

CS150 Homework 2
due Monday, October 25th 5:00 PM

Problem 1.

a) Compute the e-closure of each state.

$$\text{ECLOSE}(p) = \{p, q, r\}$$

$$\text{ECLOSE}(q) = \{q\}$$

$$\text{ECLOSE}(r) = \{r\}$$

b) Give all the strings of length three or less accepted by the automaton.

All possible strings of length 3 or less from alphabet $\{a,b,c\}$ are accepted by this automaton, including ϵ **EXCEPT** bba, bbb, bbc!

c) Convert the automaton to a DFA.

DFA			
	a	b	c
$\rightarrow * \{p, q, r\}$	$\{p, q, r\}$	$\{q, r\}$	$\{p, q, r\}$
$* \{q, r\}$	$\{p, q, r\}$	$\{r\}$	$\{p, q, r\}$
$* \{r\}$	ϕ	ϕ	ϕ
ϕ	ϕ	ϕ	ϕ

Problem 2. Write regular expressions for the following languages:

a) The set of strings of 0's and 1's whose 10th symbol from the right is a 0.
ANS: $(0 + 1)^* 0 (0 + 1)^9$

b) The set of strings of 0's and 1's with at most one pair of consecutive 0's.
ANS: $(1 + 01)^* (00 + 0 + \epsilon) (1 + 10)^*$

Problem 3.

DFA		
	0	1
$\rightarrow *a$	b	c
b	a	d
c	d	a
*d	c	b

Make a new initial state "s" that epsilon transitions to state "a". State "a" is no longer the initial state

Make a new final state "f" that has epsilon transitions going to it from states "a" and "d". States a and d are no longer final states.

1) Eliminate state b

a to d – (01)

a to a – (00)

d to a – (10)

d to d – (11)

2) Eliminate state c

a to d – (10)

a to a – (11)

d to a – (01)

d to d – (00)

3) Eliminate state d

a to a (from b and c eliminations)– (00) + (11)

a to a (from d elimination)– (01 + 10)(00 + 11)*(10 + 01)

a to f (from d elimination)– (01 + 10)(00 + 11)*

4) Eliminate state a

s to f – ((00) + (11) + (01 + 10)(00 + 11)*(10 + 01))* + (01 + 10)(00 + 11)*

Regex (simply s to f)–

((00) + (11) + (01 + 10)(00 + 11)*(10 + 01))* + (01 + 10)(00 + 11)*

Problem 4.

- c) The set of prefixes of strings in L
 d) The union of the set of prefixes of strings in L (part c) and the set of suffixes of strings in L (part b)

Problem 5.

e) $(R+S)T = RT + ST$.

Replace R, S, and T with $\{a\}$, $\{b\}$, and $\{c\}$ respectively, + with \cup , and \cdot with x

$$(\{a\} \cup \{b\})x\{c\} = \{a\}x\{c\} \cup \{b\}x\{c\}$$

LHS: $(\{a\} \cup \{b\})x\{c\} = \{a, b\}x\{c\}$

LHS: $\{a, b\}x\{c\} = \{\mathbf{ac}, \mathbf{bc}\}$

RHS: $\{ac\} \cup \{bc\}$

RHS: $\{\mathbf{ac}, \mathbf{bc}\}$

Both LHS and RHS are equal, the identity $(R+S)T = RT + ST$ holds. Q.E.D.

g) $(\epsilon+R)^* = R^*$.

Replace ϵ with $\{\epsilon\}$ and R with $\{a\}$, + with \cup

$$(\{\epsilon\} \cup \{a\})^* = \{a\}^*$$

LHS: $\{\epsilon, a\}^*$

This is simply the set of all strings of the empty string (ϵ) and a's, which is simply the set of all strings of a's. Because no matter how many ϵ are concatenated to a's, the string remains a string of a's.

The set of all strings of a's is : LHS: $\{\mathbf{a}\}^*$

RHS: $\{\mathbf{a}\}^*$

Both LHS and RHS are equal, the identity $(\epsilon+R)^* = R^*$ holds. Q.E.D.