UNIT 2

1. Regular Expressions and Identities

- **Regular Expression (RE)**:
  - A symbolic representation to define regular languages.
  - Operators:
    - **Union (A ∪ B)**: Denoted as A+BA + BA+B or A|BA|BA|B.
    - **Concatenation (AB)**: Denoted as A·BA \cdot BA·B or simply ABABAB.
    - **Kleene Star (A)*: Zero or more repetitions of AAA.
- **Identities**:
  - R+ϕ=RR + \phi = RR+ϕ=R (Union with empty set is RRR).
  - R·ϵ=RR \cdot \epsilon = RR·ϵ=R (Concatenation with epsilon is RRR).
  - (R∗)∗=R∗(R^*)^* = R^*(R∗)∗=R∗ (Double Kleene star is the same as one).
  - R+R=RR + R = RR+R=R (Union is idempotent).

2. Finite Automata and Regular Expressions

- **Transition System with Null Moves**:
  - NDFA allows **null (ε) transitions**, enabling state changes without input.
- **NDFA with Null Moves to DFA**:
  - Eliminate ε-transitions using **ε-closure** (all reachable states including null transitions).
- **Finite Automata and RE Equivalence**:
  - Any regular expression can be converted to an equivalent finite automaton, and vice versa.

3. Conversion Methods

- **Arden's Theorem**:
  - Used to solve regular expression equations.
  - If R=Q+RPR = Q + RPR=Q+RP, then R=QP∗R = QP^*R=QP∗ (provided PPP doesn't contain ε).
- **Construction of Finite Automata from RE**:
  - Steps:
    1. Create NDFA from the given RE.
    2. Convert NDFA to DFA if needed.

4. Equivalence of Two Finite Automata and Regular Expressions

- **Finite Automata Equivalence**:
  - Two finite automata are equivalent if they accept the same language.
- **Regular Expression Equivalence**:
  - Two REs are equivalent if they describe the same language.

5. Closure Properties of Regular Sets

- **Regular languages are closed under**:
  - **Union**: L1∪L2$L_1 \cup L_2$L1∪L2.
  - **Intersection**: L1∩L2$L_1 \cap L_2$L1∩L2.
  - **Complement**: Σ∗\L1Σ* \setminus L_1Σ∗\L1.
  - **Concatenation**: L1·L2L_1 \cdot L_2L1·L2.
  - **Kleene Star**: L1∗L_1^*L1∗.

6. Pumping Lemma for Regular Sets

- **Statement**:
  - If LLL is a regular language, there exists a pumping length ppp such that any string w∈Lw \in Lw∈L with |w|≥p|w| ≥ p|w|≥p can be divided into w=xyzw = xyzw=xyz satisfying:
    1. |xy|≤p|xy| ≤ p|xy|≤p,
    2. |y|>0|y| > 0|y|>0,
    3. xynz∈Lx y^n z \in Lxynz∈L for n≥0n ≥ 0n≥0.
- **Application**:
  - Used to prove that certain languages are **not regular**.

7. Myhill-Nerode Theorem

- **Theorem**:
  - A language LLL is regular if and only if it has a finite number of equivalence classes under the relation x∼Lyx \sim_L yx∼Ly (indistinguishability of strings with respect to LLL).
- **Significance**:
  - Helps in proving language regularity and minimizing finite automata.

8. Properties of Regular Languages

- **Decidability**:
  - Determining emptiness, finiteness, and equivalence is decidable for regular languages.
- **Non-Regularity Proofs**:
  - Use Pumping Lemma or Myhill-Nerode Theorem.

Important MCQ

1. Regular Expressions Basics

1. What does the regular expression a∗a^*a∗ denote?
   a) Zero or more occurrences of aaa
   b) At least one aaa
   c) Exactly one aaa
   d) None of the above
   Answer: a

2. Which regular expression represents all strings over {a,b}\{a, b\}{a,b}?
   a) (a+b)∗(a + b)^*(a+b)∗
   b) (ab)∗(ab)^*(ab)∗
   c) a+ba + ba+b
   d) None of the above
   Answer: a

3. What does (a|b)+(a|b)^+(a|b)+ represent?
   a) All strings with at least one aaa or bbb
   b) Empty string
   c) Strings with zero or more aaa and bbb
   d) None of the above
   Answer: a

2. Regular Languages

4. Which of the following is NOT a regular language?
   a) {an|n≥0}\{a^n | n ≥ 0\}{an|n≥0}
   b) {anbn|n≥0}\{a^n b^n | n ≥ 0\}{anbn|n≥0}
   c) (a+b)∗(a + b)^*(a+b)∗
   d) {ε}\{ε\}{ε}
   Answer: b

5. Regular languages are closed under which operation?
   a) Union
   b) Concatenation
   c) Kleene Star
   d) All of the above
   Answer: d

