Fullerton + Rogers calibrate Cons

From Stone-Geary util, F&C + B.C. suggests demand as:

→ not this differs slightly from theory section but this seems more correct

$$① \quad c_{it} = b_{it} + \frac{\beta_{it}}{p_i}\left(\tilde{p}_t \tilde{c}_t - \sum_{i=1}^{N} p_i b_{it}\right)$$

In data, don't observe prices, so transform ① to

$$② \quad p_i c_{it} = p_i b_{it} + \beta_{it} \tilde{p}_t \tilde{c}_t - \beta_{it} \sum_{i=1}^{N} p_i b_{it}$$

$p_i c_{it}$ → total exp on good i } obs both in data

$\tilde{p}_t \tilde{c}_t$ → total cons. exp.
→ call this m_t

$p_i b_{it}$ = min expend on good i at age t → unobs.
→ can be negative

rewrite:

$$②' \quad p_i c_{it} = exp_{it} = p_i b_{it} + \beta_{it}\left(m_t - \sum_{i=1}^{N} p_i b_{it}\right)$$

rewrite as:

$$③ \quad exp_{it} = k_{it} + \beta_{it} m_t + \varepsilon_{it}$$

$$\varepsilon_{it} \sim N(0, \sigma^2)$$

$$④ \quad k_{it} = p_i b_{it} - \beta_{it} \sum_{j=1}^{N} p_j b_{jt}$$

OLS to estimate ③ ~~Equation~~ for each
~~sep~~ → use population weights
→ use k_{it} estimates (the constants) ~~to~~ and β_{it} to solve for $p_i b_{it}$

→ Note → if include married/single status in model, may want to estimate separately for these

* age groups in F+R
~~18 ages~~ → 12 age groups
→ should we do
→ prob yes to smooth & so more obs by group
↓
even w/ this
12 × 17 + 2 params
(age groups) (cons categ) (b_{it}, β_{it})

note that it's SUR, but unbiased + efficient b/c indep. vars the same across regressions

$$\Rightarrow \lambda p_i = \frac{\beta_{it}}{c_{it}-b_{it}} \prod_{i=1}^{N} (c_{it}-b_{it})^{\beta_{it}} \quad \forall i$$

$$\Rightarrow \lambda = \frac{\beta_{it}}{p_i(c_{it}-b_{it})} \prod_{i=1}^{N} (c_{it}-b_{it})^{\beta_{it}}, \quad \forall i$$

$$\Rightarrow \frac{\beta_{it}}{p_i(c_{it}-b_{it})} = \frac{\beta_{jt}}{p_j(c_{jt}-b_{jt})}, \quad \forall i,j$$

$$\Rightarrow c_{it}-b_{it} = \frac{\beta_{it} \; p_j(c_{jt}-b_{jt})}{\beta_{jt} \, p_i}$$

$$\Rightarrow c_{it} = \frac{\beta_{it} \; p_j(c_{jt}-b_{jt})}{\beta_{jt} \, p_i} + b_{it}$$

$$c_{it} = \frac{\beta_{it}}{p_i}\left[\frac{p_j(c_{jt}-b_{jt})}{\beta_{jt}}\right] + b_{it}$$

plug this into:

$$\tilde{p}_t \tilde{c}_t = \sum_{i=1}^{N} p_i(c_{it}-b_{it})$$

$$\cancel{\tilde{p}_t \tilde{c}_t = \sum_{i=1}^{N} \frac{\beta_{it}}{p_i}\left(\frac{p_j(c_{jt}-b_{jt})}{\beta_{jt}}\right)}$$

$$\cancel{\tilde{p}_t \tilde{c}_t = \left(\frac{p_j(c_{jt}-b_{jt})}{\beta_{jt}}\right) \sum_{i=1}^{N} \frac{\beta_{it}}{p_i}}$$

$$\cancel{\frac{p_i(c_{it}-b_{it})}{\beta_{it}} = \prod_{i=1}^{N} (c_{it}-b_{it})^{\beta_{it}} = \tilde{c}_t}$$

$$\cancel{\frac{\tilde{c}_t \beta_{it}}{\lambda p_i} + b_{it} = c_{it}}$$

$$\cancel{\lambda = \frac{1}{\tilde{p}_t}\ ?}$$

note, $\sum_i \beta_{it} = 1$

$$\tilde{p}_t \tilde{c}_t = \sum_{i=1}^{N} p_i c_{it} - \sum_{i=1}^{N} p_i b_{it}$$

