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CS111 Homework 1

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Problem 1.
(a) $g(n)$ represented in summation notation is as follows.

$$(\sum_{j=1}^{2n+3}(\sum_{k=1}^{2j}1))+(\sum_{j=1}^{n}(\sum_{k=1}^{j^2}1))$$

(b) Using $\sum_{i=m}^{n} 1 = n - m + 1$**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**, and** $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$:

$$\begin{aligned} g(n) &= (\sum_{j=1}^{2n+3} 2j) + (\sum_{j=1}^{n} j^2) \\ &= (2n+3)(2n+4) + \frac{n(n+1)(2n+1)}{6} \\ &= n^3/3 + 9n^2/2 + 85n/6 + 12 \end{aligned}$$

(c) $g(n) = \Omega(n^3)$, since $n^3/3 + 9n^2/2 + 85n/6 + 12 \geq \frac{n^3}{3}$ for $n \geq 0$
$g(n) = O(n^3)$, since $n^3/3 + 9n^2/2 + 85n/6 + 12 \leq \frac{85}{6}(n^3 + n^3 + n^3 + n^3) = \frac{340}{6}n^3$ for $n \geq 1$
We conclude that $f(n) = \Theta(n^3)$.

Problem 2. (a) Use mathematical induction to prove that $3^n \geq 2^n + 2n^2$ for all integers $n \geq 3$:

- Base case: $n = 3 : 27 \geq (6 + 18); 27 \geq 24$. So our base case passes.
- Inductive Step: Assume the identity holds for some for $n = k$ that is:

$$3^k \geq 2^k + 2k^2$$

Prove that it is true for $n = k + 1$: $\forall k \geq 3$

$$3^{k+1} \geq 2^{k+1} + 2(k+1)^2$$

LHS $= \sum_{i=0}^{k+1} a_i = \sum_{i=0}^{k} a_i + a^{k+1}$ (separate last term from the sum)
$= \frac{a^{k+1}-1}{a-1} + a^{k+1}$ (apply inductive assumption)
$= \frac{a^{k+1}-1+a^{k+1}(a-1)}{a-1}$
$= \frac{a \cdot a^{k+1}-1}{a-1} = \frac{a^{k+2}-1}{a-1} =$ RHS

- Conclusion: The claim holds for $n = k+1$. From the base case and the inductive step, it holds for $n \geq 0$

Problem 3. Give asymptotic values for this function using Θ-notation: $f(n) = \dfrac{n^2 3^n}{4} + n^4 2^n$

- $\frac{1}{4}n^2 3^n + n^4 2^n \geq \frac{1}{4}n^2 3^n$ for $n \geq 0$, so $f(n) = \Omega(n^2 3^n)$
- We also have:

$$\begin{aligned} f(n) &= \frac{1}{4}n^2 3^n + n^4 2^n \\ &= O(n^2 3^n) + n^2 \cdot n^2 \cdot 2^n \\ &= O(n^2 3^n) + n^2 \cdot O(1.5^n) \cdot 2^n \quad \text{(because } n^2 = O(1.5^n)\text{)} \\ &= O(n^2 3^n) + O(n^2 3^n) = O(n^2 3^n) \end{aligned}$$

We have shown that $f(n) = \Omega(n^2 3^n)$ and $f(n) = O(n^2 3^n)$. Therefore $f(n) = \Theta(n^2 3^n)$

Academic integrity declaration.
Anand Mahadevan and Husam Chekfa completed this assignment together, and did not use external websites.