CS130 - Parametric functions

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Given a parametric surface parameterized as $f(u,v) = \begin{pmatrix} u^2 - v^2 \\ 2uv \\ u^2 + v^2 \end{pmatrix}$ and a ray with endpoint $(-5, 1, 7)$ and direction $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$.

1. Normalize the ray's direction.

$$\begin{pmatrix} 2/3 \\ 1/3 \\ -2/3 \end{pmatrix}$$

2. Compute the intersection location and distance along the ray. *Hint: note that $u \to -u$ and $v \to -v$ results in the same point, so we may assume that $u > 0$. Solve for u^2 to find u. Then, eliminate v.*

Observation: System of equations to solve
$u^2 - v^2 = -5 + 2t/3$ ... (1)
$2uv = 1 + t/3$ ... (2)
$u^2 + v^2 = 7 - 2t/3$ ... (3)

$u^2 = v^2 - 5 + 2t/3$ ... (1a)
$t = 6uv - 3$ ... (2a)
(1a) plugged in (3)...
$v^2 - 5 + 2t/3 + v^2 = 7 - 2t/3$ ... (4)
$v^2 = 6 - 2t/3$ ... (4a)
$t = 9 - 3v^2/2$ ... (4b)
(2a) plugged in (4a)...
$v^2 = 8 - 4uv$ ... (5)
$u = (8 - v^2)/4v$ ... (5a)
Plugging both (4b) and (5a) into (1)...
$((8 - v^2)/4v)^2 - v^2 = 1 - v^2$ ... (6)
$64 - 16v^2 + v^4 = 16v^2$ ... (6a)

$v^4 - 32v^2 + 64 = 0$ ... (6b)
$v^2 = 16 \pm 8 * \sqrt{3}$ ... (7)
Plugging (7) into (4b)...
$t = -15 + 12 * \sqrt{3}$ ... (8) (Negative solution for t results in negative t)
Distance along ray $= -15 + 12 * \sqrt{3}$

Intersection location $= \begin{pmatrix} -5 \\ 1 \\ 7 \end{pmatrix} + \begin{pmatrix} 2/3 \\ 1/3 \\ -2/3 \end{pmatrix} * (-15 + 12 * \sqrt{3}) =$

$$\begin{pmatrix} -15 + 8 * \sqrt{3} \\ -4 + 4 * \sqrt{3} \\ 17 - 8 * \sqrt{3} \end{pmatrix}$$

3. Compute the normal direction for the surface at the intersection location.

Square root of (7)...
$v_i = \sqrt{16 - 8 * \sqrt{3}}$
Plugging v_i into (5a)...
$u_i = (-2 + 2 * \sqrt{3})/\sqrt{16 - 8 * \sqrt{3}} = 1$

$$\frac{\partial f}{\partial u} \times \frac{\partial f}{\partial v} = \begin{pmatrix} 2u \\ 2v \\ 2u \end{pmatrix} \times \begin{pmatrix} -2v \\ 2u \\ 2v \end{pmatrix} = \begin{pmatrix} 4v^2 - 4u^2 \\ -8uv \\ 4u^2 + 4v^2 \end{pmatrix}$$

At point (u_i, v_i)...

$$\begin{pmatrix} 60 - 32 * \sqrt{3} \\ -8\sqrt{16 - 8 * \sqrt{3}} \\ 68 - 32 * \sqrt{3} \end{pmatrix}$$