$$\tilde{p}_t \tilde{c}_t = \sum_{i=1}^{N} p_i \frac{\beta_{it}}{p_i}\left[\frac{p_j(c_{jt}-b_{jt})}{\beta_{jt}}\right] + p_i b_{it} - \sum_{i=1}^{N} p_i b_{it}$$

$$= \sum_{i}\left[\frac{p_j(c_{jt}-b_{jt})}{\beta_{jt}}\right] \underbrace{\sum_{i=1}^{N} \beta_{it}}_{=1} + \cancel{\sum_{i=1}^{N} p_i b_{it}} - \cancel{\sum_{i=1}^{N} p_i b_{it}}$$

$$\Rightarrow \tilde{p}_t \tilde{c}_t = \frac{p_j(c_{jt}-b_{jt})}{\beta_{jt}}$$

$$\Rightarrow c_{jt} = \frac{\tilde{p}_t \tilde{c}_t \beta_{jt}}{p_j} + b_{jt}$$

how determine cons. basket?

→ β_{it} estimated

→ b_{it} estimated

→ $\tilde{c}_t$ from individual optimization

→ $\tilde{p}_t = ?$ aggregate price level? but varies w/ age — "price of age t composite cons. good"

→ $p_j =$ ~~from producers' problem~~ ? from fixed coeff ("transition matrix") mapping production goods to cons. goods

$$p_j = \sum_{i=1}^{M} p_i^{0} z_{ji}$$

(deter. earlier; trans)

$$\cancel{c_{it} = b_{it} + \frac{\tilde{p}_t \tilde{c}_t \beta_{it}}{p_i}}$$

$$\cancel{\sum_i c_{it} = \tilde{c}_t = \sum_i b_{it} + \sum_i \frac{\tilde{p}_t \tilde{c}_t \beta_{it}}{p_i}}$$

$$\cancel{\tilde{c}_t = \underbrace{\sum_i b_{it}}_{\tilde{B}_t} + \tilde{p}_t \tilde{c}_t \sum_{i=1}^{N} \frac{\beta_{it}}{p_i}}$$

$$\cancel{\tilde{c}_t - \tilde{B}_t = \tilde{p}_t \tilde{c}_t \sum \frac{\beta_{it}}{p_i}}$$

$$\cancel{\frac{\tilde{c}_t - \tilde{B}_t}{\tilde{c}_t \sum \frac{\beta_{it}}{p_i}} = \tilde{p}_t}$$ → ~~could find this given info above~~

$$\cancel{\sum_{i=1}^{N} p_i (c_{it} - b_{it}) = \tilde{p}_t \prod_{i=1}^{N} (c_{it} - b_{it})^{\beta_{it}}}$$

$$\tilde{c}_t = \prod_{i=1}^{N} \left(\frac{\tilde{p}_t \tilde{c}_t \beta_{it}}{p_i} + \cancel{b_{it}} - \cancel{b_{it}} \right)^{\beta_{it}}$$

$$\cancel{\tilde{c}_t} = \cancel{\tilde{p}_t \tilde{c}_t} \prod_{i=1}^{N} \left(\frac{\beta_{it}}{p_i} \right)^{\beta_{it}} \Rightarrow \frac{1}{\tilde{p}_t} = \prod_{i=1}^{N} \left(\frac{\beta_{it}}{p_i} \right)^{\beta_{it}}$$ ✓

$$\Rightarrow \tilde{p}_t = \prod_{i=1}^{N} \left(\frac{p_i}{\beta_{it}} \right)^{\beta_{it}}$$

Consumer prefs over ~~produced~~ production goods

$$(3)\quad \tilde{Q}_j = \left[\gamma_j^{1/\varepsilon_3} Q_j^{c\,(\varepsilon_3-1)/\varepsilon_3} + (1-\gamma_j)^{1/\varepsilon_3} \left(Q_j^{nc}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} \right]^{\varepsilon_3/(\varepsilon_3-1)}$$

$\uparrow$ total cons. of production good j

$$\text{Max } (3) \text{ s.t.: } P_j^c Q_j^c + P_j^{nc} Q_j^{nc} = P_j^Q Q_j$$

P_j^c = price of prod good j from corp

$$\Rightarrow \mathcal{L} = \left[\gamma_j^{1/\varepsilon_3} Q_j^{c\,\frac{\varepsilon_3-1}{\varepsilon_3}} + (1-\gamma_j)^{1/\varepsilon_3} \left(Q_j^{nc}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} \right]^{\varepsilon_3/(\varepsilon_3-1)}$$

