Capital Taxation with Entrepreneurial Risk

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Abstract

This paper studies the dynamic macroeconomic and welfare effects of capital taxation in a general equilibrium heterogeneous agent economy with uninsurable idiosyncratic investment or capital-income risk. Unlike either complete markets or Bewley-type models, the surprising result here is that the effect of capital taxation on capital accumulation is theoretically ambiguous, so that steady state capital might actually be increasing in the tax. This novel theoretical possibility emerges due to the general equilibrium effects of the tax on asset returns. In particular, capital taxation provides partial insurance against idiosyncratic risk, thereby reducing the demand for precautionary saving and increasing the equilibrium risk-free rate. In turn, this increase in the risk-free rate potentially allows for increased saving, capital and wealth accumulation in the long run. If these general equilibrium effects are strong enough, they outweigh the standard negative effect of the capital tax on investment. For example, in the preferred calibration, the steady state capital stock is maximized when the capital tax is 50%, at which point output per worker is 7% higher than it would have been, had the tax rate been zero.

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1 Introduction

This paper studies the macroeconomic and welfare effects of capital taxation in an environment where agents face uninsurable idiosyncratic investment or capital-income risk. Such risk is empirically important for all investment decision makers, whether they are entrepreneurs and private business owners, or managers of publicly traded firms holding a stake in their firm. In this context, capital taxation raises an interesting tradeoff: On the one hand, it comes at the usual cost, as it distorts agents' saving decisions. On the other hand, it has benefits, as it provides agents with partial insurance against idiosyncratic investment risk. This suggests that a positive tax on capital income could be welfare improving, even if it reduced capital accumulation.

Most surprisingly though, it is shown that a positive tax on capital income might actually stimulate capital accumulation. In other words, the effect of capital taxation on the steady state capital stock is theoretically ambiguous. This novel theoretical possibility emerges due to the dynamic general equilibrium effects of capital taxation on asset returns. Specifically, by reducing the variance of business returns, the capital tax provides direct insurance against idiosyncratic risk. This reduces the demand for precautionary saving and increases the risk-free rate or interest rate in general equilibrium, with opposing income and substitution effects on the level of saving. If the substitution effect is sufficiently strong, the net outcome will eventually be an increase in saving and capital accumulation. Essentially, the increase in the interest rate allows agents to accumulate more wealth in the long run, so that, with decreasing absolute risk aversion, they can potentially increase their investment in risky capital. This result stands in stark contrast to the effects of capital taxation both in complete markets, and in incomplete markets with uninsurable labor-income risk alone. In these models, capital taxation necessarily discourages capital accumulation, irrespectively of whether it is welfare improving or not.

This paper represents the first attempt to study the effects of capital-income taxation in a general equilibrium incomplete markets economy, where agents are exposed to uninsurable idiosyncratic capital-income risk. The framework builds on Angeletos (2007), who develops a variant of the neoclassical growth model that allows for idiosyncratic investment risk, and studies the effects of such risk on macroeconomic aggregates. In particular, agents own privately held businesses operating under constant returns to scale. These businesses are subject to idiosyncratic risk that the agents cannot diversify away. However, agents are not exposed to labor-income risk, and they can freely borrow and lend in a riskless bond. Abstracting from borrowing constraints, labor-income risk, and other market frictions, isolates the impact of the idiosyncratic investment risk, and preserves tractability of the model. Finally, there is a government, imposing proportional taxes on capital- and labor-income, along with a non-contingent lump sum tax or transfer.

The main theoretical result of the paper is that an increase in capital-income taxation may actually stimulate capital accumulation, due to the general equilibrium effects of the tax, operating through the interest rate. In fact, it is shown that, when the interest rate is exogenously fixed, as would be the case in a "small open economy" version of the model, an increase in the capital tax unambiguously lowers the steady state capital stock. Clearly then, the fact that a higher tax always directly lowers the variance of idiosyncratic risk is not on its own a force strong enough to induce the possibility that capital increases with the tax. For that result to emerge, the interest rate needs to adjust to the tax increase, with subsequent dynamic effects on wealth and capital accumulation. This could be termed an indirect or general equilibrium insurance aspect of capital taxation. Therefore, the present paper indicates that the appropriate setting for discussing issues of capital taxation when investment is risky has to consider the dynamic aspect of investment decisions as well as the general equilibrium effects of the adjustment of asset returns.

Turning to the quantification of the main theoretical result, in the preferred calibration of the model, steady state capital is maximized when the capital tax is 50%. At that level of the tax, output per work-hour is 7% higher than it would have been, had the tax rate been zero, whereas with complete markets output per work-hour is 21% lower. The result that capital is inversely U-shaped with respect to the tax is robust for a wide range of empirically plausible parameter values. However, even if it turned out that capital fell all the way with the tax, the theoretical mechanism would still be robust, as capital would nonetheless fall less than under complete markets, due to the general equilibrium insurance aspect of capital taxation identified here but absent in complete markets.

In addition, the paper quantitatively examines the dynamic aggregate and welfare effects of eliminating the capital-income tax, both at the time of the reform, while taking into account the entire transitional dynamics of the economy to the new steady state with the zero tax, and across steady states. When the tax is eliminated, investment falls immediately, as well as in the long run, the exact opposite of what happens with complete markets. Furthermore, most agents are hurt by the elimination of the tax, whether in current or in future generations, as they value its insurance aspect and its effect on the aggregates. One exception are the future rich, who prefer the elimination of the distortion on the mean investment return, and who are wealthy enough to bear the loss of insurance provision.

2 Related Literature

This paper focuses on uninsurable investment or capital-income risk, because such risk is in fact empirically relevant for investment decisions, even in a financially developed country like the United States. First, Moskowitz and Vissing-Jorgensen (2002), among others, find that about 80% of all *private equity* is owned by agents who are actively involved in the management of their own firm, and for whom such investment constitutes at least half of their total net worth.[1] It seems plausible, then, that entrepreneurial risk must be even more prevalent in less developed economies. Second, Panousi and Papanikolaou (2011) show that the negative relationship between idiosyncratic risk and the investment of *publicly traded* firms in the United States is stronger in firms where the mangers hold a larger fraction of the firm's shares. Combined, these findings strengthen the empirical applicability of our model setup, because they demonstrate that a large fraction of total investment in the United States is sensitive to idiosyncratic risk, essentially through the poor diversification and risk aversion of the agents making the investment decisions. In that sense, for purposes of the present paper, the terms "entrepreneurial risk" and "entrepreneurs" should be interpreted more broadly, as referring to all types of uninsurable investment risk and to all agents involved in making investment decisions.

The idea that proportional income taxation may provide insurance has been proposed by the public finance literature, for example in Eaton and Rosen (1980), Varian (1980), Kanbur (1981), and Mirrlees (1990). The notion that taxation of a risky asset may increase risk taking and investment in that asset, also has its roots in public finance, including Domar and Musgrave (1944), Stiglitz (1969), Ahsan (1976), and Sandmo (1977).[2] In particular, Ahsan (1976) showed that, in a two-period economy with exogenously fixed interest rate, the effect of capital-income taxation on capital investment is theoretically ambiguous. However here, in the "small open economy" version of the present model, which differs from Ahsan's in that the time horizon is infinite, it is shown that the steady state capital stock is unambiguously decreasing in the capital-income tax. This finding highlights that restoring the original public finance ambiguous effect of the tax on capital requires novel in the literature general equilibrium effects associated with the interest rate, which only arise due to the dynamic nature of the capital accumulation problem.[3]

[1] Additional evidence for poor entrepreneurial diversification is provided in Carroll (2000), Quadrini (2000), Gentry and Hubbard (2000), and Cagetti and De Nardi (2006).

[2] Bruce (2000), Schuetze (2000), Bird (2001), Cullen and Gordon (2002), and Bustos, Engel and Galetovic (2004) provide some empirical evidence that taxation may encourage investment and entrepreneurial activity.

[3] The general equilibrium effect of higher interest rates also plays a central role in Goldberg (2011) and Moll

This paper also relates to the strand of the macroeconomic literature discussing the aggregate and welfare effects of taxation. In his seminal paper, Judd (1985) showed that in the neoclassical model of complete markets, an increase in capital taxation reduces steady state capital and welfare. The recent literature on entrepreneurs has examined some policy questions, but always finds that, though taxation may sometimes yield welfare gains, it nonetheless unambiguously reduces capital investment. A non-exhaustive list includes Li (2002), Domeij and Heathcote (2004), Meh (2005), Kitao (2007), and Cagetti and DeNardi (2009). Furthermore, this literature focuses entirely on the implications of labor income risk for fiscal policy, whereas the present paper takes on the possibility of capital-income risk.[4]

Although this paper does not examine optimal taxation, it is related to the macro literature that does.[5] In the Ramsey tradition of exogenous market structure and policy instruments, Chamley (1986), and Atkeson, Chari and Kehoe (1999) have established the result of zero optimal capital taxation when markets are complete. Aiyagari (1995) extends the complete-markets framework to include uninsurable labor-income risk, and finds that the optimal capital tax is positive in the long run, as labor risk leads to overaccumulation of capital, compared to complete markets. In the Mirrlees tradition of endogenous market incompleteness and policy instruments, the literature mostly focuses on labor-income risk and shows that, if insurance is limited due to asymmetric information, then it may be best to restrict free access to savings. This result has been interpreted as a justification for positive optimal capital taxation. Some examples here include Diamond and Mirrlees (1978), Golosov, Kocherlakota and Tsyvinski (2003), Albanesi and Sleet (2006), and Farhi and Werning (2010). Two papers in the Mirrlees tradition that actually incorporate idiosyncratic capital-income risk in optimal policy questions are Albanesi (2006) and Shourideh (2011). However, in these papers, the dynamic general equilibrium mechanisms that here lead to the novel result that capital may be increasing in the tax are absent. Finally, the overlapping-generations literature has found support for positive optimal capital taxes. Conesa, Kitao and Krueger (2009) quantitatively characterize the optimal capital tax in a setting with permanent productivity differences and uninsurable income shocks, and find that the optimal capital tax is significantly positive at 36%, mainly due to the life-cycle structure. Erosa and Gervais (2002) and Garriga (2003) find that a positive capital tax may be optimal when labor taxes cannot be conditioned on age.

(2012), where it enhances wealth accumulation and helps alleviate collateral constraints (such constraints are absent in the present paper).

[4]Benabou (2002) and Erosa and Koreshkova (2007) study taxation in models with human capital.

[5]See Panousi and Reis (2012) for the optimal taxation analysis in a model framework similar to the one in the present paper.

The theoretical framework within which the main result emerges builds on a continuous-time variant of Angeletos (2007), but in addition it introduces a government, imposing proportional taxes on capital and labor income (in the Ramsey tradition), along with a non-contingent lump-sum tax or transfer. Subsequently, for purposes of a better calibration, the framework extends Angeletos's to include endogenous labor supply, finite lives, and two types of agents. In a framework like the one in Angeletos (2007), Angeletos and Panousi (2009) examine the effects of government spending on aggregates, when government spending is financed exclusively through lump-sum taxation, while Angeletos and Panousi (2011) examine the effects of financial integration for welfare and current-account dynamics.

The rest of the paper is organized as follows. Section 3 presents the basic model. Section 4 characterizes individual behavior, the general equilibrium, and the steady state. Section 5 discusses the main theoretical result of the paper. Section 6 augments the theoretical model for purposes of a better quantification of the theoretical result and presents the benchmark parameter choices for the preferred calibration. Section 7 quantifies the effects of capital taxation in steady state, as well as the dynamic macroeconomic and welfare effects of eliminating the capital tax. Section 8 concludes. Proofs and additional material are delegated to the on line appendix.

