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CS111 Homework 4

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Problem 1.
The floor and/or ceiling functions do not affect the results of the Master Theorem.

a)

Starting from else, print("A") happens n^2 times, and PrintAs($n/2$) happens 5 times.
$A(n) = 5 * A(n/2) + n^2$
Master Theorem - a = 5, b = 2, d = 2
Case: $a > b^d$
Solution: $A(n) = \Theta(n^{log_2 5}) \approx \Theta(n^{2.322...})$

b)

Starting from if $n \geq 3$, print("B") happens n^2 times, PrintBs($n/3$) happens 3 times, and PrintBs($n/3$) happens 6 times.
$B(n) = 9 * B(n/3) + n^2$
Master Theorem - a = 9, b = 3, d = 2
Case: $a = b^d$
Solution: $B(n) = \Theta(n^2 * log(n))$

c)

Starting at else: print("C") happens n times. Then PrintCs(n/3) happens twice.
$C(n) = 2 * C(n/3) + n$
Master Theorem - a = 2, b = 3, d = 1
Case: $a < b^d$
Solution: $C(n) = \Theta(n^1) = \Theta(n)$

d)

Start: In the Outer if: every PrintDs call with $n \geq 4$ will print D once; then enter either the inner if or else branch depending on the initial value of x. Either way, PrintDs(n/4) is run twice. Then, for our purposes, x is made even from odd, and odd from even.
The initial value of x does not affect the number of Ds printed because both inner branches have the same two PrintDs(n/4) statements.
$D(n) = 2 * D(n/4) + 1$
Master Theorem - a = 2, b = 4, d = 0
Case: $a > b^d$
Solution: $D(n) = \Theta(n^{log_4 2}) = \Theta(\sqrt{n})$

Problem 2.
$m = 23$
$k = 3$
1. **At least** 2 dogs of each breed
2. **At most** 9 Akitas
3. **At most** 10 Boxers
4. **At most** 7 Corgis
How many possible lists?

First things first, let's focus on statement 1.
We can subtract 2*3 = 6 from m, and 2 from each of statements 2-4 because those 6 dogs are already "set in stone". So we can remove them from our calculations of possible lists.
Let a represent the number of Akitas.
Let b represent the number of Boxers.
Let c represent the number of Corgis.
So, out new list is
$m = 17$
$k = 3$
1. $a \leq 7$
2. $b \leq 8$
3. $c \leq 5$

Using the Inclusion-Exclusion Principle...
$S(a \leq 7 \wedge b \leq 8 \wedge c \leq 5) = S() - S(a \geq 8 \vee b \geq 9 \vee c \geq 6)$
$= \binom{17+3-1}{3-1} - (S(a \geq 8) + S(b \geq 9) + S(c \geq 6) - S(a \geq 8 \wedge b \geq 9) - S(a \geq 8 \wedge c \geq 6) - S(b \geq 9 \wedge c \geq 6) + S(a \geq 8 \wedge b \geq 9 \wedge c \geq 6))$
$= \binom{19}{2} - (\binom{17-8+3-1}{3-1} + \binom{17-9+3-1}{3-1} + \binom{17-6+3-1}{3-1} - \binom{17-(8+9)+3-1}{3-1} - \binom{17-(8+6)+3-1}{3-1} - \binom{17-(9+6)+3-1}{3-1} + \binom{17-(8+9+6)+3-1}{3-1})$
$= 171 - (\binom{11}{2} + \binom{10}{2} + \binom{13}{2} - \binom{2}{2} - \binom{5}{2} - \binom{4}{2} + \binom{-4}{2})$
(When $n < 0$ in the binomial coefficient, it reduces to 0)
$= 171 - (55 + 45 + 78 - 1 - 10 - 6 + 0)$
$= 10$

So there are 10 possible lists that meet all the requirements stated in the problem.

Problem 3.
Using the Inclusion-Exclusion Principle...
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
Plugging in all the four given properties for $|A|$,$|C|$,$|A \cup B \cup C|$, and $|B|$

$$
\begin{aligned}
|A| &= |B| + 3 \\
|C| &= |A \cap B| + |A \cap C| + |B \cap C| \\
|A \cup B \cup C| &= 2 \cdot |A| \\
|B| &= 4 \cdot |A \cap B \cap C| + 2
\end{aligned}
$$

Results in...
$(2 \cdot |A|) = (|B|+3)+(4 \cdot |A \cap B \cap C|+2)+(|A \cap B|+|A \cap C|+|B \cap C|)-|A \cap B|-|A \cap C|-|B \cap C|+|A \cap B \cap C|$
Reducing and plugging in $|A| = |B| + 3, |B| = 4 \cdot |A \cap B \cap C| + 2$
$(2 * (|B| + 3)) = (4 \cdot |A \cap B \cap C| + 2 + 3) + (4 \cdot |A \cap B \cap C| + 2) + |A \cap B \cap C|$
Plugging in $|B| = 4 \cdot |A \cap B \cap C| + 2$ and reducing the right side...
$2 * (4 \cdot |A \cap B \cap C| + 2 + 3) = 9 \cdot |A \cap B \cap C| + 7$
$8 \cdot |A \cap B \cap C| + 10 = 9 \cdot |A \cap B \cap C| + 7$ ... (1)
$|A \cap B \cap C| = 3$
Since the left side was always equal to $|A \cup B \cup C|$, we can use (1) to solve for it easily
$|A \cup B \cup C| = 9 \cdot |A \cap B \cap C| + 7$
$|A \cup B \cup C| = 9 * 3 + 7$

$$|A \cup B \cup C| = 34$$

Academic integrity declaration.
Anand Mahadevan and Husam Chekfa completed this assignment together, and did not use external websites.