$$+ \lambda \left(P_j^Q Q_j - P_j^c Q_j^c - P_j^{nc} Q_j^{nc} \right)$$

$$(1)\quad \frac{\partial \mathcal{L}}{\partial Q_j^c} = \left(\frac{\varepsilon_3}{\varepsilon_3-1}\right) \left[\gamma_j^{1/\varepsilon_3} Q_j^{c\,\frac{\varepsilon_3-1}{\varepsilon_3}} + (1-\gamma_j)^{1/\varepsilon_3} \left(Q_j^{nc}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3-1}-1}$$

$$\left(\frac{\varepsilon_3-1}{\varepsilon_3} \gamma_j^{1/\varepsilon_3} Q_j^{c\,\frac{\varepsilon_3-1}{\varepsilon_3}-1} \right) - \lambda P_j^c = 0$$

$$(2)\quad \frac{\partial \mathcal{L}}{\partial Q_j^{nc}} = \left(\frac{\varepsilon_3}{\varepsilon_3-1}\right) \left[\gamma_j^{1/\varepsilon_3} Q_j^{c\,\frac{\varepsilon_3-1}{\varepsilon_3}} + (1-\gamma_j)^{1/\varepsilon_3} \left(Q_j^{nc}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3-1}-1}$$

$$\left[\left(\frac{\varepsilon_3-1}{\varepsilon_3}\right) (1-\gamma_j)^{1/\varepsilon_3} \left(Q_j^{nc}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}-1} \right] - \lambda P_j^{nc} = 0$$

$$(3)\quad \frac{\partial \mathcal{L}}{\partial \lambda} = P_j^Q Q_j - P_j^c Q_j^c - P_j^{nc} Q_j^{nc} = 0$$

① can be rewritten as:

$$\left[\gamma_j^{1/\varepsilon_3} Q_j^{c\,\frac{\varepsilon_3-1}{\varepsilon_3}} + (1-\gamma_j)^{1/\varepsilon_3} (Q_j^{nc})^{\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{\cancel{\varepsilon_3}-(\cancel{\varepsilon_3}-1)}{\varepsilon_3-1}}$$

$$\left(\gamma^{1/\varepsilon_3} Q_j^{c\,\frac{\cancel{\varepsilon_3}-1-\cancel{\varepsilon_3}}{\varepsilon_3}}\right) = \lambda P_j^c$$

③ as:

$$\left[\gamma_j^{1/\varepsilon_3} Q_j^{c\,\frac{\varepsilon_3-1}{\varepsilon_3}} + (1-\gamma_j)^{1/\varepsilon_3} Q_j^{nc\,\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{1}{\varepsilon_3-1}}$$

$$\left[(1-\gamma_j)^{1/\varepsilon_3} (Q_j^{nc})^{\frac{\cancel{\varepsilon_3}-1-\cancel{\varepsilon_3}}{\varepsilon_3}}\right. = \lambda P_j^{nc}$$

$$\frac{①}{③} = \frac{\gamma^{1/\varepsilon_3} Q_j^{c\,-1/\varepsilon_3}}{(1-\gamma)^{1/\varepsilon_3} Q_j^{nc\,-1/\varepsilon_3}} = \frac{\cancel{\lambda} P_j^c}{\cancel{\lambda} P_j^{nc}}$$

$$\Leftrightarrow \frac{\gamma^{1/\varepsilon_3} Q_j^{nc\,1/\varepsilon_3}}{(1-\gamma)^{1/\varepsilon_3} Q_j^{c\,1/\varepsilon_3}} = \frac{P_j^c}{P_j^{nc}}$$

$$\Leftrightarrow \frac{\gamma\, Q_j^{nc}}{(1-\gamma\; Q_j^c} = \left(\frac{P_j^c}{P_j^{nc}}\right)^{\varepsilon_3}$$

$$\Rightarrow Q_j^{nc} = \left(\frac{P_j^c}{P_j^{nc}}\right)^{\varepsilon_3} \cdot \frac{(1-\gamma)}{\gamma} Q_j^c$$

