**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:- [Input] # Mean and standard deviation of the service time

mean\_service\_time = 45 # minutes

std\_dev\_service\_time = 8 # minutes

# Time limit for servicing

time\_limit = 60 # minutes

# Time buffer before starting service

buffer\_time = 10 # minutes

# Calculate the probability of service time exceeding the time limit

# We subtract the buffer time from the time limit to account for the delay in starting service

prob\_not\_meeting\_commitment = 1 - norm.cdf(time\_limit - buffer\_time, loc=mean\_service\_time, scale=std\_dev\_service\_time)

print("Probability that the service manager cannot meet his commitment:", prob\_not\_meeting\_commitment)

[Output] :- Probability that the service manager cannot meet his commitment: 0.2659855904870054

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:- [A] :- [Input]:-

mean\_age = 38

std\_dev\_age = 6

total\_employees = 400

prob\_older\_than\_44 = 1 - norm.cdf(44, loc=mean\_age, scale=std\_dev\_age)

prob\_between\_38\_and\_44 = norm.cdf(44, loc=mean\_age, scale=std\_dev\_age) - norm.cdf(38, loc=mean\_age, scale=std\_dev\_age)

statement\_a = prob\_older\_than\_44 > prob\_between\_38\_and\_44

print("Statement A:", statement\_a)

[Output]:- False

[B]:- prob\_under\_30 = norm.cdf(30, loc=mean\_age, scale=std\_dev\_age)

expected\_employees\_under\_30 = total\_employees \* prob\_under\_30

statement\_b = np.isclose(expected\_employees\_under\_30, 36, atol=1)

print("Statement B:", statement\_b)

[Ouput]:- True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans- [Input]:-

mu = 10

sigma\_squared = 4

np.random.seed(42) # for reproducibility

X1 = np.random.normal(mu, np.sqrt(sigma\_squared), 10000)

X2 = np.random.normal(mu, np.sqrt(sigma\_squared), 10000)

two\_X1 = 2 \* X1

sum\_X1\_X2 = X1 + X2

mean\_2X1 = np.mean(two\_X1)

variance\_2X1 = np.var(two\_X1)

mean\_X1\_X2 = np.mean(sum\_X1\_X2)

variance\_X1\_X2 = np.var(sum\_X1\_X2)

print("Mean of 2X1:", mean\_2X1)

print("Variance of 2X1:", variance\_2X1)

print("Mean of X1 + X2:", mean\_X1\_X2)

print("Variance of X1 + X2:", variance\_X1\_X2)

[Output]:-

Mean of 2X1: 19.991456066526293

Variance of 2X1: 16.109376886570377

Mean of X1 + X2: 20.02279613939292

Variance of X1 + X2: 7.96630257197117

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:- [Input]

mean = 100

std\_dev = 20

z\_score = norm.ppf(0.995)

a = mean - z\_score \* std\_dev

b = mean + z\_score \* std\_dev

print("Values of a and b:", a, b)

[Output]

Values of a and b: 48.48341392902199 151.516586070978

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:- [A]

[Input]: -

conversion\_rate = 45

mean\_profit1 = 5

variance\_profit1 = 32

mean\_profit2 = 7

variance\_profit2 = 42

mean\_total\_usd = mean\_profit1 + mean\_profit2

std\_dev\_total\_usd = np.sqrt(variance\_profit1 + variance\_profit2)

# Convert to Rupees

mean\_total\_rupees = mean\_total\_usd \* conversion\_rate

std\_dev\_total\_rupees = std\_dev\_total\_usd \* conversion\_rate

# A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company

lower\_bound = mean\_total\_rupees - 1.96 \* std\_dev\_total\_rupees

upper\_bound = mean\_total\_rupees + 1.96 \* std\_dev\_total\_rupees

print("95% probability range for annual profit of the company (in Rupees):", (lower\_bound, upper\_bound))

[Output]:-

95% probability range for annual profit of the company (in Rupees): -218.7250885531597 , 1298.7250885531598

[B]:-

[Input]:-

# B. Specify the 5th percentile of profit (in Rupees) for the company

fifth\_percentile = norm.ppf(0.05, loc=mean\_total\_rupees, scale=std\_dev\_total\_rupees)

print("5th percentile of profit for the company (in Rupees):", fifth\_percentile)

[Output]:-

5th percentile of profit for the company (in Rupees): -96.73046620701132

[C]:-

[Input]:-

# C. Determine which of the two divisions has a larger probability of making a loss in a given year

# Calculate the z-scores for each division

z\_score\_profit1 = (0 - mean\_profit1) / np.sqrt(variance\_profit1)

z\_score\_profit2 = (0 - mean\_profit2) / np.sqrt(variance\_profit2)

# Calculate the probabilities of making a loss for each division

prob\_loss\_profit1 = norm.cdf(z\_score\_profit1)

prob\_loss\_profit2 = norm.cdf(z\_score\_profit2)

# Compare the probabilities

if prob\_loss\_profit1 > prob\_loss\_profit2:

print("Profit1 has a larger probability of making a loss in a given year.")

elif prob\_loss\_profit1 < prob\_loss\_profit2:

print("Profit2 has a larger probability of making a loss in a given year.")

else:

print("Both divisions have the same probability of making a loss in a given year.")

[Output]:-

Profit1 has a larger probability of making a loss in a given year.