set equal to 47%, which is an average for OECD countries of the marginal tax rates on labor income (marginal personal income tax plus employee's and employer's social security contributions) for workers with the average wage in year 2006 (see the OECD Tax Database). We choose $\delta = 0.29$, that is the share of capital income in national product amounts to 29%. This value is in line with European data over the last twenty years (see Bouzahzah, de la Croix, and Docquier 2002). The intertemporal discount factor θ is set to 0.37 (the quarterly discount factor is 0.99, which is the standard value used in the Real Business Cycle literature). The world interest rate r is set to 4.5%.

The remaining parameters are obtained through the following calibration procedure. We choose i_j with $j = 1, 2, 3$ in order to generate a realistic allocation of time between labor, child care and leisure. As far as the time devoted to child care is concerned, we set it equal

[10] It is obviously possible to perform alternative policy experiments, namely, to change the tax rate τ by a higher amount. However, the assumption that structural parameters of the economy (such as the preferences for consumption, leisure and child care) are constant is more grounded when one considers small deviations from the data on which the model is calibrated. Nonetheless, we have computed the effects of reducing τ to 0 and the main message of the paper is not affected.

[11] This package implements a Newton-Raphson relaxation method put forward by Laffargue (1990) and Boucekkine (1995) for solving dynamic nonlinear models with perfect foresight.

to 13 hours per week, which is compatible with Cardia and Ng (2003) and with the references quoted therein. As to the time devoted to work, we make a standard assumption that parents devote to it about 36 hours per week. Assuming, as in Juster (1985), that non-personal time available for discretionary use amounts to 100 hours per week, we have: $l = 36\%$, $n = 13\%$, $z = 51\%$. The parameter q of the human capital production function (2) is chosen in order to obtain an annual growth rate equal to 1.5%. The parameter λ is set in order to match a ratio between total expenditure on formal schooling and GDP equal to 6.2%, which is consistent with what we observe in OECD countries (see OECD 2007). The parameter σ is chosen to match the share of parents' income devoted to child care expenditures, which amounts to 3% according to data reported by Cardia and Ng (2003).

The assumptions underlying the numerical simulation of this and the following Sections are summarised in Table 1.

[Table 1 about here.]

Simulation's results The decrease of labour income taxation determines an increase in child care expenditure as a ratio of human capital $\widetilde{\varphi}$. This increase does not compensate the reduction in parental time and therefore $\widetilde{x}$ decreases. Overall, the impact of a tax reduction on the growth rate is positive, thanks to the positive impact on formal schooling. If compared to a model where child care is absent, the impact on growth is here smaller because the reduction in taxation deteriorates early environments, with negative repercussions on human capital accumulation.[12]

4.2 Extensions

We here relax the restrictions imposed in the benchmark case. The main purpose is to test the validity of the results there obtained.

We will first perform a sensitivity analysis on some parameters characterising the human capital and early environments production functions, namely (ν, ρ). We will then analyse the effects of having fully flexible factor prices: for this purpose we assume that the economy is closed.

[12]The calibration procedure described above implies $i_1 = 0.245$, $i_2 = 0.58$, $i_3 = 0.175$, $q = 38.8$, $\lambda = 0.34$, and $\sigma = 0.133$ for the benchmark case we are considering here. As a consequence we have that the conditions in Propositions 1, 2 and 3 are, respectively: $\Gamma_1 = -0.17 < 0$, $\Gamma_2 = 0.09 > 0$ and $\Gamma_3 = -0.12 < 0$.

Once we abandon the restriction imposed in the previous Section, we cannot solve the model analytically and we therefore provide numerical examples on the working of the model. The values of τ, δ, θ and r are the same as in Section 4.1.2.[13] The parameters i_j, q, λ and σ are calibrated following the procedure described in the benchmark case: for each case we are going to examine, they will change to match the target values of the endogenous variables there mentioned.[14]

As we will show, in the majority of the cases we analyse, the response of early environments to taxation is qualitatively the same as the one identified in the benchmark model.