Plug into (3):

$$P_j^Q Q_j = P_j^c Q_j^c + P_j^{nc} \left(\frac{P_j^c}{P_j^{nc}}\right)^{\varepsilon_3} \cdot \frac{(1-\gamma)}{\gamma} Q_j^c$$

$$\gamma \cdot P_j^Q Q_j = \left\{Q_j^c \left[P_j^c + P_j^{nc\;1-\varepsilon_3} P_j^{c\;\varepsilon_3} \left(\frac{1-\gamma}{\gamma}\right)\right]\right\} \cdot \gamma$$

$$\gamma P_j^Q Q_j = Q_j^c \left[\gamma P_j^c + (1-\gamma) P_j^{c\;\varepsilon_3} P_j^{nc\;1-\varepsilon_3}\right]$$

$$\Rightarrow Q_j^c = \frac{\gamma P_j^Q Q_j}{\left[\gamma P_j^c + (1-\gamma) P_j^{c\;\varepsilon_3} P_j^{nc\;1-\varepsilon_3}\right]}$$

$$\Rightarrow Q_j^c = \frac{\gamma P_j^Q Q_j}{P_j^{c\;\varepsilon_3}\left[\gamma P_j^{c\;1-\varepsilon_3} + (1-\gamma) P_j^{nc\;1-\varepsilon_3}\right]}$$

6

now find Q^{nc}_j :

$$Q^{nc}_j = \left(\frac{P^c_j}{P^{nc}_j}\right)^{\varepsilon_3} \cdot \left(\frac{1-\gamma}{\gamma}\right) Q^c_j$$

$$= \left(\frac{P^c_j}{P^{nc}_j}\right)^{\varepsilon_3} \frac{(1-\gamma)}{\cancel{\gamma}} \left[\frac{\cancel{\gamma} P^Q_j Q_j}{\cancel{P^{c\,\varepsilon_3}_j}\left[\gamma P^{c\,1-\varepsilon_3}_j + (1-\gamma) P^{nc\,1-\varepsilon_3}_j\right]}\right]$$

$$Q^{nc}_j = \frac{(1-\gamma) P^Q_j Q_j}{P^{nc\,\varepsilon_3}_j\left[\gamma P^{c\,1-\varepsilon_3}_j + (1-\gamma) P^{nc\,1-\varepsilon_3}_j\right]}$$

$P^Q_j Q_j =$

$P^{nc}_j Q^{nc}_j + P^c_j Q^{nc}_j$

$= P^{nc}_j \dfrac{(1-\gamma) P^Q_j Q_j}{P^{nc\,\varepsilon_3}_j\left[\gamma P^{c\,1-\varepsilon_3}_j + (1-\gamma) P^{nc\,1-\varepsilon_3}_j\right]}$

$+ P^c_j \dfrac{\gamma P^Q_j Q_j}{P^{c\,\varepsilon_3}_j\left[\gamma P^{c\,1-\varepsilon_3}_j + (1-\gamma) P^{nc\,1-\varepsilon_3}_j\right]}$

$\Rightarrow P^Q_j =$

How determine amounts?

$\rightarrow P^Q_j =$ price of composite output → use same coeff matrix as below and prices of cons goods?

$\rightarrow Q_j =$ after determine demands for each cons. good, use transition matrix to map how these demands for cons. goods map into demands for production goods

~~Q~~ ok

see pg 8

$\rightarrow P^{nc}_j =$ ~~from production problem~~? { see pg 9 → from firm problem

$\rightarrow P^c_j =$

To determine P^Q_j →

→ note that $U(x,y)$ is linearly homogenous:

~~$U(\lambda x, \lambda$~~ $\quad U(\lambda Q^c_j, \lambda Q^{nc}_j) = \lambda U(Q^c_j, Q^{nc}_j)$ /

~~$U(Q^c_j, Q^{nc}_j)$~~ =

→ Because $U(\cdot,\cdot)$ is linearly homogenous, we know indirect utility, V, is homogenous of degree one in Q:

$$V(p^c_j, p^{nc}_j, \lambda Q_j) = \lambda V(p^c_j, p^{nc}_j, Q_j)$$

and V is homogenous of degree −1 in P:

$$V(\lambda p^c_j, \lambda p^{nc}_j, Q_j) = \frac{V(p^c_j, p^{nc}_j, Q_j)}{\lambda}$$

also know that when utility is linearly homogenous; ~~[illegible]~~ .

$$V(p^c_j, p^{nc}_j, Q_j) = \frac{P^Q_j Q_j}{e(p^c_j, p^{nc}_j)}, \text{ where}$$

$e(p^c_j, p^{nc}_j)$ = the min cost for a unit of ~~[illegible]~~ utility

$$\Rightarrow e(p^c_j, p^{nc}_j) = \frac{P^Q_j Q_j}{V(p^c_j, p^{nc}_j, Q_j)}$$

$$= \frac{Q_j}{\left[\gamma_j^{1/\varepsilon_3}\left(\frac{\gamma P^Q_j Q_j}{p^{c\,\varepsilon_3}_j [\cdot]}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + (1-\gamma_j)^{1/\varepsilon_3}\left(\frac{(1-\gamma) P^Q_j Q_j}{p^{nc\,\varepsilon_3}_j [\cdot]}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3-1}}}$$

