Assignment 2
CS 3200

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(1a) the approximation to the first derivative of of some function f(x) using the polynomial interpolation approach at 3 points.

$$
\begin{aligned}
p(x) &= y_0 l_0(x) + y_1 l_1(x) + y_2 l_2(x) \\
p'(x) &= y_0 l_0'(x) + y_1 l_1'(x) + y_2 l_2'(x)
\end{aligned}
$$

$$
\begin{aligned}
l_0(x) &= (\frac{x - x_1}{x_0 - x_1})(\frac{x - x_2}{x_0 - x_2}) \\
l_1(x) &= (\frac{x - x_0}{x_1 - x_0})(\frac{x - x_2}{x_1 - x_2}) \\
l_2(x) &= (\frac{x - x_1}{x_2 - x_1})(\frac{x - x_0}{x_2 - x_0})
\end{aligned}
$$

$$
\begin{aligned}
l_0(x)' &= \frac{2x - x_2 - x_1}{(x_1 - x_0)(x_2 - x_0)} \\
l_1(x)' &= \frac{-2x - x_2 - x_0}{(x_1 - x_0)(x_1 - x_2)} \\
l_2(x)' &= \frac{2x - x_1 - x_0}{(x_2 - x_0)(x_2 - x_1)}
\end{aligned}
$$

$$
\begin{aligned}
l_0(x)' &= \frac{2x - x_2 - x_1}{2h^2} \\
l_1(x)' &= \frac{-2x - x_2 - x_0}{h^2} \\
l_2(x)' &= \frac{2x - x_1 - x_0}{2h^2}
\end{aligned}
$$

$$
f'(x) = \frac{1}{2h^2}[y_0(2x - x_2 - x_1) - 2y_1(2x - x_2 - x_0) + y_2(2x - x_1 - x_0)]
$$

the finite difference formula at each point is as follows:

$$
\begin{aligned}
f'(x_0) = \frac{1}{2h^2}[y_0(2x_0 - x_2 - x_1) - 2y_1(2x_0 - x_2 - x_0) + y_2(2x_0 - x_1 - x_0)] \\
= \frac{1}{2h}(-3y_0 + 4y_1 - y_2) \\
f'(x_1) = \frac{1}{2h^2}[y_0(2x_1 - x_2 - x_1) - 2y_1(2x_1 - x_2 - x_0) + y_2(2x_1 - x_1 - x_0)] \\
= \frac{1}{2h}(-y_0 + y_2) \\
f'(x_2) = \frac{1}{2h^2}[y_0(2x_2 - x_2 - x_1) - 2y_1(2x_2 - x_2 - x_0) + y_2(2x_2 - x_1 - x_0)] \\
= \frac{1}{2h}(y_0 - 4y_1 + 3y_2)
\end{aligned}
$$

(1b) show that the error for the above is $O(h^2)$

$$
\begin{aligned}
E_n(x) &= \frac{f^{N+1}(x)}{(N+1)!}[(x - x_0)(x - x_1)(x - x_2)] \\
&= \frac{f^3(x)}{3!}[(x - x_0)(x - x_1)(x - x_2)] \\
&= \frac{[(x - x_0)(x - x_1)(x - x_2)]}{3!}f'''(x) \\
\\
&= \frac{1}{6}\frac{df}{dx}(x - x_0)(x - x_1)(x - x_2) \\
&= (x - x_1) * (x - x_2) + (x - x_0) * (x - x_2) + (x - x_0) * (x - x_1) \\
&= \frac{1}{6}3x^2 - 2xx_0 - 2xx_1 - 2xx_2 + x_1x_2 + x_0x_2 + x_0x_1
\end{aligned}
$$

polynomial is 2nd order

(1c) see matlab files FirstDer.m and DriverDer.m

(1d) Error Plot - see file plot.jpg.
it follows the derivation from part 1(b)

(2a) Generate an approximation to $\int_a^b f(x)dx$ using quadratic polynomial interpolant of f.

$$\int_a^b f(x)dx$$

$$x_0 = a, x_1 = (a+b)/2, x_2 = b$$

$$l_0(x) = \frac{(x - \frac{a+b}{2})(x-b)}{(a-\frac{a+b}{2})(a-b)}$$

$$l_1(x) = \frac{(x-a)(x-b)}{-(\frac{a+b}{2}-a)(\frac{(a+b)}{2}-b)}$$

$$l_2(x) = \frac{(x-a)(x-\frac{a+b}{2})}{(b-a)(b-\frac{a+b}{2})}$$

$$\int_a^b f(x)dx = f(a)[\frac{2(x-\frac{a+b}{2})(x-b)}{(a-b)^2}]$$

$$-f(\frac{a+b}{2})[\frac{4(x-a)(x-b)}{(b-a)^2}] + f(b)[\frac{2(x-a)(x-\frac{a+b}{2})}{(b-a)^2}]$$

(2b) Derive the corresponding error for this quadrature rule.

$$E_n = \int_{x_0}^{x_2} f(x)dx$$

$$= \int_{x_0}^{x_2} (x-x_0)(x-x_1)(x-x_2)dx$$

$$= \frac{(x_2-x_0)^3(x_2-2x_1+x_0)}{12}$$

$$= \frac{2h^3 * 0}{12}$$

$$= 0$$

(2c) what is the problem with using equispaced points to generate quadrature rules as the number of points increase? how do you solve this problem?

The problem with equally spaced points is that you will run into the Runge's Phenomenon which is a problem of oscillation at the edges of an interval. Runge's Phoenomenon says that going to higher degree does not always improve accuracy when using equally spaced points. to solve this issue you should use points that are not equally spaced, for example, Chebyshev Extrema points.