4.2.1 Removing parametric restrictions

We here study the effects of changing, with respect to the benchmark case, the parameters ν and ρ, which respectively determine the elasticity of substitution between child care expenditure and parental time $\zeta_\nu = \frac{1}{1-\nu}$ and the elasticity of substitution between formal schooling and early environments $\zeta_\rho = \frac{1}{1-\rho}$.[15]

As far as we know, there are no available estimates of ν. However, given that child care expenditure not only includes expenditures on goods (e.g. books and toys) but also expenditures on child care services (e.g. day care centers), it seems reasonable to consider $0 < \nu < 1$, that is to explore the possibility that the two inputs of the production of early environments, though they are not perfect substitutes, show less complementarity than implied by the Cobb-Douglas case.

As far ρ is concerned, there are reasons to think that these two inputs have a low degree of substitutability (see the references in Section 2), though there are no well established estimates. Cuhna (2006), for example, argues that the degree of complementarity between early and later investment is slightly higher than the one implied by a Cobb-Douglas.

[13]Notice, however, that when we consider in Section 4.2.2 the flexible factor price case, r is obviously no longer exogenously fixed and thus θ is calibrated to have $r = 4.5\%$ as in Section 4.1.2.

[14]In other terms, we always compare economy which are observationally equivalent (and differ in the underlying parameters).

[15]In the utility function (1) it is assumed that the value of the intertemporal elasticity of leisure is 1, which is within the wide range of estimates that can be found in the literature; indeed, though all the earlier estimates suggest values (much) below 1, values higher than 1 can be found in the most recent literature (see the references reported by Fuster, Imrohoroglu, and Imrohoroglu 2007). We have performed a sensitivity analysis - not reported here, but available upon request- with respect to the intertemporal elasticity of leisure. In particular, values of this elasticity which are lower than 1 strengthen the results we obtain.

We explore the following possibilities: $\nu = 0$, $\nu = 0.4$, $\nu = 0.8$, $\nu = 0.95$; $\rho = 0$, $\rho = -0.2$, $\rho = -0.4$, $\rho = -0.6$. The results of our numerical analysis are presented in Tables 2, 3, 4, which show respectively the percentage change in $\widetilde{e}$, $\widetilde{x}$, g when the tax rate is reduced - as in Section 4.1.2- by 10%.

We can notice that in most cases the sign of the change in $\widetilde{x}$ is the same as in Section 4.1.2: $\widetilde{x}$ does not decrease only when ν is very high. The intuition is that the higher the degree of substitution between child care expenditure and parental time, the easier it is for the parents to replace their reduced availability with goods and services. For a given value of ν, the higher ρ - in absolute value -, the smaller the increase in $\widetilde{e}$: the higher the degree of complementarity between formal schooling and early environments, the more early environments constrain the choices on formal schooling. We also see that here are instances where the combined effect of taxation on $\widetilde{e}$ and $\widetilde{x}$ is such that the growth rate decreases.

[Table 2 about here.]

[Table 3 about here.]

[Table 4 about here.]

4.2.2 Flexible factor prices

Finally, we explore the role of having fully flexible factor prices; to this end, and differently from what we have assumed in the previous sections, we consider the case of a closed economy.

We perform the same analysis described in Section 4.2.1. The results are reported in Tables 5, 6, 7 and 8.

First we can notice that $\widetilde{x}$ decreases also when ν is very high: the effects of early environments and of formal schooling on the accumulation of human capital are always of opposite sign. Second, the increase in $\widetilde{e}$ is lower than in a small open economy. The reason is related to the behaviour of the ratio between the physical capital stock and the labour supply in efficiency units K_t/L_t. In (almost) all the cases considered in Table 8, K_t/L_t decreases; thus the interest rate r, which affects the cost of the investment in formal schooling, rises and the gross wage, which influences the benefit of investing in $\widetilde{e}$, decreases (while the net wage $(1-\tau)w$ rises). All together, the overall increase in the growth rate is more modest than in a small open economy; there are even parameter configurations where, while g rises when factor prices are fixed, it decreases within a closed economy environment.