$\gamma^{\frac{\varepsilon_3-1}{\varepsilon_3}+\frac{1}{\varepsilon_3}} = \gamma^{\frac{\varepsilon_3}{\varepsilon_3}} = \gamma$

$$= \frac{Q_j}{\left[\gamma_j^{1/\varepsilon_3}\left(\frac{\gamma P^Q_j Q_j}{[\cdot]}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} p^{c\,-\varepsilon_3\left(\frac{\varepsilon_3-1}{\varepsilon_3}\right)}_j + (1-\gamma_j)^{1/\varepsilon_3}\left(\frac{(1-\gamma) P^Q_j Q_j}{[\cdot]}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} p^{nc}_j\right]^{\frac{\varepsilon_3}{\varepsilon_3-1}}}$$

$$e(p^Q_j, p^{nc}_j, Q_j) \equiv \frac{\cancel{Q_j}}{\frac{p^Q_j \cancel{Q_j}}{[\cdot]}\left[\gamma\, p_j^{c\ 1-\varepsilon_3} + (1-\gamma_j)\, p_j^{nc\ 1-\varepsilon_3}\right]^{\varepsilon_3/\varepsilon_3-1}}$$

$$= \frac{[\cdot]}{p^Q_j\,[\cdot]^{\varepsilon_3/\varepsilon_3-1}}$$

$$= \frac{[\cdot]^{1-\frac{\varepsilon_3}{\varepsilon_3-1}}}{p^Q_j} = \frac{[\cdot]^{\frac{\varepsilon_3-1-\varepsilon_3}{\varepsilon_3-1}}}{p^Q_j} = \frac{[\cdot]^{-\frac{1}{\varepsilon_3-1}}}{p^Q_j}$$

Indirect Util:

$$V(p^c_j, p^{nc}_j, \cancel{\ }\ p^Q_j, Q_j) = p^Q_j Q_j [\cdot]^{\frac{\varepsilon_3-\varepsilon_3+1}{\varepsilon_3-1}} = p^Q_j Q_j [\cdot]^{\frac{1}{\varepsilon_3-1}}$$

$$e(p^c_j, p^{nc}_j) = \text{price of comp} = p^Q_j = \frac{p^Q_j Q_j}{V(p^c_j, p^{nc}_j, p^Q_j Q_j)}$$

$$= \frac{\cancel{p^Q_j Q_j}}{\cancel{p^Q_j Q_j}\,[\cdot]^{\frac{1}{\varepsilon_3-1}}}$$

$$= [\cdot]^{-\frac{1}{\varepsilon_3-1}} = [\cdot]^{\frac{1}{1-\varepsilon_3}}$$

9

How get P_j^{nc}, P_j^c?

Zero profit condition?

→ I think yes

→ final price$_j$ = mc_j
where mc_j is the cost of producing one unit of output of good j given optimal mix of capital and labor
→ this a function of r, w

→ I haven't written this for supply side of model, but think I can do similar to what's done in Tax DSGE paper

→ so guess w, r ⟶ then all other prices fall out

after aggregates, everything else here is static

⇒ P_j^{nc}, P_j^c

→ P_j^{nc}, P_j^c ⇒ P_j^Q by CES utils from goods across ~~goods~~ sectors

→ P_j^Q ⇒ P_i by transition matrix (fixed coeffs)

→ P_i ⇒ $\tilde{P}_t = \prod_{i=1}^{N} \left(\frac{P_i}{\beta_{it}}\right)^{\beta_{it}}$

$\tilde{P}_t$ ~~agg price level~~ price of composite cons. good for ~~ags~~ age + indiv

except does seem like need to think @ how consumers forecast how $\tilde{P}_t$ varies over time when make intertemporal cons/save decision → ok, but have whole path of $\tilde{P}_t$ when guess w, r paths
→ ~~[illegible]~~

What about quantities across levels of cons/prod?

→ $Q^c_{jt} + Q^{nc}_{jt} = Q_{jt}$

→ $Q^c_{jt} \Rightarrow c_{it}$ by "trans" matrix (Note Q_{jt} also needs to be mapped into capital (FA's, Land, inventories))

→ $\sum_t \cancel{c_{it} = c_i}$

→ $\sum_i c^d_{it} = \tilde{c}_t$ (right?) ~~or not~~ ~~mix cons~~?

→ $\sum_t \tilde{c}_t = c$

invest i

→ $\cancel{\sum k_t} = \tilde{K}$

and so for gov't cons

$c + i + g = y$