CS130 - Barycentric coordinates

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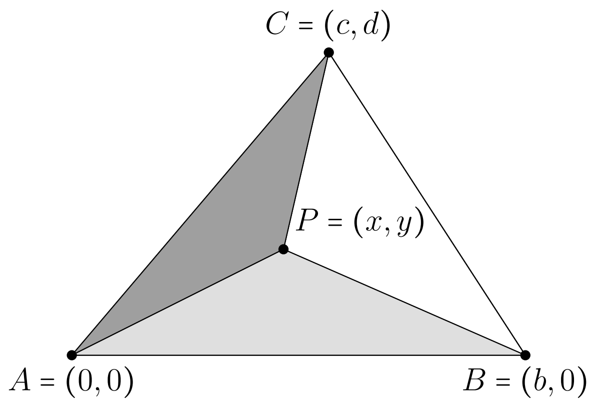
1. In class, we formulated the barycentric coordinates through ratios of triangle areas. We then implicitly assumed that $P = \alpha A + \beta B + \gamma C$. That is, the barycentric coordinates have the property that they interpolate the vertices of the triangle to the point P. In this problem, you will prove this property.

For the triangle ABC illustrated, show that when the barycentric coordinates are determined through ratios of triangle areas, they interpolate the vertices to the point P. That is, show that $P = \alpha A + \beta B + \gamma C$.



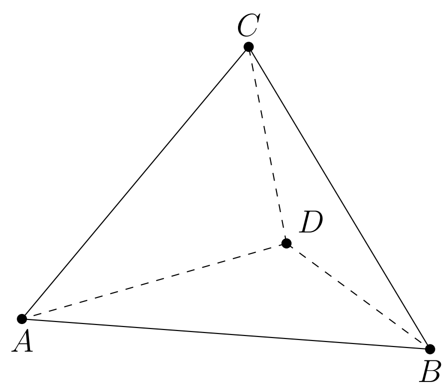
Plugging in each of the 3 points into $P = \alpha A + \beta B + \gamma C$
$P = \beta * (b, 0) + \gamma * (c, d)$ ... (1)
Area Triangle $= a_t = \frac{1}{2} * b * d$
$\beta = \frac{1}{2} * ((x-0)(d-0) - (c-0)(y-0))/a_t = (xd - cy)/bd$
$\gamma = \frac{1}{2} * ((b-0)(y-0) - (x-0)(0-0))/a_t = by/bd = y/d$
Plugging into (1)...
$P = (xd - cy)/bd * (b, 0) + y/d * (c, d)$
$P = ((xd - cy)/d, 0) + (cy/d, y)$
$P = ((xd - cy + cy)/d, y)$
$P = (x, y)$
This proves that barycentric coordinates, determined through the ratios of triangle areas,

interpolate the vertices to the point P.

2. The transformation $\mathbf{x} \rightarrow \mathbf{M}\mathbf{x} + \mathbf{b}$ is called an *affine* transformation, where $\mathbf{M}$ is a matrix and $\mathbf{b}$ is a vector. Let P be a point inside triangle ABC. The transformed point $P' = \mathbf{M}P + \mathbf{b}$ is inside the triangle with vertices $A' = \mathbf{M}A + \mathbf{b}$, $B' = \mathbf{M}B + \mathbf{b}$, and $C' = \mathbf{M}C + \mathbf{b}$. Show that P' has the same barycentric coordinates (in $A'B'C'$) as P (in ABC). That is, barycentric coordinates are preserved under affine transformations.

Let's look at the barycentric weight for A', which we'll call α'
$\alpha' = area(P'B'C')/area(A'B'C') =$
$((C' - B') * P')/((C' - B') * A') = P'/A'$
$\alpha = P/A$
Both α and α' are the same barycentric coordinates for their respective triangles.
This above statement applies to β and γ as the same calculations are resolved.
Thus, barycentric coordinates are preserved under affine transformations.

3.



How might one formulate barycentric coordinates for a tetrahedron? Suggest formulas for computing them.

Imagine Point P somewhere inside of the tetrahedron
We already know barycentric coordinates for triangle ADB because it's the same 2D version we know.
All that's left is how far towards C the Point P is. Call its barycentric weight κ
Each barycentric weight is the volume of the tetrahedron with P as a vertex divided by the full tetrahedron
FORMULA:
$\alpha + \beta + \gamma + \kappa = 1$
$\alpha = volume(BCDP)/volume(ABDC)$
$\beta = volume(ACDP)/volume(ABDC)$
$\gamma = volume(ABCP)/volume(ABDC)$

$\kappa = volume(ABDP)/volume(ABDC)$