[Table 5 about here.]

[Table 6 about here.]

[Table 7 about here.]

[Table 8 about here.]

5 Conclusions

Human capital is produced over a lifetime not just by genetic heritage, but also by families and schools. Recently, the identification of critical phases in the learning process and, in particular, the analysis of how early environments in which children grow up are shaped and of how important they are in affecting the individual's skill formation process have received increasing attention. Here we develop a three-period general equilibrium OLG growth model where human capital is formed both via formal schooling and via early environments. Parents can affect the early environments where children grow up via child care, that is parental time and child care expenditure. In this framework we assess the impact on growth of a policy reform which modifies the tax on labour income. On the one hand, such a reform affects the returns to education and it therefore impinges on the decision to invest in formal schooling: this is the standard effect studied in the literature. On the other hand, it influences the time parents devote to child care and the amount of child care expenditure: this is the new channel we identify.

Theoretically, the overall effects of the different forces at work in our set-up are indeterminate, depending on the structural parameters of the model. The calibration and the sensitivity analysis show that in most cases a reduction in taxation determines a deterioration of the process of acquisition of skills taking place in childhood. In this respect, the introduction of early environments in the skill formation process weakens the negative relationship between taxation and growth that is usually obtained in most part of the literature. There can also be instances where the relationship between taxation on labour income and growth becomes positive.

Our analysis is based on the assumption that agents are homogeneous and our computational experiment is calibrated on average data. Thus, they focus on the average effects of a generalized reduction of the tax rate on labour income. This is a first step in exploring the impact of taxes on growth when the role of child care for human capital production is explicitly considered. Our investigation is however silent on issues related to the use of the tax instrument to increase the labor market participation of specific groups of the population. In some countries, like Italy for instance, the debate on the need to encourage women to enter the labour force is heated. This issue cannot be properly tackled without abandoning the intragenerational homogeneity assumption. By the same token, the introduction of early environments in the technology of skill formation is relevant not only to address issues related to the growth potential of an economy but also questions concerning the degree of inequality and mobility which characterises it. Policies which affect child care inputs have relevant redistributive dimensions which cannot be addressed here. These important issues are left for future research.

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Appendix A

In this Appendix we first derive the first order conditions for the agent's and firm's optimization problem. Then we report the stationarized version of the model.

Individual optimization problem Agents maximize (1) subject to (2), (3), and to the budget constraints (5)-(7). After some simple manipulations, first order conditions can be written as:[16]

$$e_t : (1+r_{t+1}) = (1-\tau_{t+1})w_{t+1}l_{t+1}\frac{\lambda h_{t+1}}{[\lambda(e_t)^\rho + (1-\lambda)(x_t)^\rho]}\frac{(e_t)^\rho}{e_t} \tag{14}$$

$$s_{t+1} : i_1\frac{1}{c^y_{t+1}} = (1+r_{t+2})\theta\frac{1}{c^o_{t+2}} \tag{15}$$

$$l_{t+1} : i_2\frac{1}{z_{t+1}} = i_1\frac{1}{c^y_{t+1}}(1-\tau_{t+1})w_{t+1}h_{t+1} \tag{16}$$

$$n_{t+1} : i_2\frac{1}{z_{t+1}} = i_3\frac{(1-\sigma)(n_{t+1}h_{t+1})^\nu}{[\sigma(\varphi_{t+1})^\nu + (1-\sigma)(n_{t+1}h_{t+1})^\nu]}\frac{1}{n_{t+1}} \tag{17}$$

$$\varphi_{t+1} : i_1\frac{1}{c^y_{t+1}} = i_3\frac{\sigma(\varphi_{t+1})^\nu}{[\sigma(\varphi_{t+1})^\nu + (1-\sigma)(n_{t+1}h_{t+1})^\nu]}\frac{1}{\varphi_{t+1}} \tag{18}$$

Firms' optimization problem Full depreciation of capital is assumed. Profit maximizing behavior of the competitive firms implies that the interest rate is:

$$1+r_{t+1} = \delta\left(\frac{K_{t+1}}{L_{t+1}}\right)^{\delta-1} \tag{19}$$

and that the wage in efficiency units is:

$$w_{t+1} = (1-\delta)\left(\frac{K_{t+1}}{L_{t+1}}\right)^{\delta} \tag{20}$$

which are the standard conditions.