3 The Basic Model

Time is continuous and indexed by $t \in [0, \infty)$. There is a continuum of infinitely lived households distributed uniformly over $[0, 1]$. Each household is endowed with one unit of labor, supplied inelastically in a competitive labor market, and it also owns and runs a privately-held firm. Each firm employs labor in the competitive labor market, but can only use the capital stock invested by the particular household. Each firm is hit by idiosyncratic shocks, which the household can only partially diversify, as it cannot invest in other households' firms. However, each household can freely save or borrow in a riskless bond (up to a natural borrowing constraint), which is in zero net supply. In terms of timing for the firm's problem, first capital is installed, then the idiosyncratic shock is realized, and lastly the labor choice is made. All uncertainty is purely idiosyncratic, and therefore aggregates are deterministic. Finally, the government imposes proportional taxes on savings or capital income and on labor income, and balances the budget by giving back to agents, in the form of lump-sum transfers, the proceeds of taxation minus any government consumption.

3.1 Households, firms, and idiosyncratic risk

Preferences are assumed to have an Epstein-Zin specification, with constant elasticity of intertemporal substitution (CEIS) and constant relative risk aversion (CRRA). Given a consumption process, the utility process is defined by the solution to the following integral equation:

$$U_t = E_t \int_t^\infty z(c_s, U_s)\, ds\,, \tag{1}$$

where

$$z(c, U) \equiv \frac{\rho}{1 - 1/\theta} \left[\frac{c^{1-1/\theta}}{((1-\gamma)U)^{\frac{-1/\theta+\gamma}{1-\gamma}}} - (1-\gamma)U \right]. \tag{2}$$

Here, $\rho > 0$ is the discount rate, $\gamma > 0$ is the coefficient of relative risk aversion, and $\theta > 0$ is the elasticity of intertemporal substitution. For $\theta = 1/\gamma$, this reduces to the case of standard expected utility.[6]

The financial wealth of a household i, denoted by a_t^i, is the sum of its asset holdings in private capital, k_t^i, and in the riskless bond, b_t^i, so that $a_t^i = k_t^i + b_t^i$. The evolution of a_t^i is given by the household budget:

$$da_t^i = (1 - \tau_t^K)\, dp_t^i + [\, (1 - \tau_t^K)\, R_t\, b_t^i + (1 - \tau_t^L)\, w_t + T_t - c_t^i \,]\, dt\,, \tag{3}$$

where dp_t^i are the profits from the firm the household operates or the household's capital income, R_t is the interest rate on the riskless bond or the risk-free rate, w_t is the wage rate in the aggregate economy, c_t^i is consumption, τ_t^K is the proportional savings or capital-income tax, applied to the income from capital and bond alike, τ_t^L is the proportional labor-income tax, and T_t are non-contingent lump-sum transfers received from the government. A no-Ponzi-game condition is also imposed.

Firm profits are subject to undiversifiable idiosyncratic risk:

$$dp_t^i = [\, F(k_t^i, l_t^i) - w_t\, l_t^i - \delta\, k_t^i \,]\, dt \,+\, \sigma\, k_t^i\, dz_t^i\,. \tag{4}$$

Here, F is a constant-returns-to-scale neoclassical production function, assumed to be Cobb-Douglas for simplicity, $F(k, l) = k^\alpha l^{1-\alpha}$ with $\alpha \in (0, 1)$, where l_t^i is the amount of labor the firms hires in the competitive labor market, and δ is the mean depreciation rate in the economy. Idiosyncratic risk is introduced through dz_t^i, a standard Wiener process that is

[6] The distinction between θ and γ, though not crucial for the theoretical results of this paper, nonetheless facilitates a better clarification of the qualitative properties of the steady state, and also a better calibration.

i.i.d. across agents and across time. Literally taken, dz_t^i represents a stochastic depreciation shock. However, these shocks can also be modelled or interpreted as stochastic productivity shocks: All the matters, for purposes of the present analysis, is that the idiosyncratic shocks generate risk in the rate of return to investment endeavors. The scalar σ measures the amount of undiversified idiosyncratic risk, and it is an index of market incompleteness, with higher σ corresponding to a lower degree of risk-sharing, and with $\sigma = 0$ corresponding to complete markets.

3.2 Government

At each point in time the government taxes capital and bond income at the rate τ_t^K, and it taxes labor income at the rate τ_t^L. Part of the tax revenue is used by the government for own consumption at the deterministic rate G_t. Government spending does not affect the utility from private consumption or the production technology. The remaining tax proceeds are then distributed back to the households in the form of non-contingent lump-sum transfers, T_t. The government budget constraint is therefore:

$$0 = [\ \tau_t^L\, F_{L_t}(\int_i k_t^i, 1) + \tau_t^K\ (\ F_{K_t}(\int_i k_t^i, 1) - \delta\ )\ \int_i k_t^i - G_t\ - T_t\ ]\ dt\,, \tag{5}$$

where $F_{K_t}(\int_i k_t^i, 1)$ is the marginal product of capital in the aggregate economy, $F_{L_t}(\int_i k_t^i, 1)$ is the marginal product of labor, and $\int_i l_t^i = 1$.

4 Equilibrium and Steady State

This section characterizes the equilibrium and the steady state of the economy. It first solves for households' optimal plans, given the sequences of prices and policies. Then it aggregates across households to derive the general equilibrium dynamics, and it characterizes the steady state as the fixed point of the general equilibrium dynamic system.

4.1 Individual behavior

Entrepreneurs choose employment after their capital stock has been installed and their idiosyncratic shock has been observed. Hence, since their production function exhibits constant returns to scale, optimal firm employment and profits are linear in own capital:

$$l_t^i = l(w_t)\, k_t^i \qquad \text{and} \qquad dp_t^i = r(w_t)\, k_t^i\, dt + \sigma\, k_t^i\, dz_t^i\,, \tag{6}$$

where $l(w_t) \equiv \arg\max_l [\, F(1,l) - w_t\, l \,]$ and $r(w_t) \equiv \max_l [\, F(1,l) - w_t\, l \,] - \delta$. Here, $r_t \equiv r(w_t)$ is an entrepreneur's expectation of the return to his capital prior to the realization of his idiosyncratic shock, as well as the mean of the realized returns in the cross-section of firms, since there is no aggregate uncertainty. As in Angeletos (2007), the key result here is that entrepreneurs face linear, albeit risky, returns to their investment. To see how this translates to linearity of wealth in assets, let h_t denote a household's human wealth, namely the present discounted value of net-of-taxes labor endowment plus government transfers:

$$h_t = \int_t^{\infty} e^{-\int_t^s (1-\tau_j^K) R_j \, dj} \big((1-\tau_s^L) w_s + T_s \,\big)\, ds \,. \tag{7}$$

Next, define total effective wealth, x_t^i, as the sum of financial and human wealth:

$$x_t^i \equiv a_t^i + h_t = k_t^i + b_t^i + h_t \,. \tag{8}$$

Total effective wealth is then the only state variable relevant for the household's optimization problem. The only constraint imposed is that consumption is non-negative, which implies non-negativity of total effective wealth, so that $x_t^i \geqslant 0 \Leftrightarrow a_t^i \geqslant -h_t$. In turn, this inequality shows that agents can freely borrow the entire net present value of their future safe income, h_t, so that there is no ad hoc borrowing constraint. The evolution of total effective wealth is then described by:

$$dx_t^i = [\, (1-\tau_t^K)\, r_t\, k_t^i + (1-\tau_t^K)\, R_t\, (b_t^i + h_t) - c_t^i \,]\, dt + \sigma\, (1-\tau_t^K)\, k_t^i\, dz_t^i \,. \tag{9}$$

The first term captures the expected rate of growth of effective wealth, and it shows that wealth grows when total saving exceeds consumption expenditures. The second term captures the effect of idiosyncratic risk. This linearity of wealth in assets, together with the homotheticity of preferences, ensures that the household's consumption-saving problem reduces to a tractable homothetic optimization problem, as in Samuelson's and Merton's classic portfolio analysis. Therefore, the optimal individual policy rules will be linear in total effective wealth, for given prices and government policies, as the next proposition shows.

Proposition 1. *Let* $\{w_t, R_t, r_t\}_{t\in[0,\infty)}$ *and* $\{\tau_t^K, \tau_t^L, T_t, G_t\}_{t\in[0,\infty)}$ *be equilibrium price and policy sequences. The household maximizes preferences as described in* (1)-(2) *subject to the total effective wealth evolution constraint* (9)*. The value function for household i is:*

$$U(x_t^i, t) = D(t)\, \frac{x_t^{i,\,1-\gamma}}{1-\gamma} \,, \tag{10}$$

where $D(t)$ captures the explicit dependence of the value function on time. The optimal consumption, investment, and bond holding choices, respectively, are given by:

$$c_t^i = m_t \, x_t^i, \quad k_t^i = s_t \, x_t^i, \quad b_t^i = (1 - s_t) \, x_t^i - h_t \, . \tag{11}$$

Consumption, c_t^i, and the fraction of effective wealth invested in capital, s_t, are the solutions to the Bellman equation:

$$0 = \max_{c_t^i, \, s_t} \{ \; z(c_t^i, U) \; + \; \frac{\partial U}{\partial x_t^i} \cdot \{ \; [\; (1 - \tau_t^K) r_t \cdot s_t \; + \; (1 - \tau_t^K) R_t \cdot (1 - s_t) \;] \cdot x_t^i \; - \; c_t^i \; \} +$$

$$\frac{1}{2} \cdot \frac{\partial^2 U}{\partial x_t^{i,2}} \cdot \sigma^2 (1 - \tau_t^K)^2 \cdot s_t^2 \cdot x_t^{i,2} \; + \; \frac{\partial U}{\partial t} \; \} \, , \tag{12}$$

with z given by (2), and with $\partial U / \partial t$ the direct derivative of the value function with respect to time. In particular, the optimal portfolio allocation is given by the first order condition:

$$s_t = \frac{(1 - \tau_t^K) \, r_t - (1 - \tau_t^K) \, R_t}{\gamma \, \sigma^2 \, (1 - \tau_t^K)^2}, \tag{13}$$

and the marginal propensity to consume satisfies the following ordinary differential equation:

$$\frac{\dot{m}_t}{m_t} = m_t - \theta \rho + (\theta - 1) \, \hat{q}_t \, , \tag{14}$$

where $\hat{q}_t \equiv (1 - \tau_t^K) r_t s_t + (1 - \tau_t^K) R_t (1 - s_t) - \frac{1}{2} \gamma \sigma^2 (1 - \tau_t^K)^2 s_t^2$ denotes the risk-adjusted return to saving.

In order to understand Proposition 1, note first that, because capital is risky, agents require a positive (private) risk premium as compensation for undertaking risky investment. Therefore, in equilibrium, it has to be that the mean return to the risky asset (capital) exceeds the mean return to the safe asset (bond), or that $(1 - \tau_t^K) r_t > (1 - \tau_t^K) R_t$. Second, define q as the return to saving, namely the total or overall portfolio return for the household, so that $q_t \equiv (1 - \tau_t^K) r_t s_t + (1 - \tau_t^K) R_t (1 - s_t)$. The total return to saving is therefore a weighted average of the (net-of-tax) marginal product of capital and the (net-of-tax) risk-free rate. The risk-adjusted return to saving, $\hat{q}_t = q_t - 1/2 \cdot \gamma \sigma^2 (1 - \tau_t^K)^2 s_t^2$, is then the certainty equivalent of the overall portfolio return, and is lower than q_t because agents are risk averse and face risk in their consumption stream through the uninsurable investment shocks. It follows that, in equilibrium, it will have to be $(1 - \tau_t^K) \, R_t < \hat{q}_t < q_t < (1 - \tau_t^K) \, r_t$.

