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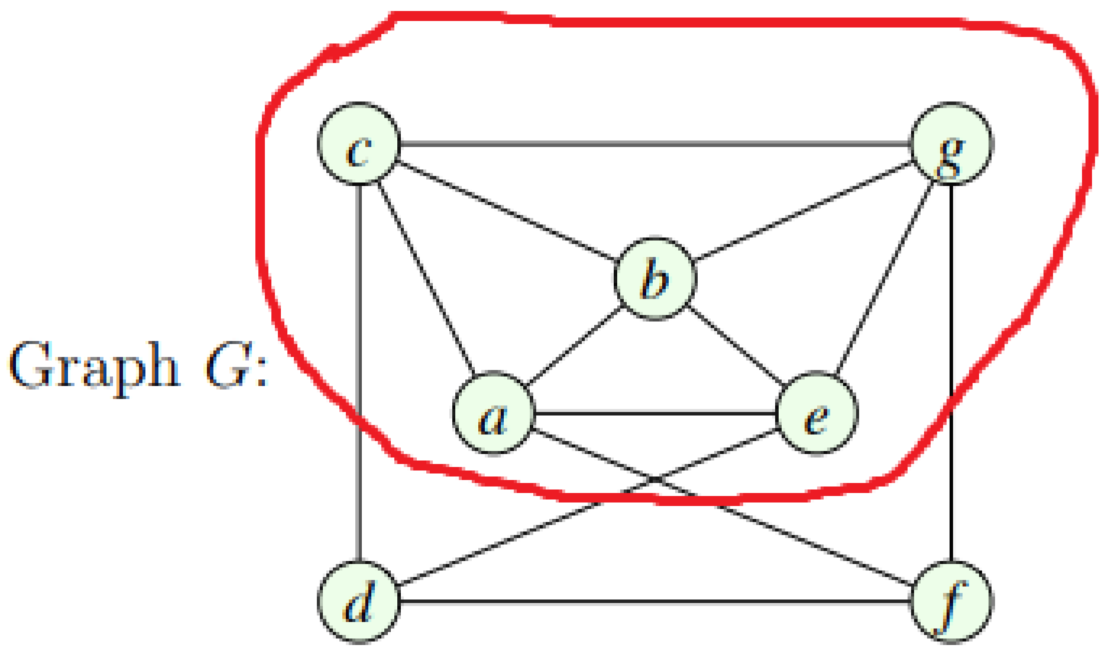
CS111 Homework 5
due Friday, May 28th 11:59 PM

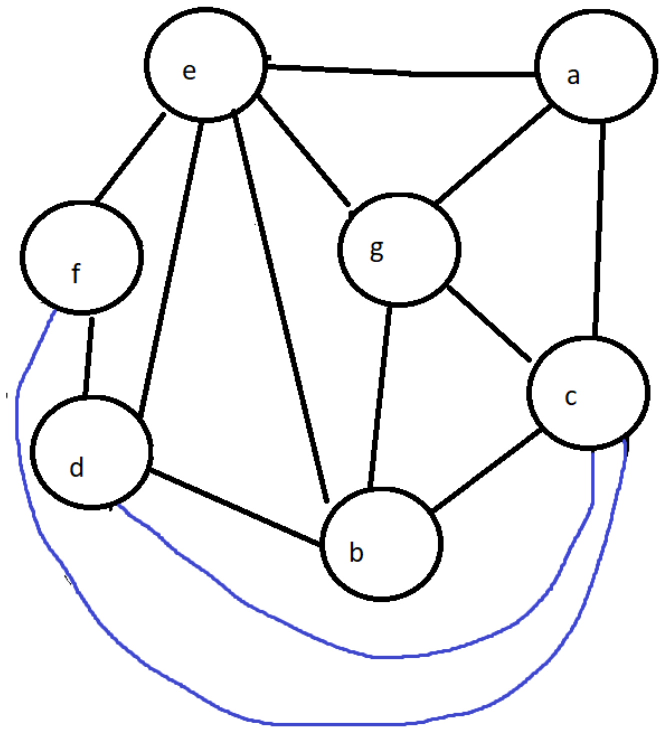
Problem 1.
Graph G:

The subgraph, highlighted in red, using the vertices: a, b, c, e, g produces a K_5 subgraph. So through Kuratowski's theorem, this graph is NOT planar.
By smoothing out vertices d and f, we create new edges C-E and A-G. This means the five remaining nodes, a, b, c, e, g each have a degree of four, which is a K_5 graph.