Stationarized model We here report the model in its stationarized version, which is used for the simulations we perform in Dynare. The variables related to the use of time are constant along a BGP and therefore no stationarization is needed. All the other variables are divided by human capital. The $\sim$ denotes the new stationarized variables.

[16] In equation (14) we assume that agents invest in education to enhance their productivity on the labour market. The fact that human capital is also relevant for the production of early environments is treated as an externality.

Stationarizing human capital production function (2) we get the following expression for the growth rate:

$$g_{t+1} = \frac{h_{t+1}}{h_t} = q\left[\lambda(\widetilde{e}_t)^\rho + (1-\lambda)(\widetilde{x}_t)^\rho\right]^{\frac{1}{\rho}} \tag{21}$$

where (using (3)):

$$\widetilde{x}_t = \frac{x_t}{h_t} = \left[\sigma(\widetilde{\varphi}_t)^\nu + (1-\sigma)(n_t)^\nu\right]^{\frac{1}{\nu}} \tag{22}$$

with $\widetilde{e}_t = \frac{e_t}{h_t}$ and $\widetilde{\varphi}_t = \frac{\varphi_t}{h_t}$.

The time constraint (5) remains unchanged, while the individual budget constraints become:

$$\widetilde{c}^y_{t+1} = (1-\tau_{t+1})w_{t+1}l_{t+1} - \widetilde{s}_{t+1} + \widetilde{T}_{t+1} - \widetilde{\varphi}_{t+1} - (1+r_{t+1})\frac{\widetilde{e}_t}{g_{t+1}} \tag{23}$$

$$\widetilde{c}^o_{t+2} = (1+r_{t+2})\widetilde{s}_{t+1} \tag{24}$$

with $\widetilde{c}^y_{t+1} = \frac{c^y_{t+1}}{h_{t+1}}$, $\widetilde{s}_{t+1} = \frac{s_{t+1}}{h_{t+1}}$, $\widetilde{T}_{t+1} = \frac{T_{t+1}}{h_{t+1}}$, $\widetilde{c}^o_{t+2} = \frac{c^o_{t+2}}{h_{t+1}}$.

The government budget constraint (4) is now:

$$\tau_{t+1}w_{t+1}l_{t+1} = \widetilde{T}_{t+1}.$$

The first order conditions (14)-(18) in stationarized terms are:

$$e_t : (1+r_{t+1}) = (1-\tau_{t+1})w_{t+1}l_{t+1}\frac{\lambda}{\left[\lambda(\widetilde{e}_t)^\rho + (1-\lambda)(\widetilde{x}_t)^\rho\right]}\frac{(\widetilde{e}_t)^\rho}{\widetilde{e}_t}g_{t+1} \tag{25}$$

$$s_{t+1} : i_1\frac{1}{\widetilde{c}^y_{t+1}} = (1+r_{t+2})\theta\frac{1}{\widetilde{c}^o_{t+2}} \tag{26}$$

$$l_{t+1} : i_2\frac{1}{z_{t+1}} = i_1\frac{1}{\widetilde{c}^y_{t+1}}(1-\tau_{t+1})w_{t+1} \tag{27}$$

$$n_{t+1} : i_2\frac{1}{z_{t+1}} = i_3\frac{(1-\sigma)(n_{t+1})^\nu}{\left[\sigma(\widetilde{\varphi}_{t+1})^\nu + (1-\sigma)(n_{t+1})^\nu\right]}\frac{1}{n_{t+1}} \tag{28}$$

$$\varphi_{t+1} : i_1\frac{1}{\widetilde{c}^y_{t+1}} = i_3\frac{\sigma(\widetilde{\varphi}_{t+1})^\nu}{\left[\sigma(\widetilde{\varphi}_{t+1})^\nu + (1-\sigma)(n_{t+1})^\nu\right]}\frac{1}{\widetilde{\varphi}_{t+1}}. \tag{29}$$