Turning to (11), optimal consumption is a linear function of total effective wealth, where the marginal propensity to consume, m_t, does not depend on the level of wealth. Similarly, the fraction of wealth invested in the risky asset, s_t, is the same across all agents. In addition, s_t is increasing in the risk premium, and decreasing in risk aversion, γ, and the effective variance of risk, $\sigma(1-\tau_t^K)$. Equation (14) follows from substituting for the optimal c_t^i and s_t into the Bellman equation and using the envelope condition, and it is the continuous time analog of an Euler equation: It describes the growth rate of the marginal propensity to consume as a function of the anticipated path of risk-adjusted returns to saving. Whether higher risk-adjusted returns increase or decrease the marginal propensity to consume depends on the elasticity of intertemporal substitution, θ, through the familiar tension between the income and substitution effect.

4.2 General equilibrium

The initial position of the economy is given by the cross-sectional distribution of (k_0^i, b_0^i) across households. Households choose plans $(\{c_t^i, l_t^i, k_t^i, b_t^i\}_{t\in[0,\infty)})$ for $i \in [0,1]$, contingent on the history of their idiosyncratic shocks, and given the price sequence and the government policy, so as to maximize their lifetime utility. Idiosyncratic risk washes out in the aggregate. An equilibrium is then defined as a deterministic sequence of prices $\{w_t, R_t, r_t\}_{t\in[0,\infty)}$, policies $\{\tau_t^K, \tau_t^L, T_t, G_t\}_{t\in[0,\infty)}$, and macroeconomic variables $\{C_t, K_t, Y_t\}_{t\in[0,\infty)}$, along with a collection of individual contingent plans $(\{c_t^i, l_t^i, k_t^i, b_t^i\}_{t\in[0,\infty)})$ for $i \in [0,1]$, such that the following conditions hold: (i) given the sequences of prices and policies, the plans are optimal for the households; (ii) the labor market clears, $\int_i l_t^i = 1$, in all t; (iii) the bond market clears, $\int_t b_t^i = 0$, in all t; (iv) the government budget constraint (5) is satisfied in all t; and (v) the aggregates are consistent with individual behavior, $C_t = \int_i c_t^i$, $L_t = \int_i l_t^i = 1$, $K_t = \int_i k_t^i$, $Y_t = \int_i F(k_t^i, l_t^i) = F(\int_i k_t^i, 1)$, $X_t = \int_i x_t^i = \int_i k_t^i$, in all t. Note that the aggregates do not depend on the extend of wealth inequality, because individual consumption and investment are linear in individual wealth.

Define $f(K) \equiv F(K,1) = K^\alpha$. From Proposition 1, the equilibrium ratio of capital to effective wealth and the equilibrium total return to savings are identical across agents and can be expressed as functions of the capital stock and risk-free rate, namely $s_t \equiv s(K_t, R_t)$ and $q_t \equiv q(K_t, R_t)$. Similarly, the wage is $w_t \equiv w(K_t) = f(K_t) - f'(K_t)K_t = (1-\alpha)f(K_t)$. Using this, aggregating the policy rules across agents, and imposing bond market clearing, we arrive at the following characterization of the general equilibrium.

Proposition 2. *In equilibrium, the aggregate dynamics satisfy:*

$$\frac{\dot{X}_t}{X_t} = q_t - m_t \tag{15}$$

$$\dot{H}_t = (1-\tau_t^K)R_t\, H_t - (1-\tau_t^L)\, w_t - (\, \tau_t^L\, w_t + \tau_t^K\, (\, F_{K_t} - \delta\,)\, K_t - G_t\,) \tag{16}$$

$$K_t = \frac{s_t}{1-s_t}\, H_t\ , \tag{17}$$

along with the Euler condition for the marginal propensity to consume (14)*, and with* $q_t = q(K_t, R_t)$*,* $w_t = w(K_t)$*, and* $s_t = s(K_t, R_t)$*.*

Condition (15) follows from aggregating the individual wealth evolution constraints from (9) across agents, and using (11) and the definition of q. It captures the evolution of total effective wealth, showing that wealth grows when the total return to saving, q_t, exceeds the marginal propensity to consume, m_t. It can be shown that (15) also implies $\dot{C}_t/C_t = \theta(\hat{q}_t - \rho) + 1/2 \cdot \gamma\sigma^2(1-\tau_t^K)^2 s_t^2$, so that, compared to complete markets, the precautionary saving motive here introduces a positive drift in consumption growth, represented by the risk-adjustment term $1/2 \cdot \gamma\sigma^2(1-\tau_t^K)^2 s_t^2$. Condition (16) is the evolution of the present value of aggregate net-of-taxes labor income plus transfers, from the definition of human wealth in (7) combined with factor market clearing and with the intertemporal government budget from (5). Condition (17) represents bond market clearing and follows from aggregation of (11) and imposing $B_t{=}0$.

4.3 Characterization of steady state aggregates

A steady state is a competitive equilibrium as defined in section 4.2, where prices, policies, and aggregates are time-invariant. The steady state is then the fixed point of the dynamic system in Proposition 2. The following proposition characterizes the steady state.

Proposition 3. *(i) The steady state always exists and is unique. (ii) In steady state, the capital stock,* K*, and the interest rate,* R*, are the solution to:*

$$f'(K) - \delta = R + \sqrt{\frac{2\,\theta\,\gamma\,\sigma^2}{\theta+1}\,(\,\rho - (1-\tau^K)\,R\,)} \tag{18}$$

$$K = \frac{s(K,R)}{1-s(K,R)}\ \frac{w(K) + \tau^K\,(\,f'(K)-\delta\,)\,K\ - G}{(1-\tau^K)\,R}\ , \tag{19}$$

where $f'(K)$ *is the marginal product of capital and* $w(K)$ *is the wage rate.*

From (14) and (15) in steady state, and using the definition of s, we get equation (18). This condition gives the combinations of K and R that are consistent with wealth and consumption stationarity. From (18), note that the difference from complete markets is the presence of the square-root term, which captures the risk premium agents require in order to invest in privately risky capital. In addition, this equation can be manipulated to show that $\hat{q} = \rho - 1/2 \cdot \gamma/\theta \cdot \sigma^2(1-\tau^K)^2 s^2$. In other words, for a steady state to exist, the risk-adjusted return to saving, $\hat{q}$, must be just low enough so that the resulting negative intertemporal substitution effect exactly offsets the precautionary saving motive. Since $(1-\tau^K)R < \hat{q}$, it follows that $(1-\tau^K)R < \rho$, namely in steady state the after-tax interest rate is lower than the discount rate, and hence lower that it would have been under complete markets. This is also the case in Bewley-type models, and for a similar reason: If $(1-\tau^K)R > \rho$, then savings and wealth would explode, due to the presence of the precautionary saving motive. Using (16) and (17) in steady state yields equation (19). This condition gives the combinations of K and R that are consistent with stationarity of human wealth and bond market clearing.

Equations (18) and (19) therefore show that the steady state can be characterized as a function of only two variables, the capital stock, K, and the interest rate, R. Equation (18) determines steady steady capital as a function of the interest rate, for each capital tax. Plugging this into (19) yields an equation to be solved for R, as a function of the tax. The next section further examines the implications of these two equations for the steady state effects of capital-income taxation.

5 Steady State Effects of Capital-Income Taxation

This section presents the main result of the paper, namely the theoretical possibility that steady state capital may be increasing in the capital-income tax. This novel and surprising theoretical ambiguity is absent both in complete markets models and in incomplete markets models of the Bewley type with labor-income risk alone. In all of those models, steady state capital is unambiguously falling with the capital tax, regardless of whether the tax is welfare improving or not. The following proposition states the main theoretical result of the paper.

Proposition 4. *The effect of an increase in the capital-income tax on the steady state capital stock is theoretically ambiguous.*

The remainder of this section provides the proof, the intuition, and a discussion for the main theoretical result.

5.1 Proof of main theoretical result

Recall that the steady state is characterized by equations (18) and (19). From (18), let $K^o(R;\tau^K)$ be the steady state capital stock as a function of the steady state interest rate, for a given capital-income tax. From (19), let $R^c(\tau^K)$ be the solution for the steady state interest rate. By plugging $R^c(\tau^K)$ into $K^o(R;\tau^K)$, we then get the steady state capital stock as a function of the capital tax alone, denoted by $K^c(\tau^K)$. In other words, using this notation, we can tautologically write the steady state capital stock as $K^c(\tau^K) \equiv K^o(\tau^K,\ R^c(\tau^K))$.[7] It follows that the effect of the capital tax on steady state capital can be decomposed as:

$$\frac{dK^c}{d\tau^K} = \frac{\partial\, K^o}{\partial\, \tau^K} + \frac{\partial\, K^o}{\partial\, R}\,\frac{d\, R^c}{d\, \tau^K}\,. \tag{20}$$

The term $\partial\, K^o/\partial\, \tau^K$ is the derivative of steady state capital with respect to the tax (for a given interest rate) from equation (18). Since the right-hand side of that equation is increasing in the tax, it immediately follows that capital is decreasing in the tax. Therefore, $\partial\, K^o/\partial\, \tau^K < 0$ always.

The term $\partial\, K^o/\partial\, R$ is the derivative of steady state capital with respect to the interest rate (for a given tax) from equation (18). Taking the total differential of that equation with respect to K and R and using the definition of $\hat{q}$, yields that $\partial K/\partial R > 0 \Leftrightarrow \theta > s/(1-s)$. Hence, the term $\partial\, K^o/\partial\, R$ could be either positive or negative.

The term $dR^c/d\tau^K$ is the derivative of the steady state interest rate with respect to the capital tax from equation (19). It is easy to see that, for a given level of the capital stock, the right-hand side of that equation is increasing in the tax, τ^K, and decreasing in the interest rate, R. Now suppose that the economy starts with a historically given level of capital. Then, if τ^K increases, the left-hand side of (19) will not change. For the right-hand side to also remain unchanged, it has to be that the interest rate increases with the tax. In other words, for a given capital stock, an increase in the capital tax increases the steady state interest rate, so that $dR^c/d\tau^K > 0$.

This completes the proof of the main theoretical result in Proposition 4: In equation (20), the first term is negative, the second might be positive or negative, and the third is positive. Therefore, it is *theoretically* possible that the effect of the capital tax on steady state capital is positive. This theoretical possibility is unique to the present paper, since in all other models in the literature the theoretical result is always that steady state capital unambiguously falls with the capital-income tax.

[7] This notation is simply a re-writing of equations (18) and (19), which characterize the steady state. The reason for the particular superscripts used here will become apparent in section 5.3.

