

Chapter 2 Question 10

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Question

Write the differential equation that is mathematically equivalent to the block diagram shown in Figure P2.2. Assume that $r(t) = 3t^3$.

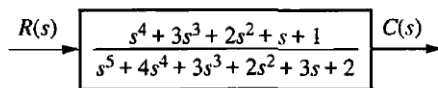


FIGURE P2.2

$$\text{Transfer function} = \frac{s^4 + 3s^3 + 2s^2 + s + 1}{s^5 + 4s^4 + 3s^3 + 2s^2 + 3s + 2} = \frac{C(s)}{R(s)}.$$

Cross multiplying,

$$C(s) (s^5 + 4s^4 + 3s^3 + 2s^2 + 3s + 2) = R(s) (s^4 + 3s^3 + 2s^2 + s + 1)$$

Assuming zero initial conditions and
taking the inverse Laplace transform,

Solution

$$\frac{d^5 c}{dt^5} + 4\frac{d^4 c}{dt^4} + 3\frac{d^3 c}{dt^3} + 2\frac{d^2 c}{dt^2} + 3\frac{dc}{dt} + 2c = \frac{d^4 r}{dt^4} + 3\frac{d^3 r}{dt^3} + 2\frac{d^2 r}{dt^2} + \frac{dr}{dt} + r \quad (1)$$

$$\text{Given, } r(t) = 3t^3$$

Substitute $r(t)$ in the above equation.

We get,

$$\frac{d^5 c}{dt^5} + 4\frac{d^4 c}{dt^4} + 3\frac{d^3 c}{dt^3} + 2\frac{d^2 c}{dt^2} + 3\frac{dc}{dt} + 2c = (3t^3 + 9t^2 + 36t + 54)u(t) + 18\delta(t) \quad (2)$$

This is the required mathematical equation.