Graph H:

The given graph H has two edges which can be altered to produce the graph as planar. These two edges are C-D and C-F. By placing these two edges, drawn in blue, outside of the graph, graph H becomes planar, as seen below.



Problem 2.

Graph G: By using Hall's Theorem, we can prove this bipartite graph does not have a perfect matching by finding a set of vertices in set X where the cardinality of the set of neighbor vertices of all vertices in X is less than the cardinality of X.

Let X be the set of green vertices: $1, 3, 4$. Let Y be the set of neighbors of the vertices in X and be highlighted red: b, e.

Since the cardinality of Set X is 3 and the cardinality of Set Y is 2 and $3 > 2$, we have found a contradiction as it relates to Hall's Theorem. Thus, Graph G cannot have a perfect matching.

Graph H: As shown below in Figure 1, there is a perfect matching with the edges highlighted in green.

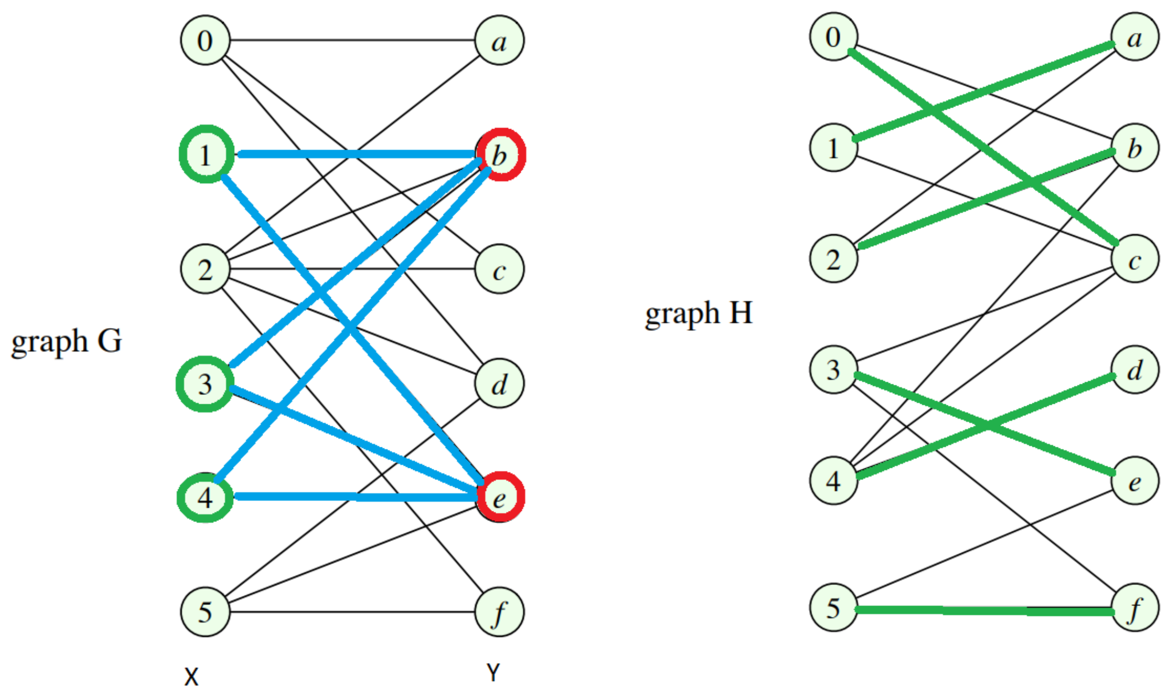
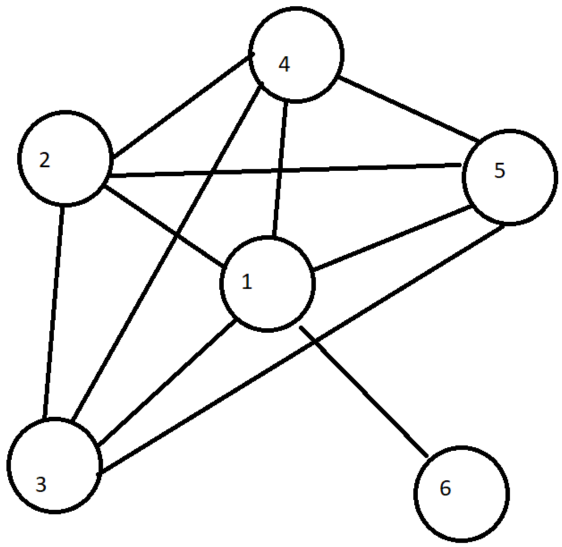