5.2 Intuition for main theoretical result

The intuition for the main theoretical result proceeds as follows. On the one hand, a higher capital tax directly reduces the return to saving, pointing to lower capital accumulation. This is captured by the term $\partial K^o/\partial \tau^K$, which is negative and represents the familiar negative effect of the tax on capital, namely the standard effect identified in the rest of the literature. To see this, note that equation (18) combines consumption and wealth stationarity from (14) and (15), and therefore is simply $q = \rho$, where q is the total return to saving and ρ is the discount rate in preferences. Using the definition of q, we can write this as $(1-\tau^K)(r-R)s + (1-\tau^K)R = \rho$, where $r = f'(K) - \delta$. If markets were complete, the risky and the riskless return would be equal, or $r = R$, and therefore this condition would become $(1-\tau^K)r = \rho$. But this is exactly the familiar steady state condition that captures the negative relationship between capital and the capital tax in complete markets.

On the other hand, a higher capital tax directly reduces the effective variance of risk, $\sigma(1-\tau^K)$, thereby reducing the overall riskiness of saving and weakening agents' precautionary saving motive. Consequently, the steady state after-tax interest rate increases. To see that this is a manifestation of the precautionary saving motive, note that this increase in the after-tax interest rate is possible here because, in steady state, $(1-\tau^K)R < \rho$. By contrast, under complete markets, $(1-\tau^K)R = \rho$ in steady state. Therefore, though both here and in complete markets the increase in the tax increases the before-tax interest rate, it is only here that this increase can outweigh the increase in the tax, so that the *after-tax* interest rate increases. This brings the after-tax interest rate closer to the discount rate, and is reflected in the term $dR^c/d\tau^K$, which is positive.

Next, as a result of the increase in the interest rate, the overall return to saving also increases.[8] But then, as is standard in any deterministic savings problem, this increase in the return to saving will have opposing substitution and income effects on the level of saving: The substitution effect points to higher saving, while the income effect points to lower saving. If the elasticity of intertemporal substitution, θ, is sufficiently high, then the substitution effect will dominate and hence saving/capital will increase with the increase in the saving return/interest rate. The term $\partial K^o/\partial R$ reflects precisely the strength of the substitution versus the income effect of an increase in the return R on the level of saving K. The only difference from a more familiar model is that, in the standard problem, it is the comparison of θ with 1 that determines which effect dominates, whereas here it is the comparison of θ

[8] Recall that the overall or total return to saving, q, is a weighted average of the marginal product of capital and the risk-free rate.

with the endogenous threshold $s/(1-s)$, where s is the fraction of wealth invested in the risky asset.

To further understand this point, note that here, as usual, the strength of the substitution effect is governed by the elasticity of intertemporal substitution, θ: The higher is θ, the stronger is the substitution effect. The strength of the income effect depends positively on the fraction of total wealth invested in capital, s, and therefore negatively on $(1-s)$, which captures safe income as a fraction of wealth: Following an increase in the return to saving, agents will be more comfortable saving less if they already have a lot of safe income from other sources. Therefore, the strength of the income effect is attenuated when $(1-s)$ is high or, equivalently, when s is low. In that case, it will be more likely that θ will be above the threshold $s/(1-s)$, and therefore more likely that $\partial K^o/\partial R > 0$. This mechanism is closely related to the role of an increase in the rate of return in simple decision-theoretic or AK models, with the only difference lying in the equilibrium determination of s. In particular, in AK models, risky capital accounts for the entire household wealth, so that $s = 1$. Here, instead, risky capital is only a fraction of effective wealth, so that $s < 1$. In particular, $s \to 1$ as $\alpha \to 1$, but $s < \alpha$ for any $\alpha < 1$, where α is the share of capital in production. In other words, it is the existence of labor or other "safe" income that explains why the critical threshold for θ is lower here than in AK models.

5.3 Discussion: Importance of result and ties to public finance

The theoretical mechanism identified in this paper builds a bridge between the public finance and the macro taxation literatures. In particular, this is the first paper to emphasize the role of general equilibrium transmission mechanisms for tax policy. Such mechanisms are absent in the public finance two-period models, and they are also absent in two-period macro models dealing with optimal taxation. In addition, this paper revisits and clarifies the old public finance idea that capital taxation (in general, taxation of a risky asset) may be used by the government as a means of insurance provision.[9] In particular, it shows that taxation-as-insurance issues and the corresponding policy-making recommendations may depend critically on dynamic and general equilibrium considerations. In other words, there are two relevant dimensions that differentiate the present model from the rest of the literature and that help generate the main theoretical result: The time horizon and the endogenous adjustment of the interest rate.

[9] A non-exhaustive list includes Domar and Musgrave (1944), Stiglitz (1969), Ahsan (1976), Sandmo (1977), Eaton and Rosen (1980), Varian (1980), Kanbur (1981), Bulow and Summers (1984), Mirrlees (1990), Bird (2001), and Gentry and Hubbard (2004).

First, let's fix the interest rate and examine how the time horizon matters, namely what the results look like in two periods versus in infinite horizon. With the interest rate exogenously fixed, we essentially get a "small open economy" version of the more general model.[10] In a two-period setup of that small open economy, Ahsan (1976) finds that an increase in the tax on the risky asset may increase investment in the risky asset. When the horizon is infinite, we see from (20) that the exogeneity of the interest rate implies a zero third term, since the interest rate cannot respond to the change in the tax, and hence $dK^c/d\tau^K = \partial K^o/\partial \tau^K < 0$. In other words, Ahsan's theoretical ambiguity disappears and capital necessarily falls with the tax. Therefore, when the interest rate is fixed, going from two periods to infinite horizon drastically changes the implications of capital taxation for the steady state capital stock. Hence, the standard public finance result does not carry over to infinite horizon, which implies that the dynamic nature of capital and wealth accumulation is of crucial importance when assessing the effects of capital taxation. Incidentally, the public finance literature attributes its two-period result to a "direct" insurance effect of the tax, namely that a higher tax directly reduces the effective variance of risk, $\sigma(1-\tau^K)$. Clearly, when the horizon is infinite, the direct insurance effect is not strong enough to yield the theoretical ambiguity; instead, the standard distortion effect of the tax on capital outweighs the direct insurance aspect of the tax.

Second, let's allow the interest rate to endogenously adjust to tax changes, and again examine how the time horizon matters. The on line appendix shows that in a two-period version of the general (i.e. with endogenous interest rate) model, an increase in the capital tax might increase capital accumulation. However, it is shown that the reason for this theoretical ambiguity is solely related to income and substitution effects similar, but not identical, to the ones behind the term $\partial K^o/\partial R$ in equation (20).[11] In other words, the two-period outcome, though of a similar flavor, is yet due to only part of the overall infinite horizon dynamic mechanism, which includes all three terms in (20). The fact that the two-period result generalizes in infinite horizon, when the interest rate is endogenous, is then purely coincidental, and also the intuition behind each result is in fact very different: By

[10] This is an economy with the same preferences, technologies, and risks, but which is open to an international market for the riskless bond, thus facing an exogenously fixed interest rate. In this economy, risky capital continues to be a non-traded asset. However, the bond is not in zero net supply anymore, and therefore equation (19) is not relevant for the characterization of the steady state; instead, it simply captures the net foreign asset position of the country. In addition, equation (18) then expresses the open economy steady state capital as a function of the tax and interest rate parameters, hence the superscript in the notation $K^o(\tau^K, R)$.

[11] In two-periods of the standard neoclassical growth model, an increase in the capital tax also has an ambiguous effect on capital, due precisely to income and substitution effects. However, this result disappears in infinite horizon, where the income effect never dominates.

definition, the two-period problem is not of a dynamic nature, and therefore cannot capture the dynamic effects of capital taxation on the interest rate and on capital accumulation.[12] By contrast, the general mechanism here requires the initial or public finance "direct" insurance aspect of the tax to interact with the strength of the precautionary saving motive, which is a motive only arising in a dynamic framework, and to increase the equilibrium interest rate. This increase in the interest rate will then induce income and substitution effects, but these effects are indirect (i.e. they operate through the effect of the tax on the interest rate, rather than from the tax directly on the mean return to saving) and their strength also depends on dynamically endogenous considerations (recall that the threshold $s/(1-s)$ is endogenous).

The conclusion from this discussion is then that, regardless of whether the interest rate is exogenous or endogenous, two-period outcomes of similar flavor as the main theoretical result here either do not generalize when the horizon is infinite, or they are due to a different mechanism. In fact, the general equilibrium mechanism of this paper could not possibly have emerged in two periods and is specific to the dynamic nature of wealth and capital accumulation (though the two-period analysis does provide some intuition for part of the overall result). This dynamic nature of the framework is especially crucial in a context of capital-income risk, where agents make risky investment decisions over time and can endogenously adjust their capital accumulation and their exposure to investment risk. Therefore, the relevant intuition for the effects of capital taxation when investment is risky has to come from a dynamic model with endogenous asset returns, and has to account for all three effects identified in equation (20). Another way of putting this is to say that the present theoretical mechanism can only emerge in its entirety in infinite horizon, which is the only appropriate setting for issues of investment risk and capital accumulation, and then in infinite horizon the original public finance theoretical ambiguity can only re-emerge if the interest rate is allowed to endogenously adjust in general equilibrium. In such a framework, a novel "indirect"insurance aspect of taxation emerges, namely one that operates through the effects of the tax on the interest rate, and from there on savings, wealth, and capital accumulation, rather than through directly reducing the variance of risk.

[12] This point also differentiates the present model from two-period macro models of optimal taxation, such as Albanesi (2006). Note also, however, that these models do not generate the theoretical ambiguity that capital might increase with the tax.

6 Calibration Framework

In this section, the basic model is first augmented along several dimensions, so as to ensure a better calibration and quantification of the effects of capital taxation. The on line appendix presents the technical details and proofs for all extensions, and it also shows that the main theoretical result continues to hold in the extended model. Next, this section discusses the parameter choice for the preferred benchmark calibration.

6.1 Model extension: Endogenous labor supply

This section endogenizes labor supply in the economy, without compromising the tractability of the model. In particular, it is assumed that preferences are homothetic between consumption, c and leisure, n, where $\psi \in (0,1)$ is the scalar homotheticity coefficient in preferences. The homotheticity of the household's optimization problem is then preserved and the equilibrium analysis proceeds in a similar fashion as in the benchmark model. The only essential novelty is that aggregate employment is now given by $N_t = 1 - L(w_t, C_t) = 1 - (\psi/(1-\psi)) \cdot C_t/((1-\tau_t^L)w_t)$.

6.2 Model extension: Finite lives and two types of agents

Equation (9) indicates that, at steady state prices, individual wealth follows a geometric random walk. As a result, though the steady state is well-behaved in terms of the aggregates, there is however no stationary distribution for the cross section. To ensure the existence of a stationary wealth distribution across agents, it is now assumed that all households face a constant probability of death, with Poisson arrival rate $\nu\, dt$ at every instant in time. There is no intergenerational altruism linking a household to its descendants, and utility is zero after death.[13,14] In order to isolate the effects of capital-income risk, it is assumed that there exist annuity firms permitting all agents to get insurance against mortality risk, by freely borrowing the entire net present value of their future labor income. As a result, the human

[13] The discount rate in preferences can then be reinterpreted as $\rho = \tilde{\rho} + \nu$, where $\tilde{\rho}$ is the psychological or subjective discount rate and ν is the probability of death.