The stationarized capital market equilibrium condition (9) at $t+2$ can be written as:

$$\widetilde{K}_{t+2} = \frac{K_{t+2}}{h_{t+2}} = \frac{s_{t+1}}{g_{t+2}} \tag{30}$$

The first order conditions (19) and (20) for the firm can be expressed as:

$$1 + r_{t+1} = \delta\left(\frac{\widetilde{K}_{t+1}}{\widetilde{L}_{t+1}}\right)^{\delta-1} \tag{31}$$

$$w_{t+1} = (1-\delta)\left(\frac{\widetilde{K}_{t+1}}{\widetilde{L}_{t+1}}\right)^{\delta} \tag{32}$$

where $\widetilde{K}_{t+1} = \frac{K_{t+1}}{h_{t+1}}$ and $\widetilde{L}_{t+1} = \frac{L_{t+1}}{h_{t+1}} = l_{t+1}$.

As we stress in Section 3.2, a balanced growth path is, by definition, the steady state of such stationarized system. For the existence of this steady state we need to assume that the tax rate is constant, that is $\tau_t = \tau$.

Appendix B

Under the restrictions imposed in Section 4.1 it is possible to give a closed form solution of the stationarized system of equations presented in Appendix A.

Here we only report the features of this analytical solution that are more useful to derive the results presented in Section 4.1.1. In particular it is possible to derive the following equations (written for a balanced growth path):

$$g = q(\widetilde{e})^{\lambda}(\widetilde{x})^{1-\lambda} \tag{33}$$

$$\widetilde{e} = \lambda \frac{w}{(1+r)}(1-\tau)lg \tag{34}$$

$$\widetilde{x} = q(\widetilde{\varphi})^{\sigma}(n)^{1-\sigma} \tag{35}$$

$$\widetilde{\varphi} = \frac{\sigma i_3}{\theta + i_1 + \sigma i_3}[1-(1-\tau)\lambda]wl \tag{36}$$

where:

$$l = \frac{i_1(1-\tau)}{i_1(1-\tau) + A[i_2 + i_3(1-\sigma)][1-(1-\tau)\lambda]} \tag{37}$$

$$n = A\frac{i_3(1-\sigma)}{i_1(1-\tau) + A[i_2 + i_3(1-\sigma)][1-(1-\tau)\lambda]}[1-(1-\tau)\lambda] \tag{38}$$

with $A = \frac{i_1}{\theta + i_1 + \sigma i_3}$.

Substituting (34) in (33) and differentiating with respect to τ we have:

$$\frac{\partial g}{\partial \tau} = g\left\{\lambda\left(-\frac{1}{1-\tau} + \frac{\partial l}{\partial \tau}\frac{1}{l} + \frac{\partial g}{\partial \tau}\frac{1}{g}\right) + (1-\lambda)\frac{\partial \widetilde{x}}{\partial \tau}\frac{1}{\widetilde{x}}\right\} \tag{39}$$

Solving for $\frac{\partial g}{\partial \tau}$ we obtain equation (11) of Section 4.1.1.

Using equations (35) and (36), we obtain respectively equations (12) and (13) of Section 4.1.1.

