Firms Problem with CES Production and Ownership of Capital

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Denote the stock of bonds issued by the firm as b^F. T^F denotes taxes on the firm. Firm ι's maximization problem is:

$$\begin{aligned} & V_\iota(k^F_{\iota,t}, b^F_{\iota,t}, \Omega_t) \\ & = \max_{n^F_{\iota,t}, k^F_{\iota,t+1}, b^F_{\iota,t+1}} e^F_{\iota,t} + \frac{1}{1+r_{t+1}} V_\iota(k^F_{\iota,t+1}, b^F_{\iota,t+1}, \epsilon^F_{\iota,t}, \Omega_{t+1}) \end{aligned} \tag{1}$$

$$e^F_{\iota,t} = p_{\iota,t} y^F_{\iota,t} - w_t n^F_{\iota,t} + (1-\delta)k^F_{\iota,t} - (1+r_t)b^F_{\iota,t} - k^F_{\iota,t+1} + b^F_{\iota,t+1} - T^F_{\iota,t} \tag{2}$$

$$y^F_{\iota,t} = f(z_t, k^F_{\iota,t}, n^F_{\iota,t}) = z_t \left[\zeta (k^F_{\iota,t})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1}{\eta}} \tag{3}$$

The first-order conditions are:

$$p_{\iota,t} z_t \left[\zeta (k^F_{\iota,t})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1-\eta}{\eta}} (1-\zeta)(n^F_{\iota,t})^{\eta-1} - w_t - \frac{\partial T^F_{\iota,t}}{\partial n^F_{\iota,t}} = 0$$

$$-1 - \frac{\partial T^F_{\iota,t}}{\partial k^F_{\iota,t+1}} + \frac{1}{1+r_{t+1}} \frac{\partial V_\iota}{\partial k^F_\iota}(t+1) = 0$$

$$1 - \frac{\partial T^F_{\iota,t}}{\partial b^F_{\iota,t+1}} + \frac{1}{1+r_{t+1}} \frac{\partial V_\iota}{\partial b^F_\iota}(t+1) = 0$$

Envelope conditions are:

$$\frac{\partial V_\iota}{\partial k^F_\iota}(t) = p_{\iota,t} z_t \left[\zeta (k^F_{\iota,t})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1-\eta}{\eta}} \zeta (k^F_{\iota,t})^{\eta-1} + 1 - \delta - \frac{\partial T^F_{\iota,t}}{\partial k^F_{\iota,t}}$$

$$\frac{\partial V_\iota}{\partial b^F_\iota}(t) = -\left(1 + r_t + \frac{\partial T^F_{\iota,t}}{\partial b^F_{\iota,t}}\right)$$

Euler equations are:

$$p_{\iota,t} z_t \left[\zeta (k^F_{\iota,t})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1-\eta}{\eta}} (1-\zeta)(n^F_{\iota,t})^{\eta-1} = w_t + \frac{\partial T^F_{\iota,t+1}}{\partial n^F_{\iota,t+1}} \tag{4}$$

$$\begin{aligned} & p_{\iota,t+1} z_{t+1} \left[\zeta (k^F_{\iota,t+1})^\eta + (1-\zeta)(n^F_{\iota,t})^\eta\right]^{\frac{1-\eta}{\eta}} \zeta (k^F_{\iota,t+1})^{\eta-1} \\ & = \left(1 + \frac{\partial T^F_{\iota,t+1}}{\partial k^F_{\iota,t+1}}\right)(1+r_{t+1}) - 1 - \delta - \frac{\partial T^F_{\iota,t}}{\partial k^F_{\iota,t+1}} \end{aligned} \tag{5}$$

$$r_{t+1} = -\frac{\frac{\partial T^F_{\iota,t+1}}{\partial b^F_{\iota,t+1}} + \frac{\partial T^F_{\iota,t}}{\partial b^F_{\iota,t+1}}}{\frac{\partial T^F_{\iota,t}}{\partial b^F_{\iota,t+1}}} \tag{6}$$

Starting with a values for $k^F_{\iota,1}$, $b^F_{\iota,1}$, w_t, $p_{\iota,t}$ and z_t, we get $n^F_{\iota,t}$ from equation (4). Equations (5) and (6) then give $k^F_{\iota,t+1}$ and $b^F_{\iota,t+1}$using the known value of r_{t+1}. This allows us to iteratively solve for labor hired, capital and outstanding debt for firm ι over time.

WE NEED TO ADD EQUITY SHARES CHOICE