[14] In general, with finite lives and no altruism, Ricardian equivalence might fail, since some of the tax burden associated with the current issue of a bond is borne by agents who are not alive when the bond is issued. It might then be the case that the dynamic effects of time-varying policy changes possibly depend on the validity of Ricardian equivalence. Nonetheless, for the purposes of this paper, the government budget constraint will be written as in (5) for ν positive but small.

wealth of an agent is now given by a variant of (7):

$$h_t = \int_t^{\infty} e^{-\int_t^s ((1-\tau_j^K) R_j + \nu)\, dj} \big((1-\tau_s^L) w_s + T_s \big)\, ds \,.$$

In addition, this section takes the model a step closer to capturing the significant heterogeneity in wealth and investment choices in the cross section, by separating the population in two groups, namely entrepreneurs and laborers. Specifically, at each point in time, an agent can be either an entrepreneur, denoted by E, or a laborer, denoted by L. The entrepreneurs are agents who behave as described so far, namely agents making risky investment decisions. The laborers are in all aspects similar to entrepreneurs, except that they cannot invest in risky capital. In other words, the modified version of (9) that describes the wealth evolution for a laborer is:

$$dx_t^i = [\, (1-\tau_t^K)\, R_t\, (b_t^i + h_t) - c_t^i \,]\, dt \,.$$

The probabilities of switching between these two types of agents are stochastic but exogenous. In particular, the probability of switching from entrepreneur to laborer is $p_{EL}\, dt$, and the probability of switching from laborer to entrepreneur is $p_{LE}\, dt$. It is assumed that capital is fully fungible upon exit from entrepreneurship. Sections 6.2.1 and 6.2.2 summarize the implications of the existence of two types of agents and of finite lives for the equilibrium and the steady state.

6.2.1 Equilibrium implications

The characterization of individual behavior is now given by the following modified version of Proposition 1.

Proposition 5. *Let $\{w_t, R_t, r_t\}_{t\in[0,\infty)}$ and $\{\tau_t^K, \tau_t^L, T_t, G_t\}_{t\in[0,\infty)}$ be equilibrium price and policy sequences. If an agent i is an entrepreneur, his optimal consumption, investment, portfolio, and bond holding choices, respectively, are given by:*

$$c_t^i = m_t^E\, x_t^i, \quad k_t^i = s_t\, x_t^i, \quad s_t = \frac{(1-\tau_t^K)\, r_t - (1-\tau_t^K)\, R_t}{\gamma\, \sigma^2\, (1-\tau_t^K)^2}, \quad b_t^i = (1-s_t)\, x_t^i - h_t \,.$$

If an agent i is a laborer, his optimal consumption and bond holdings are:

$$c_t^i = m_t^L\, x_t^i, \quad b_t^i = x_t^i - h_t \,.$$

The marginal propensities to consume satisfy the following system:

$$\frac{\dot{m}_t{}^E}{m_t^E} = m_t^E - \theta\rho + (\theta - 1)\,\hat{q}_t + \frac{\theta - 1}{1-\gamma}\; p_{EL}\; [\; (\frac{m^L}{m^E})^{\frac{1-\gamma}{1-\theta}} - 1\;]$$

$$\frac{\dot{m}_t{}^L}{m_t^L} = m_t^L - \theta\rho + (\theta - 1)(1 - \tau_t^K)\, R_t + \frac{\theta - 1}{1-\gamma}\; p_{LE}\; [\; (\frac{m^E}{m^L})^{\frac{1-\gamma}{1-\theta}} - 1\;].$$

Here, all entrepreneurs share a common marginal propensity to consume, m_t^E, and all laborers share a common marginal propensity to consume, m_t^L. The differential equation system shows that the marginal propensities to consume now also depend on the probability that the agent might switch types. The dimensionality of the aggregate dynamic system will now increase by one, because we need to keep track of the allocation, v_t, of total wealth between the two groups of agents, where $v_t \equiv X_t^E / X_t$. Hence, in equilibrium, the aggregate dynamics are described by the following modified version of Proposition 2.

Proposition 6. *In equilibrium, the aggregate dynamics satisfy:*

$$\begin{aligned}
\dot{X}_t / X_t &= v_t(q_t - m_t^E) + (1 - v_t)(\,(1-\tau_t^K)R_t - m_t^L\,) \\
\dot{v}_t / v_t &= (1 - v_t)s_t(1-\tau_t^K)(r_t - R_t) + (1-v_t)(m_t^L - m_t^E) + p_{LE}(\frac{1}{v_t} - 1) - p_{EL} \\
\dot{H}_t &= (\,(1-\tau_t^K)R_t + \nu\,)H_t - (1-\tau_t^L)\,w_t - (\,\tau_t^L\,w_t + \tau_t^K\,(\,F_{K_t} - \delta\,)\,K_t - G_t\,) \\
K_t &= \frac{s_t\,v_t}{1 - s_t\,v_t}\,H_t\;,
\end{aligned}$$

along with the differential equations in m_t^E and m_t^L from Proposition 5.

The first equation in Proposition 6 shows that the evolution of total effective wealth is now a weighted average of two terms. The first term is positive when the mean net-of-tax return to saving for entrepreneurs, q_t, exceeds their marginal propensity to consume, m_t^E, and is weighted by the fraction of total wealth the entrepreneurs hold, v_t. The second term is the corresponding term for laborers, with a similar intuition. The second equation shows that the evolution of the relative distribution of wealth depends on the following. First, on the differential excess return the entrepreneurs face on their saving, $s_t(1-\tau_t^K)(r_t - R_t)$. Second, on the difference in the level of saving between entrepreneurs and laborers, $m_t^L - m_t^E$. Third, on the adjustment made for the transition probabilities. The third equation is a modified version of (16). The last equation is bond market clearing, a variant of (17).

6.2.2 Steady state implications

The on line appendix presents the modified version of Proposition 3 that now characterizes the steady state aggregates. Furthermore, it is assumed that, at each point in time, the dying agents are replaced by newborn agents, who are endowed with the wealth of the exiting agents. Hence, from a law of large numbers, each agent starts life with the sum of his human wealth plus the mean aggregate wealth. Let $u_t^i \equiv x_t^i/X_t$ denote the distance between individual and aggregate effective wealth. Let Φ_L and Φ_E be the conditional invariant distributions for laborers and entrepreneurs, respectively. The following proposition characterizes the invariant distributions.

Proposition 7. *The conditional invariant distributions Φ_L and Φ_E are characterized by the following second-order linear differential system:*

$$0 = \kappa_1 \, u \, \frac{\partial \Phi_L}{\partial u} \; + \; \kappa_2 \, \Phi_L \; + \; p_{EL} \, \Phi_E,$$

$$0 = \kappa_3 \, u^2 \, \frac{\partial^2 \Phi_E}{\partial u^2} \; + \; \kappa_4 \, u \, \frac{\partial \Phi_E}{\partial u} \; + \; \kappa_5 \, \Phi_E \; + \; p_{LE} \, \Phi_L \, ,$$

where $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5$ are constants determined by steady state aggregates.

The point to note here is that the tractability of the model allows for a very detailed characterization of the invariant distributions. This is particularly useful for the case of entrepreneurs, since it is reasonable to expect that the distribution of wealth over entrepreneurs will be, to a large extent, determined by the realization of entrepreneurial returns.[15]

6.3 Parameter choice

The dynamic system described in Proposition 6 is highly tractable compared to other incomplete-markets models, as the wealth distribution is not a relevant state variable for aggregate equilibrium dynamics. The on line appendix provides the technical details for the algorithms used. The economy is parameterized by $(\alpha,\, \rho,\, \gamma,\, \delta,\, \theta,\, \sigma,\, \psi,\, \nu,\, p_{EL},\, p_{LE},\, \tau^K,\, \tau^L,\, G)$. Table 1 presents the parameter choices for the preferred benchmark model calibration. The parameter values refer to annual data from the United States.

The income share of capital is $\alpha = 0.40$. The preference parameter is $\psi = 0.75$, as is standard in the macro literature. The depreciation rate is $\delta = 0.05$. The probability of

[15] Whereas the tractability of the aggregates follows from Angeletos (2007), the result about the tractability of the invariant distributions is novel to the present paper.

exiting entrepreneurship is $p_{EL} = 0.18$, and the probability of entering entrepreneurship is $p_{LE} = 0.025$, as estimated by Quadrini (2001).[16] The probability of death is $\nu = 1/150$. The elasticity of intertemporal substitution is $\theta = 1$, consistent with the US micro data, see Angeletos (2007) for a detailed discussion. The discount rate is set to $\rho = 0.025$, so as to match the equilibrium risk-free rate. For the period 1985-2011, using data from the Monthly Treasury Statements (MTS), the nominal interest rate can be calculated as the ratio of interest payments over debt held by the public. Depending on the measure of inflation used, we then get an estimated average real interest rate between 2-3.5%. Bohn (2005) reports that the average real interest rate for the period 1915-2003 is 1.3%. For the period 1997-2007, the average real rate on ten-year Treasury notes, used by the Congressional Budget Office (CBO) as a proxy for the real long term risk-free rate, is between 2-3%, depending on measures of inflation expectations. In the benchmark calibration here, the discount rate is chosen to match a steady state risk-free rate of $R = 2.5\%$, in the middle of these empirical estimates.

The proportional tax on capital income is $\tau^K = 0.25$. The CBO Background Paper of December 2006 reports that the average marginal rates at which corporate and non-corporate profits are taxed are 35% and 27%, respectively. The CRS Report for Congress (October 2003) states that, if bonus depreciation is taken into account, the effective marginal capital-income taxes are between $25 - 30\%$ for corporate businesses, and between $20 - 25\%$ for non-corporate businesses. Altig et al. (2001) report a proportional capital-income tax of 20% at the federal level, but they also subject capital income to a 3.7% state tax. The proportional tax on labor income is $\tau^L = 0.35$. The CBO Background Paper of December 2006 reports that the median effective marginal tax rate on labor income is 32%, inclusive of federal, state, and payroll taxes. This number is also reported by Jokisch and Kotlikoff (2006). The level of government spending, G, is chosen so that the steady state government-spending-to-GDP ratio is 20% in the benchmark calibration. This is the average of the ratio of government consumption expenditures to GDP in the National Income and Product Accounts (NIPAs) during 1995-2011.[17]

The coefficient of relative risk aversion is $\gamma = 5$. Using the Consumer Expenditure

[16] This choice determines the steady state fraction of entrepreneurs at 12%, see on line appendix.

[17] Government spending is kept fixed at the benchmark level in calibrations where the capital tax varies, and the government budget is then balanced by corresponding changes in lump-sum taxes. This is done so as not to conflate the results with those from Angeletos and Panousi (2009), who show, in a similar framework, that an increase in government spending may reduce steady state aggregates, even when financed solely by lump-sum taxes. None of the theoretical results of this paper depend on the particular way of balancing the government budget, though the choice here does, albeit only slightly, quantitatively boost the insurance aspect of an increase in the capital tax, because it reduces the threshold $s/(1-s)$, all else equal.

Survey (CEX), Vissing-Jørgensen and Attanasio (2003) find estimates of risk aversion for stockholders in the range of $5-10$. Alan and Browning (2010) use the Panel Study of Income Dynamics (PSID) data to structurally estimate the joint distribution of discount factors and relative risk aversion coefficients, and find that the median risk aversion is 6.2 and 8.4 for less and more risk averse households, respectively. Barsky et al. (1997) measure risk aversion based on survey responses in the Health and Retirement Study (HRS), and find that most individuals have mean relative risk aversion of 16. Cohen and Einav (2005), using data on individuals' deductible choices in auto-insurance contracts, find that the 82nd percentile in the distribution of the coefficient of relative risk aversion is about $13-15$. Guiso and Paiella (2005), using data from the Bank of Italy Survey on Household Income and Wealth, find that the median relative risk aversion is 6, if consumers have a one-year horizon, and 16, if they have a lifetime horizon. Dohmen et al. (2005), using survey questions backed by field experiments in Germany, find that most of the mass in the γ-distribution is located between $1-10$. Estimates of relative risk aversion in the range of 10 have also been reported by Palsson (1996), who uses Swedish cross-sectional data from tax returns. The value for γ chosen here is at the low end of the available empirical estimates, in line with the macro and finance literature standard choices.