Starting from equations (37) and (38) we can write:

$$\frac{\partial l}{\partial \tau}\frac{1}{l} = -\frac{1}{(1-\tau)}\frac{A\left[i_2 + (1-\sigma)i_3\right]}{\left\{i_1(1-\tau) + A\left[i_2 + (1-\sigma)i_3\right]\left[1-(1-\tau)\lambda\right]\right\}} \tag{40}$$

$$\frac{\partial n}{\partial \tau}\frac{1}{n} = \frac{1}{\left[1-(1-\tau)\lambda\right]}\frac{i_1}{\left\{i_1(1-\tau) + A\left[i_2 + (1-\sigma)i_3\right]\left[1-(1-\tau)\lambda\right]\right\}} \tag{41}$$

Substituting equation (40) in equation (13) we have:

$$\frac{\partial \widetilde{\varphi}}{\partial \tau}\frac{1}{\widetilde{\varphi}} = \frac{\overbrace{\lambda i_1(1-\tau)^2 - \left[1-(1-\tau)\lambda\right]^2 A\left[i_2 + (1-\sigma)i_3\right]}^{\equiv \Gamma_1}}{\left[1-(1-\tau)\lambda\right](1-\tau)\left\{i_1(1-\tau) + \left[1-(1-\tau)\lambda\right]A\left[i_2 + (1-\sigma)i_3\right]\right\}} \tag{42}$$

that proves Proposition 1.

Using equations (41) and (42) in equation (12) we obtain:

$$\frac{\partial \widetilde{x}}{\partial \tau}\frac{1}{\widetilde{x}} = \frac{\overbrace{(1-\sigma)i_1(1-\tau)+\sigma\Gamma_1}^{\equiv\Gamma_2}}{\left[1-(1-\tau)\lambda\right](1-\tau)\left\{i_1(1-\tau)+\left[1-(1-\tau)\lambda\right]A\left[i_2+(1-\sigma)i_3\right]\right\}} \tag{43}$$

that proves Proposition 2.

Finally we can prove Proposition 3, using equations (43) and (40) in equation (11):

$$\frac{\partial g}{\partial \tau} = \frac{g}{1-\lambda}\frac{\overbrace{-\lambda\left[1-(1-\tau)\lambda\right]\left\{i_1(1-\tau)+\left[2-(1-\tau)\lambda\right]A\left[i_2+(1-\sigma)i_3\right]\right\}+(1-\lambda)\Gamma_2}^{\equiv\Gamma_3}}{\left[1-(1-\tau)\lambda\right](1-\tau)\left\{i_1(1-\tau)+\left[1-(1-\tau)\lambda\right]A\left[i_2+(1-\sigma)i_3\right]\right\}}$$

From equations (42) and (43) we can see that the higher τ is, the more likely it is that $\frac{\partial \widetilde{\varphi}}{\partial \tau}\frac{1}{\widetilde{\varphi}} < 0$ and $\frac{\partial \widetilde{x}}{\partial \tau}\frac{1}{\widetilde{x}} < 0$.

As to the condition $\Gamma_3 < 0$, it can be written as:

$$1 < \left\{\left[\lambda+(1-\lambda)\sigma\right]\left[1-(1-\tau)\lambda\right]+\lambda\right\}\left\{1+\frac{i_2+(1-\sigma)i_3}{\theta+i_1+\sigma i_3}\left[\frac{1}{1-\tau}-\lambda\right]\right\}$$

from which we can conclude that the higher τ is, the more likely it is that $\frac{\partial g}{\partial \tau} < 0$.

