CS130 - Reflections and transparency

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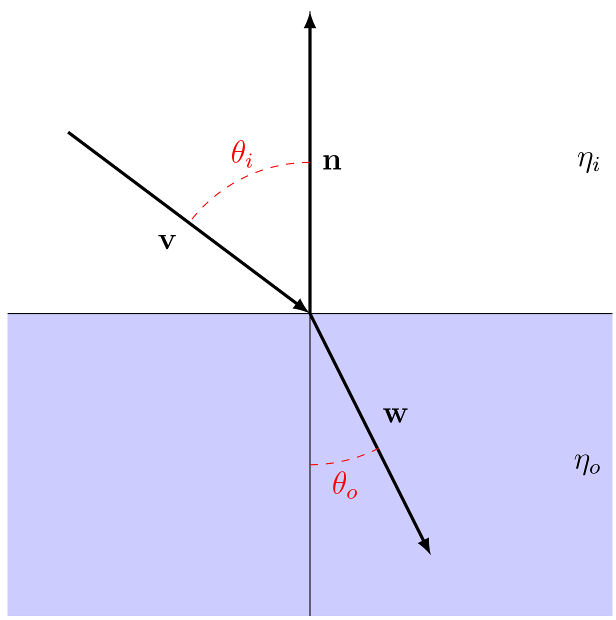
1. Given the vector $\mathbf{u} = \langle 1, 4 \rangle$ and the unit vector $\mathbf{n} = \langle -\frac{3}{5}, \frac{4}{5} \rangle$, decompose $\mathbf{u}$ into component $\mathbf{u}_\perp$ perpendicular to $\mathbf{n}$ and component $\mathbf{u}_\parallel$ parallel to $\mathbf{n}$ such that $\mathbf{u} = \mathbf{u}_\perp + \mathbf{u}_\parallel$.

$\|n\| = 1$
$u \cdot n = \frac{13}{5}$
$u_\parallel = (u \cdot n) * n = \langle -\frac{39}{25}, \frac{52}{25} \rangle$
$u_\perp = u - u_\parallel = \langle \frac{64}{25}, \frac{48}{25} \rangle$



In the figure above, a ray originally in the air (index of refraction η_i) enters a transparent material (index of refraction η_o). The ray enters along direction $\mathbf{v}$ and leaves along direction $\mathbf{w}$. You are given that $\|\mathbf{v}\| = 1$ and $\|\mathbf{n}\| = 1$. You will construct $\mathbf{w}$ such that $\|\mathbf{w}\| = 1$. $\mathbf{w}$ lies in the same plane as $\mathbf{n}$ and $\mathbf{v}$.

2. Snell's law states that $\eta_i \sin\theta_i = \eta_o \sin\theta_o$. Express this equation in terms of the vectors $\mathbf{v}$, $\mathbf{n}$, and $\mathbf{w}$ using cross products (no dot products).

$\eta_i * \| - v \times n\| = \eta_o * \|w \times -n\|$

3. Taking advantage of the fact that **w**, **n**, and **v** lie in the same plane, we can write $\mathbf{w} = a\mathbf{v} + b\mathbf{n}$. Using your result from the previous problem, solve for a. Note that you will only be able to solve for a up to a sign.

$\eta_i * \| - v \times n\| = \eta_o * \|(a * v + b * n) \times -n\|$
$\eta_i/\eta_o * \| - v \times n\| = \|(a * v + b * n) \times -n\|$
$a = \eta_i/\eta_o$

4. Let **t** be a vector orthogonal to **n** as shown in the figure. Taking the dot product of $\mathbf{w} = a\mathbf{v} + b\mathbf{n}$ by **t**, deduce the sign of a.

$(a \cdot t) * v + (b \cdot t) * n = 0$
a should be positive, as if it's 0 or negative, there is no vector left that can go in the x direction towards w.

5. Using $\|\mathbf{w}\|^2 = 1$ to derive a quadratic equation in b. Solve this for b, which should give you two solutions. We will select the solution we want later.

$b = \eta_i/\eta_o * (n \cdot v) \pm (\sqrt{1 - (\eta_i/\eta_o)^2 * (1 - (n \cdot v)^2)}$

6. If $\mathbf{v} = -\mathbf{n}$, then we should get $\mathbf{w} = \mathbf{v}$ as our solution. Use this special case to deduce the correct sign for b. Using the a and b you derived, write out **w**.

$w = \eta_i/\eta_o * v + (\eta_i/\eta_o * (n \cdot v) - (\sqrt{1 - (\eta_i/\eta_o)^2 * (1 - (n \cdot v)^2)})) * n$

7. Based on your formula for **w**, deduce the conditions under which complete internal reflection occurs.

When the term under the square root is less than 0.

8. What happens as the index of refraction of the sphere in `??.txt` is made closer to the index of refraction of the air? Support your conclusion by showing a sequence of renders. What happens when they are equal?

As the index of refraction goes closer to air, the sphere continually becomes more of an exact image of what's behind it, rather than refracting what's behind it. The following 3 renders are of decreasing index of refraction. (08.png, 07.png, 06.png). When they are equal, the sphere shows simply what's behind it.