📘 Day 9 – Hypothesis Testing + Probability Foundations

🔷 1. Probability Basics

Probability = likelihood of an event happening.

- Formula:

  $$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total possible outcomes}}$$

📍 Example: Toss a coin

- Sample space S = {H, T}
- P(H) = 1/2 = 0.5

📍 Example: Roll a die

- S = {1,2,3,4,5,6}
- P(odd) = 3/6 = 0.5

🔷 2. Types of Events

- **Independent**: One event does not affect the other.

  Example: Tossing 2 coins.

  P(H on 1st AND H on 2nd) = 0.5 × 0.5 = 0.25

- **Mutually exclusive**: Both cannot happen at the same time.

  Example: Drawing a King and Queen at the same time from 1 card = 0.

🔷 3. Conditional Probability

$$P(A \mid B) = P(A \cap B)P(B)P(A|B) = \frac{P(A \cap B)}{P(B)}$$

📍 Example: Deck of 52 cards

- P(King | Face card) = King face cards / total face cards = 4/12 = 1/3

🔷 4. Law of Total Probability

If events B1,B2,...BnB_1, B_2, ... B_n partition the sample space:

$$P(A) = \sum i = 1nP(A \mid Bi) \cdot P(Bi)P(A) = \sum_{i=1}^{n} P(A|B_i) \cdot P(B_i)$$

📍 Example:

Factory has 3 machines: A(40%), B(35%), C(25%).

Defective probability: A(2%), B(3%), C(4%).

Overall defect rate = (0.4×0.02) + (0.35×0.03) + (0.25×0.04) = **0.0295 = 2.95%**

🔷 5. Bayes' Theorem

$$P(A \mid B) = P(B \mid A)P(A)P(B)P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

📍 Example: Medical Testing

- Disease prevalence = 1%
- Test detects correctly = 99%
- False positive = 5%

If test is positive, what is P(person has disease)?

$$P(Disease \mid Positive) = 0.99 \times 0.010.99 \times 0.01 + 0.05 \times 0.99 \approx 0.167P(Disease|Positive) = \frac{0.99 \times 0.01}{0.99 \times 0.01 + 0.05 \times 0.99} \approx 0.167$$

👉 Even if test says "Positive", probability is only **16.7%** (because disease is rare).

🔷 6. Hypothesis Testing

- Hypothesis = claim about a population.
- **Null Hypothesis (H_0)** → No effect, no difference.
- **Alternative Hypothesis (H_1)** → There is effect, difference.

📍 Example: Average student height = 170 cm.

- H_0: μ = 170
- H_1: μ ≠ 170

🔷 7. Errors in Testing

- **Type I error (α)** → Rejecting H_0 when true (false alarm).
- **Type II error (β)** → Not rejecting H_0 when false (missed detection).

👉 α = significance level (usually 0.05 = 5%).

🔷 8. Test Statistics

- **Z-test** → population variance known, large n (>30).

$$Z = x^{-} - \mu\sigma/nZ = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

- **t-test** → population variance unknown, small n (<30).

$$t = x^{-} - \mu s/nt = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

📍 Example: Company claims avg. salary = ₹50,000.

Sample of 25 employees → mean = ₹48,000, s = ₹4,000.

$$t = 48000 - 500004000/25 = -2000800 = -2.5t = \frac{48000-50000}{4000/\sqrt{25}} = \frac{-2000}{800} = -2.5$$

Compare with t-table (df=24). If |t| > critical value → reject H_0.

🔷 9. Confidence Intervals

CI gives range of plausible values for mean.

$$CI = x^{-} \pm Z\alpha/2 \cdot \sigma nCI = \bar{x} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

📍 Example: Mean = 100, σ=15, n=36, 95% CI.

$$CI = 100 \pm 1.96 \cdot 156 = 100 \pm 4.9CI = 100 \pm 1.96 \cdot \frac{15}{6} = 100 \pm 4.9$$

CI = (95.1, 104.9)

✅ Summary of Day 9

- Probability basics → events, independence, conditional, total probability, Bayes.
- Hypothesis testing → Null vs Alternative, errors, p-values.
- Z-test, t-test.
- Confidence Intervals.