Figure 1: Graphs G and H of Problem 2. Graph G has no perfect matching, while Graph H does.

Problem 3.
a)
a1)
The sum of the degree sequence is odd, so this graph is not possible to draw by the Handshaking Lemma.
a2)
This graph is possible.



a3)
This graph is not possible because there are two vertices with degree 5, and one vertex with degree 1, when this sequence demands a minimum degree of two.

b)
b1) As seen below in Figure 2, there exists a graph of 6 nodes with the nodes having degrees of 5, 5, 4, 4, 3, 3, respectively.
b2) As seen below in Figure 3, there exists a K_5 sub-graph, highlighted in green, within this graph. So by Kuratowski's Theorem, there cannot exist a planar graph for this particular degree sequence.

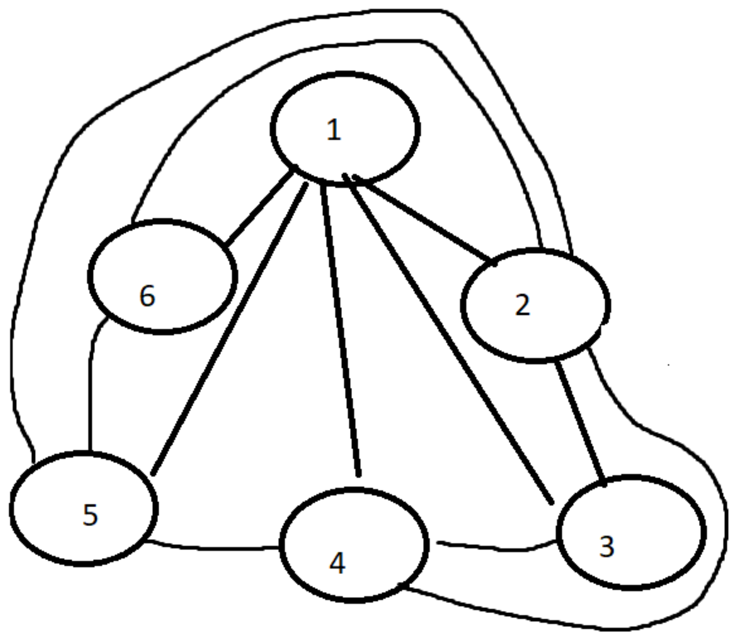


Figure 2: Problem 3 - b1 - Planar graph possible

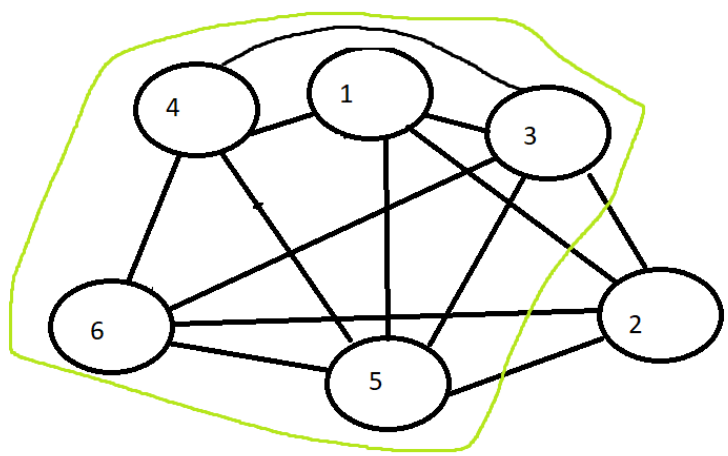


Figure 3: Problem 3 - b2 - K_5 sub graph shown in green - No planar graph possible

Academic integrity declaration.
Anand Mahadevan and Husam Chekfa completed this assignment together, and did not use external websites.