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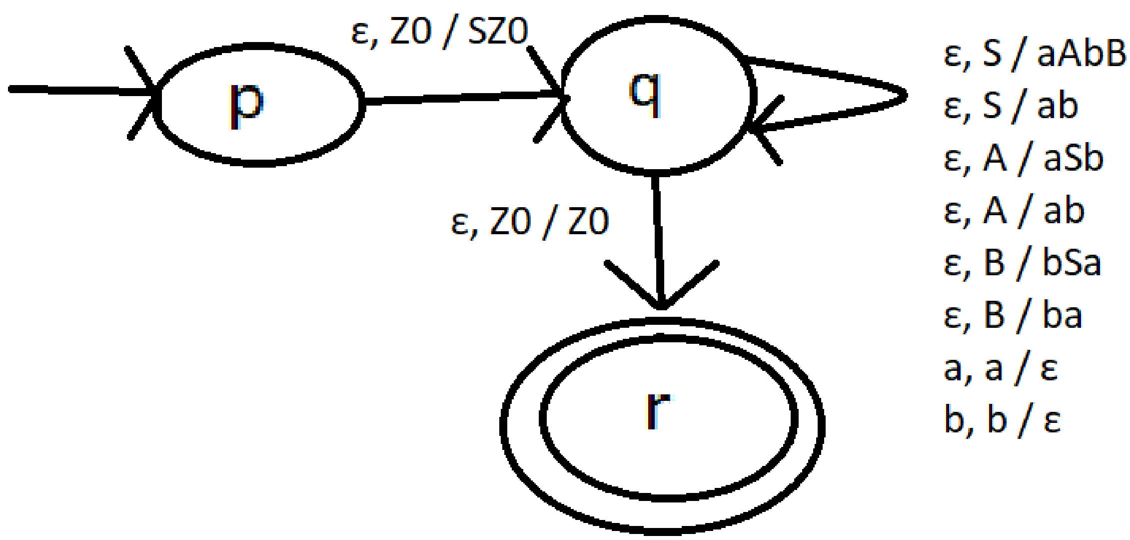
CS150 Homework 5

due Wednesday, December 1st 5:00 PM

Problem 1.

Q1 [10 pts] Convert the grammar to a PDA that accepts the same language by final state. Please represent the PDA as a transition diagram.

$S \to aAbB \mid ab$
$A \to aSb \mid ab$
$B \to bSa \mid ba$



Define PDA as

({p, q, r}, {a,b}, {S, A, B, a, b}, δ, p, Z0, {r})

Problem 2.

Let us pretend the PDA of Exercise 6.1.1 (on P.233-234 or P.228 in the 2nd ed) is an empty-stack PDA. Convert it to a CFG.

1. S $\rightarrow$ [qZ_0q] | [qZ_0p]

From transition 1, $\delta(q, 0, Z_0) = (q, XZ_0)$

2. [qZ_0q] $\rightarrow$ 0 [qXq][qZ_0q]
3. [qZ_0q] $\rightarrow$ 0 [qXp][pZ_0q]
4. [qZ_0p] $\rightarrow$ 0 [qXq][qZ_0p]
5. [qZ_0p] $\rightarrow$ 0 [qXp][pZ_0p]

From transition 2, $\delta(q, 0, X) = (q, XX)$

6. [qXq] $\rightarrow$ 0 [qXq][qXq]
7. [qXq] $\rightarrow$ 0 [qXp][pXq]
8. [qXp] $\rightarrow$ 0 [qXq][qXp]
9. [qXp] $\rightarrow$ 0 [qXp][pXp]

From transition 3, $\delta(q, 1, X) = (q, X)$

10. [qXq] $\rightarrow$ 1 [qXq]
11. [qXp] $\rightarrow$ 1 [qXp]

From transition 4, $\delta(q, \epsilon, X) = (p, \epsilon)$

12. [qXp] $\rightarrow$ ϵ

From transition 5, $\delta(p, \epsilon, X) = (p, \epsilon)$

13. [pXp] $\rightarrow$ ϵ

From transition 6, $\delta(p, 1, X) = (p, XX)$

14. [pXq] $\rightarrow$ 1 [pXq][qXq]
15. [pXq] $\rightarrow$ 1 [pXp][pXq]
16. [pXp] $\rightarrow$ 1 [pXq][qXp]
17. [pXp] $\rightarrow$ 1 [pXp][pXp]

From transition 7, $\delta(p, 1, Z_0) = (p, \epsilon)$

18. [pZ_0p] $\rightarrow$ 1

Problem 3.
P.277 (or P.271 in the 2nd ed) Ex.7.1.4 Repeat Exercise 7.1.2 for the following grammar.
S $\rightarrow$ AAA | B
A $\rightarrow$ aA | B
B $\rightarrow$ ϵ

a) Eliminate ϵ productions

{S, A, B} are all nullable
S $\rightarrow$ AAA | AA | A
A $\rightarrow$ aA | a

b) Eliminate any unit productions in the resulting grammar.

Only unit production is S $\rightarrow$ A
Simply expand A with its productions resulting in the following grammar.
S $\rightarrow$ AAA | AA | aA | a
A $\rightarrow$ aA | a

c) Eliminate any useless symbols in the resulting grammar.