The volatility coefficient is $\sigma = 0.30$. Unfortunately, there is no direct measure of the rate-of-return risk faced by the "typical" investor in the US economy. However, there are various indications that idiosyncratic investment risks are significant. For instance, the probability that a privately-held firm survives five years after entry is less than 40%. Furthermore, even conditional on survival, the risks faced by entrepreneurs appear to be very large: As Moskowitz Vissing-Jørgensen (2002) document, not only is there a dramatic cross-sectional variation in the returns to private equity, but also the volatility of the book value of an index of private firms is twice as large as that of the index of public firms. Note then that the standard deviation of annual returns is about 15% per annum for the entire pool of public firms; it is over 50% for a single public firm (which gives a measure of firm-specific risk); and it is about 40% for a portfolio of the smallest public firms (which are likely to be similar to large private firms). A value of $\sigma = 0.5$ is preferred in Moskowitz and Vissing-Jørgensen (2002), Bitler, Moskowitz and Vissing-Jørgensen (2005), Benhabib and Zhou (2008), and Roussanov (2010). Nonetheless, it is possible that entrepreneurs will actually be able to diversify away part of the idiosyncratic risk they face, so that the remaining uninsurable component has $\sigma < 0.5$. Although the choice of $\sigma = 0.3$ here is somewhat arbitrary, it is reassuring that the volatility of individual consumption generated by the model is comparable to its empirical counterpart. For instance, using the Consumer Expenditure Survey (CEX), Malloy, Moskowitz and Vissing-Jørgensen (2009) estimate the standard deviation of consumption growth to be about 8% for stockholders. Similarly, using data that include consumption of luxury goods, Aït-Sahalia, Parker and Yogo (2005) get estimates between 6% and 15%. In the benchmark calibration here, the standard deviation of individual consumption growth is 7% per annum along the steady state.

7 Quantitative Implications of Capital Taxation

This section presents the implications of the model for steady state aggregates and distributions, quantifies the steady state effects of capital-income taxation, and examines the macroeconomic and welfare implications of eliminating the capital tax.

7.1 Aggregates and distributions

Table 2 presents the implications of the model for steady state aggregates, and compares them to the data from the US economy. The empirical analog of the K/Y ratio is the ratio of financial wealth to total income. In the Survey of Consumer Finances (SCF), financial wealth is defined as the value of the sum of corporate equity holdings, bond holdings, the household's share in non-corporate businesses, and the household's deposits in transaction accounts. Total income is GDP from the NIPAs. Using the 1989 SCF, the ratio of total financial wealth to GDP was about 2.5, and it is a bit higher but in the same range in other years, hence the value of 2.7 matched in the model's benchmark calibration.[18] The empirical analog of the I/Y ratio is the average ratio of private domestic investment to GDP from the NIPAs (the period used is 1970-2011). The empirical analogs for the ratio of government spending to GDP, G/Y, and the risk-free rate, R, have already been discussed in section 6.3. Of particular interest is the fraction of financial wealth held by entrepreneurs, v, as all other aggregates could have been matched by a standard neoclassical model. Using the SCF and the aforementioned measure of financial wealth, business owners, broadly defined to include owners of active and non-active businesses, hold about $35-40\%$ of financial wealth, depending on the year. In the model, this ratio is 38%, in the middle of this range of estimates. Because the choice of the risk aversion, γ, and the volatility of risk, σ, is crucial for the determination of v, the robustness of the results to these two parameters will be

[18] Note, in particular, that the relevant measure of financial wealth for this model's calibration should and does exclude the value of housing and consumer durables.

examined in section 7.2.

Table 3 examines the wealth distribution generated by the model. The first two rows present the wealth (net worth) percentiles from the 1989 SCF and the PSID, as reported in Quadrini (2000). The last row is the financial wealth distribution of the benchmark calibration, conditional on wealth being positive. The model demonstrates how the random-walk component introduced in wealth by investment risk helps generate a fatter right tail in the wealth distribution, compared to models with labor-income risk alone. For example, the benchmark calibration predictions of Aiyagari (1994) for the wealth holdings of the top 5% and the top 1% of the population are 13.1% and 3.2%, respectively.[19] Figure 1 presents the model's conditional wealth distributions for entrepreneurs and laborers. Consistent with the data, the distribution of wealth for the population of entrepreneurs displays a fatter tail than the one for laborers, due to the random-walk component of the uninsurable investment risk. Furthermore, the entrepreneurial wealth distribution is shifted to the right, due to the higher mean return of the total entrepreneurial portfolio. Finally, the distributions of wealth for both groups have significant mass of people with wealth higher than 50 times mean income. In the model, the laborers at the right tail of the wealth distribution are former successful entrepreneurs.[20]

7.2 Steady state effects of capital-income taxation

This section quantifies the main theoretical result of the paper, namely that an increase in the capital-income tax may increase the steady state capital stock. Here, the steady state is characterized by the parameters in Table 1, for varying levels of the capital-income tax. Figure 2 shows that steady state capital (panel (a)) and output (panel (b)) are inversely-U shaped with respect to the tax, and they reach a maximum when $\tau^K = 50\%$. The same is true for employment, capital per work-hour, and output per work-hour (not shown). At $\tau^K = 50\%$, steady state capital per work-hour is 16% higher and output per work-hour is 7% higher, respectively, than when the tax is zero. Panel (c) shows that the after-tax interest rate increases with the tax, and that it tends to the discount rate, $\rho = 0.025$, as $\tau^K \to 1$, a manifestation of the precautionary saving motive. Panel (d) shows that, as the tax increases

[19] A tractable extension that could improve the model's predictions about wealth concentration at the top would be to introduce a low-persistence third state, in which an agent gets to be an entrepreneur operating a very high-return or very low-risk production function. Then, the transition probabilities between the three states can be freely chosen to match desired moments of the wealth distribution.

[20] As an additional check, Figure $A1$ in the on line appendix plots the Lorenz curves for the model's aggregate wealth and consumption distributions. The model produces results in the right direction, in that the distribution of wealth over the population is much more unequal than the distribution of consumption.

and the precautionary saving motive becomes weaker, entrepreneurs are satisfied with a lower risk premium.

At this point, it is useful to compare the effects of capital-income taxation to those in complete markets, where there is no scope for insurance when agents are homogeneous, as well as to the "small open economy" version of the present model, where only the direct insurance aspect of the tax is present. In complete markets, where the calibration uses the relevant parameter values from the benchmark Table 1, at $\tau^K = 50\%$, steady state capital per work-hour and output per work-hour are 40% and 20% lower, respectively, than when the tax is zero. In the open economy with incomplete markets, and with the same after-tax interest rate as in the closed economy at the benchmark steady state, the aggregates fall all the way with the tax, but less so than under complete markets. In particular, at $\tau^K = 50\%$, steady state capital per work-hour and output per work-hour are 23% and 11% lower, respectively, than when $\tau^K = 0$, which is roughly half the complete-markets drop.

Aggregate steady state welfare (not shown) is maximized at $\tau^K = 75\%$. This tax is higher than the one that maximizes aggregates, because it also captures the direct insurance effect of the tax in reducing the effective variance of risk, $\sigma(1 - \tau^K)$. The latter effect is the only one operating in the "small open economy", where steady state welfare is maximized at $\tau^K = 45\%$. Of course, steady state welfare in complete markets is maximized when the tax is zero. Note, however, that, as is standard in the literature, the tax maximizing steady state welfare here is not the optimal tax from an ex ante welfare perspective, because it does not take into account transitional dynamic effects.[21]

Finally, Table 4 presents robustness checks with respect to risk aversion, γ, and volatility of risk, σ. When either γ or σ increases, all else equal, then the tax that maximizes the steady state capital stock increases. For example, when $\gamma = 8$, and keeping $\sigma = 0.3$, then the tax that maximizes steady state capital is $\tau^K = 0.58$. When $\sigma = 0.5$, and keeping $\gamma = 5$, the tax that maximizes capital is $\tau^K = 0.68$.[22] These comparative statics further indicate that the effects of capital taxation in this framework are related to an insurance interpretation of the tax system: When the volatility of idiosyncratic risk is higher or when risk aversion is higher, then agents have a stronger need for insurance provision, and therefore both the direct and the indirect effects of the capital tax are likely to be stronger, leading to a higher

[21] In fact, Panousi and Reis (2012), who study the fully-fledged optimal taxation problem in a similar framework, find that the optimal capital tax could in fact be negative, when agents are sufficiently impatient.

[22] If both γ and σ were implausibly low, then it is possible that the capital-maximizing tax could be zero. Even in that case, however, the main theoretical mechanism of this paper would still be robust, as the negative effects of the capital tax on the aggregates would still be less pronounced here than under complete markets. For an example, see the discussion behind figure $A2$ of the on line appendix.

tax that maximizes steady state aggregates.

7.3 Eliminating the capital-income tax

This section examines the aggregate and welfare implications of eliminating the capital-income tax. In the standard neoclassical model, the optimal capital tax is zero in the long run, as well as in most of the short run for an interesting class of preferences, and steady state welfare is also decreasing in the tax. These findings have initiated an extensive debate on the potential benefits of eliminating the capital tax. By contrast, the main result of the present paper is that an increase in the capital tax may actually stimulate capital accumulation. In light of this result, it is worthwhile revisiting the discussion surrounding the zero-tax reform. The assumption here is that the economy starts from the steady state characterized by Table 1 so that, before time $t = 0$, the steady state is characterized by $\tau^K = 0.25$. Subsequently, at $t = 0$, a policy reform is initiated that sets the capital tax to zero, unexpectedly and permanently.

7.3.1 Aggregate effects

The short-run and long-run implications of the zero-tax policy reform for the aggregate macroeconomic variables are presented in Figure 3.[23] Under complete markets, the elimination of the capital tax leads to an immediate negative jump in consumption and positive jump in investment. Capital then slowly increases, and it converges to a higher steady state value, while consumption is initially lower and increases over time. In other words, the long-run increase in investment requires an initial period of lower consumption, which in turn allows for an immediate increase in investment as well. By contrast, here, the effects are exactly reversed. In light of the main mechanism, investment is lower in the long run. This allows for an immediate increase in consumption, and therefore necessitates a fall in current investment. In particular, the investment-output ratio falls by about 5 percentage units.

7.3.2 Welfare effects

This section examines the welfare effects of eliminating the capital-income tax. The goal is to distinguishing how these effects may vary between the poor and the rich, as well as across different generations. This motivates the following two exercises. The first studies welfare at the moment the reform takes place or for generations currently alive, taking into account

[23] See also Table $A1$ of the on line appendix.

the entire transitional dynamics of the economy toward the new steady state with the zero tax. The second compares welfare across the two steady states, one with $\tau^K = 0.25$ and one with $\tau^K = 0$, namely it captures the welfare effects of the reform for future generations, who will be alive at the new steady state with the zero tax.