6. Which property is true for regular languages?
   a) Infinite memory is required
   b) Can be represented by finite automata
   c) Context-free grammar is required
   d) None of the above
   Answer: b

3. Finite Automata and Regular Expressions

7. Finite automata and regular expressions are:
   a) Equivalent in power
   b) Not equivalent
   c) Regular expressions are more powerful
   d) None of the above
   Answer: a

8. What is used to convert NDFA with null moves to DFA?
   a) Subset construction
   b) Transition table
   c) Kleene Star
   d) Pumping Lemma
   Answer: a

9. What is the output of solving R=Q+RPR = Q + RPR=Q+RP using Arden's Theorem?
   a) R=QP∗R = QP^*R=QP∗
   b) R=PQR = PQR=PQ
   c) R=Q+PR = Q + PR=Q+P
   d) None of the above
   Answer: a

4. Properties of Regular Languages

10. Which of the following is TRUE about regular languages?
    a) Closed under intersection
    b) Closed under complement
    c) Closed under union
    d) All of the above
    Answer: d

11. Which operation is NOT closed for regular languages?
    a) Union
    b) Complement
    c) Intersection
    d) Infinite repetition
    Answer: d

12. What is the smallest automaton that can recognize a regular language?
    a) DFA
    b) NDFA
    c) Turing machine
    d) Moore machine
    Answer: a

5. Pumping Lemma

13. Pumping Lemma is used to prove:
    a) A language is regular
    b) A language is not regular
    c) Closure properties of languages
    d) Equivalence of two finite automata
    Answer: b

14. Which condition is NOT part of the Pumping Lemma?
    a) |xy|≤p|xy| ≤ p|xy|≤p
    b) |y|>0|y| > 0|y|>0
    c) xynz∉Lx y^n z \notin Lxynz∈/L
    d) xynz∈Lx y^n z \in Lxynz∈L
    Answer: c

15. What does ppp in the Pumping Lemma denote?
    a) Pumping length
    b) Number of states
    c) Transition function
    d) None of the above
    Answer: a

6. Conversion Methods

16. NDFA to DFA conversion uses:
    a) Subset construction method
    b) Pumping Lemma
    c) Transition graph
    d) State minimization
    Answer: a

17. Which theorem helps in solving equations involving regular expressions?
    a) Pumping Lemma
    b) Arden's Theorem
    c) Myhill-Nerode Theorem
    d) Kleene's Theorem
    Answer: b

18. What is true about null moves in finite automata?
    a) They do not consume input symbols
    b) They add power to finite automata
    c) They make automata non-deterministic
    d) Both a and c
    Answer: d

7. Myhill-Nerode Theorem

19. Myhill-Nerode Theorem is used to:
    a) Minimize finite automata
    b) Prove regularity of a language
    c) Construct finite automata from RE
    d) Both a and b
    Answer: d

20. According to Myhill-Nerode Theorem, a language is regular if:
    a) It has a finite number of equivalence classes
    b) It is closed under union
    c) It can be represented by a Turing Machine
    d) None of the above
    Answer: a

8. Regular Expressions and Automata Equivalence

21. A finite automaton can be constructed from:
    a) A regular expression
    b) A context-free grammar
    c) Both a and b
    d) None of the above
    Answer: a

22. Which step is NOT required in constructing a DFA from an NDFA?
    a) Finding ε-closure
    b) Subset construction
    c) State minimization
    d) Adding new states
    Answer: d

23. How are two finite automata considered equivalent?
    a) When they have the same number of states
    b) When they recognize the same language
    c) When they have the same transitions
    d) None of the above
    Answer: b

9. Closure Properties

24. Regular languages are NOT closed under:
    a) Complement
    b) Union
    c) Infinite intersection
    d) Concatenation
    Answer: c

25. If L1L_1L1 and L2L_2L2 are regular languages, then L1·L2L_1 \cdot L_2L1·L2:
    a) Is not regular
    b) Is always regular
    c) Depends on L1L_1L1 only
    d) None of the above
    Answer: b

26. Which of the following is a valid closure property of regular languages?
    a) Difference
    b) Reversal
    c) Homomorphism
    d) All of the above
    Answer: d

10. Miscellaneous

27. What is the significance of Kleene Star in regular expressions?
    a) Adds complement functionality
    b) Adds infinite repetition functionality
    c) Represents union of languages
    d) None of the above
    Answer: b

28. Which of the following operations on a regular language is undecidable?
    a) Membership test
    b) Emptiness test
    c) Equality test
    d) None of the above
    Answer: d

29. What is the relationship between DFA and NDFA?
    a) DFA is a subset of NDFA
    b) NDFA is more powerful than DFA
    c) DFA and NDFA recognize the same class of languages
    d) None of the above
    Answer: c

30. What is the result of concatenating two regular languages L1L_1L1 and L2L_2L2?
    a) A regular language
    b) A context-free language
    c) An undecidable language
    d) None of the above
    Answer: a