There are no useless symbols as S and A are both generating and reachable.

d) Put the resulting grammar in Chomsky Normal Form

Add a Symbol for each Terminal that appears in a body length ≥ 2 resulting in...
S $\rightarrow$ AAA | AA | BA | a
A $\rightarrow$ BA | a
B $\rightarrow$ a

Finally, introduce variables to get all bodies with more than 2 variables down to exactly 2

S $\rightarrow$ AC | AA | BA | a
A $\rightarrow$ BA | a
B $\rightarrow$ a
C $\rightarrow$ AA

Problem 4.
Use the CFL Pumping Lemma to show each of the following language not to be context-free:
a) A = $\{a^n b^n c^i \mid i < n\}$

Suppose A is indeed context free.
So there must be a partition of a string in the language z = uvwxy s.t.
$|vwx| \leq$ n , $|vx| > 0$, and $\forall i \geq 0$, $uv^i wx^i y \in$ A
Let z = $a^n b^n c^{n-1}$

Case 1: vwx contains at least 1 c
If vwx contains a c, either v or x must have the c, or w must have the c.
- If v or x has a c, then I will pump with i = 2, such that now the number of c's is guaranteed $\geq$ n, so this string $\notin$ A.
- Else if v and x don't contain a c, then w has the c's, and the only valid option is v = b, w = c^{n-1}, and x = ϵ
In this case, I will pump i = 0, so now the number of b's is not equal to the number of a's, and this string is $\notin$ A.
Case 2: vwx does not contain a c
If vwx doesn't contain a c, v or x must contain at least 1 a or b.
I will pump i = 0, causing there to be less than n a's or b's. This is not possible, however, since the number of c's now must be strictly less than (n - 1) guaranteed, where c already is fixed at a quantity of (n - 1), causing a contradiction.
So this string is $\notin$ A.

With all possible cases of partitioning exhausted, it is evident that Language A is not context-free. Q.E.D.

b) B = {www | w is a binary string over {0,1}}
Suppose B is indeed context free.
So there must be a partition of a string in the language z = uvwxy s.t.
$|vwx| \leq$ n , $|vx| > 0$, and $\forall i \geq 0$, $uv^i wx^i y \in$ B
Let z = $0^n 0^n 0^n$

Case 1: vwx is within the first two 0^n
I will simply pump with i = 0, such that there are guaranteed less zeroes in 1 of the first two 0^n than the third 0^n, so this string $\notin$ B.
Case 2: vwx is within the last two 0^n
I will simply pump with i = 0, such that there are guaranteed less zeroes in the last two 0^n than the first 0^n, so this string $\notin$ B.

With all possible cases of partitioning exhausted, it is evident that Language B is not context-free. Q.E.D.

Problem 5.

P.297 (or P.292 in the 2nd ed) Ex.7.3.2 Consider the following two languages:
$L_1 = \{a^n b^{2n} c^m \mid \text{n, m} \geq 0\}$
$L_2 = \{a^n b^m c^{2m} \mid \text{n, m} \geq 0\}$
a) Show that both of these languages are context-free by giving grammars for each.

L_1:
S $\rightarrow$ AB
A $\rightarrow$ aAbb | abb | ϵ
B $\rightarrow$ cB | c | ϵ

L_2:
S $\rightarrow$ AB
A $\rightarrow$ aA | a | ϵ
B $\rightarrow$ bAcc | bcc | ϵ

b) Is $L_1 \cap L_2$ a CFL? Justify.

Observation: The intersection must accept double b's than a's and double c's than b's.
Result: $L_1 \cap L_2 = \{a^n b^{2n} c^{4n} \mid \text{n} \geq 0\}$

Suppose $L_1 \cap L_2$ is indeed context free.
So there must be a partition of a string in the language z = uvwxy s.t.
$|vwx| \leq$ n , $|vx| > 0$, and $\forall i \geq 0$, $uv^i wx^i y \in L_1 \cap L_2$
Let z = $a^n b^{2n} c^{4n}$

Case 1: vwx contains at least 1 c
If vwx contains a c, vwx is completely within the b's and c's.
I will pump i = 0, so now the number of b's and/or c's does not match the rule that the number of b's is double a's and c's are double of b's, so this string is $\notin L_1 \cap L_2$.

Case 2: vwx does not contain a c
If vwx doesn't contain a c, vwx is completely within the a's and b's.
I will pump i = 0, so now the number of a's and/or b's does not match the rule that the number of b's is double a's and c's are double of b's, so this string is $\notin L_1 \cap L_2$.

With all possible cases of partitioning exhausted, it is evident that $L_1 \cap L_2$ is not a CFL. Q.E.D.

Problem 6.
For the grammar G of Example 7.34 on P.306 (or P.301 in the 2nd ed), use the CYK algorithm to determine if each of the following strings is in L(G):
Grammar:
$S \rightarrow AB \mid BC$
$A \rightarrow BA \mid a$
$B \rightarrow CC \mid b$
$C \rightarrow AB \mid a$

a) bbaaa
b) aaaaa

(a)

{A,S,C}				
---	{S,A,C}			
{A}	---	{C,A,S}		
---	{S,A}	{B}	{B}	
{B}	{B}	{A,C}	{A,C}	{A,C}
b	b	a	a	a

(b)

{S,A,C}				
{B}	{B}			
{C,A,S}	{C,A,S}	{C,A,S}		
{B}	{B}	{B}	{B}	
{A,C}	{A,C}	{A,C}	{A,C}	{A,C}
a	a	a	a	a

Because the Start symbol S appears in the top-left corner of both tables, both bbaaa and aaaaa are members of the above grammar's language.