List of Tables

1 Parameterization and Calibration 24

2 Small open economy: Effect (% change) on $\widetilde{e}$ of a 10% reduction in labour income taxation. 25

3 Small open economy: Effect (% change) on $\widetilde{x}$ of a 10% reduction in labour income taxation. 26

4 Small open economy: Effect (% change) on g of a 10% reduction in labour income taxation. 27

5 Closed economy: Effect (% change) on $\widetilde{e}$ of a 10% reduction in labour income taxation. 28

6 Closed economy: Effect (% change) on $\widetilde{x}$ of a 10% reduction in labour income taxation. 29

7 Closed economy: Effect (% change) on g of a 10% reduction in labour income taxation. 30

8 Closed economy: Effect (% change) on K_t/L_t of a 10% reduction in labour income taxation. 31

Table 1: Parameterization and Calibration

Tax rate	τ	47%
Share of capital income	δ	29%
Discount factor	θ	In open economy: 0.37 In closed economy: chosen to match an interest rate of 4.5% (which is exogenous in open economy)
Weights in the utility function	i_j	chosen to match the allocation of time between labour (36%), time devoted to children (13%) and leisure (51%).
Weight of formal schooling	λ	chosen to match a ratio between total expenditure on formal schooling and GDP equal to 6.2%
Weight of childcare expenditure	σ	chosen to match a share of parents' income devoted to child care expenditures equal to 3%
Elasticity of substitution between child care expenditures and parental time	$\zeta_\nu = \frac{1}{1-\nu}$	Sensitivity analysis in the range $0 \leq \nu < 1$
Elasticity of substitution between formal schooling and early environments	$\zeta_\rho = \frac{1}{1-\rho}$	Sensitivity analysis in the range $0 \leq \rho \leq 0.6$

Table 2: Small open economy: Effect (% change) on $\widetilde{e}$ of a 10% reduction in labour income taxation.

	$\nu = 0$	$\nu = 0.4$	$\nu = 0.8$	$\nu = 0.95$
$\rho = 0$	22%	23%	25%	44%
$\rho = -0.2$	15%	16%	19%	33%
$\rho = -0.4$	12%	12%	15%	27%
$\rho = -0.6$	9%	10%	12%	23%

Table 3: Small open economy: Effect (% change) on $\widetilde{x}$ of a 10% reduction in labour income taxation.

	$\nu = 0$	$\nu = 0.4$	$\nu = 0.8$	$\nu = 0.95$
$\rho = 0$	-3%	-3%	-2%	0%
$\rho = -0.2$	-3%	-2%	-2%	0%
$\rho = -0.4$	-2%	-2%	-2%	0%
$\rho = -0.6$	-2%	-2%	-2%	0%

Table 4: Small open economy: Effect (% change) on g of a 10% reduction in labour income taxation.

	$\nu = 0$	$\nu = 0.4$	$\nu = 0.8$	$\nu = 0.95$
$\rho = 0$	13%	14%	16%	33%
$\rho = -0.2$	4%	6%	10%	24%
$\rho = -0.4$	0%	1%	5%	17%
$\rho = -0.6$	-3%	-1%	3%	14%

Table 5: Closed economy: Effect (% change) on $\widetilde{e}$ of a 10% reduction in labour income taxation.

	$\nu = 0$	$\nu = 0.4$	$\nu = 0.8$	$\nu = 0.95$
$\rho = 0$	12%	12%	12%	11%
$\rho = -0.2$	11%	11%	11%	10%
$\rho = -0.4$	10%	10%	10%	9%
$\rho = -0.6$	9%	9%	9%	8%

Table 6: Closed economy: Effect (% change) on $\widetilde{x}$ of a 10% reduction in labour income taxation.

	$\nu = 0$	$\nu = 0.4$	$\nu = 0.8$	$\nu = 0.95$
$\rho = 0$	-3%	-3%	-3%	-2%
$\rho = -0.2$	-3%	-3%	-3%	-2%
$\rho = -0.4$	-3%	-2%	-2%	-2%
$\rho = -0.6$	-2%	-2%	-2%	-1%

Table 7: Closed economy: Effect (% change) on g of a 10% reduction in labour income taxation.

	$\nu = 0$	$\nu = 0.4$	$\nu = 0.8$	$\nu = 0.95$
$\rho = 0$	5%	5%	5%	6%
$\rho = -0.2$	2%	2%	3%	4%
$\rho = -0.4$	-1%	0%	1%	3%
$\rho = -0.6$	-3%	-2%	0%	2%

Table 8: Closed economy: Effect (% change) on K_t/L_t of a 10% reduction in labour income taxation.

	$\nu = 0$	$\nu = 0.4$	$\nu = 0.8$	$\nu = 0.95$
$\rho = 0$	-5%	-6%	-7%	-11%
$\rho = -0.2$	-3%	-3%	-5%	-10%
$\rho = -0.4$	-1%	-2%	-4%	-9%
$\rho = -0.6$	0%	-1%	-3%	-9%