More precisely, the first exercise seeks to answer the following question. Suppose that the economy rests in its steady state with $\tau^K = 0.25$, and that the current generation contemplates the option to undertake a reform that would set $\tau^K = 0$. Pick a particular level of wealth. What is the minimal compensation an agent with that particular level of wealth would be willing to accept in return for the failure of the reform to take place? The second exercise seeks to answer the following question. Suppose that future generations are offered the option to be born in the steady state with $\tau^K = 0.25$, versus in the steady state with $\tau^K = 0$. Fix a particular ranking in the wealth distribution (say, the 8-th percentile). What is the minimal compensation an agent with that particular ranking would have to receive under the steady state with $\tau^K = 0.25$, in order to be as happy as an agent with the same ranking under the steady state with $\tau^K = 0$? For either of these two exercises, the corresponding compensating differential is then expressed as a fraction of the agent's permanent income.[24] The resulting number represents a welfare gain if it is positive, and a welfare loss if it is negative (see the on line appendix for a formal definition of the compensating differentials). These welfare gains and losses are then illustrated in Table 5, for each type of agent, and across the four quartiles of the steady state wealth distributions with $\tau^K = 0.25$.

On impact, poorer agents lose less than richer agents from the elimination of the capital tax, regardless of whether they are entrepreneurs or laborers. For example, entrepreneurs in the bottom quartile of the distribution actually gain 0.5%, whereas entrepreneurs at the top quartile lose 4.6%. Similarly, laborers in the bottom quartile of the distribution lose 0.6%, whereas laborers at the top quartile lose about 6%. It is possible then that some really poor agents of either type do in fact benefit from the elimination of the tax. The losses from the elimination of the tax are higher for laborers than for entrepreneurs, for each wealth level. Hence, poorer agents of either type lose less or even gain from setting the tax to zero, whereas richer agents suffer, and laborers more so than entrepreneurs. The intuition for these results is as follows. On the impact of the reform, the elimination of the tax deprives agents from some insurance against idiosyncratic risk. This increases the demand for precautionary saving and reduces the interest rate. Since the capital stock cannot adjust, the fall in the

[24] That is, a number equal to, say, 5% means that the agent must receive either a lump sum equal to 5 percent of his effective wealth or, equivalently, a perpetuity with annual dividend equal to 5 percent of his permanent income.

interest rate leads to an increase in agents' human wealth. For poor agents of either type, human wealth constitutes a larger part of their total wealth, compared to richer agents. Hence, the poor are not as adversely affected as the rich, and in fact the very poor might actually enjoy some (small) gains from the elimination of the tax. In addition, the fall in the interest rate (bond return) hurts laborers, who are more likely to be lenders, while it benefits entrepreneurs, who are more likely to be borrowers. This is why overall the losses are bigger (the gains are smaller) for laborers than for entrepreneurs.

In the long run, poorer agents, regardless of type, suffer from the elimination of the capital-income tax, the laborers more so than the entrepreneurs. For example, laborers in the bottom quartile of the distribution lose about 34%, whereas entrepreneurs in the bottom quartile of the distribution lose about 26%. By contrast, richer agents of either type lose less or even gain from the elimination of the tax, the entrepreneurs more so than the laborers. For example, entrepreneurs in the top quartile enjoy an average gain of almost 10%, while laborers in the top quartile lose about 1.4%. The intuition for these results is as follows. In the long run, the elimination of the capital-income tax reduces the interest rate, and thereby capital accumulation, in light of the main result of the paper. The consequent reduction in wages is strong enough to outweigh the fall in the interest rate, so that human wealth actually falls. In turn, this hurts all agents, but more so the poorer ones, and especially the laborers among the poor. At the same time, the reduction in wages means that private businesses now face lower labor costs, leading to an increase in the overall risk-adjusted portfolio return. This effect tends to benefit the rich, who have large amounts of financial wealth relative to human wealth, and especially the rich who are entrepreneurs. For rich agents and especially for entrepreneurs, the positive effect from higher portfolio returns is strong enough to offset the negative effect from lower human wealth, and it explains why rich agents gain whereas poorer agents lose from the elimination of the capital-income tax.

To summarize, the elimination of the capital-income tax slightly benefits the very poor agents on impact, as they temporarily enjoy higher human wealth, and it also benefits the future rich, as they benefit from higher saving returns. Both on impact and at the new steady state, these gains from the elimination of the tax are higher for entrepreneurs than for laborers. All other income groups lose from the elimination of the capital-income tax, and the consequent direct and indirect loss of insurance provision, regardless of type, and the laborers more so than the entrepreneurs.

8 Conclusions

This paper studies the aggregate and welfare effects of capital-income taxation in an environment where agents face uninsurable idiosyncratic investment risk. The novel and surprising result emerging is that an increase in the capital tax may actually stimulate capital accumulation. This effect is due to the general equilibrium insurance aspect of the tax, operating through the endogenous adjustment of the interest rate. This result stands in stark contrast to the effects of capital-income taxation in either complete markets models, or in Bewley type models, as in all of those models capital taxation necessarily discourages capital accumulation, regardless of whether it is welfare improving or not. Furthermore, the result arises in an empirically relevant framework and is quantitatively significant: For the preferred calibration, the steady state levels of the capital stock, output, and employment are all maximized for $\tau^K = 50\%$, at which point output per work-hour is 7% higher than it would have been, had the tax rate been zero, whereas it is 21% lower under complete markets. In addition, most agents would suffer welfare losses from a reform that eliminates the capital tax, with the possible exception of the future rich, who benefit from the elimination of the distortion in the mean investment return.

In order to focus on entrepreneurial risk, this paper abstracts from labor-income risk, decreasing returns to scale at the individual level, and borrowing constraints. Extending the model to include these relevant aspects of the data is important, not only to get a better quantitative evaluation of the implications of capital taxation, but also to examine whether the general equilibrium effects identified here interact with other sources of market incompleteness in an interesting way. One obvious example is the potential interaction of higher interest rates, following an increase of the capital tax, with wealth accumulation, the cost of borrowing, and collateral constraints.

Although this paper provides some useful guidance about the direction of optimal policy, it does not solve for the fully-fledged optimal policy problem, namely it does not formally perform the exercise of maximizing ex ante welfare. Panousi and Reis (2012), in a setting similar to the one in the present paper, show that the optimal tax might actually be negative, if the variance of risk is lower than the mean return to the risky asset, due to agents' impatience, and thus due to reasons similar to those dictating that the tax maximizing ex ante welfare in complete markets (zero) is different from the tax maximizing steady state welfare (negative) at the golden rule.

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Table 1: Benchmark Calibration Parameter Values

Parameters	Values
Preferences	
ρ	0.025
γ	5
θ	1
ψ	0.75
Technology	
α	0.40
δ	0.05
Probabilities	
ν	0.0067
p_{EL}	0.18
p_{LE}	0.025
Government	
τ^K	0.25
τ^L	0.35
G/GDP	0.20
Risk	
σ	0.30

Table 1 presents the parameter values for the benchmark calibration. The parameter values refer to annual data from the United States. Here, ρ is the discount rate in preferences, γ is the coefficient of relative risk aversion, θ is the elasticity of intertemporal substitution, ψ is the coefficient of homotheticity in preferences, α is the share of capital in production, δ is the average depreciation rate, ν is the probability of death, p_{EL} is the probability that an entrepreneur will become a laborer, p_{LE} is the probability that a laborer will become an entrepreneur, τ^K is the proportional capital-income tax, τ^L is the proportional labor-income tax, G/GDP is the ratio of government spending to GDP, and σ is the standard deviation of idiosyncratic capital-income risk. See section 6.3 for references.

Table 2: Steady State Aggregates

	K/Y	I/Y	G/Y	R	v
US Data	2.7	13	20	2.5	$35-40$
Model	2.7	13	20	2.5	38

Table 2 demonstrates the performance of the benchmark model calibration (from Table 1) in matching the US steady state aggregates. Here, K/Y is the ratio of financial wealth to GDP, I/Y is the ratio of private investment to GDP, G/Y is the ratio of government consumption to GDP, R is the real before-tax interest rate, and v is the fraction of financial wealth held by entrepreneurs in the economy. Except for the capital-output ratio, all numbers are in percent. See sections 6.3 and 7.1 for details on the data sources.

Table 3: Steady State Wealth Distribution

	Top Percentiles				
	30%	20%	10%	5%	1%
SCF	91.8	83.7	70.1	58.0	35.7
PSID	86.9	77.1	60.9	47.0	25.4
Model	78.0	66.2	47.7	33.0	12.8

Table 3 demonstrates the performance of the benchmark model calibration (from Table 1) in matching the wealth distribution in the United States. The data from the Survey of Consumer Finances (SCF) and the Panel Study of Income Dynamics (PSID) are for the year 1989 and are taken from Quadrini (2000). In both the SCF and the PSID, the measure of wealth is total household net worth. In the model, the relevant measure of wealth is total financial wealth, conditional on being positive.

Table 4: Steady State Capital Maximizing Tax

	Risk aversion γ					Standard deviation of risk σ				
	2	3	5	8	10	0.1	0.2	0.3	0.4	0.5
Capital maximizing tax	0.35	0.38	0.5	0.58	0.62	0.1	0.25	0.5	0.60	0.68

Table 4 shows the capital tax that maximizes the steady state capital stock for different values of the coefficient of relative risk aversion, γ, and the standard deviation of idiosyncratic risk, σ, all else equal.

Table 5: Welfare Effects of Eliminating the Capital Tax

	Entrepreneurs				Laborers			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Current generation (at time of reform)	0.5	−2.5	−3.3	−4.6	−0.6	−4.4	−5.2	−6.1
Future generations (across steady states)	−25.8	−12.0	−4.2	9.9	−34.2	−16.5	−9.3	−1.4

Table 5 summarizes the welfare effects of eliminating the capital-income tax across the quartiles of the wealth distribution with $\tau^K = 0.25$, for entrepreneurs and for laborers. Q1 is the first quartile, Q2 is the second quartile, and so on. The numbers in the cells of the table report the within-quartile averages of the compensating differentials for the current generation (at the moment the elimination of the tax takes place, but taking transitional dynamics into account) and for future generations (compares welfare across steady states). The compensating differentials are measured as percent of permanent income. The economy starts at the steady state characterized by the benchmark model calibration (from Table 1), with $\tau^K = 0.25$. Subsequently, the capital-income tax is set to zero, unexpectedly and permanently.

Figure 1: Steady State Distributions

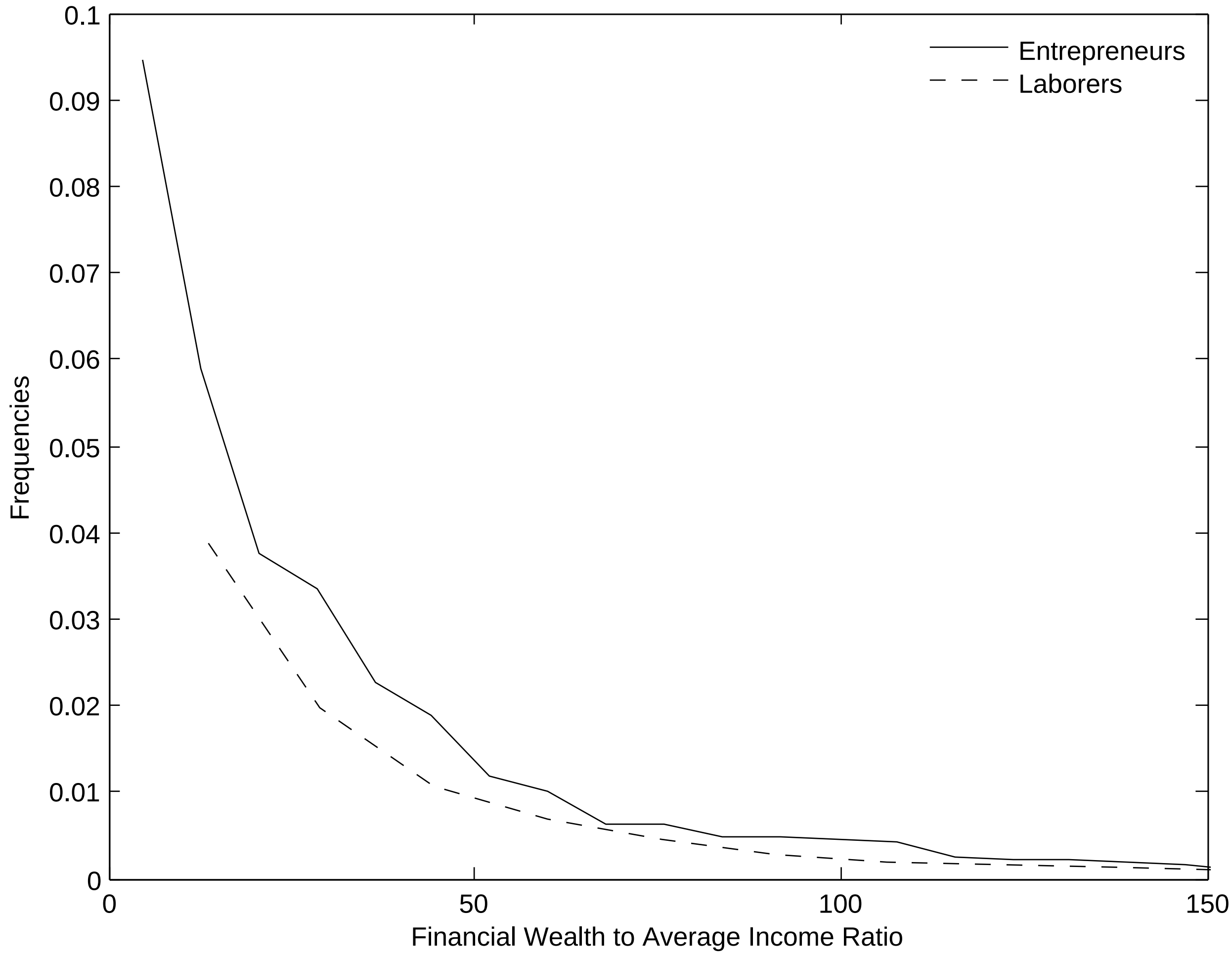
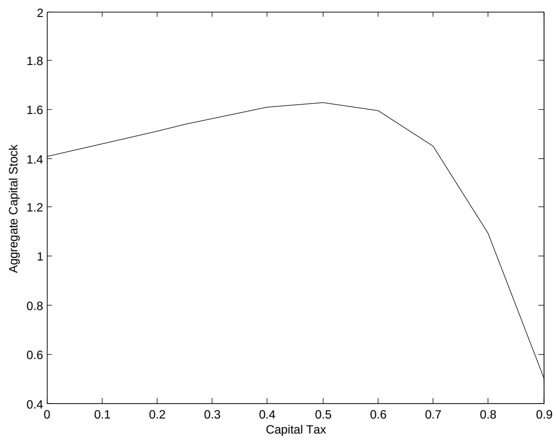
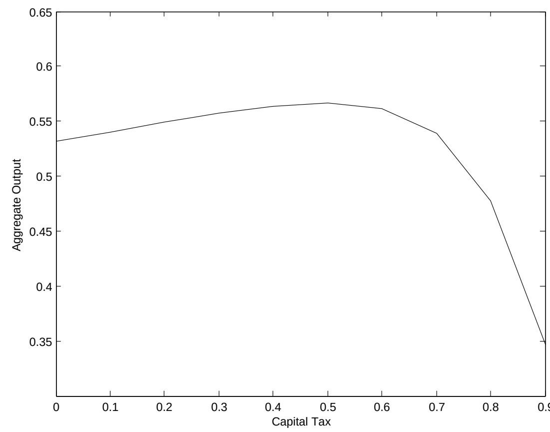


Figure 1 presents the steady state conditional wealth distributions for entrepreneurs (solid line) and laborers (dashed line) generated by the benchmark model calibration from Table 1. Financial wealth normalized by mean income is on the horizontal axis. Frequencies are on the vertical axis.

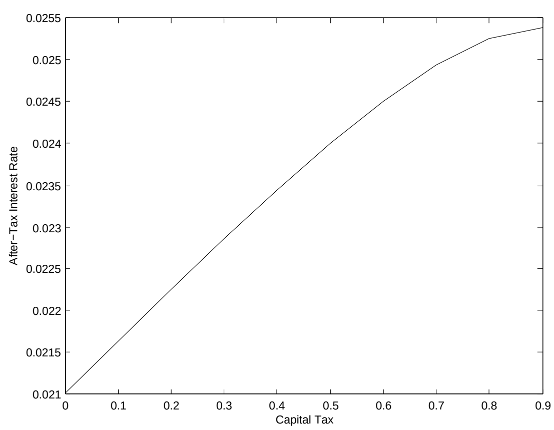
Figure 2: Steady State Aggregates and the Capital Tax



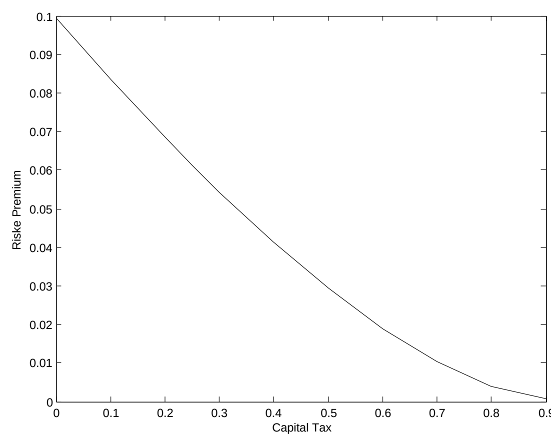
(a) Aggregate Capital Stock



(b) Aggregate Output



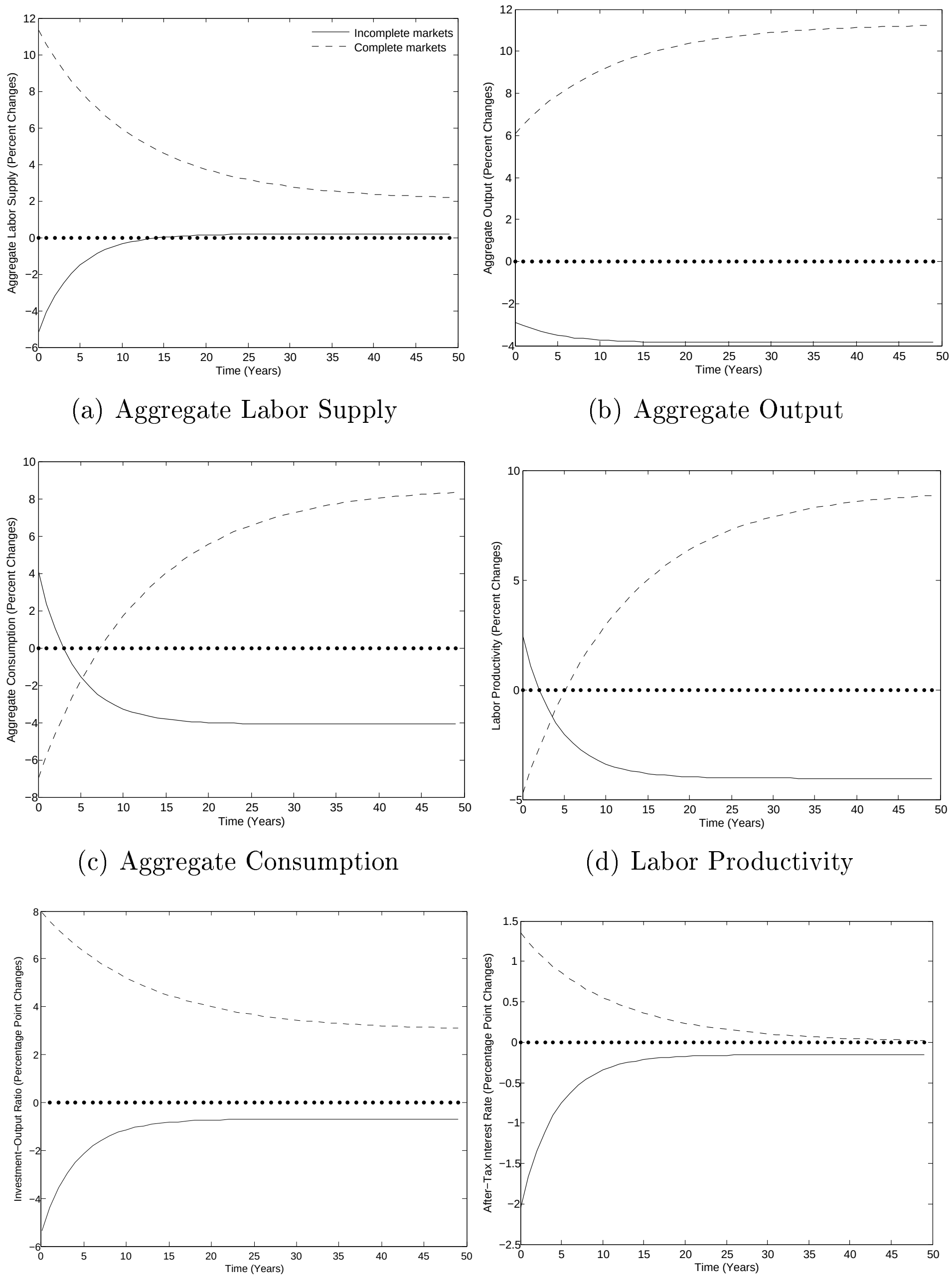
(c) After-Tax Interest Rate



(d) Risk Premium

Figure 2 presents selected steady state macroeconomic aggregates as a function of the capital-income tax. The relevant parameter values are those from the benchmark calibration in Table 1, where now the capital-income tax is allowed to vary between zero and one. The tax is on the horizontal axis and the aggregates on the vertical axis.

Figure 3: Eliminating the Capital Tax



(a) Aggregate Labor Supply (b) Aggregate Output

(c) Aggregate Consumption (d) Labor Productivity

(e) Investment-Output Ratio (f) After-Tax Interest Rate

Figure 3 presents the dynamic response of selected macroeconomic aggregates to the elimination of the capital-income tax. Time in years is on the horizontal axis, and the variables are on the vertical axis. The economy initially rests at the steady state characterized by the benchmark calibration in Table 1, with $\tau^K = 0.25$. Subsequently, the capital-income tax is set to zero, unexpectedly and permanently. The solid lines indicate the response of the variables in the present model of incomplete markets. The dashed lines indicate the response of the variables in complete markets. Aggregate labor supply, aggregate output, aggregate consumption, and labor productivity are normalized by their values at the initial steady state with the positive capital tax. The after-tax interest rate and the investment-output ratio are presented as changes from their initial steady state values.